NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Process Control: Part II- Model Predictive Control (EE6225, AY2018/19, S1)

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- Provide scholarship
- Research topic: Control and AI application
- Requirement: Refer to the NTU requirements
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APPLICATION OF MPC IN DC/DC CONVERTERS

[14/11/2019]

- Traditional PID control method of DC/DC converter
- MPC method of DC/DC converter
- Overview: MPC application in power converters
- Design issues of FCS-MPC for the power converters

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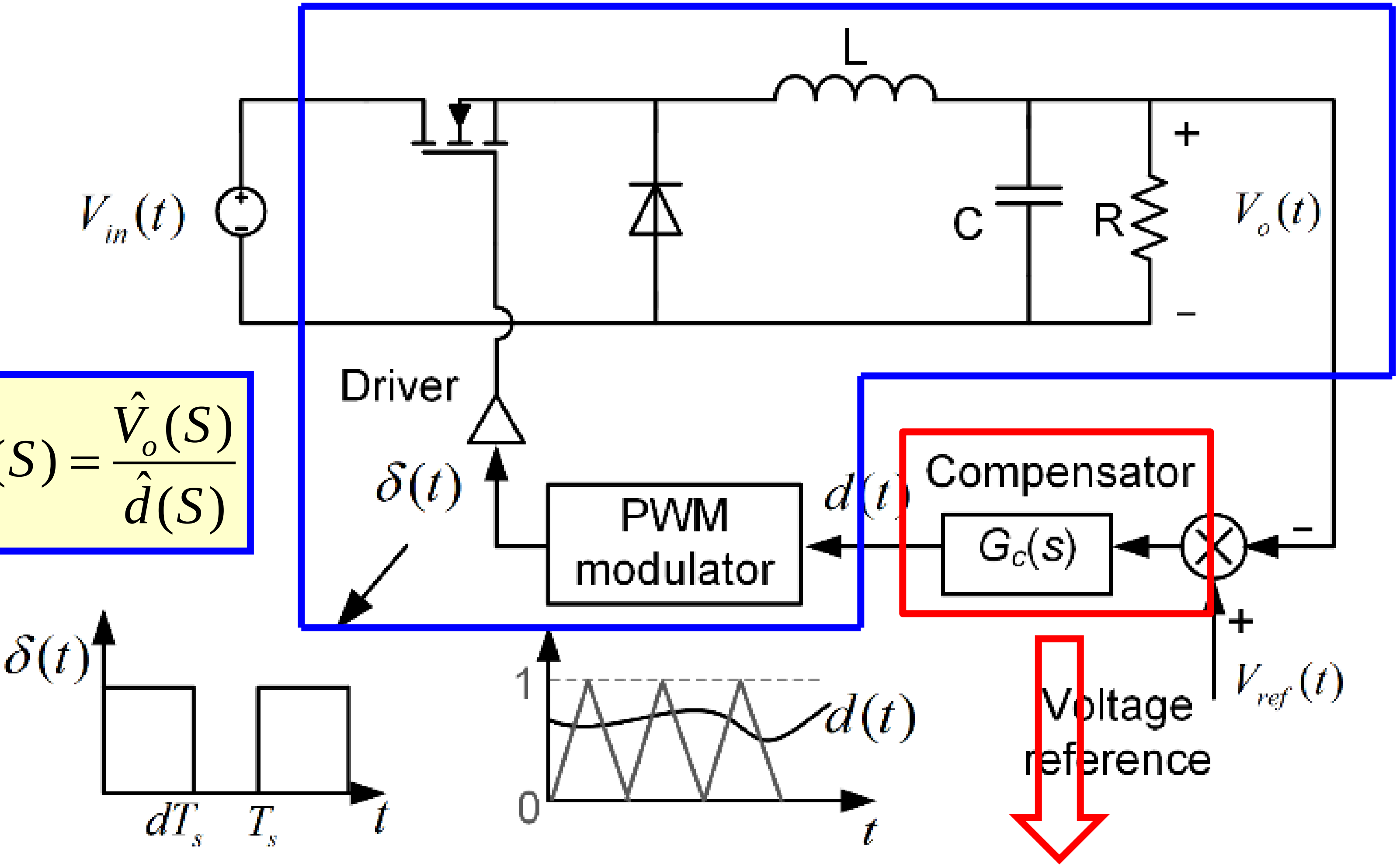
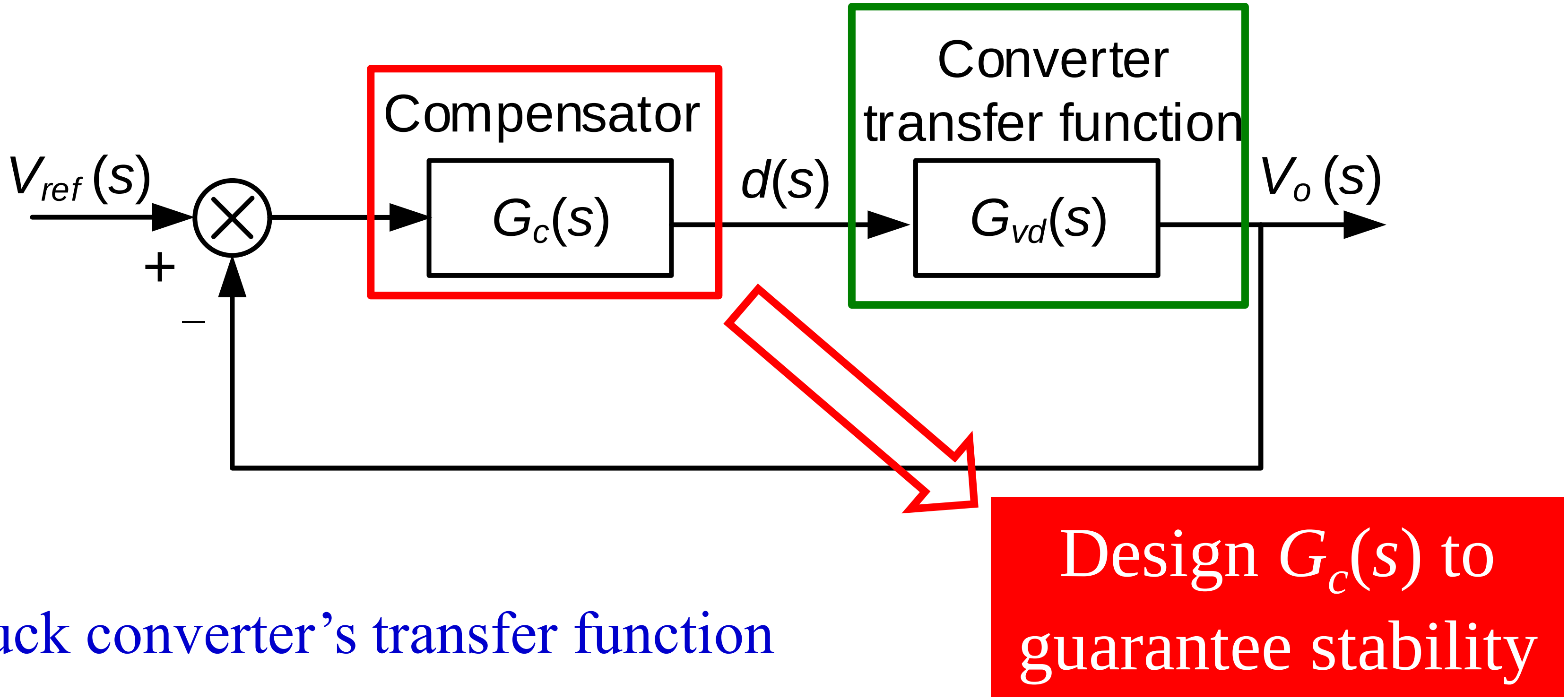


$$G_{Vd}(S) = \frac{\hat{V}_o(S)}{\hat{d}(S)}$$

Need to design compensator

Control Diagram



- Buck converter's transfer function

$$\boxed{G_{Vd}(S) = \frac{\hat{V}_o(S)}{\hat{d}(S)} = V_{in} \cdot \frac{1}{1 + S\frac{L}{R} + S^2 LC}}$$

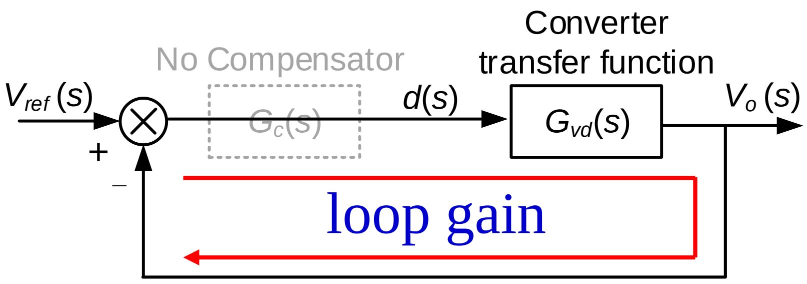
Specification of Buck converter

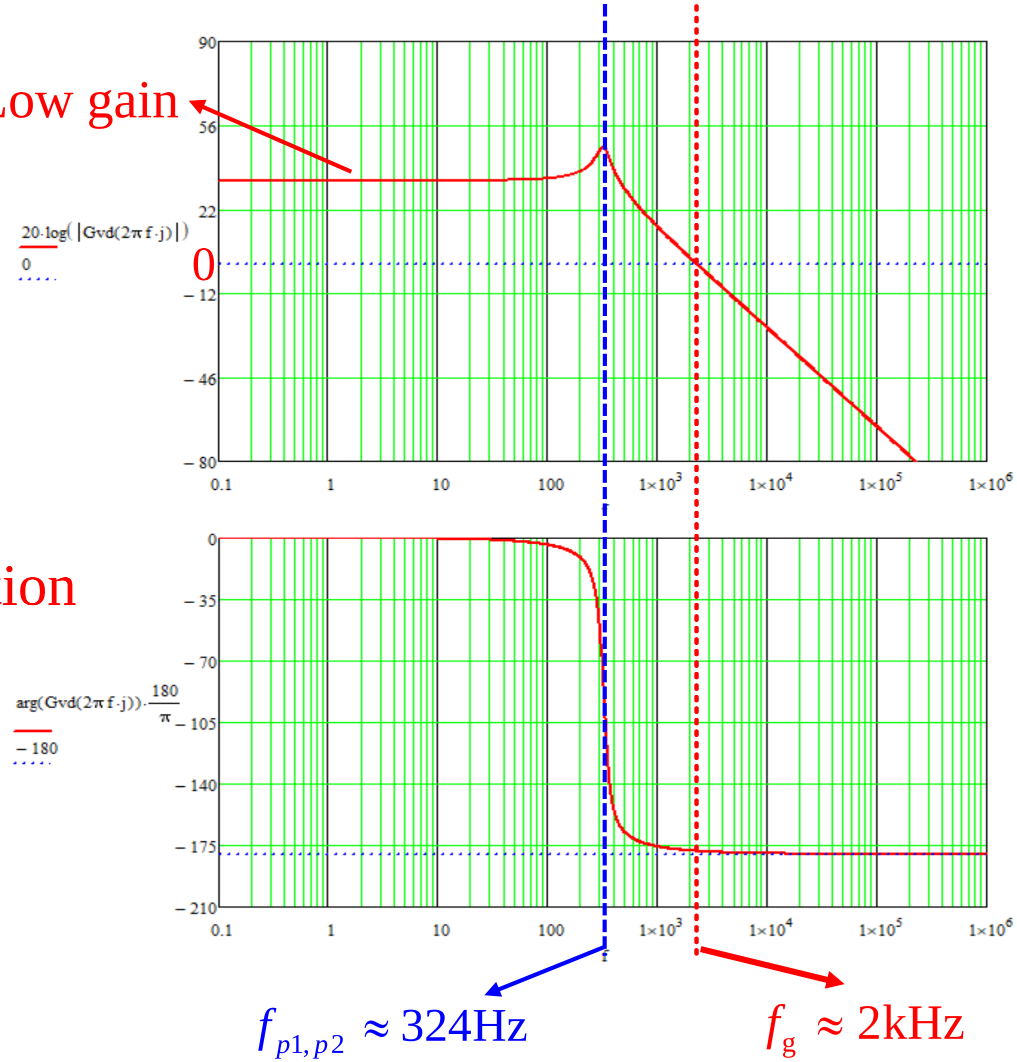
- Buck converter specification

Parameters	Values
V_{in}	48 V
V_{ref}	12 V
L	60 μH
C	4000 μF
R	0.6 Ω
Switching frequency f_s	40 kHz

$$G_{Vd}(S) = \frac{\hat{V}_o(S)}{\hat{d}(S)} = 48 \cdot \frac{1}{1 + 10^{-4} S + 2.4 \cdot 10^{-7} S^2}$$

Bode plot of the open loop buck converter





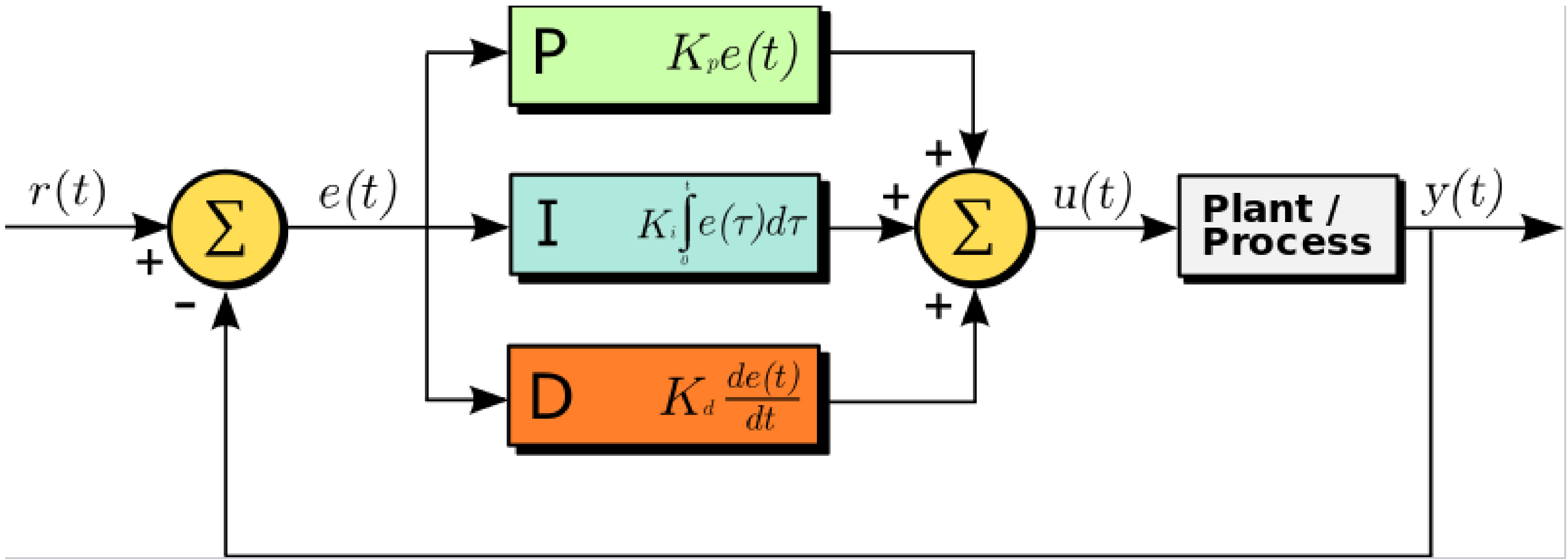
Phase margin $(PM) \approx 2^0$

- Loop gain (G_o) before compensation

$$G_o = G_{Vd}(S)$$

$$= \frac{48}{1 + 10^{-4} S + 2.4 \cdot 10^{-7} S^2}$$

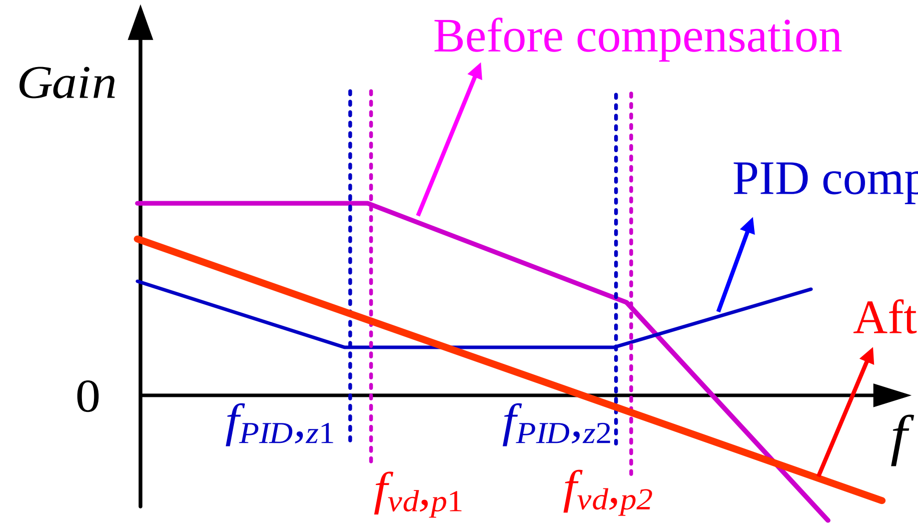
Select PID controller



$$u = \underbrace{K_p e}_{\text{Proportional Term}} + \underbrace{K_i \int_0^t e\,dt}_{\text{Integral Term}} + \underbrace{K_d \frac{d}{dt} e}_{\text{Differential Term}}$$

$$\mathrm{PID(s)} = K_P + K_i \frac{1}{s} + K_D s$$
$$= \frac{K_D s^2 + K_P s + K_i}{s}$$

Design tips of PID controller



- Pole: $f_{vd,p1} \approx f_{vd,p1} \approx 324$Hz

Cut-off frequency≈8 kHz

PID controller's zeros
=
Original poles
(zeros & poles elimination)

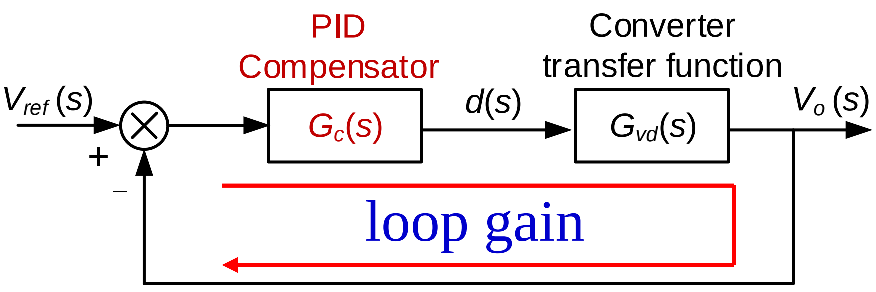
$$\mathrm{PID(s)} = K_P + K_i \frac{1}{s} + K_D s = \frac{K_D s^2 + K_P s + K_i}{s}$$

- Pole: $f_{PID,z1} \approx f_{PID,z2} \approx 324$Hz

$k_p=0.25$, $k_i=4000$, $k_d=0.001$

Loop gain with designed PID controller

- Choose PID compensator



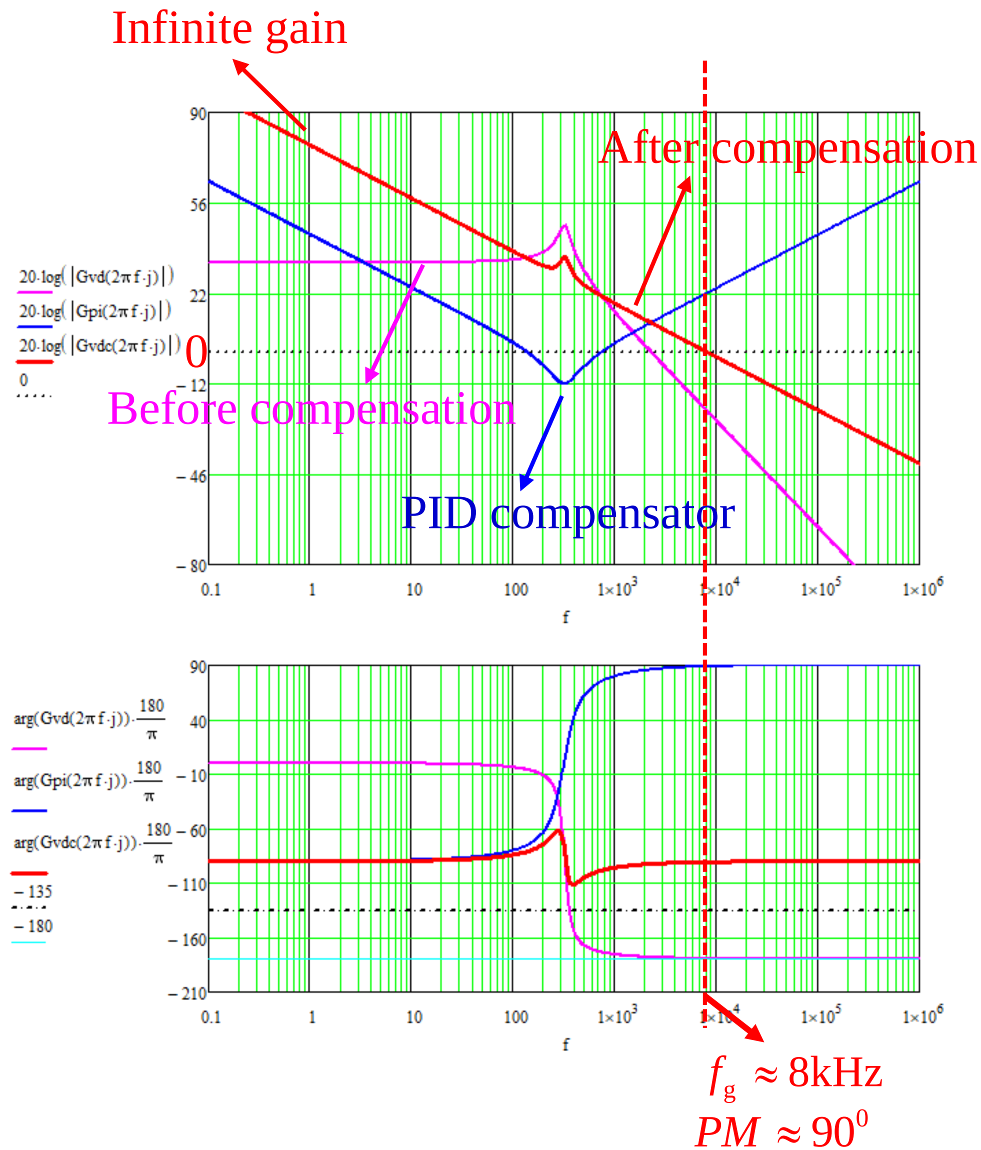
- Loop gain after compensation

$$G_{oc} = G_{Vd}(S) \cdot G_c(S)$$

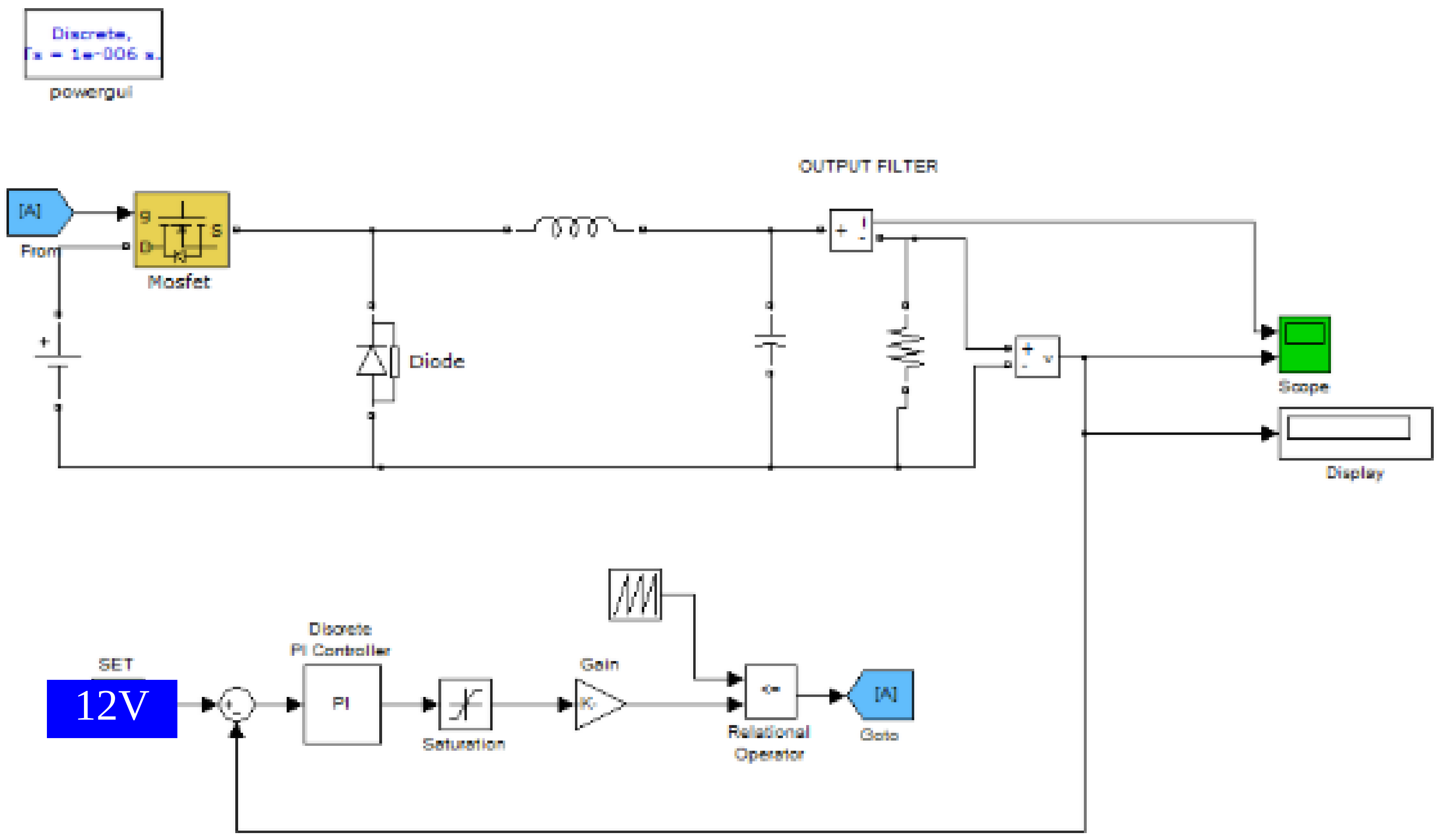
$$= \frac{48}{1+10^{-4}S+2.4\cdot 10^{-7}S^2} \cdot k_p\left(1+\frac{k_i}{S}+k_d S\right)$$

