Vacuum expectations of the high dimensional operator and their contribution in Bjorken and Ellis-Jaffe sum rules

A.G.Oganesian *
Institute of Theoretical and Experimental Physics,
RU-117259 Moscow, Russia

*E-mail: armen@vitep5.itep.ru

Introduction

It is well-known, that in some cases the contributions of the high-dimensional ($d > 6$) operators in the QCD sum rules [1] are important. For estimation of the vacuum average of high-dimensional operators usually is used factorization hypothesis [1]. According to this hypothesis it is assumed that in the high-dimensional operators expansion over intermediate states, vacuum states contributions are dominant. Formally this assumption can be written as

$$< O_1 \cdot O_2 > \simeq < O_1 > \cdot < O_2 >$$

where $< O >$ are some color scalar operators.

In [1] factorization hypothesis was used to estimate vacuum average of the 4-quarks operator ($d = 6$). Gluons operators with $d = 6$ and $d = 8$ was been calculated in [2,3] (see also discussion about it in [4]). But for operators with dimension ($d = 7$) a large number of new vacuum averages appears, and some of them can't be reduced to a product of vacuum averages of the operator with lower dimension. Nevertheless in this paper we make an attempt to estimate vacuum averages of all operators with $d = 7$. The method we offer is based on factorization hypothesis.

It is well-known, that though factorization hypothesis is confirmed in $1/N$ limit [1], but for real world with N=3 factorization hypothesis in some cases has rather bad accuracy (see for example [5]). So in this work we shall use factorization hypothesis in maximal "soft" form, i.e. we suppose:
1) If any operator can be saturated by vacuum intermediate states (i.e. if vacuum intermediate states contribution exist <u>and is not zero</u>) we estimate it by its factorized value and suppose accuracy 30%.
2) To estimate other operators one should try to express them through factorizable operators, if it is possible.
To avoid uncertainty, we also suppose:
3) Vacuum average of high-dimensional operators should not depend on the way of factorization, and, particularly, vacuum average of operators, containing derivatives should not depend on the fact are the equation of motion taken in account before or after factorization.
This assumption 3) can be treated as a condition of self-consistence of factorization hypothesis. This three assumptions we hereinafter shall call anzatz.

The main idea is to consider the vacuum average of the operator with large dimension ($d = 10$). Using fagtorization hypothesis one can express it in terms of the product of vacuum averages of operators with lower dimension ($d < 7$), some of which are known and some are unknown. According condition of self-agreement of factorization one can write a number of relations for this unknown operators and estimate them. This method will be explicitly explained in section 1. As by-product some estimations for vacuum average of the operators of dimension 10 became available. Of course one must note that this is only some rough phenomenological estimations and do not claim for high accuracy. An accuracy of our estimation are about a factor 2.

The paper is organized as follows.

In section 1 we describe the method on an example of calculation vacuum averages of the of dimension 7 operators, constructed from quark and gluons.

In section 2 we discuss vacuum averages of the 7-dimension operators with one derivatives. So it appears possible to evaluate all vacuum averages of the dimension 7. In section 3 obtained results are used to calculate high dimensional operator contribution to Bjorken [6] and Ellis-Jaffe [7] sum rules.

Section 1

In this section we'll discuss following vacuum averages of the of dimension 7 nonfactorizable operators:

$$R_d = \frac{< g^2 \bar{q} d^{nkl} \lambda^l G^n_{\mu\nu} G^k_{\mu\nu} q >}{24}; \quad R_f = \frac{< g^2 \bar{q} f^{nkl} \lambda^l G^n_{\mu\nu} G^k_{\mu\beta} \sigma^{\nu\beta} q >}{24}$$

$$S_1 = \frac{i < g^2 \bar{q} \gamma_5 G^n_{\alpha\beta} \tilde{G}^n_{\alpha\beta} \bar{q} >}{24}; \quad S_d = \frac{i < g^2 \bar{q} \gamma_5 d^{nkl} \lambda^l G^n_{\mu\nu} \tilde{G}^k_{\mu\nu} \bar{q} >}{24} \tag{1}$$

where $\tilde{G}^n_{\mu\nu} = G^n_{\mu_1\nu_1} \cdot \varepsilon^{\mu\nu\mu_1\nu_1}/2$

For convenience hereinafter following notations will be used

$$\bar{R}_{d,f} = R_{d,f} < \bar{q}q > \qquad \bar{R}_1 =< g^2 G^2 >< \bar{q}q >^2 /24$$

$$\bar{S}_{1,d} = S_{1,d} < \bar{q}q > \qquad N = (< g\bar{q}\hat{G}_{\mu\nu}\sigma^{\mu\nu} q >)^2/24 \tag{1a}$$

$$\hat{G}_{\mu\nu} = \frac{\lambda^n}{2} G^n_{\mu\nu}; \qquad G^2 = G^n_{\mu\nu} G^n_{\mu\nu}$$

Note, that at standard choose of gluon, quark and quark-gluon condensates $\bar{R} \sim N$. We assume that in our accuracy vacuum averages for u and d-quarks are the same, so, for example

$$< g^2 \bar{u} d^{nkl} \lambda^l G^n_{\mu\nu} G^k_{\mu\nu} u >=< g^2 \bar{d} d^{nkl} \lambda^l G^n_{\mu\nu} G^k_{\mu\nu} d >$$

The method of estimation R_d, R_f... is based on our anzatz. We consider some vacuum averages of the of dimension 10 operators and factorize them in two different ways. Once we use equation of motion before factorization, and other time -after. We require this two results to be the same (with accuracy ~ 30 %, which is the accuracy of factorization itself). Because of such uncertainty in order to estimate vacuum averages (1) one have to use only those equations, which are enough ($> 50\%$) sensitive to unknown values of vacuum averages (1).

