1 Reproduce Eq.(25)

First we Taylor expand the following matrix element:

$$\begin{aligned}\langle 0|\,\bar q(0)\Gamma_1 P_+\Gamma_2\, q(x)\,|0\rangle &= \langle 0|\,\bar q(0)\Gamma_1 P_+\Gamma_2 q(0)\,|0\rangle + x^\mu \langle 0|\,\bar q(0)\Gamma_1 P_+\Gamma_2 D_\mu q(0)\,|0\rangle \\ &+\frac{1}{2}x^\mu x^\nu \langle 0|\,\bar q(0)\Gamma_1 P_+\Gamma_2 D_\mu D_\nu q(0)\,|0\rangle + \cdots \end{aligned}\tag{1}$$

First term in Eq.(**??**) corresponds to the quark-antiquark condensate.

$$\begin{aligned}\langle 0|\,\bar q^i_\alpha(0)\Gamma_{1,\alpha\beta}P_{+,\beta\gamma}\Gamma_{2,\gamma\delta}\, q^j_\delta(0)\,|0\rangle &= A\delta^{ij}\delta_{\alpha\delta}\Gamma_{1,\alpha\beta}P_{+,\beta\gamma}\Gamma_{2,\gamma\delta} \\ \Leftrightarrow \langle 0|\,\bar q^i_\alpha(0)\, q^j_\delta(0)\,|0\rangle\,\delta^{ij}\delta_{\alpha\delta} &= A\delta^{ij}\delta^{ij}\delta_{\alpha\delta}\delta_{\alpha\delta} \\ \Leftrightarrow \langle \bar q q\rangle &= 4AN_c \\ \Rightarrow A &= \frac{1}{4N_c}\langle \bar q q\rangle \end{aligned}\tag{2}$$

where (i,j) are color indices and $(\alpha,\beta,\gamma,\delta)$ are spinor indices. In the second line of Eq.(**??**) we multiplied on both sides $\delta^{ij}\delta_{\alpha\delta}$. Combining this result with (**??**), we find

$$\begin{aligned}\langle 0|\,\bar q^i_\alpha(0)\Gamma_{1,\alpha\beta}P_{+,\beta\gamma}\Gamma_{2,\gamma\delta}\, q^j_\delta(0)\,|0\rangle &= \Gamma_{1,\alpha\beta}P_{+,\beta\gamma}\Gamma_{2,\gamma\delta}\cdot\langle 0|\,\bar q^i_\alpha(0)\, q^j_\delta(0)\,|0\rangle = \Gamma_{1,\alpha\beta}P_{+,\beta\gamma}\Gamma_{2,\gamma\delta}\cdot A\delta^{ij}\delta_{\alpha\delta} \\ &= \Gamma_{1,\alpha\beta}P_{+,\beta\gamma}\Gamma_{2,\gamma\delta}\cdot\frac{1}{4N_c}\langle \bar q q\rangle\,\delta^{ij}\delta_{\alpha\delta} \\ &=\frac{1}{4N_c}\cdot \mathrm{Tr}[\Gamma_1 P_+\Gamma_2]\,\langle \bar q q\rangle\,\delta^{ij}\end{aligned}\tag{3}$$

Here, there will be an additional δ^{ij} due to the summation over the color indices, which cancels the factor $\frac{1}{N_c}$.

The second term in Eq.(**??**) does not contribute since according to Dirac's equation we can rewrite the covariant derivative as:

$$\not{D} q = -im_q q. \tag{4}$$

In HQET we assume $m_q = 0$ for light quarks.

Before we consider the third term in more detail, we take a closer look at the following matrix element:

$$\begin{aligned}\langle 0|\,\bar q^i_\alpha(0) g_s G_{\mu\nu}(0) q^j_\delta(0)\,|0\rangle &= E\cdot\delta^{ij}\cdot(\sigma_{\mu\nu})_{\delta\alpha} \\ \Leftrightarrow \langle 0|\,\bar q^i_\alpha(0) g_s G_{\mu\nu}(0) q^j_\delta(0)\,|0\rangle\,\delta^{ij}(\sigma^{\mu\nu})_{\alpha\delta} &= E\cdot N_c\cdot\mathrm{Tr}[\sigma^{\mu\nu}\sigma_{\mu\nu}] \\ \Leftrightarrow E = \langle 0|\,\bar q g_s \sigma\cdot G q\,|0\rangle &\cdot\frac{1}{4N_c d(d-1)}\end{aligned}\tag{5}$$

The third term in Eq.(**??**) corresponds to the quark-antiquark-gluon condensate.

$$\langle 0|\,\bar q^i_\alpha(0) D_\mu D_\nu q^j_\delta(0)\,|0\rangle = C_1\delta^{ij}\delta_{\alpha\delta}\,g_{\mu\nu} + C_2\delta^{ij}(\sigma_{\mu\nu})_{\delta\alpha}, \tag{6}$$

where we are only interested in the symmetric part, because we will multiply this expression by $x^\mu x^\nu$ which is symmetric. Now to compute C_1 we rewrite again Eq.(**??**) using translation invariance as:

$$\begin{aligned}\langle 0|\, \bar{q}^i_\alpha(0) D_\mu D_\nu q^j_\delta(0)\, |0\rangle - \langle 0|\, \bar{q}^i_\alpha(0) D_\nu D_\mu q^j_\delta(0)\, |0\rangle &= C_2 \cdot \delta^{ij} \Big((\sigma_{\mu\nu})_{\delta\alpha} - (\sigma_{\nu\mu})_{\delta\alpha} \Big) \\ &= 2 \cdot C_2 \delta^{ij} (\sigma_{\mu\nu})_{\delta\alpha} \end{aligned} \tag{7}$$

Using the definition of the gluon field strength tensor $G_{\mu\nu} = \frac{i}{g_s}[D_\mu, D_\nu]$, we obtain

$$\begin{aligned}\langle 0|\, \bar{q}^i_\alpha(0)(-i) g_s G_{\mu\nu}(0) q^j_\delta(0)\, |0\rangle &= 2 \cdot C_2 \delta^{ij} (\sigma_{\mu\nu})_{\delta\alpha} \\ \Rightarrow C_2 &= -\frac{i}{2} \cdot E \end{aligned} \tag{8}$$

The relation between C_2 and C_1 can be obtained by using the Dirac equation (in my notes).

$$C_1 = \frac{d-1}{2} \cdot E \tag{9}$$

So the final result for the third term is:

$$\frac{x^\mu x^\nu}{2} \cdot \langle 0|\, \bar{q}^i_\alpha(0) D_\mu D_\nu q^j_\delta(0)\, |0\rangle = \frac{x^2}{2} \cdot C_1 \delta^{ij} \delta_{\alpha\delta} = \frac{x^2}{2} \delta^{ij} \delta_{\alpha\delta} \cdot \langle 0|\, \bar{q} g_s \sigma \cdot G q\, |0\rangle \cdot \frac{1}{8 N_c d} \tag{10}$$

With this expression it is possible to reproduce the desired expression.