First, we are going to derive the HQET sum rules for the parameters λ_E and λ_H by Nishikawa and Tanaka. I have also derived the sum rules for the HQET coupling constant $F(\mu)$, but since the steps are similar to the more involved three-body current calculation, i will only state the latter. In the end i am deriving the sum rules for our double three-body current considerations.
NOTICE: There is a step in the calculation that i can not explain. I will explicitly mark that point. It appears when we take the product of the matrix elements for the inserted B-meson containing two disjoint traces and somehow merge them to one trace such that we can match this to our definition of the correlation functions.
The starting point for the calculation is the correlation function:

$$\begin{aligned}\Pi_{\Gamma_1\Gamma_2,3}(\omega) &= i\int \mathrm{d}^d x e^{-i\omega v\cdot x} g_s \langle 0| T\{\bar{q}(0)\Gamma_1 G_{\mu\nu}(0) b_v(0)\bar{b}_v(x)\Gamma_2 q(x)\} |0\rangle \\ &= -\frac{1}{2}\mathrm{Tr}\{\sigma_{\mu\nu}\Gamma_1 P_+\Gamma_2\}\Pi_{3H}(\omega) - \frac{1}{2}\mathrm{Tr}\{(iv_\mu\gamma_\nu - iv_\nu\gamma_\mu)\Gamma_1 P_+\Gamma_2\}\Pi_{3S}(\omega) \qquad (1)\end{aligned}$$

Another important relation comes from the unitary condition, where the ground state B-meson is separated from the continuum excited states:

$$\begin{aligned}\frac{1}{\pi}\mathrm{Im}\Pi_{\Gamma_1\Gamma_2,3}(\omega) =& \sum_n \langle 0| \bar{q}(0)\Gamma_1 g_s G_{\mu\nu}(0) b_v(0) |n\rangle \langle n| \bar{b}_v(x)\Gamma_2 q(x) |0\rangle \,\mathrm{d}\Phi_n \cdot (2\pi)^3\delta(\omega - p_n) \\ =& \langle 0| \bar{q}(0)\Gamma_1 g_s G_{\mu\nu}(0) b_v(0) |B\rangle \langle B| \bar{b}_v(x)\Gamma_2 q(x) |0\rangle \,\delta(\omega^2 - \bar{\Lambda}^2)\Theta(\omega^0) + \\ & \rho_3^{hadr.}(\omega)\Theta(\omega^2 - s_3^h) \qquad (2)\end{aligned}$$

The matrix elements can be decomposed in the following way (see paper):

$$\langle 0| \bar{q}(0)\Gamma_1 b_v(0) |B\rangle = -\frac{i}{2}F(\mu)\mathrm{Tr}[\Gamma_1 P_+\gamma_5] \qquad (3)$$

$$\begin{aligned}\langle 0| \bar{q}(0)\Gamma_1 G_{\mu\nu}(0) b_v(0) |B\rangle = & \frac{-i}{6}F(\mu)\{\lambda_H^2(\mu)\cdot \mathrm{Tr}[\Gamma_1 P_+\gamma_5\sigma_{\mu\nu}] \\ & + [\lambda_H^2(\mu) - \lambda_E^2(\mu)]\cdot \mathrm{Tr}[\Gamma_1 P_+\gamma_5(iv_\mu\gamma_\nu - iv_\nu\gamma_\mu)]\} \qquad (4)\end{aligned}$$

The next step will be to use the standard dispersion relation (after using residue theorem, Schwartz reflection principle,...):

$$\begin{aligned}\Pi_{\Gamma_1\Gamma_2,3}(\omega) =& \frac{1}{\pi}\int_0^\infty \mathrm{d}s \frac{\mathrm{Im}\Pi_{\Gamma_1\Gamma_2,3}(\omega)}{s - \omega^2 - i0^+} \\ =& \frac{1}{\bar{\Lambda}^2 - \omega^2 - i0^+} \langle 0| \bar{q}(0)\Gamma_1 g_s G_{\mu\nu}(0) b_v(0) |B\rangle \langle B| \bar{b}_v(x)\Gamma_2 q(x) |0\rangle + \int_{s_3^h}^\infty \mathrm{d}s \frac{\rho_3^{hadr.}(s)}{s - \omega^2 - i0^+} \\ & \qquad (5)\end{aligned}$$