We shall illustrate the method on an example of the following operator with dimension 10

$$< T_1 >= - < (\bar{d}[\nabla^2 u]) \cdot (\bar{u}[\nabla^2 d]) >$$

where square of a bracket like $[\nabla^2 u]$ mean that a derivative acts only on the quark operator in bracket. From one side $< T_1 >$ can be immediately factorized in the single way

$$< T_1 >= \frac{1}{12} < \bar{u}[\nabla^2 u] >< \bar{d}[\nabla^2 d] > \tag{2}$$

Using equations of motion $\hat{\nabla} q = 0$ and the fact that

$$\nabla^2 = \hat{\nabla}\hat{\nabla} + \frac{1}{2} g \hat{G}_{\mu\nu} \sigma^{\mu\nu}, \qquad (\text{where } \hat{\nabla} = \gamma_\mu \nabla_\mu) \tag{3}$$

One can find

$$< T_1 >= \frac{1}{2} \cdot N \tag{4}$$

where N is denoted in (1a).

From other side if take into consideration equation of mouton from very beginning, one get

$$< T_1 >= -\frac{g^2}{4} < \bar{d}\frac{\lambda^k}{2}\sigma^{\alpha\beta}u \cdot \bar{u}\frac{\lambda^n}{2}\sigma^{\mu\nu}d \cdot G^n_{\alpha\beta}G^k_{\mu\nu} > \tag{5}$$

Now one can do here Fiertz transformation (for a simplicity we will write down it only for color indexes, and for Lorenz indexes it is meant)

$$< T_1 >= \frac{g^2}{4}\Big\{\frac{1}{3} < (\bar{u}_{\tau'}u_\tau) \cdot (\bar{d}_{\rho'}\lambda^k\lambda^n d_\rho)G^k_{\alpha\beta}G^n_{\varphi\varepsilon} > +$$

$$+\frac{1}{2} < (\bar{u}_{\tau'}\lambda^l u_\tau) \cdot (\bar{d}_{\rho'}\lambda^k\lambda^l\lambda^n d_\rho)G^k_{\alpha\beta}G^n_{\varphi\varepsilon} >\Big\} \cdot \frac{1}{4} \cdot \sigma^{\alpha\beta}_{\rho'\tau}\sigma^{\varphi\varepsilon}_{\tau'\rho} \tag{6}$$

Using the fact that

$$\Big(\lambda^k\lambda^l\lambda^n\Big)^{ab} = \frac{2}{3}\Big(d^{nkl} + if^{nkl}\Big)\delta^{ab} + \frac{13}{21}\Big((\lambda^k)^{ab}\delta_{ln} + (\lambda^n)^{ab}\delta_{lk}\Big) -$$

$$-\frac{5}{21}(\lambda^l)^{ab}\delta^{kn} + O^{mkln}(\lambda^m)^{ab} \tag{7}$$

(where $O^{mkln} = Tr(\lambda^m\lambda^k\lambda^l\lambda^n)/2 - \frac{13}{21}(\delta^{mk}\delta^{ln} + \delta^{mn}\delta^{kl} - \frac{5}{13}\delta^{ml}\delta^{kn})$ - is traceless matrixes by each pair of indexes) we find

$$< T_1 >= \frac{g^2}{16}\sigma^{\alpha\beta}_{\rho'\tau}\sigma^{\varphi\varepsilon}_{\tau'\rho} \cdot \Big\{\frac{1}{3} < (\bar{u}_{\tau'}u_\tau) \cdot (\bar{d}_{\rho\prime}\lambda^k\lambda^n d_\rho G^k_{\alpha\beta}G^n_{\varphi\varepsilon}) >$$

$$+\frac{1}{3} < (\bar{u}_{\tau'}\lambda^l(d^{nkl} + if^{nkl})G^k_{\alpha\beta}G^n_{\varphi\varepsilon}u_\tau) \cdot (\bar{d}_{\rho'}d_\rho) >$$

$$+\frac{26}{21}\Big[< (\bar{u}_{\tau'}\hat{G}_{\varphi\varepsilon}u_\tau) \cdot (\bar{d}_{\rho'}\hat{G}_{\alpha\beta}d_\rho) > + < (\bar{u}_{\tau'}\hat{G}_{\alpha\beta}u_\tau)(\bar{d}_{\rho'}\hat{G}_{\varphi\varepsilon}d_\rho) >\Big]$$

$$-\frac{5}{42} < (\bar{u}_{\tau'}\lambda^l u_\tau)(\bar{d}_{\rho'}\lambda^l d_\rho)G^k_{\alpha\beta}G^n_{\varphi\varepsilon} >$$

$$+\frac{1}{2} < (\bar{u}_{\tau'}\lambda^l u_\tau)(d_{\rho'}O^{mkln}\lambda^m d_\rho)G^k_{\alpha\beta}G^n_{\varphi\varepsilon} >\Big\} \tag{8}$$