- PID parameters

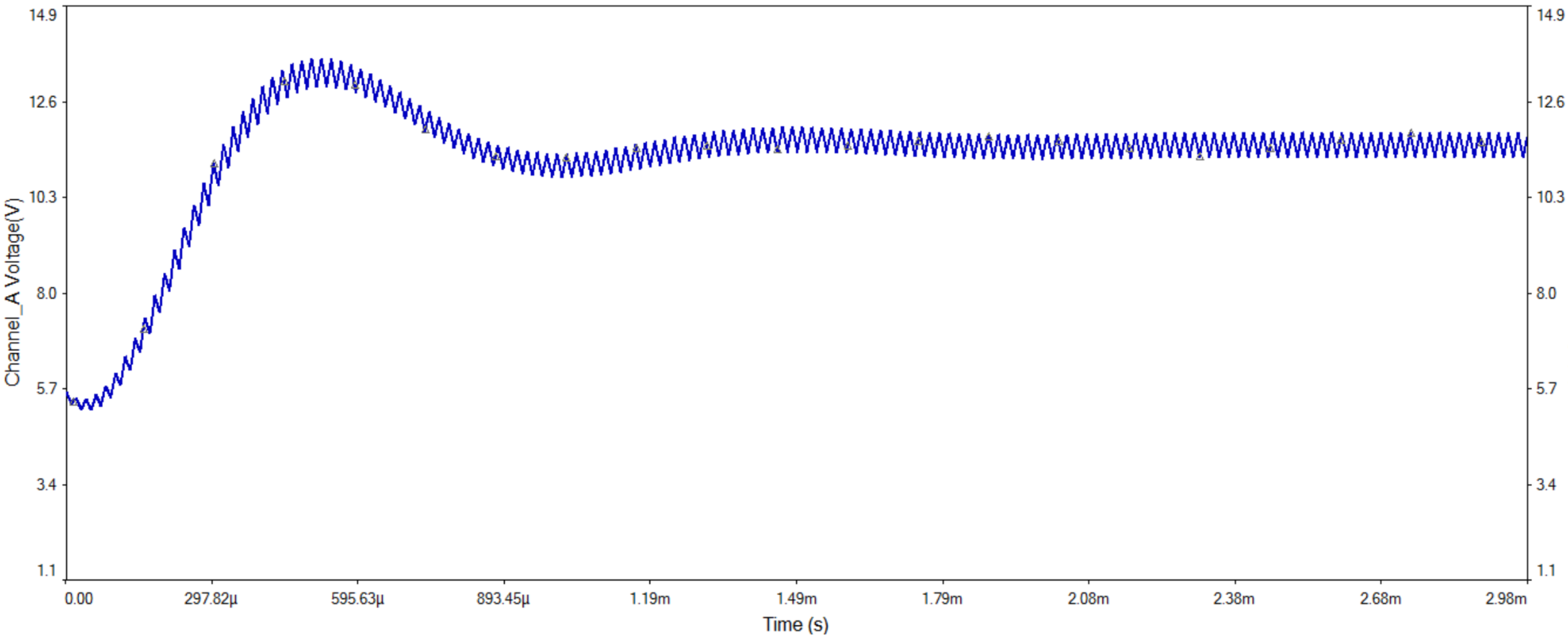
k_p=0.25, k_i=4000, k_d=0.001



Simulation


Discrete,
Ts = 1e-006 s.
powergui
OUTPUT FILTER
[A]
From
Mosfet
Diode
Scope
Display
SET
12V
Discrete
PI Controller
PI
Saturation
Gain
Relational
Operator
[A]
Goto

Simulation results



PID controller design

- Loop gain without PID:
  - Transfer function & Bode plot
- Design PID: PID controller's expression

Task 1:

- Poles & zeros of original loop gain
- Design poles & zeros of PID controller

Task 2:

- Gain of original loop gain
- Cut-off frequency of original loop gain
- Design the PID controller gain

- Need to know (lots):

Mathematical equation

Laplace transform

Transfer function

Bode plots

Poles and zeros

Not easy ⋮

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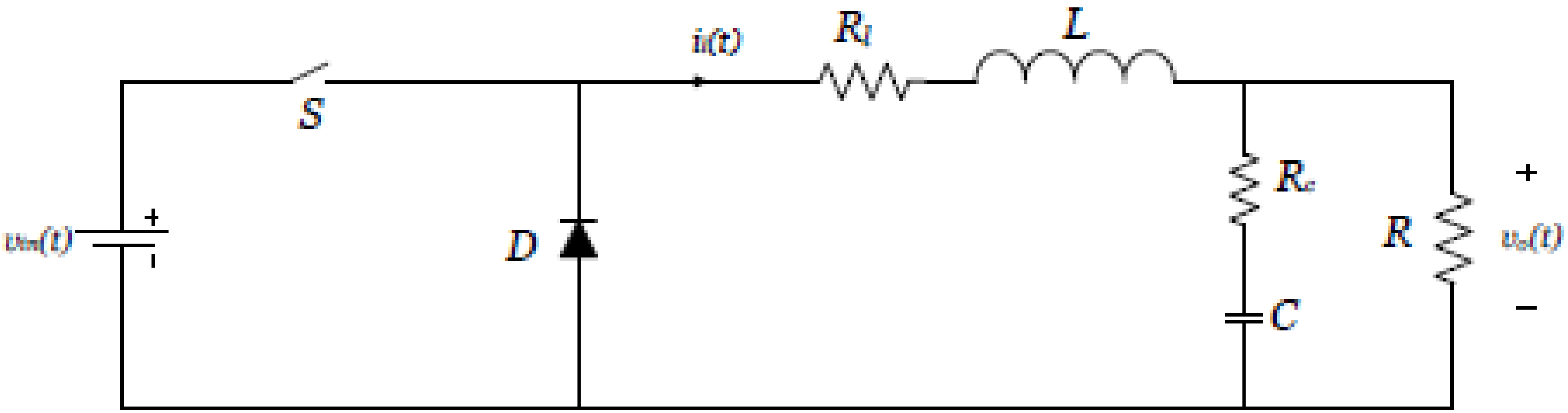
MPC FOR DC/DC CONVERTER

- PRELIMINARY: IDENTIFY CONTROL VARIABLE

Buck converter



V_{in}: Input voltage

V_{out}: Output voltage

L: Inductor R_L: Inductor resistor

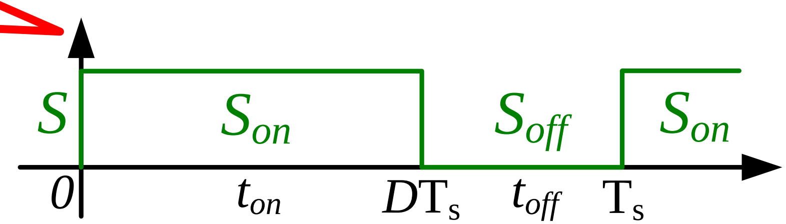
C: Capacitor R_C: Capacitor resistor

R: Load resistor

N steps Predications

2^N Possible case

- Control signal:

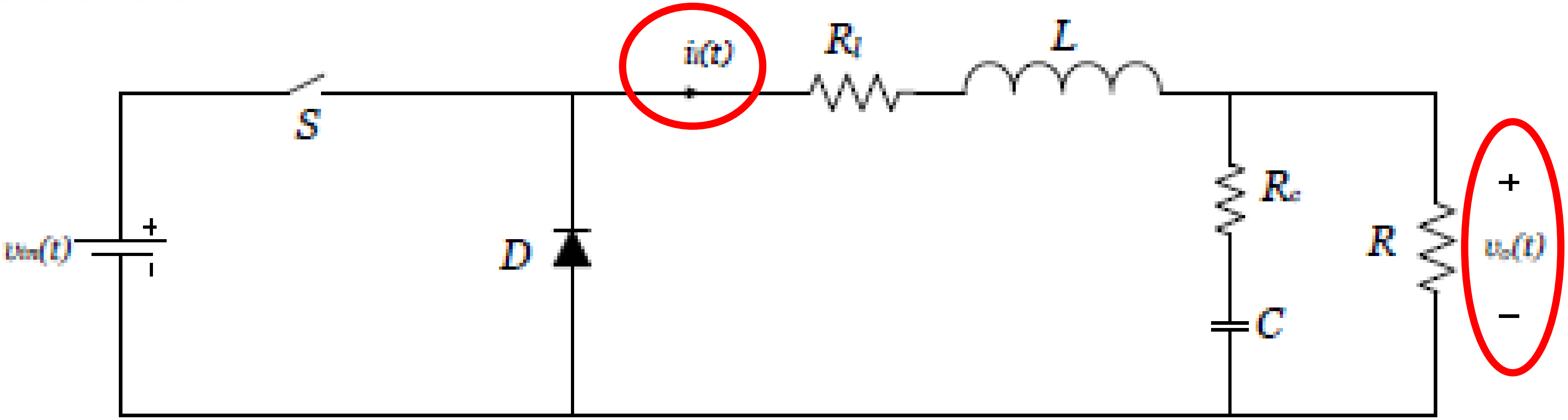


$$u(k) = \begin{cases} 1 & S1 = 1 \\ 0 & S1 = 0 \end{cases}$$

MPC FOR DC/DC CONVERTER

- STEP 1: MODEL

Continues-time model



$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = A_c x(t) + B_c u(t)$$

$$y(t) = C_c x(t)$$

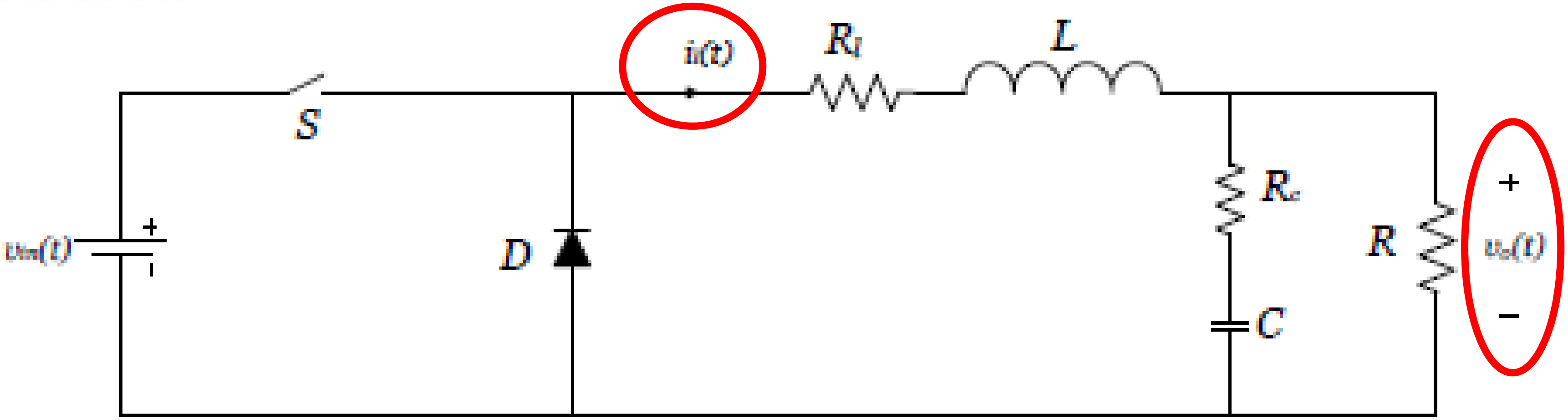
$$A_c = \begin{bmatrix} -\frac{R_l}{L} & -\frac{1}{L} \\ R\frac{L-R_cR_lC}{(R+R_c)CL} & -\frac{L+R_cRC}{(R+R_c)CL} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} i_l(t) & v_o(t) \end{bmatrix}^T$$

$$B_c = \frac{v_{in}}{L}\begin{bmatrix} 1 \\ \frac{RR_c}{R+R_c} \end{bmatrix} \quad C_c = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$y(t) = v_o(t)$$

Discrete-time model: Equations

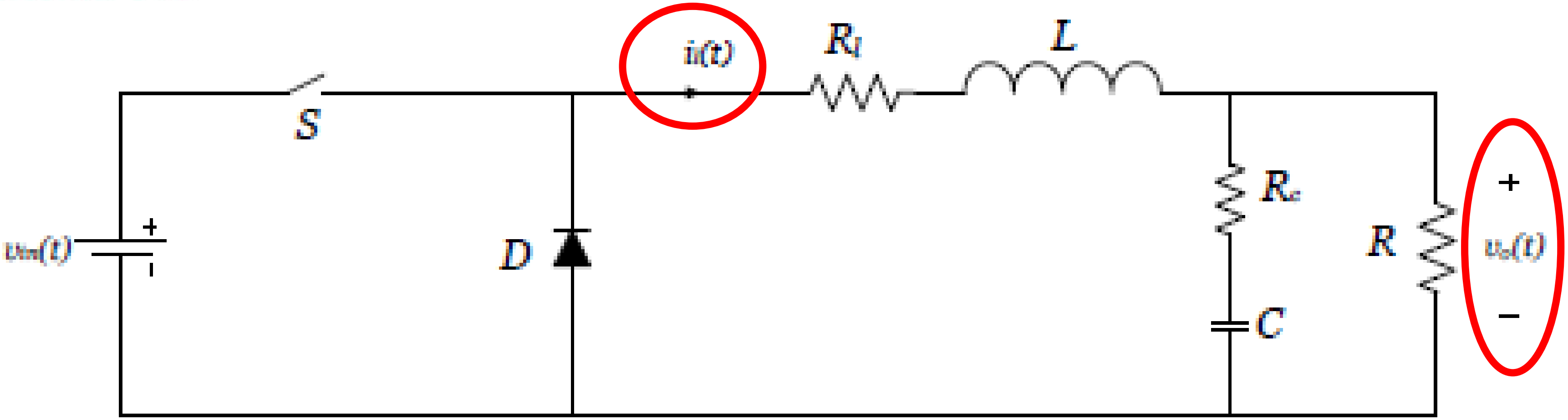


$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} \approx \frac{x(k+1) - x(k)}{T_s}$$

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = A_c x(t) + B_c u(t)$$
$$y(t) = C_c x(t)$$

$$x(k+1) = A_d x(k) + B_d u(k)$$
$$y(k) = C_d x(k)$$

Discrete-time model: Matrix



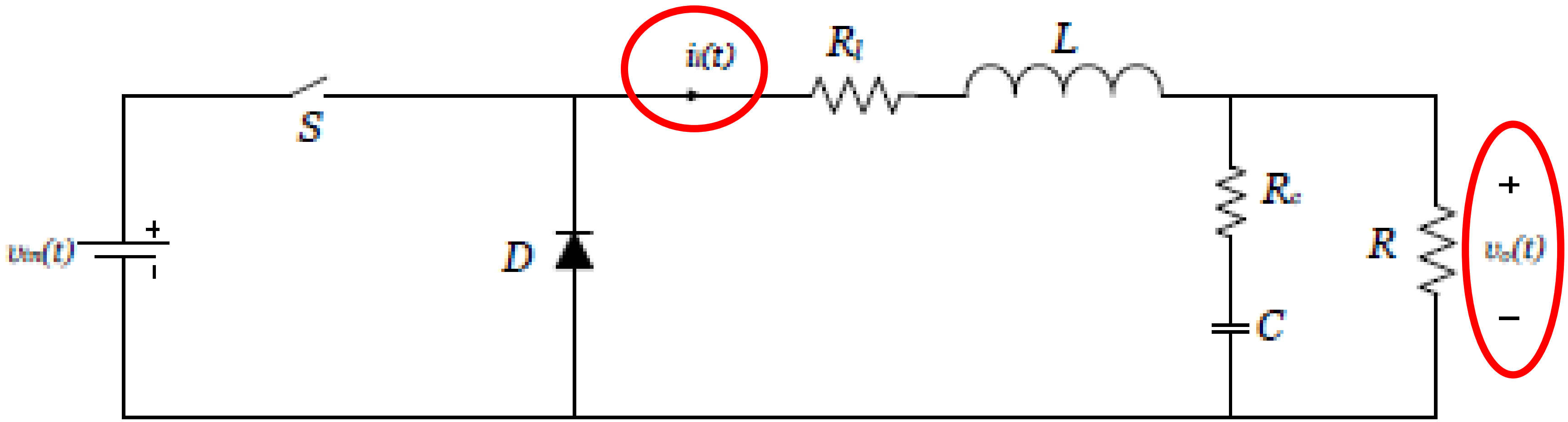
$$\boxed{\begin{aligned} x(k+1) &= \underline{A_d}\,x(k) + \underline{B_d}\,u(k) \\ y(k) &= \underline{C_d}\,x(k) \end{aligned}}$$

$$A_d = I + A_c T_s$$
$$C_d = C_c$$
$$B_d = B_c T_s$$

I: identity matrix of size two

***T*$_s$:** Sampling time

Discrete-time model: N steps predictions



$$\begin{bmatrix} x(k+1) \\ x(k+2) \\ \vdots \\ x(k+N) \end{bmatrix} = \begin{bmatrix} A_d \\ A_d{}^2 \\ \vdots \\ A_d{}^N \end{bmatrix} x(k) + \begin{bmatrix} B_d \\ B_d + A_d B_d \\ \vdots \\ \sum_{i=0}^{N-1} A_d{}^i B_d \end{bmatrix} u(k-1) + \begin{bmatrix} B_d & 0 & \cdots & 0 \\ B_d + A_d B_d & B_d & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=0}^{N-1} A_d{}^i B_d & \sum_{i=0}^{N-2} A_d{}^i B_d & \cdots & B_d \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N-1) \end{bmatrix}$$

$$\boldsymbol{Y} = \begin{bmatrix} y(k+1) \\ y(k+2) \\ \vdots \\ y(k+N) \end{bmatrix} = \begin{bmatrix} C_d & 0 & \cdots & 0 \\ 0 & C_d & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_d \end{bmatrix} \begin{bmatrix} x(k+1) \\ x(k+2) \\ \vdots \\ x(k+N) \end{bmatrix}$$

$$\boxed{\Delta u(l|k) = u(l|k) - u(l-1|k)}$$

Discrete model: N steps predictions (Simplify)

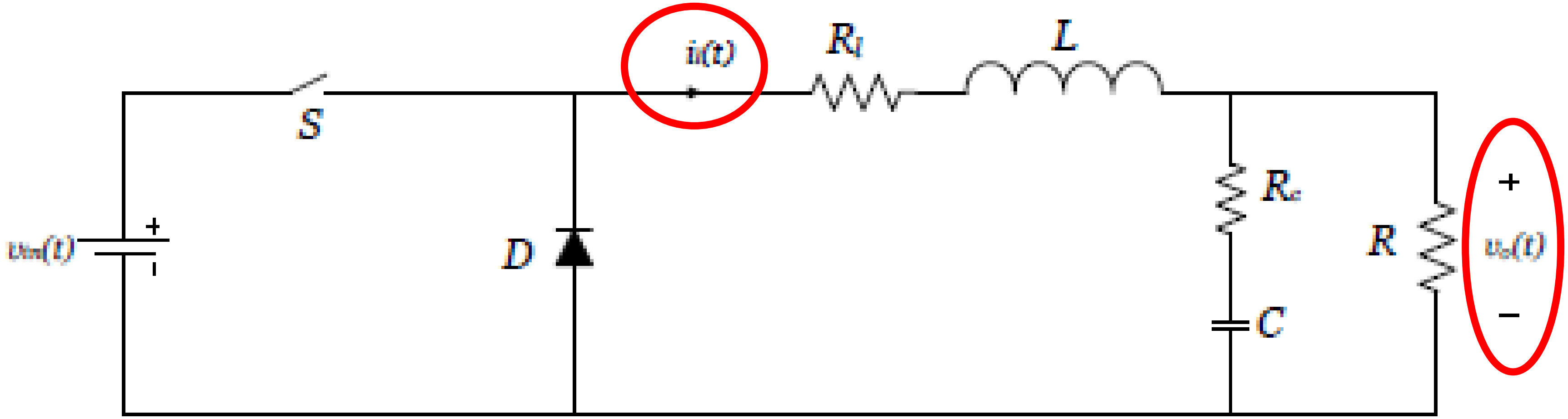
$$\boldsymbol{Y} = \boldsymbol{P}\boldsymbol{x}(k) + \boldsymbol{Q}u(k-1) + \boldsymbol{S}\Delta\boldsymbol{U}$$

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{C}_d\boldsymbol{A}_d \\ \boldsymbol{C}_d\boldsymbol{A}_d{}^2 \\ \vdots \\ \boldsymbol{C}_d\boldsymbol{A}_d{}^N \end{bmatrix} \quad \boldsymbol{Q} = \begin{bmatrix} \boldsymbol{C}_d\boldsymbol{B}_d \\ \boldsymbol{C}_d\boldsymbol{B}_d + \boldsymbol{C}_d\boldsymbol{A}_d\boldsymbol{B}_d \\ \vdots \\ \sum_{i=0}^{N-1} \boldsymbol{C}_d\boldsymbol{A}_d{}^i\boldsymbol{B}_d \end{bmatrix} \quad \Delta\boldsymbol{U} = \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N-1) \end{bmatrix}$$

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{C}_d\boldsymbol{B}_d & 0 & \cdots & 0 \\ \boldsymbol{C}_d\boldsymbol{B}_d + \boldsymbol{C}_d\boldsymbol{A}_d\boldsymbol{B}_d & \boldsymbol{C}_d\boldsymbol{B}_d & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=0}^{N-1} \boldsymbol{C}_d\boldsymbol{A}_d{}^i\boldsymbol{B}_d & \sum_{i=0}^{N-2} \boldsymbol{C}_d\boldsymbol{A}_d{}^i\boldsymbol{B}_d & \cdots & \boldsymbol{C}_d\boldsymbol{B}_d \end{bmatrix}$$

MPC FOR DC/DC CONVERTER
- STEP 2: COST FUNCTION

Cost function



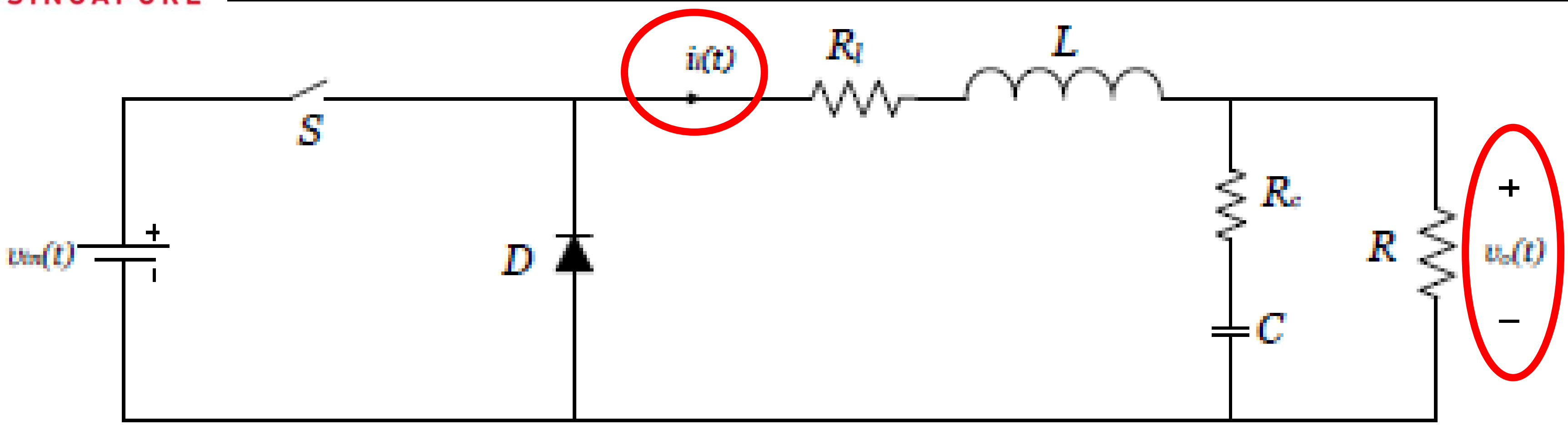
$$J(k) = \sum_{l=k}^{k+N-1} \left(\|v_o(l+1|k) - v_{o,\mathrm{ref}}\|_2^2 + \lambda \|\Delta u(l|k)\|_2^2 \right)$$

Output voltage: no error

Control changes: small

Weight factor: (0, 1)

Cost function for calculation



$$\boxed{J(k) = \sum_{l=k}^{k+N-1} \left(\|v_o(l+1|k) - v_{o,\mathrm{ref}}\|_2^2 + \lambda \|\Delta u(l|k)\|_2^2 \right)}$$

$$\sum_{l=k}^{k+N-1} \|v_o(l+1|k) - v_{o,\mathrm{ref}}\|_2^2 = \|\boldsymbol{Y} - \boldsymbol{V_{ref}}\|_2^2 = \|\boldsymbol{P}x(k) + \boldsymbol{Q}u(k-1) + \boldsymbol{S}\Delta \boldsymbol{U} - \boldsymbol{V_{ref}}\|_2^2$$

$$\sum_{l=k}^{k+N-1} \|\Delta u(l|k)\|_2^2 = \|\Delta \boldsymbol{U}\|_2^2$$

$$\boxed{J(k) = \|\boldsymbol{P}x(k) + \boldsymbol{Q}u(k-1) + \boldsymbol{S}\Delta \boldsymbol{U} - \boldsymbol{V_{ref}}\|_2^2 + \lambda \|\Delta \boldsymbol{U}\|_2^2}$$

MPC FOR DC/DC CONVERTER

- STEP 3: SELECT THE OPTIMAL SWITCH STATES

Select the best control sequence u

minimize $J(k)$

$$\boxed{J(k) = ||\boldsymbol{P}x(k) + \boldsymbol{Q}u(k-1) + \boldsymbol{S}\Delta \boldsymbol{U} - \boldsymbol{V}_{ref}||_2^2 + \lambda||\Delta \boldsymbol{U}||_2^2}$$

Algorithm Voltage MPC algorithm with Enumeration Strategy for Buck Converter

```
function BuckMPCEnum(x(k), u(k−1))
    J* = ∞, u*(k) = 0
    for all U(k) over N do
        J = 0
        for l = k to k + N − 1 do
            Δu(l) = u(l) − u(l − 1)
        end for
        J = ||Px(k) + Qu(k − 1) + SΔU − V_ref||_2^2 + λ||ΔU||_2^2
        if J < J* then
            J* = J
            u*(k) = U(1)
        end if
    end for
end function
```

