PART II

PARAMETER IDENTIFICATION FROM STEP RESPONSE

2.1 THEORETICAL STEP-RESPONSE EXPRESSIONS

A simple process model can be obtained from a transient response experiment. In such a test, a step input is injected at the process input, and the response is measured. To perform such an experiment, ***the process must be stable***. If the process includes an integrator, a pulse test may be used (will be discussed in Part IV). It is prerequisite that the process be in equilibrium when the experiment starts. There is, in principle, only one parameter that must be set a *pripori*, namely the ***amplitude of step*** signal.

The amplitude of step signal is a delicate choice. It should be chosen *sufficiently large*, so that the *response is easily visible* above the noise level. On the other hand, it should be *as small as possible* in order *not to disturb the process* more than necessary and to keep the dynamics linear.

Many methods can be used to extract process characteristics from a transient response experiment.

- The static gain can be accurately determined from a step-response experiment by comparing the stationary values of the control signal and the measurement signal before and after the step change.
- The dead time can be obtained from the starting time difference of set point change and output response
- The constant and dead time can also be obtained in several other ways.

The main advantage of the methods is that they require little prior knowledge. It is also easy to explain the methods to plant operators. The main drawback with the transient response methods is that they are sensitive to disturbances.

The theoretical step responses for the typical candidate models given above are now listed below along with the parameters that must be estimated. In each case, the magnitude of the step size is *A*.

1. First-order-plus-time-delay

$$G_p(s) = \frac{K}{\tau s + 1} e^{-Ls}$$

$$y(t) = \begin{cases} 0 & \text{t} < L \\ AK(1 - e^{-(t-L)/\tau}); & \text{t} \ge L \end{cases}$$

Parameters: K, τ, L.

2. Second-order-plus-time-delay

$$G_p(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} e^{-Ls}$$

$$y(t)=\begin{cases}0 & t<L\\ AK\left[(1-\left(\frac{\tau_1}{\tau_1-\tau_2}\right)e^{-(t-L)/\tau 1}-\left(\frac{\tau_2}{\tau_2-\tau_1}\right)e^{-(t-L)/\tau 2}\right]; & t\ge L\end{cases}$$

Parameters: K, τ_1, τ_2, L.

3. Single zero, two poles, plus time delay

$$G_p(s)=\frac{K(\xi s+1)}{(\tau_1 s+1)(\tau_2 s+1)}e^{-Ls}$$

$$y(t)=\begin{cases}0 & t<L\\ AK\left[(1-\left(\frac{\tau_1-\xi}{\tau_1-\tau_2}\right)e^{-(t-L)/\tau 1}-\left(\frac{\tau_2-\xi}{\tau_2-\tau_1}\right)e^{-(t-L)/\tau 2}\right]; & t\ge L\end{cases}$$

Parameters: K, τ_1, ξ, τ_2, L.

The time delays, time constants, and steady-state gains in these models are estimated by matching these expressions to the experimentally observed $y(t)$ data.

2.2 CLASSICAL METHODS FOR OPEN LOOP STEP TEST

Because step testing is used mostly to fit first-order-plus-time-delay models to process data; several methods will be given to show how the three parameters, K, τ and L, contained in this model may be estimated.

The theoretical step response for times greater than the time delay may be rewritten as:

$$y(t)=y_\infty(1-e^{-(t-L)/\tau})$$

where y_∞ represents the ultimate value of the response.

Given data $y(k)$, the value of the process output observed at t_k, then we have:

$$y(k)\approx y_\infty(1-e^{-(t_k-L)/\tau}) \tag{2.2.1}$$

2.2.1 LOG METHOD

A straightforward technique for estimating the unknown parameters is as follows:

Steady-State Gain, K: From the ultimate value of y_∞ and the magnitude of the given input step function A, we have:

$$y_\infty = AK$$

K, an estimate of the steady-state gain, is:

$$K=\frac{y_\infty}{A} \tag{2.2.2}$$

Time Constant τ and Time Delay L: The solution is nonlinear in the parameters Constant τ and L, but it can be rearranged as:

$$\frac{y_\infty - y}{y_\infty} = e^{-(t_k - L)/\tau} \tag{2.2.3a}$$

and:

$$\ln\left(\frac{y_\infty - y}{y_\infty}\right) = \frac{L}{\tau} - \frac{t}{\tau} \tag{2.2.3b}$$

Thus, a plot of the transformation of the step-response data against time t should be a straight line with a slope of $-1/\tau$, and an intercept on the ordinate of L/τ. Also, this straight line will meet the t-axis at the point $t = L$. We may therefore obtain estimates of these parameters graphically.

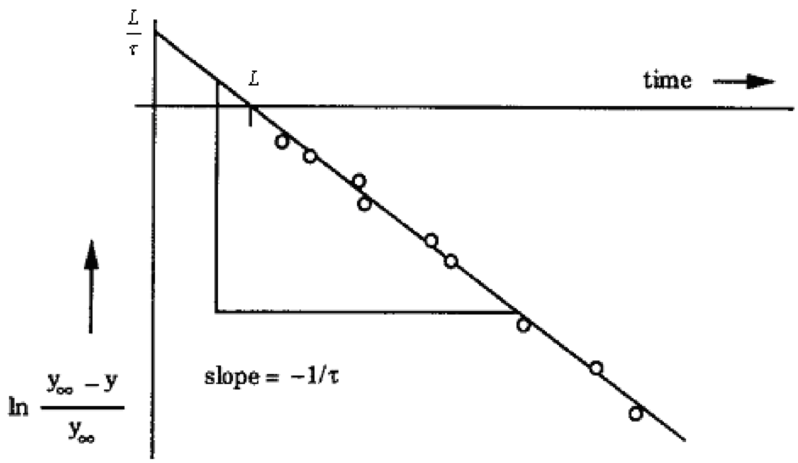

Fig. 2.2.1 Estimating time constant and time delay from step-response data.

2.2.2 TWO POINTS METHOD

Suppose the process can be approximated by the model

$$G_p(s) = \frac{K}{Ts+1}e^{-Ls}$$

with amplitude A occurs in the process input, the process response can be described by

$$y(t) = KA(1 - e^{-\frac{t-L}{T}})$$

The normally used $y(t)$ and time t are shown below

$y(\infty)$%	28.4	39.3	55	59.3	63.2	77.7	86.5
time(t)	$T/3+L$	$T/2+L$	$0.8T+L$	$0.9T+L$	$T+L$	$1.5T+L$	$2T+L$

After determining time t_1 and t_2, which are the time when the process step response has reached the value of 28.4% and 63.2% of the steady state value, respectively. Then the process time constant and dead time can be determined by

$$T = 1.5(t_2 - t_1) \tag{2.2.4a}$$

$$L = 0.5(3t_1 - t_2) \tag{2.2.4b}$$

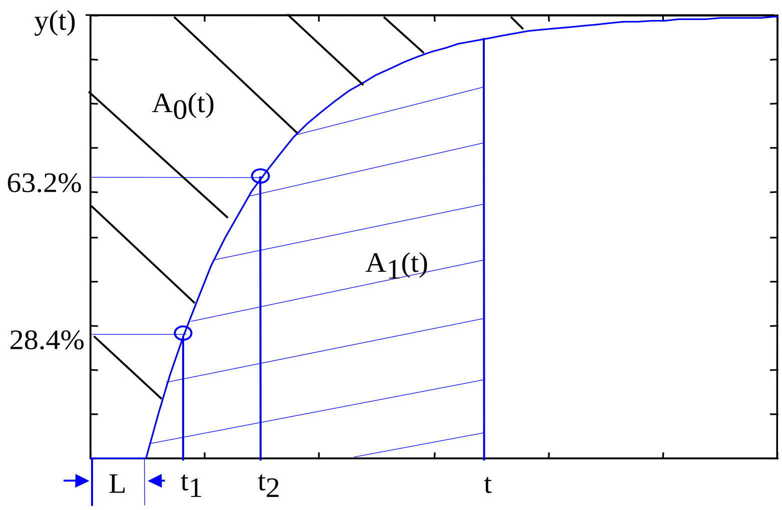

Fig. 2.2.2 Two point and Area Methods

2.2.3 AREA METHOD

Suppose that the static gain has been determined as before, and the average residence time T_{ar} is computed form the area of A_0

$$\begin{aligned} T_{ar} &= \frac{A_0}{K} = \frac{\int_0^{\infty}[y(\infty) - y(t)]dt}{K} \\ &= \frac{\int_0^{L} y(\infty)dt + \int_L^{\infty}[y(\infty) - y(t)]dt}{K} \\ &= \frac{\int_0^{L} Kdt + K\int_L^{\infty}[1-(1-e^{-(t-L)/T})]dt}{K} = T + L \end{aligned} \tag{2.2.5}$$

The area A_1 under the step up to the time T_{ar} is also measured.

