7. (i) $K = \begin{bmatrix} 22.89 & -11.64 \\ 4.689 & 5.8 \end{bmatrix}$, $K^{-T} = \begin{bmatrix} 0.0310 & -0.0250 \\ 0.0621 & 0.1222 \end{bmatrix}$

$$\Lambda = K \otimes K^{-T} = \begin{bmatrix} 0.7096 & 0.2904 \\ 0.2904 & 0.7096 \end{bmatrix}$$

$\Rightarrow$ the process is interactive, each manipulated variable affects both controlled variables.

(ii) poles: $P_1 = -\frac{1}{4.572} < 0$, $P_2 = -\frac{1}{1.801} < 0$.

$P_3 = -\frac{1}{2.174} < 0$, $P_4 = -\frac{1}{1.801} < 0$.

$\Rightarrow$ all poles lie on the left-half-plane (LHP), hence the process is open-loop stable.

(iii) $|K| = 187.34196$,

$\Rightarrow$ the inverse of gain matrix K exists, hence the process is controllable.

(iv) $e^{-Ls} \approx 1 - Ls$,

$$G(s) = \begin{bmatrix} \frac{22.89(1-0.2s)}{4.572s+1} & \frac{-11.64(1-0.4s)}{1.801s+1} \\ \frac{4.689(1-0.2s)}{2.174s+1} & \frac{5.8(1-0.4s)}{1.801s+1} \end{bmatrix}$$

$$|G(s)| = \frac{132.762(1-0.2s)(1-0.4s)}{(4.572s+1)(1.801s+1)} + \frac{54.5799(1-0.2s)(1-0.4s)}{(1.801s+1)(2.174s+1)} = 0$$

$\Rightarrow (132.762 - 79.6572s + 10.6210s^2)(2.174s+1) + (54.5799 - 32.7479s + 4.3664s^2)(4.572s+1) = 0$

$132.762 + 288.6246s - 79.6572s - 173.1748s^2 + 10.621s^2 + 23.0901s^3 + 54.5799 + 249.5393s - 32.7479s - 149.7234s^2 + 4.3664s^2 + 19.9632s^3 = 0$

$\Rightarrow 187.3419 + 425.7588s - 307.9108s^2 + 43.0533s^3 = 0$,

$\Rightarrow s_1 = 0.3481$, $s_2 = 4.9999$, $s_3 = 2.5000$

(V) Since there are zeros in the right-hand-plane, the system is not minimal phase.

(b) singular value: $\sigma_i = (\lambda_i(A^TA))^{\frac{1}{2}}$

condition number: $k = \frac{\sigma_{max}}{\sigma_{min}}$

$$A = \begin{bmatrix} 22.89 & -11.64 \\ 4.689 & 5.8 \end{bmatrix}, \quad A^T = \begin{bmatrix} 22.89 & 4.689 \\ -11.64 & 5.8 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 545.9388 & -239.2434 \\ -239.2434 & 169.1296 \end{bmatrix}$$

$$|(\lambda I - A^TA)| = \left| \begin{bmatrix} \lambda - 545.9388 & 239.2434 \\ 239.2434 & \lambda - 169.1296 \end{bmatrix} \right|$$

$$= (\lambda - 545.9388)(\lambda - 169.1296) - 57237.4044$$

$$= \lambda^2 - 715.0684\lambda + 35092.80274 = 0.$$

$\Rightarrow \lambda_1 = 662.0632.$

$\lambda_2 = 53.0052.$

$\sigma_1 = (\lambda_1)^{\frac{1}{2}} = 25.7306$

$\sigma_2 = (\lambda_2)^{\frac{1}{2}} = 7.2805$

$$k = \frac{\sigma_{max}}{\sigma_{min}} = 3.5341 < 10$$

$\Rightarrow$ the system is well-conditioned.