Find u from 2^N cases to make $J=0$

N step predictions

Calculate Δu

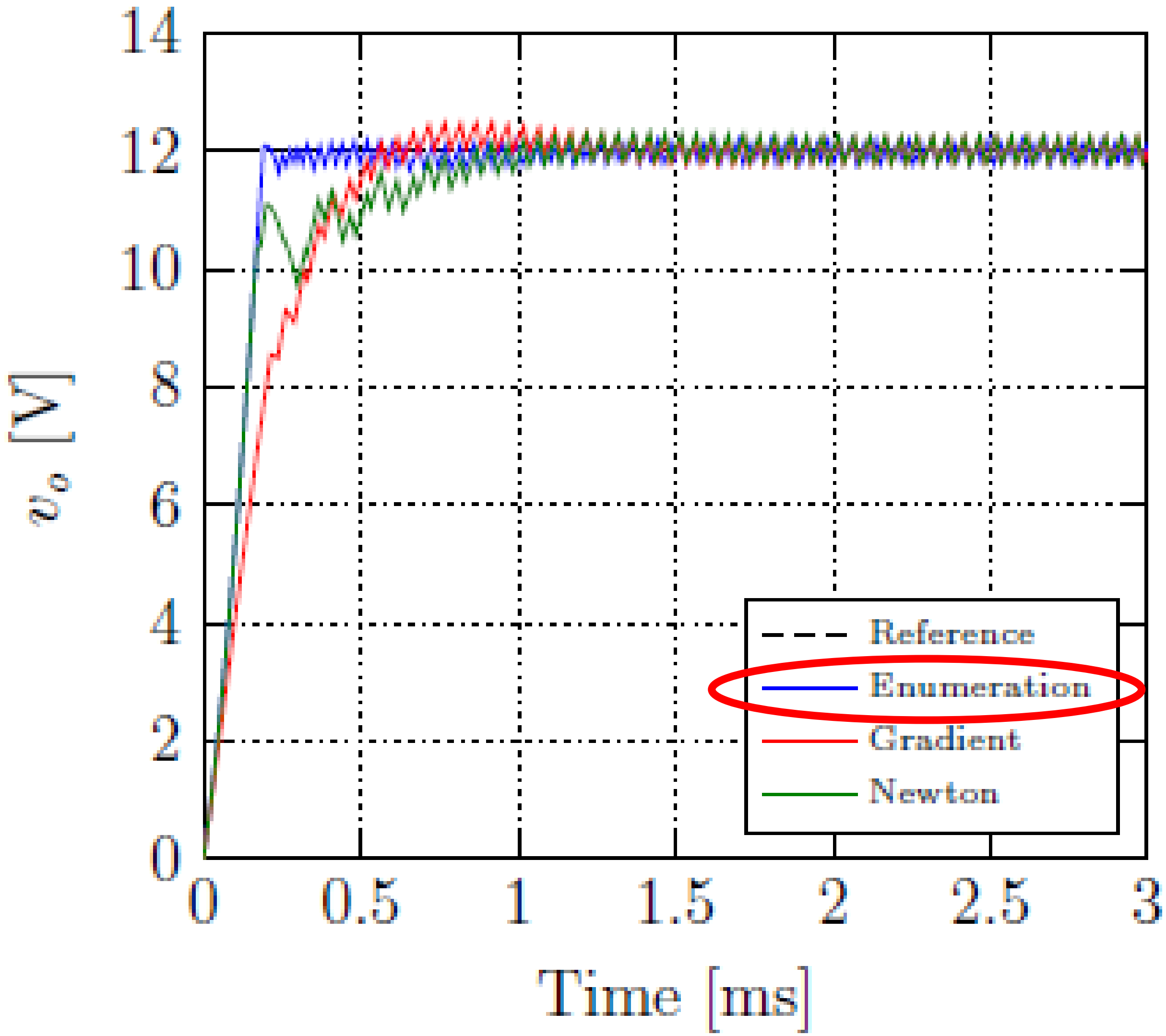
Cost function

Record the small J

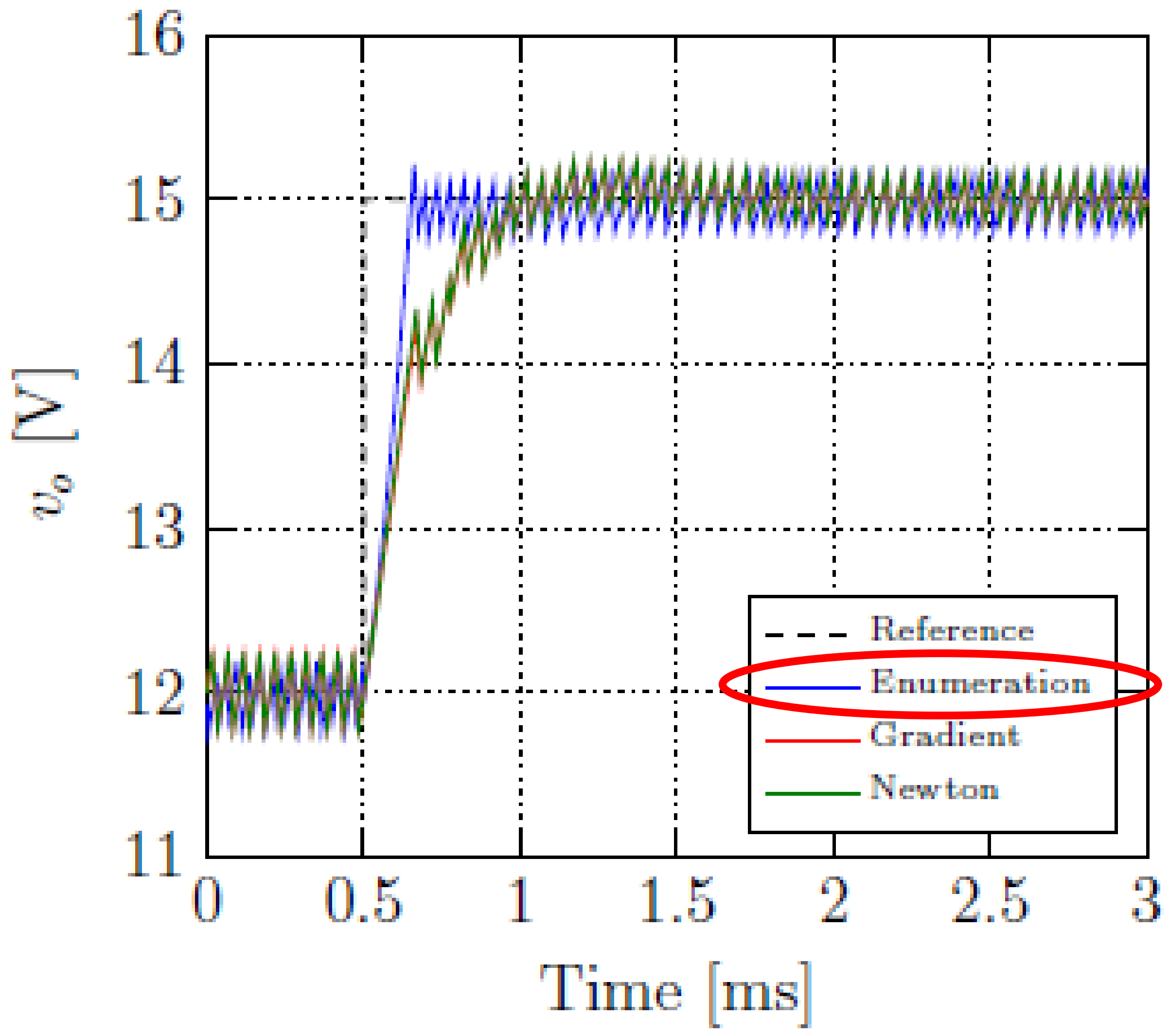
Enumeration Strategy

Simulation results

N	Enumeration
1	46.9 μs
2	55.0 μs
3	75.4 μs
4	125.2 μs
5	232.8 μs
6	487.2 μs
7	1100 μs
8	2300 μs
9	5000 μs
10	10000 μs
15	0.5s μs



Simulation results: reference change



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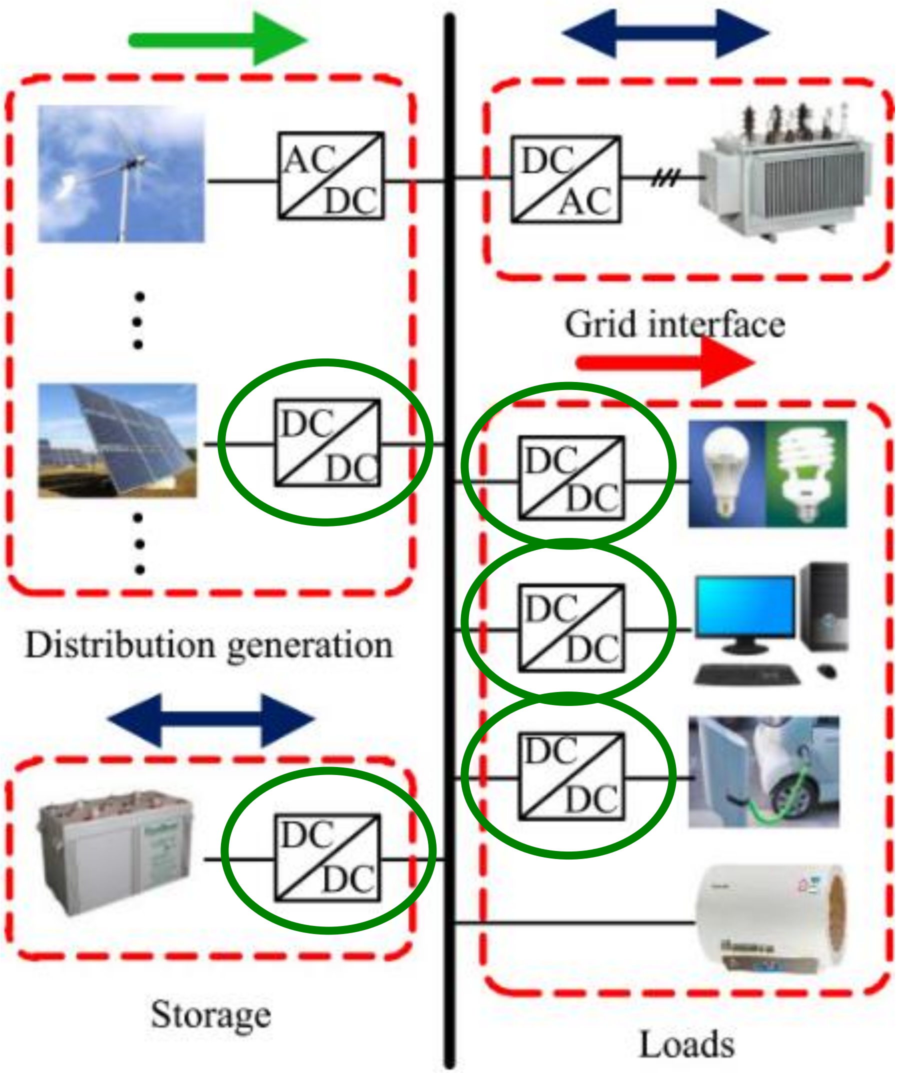
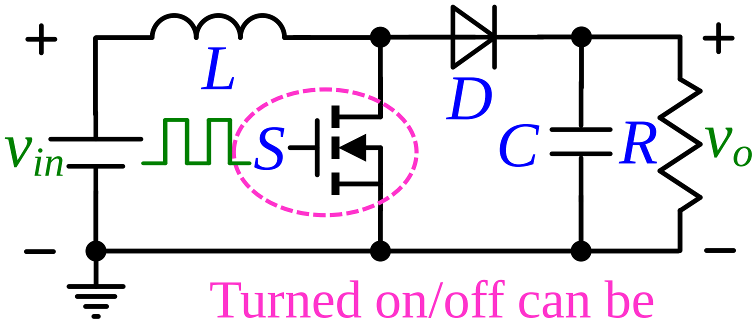
MPC FOR DC/DC CONVERTERS

- BOOST CONVERTER AS AN EXAMPLE

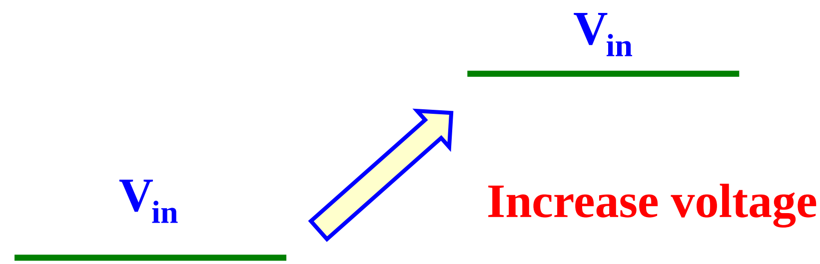
Different applications of power converters



MPC applications in DC/DC converter: Example Boost converter

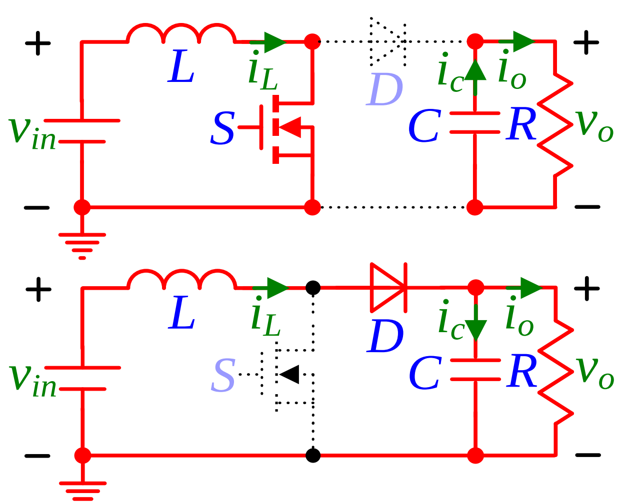


Turned on/off can be controlled by MPC controller



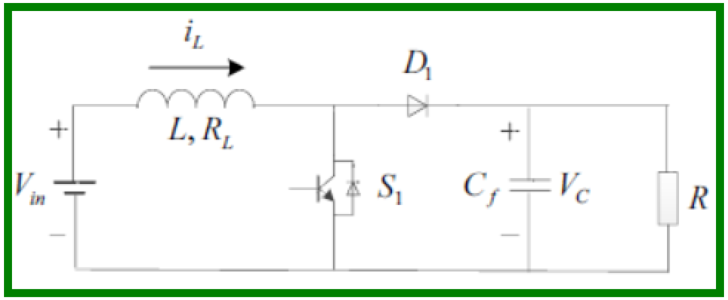
Turn on S: (t_{on}=DT$_s$)

Turn off S: (t_{off}=T$_s$-DT$_s$)



MPC applications in DC/DC converter: Example Boost converter

Circuit:



Control target: Control the output voltage V_c to reference voltage V_C*

Model:

$$i_L(k+1) = (1 - \frac{TR_L}{L})i_L(k) + (u(k)-1)\frac{T}{L}V_C(k) + \frac{T}{L}V_{in}$$

$$V_C(k+1) = \frac{T}{C_f}i_L(k) + (1 - \frac{T}{C_f R})V_C(k) - \frac{T}{C_f}i_L(k)u(k)$$

$$u(k) = \begin{cases} 1 & S1 = 1 \\ 0 & S1 = 0, \end{cases}$$

T: sampling time

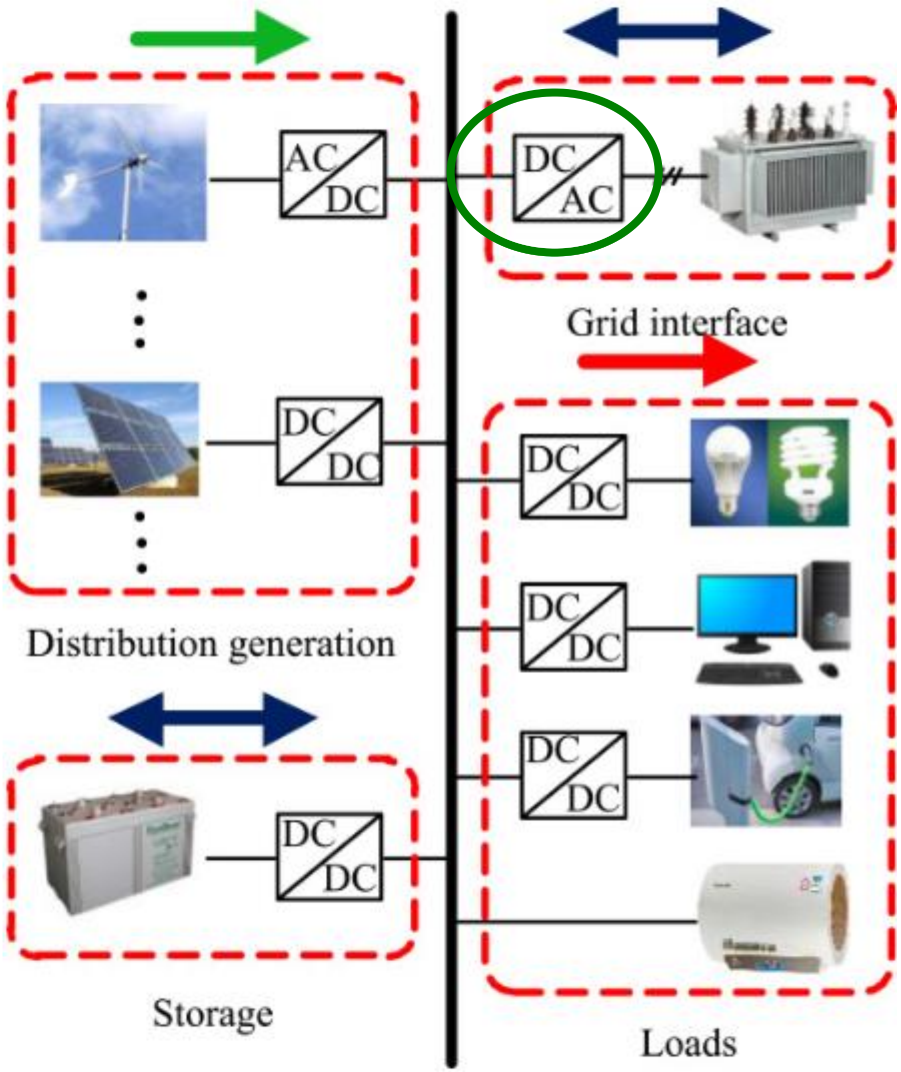
Cost function:

$$J_{DV}(k) = \sum_{k=1}^{N} (V_C(k+1) - V_C^*(k+1))^2 + \lambda|\Delta u(k)|$$

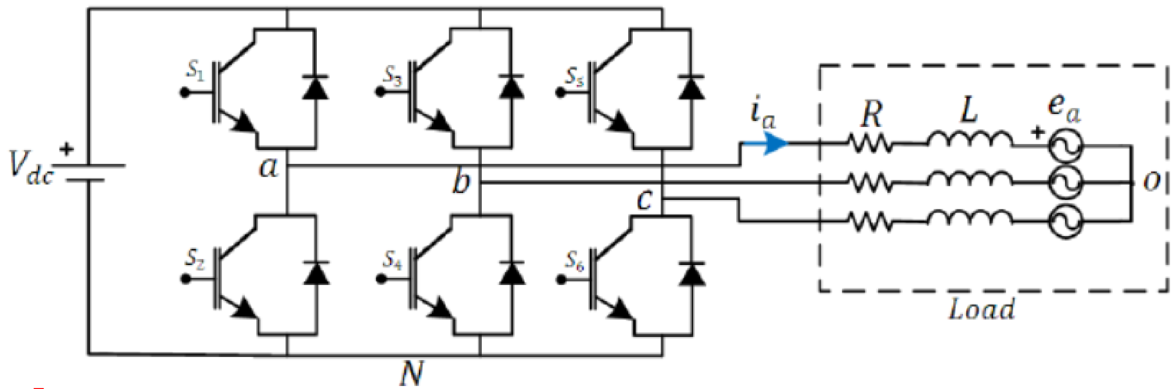
N: prediction horizon; Δu allows the controller to reduce switching frequency

MPC FOR POWER INVERTERS

Different applications of power converters

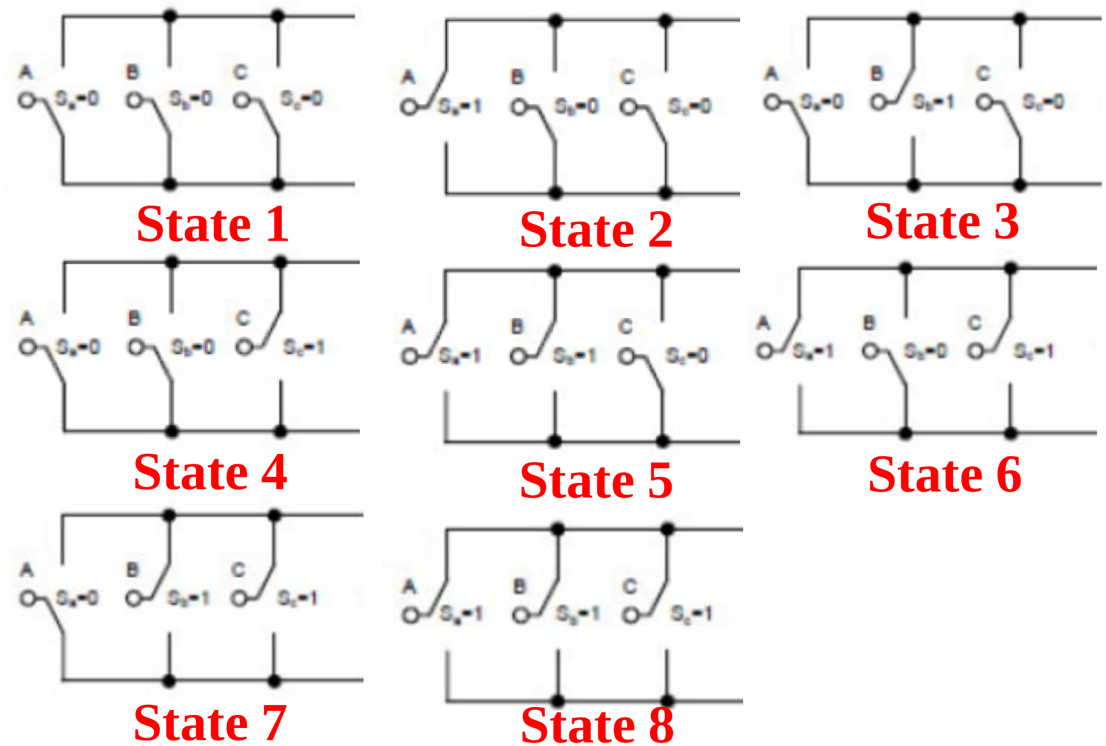


Recall: Review of 8 switch states



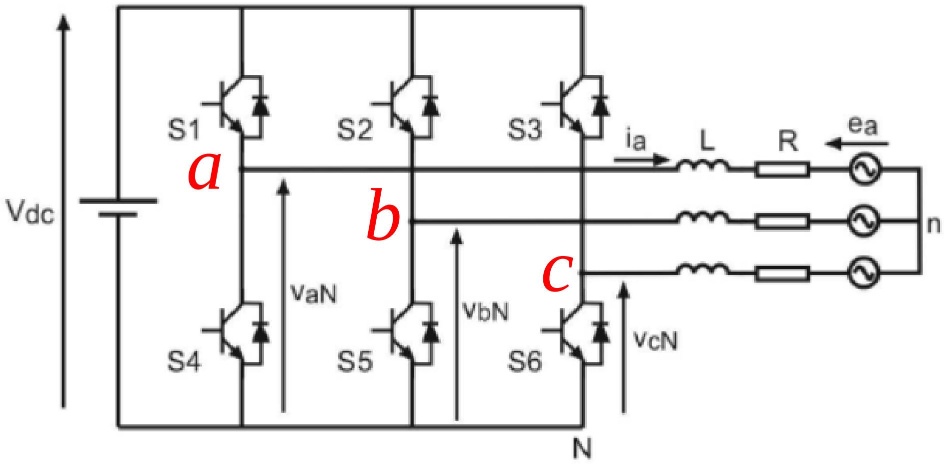
8 switch states

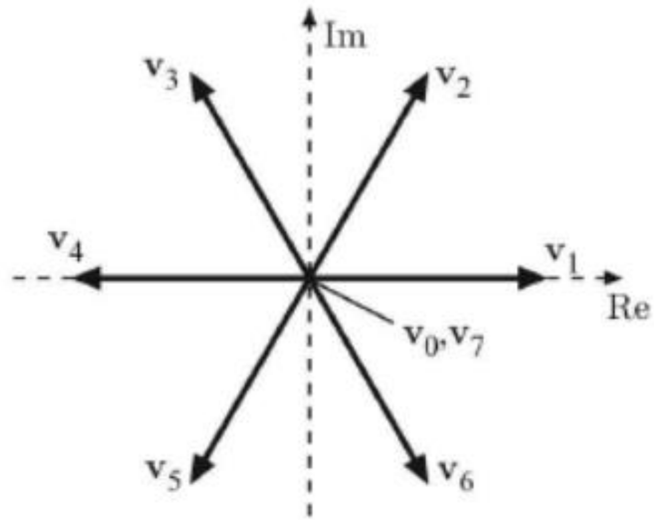
	S_1	S_2	S_3	S_4	S_5	S_6
State 1	0	1	0	1	0	1
State 2	1	0	0	1	0	1
State 3	0	1	1	0	0	1
State 4	0	1	0	1	1	0
State 5	1	0	1	0	0	1
State 6	1	0	0	1	1	0
State 7	0	1	1	0	1	0
State 8	1	0	1	0	1	0



Switching states of the example inverter

- Model of a three-phase inverter
  - Only 8 possible switching states
  - 7 different voltage vectors





$$S_a = \begin{cases} 1 & \text{if } S_1 \text{ on and } S_4 \text{ off} \\ 0 & \text{if } S_1 \text{ off and } S_4 \text{ on} \end{cases}$$

$$S_b = \begin{cases} 1 & \text{if } S_2 \text{ on and } S_5 \text{ off} \\ 0 & \text{if } S_2 \text{ off and } S_5 \text{ on} \end{cases}$$

$$S_c = \begin{cases} 1 & \text{if } S_3 \text{ on and } S_6 \text{ off} \\ 0 & \text{if } S_3 \text{ off and } S_6 \text{ on} \end{cases}$$

- Space vectors

$$\mathbf{S} = \frac{2}{3}(S_a + \mathbf{a}S_b + \mathbf{a}^2 S_c)$$

$$\mathbf{v} = \frac{2}{3}(v_{aN} + \mathbf{a}v_{bN} + \mathbf{a}^2 v_{cN})$$

$$\mathbf{v} = V_{dc}\mathbf{S}$$

Detailed switching states of example inverter

$$\mathbf{v} = \frac{2}{3}(v_{aN} + \mathbf{a}v_{bN} + \mathbf{a}^2 v_{cN}) \qquad a = e^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

S_1	S_2	S_3	S_4	S_5	S_6	Inverter terminal voltage space vector v
0	1	0	1	0	1	$v_0 = 0$
1	0	0	1	0	1	$v_1 = {}^2/_3\, V_{dc}$
0	1	1	0	0	1	$v_2 = {}^1/_3\left(-1 + j\sqrt{3}\right)V_{dc}$
0	1	0	1	1	0	$v_3 = {}^1/_3\left(-1 - j\sqrt{3}\right)V_{dc}$
1	0	1	0	0	1	$v_4 = {}^1/_3\left(1 + j\sqrt{3}\right)V_{dc}$
1	0	0	1	1	0	$v_5 = {}^1/_3\left(1 - j\sqrt{3}\right)V_{dc}$
0	1	1	0	1	0	$v_6 = -{}^2/_3\, V_{dc}$
1	0	1	0	1	0	$v_7 = 0$

Model of the example inverter

- Load model
  - Vector equation for the load current dynamics

$$\mathbf{v} = R\mathbf{i} + L\frac{d\mathbf{i}}{dt} + \mathbf{e}$$

where

$$\mathbf{v} = \frac{2}{3}(v_{aN} + \mathbf{a}v_{bN} + \mathbf{a}^2 v_{cN})$$

$$\mathbf{i} = \frac{2}{3}(i_a + \mathbf{a}i_b + \mathbf{a}^2 i_c)$$

$$\mathbf{e} = \frac{2}{3}(e_a + \mathbf{a}e_b + \mathbf{a}^2 e_c)$$

  - Discrete-time equations

$$\hat{\mathbf{i}}(k+1) = \left(1 - \frac{RT_s}{L}\right)\mathbf{i}(k) + \frac{T_s}{L}\left(\mathbf{v}(k) - \hat{\mathbf{e}}(k)\right)$$

$$\hat{\mathbf{e}}(k-1) = \mathbf{v}(k-1) - \frac{L}{T_s}\mathbf{i}(k) - \left(R - \frac{L}{T_s}\right)\mathbf{i}(k-1)$$

$$\frac{d\mathbf{i}}{dt} \approx \frac{\mathbf{i}(k+1) - \mathbf{i}(k)}{T_s}$$

Forward Euler method

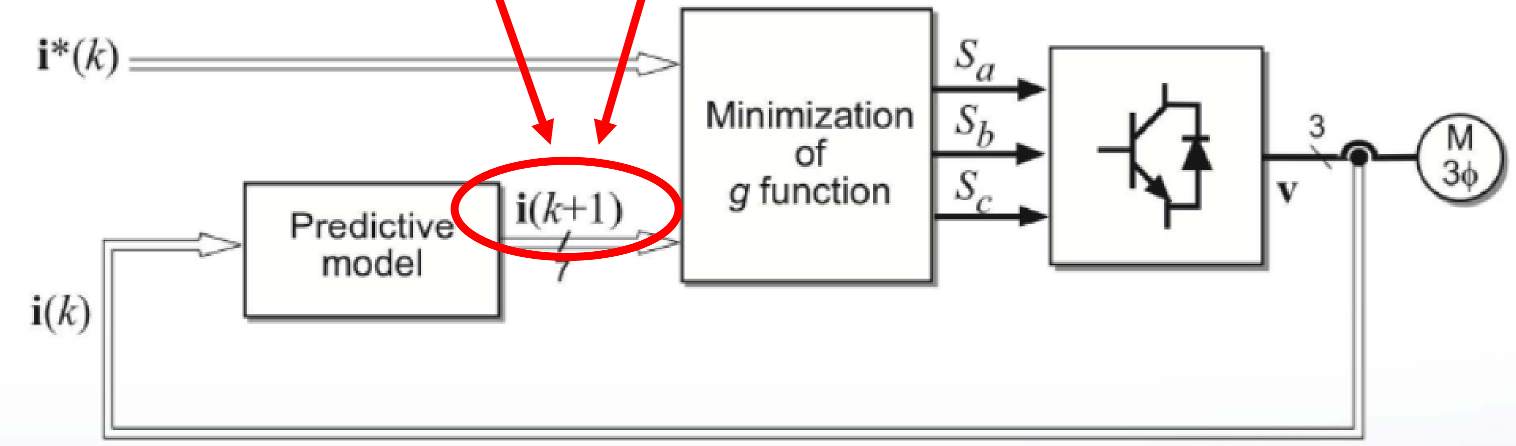
Cost function of the example inverter

- Cost function:

$$g = |i_\alpha^* - i_\alpha^p| + |i_\beta^* - i_\beta^p|$$

$i_\alpha{}^*$, $i_\beta{}^*$: reference values

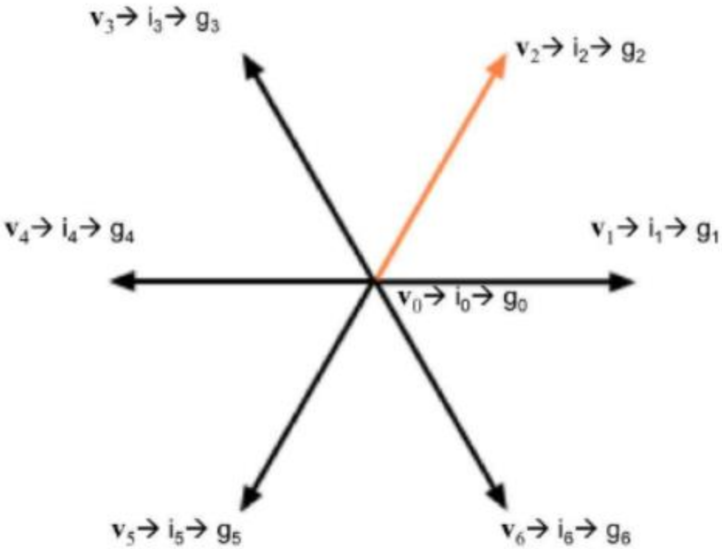
$i_\alpha{}^p$, $i_\beta{}^p$: predicted values



- No need for linear controllers !!
- No need for modulator (PWM or SVM) !!