The area A_1 can be computed by

$$A_1 = \int_0^{T_{ar}} y(t)dt = \int_0^{T} K[1-e^{-t/T}]dt = KTe^{-1} \tag{2.2.6}$$

Than T and L can be estimated as

$$\begin{aligned} T &= \frac{eA_1}{K} = \frac{e\int_0^{Tar} y(t)dt}{K} \\ L &= T_{ar} - T = \frac{A_0}{K} - \frac{eA_1}{K} \end{aligned} \tag{2.2.7}$$

The method is less sensitive to high frequency noise compare with the above two methods with which the model is determined from only a few values of the step response. However, the estimation accuracy mainly depends on the A_0. In order to have accurate A_0, One needs the testing span long enough until the process well enters the new steady state again.

Example 2.2.1: Stirred Tank

The process reaction experiment has been performed on the stirred tank process shown in Figure 2.2.3, the input and system reaction curves are given in Figures 2.2.4. Using the two points method determine parameters *K*, *T* and *L* for the first-order-plus-delay model

$$G_p(s) = \frac{K}{Ts+1}e^{-Ls}$$

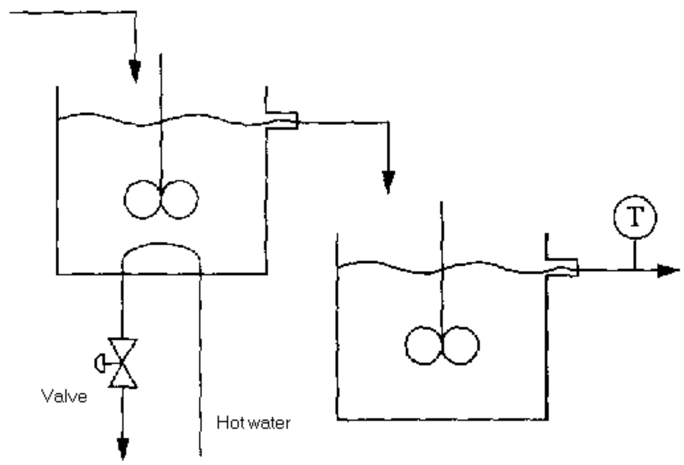

Figure 2.2.3 Stirred Tank Process

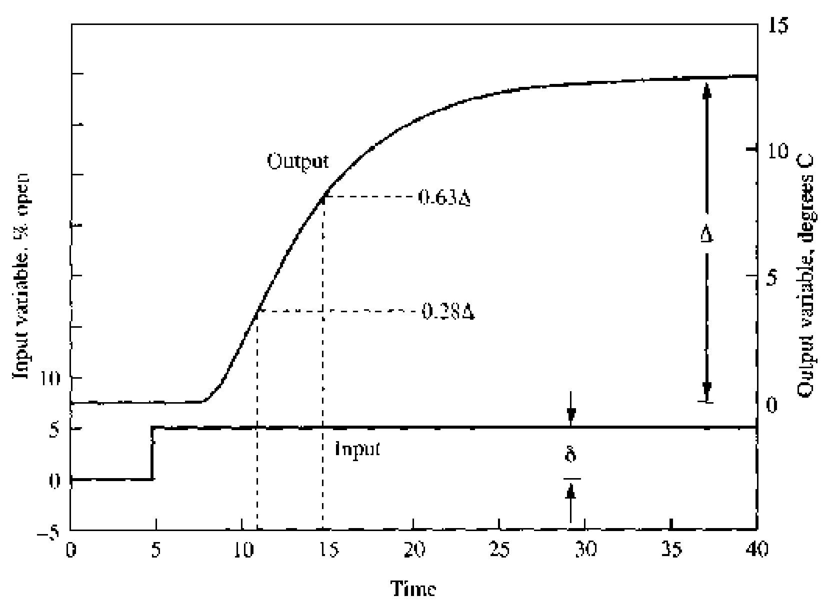

Figure 2.2.4 Reaction Curve of Stirred Tank Process

From the graphical and by two points method, the calculations are given as follows (It is ok to have some reading errors, as long as the formula is correct):

$$K = \frac{\Delta}{\delta} = \frac{13.1}{5} = 2.6^0 C / \% open$$

$$0.63\Delta = 8.3^0 C \qquad t_{0.63\%} = 9.7\min$$

$$0.28\Delta = 3.7^0 C \qquad t_{0.28\%} = 5.7\min$$

$$T = 1.5(t_{63\%} - t_{28\%}) = 1.5(9.7 - 5.7) = 6.0\min$$

$$L = t_{63\%} - T = 9.7 - 6.0 = 3.7\min$$

The model is therefore determined as

$$G_p(s) = \frac{2.6}{6s+1} e^{-3.7s}$$

#

Example 2.2.2: *Identification of Distillation Column*

The temperature of a certain industrial distillation column is regulated by the flowrate of the underfiow reflux to the tray just below it. The typical steady-state operating values for these variables are 380^{0}F and 25,000 lb/hr respectively. The result of a step test carried out on this column is presented below.

Table 2.2.1 Step Response of the Distillation Column

Time (hr)	Reflux Flowrate (R lb/hr)	Tray #16 Temp. (T ^{0}F)	*u*	*y*	*(y∞ — y) /y∞*
t <0	25,000	380.0	0	0.0	1.000
0.0	27,500	380.0	2,500	0.0	1.000
0.5	27,500	380.9	2,500	0.9	0.940
1.0	27,500	383.1	2,500	3.1	0.793
1.5	...	386.1	...	6.1	0.593
2.0	...	387.8	...	7.8	0.480
2.5	...	389.1	...	9.1	0.393
3.0	...	391.2	...	11.2	0.253
3.5	...	391.7	...	11.7	0.22
4.0	...	392.4	...	12.4	0.173
4.5	...	393.0	...	13.0	0.133
5.0	...	393.4	...	13.4	01.07
5.5	...	393.6	...	13.6	0.093
6.0	...	394.0	...	14.0	0.067
6.5	...	394.3	...	14.3	0.047
7.0	...	394.4	...	14.4	0.040
7:5	...	394.5	...	14.5	0.033
8.0	...	394.7	...	14.7	0.020
9.0	...	394.8	...	14.8	0.013
10.0	...	394.9	...	14.9	0.007
11.0	...	395.0	...	15.0	0.00
12.0	...	394.9	...	14.9	0.007
13.0	...	395.1	...	15.1	0.007
14.0	27,500	395.0	2,500	15.0	0.00

A candidate model for this process is:

$$y(s) = \frac{K}{\tau s + 1} e^{-Ls} u(s)$$

the data are given in terms of actual process variables; they must be recast in terms of deviations from initial steady-state values. Thus, from the supplied table, we obtain output *y(t)* as *T* — 3800, and input *u(t)* as *R* —25,000.

An inspection of the step-response data indicates that the *change* in the temperature as a result of the implemented reflux flow change is, on the average, 15.^{0}F, i.e.:

$$y_\infty = 15^0 F$$

Since the step input is of magnitude 2,500 (Lb/hr), (27,500 —25,000), the steady-state gain is estimated as:

$$\hat{K} = \frac{15}{2500} = 6 \times 10^{-3} F(lb/hr)$$

We next obtain the variable $(y_\infty - y)/y_\infty$ shows the data set up in the form required for estimating the model parameters.