2. (a) $G_p(s) = \frac{k}{Ts+1} e^{-Ls} = \frac{Y(s)}{U(s)}$

$\Rightarrow Y(s) = G_p(s)U(s) = \frac{b_1}{s+a_1} e^{-Ls} U(s)$, $u(t) = h \cdot 1(t)$

$y(t) = -a_1 \int_0^t y(\tau)d\tau + hb_1(t-L)$

$= -a_1 \int_0^t y(\tau)d\tau - hb_1L + thb_1$

$$\begin{cases} \gamma(t) = y(t) \\ \phi(t) = [\ -\int_0^t y(\tau)d\tau \quad -h \quad th\ ]^T \\ \theta(t) = [\ a_1 \quad b_1L \quad b_1\ ]^T \end{cases}$$

$\Rightarrow \gamma(t) = \phi^T(t)\theta(t)$, $\Gamma = \Psi\theta$, $\theta = (\Psi^T\Psi)^{-1}\Psi^T\Gamma$.

$\begin{bmatrix} a_1 \\ b_1 \\ L \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_3 \\ \theta_2/\theta_3 \end{bmatrix}$, $b_1 = k/T$, $a_1 = 1/T$

$\Rightarrow \begin{bmatrix} 1/T \\ k/T \\ L \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_3 \\ \theta_2/\theta_3 \end{bmatrix}$

(b) $h=1$, $k=2$, $L=1.25s$, $P_u = 10s$, $a = 0.5$

$w_u = \frac{2\pi}{P_u} = 0.2\pi$. $K_u = \frac{4h}{a\pi} = \frac{8}{\pi}$,

gain condition: $K_u |G_p(jw_u)| = 1$

phase condition: $\arg[G_p(jw_u)] = -\pi$

$\Rightarrow$ $K_u \left| \frac{k}{jw_uT+1} e^{-jw_uL} \right| = 1$, $K_u k = \sqrt{(w_uT)^2+1}$, $k = \frac{\sqrt{(w_uT)^2+1}}{K_u}$

$\arg\left[\frac{k}{jw_uT+1} e^{-jw_uL}\right] = -\pi$, $-w_uL - \tan^{-1}(w_uT) = -\pi$,

$\tan^{-1}(w_uT) = \pi - w_uL$

$T = \frac{\tan(\pi - w_uL)}{w_u}$

$\Rightarrow T = \frac{\tan(\pi - 0.2\pi \times 1.25)}{0.2\pi} = 0.06549$,

$k = \frac{\sqrt{(0.2\pi \times 0.06549)^2+1}}{\frac{8}{\pi}} = 0.39303$,

2. (a) $G_p(s) = \dfrac{k}{Ts+1} e^{-Ls} = \dfrac{b_1}{s+a_1} e^{-Ls}$

$Y(s) = G_p(s)U(s)$, $u(t) = h \cdot 1(t)$,

$\Rightarrow \quad y(t) = -a_1 \int_0^t y(\tau)d\tau + hb_1(t-L)$

$\quad = -a_1 \int_0^t y(\tau)d\tau - hb_1L + htb_1$;

$$\begin{cases} \gamma(t) = y(t) ; \\ \phi(t) = \left[-\int_0^t y(\tau)d\tau \quad -h \quad ht \right]^T ; \\ \theta(t) = [a_1 \quad b_1L \quad b_1]^T \end{cases}$$

$\Rightarrow \quad \gamma(t) = \phi^T(t)\theta(t)$, $\Gamma = \Psi\theta \Rightarrow \hat{\theta} = (\Psi^T\Psi)^{-1}\Psi^T\Gamma$

$$\Rightarrow \begin{bmatrix} a_1 \\ b_1 \\ L \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_3 \\ \theta_2/\theta_3 \end{bmatrix}, \quad a_1 = 1/T, \quad b_1 = k/T.$$

$$\therefore \begin{bmatrix} 1/T \\ k/T \\ L \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_3 \\ \theta_2/\theta_3 \end{bmatrix}$$

(b) $h=1$, $a=0.5$, $k=2$, $P_u=10$, $L=1.25$

$\omega_u = \dfrac{2\pi}{P_u} = 0.2\pi$, $K_u = \dfrac{4h}{a\pi} = \dfrac{8}{\pi}$;

gain condition: $K_u|G_p(j\omega_u)| = 1$.

phase condition: $\arg[G_p(j\omega_u)] = -\pi$;