Fourth term in (8) after factorization appears to be expressed in terms $< (\bar{u}\lambda^l\Gamma u) \cdot (\bar{d}\lambda^l\Gamma d) >< g^2G^2 >$ where $\Gamma = 1,\ \gamma_5,\ \gamma_\mu,\ \gamma_5\gamma_\mu,\ \sigma^{\mu\nu}$. This terms assumed to be 0, because, as was shown in [5], $< (\bar{u}\lambda^l\Gamma u) \cdot (\bar{d}\lambda^l\Gamma d) >$ are negligible small for $\Gamma = 1, \gamma^\mu$ and there is not any reasons to believe that for other Γ result will grow on an order, so, we can neglect by fourth term. The last, fifth, term in (8) appears to be 0 at factorization due to fact that O^{nklm} is traceless matrixes The first three terms in (8) allows vacuum intermediate state, so $< T_1 >$ can be factorized and after some calculations one can found

$$< T_1 >= \bar{R}_1/6 + \bar{R}_d/2 + \bar{R}_f + \frac{13}{14}N \tag{9}$$

Here we omit terms, proportional to $\bar{S}_1$, $\bar{S}_d$, because, as will be shown later, they are negligibly small. Comparing (4) and (9) we get

$$N/2 = \bar{R}_1/6 + \bar{R}_d/2 + \bar{R}_f + \frac{13}{14}N \tag{10}$$

Note that this equality is rather sensitive to unknown vacuum averages $\bar{R}_d$ and $\bar{R}_s$. Really, if we suppose $\bar{R}_d = \bar{R}_f = 0$, then left and right side differ from each other more than twice.

One more equation can be obtained, if we consider such an operator

$$< T_2 >=< \bar{d}_\lambda[L^{\alpha\mu}u]_\rho) \cdot (\bar{u}_{\tau'}u_\tau) > \cdot\delta^{\tau'\tau} \cdot (\gamma^\alpha\gamma^\mu)_{\lambda\rho} \tag{11}$$

where $L^{\alpha\mu} = (\nabla_\alpha\nabla_\beta\nabla_\mu\nabla_\beta - \nabla_\beta\nabla_\alpha\nabla_\mu\nabla_\beta)$.
$< T_2 >$ can be factorized immediately as $< T_2 >= - < \bar{u}\gamma^\alpha\gamma^\mu[\prod^{\alpha\mu} u] >< \bar{d}d > \cdot\frac{1}{12}$.
Using equations of motion after some calculations one can find

$$< T_2 >= \bar{R}_1/3 + \bar{R}_d/2 + \bar{R}_f/2 \tag{12}$$

From other side by use of equation of motion one can write (11) in the form:

$$< T_2 >= -g^2 < (\bar{d}\hat{G}_{\alpha\beta}\hat{G}_{\mu\beta}\gamma^\alpha\gamma^\mu u)(\bar{u}d) >$$

Then after Fiertz transformation just as in previous case one can get

$$< T_2 >= \bar{R}_1/3 + \bar{R}_d + \bar{R}_f + \frac{11}{21}N \tag{13}$$

From (12) and (13) we have

$$\bar{R}_1/3 + (\bar{R}_f + \bar{R}_d)/2 = \frac{\bar{R}_1}{3} + \bar{R}_d + \bar{R}_f + \frac{11}{21}N \tag{14}$$

Note, that (14) also is sensitive to $\bar{R}_d$, $\bar{R}_f$.
Using the same procedure for the operator

$$< W >= - < i(\bar{d}\gamma_5\varepsilon^{\alpha\beta\mu\nu}[\nabla_\alpha\nabla_\beta\nabla_\mu\nabla_\nu u])(\bar{u}d) > \tag{15}$$

we get such an equation:

$$\bar{S}_1/3 + \bar{S}_d/2 = \bar{S}_1/3 + \bar{S}_d - \frac{2}{21}N \tag{16}$$

From (16) we can conclude $\bar{S}_d \sim N/5 \ll N$ so we put it 0. What about $\bar{S}_1$, we assume it to be negligibly small too. Really, it is easy to show by direct calculations

$$S_1 = \frac{1}{24} < \bar{q}q \cdot G^2 > -\frac{1}{48} < \bar{q}\sigma^{\alpha\beta}\sigma^{\mu\nu}\ G^n_{\alpha\beta}G^n_{\mu\nu}q > \tag{17}$$

So S_1 is equal to difference of two factorizable vacuum averages which cancel each other after factorization. So we can expect that S_1 should be much more less then this vacuum averages (which are of order of R_1) and we can put it 0 within our accuracy.