Applying linear regression on these we obtain the following estimates for the time delay and time constant:

$$\hat{L} = 0.56\,hr \qquad \hat{\tau} = 1.91\,hr$$

A graphical display of the data and the identified model is shown in Figure below.

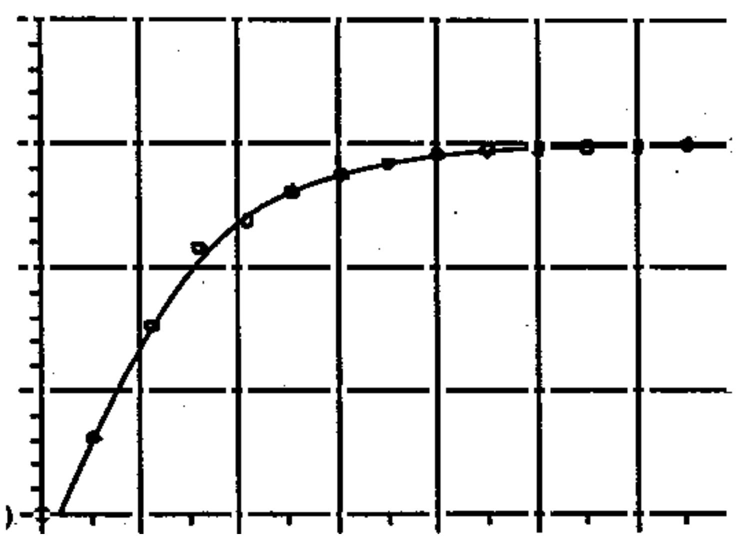

Fig. 2.2.5 Step Response of original and identified process for the Distillation Column

Example 2.2.3: SO_2 scrubber process

The result of a step test performed on an SO_2 scrubber is shown in Table 2.2.2. The input variable is the inlet water flowrate (in gallons per minute) and the output variable is the outlet concentration of SO_2 (in parts per million).

(a) Determining a suitable model to represent these data and explain the reason.

(b) Assuming the process can be represented by a three parameter model, estimate the parameters by a graphic method

TABLE 2.2.2 SO_2 Scrubber Step Test Data

Time (s)	Water Flowrate (gpm)	Outlet SO_2 Conc (ppm)
t<0	500	25.00
0.0	400	25.00
10.0	400	25.00
20.0	400	25.01
30.0	...	25.00
40.0	...	25.06
50.0	...	25.15
60.0	...	25.30
70.0	...	25.39
80.0	...	25.45
90.0	...	25.53
100.0	...	25.60
110.0	...	25.63
120.0	...	25.67
130.0		25.74
140.0	...	25.78
150.0	...	25.82
160.0	...	25.85
170.0	...	25.88
180.0	...	25.89
190.0	...	25.90
200.0	...	25.89
220.0	...	25.90
240.0	400	25.90

Try to answer your self.

2.3 TIME DOMAIN IDENTIFICATION FROM STEP TEST

2.3.1 Basic Formulation

Assume the system is stable and represented by

$$\begin{aligned} Y(s) &= g(s)U(s) \\ &= \frac{b_1 s^{n-1} + b_2 s^{n-2} \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n} e^{-Ls} U(s) \end{aligned} \tag{2.3.1a}$$

or in time domain

$$\begin{aligned} & y^{(n)}(t) + a_1 y^{(n-1)}(t) + \cdots + a_{n-1}\dot{y}(t) + a_n y(t) \\ & = b_1 u^{(n-1)}(t-L) + b_2 u^{(n-2)}(t-L) + \cdots + b_{n-1}\dot{u}(t-L) + b_n u(t-L) \end{aligned} \tag{2.3.1b}$$

For an integer $m \geq 1$, we define

$$\int_{[0,t]}^{m} f(t) = \int_0^t \int_0^{\tau_m} \cdots \int_0^{\tau_2} f(\tau_1) d\tau_1 \cdots d\tau_m$$

Integrating the transfer function in time domain *n* times, yields

$$\begin{aligned} & y(t) + a_1 \int_{[0,t]}^{(1)} y(t) + a_2 \int_{[0,t]}^{(2)} y(t) + \cdots + a_{n-1} \int_{[0,t]}^{(n-1)} y(t) + a_n \int_{[0,t]}^{(n)} y(t) \\ & = b_1 \int_{[0,t]}^{(1)} u(t-L) + b_2 \int_{[0,t]}^{(2)} u(t-L) + \cdots + b_{n-1} \int_{[0,t]}^{(n-1)} u(t-L) + b_n \int_{[0,t]}^{(n)} u(t-L) \end{aligned} \tag{2.3.2}$$

To calculate the integration from a set of sampled data, the numerical integrator can be used to transform the integration into discrete data, because such an integrator has a number of attractive features: simplicity of implementation, insensitivity to initial values of the continuous time signals, and high accuracy.

The integral of a continuous time signal, e.g. *f(t)*, over the time interval $\left[0,\ t_f\right]$ can be approximately calculated by

$$\begin{aligned} \int_0^{t_f} f(\tau) d\tau &\approx \alpha_0 f(t) + \cdots + \alpha_l f(t - lT_s) \\ &= \sum_{i=0}^{l} \alpha_i q^{-i} f(t) \end{aligned}$$

where T_s is the integration step size, chosen to be the same as the sampling interval for easy implementation; *l* is considered as the length factor of the integrator (a natural number);and q^{-1} is the unit delay operator, i.e. $q^{-1} f(t) = f(t - T_s)$. The filter coefficients α_i depend on the type of numerical integration methods to be employed. For instance, when the trapezoidal integration rule is used, the filter coefficients are given as:

$$\alpha_0 = \alpha_l = \frac{T_s}{2}, \alpha_i = T_s, i \in [1, l-1]$$

Similarly, the *s*th multiple integral of *f(t)* can be defined as follows:

$$\begin{aligned} \int_{[0,t]}^{s} f(t) &= (1 - q^{-1})(\alpha_0 + \alpha_1 q^{-1} \cdots + \alpha_l q^{-l}) f^{s-1}(t) \\ &= \sum_{i=0}^{l} \beta_i q^{-i} f^{s-1}(t) \end{aligned}$$

where, β_i is the coefficient in the polynomial.

2.3.2 Open Loop Step Test

For open loop step test, input $u(t) = A \cdot 1(t)$, there holds

$$\int_{[0,t]}^{(m)} u(t-L) = \frac{A}{m!}(t-L)^m \tag{2.3.3}$$

Therefore,

$$
\begin{aligned}
y(t) &= -a_1\int_{[0,t]}^{(1)} y(t) - a_2\int_{[0,t]}^{(2)} y(t) - \cdots - a_{n-1}\int_{[0,t]}^{(n-1)} y(t) - a_n\int_{[0,t]}^{(n)} y(t) \\
&+ Ab_1(t-L) + \frac{A}{2}b_2(t-L)^2 + \cdots + \frac{A}{(n-1)!}b_{n-1}(t-L)^{n-1} + \frac{A}{n!}b_n(t-L)^n
\end{aligned}
\tag{2.3.4}
$$

We now consider two special cases: For *n=1*, it becomes (First-Order)

$$
y(t) = -a_1\int_0^t y(\tau)d\tau + Ab_1(t-L) \tag{2.3.5}
$$

Define

$$
\begin{cases}
\gamma(t) = y(t) \\
\phi(t) = [-\int_0^t y(\tau)d\tau \quad -A \quad tA]^T \\
\theta = [a_1 \quad b_1 L \quad b_1]^T
\end{cases}
$$

we have

$$
\gamma(t) = \phi(t)^T\theta + e \tag{2.3.6}
$$

where *e* is measurement noise. Combine the two above equations and *y(L)=0*, leads to

$$
\Gamma = \Psi\theta + \Delta, \qquad \text{for } t \ge L \tag{2.3.7}
$$

where

$$
\Psi = \begin{bmatrix} \varphi(t_1) \\ \varphi(t_2) \\ \vdots \\ \varphi(t_N) \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} \gamma(t_1) \\ \gamma(t_2) \\ \vdots \\ \gamma(t_N) \end{bmatrix}
$$