$K_u \left| \dfrac{k}{j\omega_u T+1} e^{-j\omega_u L} \right| = 1 \Rightarrow K_u k = \sqrt{(\omega_u T)^2+1} \Rightarrow k = \dfrac{\sqrt{(\omega_u T)^2+1}}{K_u}$;

$\arg\left[\dfrac{k}{j\omega_u T+1} e^{-j\omega_u L}\right] = -\pi \Rightarrow -\omega_u L - \tan^{-1}(\omega_u T) = -\pi$,

$\tan^{-1}(\omega_u T) = \pi - \omega_u L$, $T = \dfrac{\tan(\pi - \omega_u L)}{\omega_u}$

2 (a) $G_p(s) = \frac{K}{Ts+1} e^{-Ls} = \frac{b}{s+a} e^{-Ls}$, $b = \frac{K}{T}$, $a = \frac{1}{T}$

$G_p(s) = \frac{Y(s)}{U(s)}$, $y(t) + a\int_0^t y(\tau)d\tau = b\int_0^t u(\tau - L)d\tau$

$\Rightarrow y(t) = -a\int_0^t y(\tau)d\tau + hb(t-L)$

$= -a\int_0^t y(\tau)d\tau - hbL + bth$

$$\begin{cases} r(t) = y(t) \\ \phi(t) = [-\int_0^t y(\tau)d\tau \quad -h \quad th]^T, \quad r(t) = \phi^T(t)\theta(t) \\ \theta(t) = [a \quad bL \quad b]^T \end{cases}$$

$\Gamma = \begin{bmatrix} r(1) \\ r(2) \\ \vdots \\ r(N) \end{bmatrix}$, $\Psi = \begin{bmatrix} \phi(1) \\ \phi(2) \\ \vdots \\ \phi(N) \end{bmatrix}$, $\Gamma = \Psi\theta$, $\theta = (\Psi^T\Psi)^{-1}\Psi^T\Gamma$

$\begin{bmatrix} a \\ b \\ L \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_3 \\ \theta_2/\theta_3 \end{bmatrix}$

(b) $P_u = 10$, $h = 1$, $a = 0.5$, $L = 1.25$, $K_u = \frac{4h}{a\pi} = \frac{8}{\pi}$,

$w_u = \frac{2\pi}{P_u} = 0.2\pi$, $G_p(jw_u) = \frac{K}{jw_uT+1} e^{-jw_uL}$,

Gain condition: $K_u|G_p(jw_u)| = 1$

$K_uK = \sqrt{(Tw_u)^2+1}$, $K = \frac{1}{K_u}\sqrt{(Tw_u)^2+1}$

phase condition: $\arg[G_p(jw_u)] = -\pi$

$-w_uL - \tan^{-1}(w_uT) = -\pi$.

$w_uT = \tan(\pi - w_uL)$. $T = \frac{1}{w_u}\tan(\pi - w_uL)$

$$\tilde{g}_{c11}(s) = \frac{-1.3535(1-0.9560s)}{1+6.692s}, \quad \tilde{g}_+(s) = (1-0.9560s).$$

$$\tilde{g}_-(s) = \frac{-1.3535}{1+6.692s}, \quad q(s) = \tilde{g}_-^{-1}(s)f(s) = \frac{1+6.692s}{-1.3535}\frac{1}{1+\lambda s}.$$

$$\lambda = 0.8L = 0.7648, \quad \therefore q(s) = \frac{1+6.692s}{-1.3535}\frac{1}{1+0.7648s}$$

$$g_c(s) = \frac{q(s)}{1-\tilde{g}(s)q(s)}, \quad 1-\tilde{g}(s)q(s) = 1-\frac{1-0.9560s}{1+0.7648s}.$$

$$\Rightarrow g_c(s) = \frac{\frac{1}{-1.3535}\frac{1+6.692s}{1+0.7648s}}{\frac{1.7208s}{1+0.7648s}} = -0.7388\frac{1+6.692s}{1.7208s}$$

