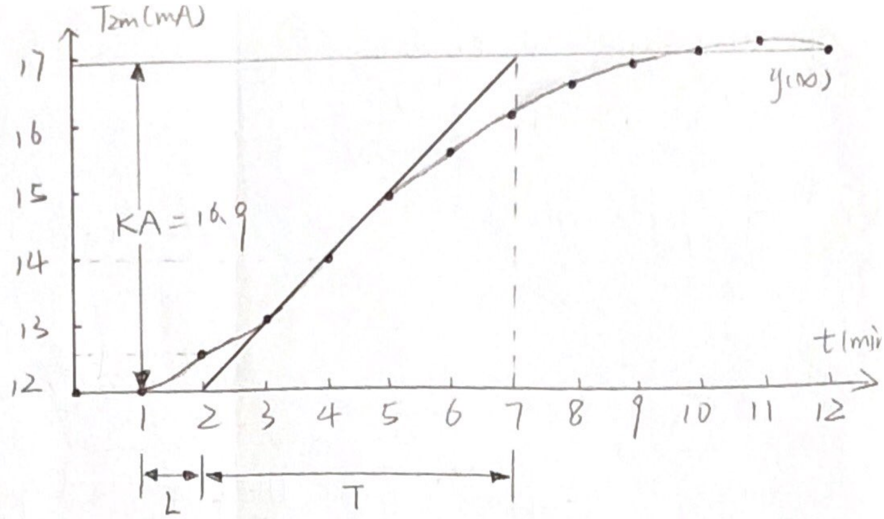
3. (a)

$$G_p(s) = \frac{K}{Ts+1} e^{-Ls},$$

$$y(t) = y_\infty \left(1 - e^{-\frac{(t-1)}{T}}\right),$$

$$K = \frac{y_\infty}{A} = \frac{17-12}{20-18} = 2.5$$



(b) $G_p(s) = \frac{b_1}{s+a_1} e^{-Ls}$, $Y(s) = G_p(s) U(s)$, $u(t) = h \cdot 1(t)$,

$$\Rightarrow y(t) = -a_1 \int_0^t y(\tau) d\tau + b_1 h (t-L)$$

$$= -a_1 \int_0^t y(\tau) d\tau - b_1 h L + h t b_1$$

$$\begin{cases} r(t) = y(t). \\ \phi(t) = \left[-\int_0^t y(\tau) d\tau \quad -h \quad ht \right]^T, \\ \theta(t) = [\, a_1 \quad b_1 L \quad b_1 \,]^T. \end{cases} \Rightarrow r(t) = \phi^T(t)\theta(t)$$

$\Gamma = \Psi \theta$ (matrix form)

$$\Rightarrow \hat{\theta} = (\Psi^T \Psi)^{-1} \Psi^T \Gamma$$

$$\begin{bmatrix} a_1 \\ b_1 \\ L \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_3 \\ \theta_2/\theta_3 \end{bmatrix}$$

(c) $\tilde{g}(s) = \frac{16.9}{5s+1} e^{-s}$, $\tilde{g}_+(s) = e^{-s}$, $\tilde{g}_-(s) = \frac{16.9}{5s+1}$,

$$\tilde{q}(s) = \tilde{g}_-^{-1}(s) = \frac{5s+1}{16.9}, \quad q(s) = \tilde{q} f(s) = \frac{5s+1}{16.9} \frac{1}{1+\lambda s} = \frac{5s+1}{16.9(1+2.75s)}.$$

$$g_c(s) = \frac{q(s)}{1 - \tilde{g}(s) q(s)}, \quad 1 - \tilde{g}(s) q(s) = 1 - \frac{e^{-s}}{1+2.75s} \approx 1 - \frac{1-s}{1+2.75s}$$

$$\Rightarrow g_c(s) = \frac{(5s+1)}{16.9(1+2.75s-1+s)} = \frac{5s+1}{46.475s} = K_c\left(1 + \frac{1}{\tau_I s}\right).$$