From (10), (14) we found

$$\bar{R}_f \sim 4/21 \cdot N - \bar{R}_1/3 \ll \bar{R}_1 \quad (\text{or} \quad N); \quad \text{so} \quad \bar{R}_f \sim 0$$

$$\bar{R}_d \sim -26/21 \cdot N + \bar{R}_1/3 \sim -N \quad (\text{or} \quad \bar{R}_d \sim -\bar{R}_1 \quad \text{because} \quad \bar{R}_1 \sim N, \quad \text{see} \quad (1))$$

Finally, we write down results of this section:

$$R_d \sim -R_1, \quad S_1 \sim S_d \sim R_f \approx 0 \tag{18}$$

Section 2

In this section we will discuss vacuum averages of the four-quark operators with one derivative, such us:

$$X_1 =< (\bar{q}[\nabla_\alpha q])(\bar{q}\gamma^\alpha q) >; \quad \bar{X}_1 =< (\bar{q}\gamma_5[\nabla_\alpha q])(\bar{q}\gamma_5\gamma^\alpha q) >$$

$$X_2 =< (\bar{q}\lambda^k[\nabla_\alpha q])(\bar{q}\lambda^k\gamma^\alpha q) >; \quad \bar{X}_2 =< (\bar{q}\lambda^k\gamma_5[\nabla_\alpha q])(\bar{q}\lambda^k\gamma_5\gamma^\alpha q) >$$

$$Y_1 = i < (\bar{q}\gamma^\tau[\nabla_\varepsilon q])(\bar{q}\sigma^{\tau\varepsilon}q) >; \quad \bar{Y}_1 = i < (\bar{q}\gamma_5\gamma^\tau[\nabla_\varepsilon q])(\bar{q}\gamma_5\sigma^{\tau\varepsilon}q) >$$

$$Y_2 = i < (\bar{q}\lambda^k\gamma^\tau[\nabla_\varepsilon q])(\bar{q}\lambda^k\sigma^{\tau\varepsilon}q) >; \quad \bar{Y}_2 = i < (\bar{q}\lambda^k\gamma_5\gamma^\tau[\nabla_\varepsilon q])(\bar{q}\lambda^k\gamma_5\sigma^{\tau\varepsilon}q) > \tag{19}$$

Note, that one can't factorize this operators immediately (after proper Fiertz transformation), because due to equation of motion they became zero after factorization (see point 1,2 of our anzatz). Note also, that all other vacuum averages of the four-quark operators with one derivative easily can be expressed by this eight, by help of equations of motions. Of course, this eight operators aren't independent. First it can easily be shown, that $Y_1 = -X_1$. Really, due to C-parity

$$Y_1 = \frac{i}{2} < (\bar{q}\gamma^\tau[\nabla_\varepsilon q] + [\bar{q}\nabla_\varepsilon]\gamma^\tau q) \cdot (\bar{q}\sigma^{\tau\varepsilon}q) >= \frac{i}{2} < [\partial_\varepsilon(\bar{q}\gamma^\tau q)] \cdot (\bar{q}\sigma^{\tau\varepsilon}q) > \tag{20}$$

Neglecting the full derivatives (and all possible anomalies) one can write

$$Y_1 = -\frac{i}{2} < (\bar{q}\gamma^\tau q) \cdot ([\bar{q}\nabla_\varepsilon]\sigma^{\tau\varepsilon}q + \bar{q}\sigma^{\tau\varepsilon}[\nabla_\varepsilon q]) >$$

Now, using equations of mouton $\hat{\nabla}q = 0$ it is easy to find

$$Y_1 = -\frac{i}{2} < (\bar{q}\gamma^\tau q) \cdot ([\bar{q}\nabla_\tau]q - \bar{q}[\nabla_\tau)q]) > \tag{21}$$

Finally from C-parity it is clear that

$$Y_1 = -X_1 \tag{22}$$

In the same way, if we neglect anomalies, one can show

$$\bar{X}_1 = - < (\bar{q}\gamma_5\gamma^\varepsilon q) \cdot \frac{1}{2}([\bar{q}\nabla_\varepsilon]\gamma_5 q + \bar{q}\gamma_5[\nabla_\varepsilon q]) >=$$
$$= -\frac{1}{2} < ([\bar{q}\nabla_\varepsilon]\gamma_5\gamma^\varepsilon q + \bar{q}\gamma_5\gamma^\varepsilon[\nabla_\varepsilon)q])(\bar{q}\gamma_5 q) >= 0 \tag{23}$$

Using Fiertz transformation both by color and scalar indexes, one can express vacuum averages (19) through each other. Finally a system of exact equations can be found, which solution, taking in account (22,23), is:

$$Y_2 = -\bar{Y}_2 = -X_2 = \frac{8}{3}X_1; \quad Y_1 = -\bar{Y}_1 = -X_1; \quad \bar{X}_1 = \bar{X}_2 = 0 \tag{24}$$

So we see, that all vacuum averages of operators, constructed from four quarks and one derivative are expressed through X_1. One must emphasize, that (24) is exact statement, don't based on factorization hypothesis.