不是PID controller，应用 $e^{-Ls} \approx \frac{1-\frac{L}{2}s}{1+\frac{L}{2}s}$ 近似

controller design 2：

$$\hat{g}_{22}(s) = \frac{4.3e^{-0.35s}}{9.2s+1} \checkmark, \quad \tilde{g}_{c22}(s) = \frac{2.6455e^{-0.3346s}}{8.7952s+1} \times$$ 应须考虑 integrity

$$= \frac{4.3(1-0.175s)}{(1+9.2s)(1+0.175s)}, \quad \tilde{g}_+(s) = (1-0.175s), \quad \tilde{g}_-(s) = \frac{4.3}{(1+9.2s)(1+0.175s)}$$

$$q(s) = \tilde{g}_-^{-1}(s)f(s) = \frac{(1+9.2s)(1+0.175s)}{4.3(1+0.28s)} \quad (\lambda = 0.35\times 0.8 = 0.28$$

$$g_c(s) = \frac{q(s)}{1-\tilde{g}(s)q(s)}, \quad 1-\tilde{g}(s)q(s) = 1-\frac{1+0.175s}{1+0.28s}.$$

$$\Rightarrow g_c(s) = \frac{(1+9.2s)(1+0.175s)}{4.3[(1+0.28s)-(1+0.175s)]} = \frac{1+9.375s+1.61s^2}{0.4515s}$$

$$= K_c\left(1+\frac{1}{T_i s}+T_d s\right);$$

(C) $$G_R(s) = \begin{bmatrix} \frac{-2.2e^{-s}}{7s+1} & 0 \\ 0 & \frac{4.3e^{-0.35s}}{9.2s+1} \end{bmatrix}, \quad \hat{G}^T = \begin{bmatrix} \frac{-2.2e^{-s}}{7s+1} & \frac{-2.8e^{-1.8s}}{9.5s+1} \\ \frac{1.3e^{-0.35s}}{7s+1} & \frac{4.3e^{-0.35s}}{9.2s+1} \end{bmatrix}$$

$$G_I(s) = G^{-1}(s)G_R(s) = \hat{G}^T(s)G_R(s) =$$

$$= \begin{bmatrix} 1 & \frac{-1.5357(9.5s+1)}{(9.2s+1)}e^{1.45s} \\ -1.6923e^{-0.7s} & 1 \end{bmatrix}$$

3. (a) $K = G(0) = \begin{bmatrix} -2.2 & 1.3 \\ -2.8 & 4.3 \end{bmatrix}$, $K^{-1} = \dfrac{\begin{bmatrix} 4.3 & -1.3 \\ 2.8 & -2.2 \end{bmatrix}}{-5.82}$

$= \begin{bmatrix} -0.7388 & 0.2234 \\ -0.4811 & 0.3780 \end{bmatrix}$.

$\Lambda = K \otimes K^{-T} = \begin{bmatrix} 1.6254 & -0.6254 \\ -0.6254 & 1.6254 \end{bmatrix}$,

select 1-1/2-2 pairing. $NI = \dfrac{|G(0)|}{g_{11}(0) \cdot g_{22}(0)} = \dfrac{-5.82}{-2.2 \times 4.3} = 0.6152 > 0$

∴ closed-loops are stable.

Controller design 1: for $g_{11}(s)$: $K_c^* = \dfrac{K_c}{\lambda} = -1.7618$

$K_c = \dfrac{0.9T}{KL} = \dfrac{0.9 \times 7}{-2.2 \times 1} = -2.8636$, $T_i = \dfrac{L}{0.3} = 3.3333$.

$\Rightarrow g_c(s) = K_c(1 + \dfrac{1}{T_i s}) = \cancel{-2.8636}(1 + \dfrac{1}{3.3333s})$;

$K_c^* = \dfrac{K}{\lambda} = -1.7618$

Controller design 2: for $g_{22}(s)$:

$K_c = \dfrac{0.9T}{KL} = \dfrac{0.9 \times 9.2}{4.3 \times 0.35} = 5.5017$, $T_i = \dfrac{L}{0.3} = 1.1667$.