Here, $\bar{\Lambda}$ is defined to be $m_B - m_b$ and a typically chosen HQET parameter. At this stage we assume that we can rewrite the hadronic continuum spectrum as:

$$\begin{aligned}\rho_3^{hadr.}(s)\Theta(s-s_3^h) :=& -\frac{1}{2}\mathrm{Tr}\{\sigma_{\mu\nu}\Gamma_1 P_+\Gamma_2\}\rho_{3,\sigma}^{hadr.}(s)\Theta(s-s_{3,\sigma}^h)- \\ &\frac{1}{2}\mathrm{Tr}\{(iv_\mu\gamma_\nu - iv_\nu\gamma_\mu)\Gamma_1 P_+\Gamma_2\}\rho_{3,v}^{hadr.}(s)\Theta(s-s_{3,v}^h)\end{aligned} \tag{6}$$

The matrix elements can also be evaluated:

$$\begin{aligned}&\langle 0|\,\bar{q}(0)\Gamma_1 g_s G_{\mu\nu}(0) b_v(0)\,|B\rangle\,\langle B|\,\bar{b}_v(x)\Gamma_2 q(x)\,|0\rangle \\ =&\frac{-i}{6}F(\mu)\{\lambda_H^2(\mu)\cdot\mathrm{Tr}[\Gamma_1 P_+\gamma_5\sigma_{\mu\nu}] + [\lambda_H^2(\mu)-\lambda_E^2(\mu)]\cdot\mathrm{Tr}[\Gamma_1 P_+\gamma_5(iv_\mu\gamma_\nu - iv_\nu\gamma_\mu)]\}\cdot \\ &\frac{-i}{2}F^\dagger(\mu)\mathrm{Tr}[\gamma_5 P_+\Gamma_2] \\ \overset{?}{=}&\frac{-1}{12}F(\mu)^2\left[\lambda_H^2\mathrm{Tr}\{\sigma_{\mu\nu}\Gamma_1 P_+\Gamma_2\} + (\lambda_H^2-\lambda_E^2)\mathrm{Tr}\{(iv_\mu\gamma_\nu - iv_\nu\gamma_\mu)\Gamma_1 P_+\Gamma_2\}\right]\end{aligned} \tag{7}$$

In the first line we took the complex conjugate of eq. (3) and got an additional minus sign due to the permuting a γ^0 with γ_5. Note that the last line represents the crucial step in the calculation which is not clear to me. I am using this notation, because it works for both the coupling constant $F(\mu)$ as well for the HQET parameters. In our case it seems to work as well (at least up to now).
Using all these relations, we can obtain expressions for Π_{3H} and Π_{3S}:

$$\Pi_{3H}(\omega) = \frac{1}{6}F(\mu)^2\lambda_H^2\frac{1}{\bar{\Lambda}^2-\omega^2-i0^+} + \int_{s_{3,\sigma}^h}^\infty \mathrm{d}s\frac{\rho_{3,\sigma}^{hadr.}(s)}{s-\omega^2-i0^+} \tag{8}$$

$$\Pi_{3S}(\omega) = \frac{1}{6}F(\mu)^2(\lambda_H^2-\lambda_E^2)\frac{1}{\bar{\Lambda}^2-\omega^2-i0^+} + \int_{s_{3,v}^h}^\infty \mathrm{d}s\frac{\rho_{3,v}^{hadr.}(s)}{s-\omega^2-i0^+} \tag{9}$$

Since we do not know any concrete information about the hadronic spectral density, we make use of the global and semi-local quark-hadron duality (QHD) in order to connect the hadronic spectral density with the spectral density which is described by the OPE:
Global QHD:

$$\Pi_X^{hadr.} = \Pi_X^{OPE} \quad X\in\{3H,3S\} \tag{10}$$

Semi-local QHD:

$$\int_{s_{3,X}^h}^\infty \mathrm{d}s\frac{\rho_{3,X}^{hadr.}(s)}{s-\omega^2-i0^+} = \int_{s_{3,X}^{th}}^\infty \mathrm{d}s\frac{\rho_{3,X}^{OPE}(s)}{s-\omega^2-i0^+} \quad X\in\{3H,3S\} \tag{11}$$

