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VACUUM CORRECTIONS TO QCD CHARMONIUM SUM RULES: Basic formalism and $O(G^3)$ results

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We present a detailed description of a new method for computing higher gluonic power corrections to QCD charmonium sum rules. The method is equivalent to using the Schwinger gauge $(x^\mu - z_0{}^\mu)A_\mu(x) = 0$ for vacuum gluons. Results for $O(G^2)$ and $O(G^3)$ corrections are presented.

1. Introduction

The QCD sum rule approach [1,2] was extremely successful in explaining many properties of hadrons. The basic idea of the approach is that at large momenta one can rely on perturbation theory (PT) whereas the deviations from PT at moderate and small momenta can be described and/or parametrized by non-vanishing vacuum matrix elements of quark and gluon local operators, such as

$$\langle g^2 G^a_{\mu\nu} G^a_{\mu\nu} \rangle\,,\ \langle \bar\psi\psi \rangle\,,\ \langle g^3 f_{abc} G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu} \rangle\,,\ \text{etc.}\ .$$

In the absence of a complete theory of the QCD vacuum these vacuum averages play the role of fundamental constants characterizing the quark-gluon interactions at long distances. As is well-known, the magnitude of the quark condensate term $\langle \bar\psi\psi \rangle$ can be extracted from the hadronic spectrum by the current algebra analysis [2–4]:

$$\langle \bar u u \rangle = \langle \bar d d \rangle = \langle \bar s s \rangle = -(0.24\ \text{GeV})^3\,. \tag{1.1}$$

The gluon condensate term $\langle g^2 G^2 \rangle \equiv \langle g^2 G^a_{\mu\nu} G^a_{\mu\nu} \rangle$ was first estimated by Shifman, Vainshtein and Zakharov [2] by analysing the QCD sum rule for charmonium systems with $\bar c \gamma_\mu c$ quantum numbers. They obtained

$$\langle g^2 G^2 \rangle = (0.83\ \text{GeV})^4\,. \tag{1.2}$$

A more extensive analysis (including sum rules related to other $L = 0, 1$ charmonium states) performed by Reinders, Rubinstein and Yazaki [5] yielded essentially the same result.

Calculating higher power corrections and comparing the results obtained with the curves based on experimental data one can, in principle, estimate also the vacuum averages of higher dimension operators $g^3 fG^3 \equiv g^3 f_{abc} G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu}$, $g^4 G^4$, etc.; i.e. extract more detailed information about the QCD vacuum structure. Unfortunately, the algorithm used so far to compute the vacuum corrections [2,5] requires a considerable effort even for calculating the simplest $O(G^2)$ correction. In essence, the complications are due to the fact that the standard Feynman rules for the quark-gluon vertices are formulated for the gauge-dependent vector field A^a_μ (potential) while the final result should be expressed in terms of the gauge-invariant operators (e.g., $G^a_{\mu\nu} G^a_{\mu\nu}$) constructed from the field strength tensor $G_{\mu\nu}$.

In a recent letter [6] we proposed a new method* that enables one to factor the "A-dependent" terms into path-ordered exponentials. The latter can be easily shown to cancel with each other for the gauge-invariant amplitudes $\Pi(q)$ studied within the QCD sum rule approach. The remaining terms depend, just as desired, only on $G_{\mu\nu}$ and its covariant derivatives.

In the course of our computations we realized that our basic representation [see eq. (3.4) below] is in fact a gauge transformation relating the quark propagator $S^c(x, y, A)$ calculated in an arbitrary gauge with that in the Schwinger gauge [8]**

$$(x^\mu - z_0^\mu) A_\mu(x) = 0. \tag{1.3}$$

In this gauge one can express A_μ just in terms of $G_{\mu\nu}$ and its covariant derivatives (see [11] and eqs. (3.12), (3.17) below), and as a result, the computation of the gluonic vacuum corrections is considerably simplified. After completing our computations of the $O(G^3)$ corrections we received papers [14–16] where the gluonic power corrections to some amplitudes were calculated just with the help of the Schwinger gauge $x^\mu A_\mu(x) = 0$. It should be noted, however, that refs. [15,16] deal with polarization operators related to massless quarks. The massive quark case (i.e., the situation we are interested in here) is analysed in ref. [14], but only the calculation of the $O(G^2)$ terms (obtained earlier [2] by straightforward methods) is described in that paper. There exist also some minor differences between our algorithm (constructed so as to be the most suitable for computer calculations) and that proposed in ref. [14].

In the present paper we give a more detailed description of our method concentrating on the calculation of the $O(G^3)$ corrections to the QCD charmonium sum rules. The paper is organized as follows. In sect. 2 we outline a general algorithm for

* This method is really a refined version of a technique developed previously by one of the authors (A.R.) to study the factorization in arbitrary gauge at the leading twist level (see ref. [7]).

** To the best of our knowledge, this gauge was first incorporated in QED by Schwinger [8]. Later it was rediscovered by many other authors [9–12] who used various names (e.g., coordinate gauge [11], fixed-point gauge [12], etc.) for it. A similar gauge ("normal" gauge [13]) is useful in the (super)gravity theory, where it enables one to express the metrics $g_{\mu\nu}(x)$ in terms of the Riemann tensor $R_{\alpha\beta\gamma\delta}(x)$ (for details see [13]).

computing vacuum gluonic corrections. In sect. 3 we describe a technique of extracting the A-dependent terms into the path-ordered exponentials and discuss the relation of our method to the Schwinger gauge technique. Our algorithm for computing the gluonic power corrections is presented in sect. 4. Applications of the results to the charmonium analysis within the QCD sum rule approach are discussed in sect. 5. Some formulas used in the $O(G^3)$ calculations are presented in appendix A. Appendix B contains our results concerning the $O(G^2)$ and $O(G^3)$ contribution for c-quark currents $j^{(\Gamma)} = \bar{c}\Gamma c$.

2. Polarization operator in the vacuum gluonic field

2.1. GENERAL DISCUSSION

The basic amplitude $\Pi^{\Gamma}(q)$ analysed within the QCD sum rule approach is the polarization operator induced by a particular current j^{Γ}

$$\Pi^{\Gamma}(q) = i\int \mathrm{d}^4x\, \mathrm{e}^{iqx}\langle 0|\mathrm{T}\{ j^{\Gamma}(x)\, j^{\Gamma}(0)\}|0\rangle . \tag{2.1}$$

In the present paper we restrict our attention to the simplest $\bar{c}\Gamma c$ currents relevant to QCD charmonium sum rules ($\Gamma = 1, \gamma_5, \gamma_\mu, (q_\mu q_\nu/q^2 - g_{\mu\nu})\gamma^\nu\gamma_5$). As explained by Shifman et al. [2], one should calculate the amplitude $\Pi(q)$ in the region $q^2 < 0$ where the corrections to the simplest contribution (fig. 1a) are small because of asymptotic freedom.

First, there exist perturbative corrections, the simplest of which are shown in figs. 1b, c. As is well-known, their contribution for the vector current can be extracted from Schwinger's book [17]. For other currents diagrams 1b, c have been computed by Reinders et al. [5] (see also ref. [18]). As for the next order (i.e., 3-loop) diagrams, their complete evaluation for fixed non-zero quark masses is, to the best of our knowledge, far beyond the capacities of any existing computational technique. However, since after the renormalization group improvement of perturbative expansion the contributions of higher-order diagrams are damped by a factor $\alpha_s/\pi \lesssim 0.1$ per loop, one can usually ignore higher perturbative corrections.

Second, in QCD one should also take into account the corrections due to fluctuating vacuum fields of non-perturbative origin. As emphasized in ref. [2], it is these corrections (not higher terms of the perturbative α_s series) that destroy

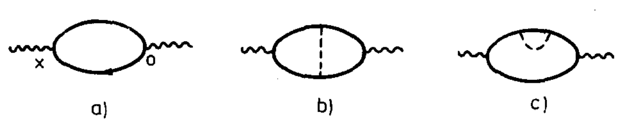


Fig. 1. Perturbative contributions to the polarization operator.