As the three rows in φ are independent to each, $\Psi^T\Psi$ is nonsingular. Thus the ordinary least squares method can be used to find its solution

$$
\hat{\theta} = \left(\Psi^T\Psi\right)^{-1}\Psi^T\Gamma \tag{2.3.8a}
$$

Than the process parameters can be recovered from

$$
\begin{bmatrix} a_1 \\ b_1 \\ L \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_3 \\ \theta_2/\theta_3 \end{bmatrix} \tag{2.3.8b}
$$

For n=2, it becomes

$$
\begin{aligned}
y(t) &= -a_1\int_0^t y(\tau)d\tau - a_2\int_0^t\int_0^\tau y(\tau)d\tau d\tau_1 \\
&+ Ab_1(t-L) + \frac{1}{2}Ab_2(t-L)^2
\end{aligned}
\tag{2.3.9a}
$$

Rearrange according to ascent power of *t* as

$$y(t) = -a_1\int_0^t y(\tau)d\tau - a_2\int_0^t\int_0^\tau y(\tau)d\tau d\tau_1 + \left(-b_1L + \frac{1}{2}b_2L^2\right)A + (b_1 - b_2L)tA + \frac{1}{2}b_2t^2A \tag{2.3.9b}$$

Again, it can be written as

$$\gamma(t) = \phi(t)^T\theta + e$$

where

$$\begin{cases} \gamma(t) = y(t) \\ \phi(t) = [-\int_0^t y(\tau)d\tau \quad -\int_0^t\int_0^\tau y(\tau)d\tau d\tau_1 \quad A \quad tA \quad \frac{1}{2}t^2A]^T \\ \theta = \left[a_1 \quad a_2 \quad -b_1L + \frac{1}{2}b_2L^2 \quad b_1 - b_2L \quad b_2\right]^T \end{cases}$$

and e is measurement noise. It again have

$$\Gamma = \Psi^T\theta + \Delta, \qquad \text{for } t \ge L$$

where

$$\Psi = \begin{bmatrix} \varphi(t_1) \\ \varphi(t_2) \\ \vdots \\ \varphi(t_N) \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} \gamma(t_1) \\ \gamma(t_2) \\ \vdots \\ \gamma(t_N) \end{bmatrix}$$

As the three rows in φ are independent to each, $\Psi^T\Psi$ is nonsingular. Thus the ordinary least squares method can be used to find its solution

$$\hat{\theta} = \left(\Psi^T\Psi\right)^{-1}\Psi^T\Gamma$$

Than the process parameters can be recovered from

$$\begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \\ L \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \beta \\ \theta_5 \\ \dfrac{\beta - \theta_4}{\theta_5} \end{bmatrix} \tag{2.3.10a}$$

where

$$\beta = \pm\sqrt{\theta_4^2 - 2\theta_5\theta_3} \tag{2.3.10b}$$

Suppose $G(0)>0$, a native β corresponds to a non-mimimum phase process and will cause inverse response. Thus

$$\beta = \begin{cases} -\sqrt{\theta_4^2 - 2\theta_5\theta_3} & \text{If reverse response} \\ \sqrt{\theta_4^2 - 2\theta_5\theta_3} & \text{Otherwise} \end{cases} \tag{2.3.10c}$$

If *G(0)<0*, reverse the sign of β accordingly.

2.3.3 Closed-Loop Step Test

The closed-loop transfer function of Fig. 2.3.1 is given as

$$\frac{Y(s)}{R(s)} = G_{cl}(s) = \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} \tag{2.3.11}$$

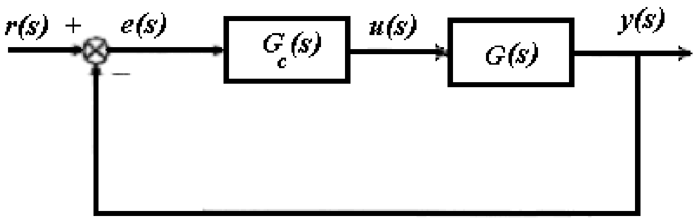

Fig. 2.3.1 Closed-Loop System

where $G_c(s)$ is the controller transfer function and $G_p(s)$ is the process transfer function.

Introduce a unit step set point change, ($r(s) = 1/s$). By measuring the controller output *u(s)*, the relation between process input and output can be expressed by (2.3.1a). Using first-order *Taylor* expansion for the unknown time delays, *i.e.* $e^{-Ls} \approx 1 - Ls$, we have:

$$y(t) + a_1 \int_{[0,t]}^{(1)} y(t) + a_2 \int_{[0,t]}^{(2)} y(t) + \cdots + a_{(n-1)} \int_{[0,t]}^{(n-1)} y(t) + a_n \int_{[0,t]}^{(n)} y(t)$$

$$= b_1 \int_{[0,t]}^{(1)} u(t) + b_2 \int_{[0,t]}^{(2)} u(t) + \cdots + b_{(n-1)} \int_{[0,t]}^{(n-1)} u(t) + b_n \int_{[0,t]}^{(n)} u(t) \tag{2.3.12}$$

$$- Lb_1 u(t) - Lb_2 \int_{[0,t]}^{(1)} u(t) + - \cdots - Lb_{(n-1)} \int_{[0,t]}^{(n-2)} u(t) - Lb_n \int_{[0,t]}^{(n-1)} u(t)$$

Similar to open loop identification, the solutions for $n = 1$ and $n = 2$ under the framework of Equation (2.3.12) are given as follows.

For $n = 1$, Equation (2.3.12) becomes:

$$y(t) = -a_1 \int_0^t y_{ij}(\tau)d\tau - Lb_1 u(t) + b_1 \int_0^t u(\tau)d\tau \tag{2.3.13}$$

which can be written into the compact form

$$\gamma^1(t) = [\phi^1(t)]^T \cdot \theta^1 \tag{2. 3.14}$$

where

$$\begin{cases} \gamma^1(t) = y(t) \\ [\phi^1(t)] = \begin{bmatrix} -\int_0^t y(\tau)d\tau & \int_0^t u(\tau)d\tau & -u(t) \end{bmatrix}^T \\ \theta^1 = \begin{bmatrix} a_1 & b_1 & Lb_1 \end{bmatrix}^T \end{cases} \tag{2. 3.15}$$

For $n = 2$, Equation (2.3.12) becomes:

$$y(t) = -a_1\int_0^t y(\tau)d\tau - a_2\int_0^t\int_0^\tau y(\tau_1)d\tau_1 d\tau - Lb_1 u(t) + \left(b_1 - Lb_2\right)\int_0^t u(\tau)d\tau + b_2\int_0^t\int_0^\tau u(\tau_1)d\tau_1 d\tau \tag{2. 3.16}$$

It is again expressed as:

$$\gamma^2(t) = [\phi^2(t)]^T \cdot \theta^2 \tag{2. 3.17}$$

where

$$\begin{cases} \gamma^2(t) = y(t) \\ [\phi^2(t)]^T = \begin{bmatrix} -\int_0^t y(\tau)d\tau & -\int_0^t\int_0^\tau y(\tau_1)d\tau_1 d\tau & \int_0^t u(\tau)d\tau & \int_0^t\int_0^\tau u(\tau_1)d\tau_1 d\tau & -u(t) \end{bmatrix} \\ \theta^2 = \begin{bmatrix} a_1 & a_2 & b_1 - Lb_2 & b_2 & Lb_1 \end{bmatrix} \end{cases} \tag{2. 3.18}$$

Equations (2.3.15) and (2.3.18) can be solved by the Least Squares methods for each transfer functions $i, j = 1, 2$ and $k = 1, 2$, to form the regression form:

$$\Gamma = \Psi\Theta \tag{2. 3.19}$$

where $\boldsymbol{\Gamma}_{ij}$ $\boldsymbol{\Gamma} = \left[\gamma(t_1), \quad \gamma(t_2), \quad \cdots \quad \gamma(t_N)\right]^T$, $\boldsymbol{\Psi} = \left[\phi(t_1), \quad \phi(t_2), \quad \cdots \quad \phi(t_N)\right]^T$. Its Least Squares estimation for $\boldsymbol{\Theta}$ are:

$$\tilde{\Theta} = \left(\Psi^T\Psi\right)^{-1}\Psi^T\Gamma \tag{2. 3.20}$$

Once $\tilde{\boldsymbol{\Theta}}$ are found from equation (2.3.20), a_1, b_1 and L can be recovered from

$$\begin{bmatrix} a_1 \\ b_1 \\ L \end{bmatrix} = \begin{bmatrix} \theta_1^1 \\ \theta_3^1 \\ \theta_2^1/\theta_3^1 \end{bmatrix} \tag{2. 3.21}$$

for $n = 1$, and a_1, a_2, b_1, b_2 and L from

$$\begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \\ L \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \beta \\ \theta_5 \\ \theta_5/\beta \end{bmatrix} \tag{2.3.22a}$$

where

$$\beta = \frac{\theta_3 \pm \sqrt{\theta_3^2 + 4\theta_4\theta_5}}{2} \tag{2.3.22b}$$

for $n = 2$, respectively.