To estimate X_1, we use factorization hypothesis analogously to what we have done in sect. 1. Let us consider vacuum averages of the operator Z_1

$$Z_1 =< 2(\bar{q}\lambda^n(D_\alpha G_{\alpha\beta})^n q) \cdot (\bar{q}[\nabla_\beta q]) > \tag{25}$$

which can be factorized as

$$Z_1 = -\frac{1}{6} < (\bar{q}\lambda^n(D_\alpha G_{\alpha\beta})^n[\nabla_\beta q]) >< \bar{q}q > \tag{26}$$

Now we can use equations of motion $D_\alpha G^n_{\alpha\beta} = -g\bar{q}(\lambda^n/2)\gamma^\beta$ (for simplicity we shall limit us by case with one flavor, two flavor case is similar). Then, taking into account (24), we get:

$$Z_1 = \frac{g}{12} < (\bar{q}\lambda^n[\nabla_\beta q])(q\lambda^n\gamma^\beta q) >< \bar{q}q >= \frac{g}{12}X_2 < \bar{q}q >= -\frac{2}{9}X_1 < \bar{q}q > \cdot g \tag{27}$$

On the other hand, using equations of motions, (25) can be rewritten as

$$Z_1 = - < (\bar{q}\lambda^n q)(\bar{q}\gamma^\beta\lambda^n q)(\bar{q}[\nabla_\beta q]) > \cdot g \tag{28}$$

Now, using Fiertz transformation and also use C-parity and neglecting full derivatives (in the same way as in (21-22)), we can write

$$Z_1 = \frac{1}{2} \cdot \left\{ \frac{7}{3} < (\bar{q}q)(\bar{q}\gamma^\beta q)(\bar{q}[\nabla_\beta q]) > + < (\bar{q}\gamma_5\gamma^\alpha\varepsilon^{\alpha\beta\tau\varepsilon}q) \cdot (\bar{q}\sigma^{\tau\varepsilon}q) \cdot (\bar{q}[\nabla_\beta q]) > \right\} \tag{29}$$

The first term here can be factorized, so according our anzatz we have

$$Z_1 = \frac{7}{6} < \bar{q}q > \cdot X_1 \cdot g \tag{30}$$

Comparing (27) and (30) we get $\frac{7}{6}\ X_1 = -\frac{2}{9}\ X_1$, So

$$X_1 = 0 \tag{31}$$

Then from (24) we can conclude, that all vacuum averages (19) are zero. One can easily see, that every vacuum averages of the operators with dimension 7 can be expressed through a set vacuum averages, discussed in this two sections. Thus, results, obtained in sections 1,2 allows one to give estimations for all possible vacuum averages of the operators with dimension 7. This estimation may be significant in a large range of problem, where contribution of high dimension operators became necessary.

An example of such problem is the calculation of Bjorken [6] and Ellis -Jaffe [7] sum rules in the framework of QCD sum rules, offered in [8] we are going to discuss in next section

Section 3

In this section we use results obtained in the previous sections for the analysis of the power corrections to the first moment of the structure function of a polarized nucleon. On importance of the power corrections $1/Q^2$ to structure functions of polarized nucleon was indicated in [9,10], where necessity of their contribution was considered to satisfy with experimental data on deep inelastic scattering on a polarized [11] nucleon, on the one hand, and Gerasimov - Drell - Hearn sum rule [12,13] - with other (see also discussion of this problem in connection with "spin-crisis" problem in review [14,15]). By the most natural candidate for this role seems the contribution of the operators of twist 4. The contribution of these operators to deep inelastic scattering on a polarized nucleon was calculated in [16].

$$M^{S(NS)} \equiv \int dx\ g_1^{p+n,(p-n)}(x,Q^2) =$$

$$= K^{S(NS)} \cdot \left\{ g_A^{S(NS)}(1-\alpha_s(Q^2)/\pi) - \frac{8}{9}\frac{\ll U^{S(NS)} \gg}{Q^2} \right\} +$$

$$+\frac{4}{3}\frac{m_N^2}{Q^2}\int dx x^2 \left(g_2^{p+n,(p-n)}(x) + \frac{5}{6} g_1^{p+n,(p-n)}(x) \right) + O(1/q^4) \tag{32}$$

Here:

$$K^{S(NS)} = \frac{5}{18}(\frac{1}{6}); \qquad g_A^{NS} = \left|\frac{G_A}{G_V}\right| = 1.25; \qquad g_A^S = 0.1 \pm 0.04 \quad \text{(see [14])}$$

$\ll U^{S(NS)} \gg$ are defined as $< N \mid U_\mu^{S(NS)} \mid N > = S_\mu \ll U^{S(NS)} \gg$

$$U^S = \bar{u}\ \hat{\tilde{G}}_{\mu\nu}\gamma_\nu u + (u \to d) + \frac{18}{5}(u \to s)$$

$$U^{NS} = \bar{u}\ \hat{\tilde{G}}_{\mu\nu}\gamma_\nu u - (u \to d)$$

and $S_\mu = \bar{N}\gamma_\mu\gamma_5 N$; N be a nucleon spinor. $\ll U^{S(NS)} \gg$ are matrix element of twist four, we are interest in this paper. $\ll U \gg$ has been calculated in [8] from sum rules for 3-point correlator

$$\Gamma_\mu(p) = i^2 \int dx e^{ipx} \int dy < T[\eta(x) U_\mu^{S(NS)}(y) \bar{\eta}(0)] >= -2p_\mu \hat{p}\gamma_5 \frac{\lambda_p^2 \ll U^{S(NS)} \gg}{(m_N^2 - p^2)^2} + ... \quad (33)$$

Where λ_p is proton coupling and η is proton current [17]

$$\eta = \varepsilon^{abc}(u^a C\gamma_\lambda u^b)\gamma_5\gamma_\lambda d^c$$

As always in QCD sum rules, the correlator (33) is considered at large negative p^2. However in this case, in difference from usual 3-point correlator, though $x \sim 1/p$ is small, but there are no limitations on y and, therefore, it is necessary to take into account region $y \gg x \sim 1/p$ too. This lead to the fact, that except usual vacuum expectations (local operators) in operator expansion also appears field induced vacuum expectations - 2-point (bilocal) correlators of the type

$$i \int d^4y < T\{O_\mu(y) O^\mu(0)\} >$$

(see papers [18 - 21], where this approach was offered and discussed).