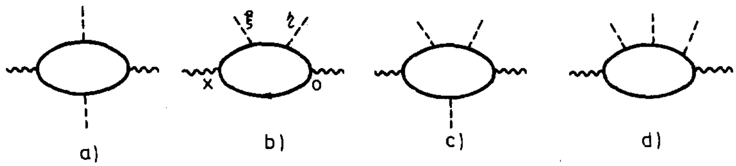


Fig. 2. 1-loop diagrams describing interactions with the vacuum gluonic field.

asymptotic freedom for q^2 close to the $\bar{c}c$ threshold. Some of the diagrams describing the interactions due to the vacuum fields are shown in fig. 2. In addition to the two external lines corresponding to the currents j^Γ entering into eq. (2.1), these diagrams possess also the external lines related to light particles (i.e., u, d, s quarks and gluons) absorbed and/or emitted by the vacuum fields [2] (see also [5]).

In this paper we shall concentrate on the calculation of the most important 1-loop diagrams, i.e., on those corresponding to the vacuum fields corrections to the lowest-order perturbative diagram.

2.2. EXTERNAL FIELD METHOD

As seen from fig. 2 the problem is to calculate the polarization operator $\Pi^\Gamma(q)$ in the presence of the external vacuum gluonic field A_μ. Using the standard Feynman rules for the quark-gluon vertices and c-quark propagators, one can write down the contribution of any given diagram of fig. 2 type. For instance, the contribution of fig. 2b for the vector current $j_\mu = \bar{c}\gamma_\mu c$ in the coordinate representation is

$$\Pi^{(2)}_{\mu\nu}(q, A) = \int e^{iqx} \operatorname{Tr}\{\gamma_\mu S^c(x)\gamma_\nu S^c(-\eta)\gamma^{\alpha_2}\tau_{a_2} S^c(\eta-\xi)\gamma^{\alpha_1}\tau_{a_1} S^c(\xi - x)\}$$

$$\times \langle 0|A^{a_1}_{\alpha_1}(\xi) A^{a_2}_{\alpha_2}(\eta)|0\rangle \, d^4x \, d^4\xi \, d^4\eta , \tag{2.2}$$

where τ_a are matrices of the gauge group SU(3) in the quark (fundamental) representation related to the Gell-Mann matrices λ^a by $\tau^a = \frac{1}{2}\lambda^a$.

Performing the Taylor expansion of the A^a_α fields at some spatial point (say, at zero)

$$A^a_\alpha(\xi) = \sum_{n=0}^{\infty} \frac{1}{n!} \xi^{\mu_1} \dots \xi^{\mu_n} \left(\partial_{\mu_1} \dots \partial_{\mu_n} A^a_\alpha(0)\right), \tag{2.3}$$

one obtains the expansion of $\Pi^{(2)}_{\mu\nu}(q, A)$ in terms of the vacuum-to-vacuum matrix elements of local operators constructed from the A-fields and their derivatives.

In charmonium calculations (as well as in all cases when the quark masses cannot be neglected) it is convenient to proceed further using the momentum representation. Then, e.g., the contribution of fig. 2b reads

$$\Pi^{(2)}_{\mu\nu}(q,A)=\sum_{n=0}^{\infty}\frac{i^n}{n!}\sum_{B=0}^{\infty}\frac{i^l}{l!}\int\frac{d^4k}{(2\pi)^4}\operatorname{Tr}\left\{\gamma_\mu\frac{\not{k}+\not{q}+m}{(k+q)^2-m^2}\gamma_\nu\frac{\not{k}+m}{k^2-m^2}\gamma^{\alpha_2}\right.$$

$$\left.\times\frac{\partial}{\partial k_{\mu_1}}\cdots\frac{\partial}{\partial k_{\mu_n}}\left[\frac{\not{k}+m}{k^2-m^2}\gamma^{\alpha_1}\frac{\partial}{\partial k_{\nu_1}}\cdots\frac{\partial}{\partial k_{\nu_l}}\left(\frac{\not{k}+m}{k^2-m^2}\right)\right]\right\}\operatorname{Tr}(\tau_{a_1}\tau_{a_2})$$

$$\times\langle 0|\left(\partial_{\mu_1}\ldots\partial_{\mu_n}A^{a_1}_{\alpha_1}(0)\right)\left(\partial_{\nu_1}\ldots\partial_{\nu_l}A^{a_2}_{\alpha_2}(0)\right)|0\rangle\,. \tag{2.4}$$

Of course, not all terms on the r.h.s. of eq. (2.4) are equally important. In the final result, any matrix element $\langle O_i\rangle$ will be accompanied by a factor like $m_c^{-d_i}$, where d_i is the mass dimension of the O_i operator. Thus, one should calculate first the contribution of the lowest dimension operators, then the next power correction, next-to-next, etc. To get operators of higher dimensions, one has to increase either the number of derivatives or the number of the A-fields. It is clear that if the number of the derivatives and/or A-fields is large, then it is difficult to calculate the integrand of eq. (2.4) by hand. Fortunately, all the necessary manipulations can be easily performed by a computer with the help of, say, the SCHOONSCHIP program written by Veltman [19]. The resulting 1-loop integrals are standard (see subsect. 4.1) and this step can be also performed at the computer.

At the last step one observes, however, that there appear numerous cancellations between contributions of different diagrams. In particular, all terms related to operators without derivatives (or with a single derivative) disappear after summation over all relevant diagrams. It is easy to realize that these cancellations are due to gauge invariance: only gauge-invariant combinations of the local operators constructed from the $(\partial\ldots\partial A)$ fields should appear in the final result. Thus, one has to express the final result in terms of the operators containing only the gluon field strength* $G^a_{\mu\nu}$:

$$G^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+gf_{abc}A^b_\mu A^c_\nu\,, \tag{2.5}$$

and its covariant derivatives $G_{\mu\nu;\alpha_1\ldots\alpha_n}$

$$G^a_{\mu\nu;\alpha_1\ldots\alpha_n}=\left(\tilde{\mathfrak{D}}_{\alpha_n}\right)^{aa_1}\left(\tilde{\mathfrak{D}}_{\alpha_{n-1}}\right)^{a_1a_2}\cdots\left(\tilde{\mathfrak{D}}_{\alpha_1}\right)^{a_{n-1}b}G^b_{\mu\nu}\,, \tag{2.6}$$

where $\tilde{\mathfrak{D}}_\alpha=\partial_\alpha-ig\tilde{A}_\alpha$ is the covariant derivative acting on the gluon field, $\tilde{A}_\alpha=A^a_\alpha\sigma_a$

* Note that in ref. [6] we used another definition of $G_{\mu\nu}$ that differs by sign from the standard one given by eq. (2.5).

and σ_a is the gauge group matrix in the gluonic (adjoint) representation

$$(\sigma_a)_{bc} = -if_{abc}. \tag{2.7}$$

The re-expansion of the operators $(\partial \ldots \partial A) \ldots (\partial \ldots \partial A)$ over the operators $(\mathcal{D} \ldots \mathcal{D}G) \ldots (\mathcal{D} \ldots \mathcal{D}G)$ is, in fact, not a trivial problem especially in a non-abelian theory where the operator $G_{\mu_1\nu_1}G_{\mu_2\nu_2}$, say, comes from diagrams with 2, 3 and 4 external gluon lines. What is still worse, it is rather difficult to computerize this step.