4. (a) poles: $P_1 = -\frac{1}{15} < 0$. $P_2 = -\frac{1}{10} < 0$. $P_3 = -\frac{1}{10} < 0$, $P_4 = -\frac{1}{15} < 0$.

zeros: $|G(s)| = \frac{9}{(15s+1)^2} - \frac{16}{(10s+1)^2} = \frac{9(10s+1)^2 - 16(15s+1)^2}{(15s+1)^2(10s+1)^2} = 0$

$\Rightarrow 9(10s+1)^2 - 16(15s+1)^2 = 0$. $900s^2 + 180s + 9 - 3600s^2 - 480s - 16 = 0$

$-2700s^2 - 300s - 7 = 0$,

$S_1 = -\frac{1}{30}$

$S_2 = -\frac{7}{90}$

Since all the poles lie on the left-hand-plane, the system is open-loop stable.

$K = G(0) = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$, $|K| = 9 - 16 = -7 \neq 0$.

Since the inverse of the gain matrix exists, the system is controll-able

(b) $\sigma = (\lambda_i(K^TK))^{\frac{1}{2}}$, $K^T = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$, $K^TK = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 25 & 24 \\ 24 & 25 \end{bmatrix}$

$|\lambda I - K^TK| = \begin{vmatrix} \lambda-25 & -24 \\ -24 & \lambda-25 \end{vmatrix} = (\lambda-25)^2 - 24^2$

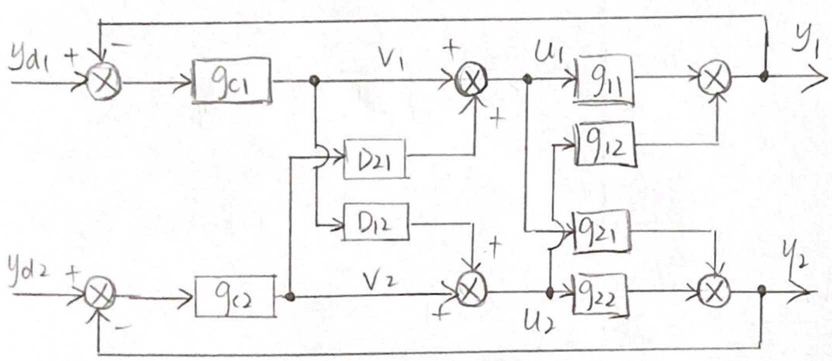
$= \lambda^2 - 50\lambda + 25^2 - 24^2 = \lambda^2 - 50\lambda + 49$.

$\lambda_1 = 49$, $\sigma_1 = \lambda_1^{\frac{1}{2}} = \sqrt{49} = 7$.

$\lambda_2 = 1$ $\sigma_2 = \lambda_2^{\frac{1}{2}} = 1$.

condition number $k = \frac{\sigma_{max}}{\sigma_{min}} = 7$

(c)



$D_{12} = -\frac{g_{12}}{g_{11}} =$

$= -\frac{3(10s+1)}{4(15s+1)} e^s$.

$D_{21} = -\frac{g_{21}}{g_{22}}$

$= -\frac{4(15s+1)}{3(10s+1)} e^s$,

not reliazable, add delay terms.

$u_1 = v_1 + D_{12} v_2$

$u_2 = v_2 + D_{21} v_1$

$D(s) = \begin{bmatrix} 1 & 0 \\ 0 & e^{-s} \end{bmatrix}$

(c) $G_p(s) = \frac{2.5}{3.3s+1} e^{-2.2s}$,

using pade approximation: $e^{-2.2s} = \frac{1-1.1s}{1+1.1s}$

$\Rightarrow \quad G_p(s) = \frac{2.5(1-1.1s)}{(3.3s+1)(1+1.1s)} = \tilde{g}$. $\tilde{g}_+ = (1-1.1s)$, $\tilde{g}_- = \frac{2.5}{(3.3s+1)(1+1.1s)}$

$\tilde{q} = \tilde{g}_-^{-1} = \frac{(1+3.3s)(1+1.1s)}{2.5}$, $q = \tilde{q} f = \frac{(1+3.3s)(1+1.1s)}{2.5(1+2.75s)}$