In some cases these bilocal operators can be reduced to local ones, using low -energy relations (see, for example [18,22]), in other cases one should consider corresponding 2-point sum rules to estimate this bilocal operators (see [20,21], for example).

In the [8] the correlator (33) was calculated and the power correction in $1/p^2$ up to contribution of the operators with dimension $d = 8$ was accounted. After borelization the result, obtained in [8], has the form

$$\ll U \gg + R \cdot M^2 = -\frac{exp(m_N^2/M^2)}{2\lambda_p^2}\Bigg(2AM^2 \int_0^{s_0} ds\ s^2 e^{-s/M^2} ln\ \mu^2/s +$$
$$+ BM^4(1 - exp(-s_0/M^2)) + CM^2 + D\Bigg) \quad (34)$$

A, B, C, D correspond to loop contribution (A) and power correction of operators of dimension 4,6,8[1]. (Here, and also in expressions (35, 36,40) we for simplification omit subscript S(NS) in all cases, where it is obvious). Note, that (34) depend on ultraviolet cut-off parameter μ^2, even after borelization. This is consequence of a simple, but incorrect model of continuum accounting in [8], based on ordinary dispersion relation, as was noted in [14]. In [14] was offered the method, how one should correctly exclude continuum, using double dispersion relations and was shown, that in this method dependence of unphysical cut-off parameter μ^2 disappear. The procedure, offered in [14] lead to following sum rules (instead of (34))

[1]For the scalar case $\ll U^S \gg$ contribution of s-quark in [8] was neglected.

$$\ll U \gg + R\cdot M^2 = -\frac{exp(m_N^2/M^2)}{2\bar\lambda_p^2}\Bigg\{2AM^2\int\limits_0^{s_0} ds\ s\cdot$$

$$\cdot\, e^{-s/M^2}(s_0 + s\ ln\ s_0/s) + BM^4\left(1-(1+\frac{s_0}{M^2})e^{-s_0/M^2}\right) + CM^2 + D\Bigg\} \qquad (35)$$

Here:

$$A^S = \alpha_s/\pi\cdot 4/5;\ B^S = -32/3\cdot\pi^2 f_\pi^2\delta^2$$

$$A^{NS} = \alpha_s/\pi\cdot 4/9;\ B^{NS} = -\frac{< g^2G^2 >}{9}$$

$$C^S = (ln\ s_0/M^2 + 0.5)\cdot 32/27\cdot\alpha_s/\pi\cdot a^2 + 8/9\cdot\pi^2\cdot\Pi$$

$$C^{NS} = (ln\ s_0/M^2 + 1)\cdot 32/27\cdot\alpha_s/\pi\cdot a^2 + 8/9\cdot\pi^2\cdot\Pi$$

$$D^S = -1/9\cdot m_0^2\cdot 2\cdot a^2$$

$$D^{NS} = -1/3\cdot m_0^2\cdot 2\cdot a^2$$

$$a = -4\pi^2 < \bar\psi\psi >$$

R correspond to the contribution of single-pole terms, Π is bilocal power correction, which was estimated in [8] as $\Pi = 3.10^{-3}\mathrm{GeV}^6$; other parameters are standard: $f_\pi = 0.133\mathrm{GeV}$, $\delta^2 = 0.2\ \mathrm{GeV}^2$ [23], $m_0^2 \simeq 0.8\mathrm{GeV}^2$ [17]; $\alpha_s(1\mathrm{GeV}) \sim 0.37$, $<\bar\psi\psi>= -0.014\mathrm{GeV}^3$, $< g^2G^2 >= 0.5\mathrm{GeV}^2$; $\bar\lambda_p^2 = 32\pi^4\lambda_p^2 = 2.1\ \mathrm{GeV}^6$, and continuum threshold $s_0 = 2.25\mathrm{GeV}^2$ ([17, 21], see also [14]). Hereafter we will use this correct result for $\ll U \gg$ (35), but one should note, that results and conclusions we will present at the end of this section, are similar both for (34) and (35).