$\Rightarrow g_c(s) = K_c(1 + \dfrac{1}{T_i s}) = \cancel{5.5017}(1 + \dfrac{1}{1.1667s})$; $K_c^* = \dfrac{K}{\lambda} = -3.3848$

(b)

$K_N = \begin{bmatrix} \dfrac{-2.2}{7+1} & \dfrac{1.3}{7+0.3} \\ \dfrac{-2.8}{9.5+1.8} & \dfrac{4.3}{9.2+0.35} \end{bmatrix} = \begin{bmatrix} -0.275 & 0.1781 \\ -0.2478 & 0.4503 \end{bmatrix}$.

$K_N^{-1} = \dfrac{\begin{bmatrix} 0.4503 & -0.1781 \\ 0.2478 & -0.275 \end{bmatrix}}{-0.07969} = \begin{bmatrix} -5.6506 & 2.2349 \\ -3.1095 & 3.4509 \end{bmatrix}$.

$\Lambda_N = K_N \otimes K_N^{-T} = \begin{bmatrix} 1.5539 & -0.5539 \\ -0.5539 & 1.5539 \end{bmatrix}$, $\hat{K} = K \odot \Lambda = \begin{bmatrix} -1.3535 & -2.0787 \\ 4.4771 & 2.6455 \end{bmatrix}$

$\Gamma = \Lambda_N \odot \Lambda = \begin{bmatrix} 0.9560 & 0.8857 \\ 0.8857 & 0.9560 \end{bmatrix}$. $\hat{T} = T \otimes \Gamma = \begin{bmatrix} 6.692 & 6.1999 \\ 8.4141 & 8.7952 \end{bmatrix}$.

$\hat{L} = L \otimes \Gamma = \begin{bmatrix} 0.9560 & 0.2657 \\ 1.5943 & 0.3346 \end{bmatrix}$;

Controller design 1: $\tilde{g}_{c11}(s) = \dfrac{-1.3535 e^{-0.9560s}}{6.692s + 1} = \dfrac{-1.3535(1 - 0.9560s)}{6.692s + 1}$

(c) $G_I = G^{-1} G_R = \hat{G}^T G_R$

$$\hat{G}^T(s) = \begin{bmatrix} \dfrac{0.692s+1}{-1.3535} e^{0.956s} & \dfrac{8.4142s+1}{4.4771} e^{1.5943s} \\ \dfrac{0.1999s+1}{-2.0787} e^{0.2657} & \dfrac{8.7952s+1}{2.6455} e^{0.3346s} \end{bmatrix}$$

$$G_R(s) = \begin{bmatrix} \dfrac{2.0787 e^{-0.956s}}{0.692s+1} & 0 \\ 0 & \dfrac{4.4771 e^{-1.5943s}}{8.7952s+1} \end{bmatrix}$$

$$G_I(s) = \hat{G}^T(s) G_R(s)$$

$$= \begin{bmatrix} -1.5358 & \dfrac{8.4142s+1}{8.7952s+1} \\ (-1)\dfrac{0.1999s+1}{0.692s+1} e^{-0.6903s} & 1.6923 e^{-1.2597s} \end{bmatrix}$$

3. (a)

$$K = G(0) = \begin{bmatrix} -2.2 & 1.3 \\ -2.8 & 4.3 \end{bmatrix}, \quad K^{-T} = \begin{bmatrix} -0.7388 & -0.4811 \\ 0.2234 & 0.3780 \end{bmatrix}$$

$$\Lambda = K \otimes K^{-T} = \begin{bmatrix} 1.6254 & -0.6254 \\ -0.6254 & 1.6254 \end{bmatrix}, \quad \text{select 1-1/2-2 pairing.}$$

$$NI = \frac{|G(0)|}{g_{11} \times g_{22}} = \frac{-5.82}{-9.46} = 0.6152 > 0.$$

Controller design:

$K_{c1}^{*} = (\lambda - \sqrt{\lambda^2 - \lambda})K = 0.6172K = -1.7673$

$$K_{c1} = \frac{0.9T}{KL} = \frac{0.9 \times 7}{-2.2 \times 1} = -2.8636, \quad \underline{K_{c1}^{*} = \frac{K}{\lambda}} = \underline{-1.7618}$$

$$T_i = \frac{L}{0.3} = \frac{1}{0.3} = 3.3333 \Rightarrow g_{c11} = K_c\left(1 + \frac{1}{T_i s}\right) = -\underline{1.7618}\left(1 + \frac{1}{3.3333s}\right)$$

$$K_{c2} = \frac{0.9T}{KL} = \frac{0.9 \times 9.2}{4.3 \times 0.35} = -5.5017, \quad \underline{K_{c2}^{*} = \frac{K}{\lambda}} = -\underline{3.3848}, -3.3955$$

$$T_i = \frac{L}{0.3} = \frac{0.35}{0.3} = 1.1667 \Rightarrow g_{c22} = -\underline{3.3848}\left(1 + \frac{1}{1.1667s}\right)$$

(b)
$$K_N = \begin{bmatrix} -0.275 & 0.1781 \\ -0.2478 & 0.4503 \end{bmatrix}, \quad \Lambda_N = \begin{bmatrix} 1.5539 & -0.5539 \\ -0.5539 & 1.5539 \end{bmatrix},$$

$$\hat{K} = \begin{bmatrix} -1.3535 & -2.0787 \\ 4.4771 & 2.6453 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0.9560 & 0.8857 \\ 0.8857 & 0.9560 \end{bmatrix}, \quad \hat{\Gamma} = \begin{bmatrix} 0.692 & 0.1999 \\ 8.4141 & 8.7952 \end{bmatrix}$$

$$\hat{g}_{11} = \frac{-2.2e^{-s}}{7s+1}, \quad \hat{g}_{22}(s) = \frac{4.3e^{-0.35s}}{9.2s+1}, \quad e^{-s} = \frac{1 - \frac{L}{2}s}{1 + \frac{L}{2}s}$$

$$= \frac{-2.2(1-0.5s)}{(7s+1)(1+0.5s)}, \quad \tilde{g}_{11+} = (1-0.5s), \quad \tilde{g}_{11-} = \frac{-2.2}{(7s+1)(1+0.5s)}$$

$$\tilde{q} = \tilde{g}_{11-}^{-1} = \frac{-(7s+1)(1+0.5s)}{2.2}, \quad q = \tilde{q}f = \frac{-(7s+1)(1+0.5s)}{2.2(1+0.8s)}, \quad \lambda = 0.8L$$

$$g_c(s) = \frac{q}{1 - \tilde{g}_{11}q}, \quad 1 - \tilde{g}_{11}q = 1 - \frac{-2.2(1-0.5s)}{\cancel{(7s+1)}\cancel{(1+0.5s)}} \times \frac{-\cancel{(7s+1)}\cancel{(1+0.5s)}}{\cancel{2.2}(1+0.8s)}$$

$$= 1 - \frac{1-0.5s}{1+0.8s} = \frac{1.3s}{1+0.8s}$$

$$g_c(s) = \frac{-(7s+1)(1+0.5s)}{2.2\cancel{(1+0.8s)}} \cdot \frac{\cancel{1+0.8s}}{1.3s} = -\frac{1}{2.86s}(7s+1)(1+0.5s)$$

$$= -0.3497\frac{7s + 3.5s^2 + 1 + 0.5s}{s}$$

$$= -0.3497\left(3.5s + \frac{1}{s} + 7.5\right)$$

$$= -2.6227\left(1 + 0.1333\frac{1}{s} + 0.4667s\right)$$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2017-2018

EE6225 – PROCESS CONTROL

November/December 2017 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 5 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

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1. The transfer function matrix of a Two-Input Two-Output (TITO) Industrial-scale Polymerization Reactor (IPR) is given as:

$$G(s)=\begin{bmatrix} \dfrac{22.89}{4.572s+1}e^{-0.2s} & \dfrac{-11.64}{1.801s+1}e^{-0.4s} \\ \dfrac{4.689}{2.174s+1}e^{-0.2s} & \dfrac{5.8}{1.801s+1}e^{-0.4s} \end{bmatrix} \quad (1)$$

(a) Answer the following questions with justifications.

i) Is the process interactive?
ii) Is the process open loop stable?
iii) Is the process controllable at steady state?
iv) Find the system poles and zeros.
v) Is the process minimal phase?

(10 Marks)

(b) For the transfer function matrix given in Equation (1), determine the singular values and the condition number. Is the system well-conditioned?