3. Improved external field method

3.1. EXPONENTIATION OF THE A DEPENDENCE

To analyse the gauge-invariance structure of $\Pi^\Gamma(q)$ it is convenient to write the sum of the 1-loop diagrams as a convolution of two quark propagators in the external vacuum field A:

$$\Pi^\Gamma(q) = i\int \mathrm{d}^4x\, \mathrm{e}^{iqx} \mathrm{Tr}\{\Gamma_1 S^c(x, y; A)\Gamma_2 S^c(y, x; A)\}. \tag{3.1}$$

By definition $S^c(x, y; A)$ is the perturbative solution to the Dirac equation

$$\left[i\gamma^\mu\left(\frac{\partial}{\partial x^\mu} - ig\hat{A}_\mu(x)\right) - m\right] S^c(x, y; A) = -\delta^4(x-y), \tag{3.2}$$

where $\hat{A} \equiv A^a\tau_a$.

Note that $S^c(x, y; A)$ in contrast to $\Pi^\Gamma(q)$ is not a gauge-invariant quantity. It changes under the $\mathrm{SU}(3)_c$ gauge transformations in the same way as the path-ordered exponential $\hat{P}(x, y; A; \mathrm{C})$,

$$\hat{P}(x, y; A; \mathrm{C}) = \mathrm{P}\exp\left(ig\int_\mathrm{C} \hat{A}_\mu(z)\,\mathrm{d}z^\mu\right) \tag{3.3}$$

(recall that $\bar{\psi}(x)\hat{P}(x, y; A)\psi(y)$ is the standard gauge-invariant bilocal operator). Here, C is some path connecting x and y.

The idea is to pick out the $\hat{P}$ factor from $S^c(x, y; A)$ with the hope that the remaining factor $\mathbb{S}$ will depend on the vacuum field only through $G_{\mu\nu}$ and its covariant derivatives. To begin with, one should decide whether S^c will be represented as $P\mathbb{S}$ or $\mathbb{S}P$ (in QCD this problem is not trivial because both P and $\mathbb{S}$ are matrices). One should also specify the path C.

After checking several possibilities we observed that the most convenient is the symmetric representation

$$S^c(x, y; A) = \hat{E}(x, z_0; A)\mathbb{S}^c(x, y; A, z_0)\hat{E}(z_0, y; A), \tag{3.4}$$

where z_0 is some fixed (i.e., not depending on x, y) spatial point and $\hat{E}(x, y; A)$ is the path-ordered exponential corresponding to the straight-line path

$$\hat{E}(x, z_0; A) = \mathrm{P}\exp\left[ig(x^\nu - z_0^\nu)\int_0^1 \mathrm{d}t\, \hat{A}_\nu(z_0 + t(x - z_0))\right]. \tag{3.5}$$

It is straightforward to derive that eq. (3.2) is satisfied only if $\mathbb{S}^c(x, y; A, z_0)$ is a solution to the modified Dirac equation

$$\left[i\gamma^\mu\left(\frac{\partial}{\partial x^\mu} - ig\hat{\mathscr{A}}_\mu(x, z_0)\right) - m\right]\mathbb{S}^c(x, y; A, z_0) = -\delta^4(x - y), \tag{3.6}$$

that differs from eq. (3.2) only by the change

$$A_\mu^a \to \mathscr{A}_\mu^a(x, z_0) = (x^\nu - z_0^\nu)\int_0^1 t\,\mathrm{d}t\, G_{\nu\mu}^b(z)\tilde{E}^{ba}(z, z_0)|_{z = z_0 + t(x - z_0)}. \tag{3.7}$$

Here, $\tilde{E}$ is a straight-line-ordered exponential in the gluonic representation. To derive eqs. (3.6), (3.7), we used the commutation rule

$$(\tau^b)_{AB}\hat{E}_{BC}(z, z_0) = \hat{E}_{AB}(z, z_0)(\tau^a)_{BC}\tilde{E}^{ba}(z, z_0) \tag{3.8}$$

based on the well-known formula

$$e^A B e^{-A} = B + [A, B] + \frac{1}{2!}[A, [A, B]] + \cdots \tag{3.9}$$

and the relation

$$[\tau_b, \tau_a] = -(\sigma_b)_{ac}\tau^c. \tag{3.10}$$

Note now, that incorporating the Baker-Hausdorf theorem (see e.g., [20]) one can perform the Taylor expansion

$$G_{\nu\mu}^b(z)\tilde{E}^{ba}(z, z_0) = \sum_{n=0}^{\infty} G_{\nu\mu;\mu_1\ldots\mu_n}^a(z_0)\frac{1}{n!}(z - z_0)^{\mu_1}\ldots(z - z_0)^{\mu_n}, \tag{3.11}$$

and express $\mathscr{A}_\mu^a(z, z_0)$ in terms of $G_{\mu\nu}(z_0)$ and its covariant derivatives:

$$\mathscr{A}_\mu^a(x, z_0) = (x^\nu - z_0^\nu)\sum_{n=0}^{\infty}\frac{1}{n!(n+2)}(x^{\mu_1} - z_0^{\mu_1})\ldots(x^{\mu_n} - z_0^{\mu_n})G_{\nu\mu;\mu_1\ldots\mu_n}^a. \tag{3.12}$$

Further observation is that the exponentials $\hat{E}$ entering into $S^c(x, y; A)$ are precisely cancelled in eq. (3.1) by those present in $S^c(y, x; A)$; so that one can change $S^c \to \mathbb{S}^c$ in eq. (3.1). Solving eq. (3.6) perturbatively one obtains for $\Pi^\Gamma(q)$ the $(g\mathscr{A})$ expansion that has the same structure (corresponding to diagrams shown in fig. 2) as the original (gA) expansion. The contribution of each diagram can be then calculated just as described in subsect. 2.2. A very important difference, however, is that $\mathscr{A}$ is Taylor expanded just in terms of $G_{\mu\nu;\mu_1\ldots\mu_n}$ so that the final result has the desired form, and no further re-expansion is needed.

Of course, the final result may be trusted only if it does not depend on the arbitrary parameter z_0. Recalling that z_0 is some spatial point one may naively expect that the z_0 dependence is eliminated by the translation invariance. This is indeed the case for the vacuum matrix elements, since all the composite operators O_i (e.g., $G_{\mu\nu}(z_0)G_{\alpha\beta}(z_0)$) are constructed from the $G_{\mu\nu;\mu_1\ldots\mu_n}$ fields taken at the same point z_0 and

$$\langle 0|O_i(z_0)|0\rangle = \langle 0|O_i(0)|0\rangle . \tag{3.13}$$

However, the z_0 dependence is generated also by the $(x^{\mu_i} - z_0^{\mu_i})$ factor in the Taylor expansion (3.12). As an explicit calculation shows, the resulting z_0 dependence, or, more precisely, the dependence on $(y - z_0)$, disappears only after summing over all the relevant diagrams. This observation suggests that z_0 works also like a gauge parameter.

3.2. SCHWINGER GAUGE

It is easy to realize that our basic ansatz (3.4) is in fact a gauge transformation which relates the propagator $S^c(x, y; A)$ calculated in an arbitrary gauge with that in the Schwinger gauge [8]

$$(x^\mu - z_0^\mu)A_\mu(x) = 0. \tag{3.14}$$

Indeed, in this gauge $E(x, z_0; A) = 1$ and, as a result, $S^c = \mathbb{S}^c$. An equivalent form of eq. (3.14) is

$$A_\mu(z_0) = 0, \tag{3.15a}$$

$$\left(\partial_{\mu_1}\ldots\partial_{\mu_n}A_\mu(z_0)\right)_{\text{symmetrized}} = 0; \qquad n \geq 1. \tag{3.15b}$$

Performing the Taylor expansion of $A_\mu(x)$ at $x = z_0$

$$A_\mu(x) = \sum_{n=0}^{\infty} \frac{1}{n!}(x^{\mu_1} - z_0^{\mu_1})\ldots(x^{\mu_n} - z_0^{\mu_n})\left(\partial_{\mu_1}\ldots\partial_{\mu_n}A_\mu(z_0)\right), \tag{3.16}$$

and using eqs. (3.15) one obtains the expression*

$$A_\mu(x) = (x^\nu - z_0^\nu) \sum_{n=0}^{\infty} \frac{1}{n!(n+2)} (x^{\mu_1} - z_0^{\mu_1}) \dots (x^{\mu_n} - z_0^{\mu_n})$$

$$\times \partial_{\mu_1} \dots \partial_{\mu_n} \big(\partial_\nu A_\mu(z_0) - \partial_\mu A_\nu(z_0) \big), \tag{3.17}$$

which in the Schwinger gauge (3.14) is equivalent to eq. (3.12). In particular, due to eqs. (3.15) the derivatives $\partial_{\mu_1}, \dots, \partial_{\mu_n}$ in eq. (3.17) can be treated as the covariant ones (see also [11]).