2.3.4 Examples

Example 2.3.1: Consider a first-order plus dead-time process:

$$G(s) = \frac{e^{-s}}{s+1}$$

A unit step test is performed and the process input and output in the step test are recorded. Recall that in LSM the process responses do not have to fully enter the steady state. Here, responses from t=0s to t_f=4.14s are used, with y(4.14s)=0.95. Using Eqs. (2.3.8), the estimated process model is obtained as

$$G(s) = \frac{e^{-s}}{0.997s+1}$$

which is almost identical to the actual process with very small identification error. For the area identification method, if the step response from t=0s to the time when the process enters the new steady-state well, with y(8.28s)=0.999, is used in computation, then the estimated model is

$$G(s) = \frac{e^{-1.01s}}{0.99s+1}$$

However, if the response in t=0 to t_f=4.14s is used, then the estimation will deteriorate to

$$G(s) = \frac{0.95e^{-1.07s}}{0.78s+1}$$

The LSM saves a half to a third of testing time than area method and gives a more accurate result.

Example 2.3.2. Consider a 2nd-order plus dead-time process

$$G(s) = \frac{1}{(10s+1)\ (2s+1)} e^{-4s}$$

The results from LSM and the area method are both shown in Table 2.3.1 and an identification result improvement is observed with the LSM.

Example 2.3.3. The LSM is also tested on a non-minimum-phase process:

$$G(s) = \frac{1-s}{(s+1)^5}$$

The difference between the actual and the identified process is very small. The result from the area method is also listed for comparison.

Example 2.3. 4. Consider a process with multiple lag

$$G(s) = \frac{1}{(s+1)^8}$$

the results of above examples from both LSM and area method are presented in Table 2.3.1.

Table 2.3.1 Comparison of identification results from two methods

Processes $G(s)$	LSQ method			Area method		
	$\widehat{G}(s)$	ε	E (%)	$\widehat{G}(s)$	ε	E (%)
$\frac{e^{-s}}{s+1}$	$\frac{e^{-s}}{0.997s+1}$	7.5×10^{-7}	0.35	$\frac{e^{-1.01s}}{0.99s+1}$	1.8×10^{-6}	2.4
$\frac{1}{(10s+1)\ (2s+1)}e^{-4s}$	$\frac{1.03e^{-5.47s}}{11.41s+1}$	5.5×10^{-5}	10.0	$\frac{1.03e^{-5.83s}}{10.19s+1}$	2.1×10^{-5}	15.8
$\frac{1-s}{(s+1)^5}$	$\frac{1.01e^{-3.73s}}{2.45s+1}$	1.9×10^{-3}	4.8	$\frac{1.01e^{-4.00s}}{2.11s+1}$	9.0×10^{-4}	12.5
$\frac{1}{(s+1)^8}$	$\frac{1.06e^{-4.94s}}{3.81s+1}$	7.7×10^{-4}	10.1	$\frac{1.06e^{-4.3s}}{4.3s+1}$	2.2×10^{-3}	28

Note: the two relative errors are defined as follows:

1 The time-domain identification error is measured by the standard deviation:

$$\varepsilon=\frac{1}{\text{n+1}}\sum_{k=m}^{m+n}\left[y(kT_s)-\hat{y}(kT_s)\right]^2 \tag{2. 3.23}$$

where $y(kT_s)$ is the actual process output under a step change, while $\hat{y}(kT_s)$ is the response of the estimated process under the same step change.

2 The frequency-domain identification error is measured by the *worst-case error*:

$$E=\max_i\left\{\left|\frac{\widehat{G}(j\omega_i)-G(j\omega_i)}{G(j\omega_i)}\right|\times100\%, i=1,2,\cdots,M\right\} \tag{2. 3.24}$$

where $G(j\omega_i)$ and $\widehat{G}(j\omega_i)$ are the actual and estimated process frequency responses respectively. The Nyquist curve for a phase ranging from 0 to $-\pi$ is considered, since this part is the most significant for control design.

Since LSM makes use of multiple points rather than one or two points on the process response and adopts an instrumental variable, it is expected to be robust to noise. To demonstrate this, the process of Example 2.3.1 is again tested in the noisy case. In the context of system identification, the noise-to-signal ratio is usually defined as

$$\text{NSR}=\frac{\text{mean power spectrum density of noise}}{\text{mean power spectrum density of signal}} \tag{2.3.25}$$

Under measurement noise with NSR=25%, a step test is performed. The process response is shown in Fig. 2.3.2. The model obtained by the proposed method is almost the same as the actual process. The step response of the estimated model is also plotted in Fig. 2.3.2, which shows that the estimation agrees very well with the actual one.

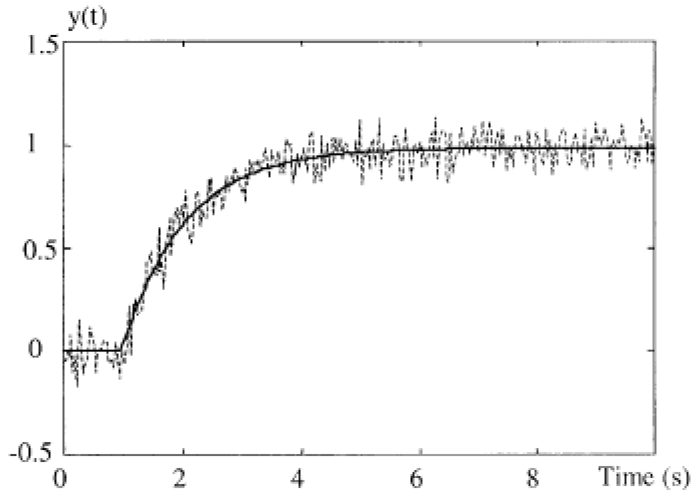

Fig. 2.3.2. The process and model responses under a step change. Actual process response under noise (......), estimated model response (—).

2.4 FREQUENCY DOMAIN IDENTIFICATION FROM STEP TEST

2.4.1 Fundamentals

The signals that vary from one steady state to another steady state are commonly encountered in the industrial processes, such as the input and output of process in the step test. The variation between two steady states includes the dynamics characteristic of interest in the frequency domain. However, the signal usually cannot be dealt directly with Fourier analysis since they are not absolutely integrable if the two steady states are different in values. To overcome this difficulty, we consider Laplace transform defined as:

$$F(s) = \int_0^\infty f(t) e^{-st} dt . \tag{2.4.1}$$

where $f(t)$ satisfies

(i) $f(t)$ is piecewise continuous function for $t \geq 0$ and zero for $t < 0$.

(ii) The integral in equation (1) has a finite value.

Decompose $f(t)$ into transient part $\Delta f(t)$

$$\Delta f(t) = f(t) - f(\infty) , \tag{2.4.2}$$

and stationary part $f_s(t)$

$$f_s(t) = f(\infty).1(t) \tag{2.4.3}$$

where $1(t)$ is unit step function. Consequently, the function $f(t)$ can be expressed as

$$f(t) = \Delta f(t) + f_s(t) , \tag{2.4.4}$$

The stationary part $f_s(t)$ is a step function and its Laplace-transform $F_s(s)$ is given by

$$F_s(s) = \int_0^\infty f_s(\infty) e^{-st} dt = \frac{f_s(\infty)}{s} . \tag{2.4.5}$$

As transient part $\Delta f(t)$ will decay to zero after certain time, the Laplace-transform $F(s)$ of $f(t)$

can then be expressed as

$$F(s) = \Delta F(s) + F_s(s) = \int_0^{\infty} \Delta f(t) e^{-st} dt + \frac{f_s(\infty)}{s}, \tag{2.4.6}$$

where $\Delta F(s)$ is the Laplace transforms of the transient part $\Delta f(t)$.