To single out $<< U >>$ itself, according to [8] the operator

$$1 - M^2\frac{d}{dM^2}$$

acting on both sides of (34) was used. Using this operator for improved result (35), we find

$$\ll U \gg = \frac{-e^{m_N^2/M^2}}{2\bar\lambda_p^2}\Bigg[2A\int\limits_0^{s_0} ds\ s(m_N^2 - s)e^{-s/M^2}(s_0 + s\ ln\ s_0/s) +$$

$$+B\left((m_N^2 - M^2)(1-(1+\frac{s_0}{m^2})e^{-s_0/M^2}) + \left(\frac{s_0}{M^2}\right)^2 e^{-s_0/M^2}\right) +$$

$$+ (Cm_N^2 + 32/27 \cdot \alpha_s/\pi \cdot a^2 \cdot M^2) + D(1 + m_N^2/M^2)\Big] \quad (36)$$

Analysis of sum rules (34), according [8], lead to $\ll U^S \gg \simeq -(0 \div 0.1)\mathrm{GeV}^2$, $\ll U^{NS} \gg \simeq 0.18$ at Borel mass $M^2 \simeq 1\mathrm{GeV}^2$ and even we take into account improved form (35,36), results change slightly (see [14]). However from (34-36) it is possible to see, that in $\ll U \gg$ just dominates the contribution of the operators of dimension 8, (both for (34) or (35,36)) that is the last accounted term of expansion. Thus, there is the problem on a reliability of results, obtained in [8], and for this purpose it is necessary to evaluate the following term in expansion, that is contribution of the operators of dimension $D = 10$. As usual, we shall consider that sum rules (36) are reliable, if contribution of the operators of dimension $D = 10$ will appear less than contribution of the operators of dimension $D = 8$.

In this section we will estimate the dimension 10 operators contribution to sum rules for $\ll U \gg$ (36). We will take into account only tree diagrams (fig.1a-1d), because all other are suppressed by loop factors like $1/4\pi^2$.

The main problem here is the estimation of vacuum averages of dimension 10 operators, and this can be done by help of results of sect.1,2. In this section we'll calculate the contribution off all local operators of D=10 (fig.1a,b) and also those bilocal operators (fig.1c), for which exact low-energy relations exists (see eq.(27) in [17], where this relation was discussed for very similar case).

In this work we don't take in account diagrams of fig.1d, because they consist bilocal operator contribution, the calculation of which is a separate problem. Nevertheless the estimations allow to hope, that the contribution of this diagram will hardly essentially change an obtained result.

It is necessary to note, that in [17] similar 3-point correlator for the current $\bar{q}\hat{\tilde{G}}_{\mu\nu}\gamma_\nu\gamma_5 q$ was considered up to dimension 10. But in our case we take into account greater number of the diagrams because, using results of the previous sections and our anzatz, we can evaluate practically all arising vacuum averages of dimension 10, and not just only those from them, which can immediately factorized to forms $< g^2G^2 >< \bar{\psi}\psi >^2$ or $(< \bar{\psi}\hat{G}_{\mu\nu}\sigma_{\mu\nu}\psi >)^2$.

Let us also do some notices about the diagram on fig.1b. On the first sight it express in terms of unfactorizible vacuum average like:

$$K =< \bar{u}_\lambda g\hat{\tilde{G}}_{\mu\nu}[\nabla_\alpha u_\rho] \cdot \bar{u}_\tau[\nabla_\beta u_\sigma] > \cdot T^{\mu\nu\alpha\beta}_{\lambda\rho\tau\sigma} \quad (37)$$

where $T^{\mu\nu\alpha\beta}_{\alpha\rho\tau\sigma}$ - any matrix, constructed from γ^ν-martrixes (and $g^{\mu\nu}$, $\varepsilon^{\mu\nu\alpha\beta}$ also). However up to full derivatives (37) can be written as

$$K = -T^{\mu\nu\alpha\beta}_{\lambda\rho\tau\sigma}\Big\{< ([\bar{u}_\lambda\nabla_\beta]\hat{\tilde{G}}_{\mu\nu}[\nabla_\alpha u_\rho]) \cdot (\bar{u}_\tau u_\sigma) > +$$

$$+ < (\bar{u}_\lambda(D_\beta\hat{\tilde{G}}_{\mu\nu})[\nabla_\alpha u_\rho]) \cdot (\bar{u}_\tau u_\sigma) >$$

$$+ < (\bar{u}_\lambda\hat{\tilde{G}}_{\mu\nu}[\nabla_\beta\nabla_\alpha u_\rho]) \cdot (\bar{u}_\tau u_\sigma) > +$$

$$+ < (\bar{u}\hat{\tilde{G}}_{\mu\nu}[\nabla_\alpha u_\rho])([\bar{u}_\tau \nabla_\beta] u_\sigma) > \Big\} \qquad (38)$$

Now one can easily see that all vacuum averages in right side of (37) can be factorized.