4. Calculations

4.1. GENERAL OUTLINE

The expansion (3.12) generates Feynman rules for vertices where a quark interacts with the $G^a_{\mu\nu;\mu_1 \dots \mu_n}(z_0)$ gluon field. It is convenient to preserve the usual graphical notation for the G-vertices indicating, in addition, also the number of covariant derivatives (see fig. 3).

Consider, as an example, the diagram shown in fig. 3a. In the coordinate representation its contribution for the vector current $j_\mu = \bar{c}\gamma_\mu c$ reads

$$\Pi^{(3a)}_{\mu\nu}(q, G) = \tfrac{1}{4} i \int e^{iqx} \langle 0 | G^a_{\alpha\beta}(z_0) G^{a'}_{\alpha'\beta'}(z_0) | 0 \rangle$$

$$\times \mathrm{Tr}\{ \gamma_\mu S^c(x-\xi) \gamma^\alpha \tau_a (\xi^\beta - z_0^\beta) S^c(\xi) \gamma_\nu S^c(-\eta) \gamma^{\alpha'}$$

$$\times \tau_{a'} (\eta^{\beta'} - z_0^{\beta'}) S^c(\eta - x) \} \, d^4x \, d^4\xi \, d^4\eta \, . \tag{4.1}$$

Incorporating the covariance properties of the vacuum matrix elements with respect to the color $SU(3)_c$ and Lorentz transformations one can write

$$\langle 0 | G^a_{\alpha\beta}(z_0) G^{a'}_{\alpha'\beta'}(z_0) | 0 \rangle = \tfrac{1}{96} \delta^{aa'} (g_{\alpha\alpha'} g_{\beta\beta'} - g_{\alpha\beta'} g_{\beta\alpha'}) \langle 0 | G^c_{\sigma\lambda} G^c_{\sigma\lambda} | 0 \rangle \, . \tag{4.2}$$

Note that due to the translation invariance [see eq. (3.13)] there is no need to specify the argument of the G-fields on the r.h.s. of eq. (4.2).

For massless quarks, the most simple way to proceed further is to use the explicit form of $S^c(x)$ and to calculate the integrand of eq. (4.1) just in the coordinate representation (see, e.g., refs. [14–16]). However, for massive quarks, which we are interested in, $S^c(x)$ contains transcendental functions, and it is much more conve-

* This derivation was suggested to us by E.A. Ivanov.

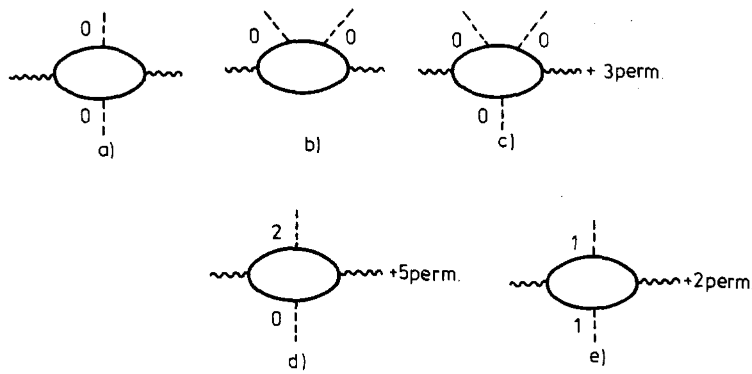


Fig. 3. 1-loop diagrams constructed according to the Feynman rules generated by eq. (3.12). (a),(b) GG diagrams; (c) GGG diagram; (d) G(DDG) diagram; (e) (DG)(DG) diagram.

nient to rewrite eq. (4.1) in the momentum representation. This can be accomplished in a straightforward way. The only complication compared to the ordinary Feynman integrals is owing to the derivatives $\partial/\partial k^{\alpha}$, $\partial/\partial k^{\beta}$ resulting from the factors ξ^{α}, η^{β} [cf. eq. (2.4)]. However, these derivatives are eliminated by using the relation

$$\frac{\partial}{\partial k^{\nu}} \frac{k^{\mu}\gamma_{\mu}+m}{k^2-m^2} = -\frac{k^{\mu}\gamma_{\mu}+m}{k^2-m^2}\gamma_{\nu}\frac{k^{\rho}\gamma_{\rho}+m}{k^2-m^2}, \tag{4.3}$$

and the resulting expression looks much like the ordinary Feynman integral. Typically, one obtains

$$\Pi_{\mu\nu}(q,G) \sim \langle gG\ldots gG\rangle \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\mathrm{Tr}\{\gamma^{\mu}\ldots(\not{k},\not{q},m)\ldots\gamma^{\nu}\ldots\}}{(k^2-m^2)^N\left[(k+q)^2-m^2\right]^M}. \tag{4.4}$$

After calculating the traces it is convenient to expand the numerator in powers of the denominator factors $((k+q)^2-m^2)$ and (k^2-m^2). As a result, the integrand is considerably simplified, and the next step is to introduce the Feynman parametrization. Then $\Pi_{\mu\nu}$ is given by a sum of integrals like

$$I_N^{mn}(Q^2,m^2) = \int_0^1 \mathrm{d}x \frac{x^n(1-x)^m}{\left[Q^2x(1-x)+m^2\right]^N}, \tag{4.5}$$

where $Q^2=-q^2$.

Note, that the denominator in eq. (4.5) is symmetric under the change $x \to 1-x$. This means that the numerator factor in eq. (4.5) can be also made symmetric with respect to this change and then re-expanded in powers of $x(1-x)$. Furthermore, using the obvious recurrence relation for $I_N^n \equiv I_N^{nn}$,

$$I_N^n = \frac{1}{Q^2}\left(I_{N-1}^{n-1} - m^2 I_N^{n-1}\right), \tag{4.6}$$

one can write $\Pi_{\mu\nu}$ as a sum of the basic integrals $\mathcal{I}_N(Q^2/m^2)$

$$\mathcal{I}_N\left(\frac{Q^2}{m^2}\right) = \int_0^1 \frac{\mathrm{d}x}{\left[1 + x(1-x)Q^2/m^2\right]^N}, \tag{4.7}$$

that can be calculated explicitly:

$$\mathcal{I}_N\left(\frac{Q^2}{m^2}\right) = \frac{(2N-3)!!}{(N-1)!}\left[\left(\frac{a-1}{2a}\right)^N \sqrt{a}\ln\frac{\sqrt{a}+1}{\sqrt{a}-1} + \sum_{k=1}^{N-1}\frac{(k-1)!}{(2k-1)!!}\left(\frac{a-1}{2a}\right)^{N-k}\right], \tag{4.8}$$

where $a = 1 + 4m^2/Q^2$.

The algorithm described above can be applied for calculating an arbitrary 1-loop diagram with external vacuum gluon lines*. The main technical problem is to construct ("by hand") a generalization of eq. (4.2) for operators of higher dimensions.