Suppose that at $t = T_f$, $f(t)$ have reached the steady-state value and $\Delta f(t)$ is approximately zero after $t = T_f$, then (6) becomes

$$F(s) \approx \int_0^{T_f} \Delta f(t) e^{-st} dt + \frac{f(\infty)}{s}. \tag{2.4.7}$$

For signals with finite initial and final values, we can always decompose them into a step change and a transient response curve, which start at zero and decay to zero after certain time. The treatment of signal decomposition although straightforward, provides a useful tool for the following system identification in the frequency domain.

At a given frequency point ω_i, the integral over the time interval $\left[0,\ t_f\right]$ can be approximately calculated by

$$\int_0^{t_f} \Delta f\left(t\right) e^{-j\omega_i t} dt = \int_0^{t_f} \Delta f\left(t\right) \cos\omega_i t dt - j\int_0^{t_f} \Delta f\left(t\right) \sin\omega_i t dt \tag{2.4.8}$$

Using the numerical integration method, given in Section 2.3, we have

$$\begin{aligned}
\int_0^{t_f} \Delta f\left(t\right) \sin\omega_i t dt &\approx \alpha_0 \Delta f(t_f) \sin\omega_i t_f + \cdots + \alpha_l \Delta f(0) \sin\omega_i t_0 = \sum_{j=0}^{l} \alpha_i q^{-i} \Delta f(t_j) \sin\omega_i t_j \\
\int_0^{t_f} \Delta f\left(t\right) \cos\omega_i t dt &\approx \alpha_0 \Delta f(t_f) \cos\omega_i t_f + \cdots + \alpha_l \Delta f(0) \cos\omega_i t_0 = \sum_{j=0}^{l} \alpha_i q^{-i} \Delta f(t_j) \cos\omega_i t_j
\end{aligned} \tag{2.4.9}$$

2.4.2 Open Loop Test

For the input $u(t)$ and output $y(t)$ of process, their respective Laplace-transform $U(s)$ and $Y(s)$ can be expressed as follows:

$$\begin{aligned}
U(s) &\approx \int_0^{T_u} \Delta u(t) e^{-st} dt + \frac{u(\infty)}{s} \\
Y(s) &\approx \int_0^{T_y} \Delta y(t) e^{-st} dt + \frac{y(\infty)}{s}
\end{aligned} \tag{2.4.10}$$

Then the transfer function of process is then obtained by

$$G(s) = \frac{Y(s)}{U(s)} \approx \frac{\int_0^{T_y} \Delta y(t) e^{-st} dt + \frac{y(\infty)}{s}}{\int_0^{T_u} \Delta u(t) e^{-st} dt + \frac{u(\infty)}{s}} = \frac{s\int_0^{T_y} \Delta y(t) e^{-st} dt + y(\infty)}{s\int_0^{T_u} \Delta u(t) e^{-st} dt + u(\infty)} \tag{2.4.11}$$

Substituting $s = j\omega$ into (2.4.11), it becomes

$$G(j\omega) \approx \frac{j\omega\int_0^{T_y} \Delta y(t)e^{-j\omega t}dt + y(\infty)}{j\omega\int_0^{T_u} \Delta u(t)e^{-j\omega t}dt + u(\infty)} \tag{2.4.12}$$

For step input *u(t)=A1(t),* $U(s) = 1/s$, $\Delta U(s) = 0$, Equation (2.4.12) becomes

$$\begin{aligned} G(j\omega) &\approx \frac{j\omega\int_0^{T_y} \Delta y(t)e^{-j\omega t}dt + y(\infty)}{A} \\ &= \frac{\left[y(\infty) + \omega\int_0^{T_y} \Delta y(t)\sin\omega_i t dt \right] + j\omega\int_0^{T_y} \Delta y(t)\cos\omega_i t dt}{A} \end{aligned} \tag{2.4.13}$$

At a given frequency point ω_i, the integral over the time interval $\left[0, t_f\right]$ can be approximately calculated by Equation (2.4.9) and $G(j\omega_i)$ by (2.4.13).

2.4.3 Closed Loop Test

The closed-loop transfer function of Fig. 2.3.1 is given as

$$\frac{Y(s)}{R(s)} = G_{cl}(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} \tag{2.4.14}$$

Introduce a unit step set point change, ($G_c(s)$ and $R(s) = 1/s$). The process transfer function $G_p(s)$ can be written as:

$$G(s) = \frac{sY(s)}{G_c(s)[1 - sY(s)]} \tag{2.4.15}$$

From the Laplace Transformation, the process transfer function can be rewritten as:

$$G(s) = \frac{s\int_0^{\infty} y(t)\,\mathrm{e}^{-st}\,dt}{G_c(s)[1 - s\int_0^{\infty} y(t)\,\mathrm{e}^{-st}\,dt]} \tag{2.4.16}$$

With introduction of steady-state value of the process output, we have

$$G(s) = \frac{y_{ss} + s\int_0^{t_f} \Delta y(t)\,\mathrm{e}^{-st}\,dt}{G_c(s)[(1 - y_{ss}) - s\int_0^{t_f} \Delta y(t)e^{-st}dt]} \tag{2.4.17}$$

Substituting $s = j\omega$ into (2.4.17) and notice that

$$\int_0^{t_f} \Delta y(t)e^{-j\omega_i t}dt = \int_0^{t_f} \Delta y(t)\cos\omega_i t dt - j\int_0^{t_f} \Delta y(t)\sin\omega_i t dt$$

$$\begin{aligned} G_c(j\omega) &= k_c[1 + (1/\, j\omega\tau_I\ ) + j\omega\tau_D] \\ &= k_c + k_I\,/\,j\omega + k_D\,j\omega \\ &= k_c + j(k_D\omega - k_I\,/\,\omega) \end{aligned}$$

we obtain

$$G(j\omega_i) = \frac{A_1(\omega_i) + jA_2(\omega_i)}{B_1(\omega_i) + jB_2(\omega_i)} \tag{2.4.18}$$

where

$$\begin{aligned} A_1(\omega_i) &= y_{ss} + \omega_i \int_0^{t_f} \Delta y(t) \sin \omega_i t dt \\ A_2(\omega_i) &= \omega_i \int_0^{t_f} \Delta y(t) \cos \omega_i t dt \\ B_1(\omega_i) &= k_c \left[1 - A_1(\omega_i)\right] + (k_D \omega - k_I / \omega) A_2(\omega_i) \\ B_2(\omega_i) &= (k_D \omega - k_I / \omega) \left[1 - A_1(\omega_i)\right] - k_c A_2(\omega_i) \end{aligned} \tag{2.4.19}$$

2.4.3 RECURSIVE SOLUTION

As the frequency range of interest is usually from zero up to the process critical frequency ω_c which corresponding to the process phase lag π, it is sufficient to calculate $G(j\omega)$ up to this frequency for control system analysis and design.

In order to obtain more accurate result over the entire frequency range of interest, let the number of the frequency response to be identified in the frequency range *(0,* ω_c) is *M*, and divide the desired phase lag π into *M* subinterval with difference of $\frac{\pi}{M-1}$ as shown in Figure 2.4.2. In practical applications, *M* may take the values of 5 to 10 to get a reasonable estimation result.

The desired phase lag ϕ^* at each iteration step is calculated by

$$\begin{aligned} \phi_i &= \arg[G(j\omega_i)] \\ &= \tan^{-1} \frac{A_2(\omega_i)}{A_1(\omega_i)} - \tan^{-1} \frac{B_2(\omega_i)}{B_1(\omega_i)} \end{aligned} \tag{2.4.20}$$

For initial values ω_0 and ϕ_0 are zeros, ω_1 is chosen to be a small value, say $\omega_1 = 10^{-3}$, and ϕ_1 is calculated by, $G_p(j\omega_i)$ along the searching direction.