All other diagrams (fig.1a, 1b) can be immediately factorized by use of results (18),(24),(31). Finally, result for sum rules (36), taking into account the contribution of operators of dimension 10 (fig.1a-1c), is:

$$\ll U \gg = \frac{-e^{m_N^2/M^2}}{2\bar{\lambda}_p^2}\Bigg[2A\int\limits_0^{s_0} ds\ s(m_N^2 - s)e^{-s/M^2}(s_0 + s\ ln\ s_0/s) +$$

$$+B\left((m_N^2 - M^2)(1-(1+\frac{s_0}{m^2})e^{-s_0/M^2}) + \left(\frac{s_0}{M^2}\right)^2 e^{-s_0/M^2}\right) +$$

$$+ (Cm_N^2 + 32/27 \cdot \alpha_s/\pi \cdot a^2 \cdot M^2) + D(1 + m_N^2/M^2) + \frac{E}{M^2}\left(1 + \frac{m_N^2}{2m^2}\right)\Bigg] \qquad (39)$$

where contribution of dimension 10 operators are

$$E^{NS} = \frac{<\bar{\psi}\psi>^2}{18}\left\{m_0^4 - \left(\frac{10}{9} < g^2G^2 > + \frac{11}{6}m_0^4\right)\right\} \simeq -\frac{1}{9}m_0^4 < \bar{\psi}\psi >^2$$

$$E^{S} = \frac{<\bar{\psi}\psi>^2}{18}\left\{m_0^4 + \left(\frac{10}{9} < g^2G^2 > + \frac{11}{6}m_0^4\right)\right\} \simeq \frac{2}{9}m_0^4 < \bar{\psi}\psi >^2 \qquad (40)$$

In (40) we omit all vacuum averages like $\bar{R}_f$, or $\bar{S}_1$ or $X_1 \cdot < \bar{\psi}\psi >$ and so on, which have been shown in sect.1,2 to be negligible small, and also take into account result for $\bar{R}_d$ from (18).

Using standard values $< g^2G^2 >, < \bar{\psi}\psi >$ one can see, that for $\ll U^{NS} \gg$ contribution of dimension 10 is only about 20-25 % of those of dimension 8 and are within the limits of permissible accuracy. So for $\ll U^{NS} \gg$ our results confirm a conclusion, made in [8], that contribution of operators of twist 4 in Bjorken sum rules (6) are small.

But for case $\ll U^S \gg$ contribution of dimension 10 became approximately more or equal that contribution of dimension 8, (at $M^2 = 1GeV^2$) so for $\ll U^S \gg$ it seems that sum rules (35,39) are inapplicable. So, most likely, for twist 4 operators contribution to M^S in (32) (i.e. in for Ellis-Jaffe [7] sum rules) it is impossible to make any predictions from QCD sum rules (at least in those approach as discussed). Note also, that some other real reasons, that also make result for $\ll U^S \gg$ doubtful are discussed in [14].

Author would like to thank B.L.Ioffe for very precious discussions and many useful advises.

This work is supported in part by CRDF grant RP2-132, INTAS 93-0283 and Schweizerishe National Fond 7SUPJ048716

References

1. M.A. Shifman., A.I. Vainstein, V.G. Zakharov, Nucl. Phys.B147 (1979)
2. S. N. Nikolaev, A.V. Radjushkin, Nucl. Phys.B213 (1983), 285.; Phys. Lett.B110 (1982) 476.
3. S. N. Nikolaev, A.V. Radjushkin, Phys. Lett.B124 (1983) 243.
4. V.A. Novikov et al., Nucl. Phys.B237 (1984), 525.
5. A.R. Zitnitsky, Yad. Fiz.41 (1985), 805, 1035, 1331.
6. J.D. Bjorken, Phys. Rev 148 (1966), 1467.
7. J. Ellis, R.L. Jaffe, Phys. Rev.D9 (1974), 1444; D10 (1974), 1669(E).
8. I.I. Balitsky, V.M. Braun, A.V. Kolesnichenko, Phys. Lett.B242 (1990), 245.; B318 (1993), 648. (E)
9. M. Anselmino, B.L. Ioffe, E. Leader, Yad. Fiz. 49 (1989), 214.
10. V.D. Burkhert, B.L. Ioffe, ZhETF 105,(1994) 1153.
11. J. Ashman et al., Nucl.Phys.B238 (1989) 1.
12. S. B. Gerasimov Yad. Fiz.2 (1965), 598
13. S.D. Drell, A.C. Hearn Phys.Rev.Lett. 16 (1966) 908
14. B.L.Ioffe Preprint ITEP 62-95, 1995; Yad. Fiz. 58 (1995), 1492.
15. B.L. Ioffe, Surveys in High Energy Physics, 1995, Vol.8. 107.
16. E.V. Shuryak, A.I. Vainstein, Nucl. Phys.B 201 (1982), 144;
    X. Ji, M. Unrau, Phys.Lett B333 (1994) 228
17. B.L. Ioffe, Nucl. Phys.B 188 (1981) 317, B191 (1981) 591. (E).
    V.M.Belyaev, B.L.Ioffe, Sov.Phys.JETP 56 (1982) 493.
18. V.M. Braun, A.V. Kolesnichenko, Nucl. Phys.B283 (1987), 723.
19. I.I. Balitsky, A.V.Yung Phys. Lett.B 129, (1983) 328.
20. V. M. Belyaev, B.L. Ioffe, Ya. I. Kogan, Phys. Lett.B151 (1985), 290.
21. B.L. Ioffe, A.V.Smilga, Nucl. Phys.B 232 (1984), 109
22. V.M. Braun, A.V. Kolesnichenko, Yad. Fiz. 44 (1986) 756.
23. V.A.Novikov et al Phys.Lett. B86 (1979) 347

Figure captions

Fig. 1. Diagrams, corresponding of the dim.10 contribution. Circles denote derivatives, dashed lines denote gluons, solid lines denote quarks.