$g_c = \frac{q}{1-\tilde{g}q}$, $\tilde{g}q = \frac{2.5(1-1.1s)}{(3.3s+1)(1+1.1s)} \frac{(1+3.3s)(1+1.1s)}{2.5(1+2.75s)} = \frac{1-1.1s}{1+2.75s}$

$1-\tilde{g}q = \frac{1+2.75s-1+1.1s}{1+2.75s} = \frac{3.85s}{1+2.75s}$, $g_c(s) = \frac{(1+3.3s)(1+1.1s)}{2.5 \times 3.85s}$

$= \frac{1}{9.625s}(1+4.4s+3.63s^2)$

$= 0.1039 \times (4.4 + \frac{1}{s} + 3.63s)$

$= 0.4571(1+0.2273\frac{1}{s} + 0.825s)$

3 (a) $G_p(s) = \frac{K}{Ts+1} e^{-Ls}$, $y(t) = y_\infty(1 - e^{-\frac{t-L}{T}})$,

$y(t_k) = y_\infty(1 - e^{-\frac{t_k - L}{T}})$, $K = \frac{\Delta y}{A}$ → step test magnitude

$A = 20 - 18 = 2$. $\Delta y = 17.0 - 12 = 5$, $K = 2.5$

① two point method:

find 28.4% point at $t_1 = 3.3$ min $(T/3 + L)$

find 63.2% point at $t_2 = 5.5$ min $(T + L)$

$t_2 - t_1 = \frac{2}{3}T = 2.2$, $T = 3.3$;

$L = t_2 - T = 2.2$;

② log method:

$y(t) = y_\infty(1 - e^{-\frac{t-L}{T}})$, $\frac{y_\infty - y}{y_\infty} = e^{-\frac{t-L}{T}}$. $\ln(\frac{y_\infty - y}{y_\infty}) = -\frac{t}{T} + \frac{L}{T}$

$y_\infty = AK$

$\Rightarrow \ln(\frac{17 - y(t)}{17}) = -\frac{t}{T} + \frac{L}{T}$

(b) $G_p(S) = \frac{b}{S+a} e^{-Ls}$. $sY(s) + aY(s) = b e^{-Ls} U(s)$, $u(t) = h \cdot 1(t)$

$y(t) = -a\int_0^t y(\tau)d\tau + bh(t-L)$

$\gamma(t) = y(t)$,

$\phi(t) = [-\int_0^t y(\tau)d\tau \quad -h \quad ht]^T$,

$\theta(t) = [a \quad bL \quad b]^T$

$\gamma(t) = \phi^T(t)\theta(t)$ $\quad \Gamma = \Psi\theta$, $\theta = (\Psi^T\Psi)^{-1}\Psi^T\Gamma$

$$\begin{bmatrix} a \\ b \\ L \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_3 \\ \theta_2/\theta_3 \end{bmatrix}$$

4 (a) Poles: $p_1 = -\frac{1}{15} = -0.0667 < 0$, $p_2 = -\frac{1}{10} = -0.1 < 0$

$$|G(s)| = \frac{9}{(15s+1)^2} - \frac{16}{(10s+1)^2} = \frac{9(10s+1)^2 - 16(15s+1)^2}{(15s+1)^2(10s+1)^2}$$

$$= \frac{9 \times (100s^2+20s+1) - 16 \times (225s^2+30s+1)}{(225s^2+30s+1)(100s^2+20s+1)} = 0$$

$$\Rightarrow 900s^2 + 180s + 9 - 3600s^2 - 480s - 16 = -2700s^2 - 300s - 7 = 0$$

$$\Rightarrow s_1 = -\frac{1}{30}, \quad s_2 = -\frac{7}{90}$$

5. (a)

$$K = G(0) = \begin{bmatrix} -0.3505 & 0.5068 \\ -0.2556 & 0.4565 \end{bmatrix}, \quad K^{-T} = \begin{bmatrix} -14.9843 & -8.3899 \\ 16.6354 & 11.5049 \end{bmatrix}$$

$$\Lambda = K \otimes K^{T} = \begin{bmatrix} 5.2520 & -4.2520 \\ -4.2520 & 5.2520 \end{bmatrix}, \quad NI = \frac{|G(0)|}{g_{11}(0)\cdot g_{22}(0)} = \frac{-0.0305}{-0.1600} > 0,$$

select 1-1/2-2 pairing.