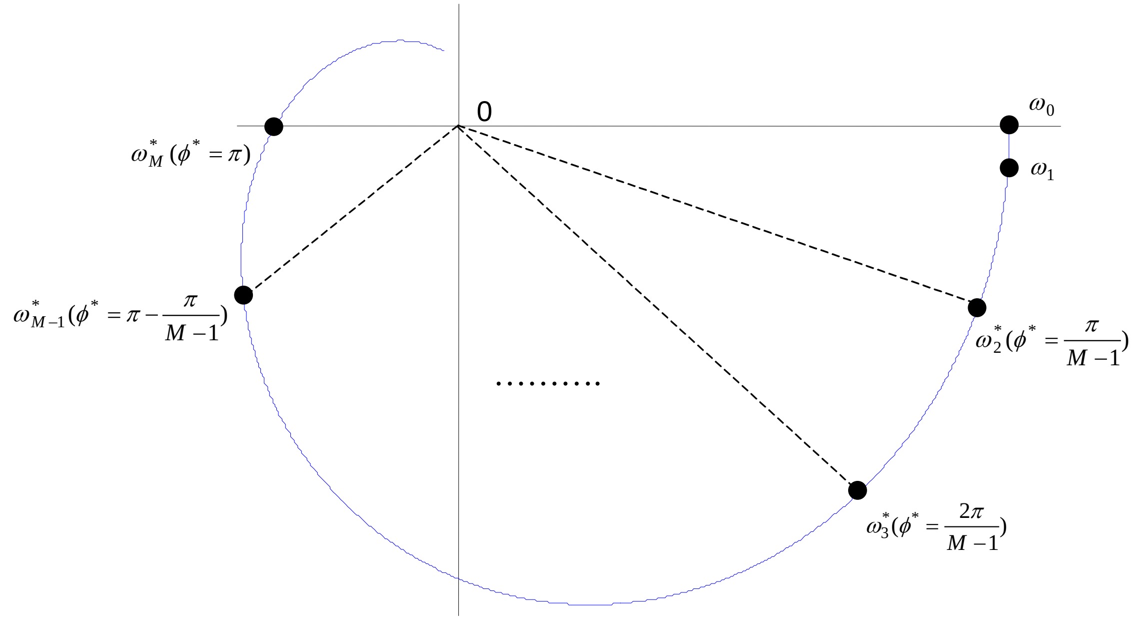

Fig. 2.4.2 The desired frequencies in the important frequency range

Then the following steps can be used to calculate $G(j\omega_i)$ near the desired discrete frequencies:

Step 1: For initial values ω_0 and ϕ_0 are zeros, ω_1 is chosen to be a value smaller than the desired frequency ω_2^*, say $\omega_1 = 10^{-3}$, and ϕ_1 is calculated by (2.4.20).

Step 2: Obtain the frequency ω_2 by

$$\omega_2 = \omega_1 - (\frac{\pi}{M-1} + \phi_1)\frac{\omega_1 - \omega_0}{\phi_1 - \phi_0}$$

which is very close to the desired frequency ω_2^* corresponding to phase lag $\frac{\pi}{M-1}$, then calculate $G(j\omega_2)$ and ϕ_2, respectively.

Step 3: For $i = 2,3,\cdots M-1$, determine ω_{i+1}, $G(j\omega_{i+1})$ and ϕ_{i+1} by

$$\omega_{i+1} = \omega_i - (\frac{i\pi}{M-1} + \phi_i)\frac{\omega_i - \omega_{i-1}}{\phi_i - \phi_{i-1}} \tag{2.4.21}$$

(2.4.19) and (2.4.20), respectively.

Note:

1. The objective of the iterative formula (2.4.21) is to determine the discrete frequencies distributed evenly with respect to phase lag over the important frequency range in order to obtain a more accurate transfer function model of the process, which will be discussed in next subsection, the closeness of the calculated discrete frequencies to their respective desired frequencies is not an important issue here.

2. ω_0, ϕ_0, ω_1 and ϕ_1 are initial conditions in calculating ω_2, they usually very close to each other, as shown in Figure 2.4.2. After all the frequency responses are obtained, one of

the two points may be dropped to reduce the computation burden without affecting the model accuracy.

3. The above development is based on the assumption of zero initial conditions. If the system has non-zero operating points of $u(0)$ and $y(0)$, then $u(t)$, $u(\infty)$, $y(t)$ and $y(\infty)$ should be replaced by $u(t)-u(0)$, $u(\infty)-u(0)$, $y(t)-y(0)$ and $y(\infty)-y(0)$, respectively.

2.4.4 TRANSFER FUNCTION MODELING

From the process frequency response $G(j\omega_i)$, $i=1,2,\cdots M$, we can obtain transfer function of the process. The second-order plus dead-time model is adopted for the transfer function

$$G'(s)=\frac{b_1 s+b_0}{a_2 s^2+a_1 s+1}e^{-Ls}, \tag{2.4.14}$$

which can represent both monotonic, oscillatory and non-minimum phase processes. Its parameters may be determined by matching $G'(j\omega)$ to $G(j\omega)$ at multiple points ω_i, $i=1,2,...,M$, that means

$$G(j\omega_i)=\frac{j\omega_i b_1+b_0}{(j\omega_i)^2 a_2+j\omega_i a_1+1}e^{-j\omega_i L}, i=1,2,...,M. \tag{2.4.15}$$

The parameters a_1, a_2, b_1, b_0 and L in (2.4.14) are determined by both magnitude and phase conditions of (2.4.15).

For the frequency ω_i, the magnitude and phase values of (2.4.15) are given as

$$-\omega_i^4|G(j\omega_i)|^2 a_2^2-\omega_i^2|G(j\omega_i)|^2(a_1^2-2a_2)+\omega_i^2 b_1^2+b_0^2=|G(j\omega_i)|^2 \tag{2.4.16a}$$

and

$$-\arg[G(j\omega_i)]+\tan^{-1}(\frac{b_1}{b_0})-\tan^{-1}(\frac{a_1\omega_i}{1-a_2\omega_i^2})=\omega_i L \tag{2.4.16b}$$

for $i=1,2,...,M$, the magnitude equation of (2.4.16a) can be arranged to a matrix form

$$\Phi\theta=\Gamma, \tag{2.4.17}$$

where

$$\Phi=\begin{bmatrix} -\omega_1^4|G(j\omega_1)|^2 & -\omega_1^2|G(j\omega_1)|^2 & \omega_1^2 & 1 \\ -\omega_2^4|G(j\omega_2)|^2 & -\omega_2^2|G(j\omega_2)|^2 & \omega_2^2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -\omega_M^4|G(j\omega_M)|^2 & -\omega_M^2|G(j\omega_M)|^2 & \omega_M^2 & 1 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} |G(j\omega_1)|^2 \\ |G(j\omega_2)|^2 \\ \vdots \\ |G(j\omega_M)|^2 \end{bmatrix}, \qquad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} a_2{}^2 \\ a_1{}^2 - 2a_2 \\ b_1{}^2 \\ b_0{}^2 \end{bmatrix}.$$

θ in (2.4.17) can be solved using the linear least-squares method

$$\theta = (\Phi^T \Phi)^{-1} \Phi^T \Gamma, \tag{2.4.18}$$

and the original model parameters a_1, a_2, b_1 and b_0 can then be recovered from θ

$$\begin{bmatrix} a_2 \\ a_1 \\ b_1 \\ b_0 \end{bmatrix} = \begin{bmatrix} \sqrt{\theta_1} \\ \sqrt{\theta_2 + 2\sqrt{\theta_1}} \\ \pm\sqrt{\theta_3} \\ \pm\sqrt{\theta_4} \end{bmatrix}. \tag{2.4.19}$$

If $G(0) > 0$, then $b_0 = \sqrt{\theta_4} > 0$. A negative b_1 corresponds to an inverse response, i.e. the output step response will first move in an opposite direction to its final value, and a positive b_1 corresponds to a minimum phase process. Thus, by observing the output time response, one is able to determine b_1 by

$$b_1 = \begin{cases} -\sqrt{\theta_3}, & \text{if a inverse response is detected;} \\ \sqrt{\theta_3}, & \text{otherwise.} \end{cases}$$

If $G(0) < 0$, reverse the signs of b_0 and b_1 accordingly.