Select the optimal state for the example inverter

Cost function minimization

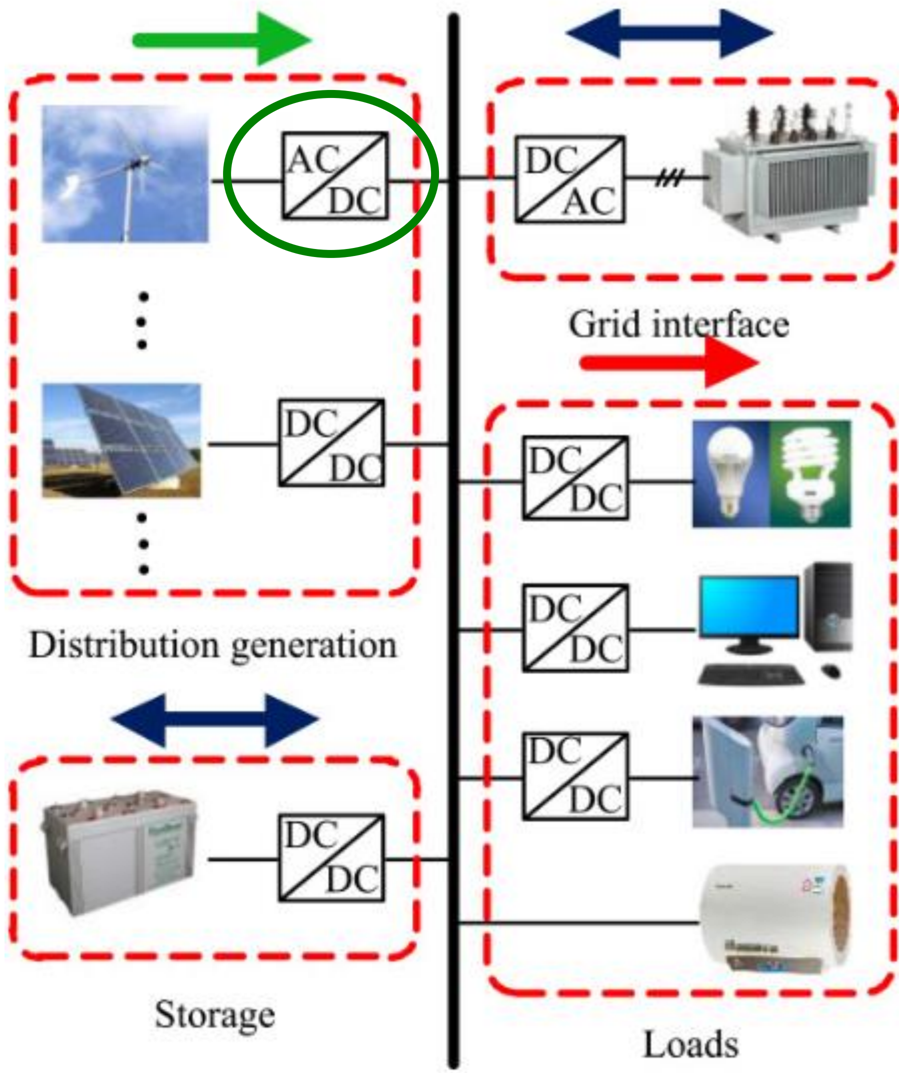


$\mathbf{v}_0$	g_0	0.60	
$\mathbf{v}_1$	g_1	0.82	
$\mathbf{v}_2$	g_2	0.24	$\leftarrow g_{min}$
$\mathbf{v}_3$	g_3	0.42	
$\mathbf{v}_4$	g_4	0.96	
$\mathbf{v}_5$	g_5	1.24	
$\mathbf{v}_6$	g_6	1.19	

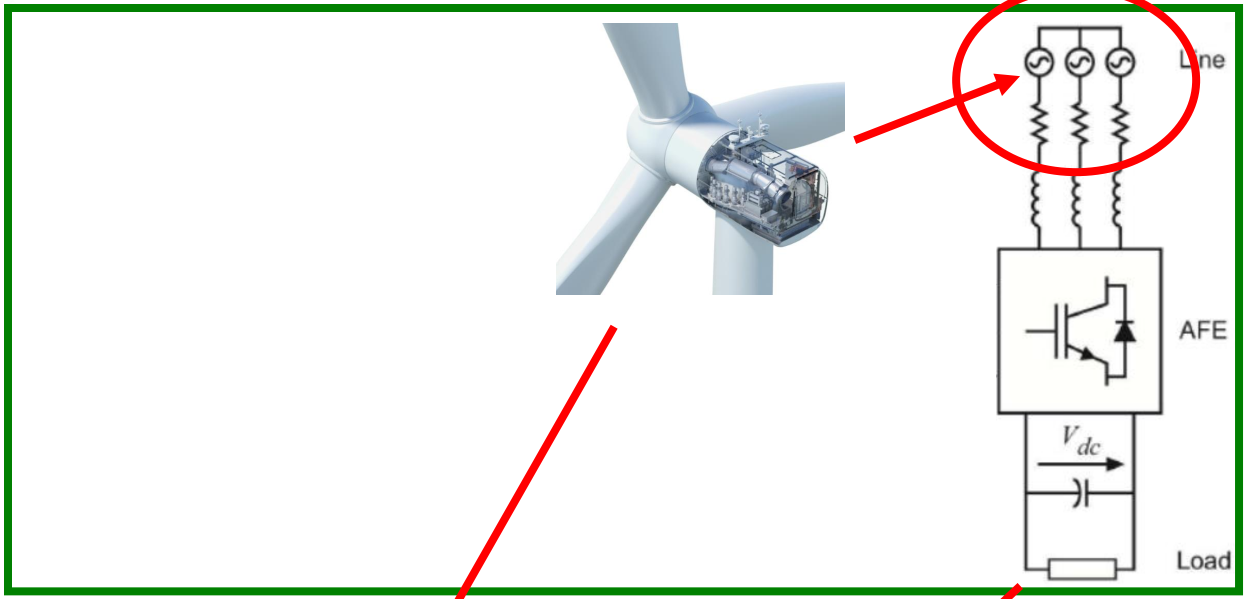
- Voltage vector $\mathbf{v}_0$ is used to predict i_0 and to calculate cost function (error) g_0.
- Voltage vector $\mathbf{v}_1$ is used to predict i_1 and to calculate cost function (error) g_1.
- …
- $g_{min} = g_2$.
- Voltage vector $\mathbf{v}_2$ is selected and will be applied during the next sampling interval.

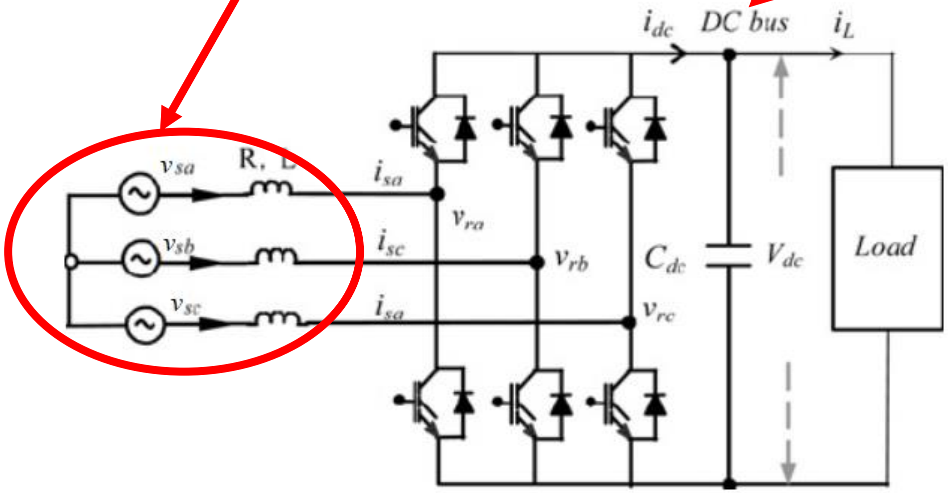
MPC FOR AC/DC RECTIFIER

Different applications of power converters



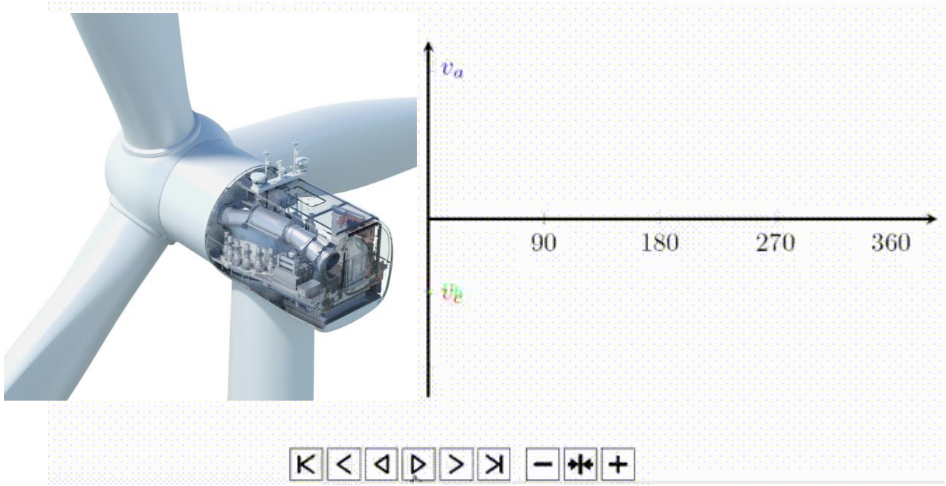
MPC application in AC/DC rectifier



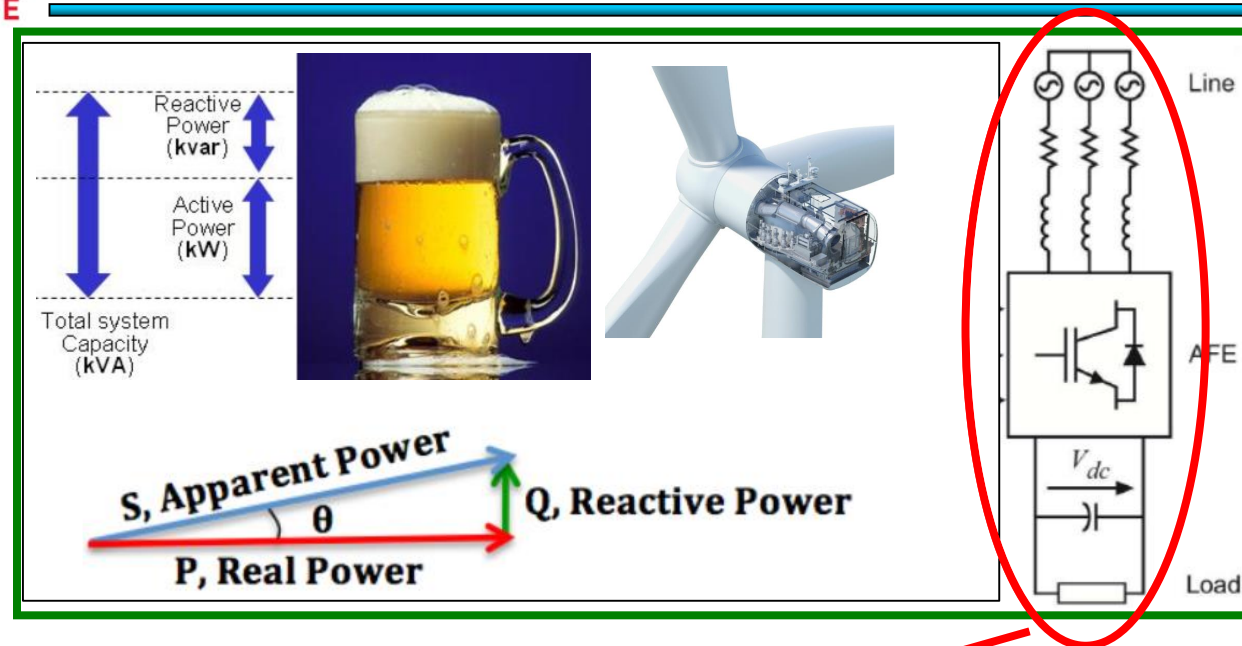


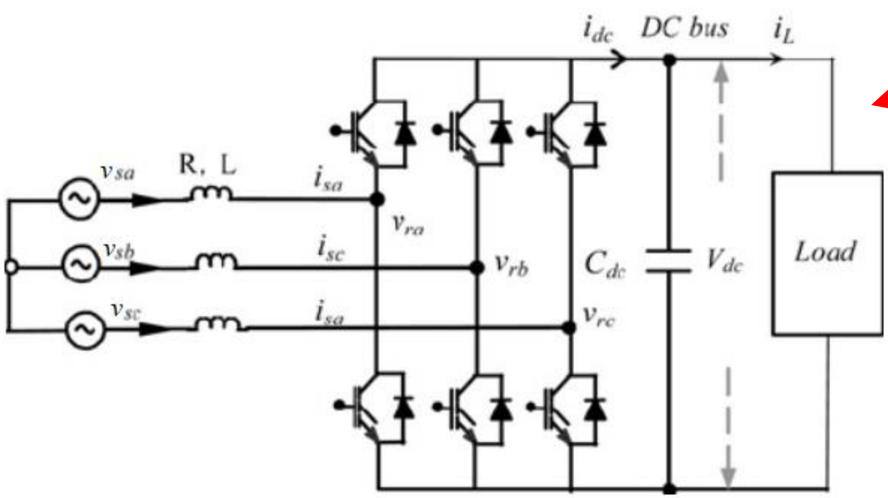
MPC application in AC/DC rectifier

MPC application in AC/DC rectifier



MPC application in AC/DC rectifier



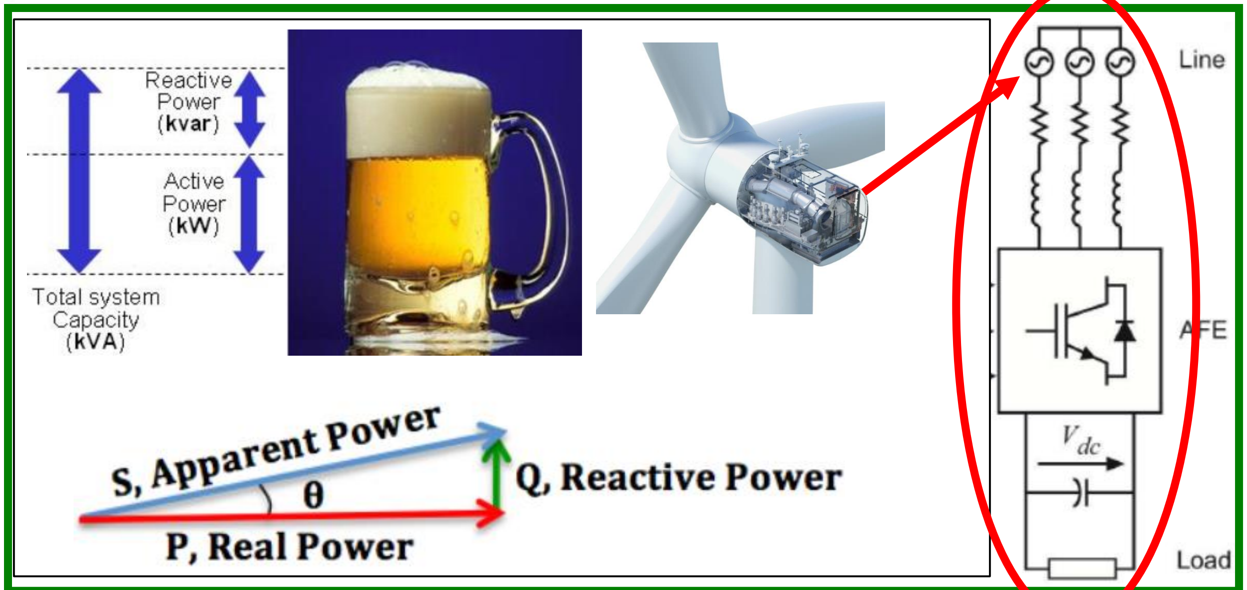


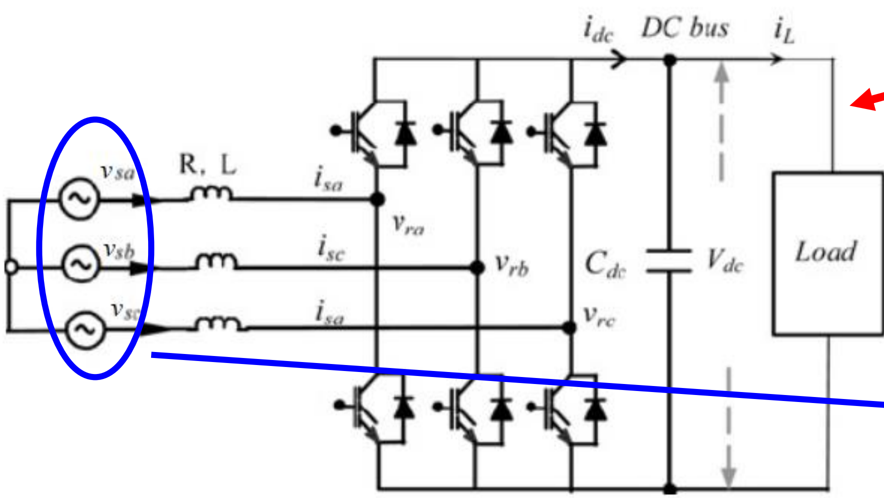
Control target:

Control active power $P_{in} = P_{in}^*$

Minimize reactive power Q_{in}

MPC application in AC/DC rectifier





Model:

$$P_{\text{in}}(k+1) = Re\{\mathbf{v}_s(k+1)\mathbf{i}_s(k+1)\} = v_{s\alpha} i_{s\alpha} + v_{s\beta} i_{s\beta}$$

$$Q_{\text{in}}(k+1) = Im\{\mathbf{v}_s(k+1)\mathbf{i}_s(k+1)\} = v_{s\beta} i_{s\alpha} - v_{s\alpha} i_{s\beta}$$

Can be measured directly

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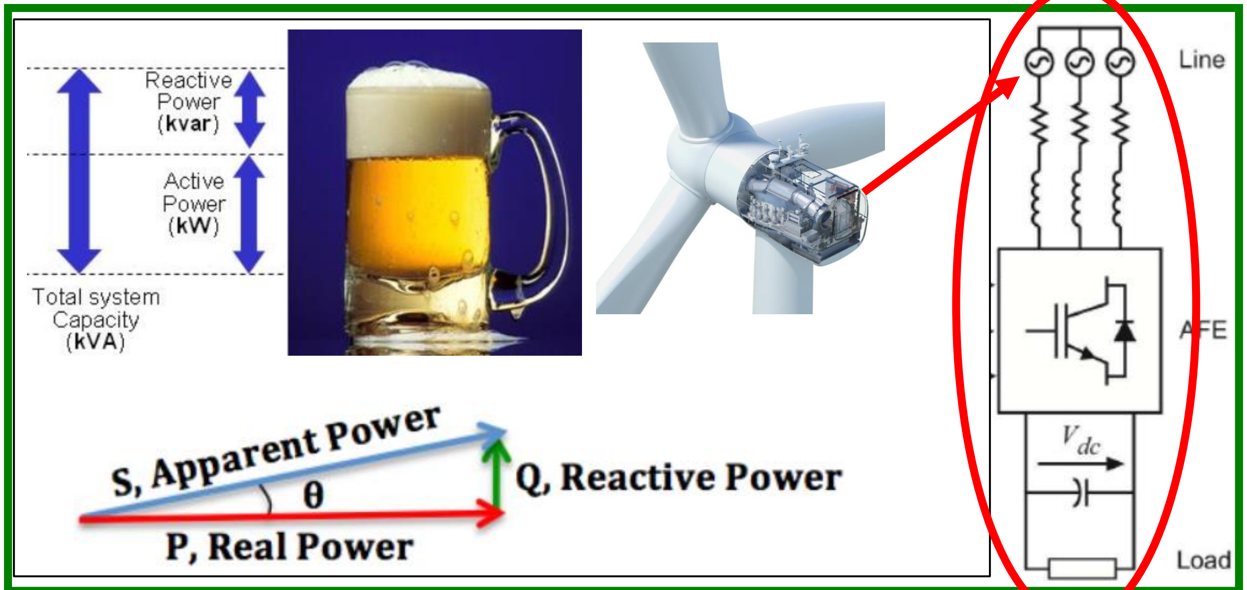
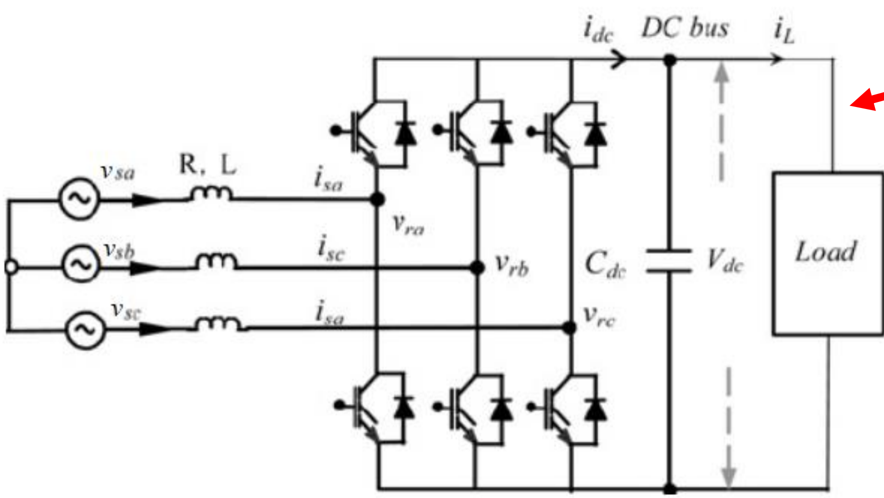
MPC application in AC/DC rectifier





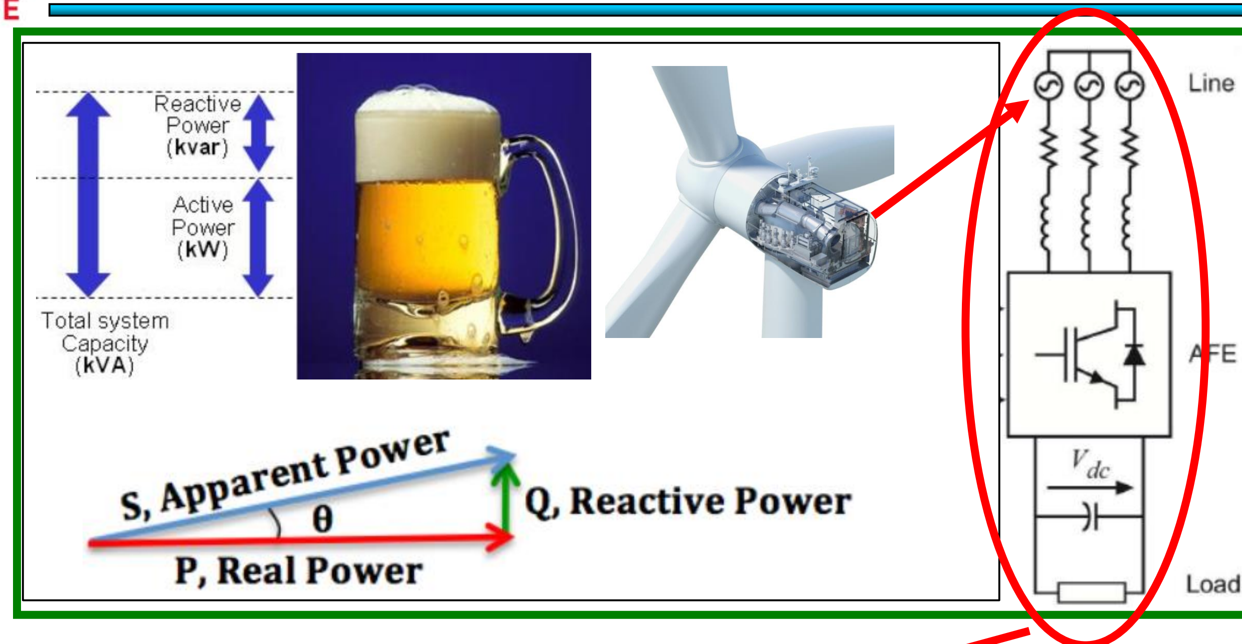
Model:

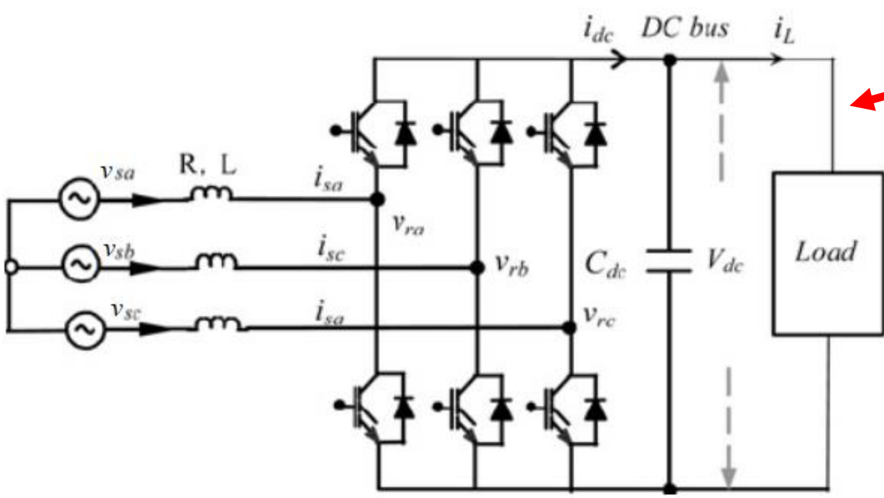
$$P_{\text{in}}(k+1) = Re\{\mathbf{v}_s(k+1)\mathbf{i}_s(k+1)\} = v_{s\alpha}i_{s\alpha} + v_{s\beta}i_{s\beta}$$

$$Q_{\text{in}}(k+1) = Im\{\mathbf{v}_s(k+1)\mathbf{i}_s(k+1)\} = v_{s\beta}i_{s\alpha} - v_{s\alpha}i_{s\beta}$$

$$\boxed{\mathbf{i}_s(k+1) = \left(1 - \frac{R_s T_s}{L_s}\right)\mathbf{i}_s(k) + \frac{T_s}{L_s}[\mathbf{v}_s(k) - \mathbf{v}_{\text{afe}}(k)]}$$

MPC application in AC/DC rectifier

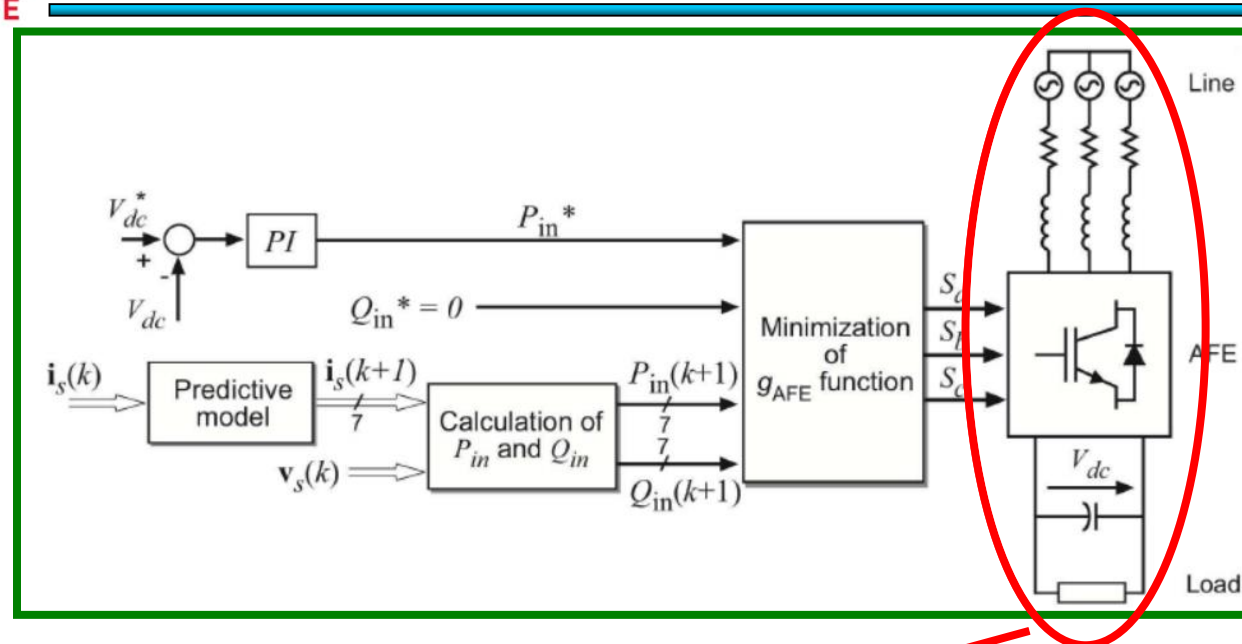


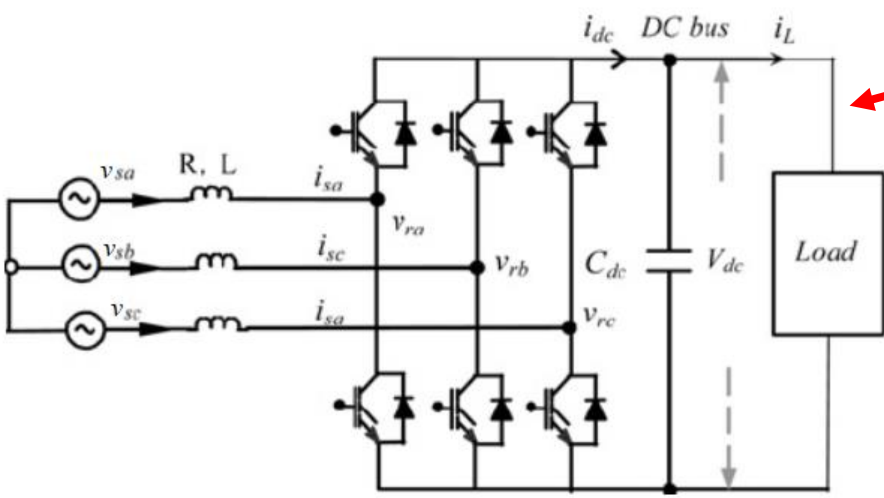


Cost function:

$$g_{\mathrm{AFE}} = |Q_{\mathrm{in}}| + |P_{\mathrm{in}}^{*} - P_{\mathrm{in}}|$$

MPC application in AC/DC rectifier





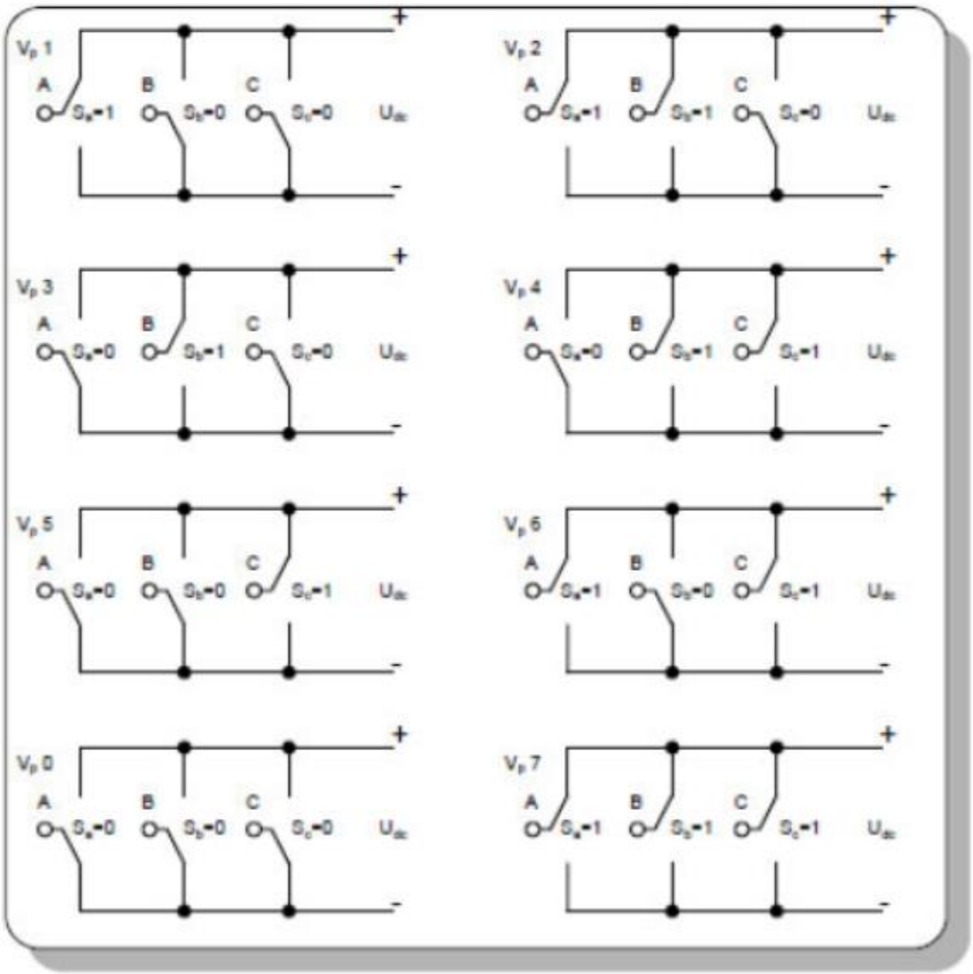
Cost function:

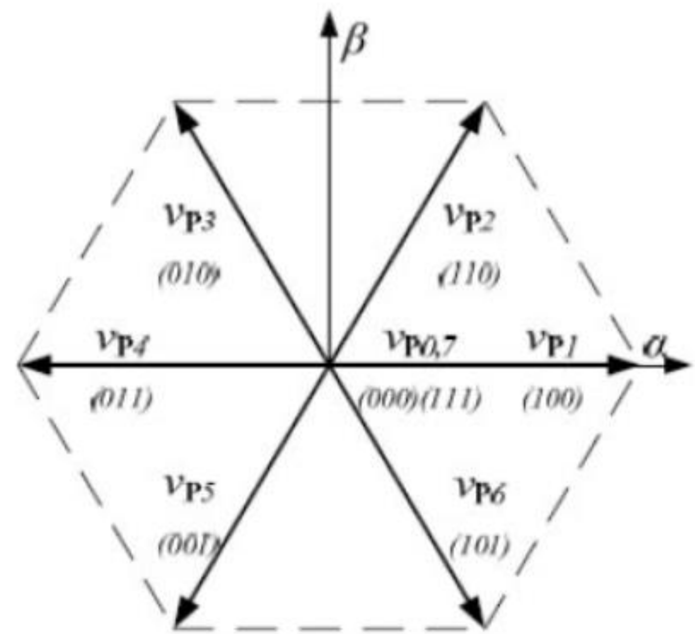
$$g_{\mathrm{AFE}} = |Q_{\mathrm{in}}| + |P_{\mathrm{in}}^* - P_{\mathrm{in}}|$$

MPC application in AC/DC rectifier

Cost function: $g_{\mathrm{AFE}} = |Q_{\mathrm{in}}| + |P_{\mathrm{in}}^{*} - P_{\mathrm{in}}|$

Possible states:





$$v_{P(n)} = \begin{cases} \frac{2}{3} U_{DC} e^{j(n-1)\frac{\pi}{3}} & n = 1 \ldots 6 \\ 0 & n = 0,7 \end{cases}$$

Application of MPC in DC/DC Converters

[14/11/2019]

- Traditional PID control method of DC/DC converter
- MPC method of DC/DC converter
- Overview: MPC application in power converters
- Design issues of FCS-MPC for the power converters

Design issues

- CONTROL DELAY
- PARAMETER ERRORS
- PRESELECTION
- EXTRAPOLATION

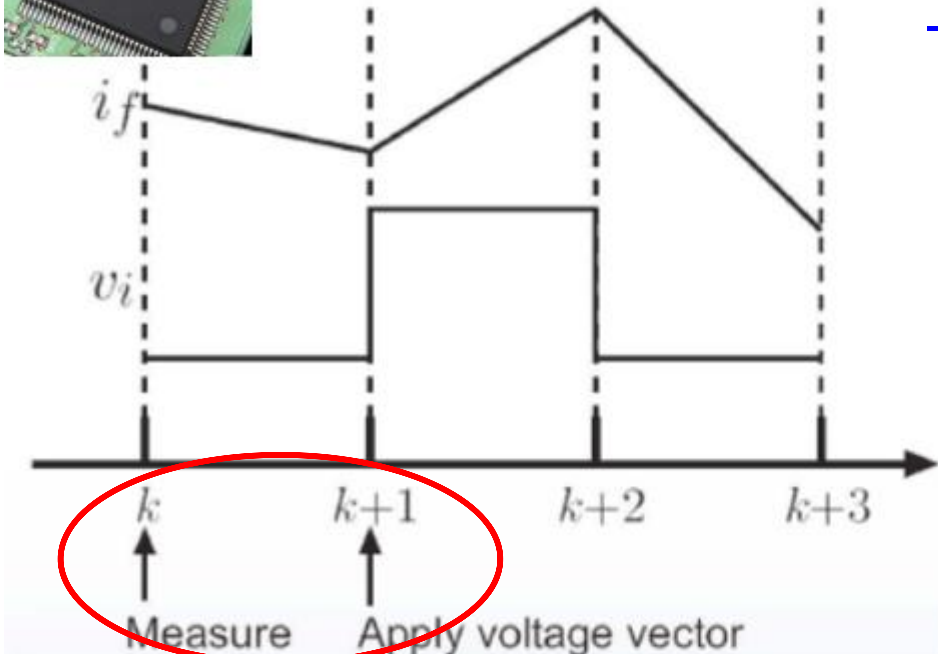
- Control delay
- Parameter errors
- Preselection
- Extrapolation

Control delay on MPC

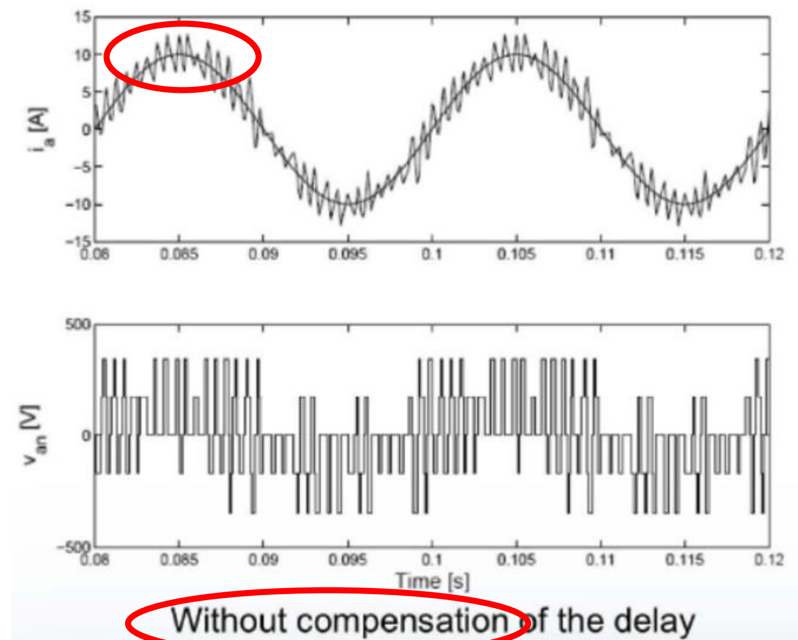
➢ Control delay:

- Delay caused by calculation time
- One sampling time delay

$$\boxed{\hat{\mathrm{i}}(k+1) = \left(1 - \frac{RT_s}{L}\right)\mathrm{i}(k) + \frac{T_s}{L}\left(\mathrm{v}(k) - \hat{\mathrm{e}}(k)\right)}$$



One sampling time delay

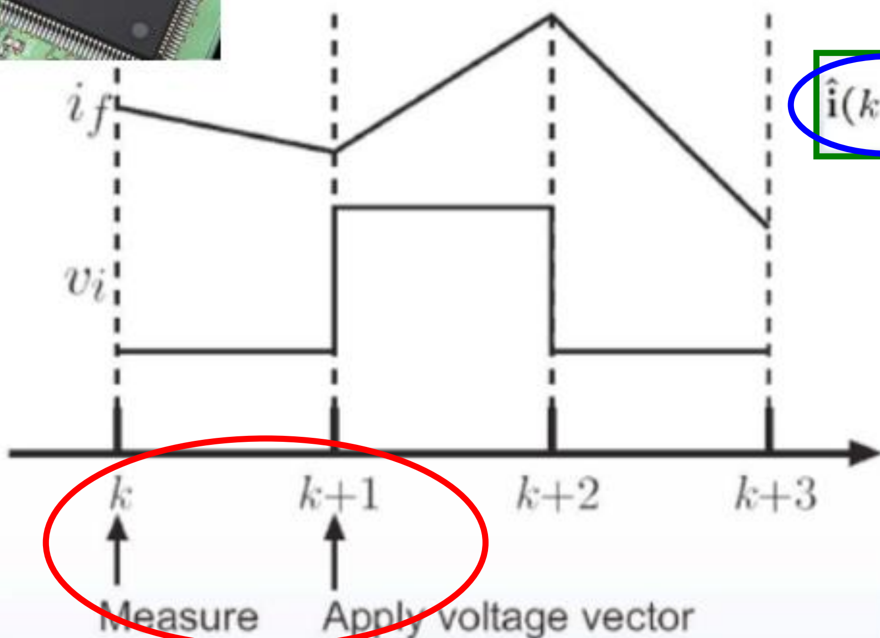


Control delay on MPC

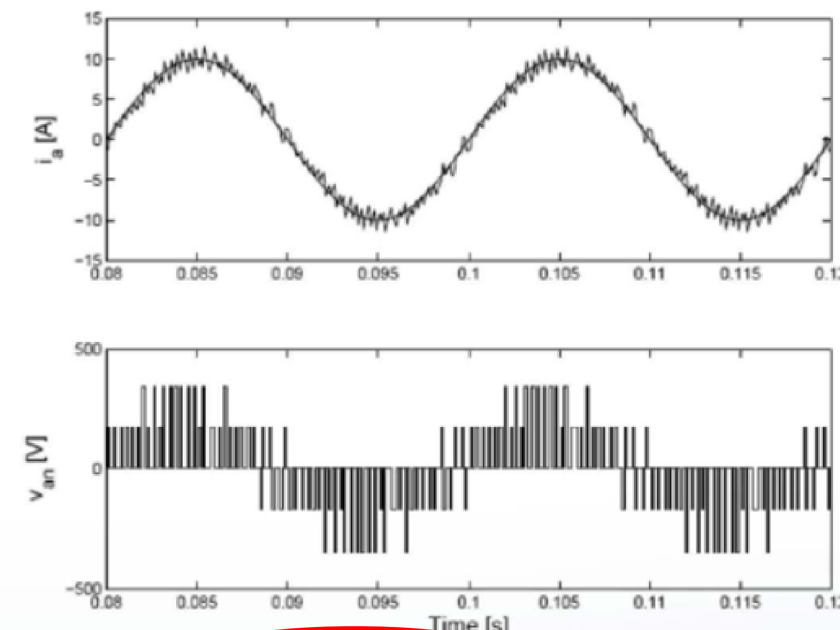
- Solve control delay problem: Two step prediction

$$\hat{i}(k+1) = \left(1 - \frac{RT_s}{L}\right) i(k) + \frac{T_s}{L}\left(v(k) - \hat{e}(k)\right)$$

$$\hat{i}(k+2) = \left(1 - \frac{RT_s}{L}\right) \hat{i}(k+1) + \frac{T_s}{L}\left(v(k+1) - \hat{e}(k+1)\right)$$



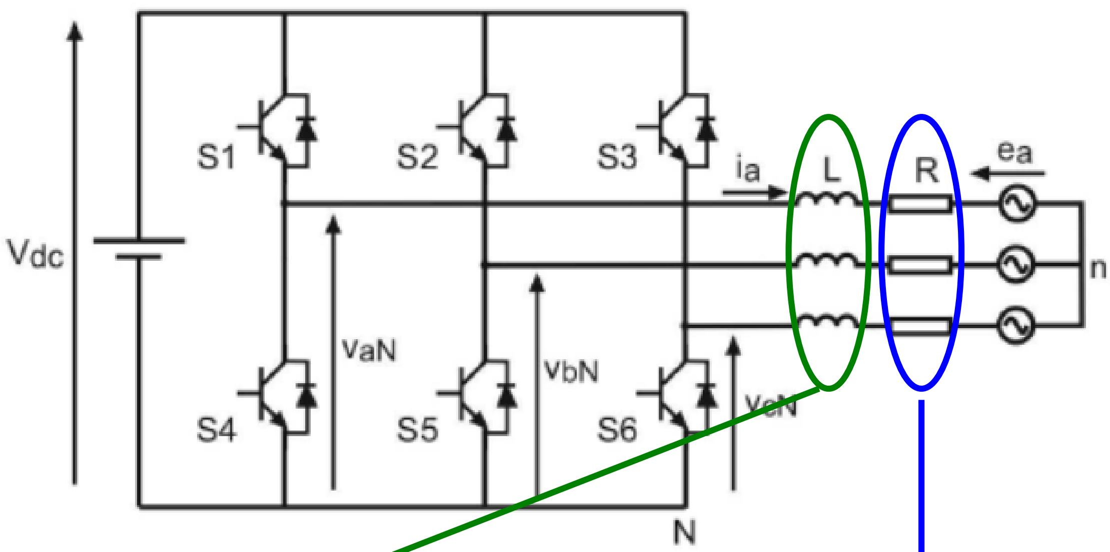
One sampling time delay



With compensation of the delay

- CONTROL DELAY
- PARAMETER ERRORS
- PRESELECTION
- EXTRAPOLATION

Example: parameter errors



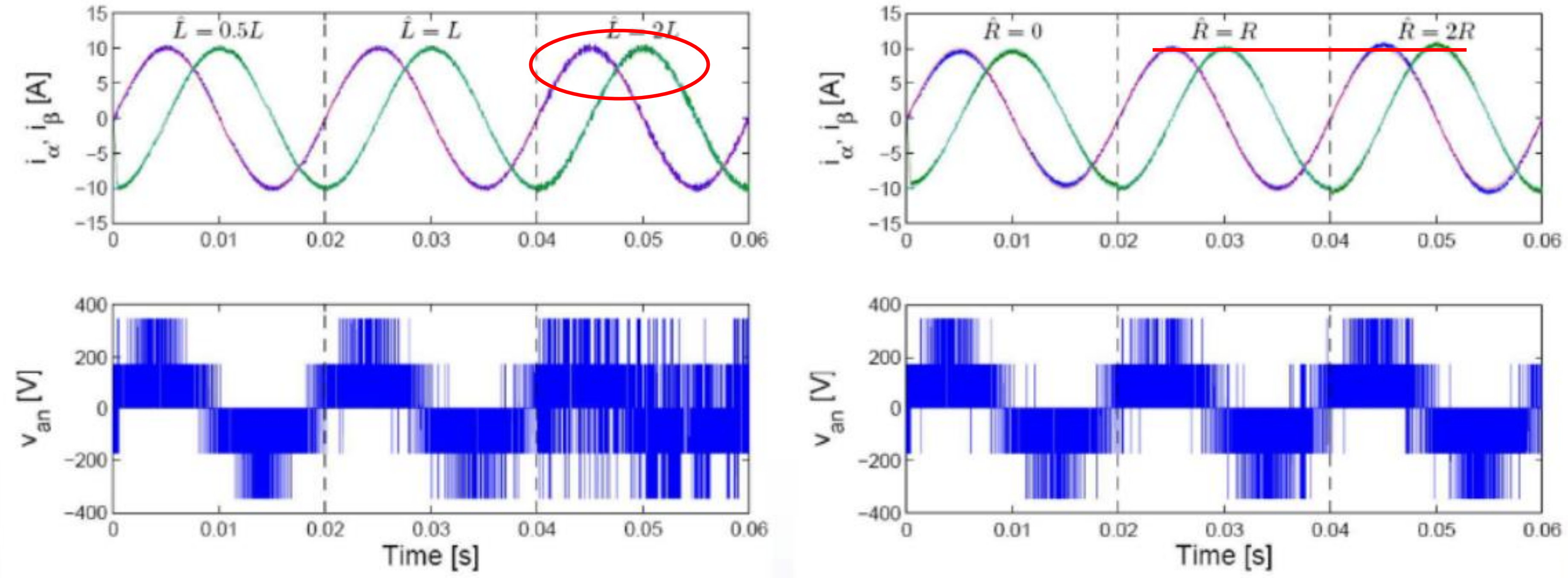
L±ΔL

C±ΔC

Effect of parameter errors on MPC

- Error in the inductance value increases the ripple.
- Error in the resistance value produces error in the current amplitude.

Stability is not affected



Robust

Current control of a three-phase inverter

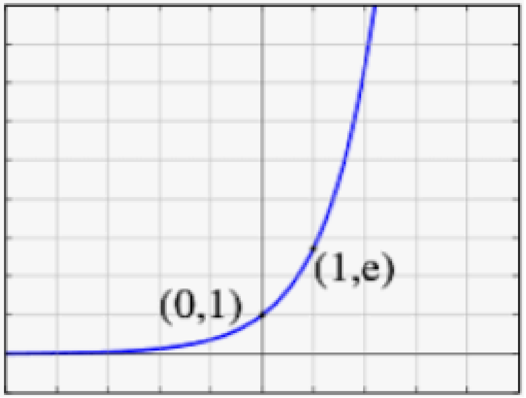
Design issues

- CONTROL DELAY
- PARAMETER ERRORS
- PRESELECTION
- EXTRAPOLATION

Reduce calculation is necessary

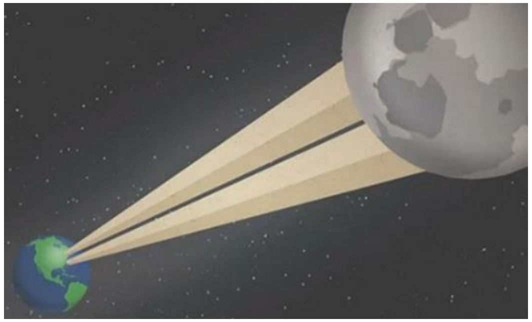
Biggest problem: Calculation effort

$$\text{switching possibilities}^{\text{prediction steps}}$$



- Problem: We have no feeling for exponential growth

Example: A sheet of paper, thickness 10 µm (1/100 mm)
Assume you could fold it 46 times (seems not much)
How long will it be? 10 µm · 2^{46} ≈ 700,000 km
=> To the moon and back!



- Exponential growth with the switching possibilities as base is even worse

=> Strategies to reduce the calculation effort are absolutely necessary!