(b)
$$K_N = \begin{bmatrix} \frac{-0.3505}{2.9727+0.24} & \frac{0.5068}{18.9322+4.47} \\ \frac{-0.2556}{29.656+8.01} & \frac{0.4565}{14.6071+2.65} \end{bmatrix} = \begin{bmatrix} -0.1091 & 0.0217 \\ -0.0068 & 0.0264 \end{bmatrix}.$$

$$K_N^{-T} = \begin{bmatrix} -9.6608 & -2.4884 \\ 7.9409 & 39.9241 \end{bmatrix}, \quad \Lambda_N = K_N \otimes K_N^{-T} = \begin{bmatrix} 1.0540 & -0.0540 \\ -0.0540 & 1.0540 \end{bmatrix}.$$

$$\hat{K} = K \odot \Lambda = \begin{bmatrix} -0.0667 & -0.1192 \\ 0.0601 & 0.0869 \end{bmatrix}, \quad \Gamma = \Lambda_N \odot \Lambda = \begin{bmatrix} 0.2007 & 0.0127 \\ 0.0127 & 0.2007 \end{bmatrix}.$$

$$\hat{g}_{11} = \frac{-0.067\, e^{-0.048s}}{0.5966s+1}, \quad \hat{g}_{12} = \frac{-0.1192\, e^{-0.0568s}}{0.2404s+1}$$

$$\hat{g}_{21} = \frac{0.0601\, e^{-0.1017s}}{0.3766s+1}, \quad g_{22} = \frac{0.0869\, e^{-0.5318s}}{0.3225s+1}$$

(c) $A_m = 3$, $\quad G(s) = \frac{b_0}{a_1 s+1} e^{-Ls}$

for $\hat{g}_{11} = \frac{-0.067 e^{0.048s}}{0.5966s+1}$

$a_1 = 0.5966$, $b_0 = -0.067$, $L = 0.048$.

$$\begin{bmatrix} k_p \\ k_i \\ k_d \end{bmatrix} = \frac{\pi}{2A_m b_0 L} \begin{bmatrix} a_1 \\ 1 \\ 0 \end{bmatrix} = \frac{\pi}{6\times(-0.067)\times 0.048} \begin{bmatrix} 0.5966 \\ 1 \\ 0 \end{bmatrix}. \quad \begin{aligned} k_p &= -97.1328 \\ k_i &= -162.8105 \\ k_d &= 0. \end{aligned}$$

$$\Rightarrow g_{c1} = -97.1328 - \frac{162.8103}{s}$$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2016-2017

EE6225 – PROCESS CONTROL

November/December 2016 | Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 4 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

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1. (a) Give a brief description of the receding horizon principle used in predictive control. Explain why, in predictive control, the sequence of future control signals calculated at time k, may not coincide with those calculated at time $k+1$. That is, $u(k+1|k)$ is, in general, not equal to $u(k+1|k+1)$, even if there is no disturbance nor plant-model mismatch.

(10 Marks)

(b) List some advantages and disadvantages of predictive control, compare and contrast predictive control with PID control. Comment on the computational load of predictive control for systems with constraints. What computations can be carried out at the design stage and what need to be computed when the controller is in operation?

(10 Marks)

2. Consider a discrete-time first order model $y(k) = ay(k-1) + bu(k-1)$ where $y(k)$ and $u(k)$ are the scalar output and the scalar input, respectively.