After model parameters a_1, a_2, b_1 and b_0 are determined, the delay L can be estimated using the phase relation in (2.4.16b):

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_M \end{bmatrix} L = \begin{bmatrix} -\arg[G(j\omega_1)] + \tan^{-1}(\frac{b_1}{b_0}) - \tan^{-1}(\frac{a_1\omega_1}{1 - a_2\omega_1{}^2}) \\ -\arg[G(j\omega_2)] + \tan^{-1}(\frac{b_1}{b_0}) - \tan^{-1}(\frac{a_1\omega_2}{1 - a_2\omega_2{}^2}) \\ \vdots \\ -\arg[G(j\omega_M)] + \tan^{-1}(\frac{b_1}{b_0}) - \tan^{-1}(\frac{a_1\omega_M}{1 - a_2\omega_M{}^2}) \end{bmatrix}, \tag{2.4.20}$$

2.4.5 APPLICATIONS AND SIMULATION EXAMPLES

Open Loop Step Testing

The step test needs minimal number of equipments and can even be performed manually, shown in Fig. 2.4.3. It is the simplest and dominant method in process control applications.

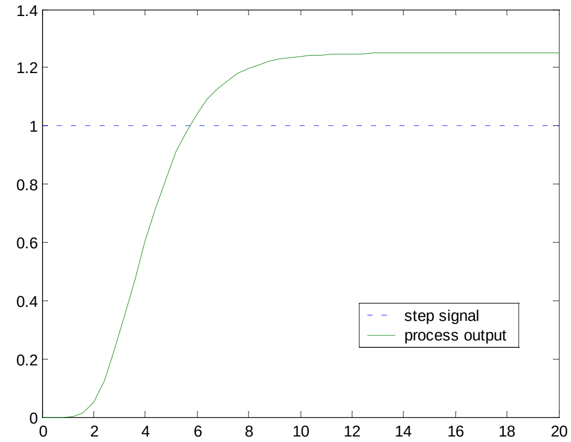

Fig. 2.4.3. Step testing

Example 2.4.1: Consider a nonminimum-phase plus dead-time process

$$G(s)=\frac{1-4s}{(3s+1)^2(s+1)}e^{-s}\ .$$

Choosing the number of the frequency response points *M =10*, the frequency responses are computed from Equations (2.5.10)-(2.5.13), and the second-order model is then calculated from (2.5.18)-(2.5.20) as

$$G'(s)=\frac{-2.9700s+1.0001}{8.2542s^2+5.3340s+1}e^{-2.8457s}$$

Nyquist curve of the estimated results are shown in Fig. 2.4.4 (Solid line: Actual process, ×: Estimated frequency responses at 20% NSR, dashed line: estimated model).

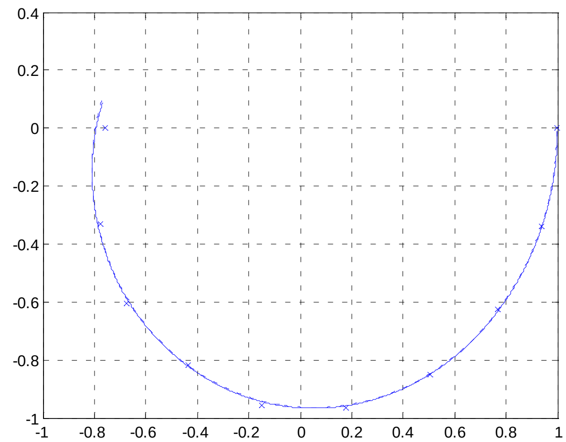

Fig. 2.4.4 Nyquist plot and estimated under noise

Closed-loop Set-point Change

Set-point changing is commonly encountered in process control, shown as Fig. 2.4.5; the process can be run continuously throughout the test, even though it may be disturbed somewhat from the nominal set point.

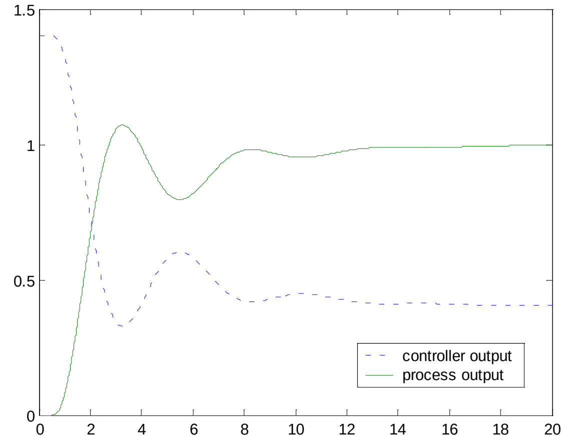

Fig.2.4.5 Set-point change in PID control loop

Example 2.4.2. Consider a high order process

$$G(s) = \frac{1}{(s+1)^8}\,.$$

We first adopted typical Ziegler-Nichols method to pre-tune the process. Choosing the number of the frequency response points *M =10*, the frequency responses are computed, and then the second-order model is then calculated as

$$G'(s) = \frac{0.4072s + 0.3318}{1.3320s^2 + 1.6322s + 1} e^{-1.9455s}$$

The frequency response identified by the proposed method is shown in Fig. 2.5.6. (Solid line: Actual process, ×: Estimated frequency responses at 20% NSR, Dashed line: Estimated model).

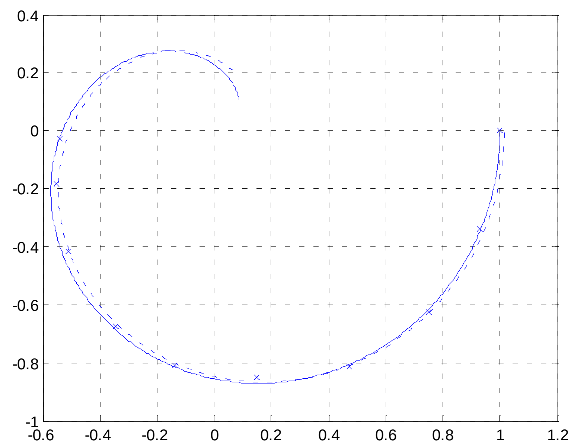

Fig. 2.4.6 Nyquist plot and estimated under noise

Example 24.3. Consider an oscillatory process given by

$$G(s) = \frac{1}{(s^2 + s + 1)(s + 3)} e^{-s} .$$

Choosing the number of the frequency response points $M = 10$, the frequency responses are computed, and then the second-order model is then calculated as

$$G'(s) = \frac{0.4072s + 0.3318}{1.3320s^2 + 1.6322s + 1} e^{-1.9455s}$$

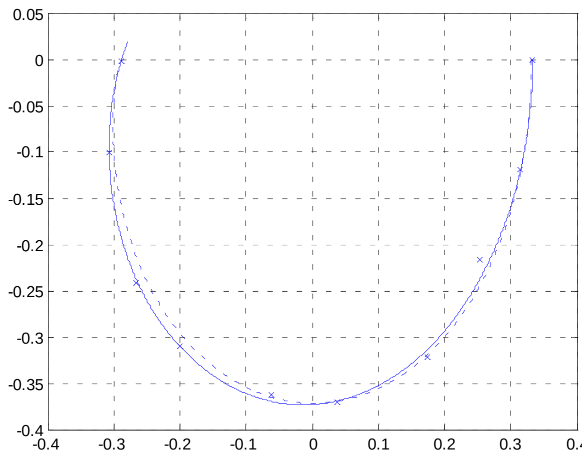

Fig. 5 Nyquist plot and estimated under noise

The frequency response identified by the proposed method is shown in Fig. 5. (Solid line: Actual process, ×: Estimated frequency responses at 20% NSR, Dashed line: Estimated model).