Tips to reduce calculation is necessary

- Calculation increase exponentially with:
  - Prediction horizon → > setting time
  - Switching possibilities → We can reduce this number

Basic idea for preselection method

Optimum integer solution of a <u>linear program</u>

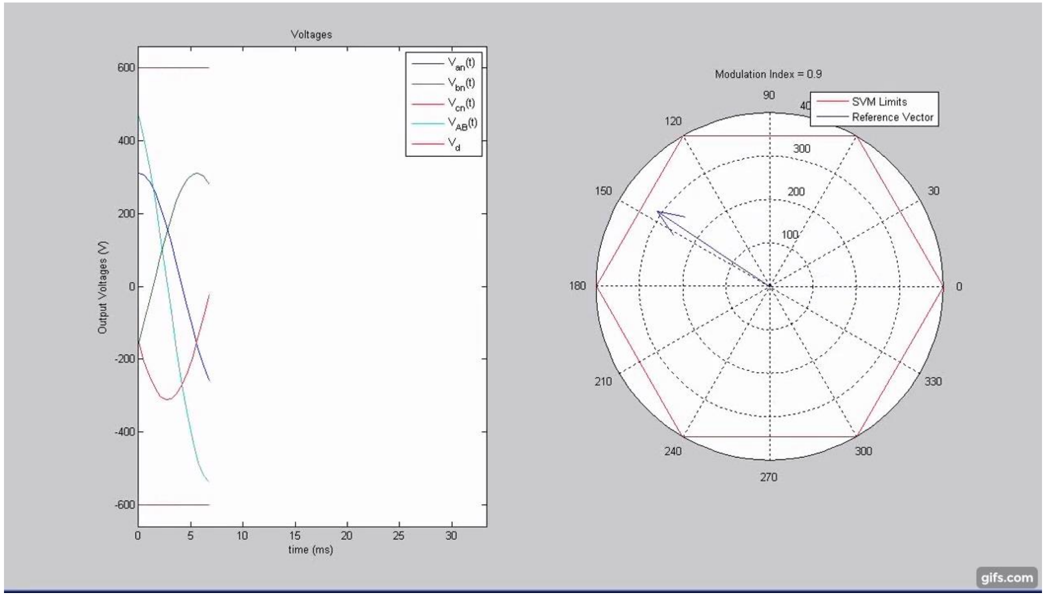
≈

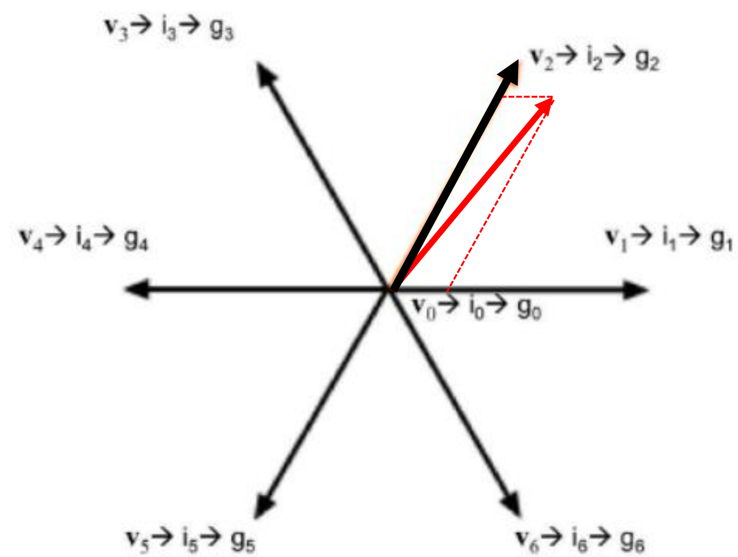
Continuous-valued solution of the <u>integer problem</u>

⇓

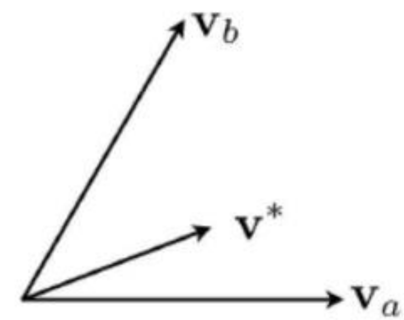
Not all integer points have to be examined

Continues optimal solution

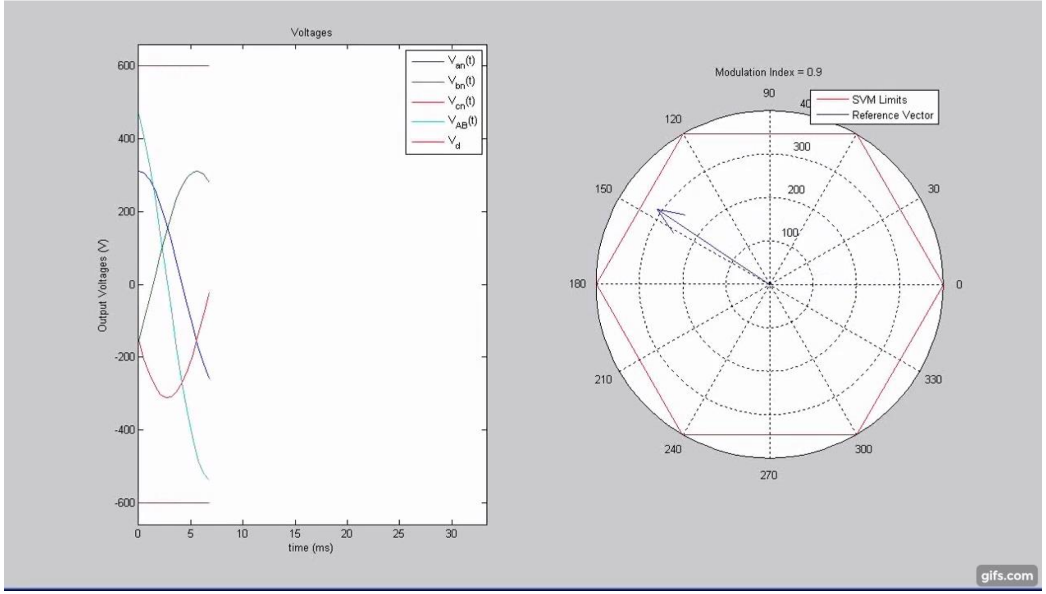


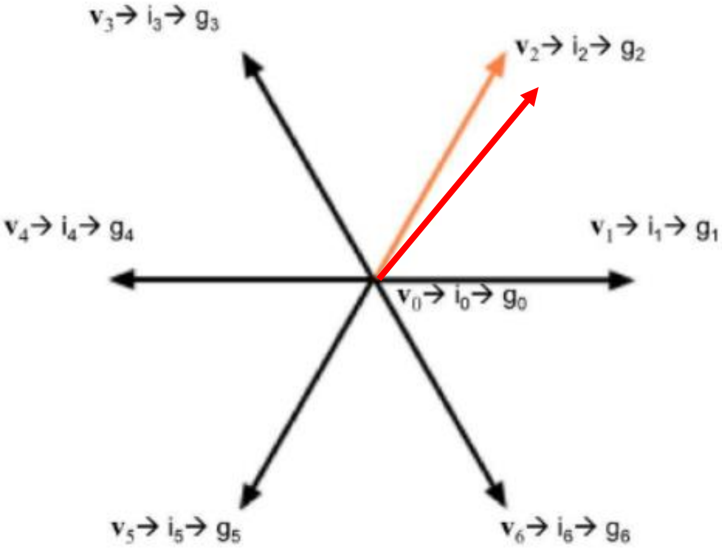


Improved FCS-MPC
in Chapter 5



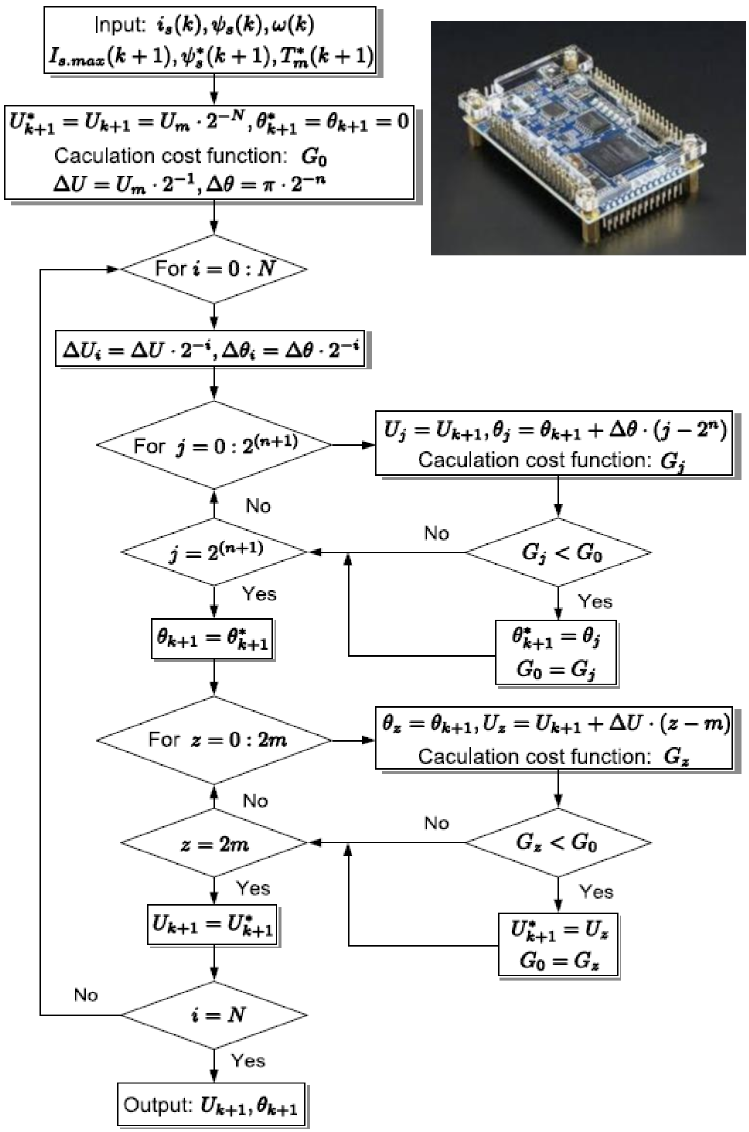
$$\mathbf{v}^* = \frac{1}{T}(\mathbf{v}_a t_a + \mathbf{v}_b t_b + \mathbf{v}_0 t_0)$$
$$t_a + t_b + t_0 = T$$





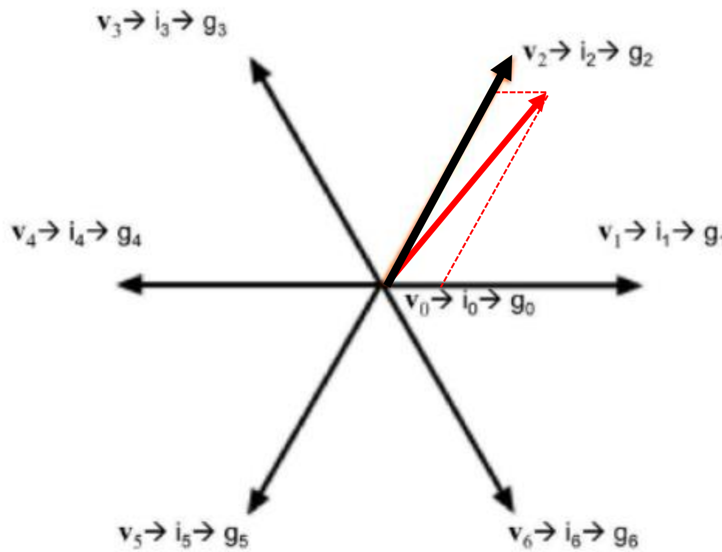
Basic FCS-MPC in Chapter 5

Example: Improved FCS-MPC and FCS-MPC

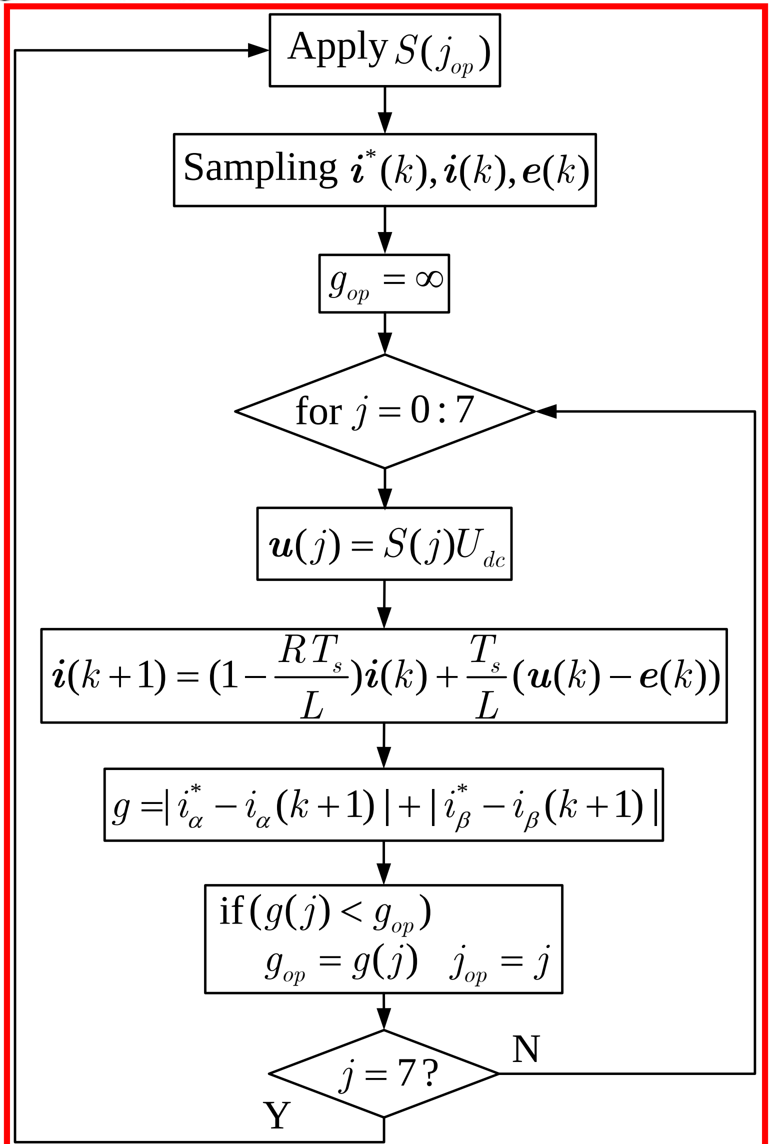


Improved FCS-MPC in Chapter 5

- Continues optimal solution
- Accurate
- **More computation load**



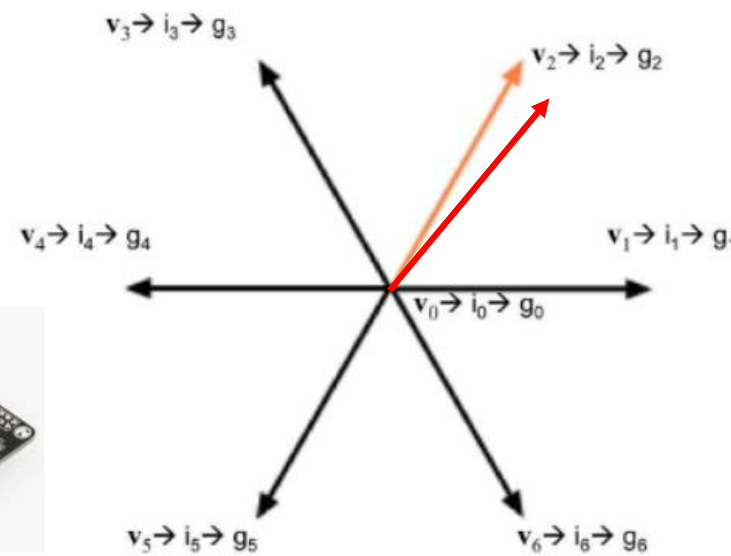
Example: Improved FCS-MPC and FCS-MPC



Basic FCS-MPC

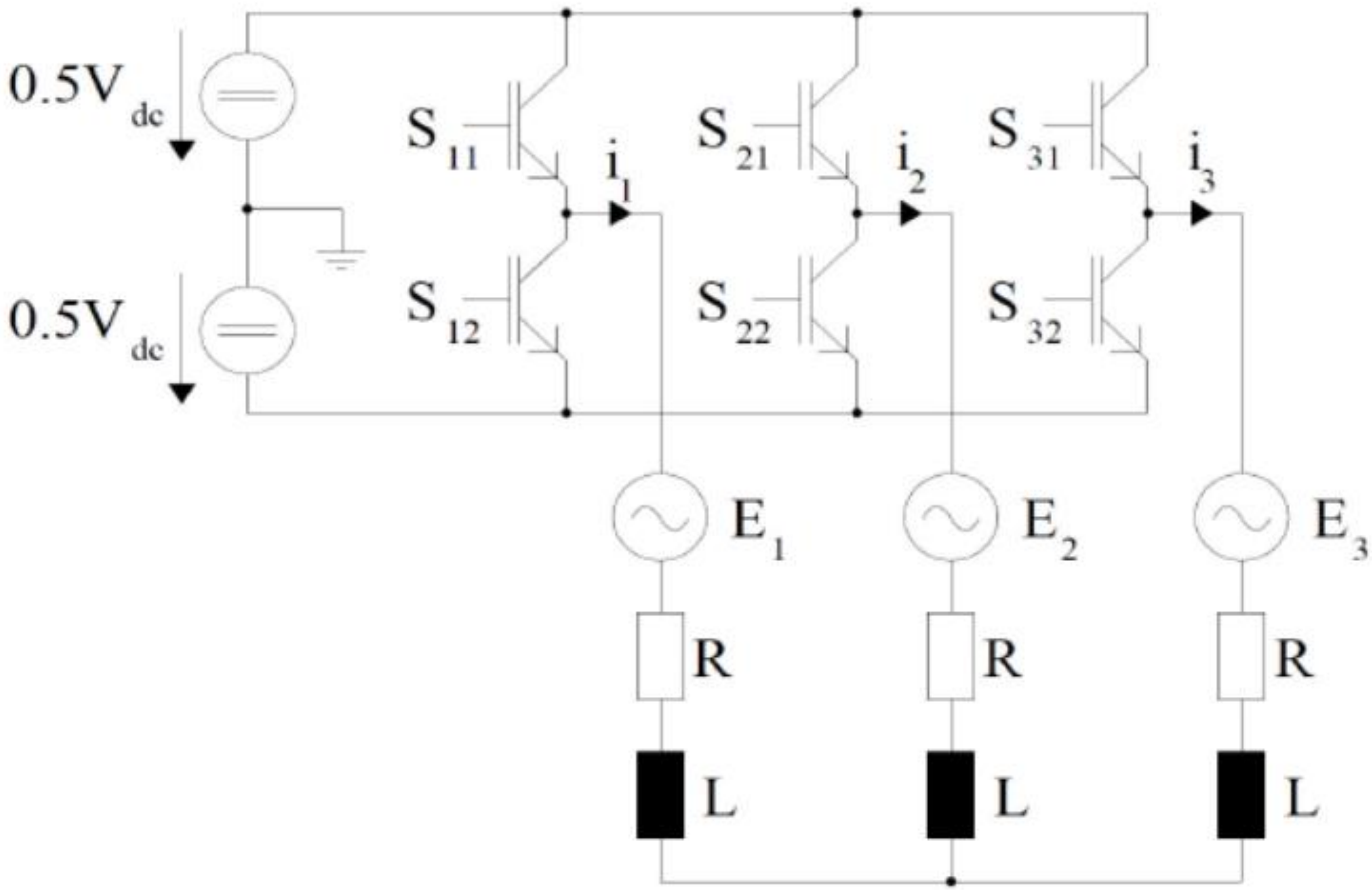
in Chapter 5

- Integer optimal solution
- Not very accurate, but fine.
- **Less computation load**



Basic FCS-MPC results (Integer optimal solution)

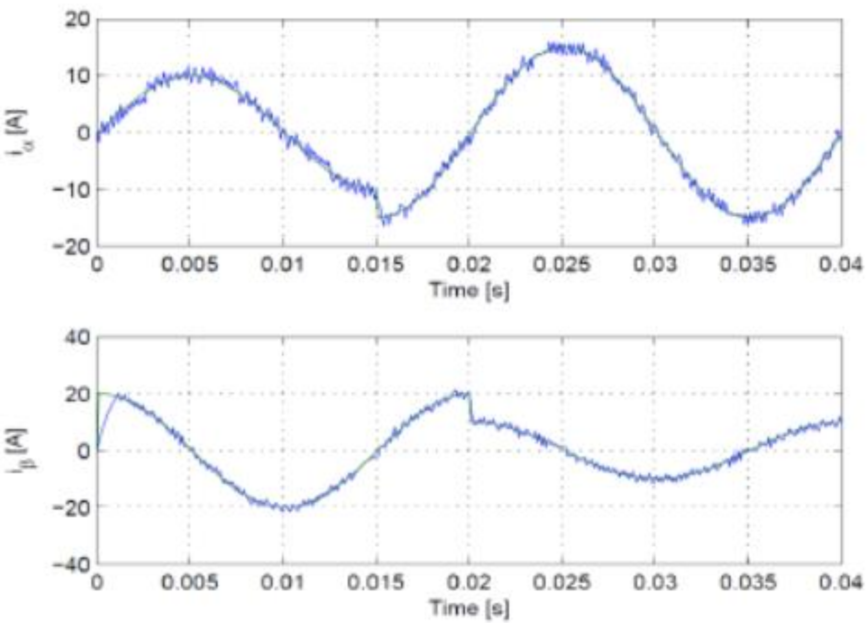
Current control of a three-phase resistive-inductive-active load



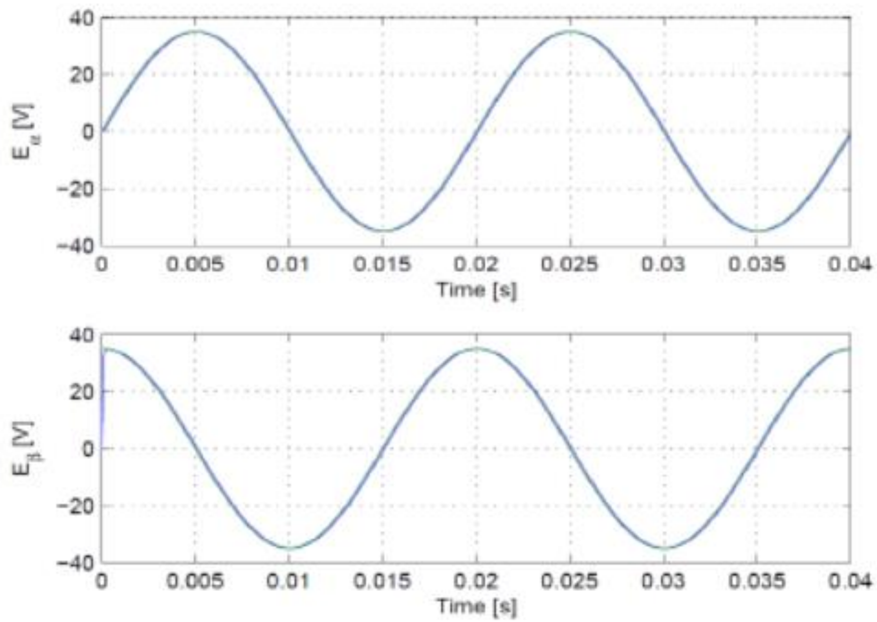
Simulation results of basic FCS-MPC

(Integer optimal solution)

Sinusoidal references



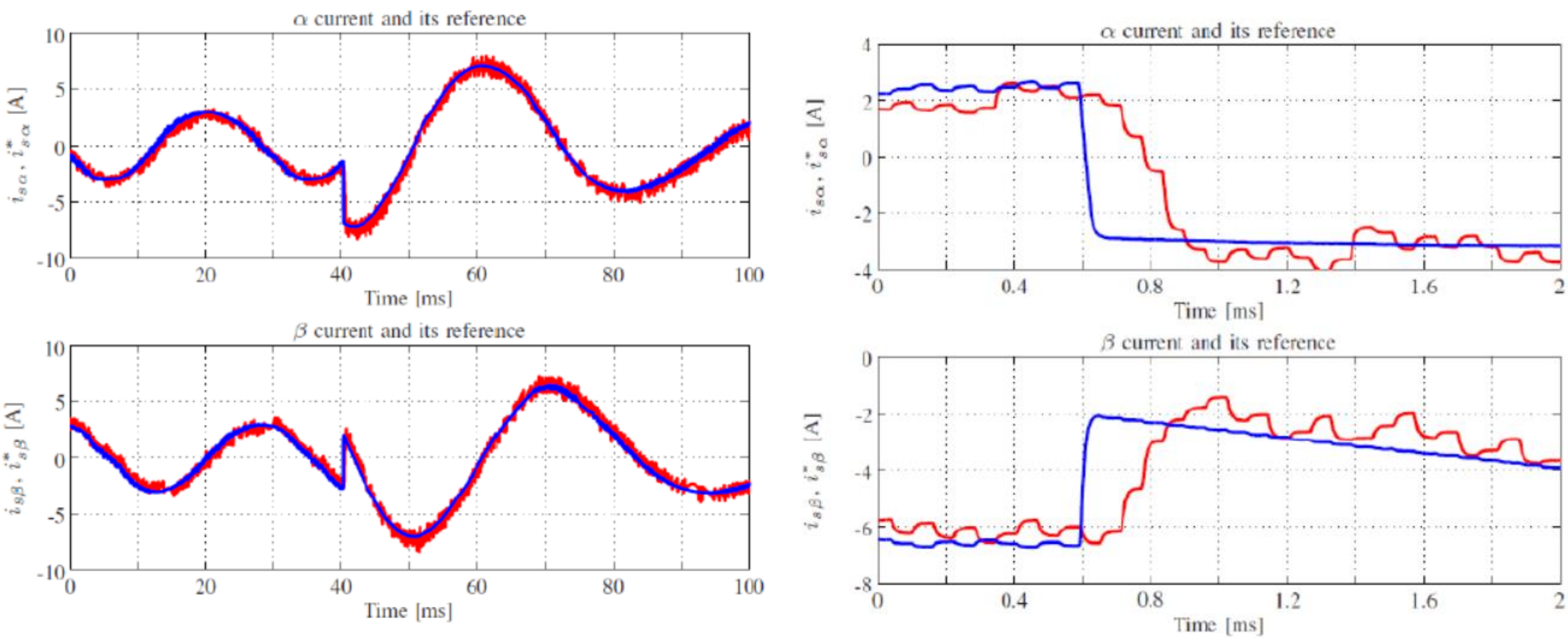
Back EMF voltages



R = 10Ω, L = 10mH, Vdc = 540V, T = 100μs

Simulation results (Zoom in)