By using (10) and (11), we obtain:

$$\frac{1}{6}F(\mu)^2\lambda_H^2\frac{1}{\bar{\Lambda}^2-\omega^2-i0^+}=\int_{s_{3,\sigma}^h}^{s_{3,\sigma}^{th}}\mathrm{d}s\frac{\rho_{3,\sigma}^{OPE}(s)}{s-\omega^2-i0^+} \tag{12}$$

$$\frac{1}{6}F(\mu)^2(\lambda_H^2-\lambda_E^2)\frac{1}{\bar{\Lambda}^2-\omega^2-i0^+}=\int_{s_{3,v}^h}^{s_{3,v}^{th}}\mathrm{d}s\frac{\rho_{3,v}^{OPE}(s)}{s-\omega^2-i0^+} \tag{13}$$

Finally we perform a Borel transformation, which removes possible substraction terms and leads to an exponential suppression of higher resonances:

$$\frac{1}{6}F(\mu)^2\lambda_H^2e^{-\frac{\bar{\Lambda}^2}{M^2}}=\int_{s_{3,\sigma}^h}^{s_{3,\sigma}^{th}}\mathrm{d}s\;\rho_{3,\sigma}^{OPE}(s)e^{-\frac{s}{M^2}}=\int_{s_{3,\sigma}^h}^{s_{3,\sigma}^{th}}\mathrm{d}s\;\frac{1}{\pi}\mathrm{Im}\Pi_{3,\sigma}^{OPE}(s)e^{-\frac{s}{M^2}} \tag{14}$$

$$\frac{1}{6}F(\mu)^2(\lambda_H^2-\lambda_E^2)e^{-\frac{\bar{\Lambda}^2}{M^2}}=\int_{s_{3,v}^h}^{s_{3,v}^{th}}\mathrm{d}s\;\rho_{3,v}^{OPE}(s)e^{-\frac{\bar{\Lambda}^2}{M^2}}=\int_{s_{3,v}^h}^{s_{3,v}^{th}}\mathrm{d}s\;\frac{1}{\pi}\mathrm{Im}\Pi_{3,v}^{OPE}(s)e^{-\frac{s}{M^2}} \tag{15}$$

These are the HQET sum rules presented in the paper. We can now move on and derive the corresponding relations for our case:

We can perform a general ansatz of the following form based on Lorentz covariance:

$$\begin{aligned}\Pi_{\Gamma_1\Gamma_2,33}&=i\int\mathrm{d}^dxe^{-i\omega v\cdot x}g_s^2\langle 0|\,T\{\bar{q}(0)\Gamma_1G_{\mu\nu}(0)b_v(0)\bar{b}_v(x)\Gamma_2G_{\rho\sigma}(x)q(x)\}\,|0\rangle\\&=-\frac{1}{2}\mathrm{Tr}\{\sigma_{\mu\nu}\Gamma_1P_+\Gamma_2\sigma_{\rho\sigma}\}\Pi_{33,\sigma}{\color{red}+}\frac{1}{2}\mathrm{Tr}\{\sigma_{\mu\nu}\Gamma_1P_+\Gamma_2(iv_\rho\gamma_\sigma-iv_\sigma\gamma_\rho)\}\Pi_{33,\sigma v}\\&\quad-\frac{1}{2}\mathrm{Tr}\{(iv_\mu\gamma_\nu-iv_\nu\gamma_\mu)\Gamma_1P_+\Gamma_2\sigma_{\rho\sigma}\}\Pi_{33,v\sigma}{\color{red}+}\frac{1}{2}\mathrm{Tr}\{(iv_\mu\gamma_\nu-iv_\nu\gamma_\mu)\Gamma_1P_+\Gamma_2(iv_\rho\gamma_\sigma-iv_\sigma\gamma_\rho)\}\Pi_{33,v}\end{aligned} \tag{16}$$

Some signs above are marked in red since i adjusted them such that there are no minus signs in the sum rule. Due to the fact that the decomposition above seems to be a definition, this should be justified.