(10 Marks)

2. Assume that a process can be represented by a first-order-plus-delay model

$$G_p(s) = \frac{K}{Ts+1}e^{-Ls} \quad = \frac{b_1}{s+a_1}e^{-Ls} \tag{2}$$

(a) Describe the least squares algorithm to find the parameters K, T and L based on a step test.

(10 Marks)

(b) From a sustained oscillation generated by a relay feedback as shown in Figure 1, find the process parameters T and L in Equation (2) assuming that $K=2$ has already been determined.

(10 Marks)

h=1

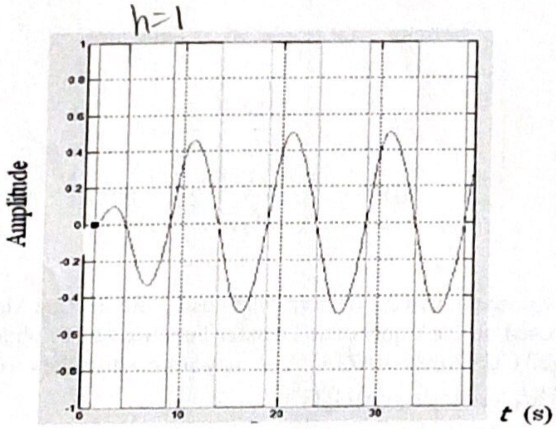

Figure 1

3. A multivariable process is represented by the following transfer function matrix:

$$\mathbf{G}(s)=\begin{bmatrix} \dfrac{-2.2e^{-s}}{7s+1} & \dfrac{1.3e^{-0.3s}}{7s+1} \\ \dfrac{-2.8e^{-1.8s}}{9.5s+1} & \dfrac{4.3e^{-0.35s}}{9.2s+1} \end{bmatrix}.$$

(a) Design two decentralized PI controllers using the Relative Gain Array (RGA) detuning method based on the Z-N tuning rules given in Table 1.

$$K_c^* = \begin{cases} (\lambda-\sqrt{\lambda^2-\lambda})K_c, & \lambda>1.0 \\ (\lambda+\sqrt{\lambda^2-\lambda})K_c & \lambda<1.0 \end{cases}$$

Table 1

Controller	Parameters		
	K_c	T_i	T_d
P	$\dfrac{T}{KL}$		
PI	$\dfrac{0.9T}{KL}$	$\dfrac{L}{0.3}$	
PID	$\dfrac{1.2T}{KL}$	$2L$	$0.5L$

(5 Marks)

(b) Design two decentralized PID controllers using the Internal Model Control (IMC) method based on the Equivalent Transfer Function (ETF) obtained from Relative Normalized Gain Array (RNGA). You may assume that the IMC tuning parameters are $\lambda_i = 0.8L_i$ for the diagonal ETFs.

(10 Marks)

(c) Design Normalized Decouplers for $\mathbf{G}(s)$ using the ETF matrix method.

(5 Marks)

4. (a) In predictive control, a sequence of future control signals: $u(k), u(k+1), u(k+2), \ldots,$ are computed at time k. At the next sampling instant $k+1$, why are these previously computed control signals not used, but instead a new sequence of control signals $u(k+1), u(k+2), \ldots,$ are computed again at time $k+1$? More specifically, why is $u(k+1)$ computed at time k not used at time $k+1$, but a new $u(k+1)$ is computed at time $k+1$?

(8 Marks)

(b) Consider the state space model of a single-input single-output system

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k)$$

where $k = 0, 1, 2, \ldots,$ is the time index, x, y and u are the state, output and input of the system respectively, and A, B, C are matrices of appropriate dimensions.

The system is subject to the following constraints:

$$-d \le y(1) \le d$$
$$-e \le y(1) + y(4) \le e$$
$$-f \le u(2) \le f$$

where d, e and f are constant parameters. Show that the constraints can be expressed in the form

$$EU \le h + Mx(k)$$

where $U = \begin{bmatrix} u(0) & u(1) & u(2) & u(3) \end{bmatrix}^T$.

Derive the expressions for the matrices E and M, and the vector h in terms of the information given.

(12 Marks)