Not accurate but accept

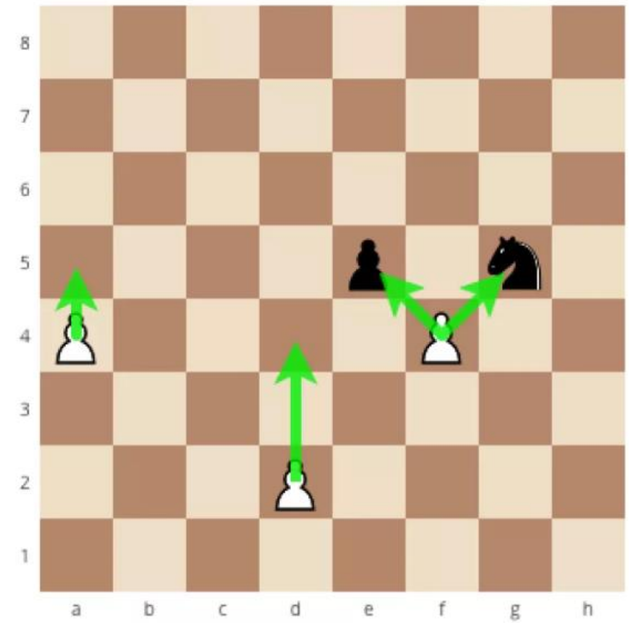


- Control delay
- Parameter errors
- Preselection
- Extrapolation

MPC likes play chess

MPC is like playing chess

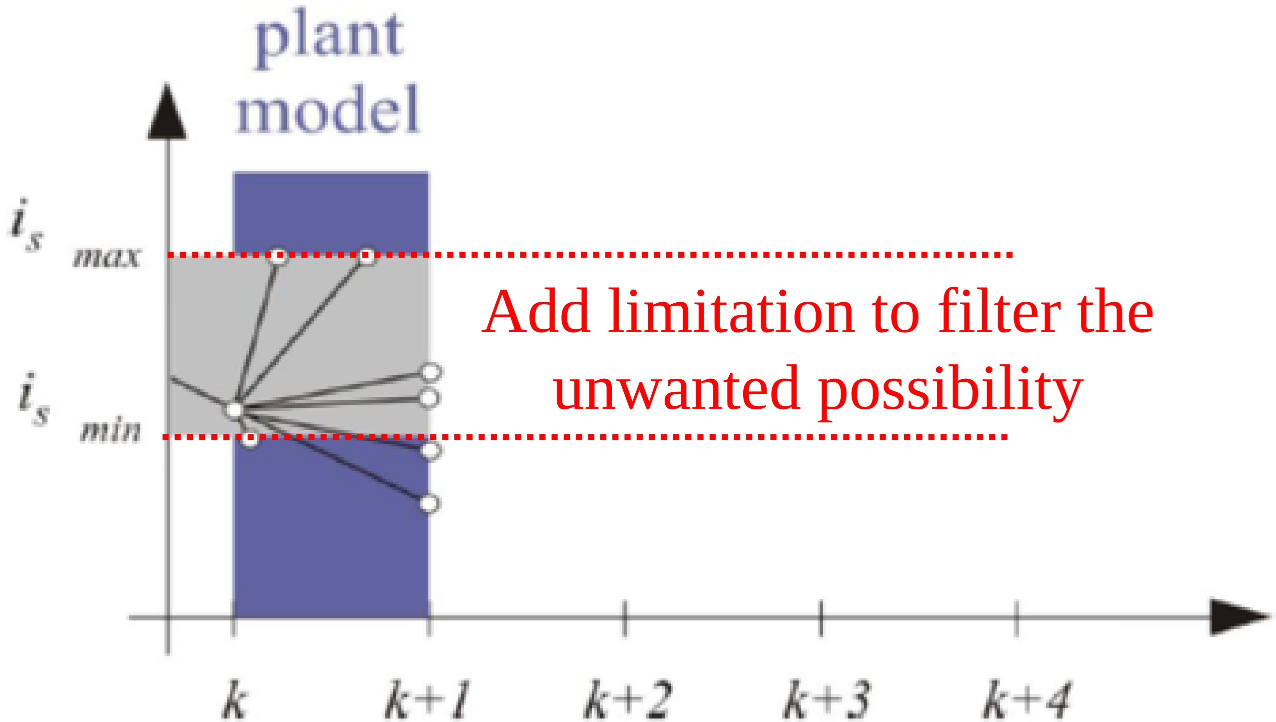
- Calculate all possible moves in prediction horizon
- Select best moves for success
- Repeat pre-calculation & optimization



Does not consider each possibility

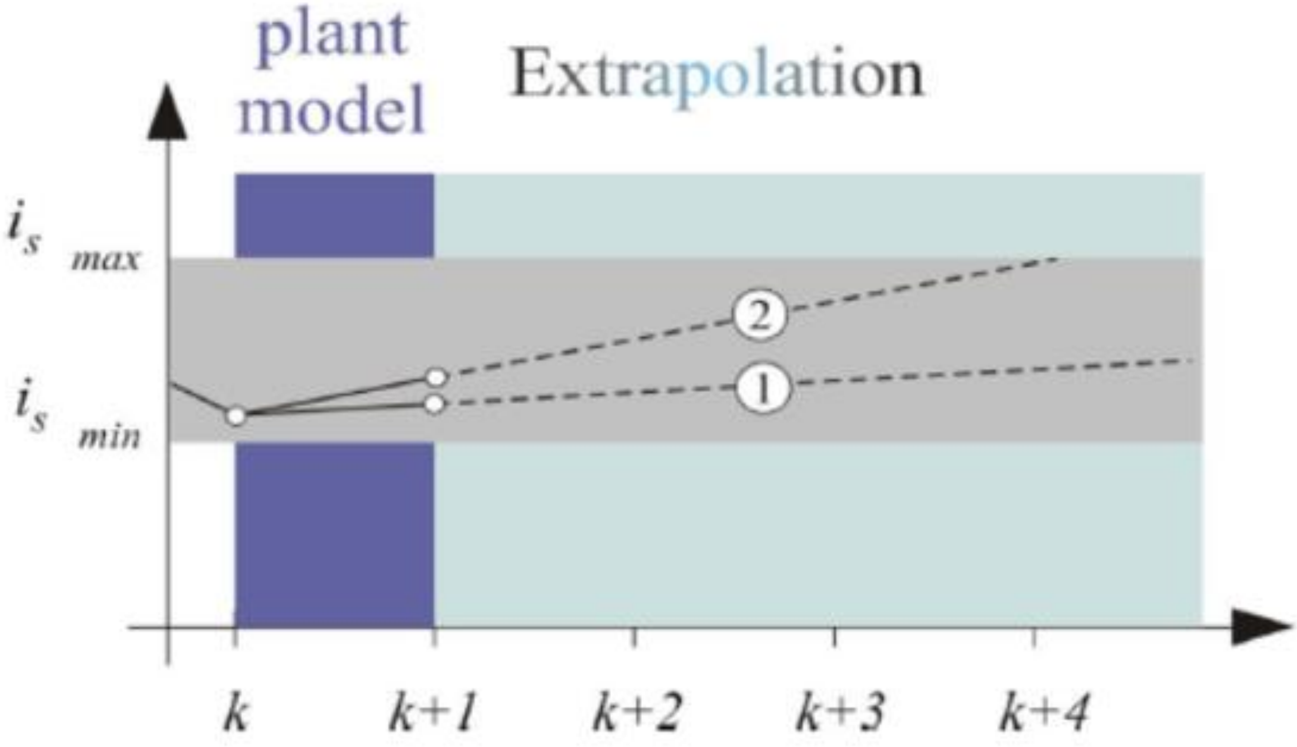
Why should we do that in predictive control? - No need

Extrapolation instead of exact calculation



Add limitation to filter the unwanted possibility

Extrapolation instead of exact calculation



Review of today's topic

- Design MPC controller for buck converter.
- Application of MPC in different power converters.
- Tips to design MPC controller for power converters.

ABOUT THE FINAL EXAMINATION

Key information of the final examination: Only for part II

- Only two questions from Part II.
- There is no math involved in Part II.
- There is no references in the previous papers before 2018 (Only for Part II).
- Test the understanding of MPC.
- Will test MPC application of power converters.

REVIEWER THE PART II OF PROCESS CONTROL (MPC)

Schedules

11/10/2019	Topic 1	• INTRODUCTION AND PRELIMINARY
17/10/2019	Topic 2	• BASIC THEORY OF MODEL PREDICTIVE CONTROL: PART-I
24/10/2019	Topic 3	• BASIC THEORY OF MODEL PREDICTIVE CONTROL: PART-II
31/10/2019	Topic 4	• QUIZ AND OPERATION PRINCIPLE OF POWER CONVERTER*
07/11/2019	Topic 5	• APPLICATION OF MODEL PREDICTIVE CONTROL IN POWER INVERTER
14/11/2019	Topic 6	• APPLICATION OF MODEL PREDICTIVE CONTROL IN DC/DC CONVERTER

*: Not compulsory, not required.

11/10/2019	Topic 1	• INTRODUCTION AND PRELIMINARY
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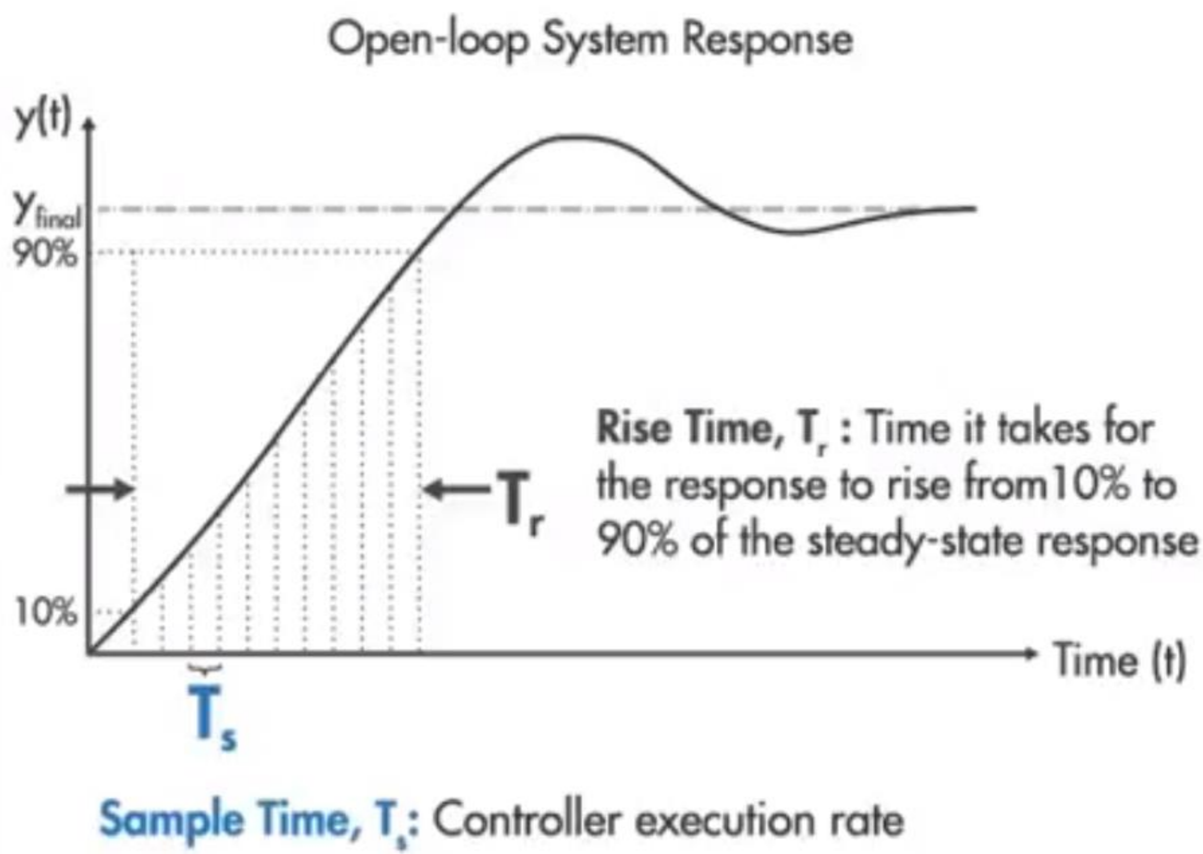
*: Not compulsory, not required.

Basic parameters of the MPC

MPC design parameters

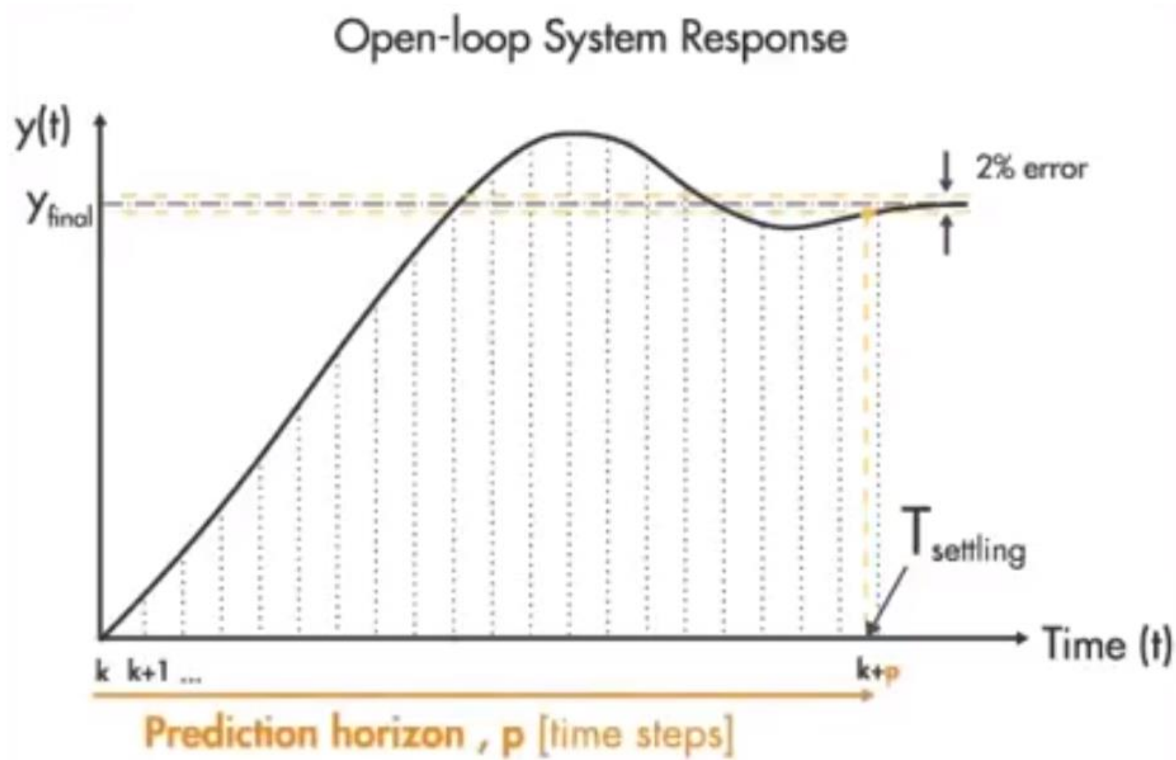
- Sampling time
- Prediction horizon
- Control horizon
- Constrains
- Weights

Basic parameters of the MPC: suitable T_s



$$\frac{T_r}{20} \leqslant T_s \leqslant \frac{T_r}{10}$$

Basic parameters of the MPC: suitable prediction horizon

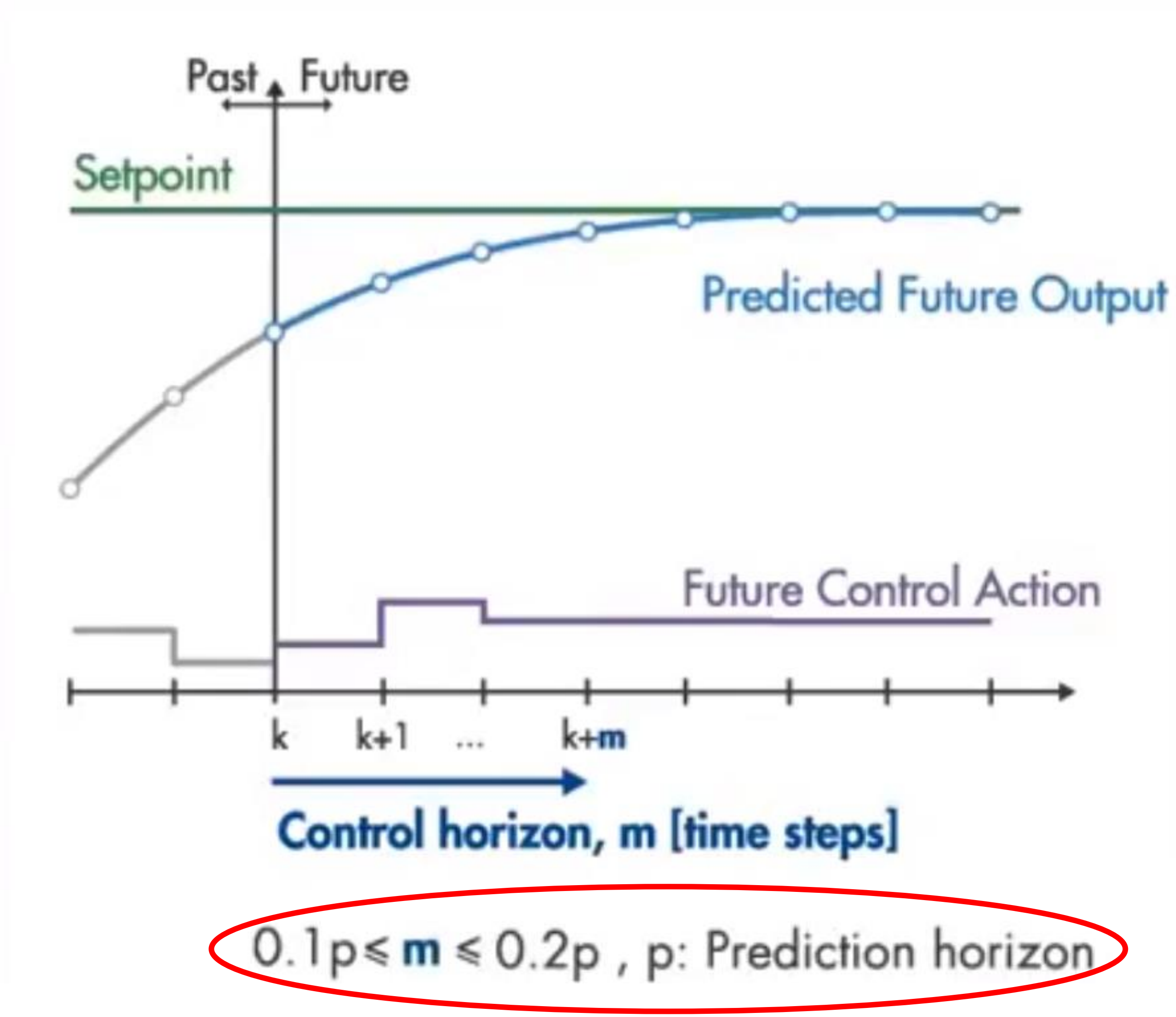


$T_{settling}$: Time it takes for the error $|y(t) - y_{final}|$ to fall to within 2% of y_{final}

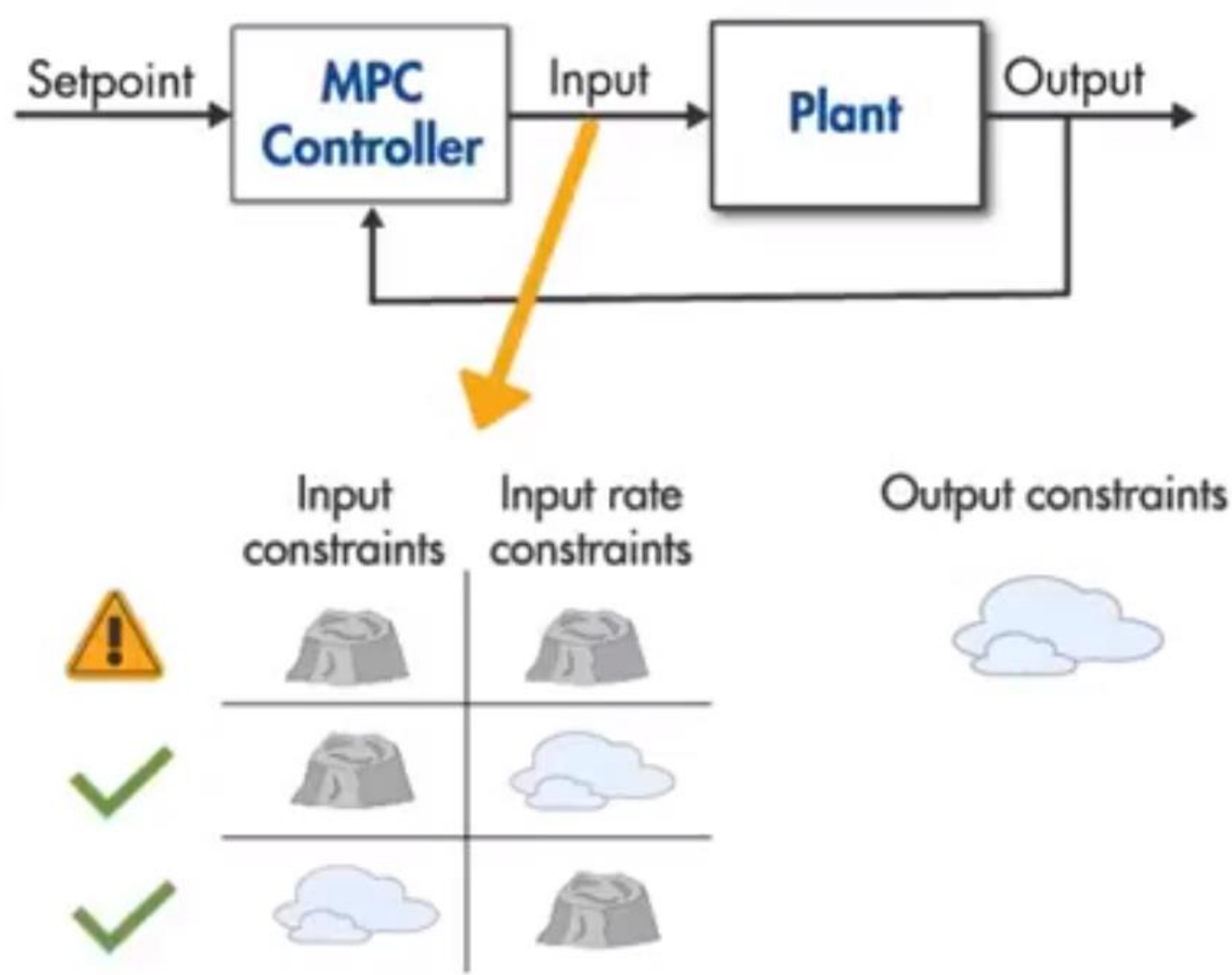
$$\frac{T_r}{20} \leqslant T_s \leqslant \frac{T_r}{10}$$, T_s: Sample time

$$p.T_s \geqslant T_{settling}$$

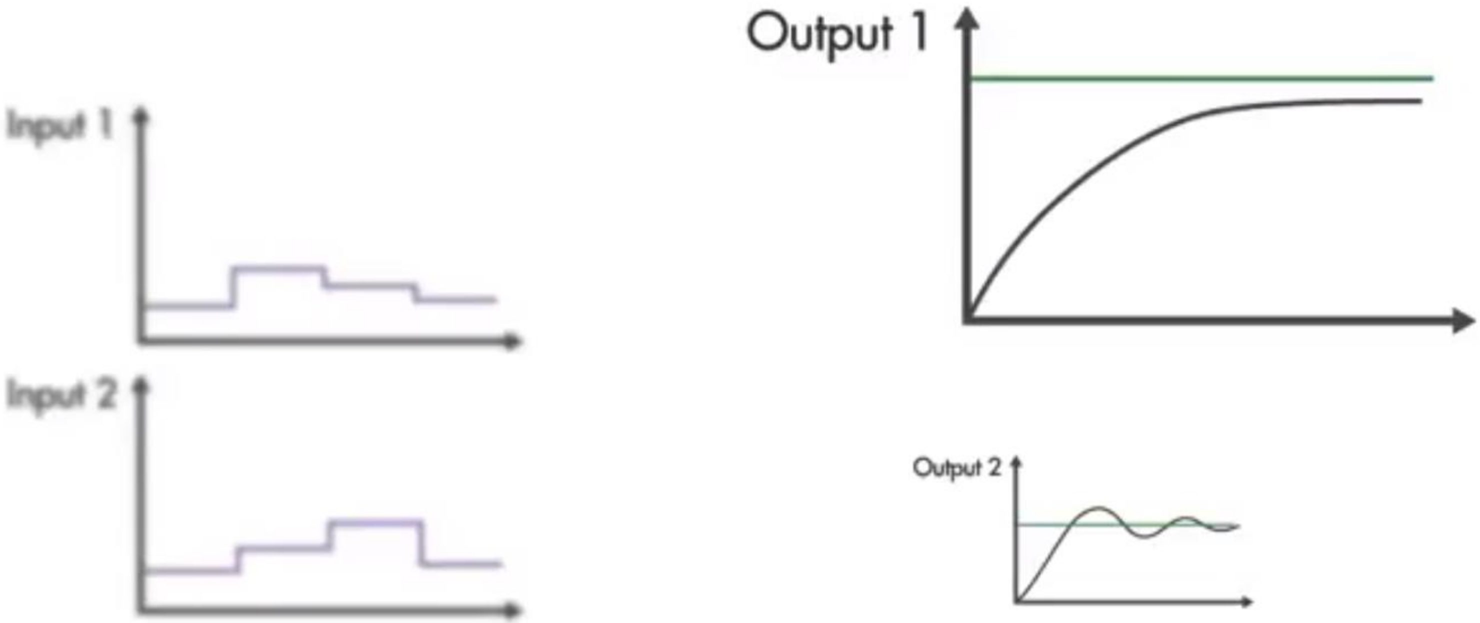
Basic parameters of the MPC: suitable control horizon



Basic parameters of the MPC: suitable constrains



Basic parameters of the MPC: weights



$$\frac{W_{output1}}{W_{output2}} > 1$$

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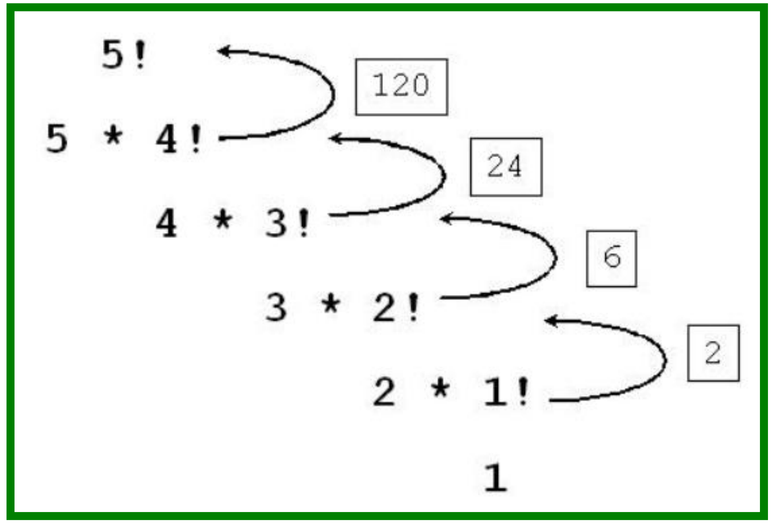
*: Not compulsory, not required.