As a final result, one obtains for $\Pi^\Gamma(q)$ the expansion in terms of vacuum averages of local gauge-invariant operators O_i:

$$\Pi^\Gamma(q) = T^\Gamma(q)\left\{C_I^\Gamma \mathbf{1} + C_{G^2}^\Gamma\langle g^2 G_{\mu\nu}^a G_{\mu\nu}^a\rangle + C_{G^3}^\Gamma\langle g^3 f_{abc} G_{\mu\nu}^a G_{\nu\lambda}^b G_{\lambda\mu}^c\rangle\right.$$
$$\left. + C_{j^2}^\Gamma\langle g^4 j_\mu^a j_\mu^a\rangle + \cdots\right\} \equiv T^\Gamma \mathcal{P}^\Gamma(q^2) \tag{4.9}$$

where $T^\Gamma(q)$ is a structure depending on the current $j^\Gamma(x)$. For a vector current, e.g., $\Pi_{\mu\nu}^{\mathrm{V}}(q)$ satisfies the transversality condition $q^\mu \Pi_{\mu\nu}^{\mathrm{V}} = q^\nu \Pi_{\mu\nu}^{\mathrm{V}} = 0$ and the usual choice is $T_{\mu\nu}^{\mathrm{V}} = q_\mu q_\nu - q^2 g_{\mu\nu}$.

To illustrate the typical structure of the coefficient functions $C_i(Q^2, m^2)$, we present below the explicit expression for the G^2 term of the expansion (4.9) for $\Pi_{\mu\nu}^{\mathrm{V}}(q)$:

$$C_{G^2}^{\mathrm{V}} = \frac{1}{48\pi^2(Q^2)^2}\left(-1 + 3\mathcal{I}_2 - 2\mathcal{I}_3\right) \tag{4.10a}$$

$$= \frac{1}{48\pi^2(Q^2)^2 4}\left[\frac{3(a-1)(a^2-1)}{2a^2\sqrt{a}}\ln\frac{\sqrt{a}+1}{\sqrt{a}-1} - \frac{3a^2-2a+3}{a^2}\right], \tag{4.10b}$$

* Of course, the necessary computer time rapidly increases with the dimension of the operator.

where $a = 1 + 4m^2/Q^2$. The final form (4.10b) coincides with that obtained in ref. [2].

It should be remarked here that the expansion (4.10a) in terms of the basic integrals (4.7) is in practice much more useful for further analysis (e.g., for calculating derivatives $\mathrm{d}^n C_i/(\mathrm{d}Q^2)^n$) than the explicit form (4.10b) (cf. subsect. 4.2 of ref. [2]).

4.2. COMPUTATION OF THE $O(G^3)$ CONTRIBUTION

Using the gluonic fields present in the generating expansion (3.12) one can construct 3 different local operators with dimension 6 (figs. 3c–e)

$$G^a_{\mu_1\nu_1} G^b_{\mu_2\nu_2} G^c_{\mu_3\nu_3}, \qquad G^a_{\mu_1\nu_1;\,\alpha\beta} G^b_{\mu_2\nu_2}, \qquad G^a_{\mu_1\nu_1;\,\alpha} G^b_{\mu_2\nu_2;\,\beta}.$$

The total number of diagrams constructed according to Feynman rules generated by eq. (3.12) is $13 = 4 + 6 + 3$ (figs. 3c–e). The most trivial step is the calculation of the color traces

$$\operatorname{Tr} \tau^a \tau^b = \tfrac{1}{2} \delta^{ab}, \tag{4.11a}$$

$$\operatorname{Tr} \tau^a \tau^b \tau^c = \tfrac{1}{4}\left(d^{abc} + i f^{abc}\right). \tag{4.11b}$$

It should be remarked from the start that the operator $d^{abc} G^a G^b G^c$ although present at intermediate stages of the calculation, disappears in the final result. This is a manifestation of the QCD Furry theorem.

The next step is to get explicit expressions for vacuum averages $\langle G^a_{\mu_1\nu_1;\,\alpha\beta} G^a_{\mu_2\nu_2} \rangle$, $\langle G^a_{\mu_1\nu_1;\,\alpha} G^a_{\mu_2\nu_2;\,\beta} \rangle$ and $\langle G^a_{\mu_1\nu_1} G^b_{\mu_2\nu_2} G^c_{\mu_3\nu_3} f^{abc} \rangle$. This is performed in a straightforward way (the result is presented in appendix A). Then we incorporate the equation of motion

$$G^a_{\nu\mu;\,\mu} = g j^a_\nu = g \sum_{\psi = u, d, s} \bar{\psi} \gamma_\nu \tau^a \psi \tag{4.12}$$

for operators containing $G_{\mu\nu;\,\mu}$. In a similar way, using the Bianchi identity

$$G_{\mu\nu;\,\alpha} + G_{\nu\alpha;\,\mu} + G_{\alpha\mu;\,\nu} = 0, \tag{4.13}$$

the commutation relation

$$G^a_{\alpha\beta;\,\mu\nu} - G^a_{\alpha\beta;\,\nu\mu} = g f^{abc} G^b_{\alpha\beta} G^c_{\mu\nu}, \tag{4.14}$$

and eq. (4.12), one can express $G^a_{\mu\nu;\,\alpha\alpha}$ in terms of $j^a_{\mu;\,\nu}$ and $f_{abc} G^b_{\mu\alpha} G^c_{\nu\alpha}$:

$$G^a_{\mu\nu;\,\alpha\alpha} = g\left(2 f_{abc} G^b_{\mu\alpha} G^c_{\nu\alpha} + j^a_{\mu;\,\nu} - j^a_{\nu;\,\mu}\right). \tag{4.15}$$

Finally, incorporating translation invariance reduces $\langle j^a_{\mu;\,\nu} G^a_{\mu\nu} \rangle$ to $\langle j^a_\mu j^a_\mu \rangle$:

$$\langle 0 | j^a_{\mu;\,\nu} G^a_{\mu\nu} | 0 \rangle = -g \langle 0 | j^a_\mu j^a_\mu | 0 \rangle . \tag{4.16}$$

As a result, the dimension-6 contribution is expressed in terms of 2 operators: $g^3 fG^3 \equiv g^3 f_{abc} G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu}$ and $g^4 j^2 \equiv g^4 j^a_\mu j^a_\mu$. For the vector current our result is

$$C^{\mathrm{V}}_{G^3} = \frac{1}{72\pi^2 (Q^2)^3} \left(\frac{2}{15} + 4\mathcal{I}_2 - \tfrac{31}{3}\mathcal{I}_3 + \tfrac{43}{5}\mathcal{I}_4 - \tfrac{12}{5}\mathcal{I}_5 - \frac{Q^2}{10m^2} \right)$$

$$= \frac{1}{72\pi^2 (Q^2)^3 \cdot 32} \left[\frac{(a-1)^2 (5a^3 + 15a^2 + 23a + 21)}{2a^4 \sqrt{a}} \ln \frac{\sqrt{a}+1}{\sqrt{a}-1} \right.$$

$$\left. - \frac{75a^5 + 25a^4 - 90a^3 + 2a^2 + 495a - 315}{15a^4 (a-1)} \right] , \tag{4.17a}$$

$$C^{\mathrm{V}}_{j^2} = \frac{1}{36\pi^2 (Q^2)^3} \left(\frac{41}{45} + \frac{2}{3}\mathcal{I}_1 - \mathcal{I}_2 - \tfrac{4}{9}\mathcal{I}_3 - \tfrac{26}{15}\mathcal{I}_4 + \tfrac{8}{5}\mathcal{I}_5 + \frac{Q^2}{3m^2}\mathcal{I}_1 - \frac{3Q^2}{5m^2} \right)$$

$$= \frac{1}{36\pi^2 (Q^2)^3 \cdot 48} \left[-\frac{5a^5 - 103a^4 - 6a^3 + 98a^2 - 79a + 21}{2a^4 \sqrt{a}} \ln \frac{\sqrt{a}+1}{\sqrt{a}-1} \right.$$

$$\left. + \frac{75a^5 - 1595a^4 + 930a^3 - 2218a^2 + 1395a - 315}{15a^4 (a-1)} \right] , \tag{4.17b}$$

where $\mathcal{I}_N$ are the basic integrals (4.7).