$$\begin{aligned}\frac{1}{\pi}\mathrm{Im}\Pi_{\Gamma_1\Gamma_2,3}(\omega)&=\sum_n\langle 0|\,\bar{q}(0)\Gamma_1g_sG_{\mu\nu}(0)b_v(0)\,|n\rangle\,\langle n|\,\bar{b}_v(x)\Gamma_2g_sG_{\rho\sigma}(x)q(x)\,|0\rangle\,\mathrm{d}\Phi_n\cdot(2\pi)^3\delta(\omega-p_n)\\&=\langle 0|\,\bar{q}(0)\Gamma_1g_sG_{\mu\nu}(0)b_v(0)\,|B\rangle\,\langle B|\,\bar{b}_v(x)\Gamma_2g_sG_{\rho\sigma}q(x)\,|0\rangle\,\delta(\omega^2-\bar{\Lambda}^2)\Theta(\omega^0)+\\&\quad\rho_{33}^{hadr.}(\omega)\Theta(\omega^2-s_{33}^h)\end{aligned} \tag{17}$$

Using this relation and remembering that the matrix elements are still the same, we get:

$$
\begin{aligned}
\Pi_{\Gamma_1\Gamma_2,33}(\omega) =& \frac{1}{\pi}\int_0^\infty \mathrm{d}s \frac{\mathrm{Im}\Pi_{\Gamma_1\Gamma_2,3}(\omega)}{s-\omega^2-i0^+} \\
=& \frac{1}{\bar{\Lambda}^2-\omega^2-i0^+}\Big[\frac{-i}{6}F(\mu)\{\lambda_H^2(\mu)\cdot \mathrm{Tr}[\Gamma_1 P_+\gamma_5\sigma_{\mu\nu}] \\
&+ [\lambda_H^2(\mu)-\lambda_E^2(\mu)]\cdot \mathrm{Tr}[\Gamma_1 P_+\gamma_5(iv_\mu\gamma_\nu - iv_\nu\gamma_\mu)]\}\cdot\frac{-i}{6}F^\dagger(\mu)\{\lambda_H^2(\mu)\cdot \mathrm{Tr}[\Gamma_1 P_+\gamma_5\sigma_{\rho\sigma}] \\
&- [\lambda_H^2(\mu)-\lambda_E^2(\mu)]\cdot \mathrm{Tr}[\Gamma_1 P_+\gamma_5(iv_\rho\gamma_\sigma - iv_\sigma\gamma_\rho)]\}\Big] + \int_{s_{33}^h}^\infty \mathrm{d}s \frac{\rho_{33}^{hadr.}(s)}{s-\omega^2-i0^+}
\end{aligned} \tag{18}
$$

After decomposing $\rho_{33}^{hadr.}(s)$ in a similar way as (6), we obtain the following relations:

$$\Pi_{33,\sigma}(\omega) = \frac{1}{18}F(\mu)^2\lambda_H^4\frac{1}{\bar{\Lambda}^2-\omega^2-i0^+} + \int_{s_{33,\sigma}^h}^\infty \mathrm{d}s\frac{\rho_{33,\sigma}^{hadr.}(s)}{s-\omega^2-i0^+} \tag{19}$$

$$\Pi_{33,\sigma v}(\omega) = +\frac{1}{18}F(\mu)^2\lambda_H^2(\lambda_H^2-\lambda_E^2)\frac{1}{\bar{\Lambda}^2-\omega^2-i0^+} + \int_{s_{33,\sigma v}^h}^\infty \mathrm{d}s\frac{\rho_{33,\sigma v}^{hadr.}(s)}{s-\omega^2-i0^+} \tag{20}$$

$$\Pi_{33,v\sigma}(\omega) = \frac{1}{18}F(\mu)^2\lambda_H^2(\lambda_H^2-\lambda_E^2)\frac{1}{\bar{\Lambda}^2-\omega^2-i0^+} + \int_{s_{33,v\sigma}^h}^\infty \mathrm{d}s\frac{\rho_{33,v\sigma}^{hadr.}(s)}{s-\omega^2-i0^+} \tag{21}$$

$$\Pi_{33,v}(\omega) = +\frac{1}{18}F(\mu)^2(\lambda_H^2-\lambda_E^2)^2\frac{1}{\bar{\Lambda}^2-\omega^2-i0^+} + \int_{s_{33,v}^h}^\infty \mathrm{d}s\frac{\rho_{33,v}^{hadr.}(s)}{s-\omega^2-i0^+} \tag{22}$$