MPC WITH STATE SPACE MODEL

Basic concepts of prediction

- The one-step ahead prediction can be used recursively to find an *n*-step ahead prediction as follows:



$$\begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k + d_k \end{cases}$$

$$\left.\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ x_{k+2} &= Ax_{k+1} + Bu_{k+1} \\ x_{k+3} &= Ax_{k+2} + Bu_{k+2} \\ x_{k+4} &= Ax_{k+3} + Bu_{k+3} \end{aligned}\right\} \Rightarrow$$

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ x_{k+2} &= A\left(Ax_k + Bu_k\right) + Bu_{k+1} \\ x_{k+3} &= A\left[A\left(Ax_k + Bu_k\right) + Bu_{k+1}\right] + Bu_{k+2} \\ x_{k+4} &= A\left\{A\left[A\left(Ax_k + Bu_k\right) + Bu_{k+1}\right] + Bu_{k+2}\right\} + Bu_{k+3} \end{aligned}$$

Compact output prediction notation

- Output predictions follow a similar method.

$$\underset{\rightarrow}{y}_{k+1} = \underbrace{\begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^n \end{bmatrix}}_{P} \cdot x_k + \underbrace{\begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{n-1}B & CA^{n-2}B & \cdots & CB \end{bmatrix}}_{H} \cdot \underbrace{\begin{bmatrix} u_{k|k} \\ u_{k+1|k} \\ \vdots \\ u_{k+n-1|k} \end{bmatrix}}_{\underset{\rightarrow}{u}_k} + \underbrace{\begin{bmatrix} d_k \\ d_k \\ \vdots \\ d_k \end{bmatrix}}_{Ld_k}$$

$$\Downarrow$$

$$\underset{\rightarrow}{y}_{k+1} = \left(P \cdot x_k + Ld_k\right) + H \cdot \underset{\rightarrow}{u}_k$$

Depends on **past**

Depends upon **decision variables**

MPC WITH CARIMA MODEL

Review of the CARIMA model based MPC

- The most common transfer function model with MPC is the so-called CARIMA model. $\underbrace{\left(a(z)\Delta\right)}_{A(z)} y_k = b(z)\Delta u_k$

$$a(z)\Delta y_k = b(z)\Delta u_k + \cancel{T(z)\varsigma_k} \Rightarrow a(z)\Delta y_k = b(z)\Delta u_k$$

$$\boxed{\underset{\rightarrow}{y}_{k+1} = C_A^{-1}C_b \cdot \Delta \underset{\rightarrow}{u}_k + \left(C_A^{-1}H_b \cdot \Delta \underset{\leftarrow}{u}_{k-1} - C_A^{-1}H_A \cdot \underset{\leftarrow}{y}_k\right)}$$

$$H = C_A^{-1}C_b \qquad P = C_A^{-1}H_b \qquad Q = C_A^{-1}H_A$$

$$\boxed{\underset{\rightarrow}{y}_{k+1} = H \cdot \Delta \underset{\rightarrow}{u}_k + \left(P \cdot \Delta \underset{\leftarrow}{u}_{k-1} - Q \cdot \underset{\leftarrow}{y}_k\right)}$$

Review of the CARIMA model based MPC: parameters of C_A, C_b, H_b, H_A

$$a(z)\Delta y_k = b(z)\Delta u_k \qquad A(z) = a(z)\Delta$$

$$\boxed{\underset{\rightarrow}{y}_{k+1} = C_A^{-1}C_b \cdot \Delta \underset{\rightarrow}{u}_k + \left(C_A^{-1}H_b \cdot \Delta \underset{\leftarrow}{u}_{k-1} - C_A^{-1}H_A \cdot \underset{\leftarrow}{y}_k \right)}$$

$$C_A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ A_1 & 1 & 0 & 0 \\ A_2 & A_1 & 1 & 0 \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$H_b = \begin{bmatrix} b_2 & b_3 & \cdots & b_{m-4} & b_{m-3} & b_{m-2} & b_{m-1} & b_m \\ b_3 & b_4 & \cdots & b_{m-3} & b_{m-2} & b_{m-1} & b_m & 0 \\ b_4 & b_5 & \cdots & b_{m-2} & b_{m-1} & b_m & 0 & 0 \\ b_5 & b_6 & \cdots & b_{m-1} & b_m & 0 & 0 & 0 \end{bmatrix}$$
$$\begin{matrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{matrix}$$

$$C_b = \begin{bmatrix} b_1 & 0 & 0 & \cdots \\ b_2 & b_1 & 0 & \cdots \\ b_3 & b_2 & b_1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$H_A = \begin{bmatrix} A_1 & A_2 & \cdots & A_{n-4} & A_{n-3} & A_{n-2} & A_{n-1} & A_n \\ A_2 & A_3 & \cdots & A_{n-3} & A_{n-2} & A_{n-1} & A_n & 0 \\ A_3 & A_4 & \cdots & A_{n-2} & A_{n-1} & A_n & 0 & 0 \\ A_4 & A_5 & \cdots & A_{n-1} & A_n & 0 & 0 & 0 \end{bmatrix}$$
$$\begin{matrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{matrix}$$

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07/11/2019	Topic 5	• Application of model predictive control in power inverter
14/11/2019	Topic 6	• Application of model predictive control in DC/DC converter

*: Not compulsory, not required.

MPC WITH IMPULSE MODEL

Relationship between the CARIMA & impulse response model based MPC

- CARIMA model: $a(z)\Delta y_k = b(z)\Delta u_k$
- Impulse response model: $1 \cdot \Delta y_k = h(z)\Delta u_k$

For CARIMA model:

$$\boxed{\underset{\rightarrow}{y}_{k+1} = C_A^{-1}C_b \cdot \Delta \underset{\rightarrow}{u}_k + \left(C_A^{-1}H_b \cdot \Delta \underset{\leftarrow}{u}_{k-1} - C_A^{-1}H_A \cdot \underset{\leftarrow}{y}_k\right)}$$

For impulse response model:

$$a(z) = 1$$
$$b(z) = h(z)$$

$$\boxed{\underset{\rightarrow}{y}_{k+1} = C_\Delta^{-1}C_h \cdot \Delta \underset{\rightarrow}{u}_k + \left(C_\Delta^{-1}H_h \cdot \Delta \underset{\leftarrow}{u}_{k-1} - C_\Delta^{-1}H_\Delta \cdot \underset{\leftarrow}{y}_k\right)}$$

$\boxed{C_A} \rightarrow \boxed{C_\Delta}$ $\boxed{C_b} \rightarrow \boxed{C_h}$ $\boxed{H_b} \rightarrow \boxed{H_h}$ $\boxed{H_A} \rightarrow \boxed{H_\Delta}$

Summary of the impulse response model based MPC

- Impulse response model:

$$\underset{\rightarrow}{y}_{k+1} = \underbrace{C_\Delta^{-1} C_h}_{H} \cdot \Delta \underset{\rightarrow}{u}_k + \left(\underbrace{C_\Delta^{-1} H_h}_{P} \cdot \Delta \underset{\leftarrow}{u}_{k-1} \underbrace{- C_\Delta^{-1} H_\Delta}_{Q} \cdot \underset{\leftarrow}{y}_k \right)$$

$$H = \begin{bmatrix} h_1 & 0 & 0 & 0 \\ h_1 + h_2 & h_1 & 0 & 0 \\ h_1 + h_2 + h_3 & h_1 + h_2 & h_1 & 0 \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

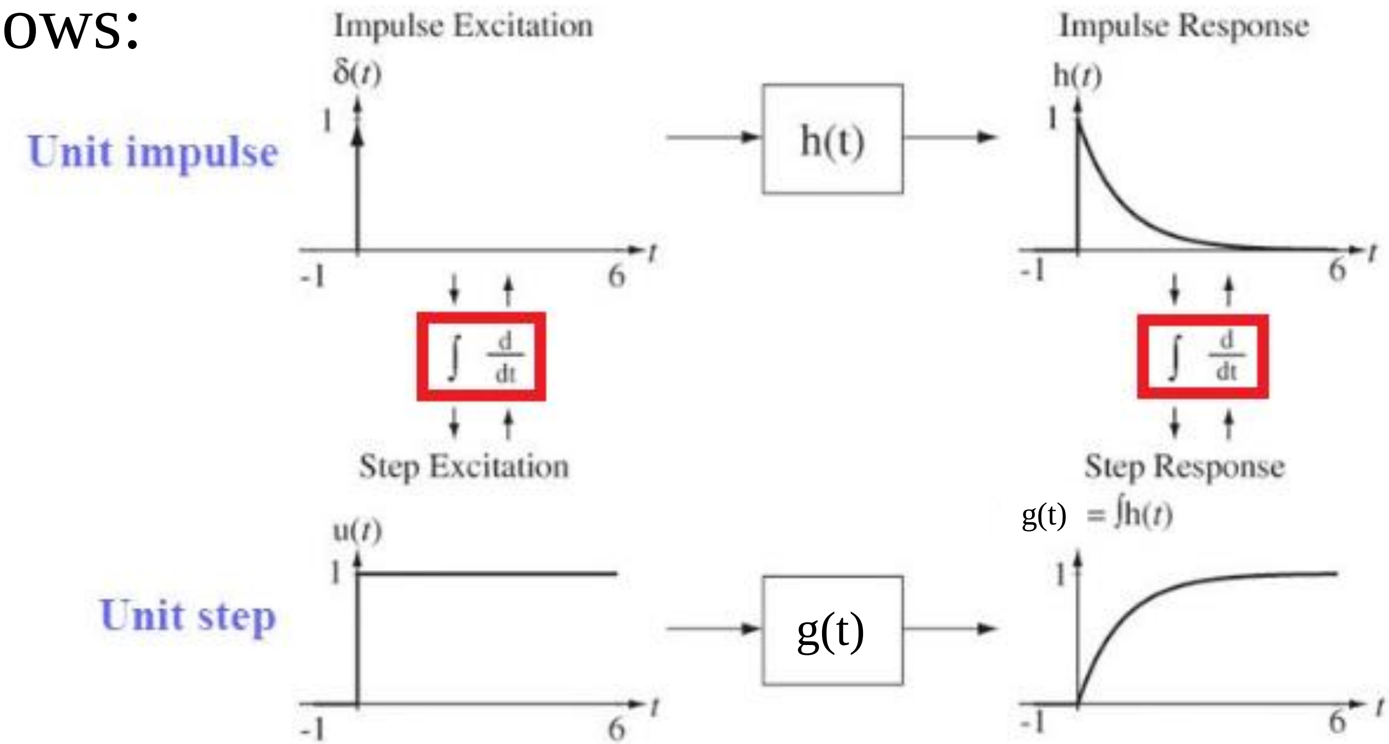
$$P = \begin{bmatrix} h_2 & h_3 & h_4 & \cdots \\ h_2 + h_3 & h_3 + h_4 & h_4 + h_5 & \cdots \\ h_2 + h_3 + h_4 & h_3 + h_4 + h_5 & h_4 + h_5 + h_6 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = L$$

MPC WITH STEP RESPONSE MODEL

Link step response to impulse response

- The step response and impulse response are linked as follows:



$$g_0 + g_1 z^{-1} + g_2 z^{-2} + \cdots = h_0 + [h_0 + h_1] z^{-1} + [h_0 + h_1 + h_2] z^{-2} + \cdots$$

$h(z)$ — Integration → $g(z)$

- Can build predictions if the impulse/step response coefficients are given.

$$\boxed{\underset{\rightarrow}{y}_{k+1} = H \cdot \Delta \underset{\rightarrow}{u}_k + P \cdot \Delta \underset{\leftarrow}{u}_{k-1} + L \cdot \underset{\leftarrow}{y}_k}$$

Impulse response:

$$H = \begin{bmatrix} h_1 & 0 & 0 & 0 \\ h_1 + h_2 & h_1 & 0 & 0 \\ h_1 + h_2 + h_3 & h_1 + h_2 & h_1 & 0 \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$P = \begin{bmatrix} h_2 & h_3 & h_4 & \cdots \\ h_2 + h_3 & h_3 + h_4 & h_4 + h_5 & \cdots \\ h_2 + h_3 + h_4 & h_3 + h_4 + h_5 & h_4 + h_5 + h_6 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Step response:

$$H = \begin{bmatrix} g_1 - g_0 & 0 & 0 & 0 \\ g_2 - g_0 & g_1 - g_0 & 0 & 0 \\ g_3 - g_0 & g_2 - g_0 & g_1 - g_0 & 0 \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$P = \begin{bmatrix} g_1 - g_0 & g_2 - g_1 & g_3 - g_2 & \cdots \\ g_2 - g_0 & g_3 - g_1 & g_4 - g_2 & \cdots \\ g_3 - g_0 & g_4 - g_1 & g_5 - g_2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$L = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

COST FUNCTION

Horizons and weights in J

- So far these slides have given the performance index in a very simplified form, using the same horizons for the inputs and outputs and scalar weights, a more generic form is as follows.

$$J=\sum_{k=1}^{n_y}\left\|W_y e_k\right\|_2^2+\sum_{k=1}^{n_u}\left\|W_u\left(u_k-u_{ss}\right)\right\|_2^2+\left\|R_u \Delta u_k\right\|_2^2$$

- Matrix weights on each term (W_y, W_u, R_u).

- Different horizons for inputs (n_y) and outputs (n_u).

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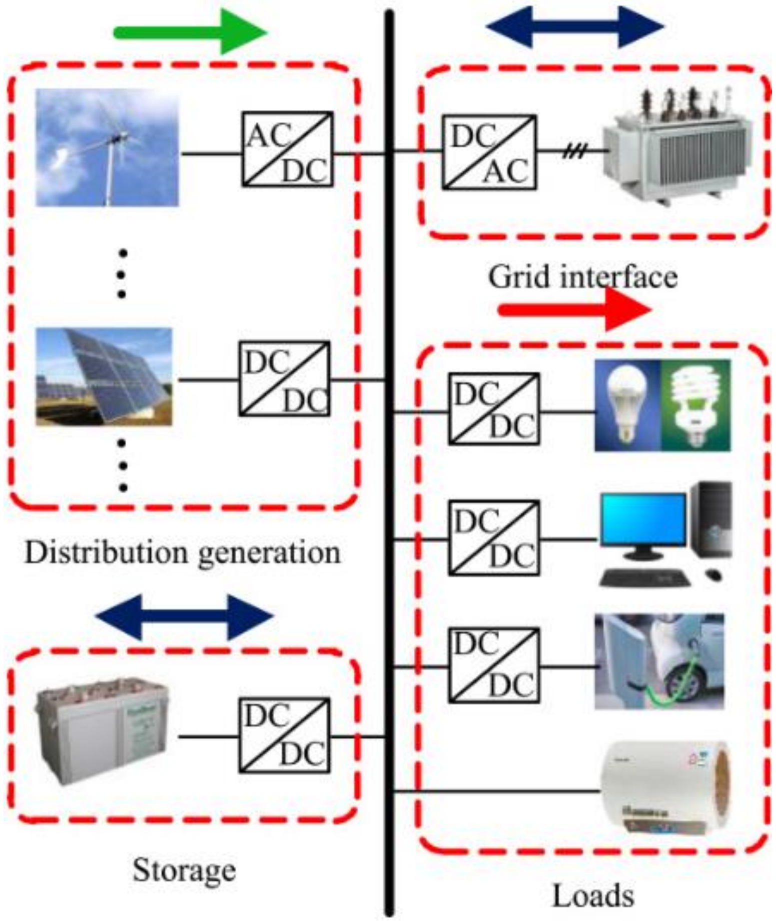
*: Not compulsory, not required.

Application of power converters



- ➢ Operation principle of the DC/DC converter
- ➢ Operation principle of the DC/AC inverter

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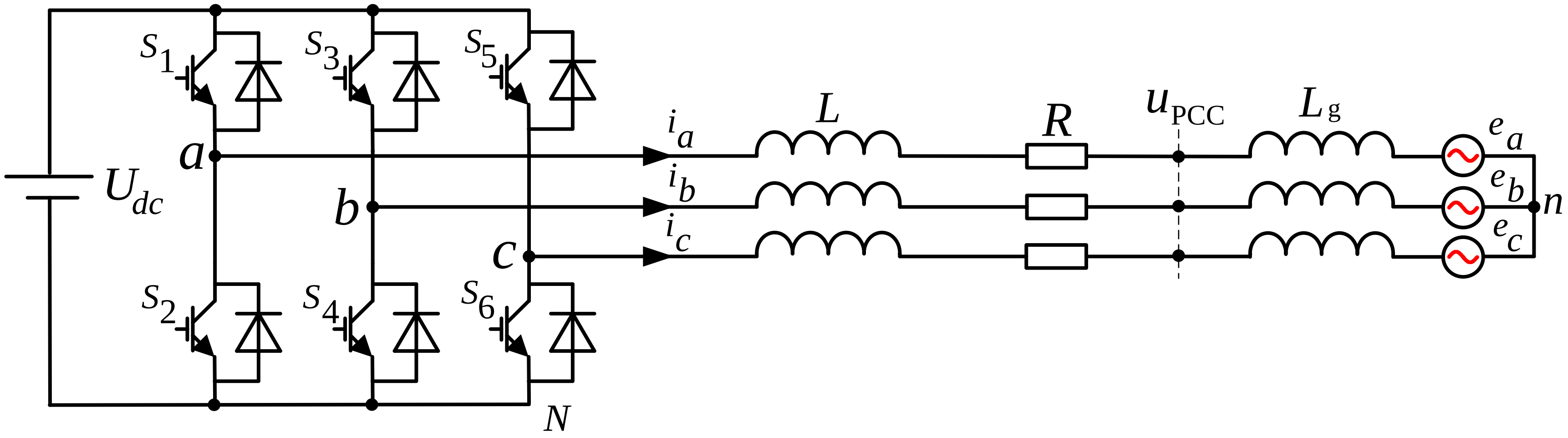
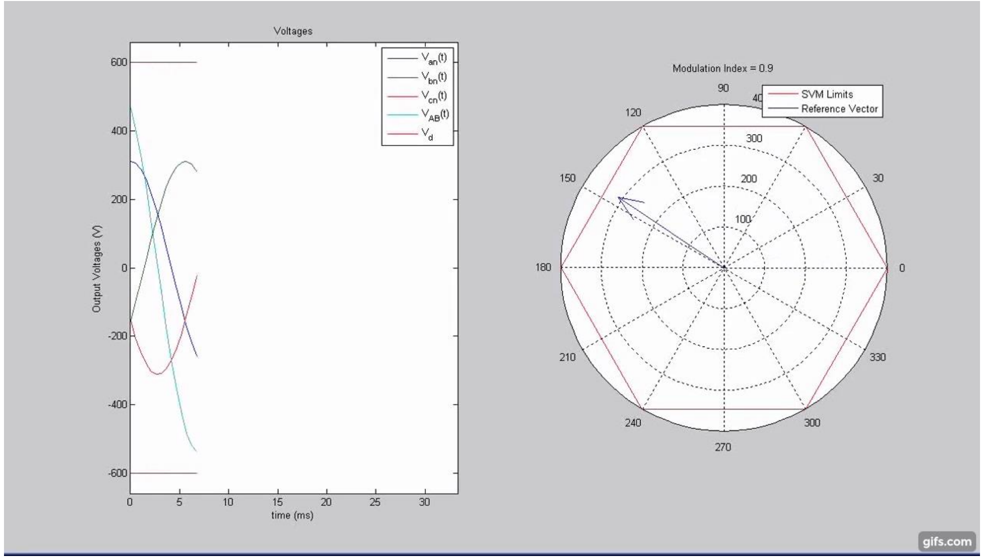
Conventional MPC for the grid connected inverter





Conventional MPC for the grid connected inverter

- Time-discrete load model:

  Euler-forwards approximation:

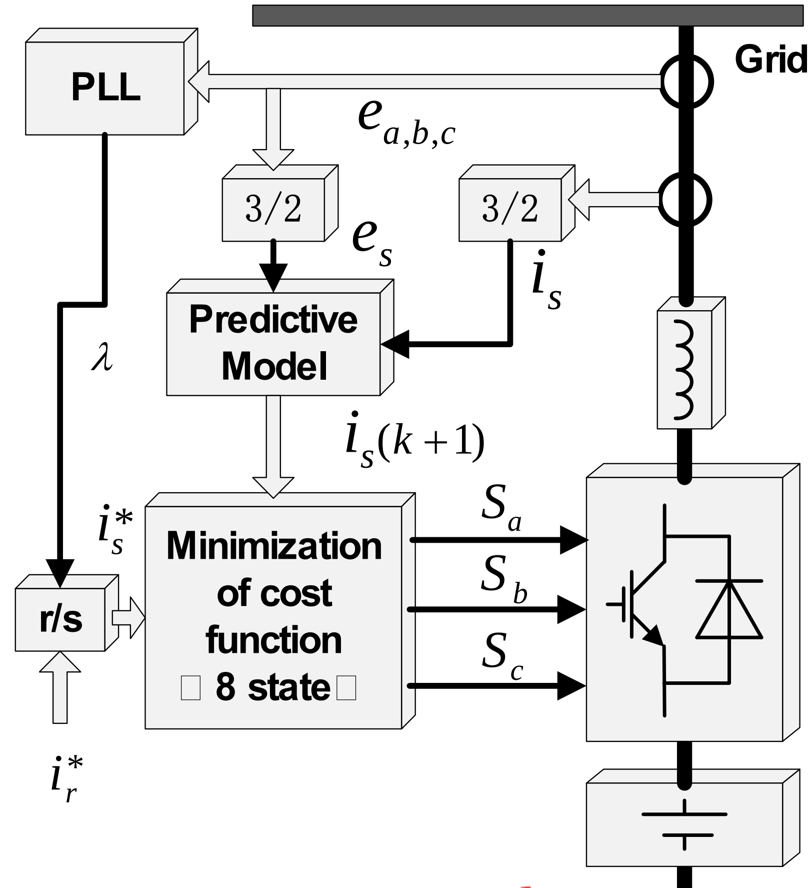
$$\frac{d\underline{i}}{dt} \approx \frac{\underline{i}(k+1) - \underline{i}(k)}{T_s}$$

- Current prediction:

$$\underline{i}(k+1) = \left(1 - \frac{RT_s}{L}\right)\underline{i}(k) + \frac{T_s}{L}\left(\underline{v}(k) - \underline{e}(k)\right)$$

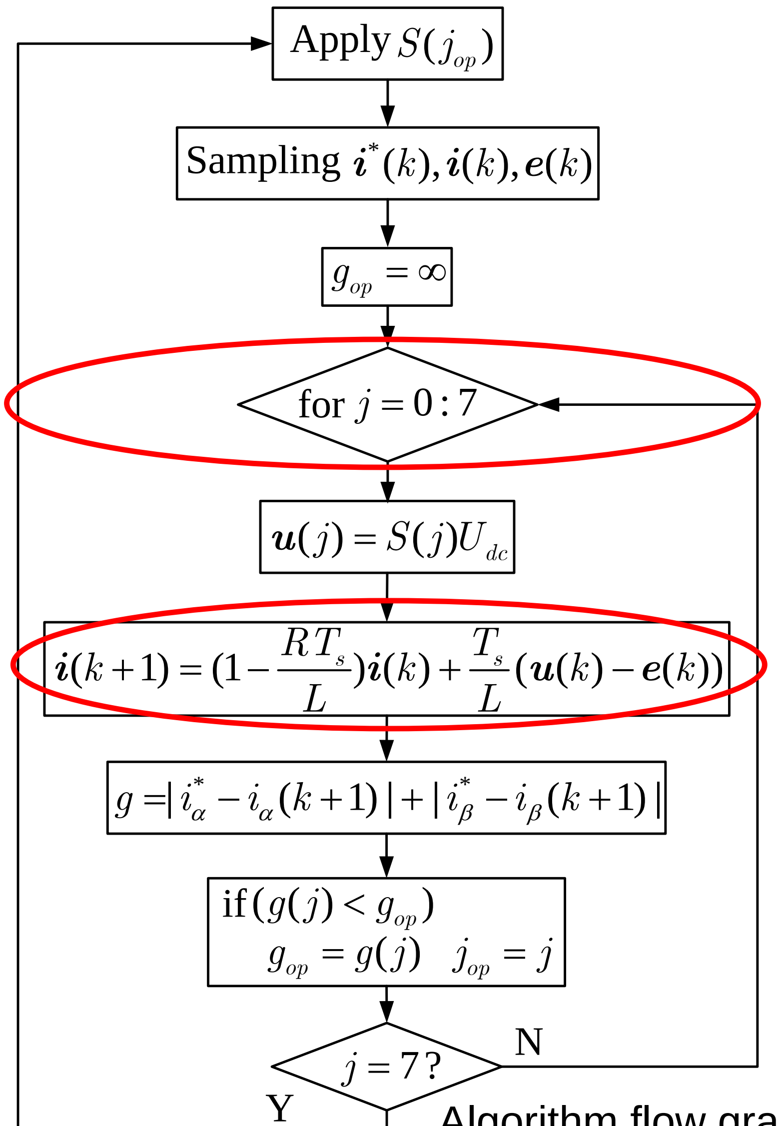
- Cost function:

$$G = \left|i_\alpha^* - i_\alpha(k+1)\right| + \left|i_\beta^* - i_\beta(k+1)\right|$$

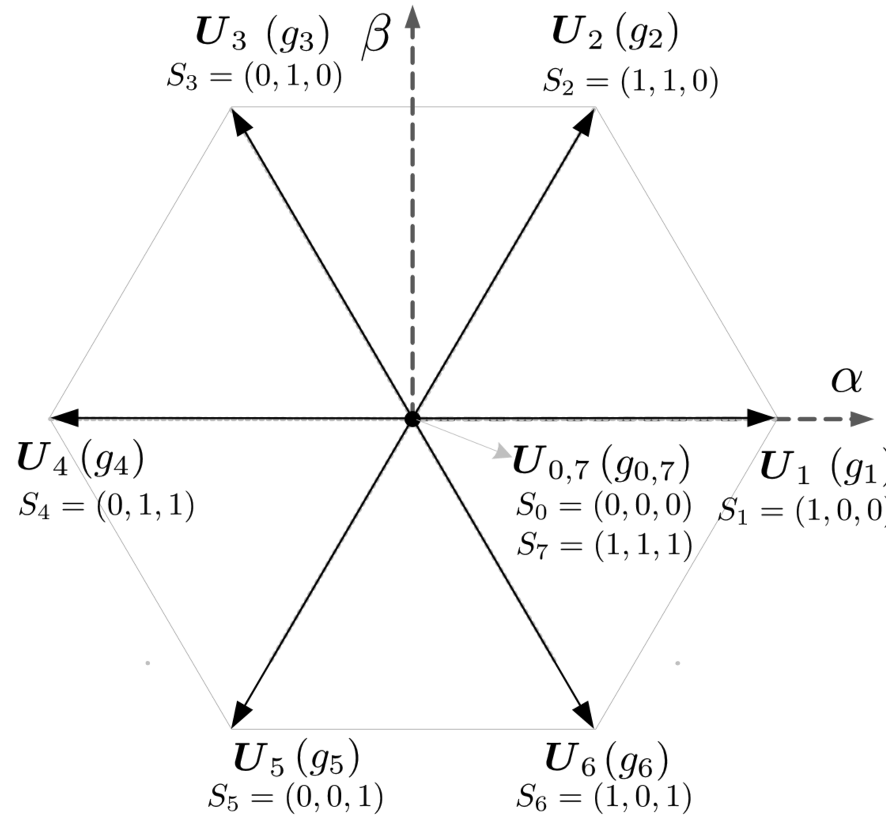


$$\underline{v} = R \cdot \underline{i} + L\frac{d\underline{i}}{dt} + \underline{e}$$

Conventional MPC for the grid connected inverter



Algorithm flow graph



Eight voltage vectors

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APPLICATION OF MPC IN DC/DC CONVERTERS

[14/11/2019]

- Traditional PID control method of DC/DC converter
- MPC method of DC/DC converter
- Overview: MPC application in power converters
- Design issues of FCS-MPC for the power converters

Thank you!