The results for other currents are given in appendix B.

4.3. TESTS ON THE COMPUTER PROGRAM

The final result should possess some general properties that we have used to check our computer program.

First, the coefficient functions related to $\langle g^3 fG^3 \rangle$ and $\langle g^4 j^2 \rangle$ should be gauge invariant (i.e., z_0 independent) for all currents. This requirement is far from being trivial since the cancellation of the z_0 dependence takes place between the contributions of different diagrams. In particular, the total contribution of the GDDG diagrams (figs. 3d, e) depends on z_0 for the G^3 part and is z_0 independent for the j^2 part. The contribution of the GGG diagrams (fig. 3c), in its turn, contains only a G^3 term which has a z_0 dependence just cancelling that due to the GDDG diagrams.

Our second check is based on the requirement that for the vector current $\Pi_{\mu\nu}(q)$ should be proportional to $(q_\mu q_\nu - q^2 g_{\mu\nu})$. Again, to get this structure, one must sum over all relevant diagrams, and a small error present in a particular diagram normally spoils the total sum.

We use the same program for all currents; hence, if the program was correct for the vector current, it should be also correct for all other currents.

Moreover, one can directly check the consistency of calculations for various currents. This check is based on the Fierz identity

$$\mathrm{Tr}(\gamma_5 \mathcal{S}^c)\,\mathrm{Tr}(\gamma_5 \mathcal{S}^c) \sim \Pi^S + \Pi^P - \Pi^V_{\mu\mu} + \Pi^{\tilde{A}}_{\mu\mu} - \Pi^T_{\mu\nu\mu\nu}, \tag{4.18}$$

where $\tilde{A} \sim \gamma_\alpha \gamma_5$, $T \sim \sqrt{\frac{1}{2}}\,\sigma^{\alpha\beta}$, and the observation that

$$\mathrm{Tr}\left(\gamma_5 \frac{\not{k}+m}{k^2-m^2}\right) = \mathrm{Tr}\left(\gamma_5 \frac{\not{k}+m}{k^2-m^2}\gamma^\mu \frac{\partial}{\partial k^\nu}\frac{\partial}{\partial k^{\alpha_1}}\cdots\frac{\partial}{\partial k^{\alpha_n}}\frac{\not{k}+m}{k^2-m^2}\right) = 0. \tag{4.19}$$

Really, from eq. (4.19) it follows that only diagrams having at least two G insertions into each bare quark line give a non-zero contribution to the l.h.s. of eq. (4.18). Thus as far as the dimension-6 contribution is concerned, the combination written down on the r.h.s. of eq. (4.18) should vanish. Our results do satisfy this requirement.

5. Applications to QCD charmonium sum rules

Using a dispersion relation one can relate $\mathcal{P}^\Gamma(Q^2)$ to its imaginary part

$$\mathcal{P}^\Gamma(Q^2) = \frac{1}{\pi}\int_{4m_c^2}^\infty \frac{\mathrm{Im}\,\mathcal{P}^\Gamma(s)}{s+Q^2}\,\mathrm{d}s. \tag{5.1}$$

In its turn, $\mathrm{Im}\,\mathcal{P}^\Gamma(s)$ is related to a cross section. In particular, for the vector current we have

$$\mathrm{Im}\,\mathcal{P}^V(s) = \frac{9}{64\pi^2\alpha^2} s\sigma(\mathrm{e}^+\mathrm{e}^- \to \mathrm{charm}), \tag{5.2}$$

where $\alpha = \frac{1}{137}$ is the fine structure constant and $\sigma(\mathrm{e}^+\mathrm{e}^- \to \mathrm{charm})$ is the total cross section of the $\mathrm{e}^+\mathrm{e}^-$ annihilation into final states with open and hidden charm. Thus, using the explicit expression for $\mathcal{P}^\Gamma(Q^2)$ in terms of the vacuum expectation values $\langle G^2\rangle, \langle G^3\rangle, \langle j^2\rangle$, etc., one can relate the parameters of the QCD vacuum to observable quantities and test the existing models of the QCD vacuum structure.

In ref. [2] it was proposed to compare with data the ratio $r_n = \mathcal{M}_n/\mathcal{M}_{n-1}$ of moments $\mathcal{M}_n$ defined by

$$\mathcal{M}_n^\Gamma = \frac{1}{\pi}\int_{4m_c^2}^\infty \frac{\mathrm{Im}\,\mathcal{P}^\Gamma(s)}{s^{n+1}}\,\mathrm{d}s. \tag{5.3}$$

The explicit expression for $\mathfrak{M}_n$ including the $\langle G^2\rangle$ vacuum correction was first obtained in ref. [2]:

$$\mathfrak{M}_n^{V} = \mathfrak{M}_n^{(0)}\left\{1 + a_n\alpha_s - \frac{(n+3)!}{(n-1)!(2n+5)}\frac{\langle g^2G^2\rangle}{9(4m_c^2)^2} + O\left(m_c^{-6}\right)\right\}, \tag{5.4}$$

where $\mathfrak{M}_n^{(0)}$ is the contribution of the simplest 1-loop diagram (fig. 1a):

$$\mathfrak{M}_n^{(0)} = \frac{3}{4\pi^2}\frac{2^n(n+1)(n-1)!}{(2n+3)!!\left(4m_c^2\right)^n}. \tag{5.5}$$

a_n are known coefficients of the 2-loop perturbative correction and m_c is the mass of the charmed quark. In fig. 4a (taken from ref. [2]) the comparison is shown between theoretical prediction for r_n based on eq. (5.4) with $m_c = 1.26$ GeV, $\alpha_s = 0.2$, $\langle g^2G^2\rangle = (0.83\text{ GeV})^4$ and experimental data. The two curves are in good agreement with each other up to $n = 8$. Adding higher $1/m_c^2$ corrections to eq. (5.4) one should presumably improve the agreement also for higher n values.

Using eq. (4.17) it is easy to calculate the explicit form of the $O(m_c^{-6})$ correction to eq. (5.4):

$$\Delta\mathfrak{M}_n^{V} = \mathfrak{M}_n^{(0)}\left\{\frac{2}{45}\frac{(n+4)!(3n^2+8n-5)}{(n-1)!(2n+5)(2n+7)}\frac{\langle g^3fG^3\rangle}{9\left(4m_c^2\right)^3}\right.$$
$$\left. - \frac{8}{135}\frac{(n+2)!(n+4)(3n^3+47n^2+244n+405)}{(n-1)!(2n+5)(2n+7)}\frac{\langle g^4j^2\rangle}{9\left(4m_c^2\right)^3}\right\}. \tag{5.6}$$

As argued by SVZ in ref. [2], the vacuum intermediate state dominates the $\langle j^2\rangle$ matrix element, and, hence, $\langle g^4j^2\rangle$ is not a free parameter:

$$\langle g^4j^2\rangle = -\tfrac{4}{3}g^4\langle\bar{u}u\rangle^2. \tag{5.7}$$

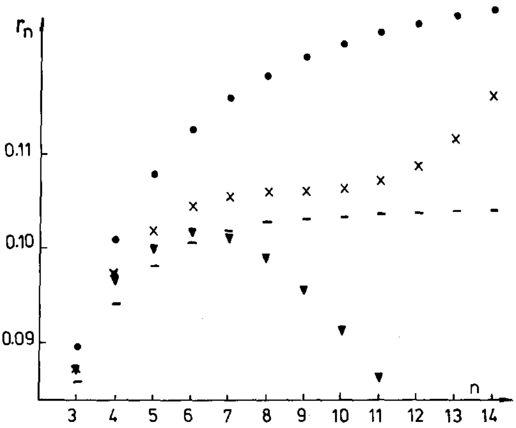
Taking $g^2 = 4\pi\cdot 0.7$ at the low normalization point $\mu = 0.2$ GeV where $\langle\bar{u}u\rangle = -(0.24\text{ GeV})^3$ we obtain the estimate

$$\langle g^4j^2\rangle = -(0.52\text{ GeV})^6 \tag{5.8}$$

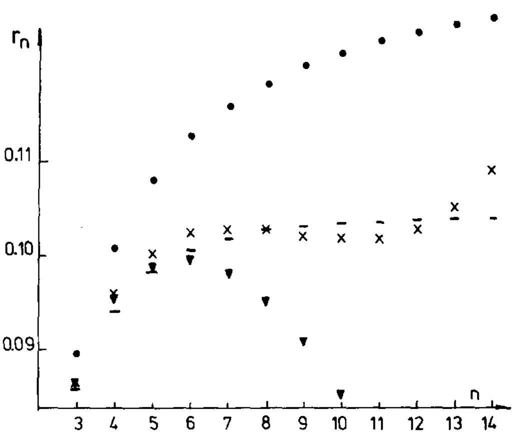
(for details see ref. [2]).