Notice that there might be some HQET symmetry which leads to a cancellation of the two mixing terms (if the ansatz only contains - signs). Now we make use of (10) and (11):

$$\frac{1}{18}F(\mu)^2\lambda_H^4\frac{1}{\bar{\Lambda}^2-\omega^2-i0^+} = \int_{s_{33,\sigma}^h}^{s_{33,\sigma}^{th}} \mathrm{d}s\frac{\rho_{33,\sigma}^{OPE}(s)}{s-\omega^2-i0^+} \tag{23}$$

$$\frac{1}{18}F(\mu)^2\lambda_H^2(\lambda_H^2-\lambda_E^2)\frac{1}{\bar{\Lambda}^2-\omega^2-i0^+} = \int_{s_{33,\sigma v}^h}^{s_{33,\sigma v}^{th}} \mathrm{d}s\frac{\rho_{33,\sigma v}^{OPE}(s)}{s-\omega^2-i0^+} \tag{24}$$

$$\frac{1}{18}F(\mu)^2\lambda_H^2(\lambda_H^2-\lambda_E^2)\frac{1}{\bar{\Lambda}^2-\omega^2-i0^+} = \int_{s_{33,v\sigma}^h}^{s_{33,v\sigma}^{th}} \mathrm{d}s\frac{\rho_{33,v\sigma}^{OPE}(s)}{s-\omega^2-i0^+} \tag{25}$$

$$\frac{1}{18}F(\mu)^2(\lambda_H^2-\lambda_E^2)^2\frac{1}{\bar{\Lambda}^2-\omega^2-i0^+} = \int_{s_{33,v}^h}^{s_{33,v}^{th}} \mathrm{d}s\frac{\rho_{33,v}^{OPE}(s)}{s-\omega^2-i0^+} \tag{26}$$

Finally, we perform a Borel transformation:

$$\frac{1}{18}F(\mu)^2\lambda_H^4 e^{-\frac{\bar{\Lambda}^2}{M^2}} = \int_{s_{33,\sigma}^h}^{s_{33,\sigma}^{th}} \mathrm{d}s\ \rho_{33,\sigma}^{OPE}(s)e^{-\frac{s}{M^2}} = \int_{s_{33,\sigma}^h}^{s_{33,\sigma}^{th}} \mathrm{d}s\ \frac{1}{\pi}\mathrm{Im}\Pi_{33,\sigma}^{OPE}(s)e^{-\frac{s}{M^2}} \tag{27}$$

$$\frac{1}{18}F(\mu)^2\lambda_H^2(\lambda_H^2-\lambda_E^2) e^{-\frac{\bar{\Lambda}^2}{M^2}} = \int_{s_{33,\sigma v}^h}^{s_{33,\sigma v}^{th}} \mathrm{d}s\ \rho_{33,\sigma v}^{OPE}(s)e^{-\frac{\bar{\Lambda}^2}{M^2}} = \int_{s_{33,\sigma v}^h}^{s_{33,\sigma v}^{th}} \mathrm{d}s\ \frac{1}{\pi}\mathrm{Im}\Pi_{33,\sigma v}^{OPE}(s)e^{-\frac{s}{M^2}} \tag{28}$$

$$\frac{1}{18}F(\mu)^2\lambda_H^2(\lambda_H^2-\lambda_E^2) e^{-\frac{\bar{\Lambda}^2}{M^2}} = \int_{s_{33,v\sigma}^h}^{s_{33,v\sigma}^{th}} \mathrm{d}s\ \rho_{33,v\sigma}^{OPE}(s)e^{-\frac{\bar{\Lambda}^2}{M^2}} = \int_{s_{33,v\sigma}^h}^{s_{33,v\sigma}^{th}} \mathrm{d}s\ \frac{1}{\pi}\mathrm{Im}\Pi_{33,v\sigma}^{OPE}(s)e^{-\frac{s}{M^2}} \tag{29}$$

$$\frac{1}{18}F(\mu)^2(\lambda_H^2-\lambda_E^2)^2 e^{-\frac{\bar{\Lambda}^2}{M^2}} = \int_{s_{33,v}^h}^{s_{33,v}^{th}} \mathrm{d}s\ \rho_{33,v}^{OPE}(s)e^{-\frac{\bar{\Lambda}^2}{M^2}} = \int_{s_{33,v}^h}^{s_{33,v}^{th}} \mathrm{d}s\ \frac{1}{\pi}\mathrm{Im}\Pi_{33,v}^{OPE}(s)e^{-\frac{s}{M^2}} \tag{30}$$