The magnitude of the $\langle g^3fG^3\rangle$ matrix element can be estimated by using the dilute instanton-gas approximation (DIGA) that gives [2]

$$\langle g^3f_{abc}G^a_{\mu\nu}G^b_{\nu\lambda}G^c_{\lambda\mu}\rangle|_{\mathrm{DIGA}} = \tfrac{12}{5}\rho_c^{-2}\langle g^2G^a_{\mu\nu}G^a_{\mu\nu}\rangle, \tag{5.9}$$



(a)



(b)

Fig. 4. Ratio $r_n = \mathfrak{M}_n/\mathfrak{M}_{n-1}$. (a) bars: experiment; circles: no power corrections; nabla's: SVZ fit $\langle g^2G^2\rangle = (0.83\ \text{GeV})^4$; crosses: SVZ fit corrected by $\langle g^4j^2\rangle$ and $\langle g^3fG^3\rangle$ contributions, the latter estimated in the DIGA, $\langle g^3fG^3\rangle = (0.60\ \text{GeV})^6$, $\langle g^4j^2\rangle = -(0.52\ \text{GeV})^6$. (b) bars: experiment; circles: no power corrections; nabla's: $\langle g^2G^2\rangle = (0.87\ \text{GeV})^4, \langle g^3fG^3\rangle = \langle g^4j^2\rangle = 0$; crosses: $\langle g^2G^2\rangle = (0.87\ \text{GeV})^4, \langle g^3fG^3\rangle = (0.62\ \text{GeV})^6, \langle g^4j^2\rangle = -(0.52\ \text{GeV})^6$.

where $\rho_c = (200\ \text{MeV})^{-1}$. If we adhere to this estimate and take the SVZ value $(0.83\ \text{GeV})^4$ for $\langle g^2G^2\rangle$, then [2]

$$\langle g^3fG^3\rangle = (0.60\ \text{GeV})^6. \tag{5.10}$$

Hence, both $O(m_c^{-6})$ corrections to $\mathfrak{M}_n$ and r_n are positive, i.e., work in the desired direction. The resulting curve is shown in fig. 4a. It goes systematically higher than the curve based on experimental data.

To improve the agreement between the two curves, one should take a larger value for $\langle g^2G^2\rangle$. In particular, multiplying $\langle g^2G^2\rangle$ and $\langle g^3fG^3\rangle$ by 1.2 one obtains a curve that is in a good agreement with experimental data up to $n = 13$ (see fig. 4b).

Moreover, treating $\langle g^2G^2\rangle$, $\langle g^3fG^3\rangle$ and $\langle g^4j^2\rangle$ as independent free parameters, one can get an even better fit to data. However, our point of view is that it is premature to attempt such a fitting without including the next, i.e., $O(G^4)$ correction. The motivation is that the $O(G^3)$ contribution has an additional numerical suppression compared to $O(G^2)$ and $O(G^4)$ ones. To illustrate the suppression, let us write down the leading large-n behaviour of $\mathcal{M}_n^V$:

$$\mathcal{M}_n^V\big|_{n\to\infty} = \mathcal{M}_n^{(0)}\left\{1 + a_n\alpha_s - \frac{n^3}{18}\left[\frac{\langle g^2G^2\rangle}{(4m_c^2)^2} - \frac{n^2}{15}\frac{\langle g^3fG^3\rangle}{(4m_c^2)^3} + \cdots\right]\right\}. \tag{5.11}$$

As is clear from eq. (5.11), the $O(G^3)$ contribution is suppressed by the factor $\frac{1}{15}$ compared to what one can expect from naive dimensional considerations. This suppression was first observed by Voloshin [21] who demonstrated that in the non-relativistic limit, i.e., for $n\to\infty$, all the G^{2n+1} contributions have additional small factors absent for the G^{2n} ones. Hence, the G^4 contribution may exceed the G^3 one even for not very large n values $n \geqslant 4$, and a reliable test of existing models of the QCD vacuum requires the calculation of the G^4 corrections.

6. Conclusion

In the present paper we described the method of computing vacuum gluonic corrections to the polarization operator $\Pi^\Gamma(q)$ of quark currents. The basic idea of the method is that one can separate the "A-dependence" from "G-dependence" by extracting the A-dependent terms into path-ordered exponentials which cancel with each other for gauge-invariant amplitudes like $\Pi^\Gamma(q)$. We demonstrated also that our technique is equivalent to the use of the Schwinger gauge $(x^\mu - z_0^\mu)A_\mu(x) = 0$ for vacuum gluonic fields. Furthermore, we described the algorithm used in computer calculations of $O(m_c^{-4})$ and $O(m_c^{-6})$ corrections to QCD charmonium sum rules and presented our results for the polarization operator related to 4 different quark currents. Comparing our results for the vector current $\bar{c}\gamma_\mu c$ with the curve based on experimental data we observed that the data favour a larger $\langle g^2G^2\rangle$ value than that used by SVZ in ref. [2]. However, as argued in sect. 5, to get a reliable estimate for $\langle g^2G^2\rangle$ and $\langle g^3fG^3\rangle$ from experimental data, one should compute also the G^4 contribution. The computations based on the approach described in the present paper are under completion now, and their results will be published elsewhere.

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Note added in proof. As we have been recently informed by N.B. Skachkov, the gauge (1.3) which is in the present paper referred to as the Schwinger gauge, was in fact proposed by V.A. Fock [22]. [See also V.A. Fock, ref. [23], in Russian].

Appendix A

In computing the coefficients C_{G^3} and C_{j^2} in the expansion (4.9) we used the following representation for the tensor structure of the basic matrix elements:

$$\langle 0|f_{abc}G^a_{\mu\nu}G^b_{\alpha\beta}G^c_{\rho\sigma}|0\rangle = \tfrac{1}{24}\langle 0|f_{abc}G^a_{\gamma\delta}G^b_{\delta\varepsilon}G^c_{\varepsilon\gamma}|0\rangle$$
$$\times\left(g_{\mu\sigma}g_{\alpha\nu}g_{\beta\rho}+g_{\mu\beta}g_{\alpha\rho}g_{\sigma\nu}+g_{\alpha\sigma}g_{\mu\rho}g_{\nu\beta}+g_{\rho\nu}g_{\mu\alpha}g_{\beta\sigma}\right.$$
$$\left.-g_{\mu\beta}g_{\alpha\sigma}g_{\rho\nu}-g_{\mu\sigma}g_{\alpha\rho}g_{\nu\beta}-g_{\alpha\nu}g_{\mu\rho}g_{\beta\sigma}-g_{\beta\rho}g_{\mu\alpha}g_{\nu\sigma}\right), \quad \text{(A.1)}$$

$$\langle 0|G^a_{\mu\nu}G^a_{\alpha\beta;\,\rho\sigma}|0\rangle = 2O^- g_{\rho\sigma}\left(g_{\mu\beta}g_{\alpha\nu}-g_{\mu\alpha}g_{\nu\beta}\right)$$
$$+O^-\left(g_{\mu\beta}g_{\alpha\sigma}g_{\rho\nu}+g_{\alpha\nu}g_{\mu\rho}g_{\beta\sigma}-g_{\alpha\sigma}g_{\mu\rho}g_{\nu\beta}-g_{\rho\nu}g_{\mu\alpha}g_{\beta\sigma}\right)$$
$$+O^+\left(g_{\mu\sigma}g_{\alpha\nu}g_{\beta\rho}+g_{\mu\beta}g_{\alpha\rho}g_{\sigma\nu}-g_{\mu\sigma}g_{\alpha\rho}g_{\nu\beta}-g_{\rho\beta}g_{\mu\alpha}g_{\nu\sigma}\right), \quad \text{(A.2)}$$

where

$$O^{\pm}=\tfrac{1}{72}\langle 0|g^2 j^a_\mu j^a_\mu|0\rangle \pm \tfrac{1}{48}\langle 0|gf_{abc}G^a_{\mu\nu}G^b_{\nu\lambda}G^c_{\lambda\mu}|0\rangle\,.$$

Appendix B

Here we present our results for the coefficients C_{G^2}, C_{G^3} and C_{j^2} in the expansion (4.9) for scalar ($\bar\psi\psi$), pseudoscalar ($\bar\psi\gamma_5\psi$), and axial-vector ($\bar\psi\eta_{\mu\nu}\gamma^\nu\gamma_5\psi$) currents. The results are presented both in terms of the basic integrals $\mathcal{I}_N$ (4.7) and in an explicit form. We denote $\eta_{\mu\nu}=q_\mu q_\nu/q^2-g_{\mu\nu}$, $a=1+4m^2/Q^2$.

(1) Scalar current. $J^{PC}=0^{++}$, $T^{\mathrm{S}}=1$:

$$C^{\mathrm{S}}_{G^2}=\frac{1}{32\pi^2Q^2}\left(-1-2\mathcal{I}_1+3\mathcal{I}_2\right)$$
$$=\frac{1}{32\pi^2Q^2\cdot 2}\left[\frac{(1-a)(a+3)}{2a\sqrt{a}}\ln\frac{\sqrt{a}+1}{\sqrt{a}-1}+\frac{a-3}{a}\right], \quad \text{(B.1)}$$

$$C^{\mathrm{S}}_{G^3}=\frac{1}{144\pi^2(Q^2)^2}\left(-\tfrac{14}{5}-9\mathcal{I}_1+42\mathcal{I}_2-44\mathcal{I}_3+\tfrac{69}{5}\mathcal{I}_4-\frac{3}{10}\frac{Q^2}{m^2}\right)$$
$$=\frac{1}{144\pi^2(Q^2)^2\cdot 16}\left[\frac{3(1-a)(a^3+5a^2+19a+23)}{2a^3\sqrt{a}}\ln\frac{\sqrt{a}+1}{\sqrt{a}-1}\right.$$
$$\left.+\frac{3a^4+10a^3-43\frac{1}{5}a^2-58a+69}{a^3(a-1)}\right], \quad \text{(B.2)}$$

$$C_{j^2}^{S}=\frac{1}{72\pi^2(Q^2)^2}\left(\tfrac{13}{15}-2\mathcal{I}_1+9\mathcal{I}_2+\tfrac{4}{3}\mathcal{I}_3-\tfrac{46}{5}\mathcal{I}_4-\frac{Q^2}{m^2}\mathcal{I}_1-\frac{9}{5}\frac{Q^2}{m^2}\right)$$

$$=\frac{1}{72\pi^2(Q^2)^2\cdot 8}\left[\frac{a^4-8a^3-90a^2+88a-23}{2a^3\sqrt{a}}\ln\frac{\sqrt{a}+1}{\sqrt{a}-1}\right.$$

$$\left.+\frac{3a^4-26a^3+436\tfrac{4}{5}a^2-310a+69}{3a^3(1-a)}\right]. \tag{B.3}$$

(2) Pseudoscalar current. $J^{PC}=0^{-+}, T^P=1$:

$$C_{G^2}^{P}=\frac{1}{96\pi^2Q^2}(-5-6\mathcal{I}_1+15\mathcal{I}_2-4\mathcal{I}_3)$$

$$=\frac{1}{96\pi^2Q^2\cdot 2}\left[\frac{3(1-a)(1+3a)}{2a^2\sqrt{a}}\ln\frac{\sqrt{a}+1}{\sqrt{a}-1}-\frac{3+7a}{2a^2}\right], \tag{B.4}$$

$$C_{G^3}^{P}=\frac{1}{144\pi^2(Q^2)^2}\left(-\tfrac{14}{5}-9\mathcal{I}_1+48\mathcal{I}_2-62\mathcal{I}_3+\tfrac{153}{5}\mathcal{I}_4-\tfrac{24}{5}\mathcal{I}_5-\frac{Q^2}{10m^2}\right)$$

$$=\frac{1}{144\pi^2(Q^2)^2\cdot 16}\left[\frac{3(1-a)(5a^3+13a^2+23a+7)}{2a^4\sqrt{a}}\ln\frac{\sqrt{a}+1}{\sqrt{a}-1}\right.$$

$$\left.+\frac{15a^4-11\tfrac{3}{5}a^3-64\tfrac{4}{5}a^2+34a+21}{a^4(a-1)}\right], \tag{B.5}$$

$$C_{j^2}^{P}=\frac{1}{72\pi^2(Q^2)^2}\left(-\tfrac{67}{15}-8\mathcal{I}_1+25\mathcal{I}_2-\tfrac{22}{3}\mathcal{I}_3-\tfrac{42}{5}\mathcal{I}_4+\tfrac{16}{5}\mathcal{I}_5+\frac{Q^2}{m^2}\mathcal{I}_1-\frac{3Q^2}{5m^2}\right)$$

$$=\frac{1}{72\pi^2(Q^2)^2\cdot 8}\left[\frac{11a^4-22a^3+36a^2+14a-7}{2a^4\sqrt{a}}\ln\frac{\sqrt{a}+1}{\sqrt{a}-1}\right.$$

$$\left.+\frac{33a^4+142\tfrac{2}{5}a^3-82\tfrac{4}{5}a^2-56a+21}{3a^4(1-a)}\right]. \tag{B.6}$$

(3) Axial-vector current. $J^{PC}=1^{++}, T_{\mu\nu}^{A}=q_\mu q_\nu-q^2g_{\mu\nu}$:

$$C_{G^2}^{A}=\frac{1}{16\pi^2(Q^2)^2}(1-\mathcal{I}_2)$$

$$=\frac{1}{16\pi^2(Q^2)^2\cdot 2}\left[\frac{(a-1)^2}{2a\sqrt{a}}\ln\frac{\sqrt{a}-1}{\sqrt{a}+1}+\frac{a+1}{a}\right], \tag{B.7}$$

$$C_{G^3}^{\mathrm{A}} = \frac{1}{72\pi^2(Q^2)^3}\left(-\tfrac{1}{15} - 5\mathcal{I}_2 + \tfrac{29}{3}\mathcal{I}_3 - \tfrac{23}{5}\mathcal{I}_4 + \frac{Q^2}{10m^2}\right)$$

$$= \frac{1}{72\pi^2(Q^2)^3 \cdot 16}\left[\frac{(a-1)^2(5a^2+12a+23)}{2a^3\sqrt{a}}\ln\frac{\sqrt{a}-1}{\sqrt{a}+1}\right.$$

$$\left. + \frac{15a^4 - 4a^3 - 70\tfrac{4}{5}a^2 + 148a - 69}{3a^3(a-1)}\right], \tag{B.8}$$

$$C_{j^2}^{\mathrm{A}} = -\frac{2}{3Q^2}C_{j^2}^{\mathrm{S}}. \tag{B.9}$$

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