Define $x(k) = \begin{bmatrix} \Delta y(k) \\ y(k) \end{bmatrix}$, and the input and output increments as

$$\Delta u(k) = u(k) - u(k-1) \text{ and } \Delta y(k) = y(k) - y(k-1).$$

(a) Construct an equivalent model of the form

$$x(k+1) = Ax(k) + B\Delta u(k)$$

and the prediction equation $X = Fx(k) + GU$ where

$$X = \begin{bmatrix} x(k+1) \\ x(k+2) \\ \vdots \\ x(k+N) \end{bmatrix} \text{ and } U = \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N-1) \end{bmatrix}$$

Express the matrices A and B in terms of a and b, and the matrices F and G in terms of A and B. What are the dimensions of F and G?

(5 Marks)

(b) A predictive controller is designed to minimise the cost function

$$J = \sum_{i=1}^{N} (r - y(k+i))^2 + \lambda \Delta u(k)^2$$

with $\Delta u(k+i) = 0, i > 0$, N is the horizon, λ is the control weighting, and r is the constant setpoint.

Derive the predictive control law and show that it can be written in the form

$$\Delta u(k) = k_1 r + k_2 y(k) + k_3 \Delta y(k)$$

Does the predictive controller ensure that the output y tracks the constant set point r at steady state? Justify your answer.

(10 Marks)

(c) For the predictive controller in part 2(b), the output predictions $y(k+i)$ are subjected to constraints

$$-d \leq y(k+i) \leq d, \quad i = 1, 2, \ldots, N$$

where d is a known constant.

Show that the constraints can be expressed as

$$E\Delta u(k) \leq h + Mx(k)$$

Derive the matrices E and M and vector h in terms of A and B. What are the dimensions of E, M and h?

(5 Marks)

3. A process stream is heated using a shell and tube heat exchanger. The exit temperature is controlled by adjusting the steam control valve shown in Figure 1.

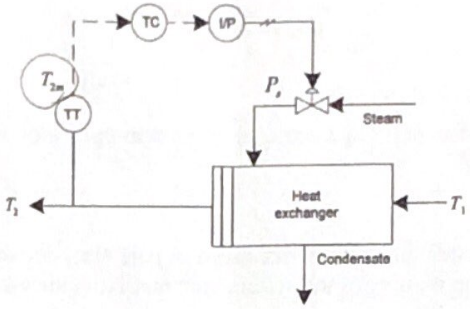

Figure 1

During an open-loop experimental test, the steam pressure P_s was suddenly changed from 18 to 20 bar and the temperature data shown as in Table 1 were obtained.

Table1 Experimental Data

t (min)	T_{2m} (mA)	t (min)	T_{2m} (mA)
0	12	7	16.1
1	12	8	16.4
2	12.5	9	16.8
3	13.1	10	16.9
4	14.0	11	17.0
5	14.8	12	16.9
6	15.4	--	--

(a) Assuming that the process can be approximated by first-order-plus-time delay model, determine the parameters by using the graphical method.

(8 Marks)

(b) For the data presented in Table 1, describe the procedures for determining the model parameters using the Least Squares Method.

(7 Marks)

(c) Design an appropriate *PID* controller for the identified model in part 3(a) using Internal Model Control (IMC) method. Assume that the tuning parameter λ is 2.75 minutes and the control valve and current-to-pressure transducers at the nominal conditions have gains of K_v = 0.9 bar/bar and K_{IP} = 0.75 bar/mA, respectively.

(5 Marks)

4. The transfer function matrix of a two-input two-output system is given as

$$G(s)=\begin{bmatrix}\dfrac{3}{15s+1} & \dfrac{4}{10s+1}\\ \dfrac{4}{10s+1} & \dfrac{3}{15s+1}\end{bmatrix}$$

(a) Find the system poles and zeros. Is the system open loop stable and controllable at steady state?

(7 Marks)

(b) For the transfer function matrix given in part 4(a), determine the singular values and condition number of the system gain matrix. Is the system well- conditioned?

(5 Marks)

(c) Assuming that now the transfer function matrix is changed to

$$G(s)=\begin{bmatrix}\dfrac{3}{15s+1}e^{-s} & \dfrac{4}{10s+1}\\ \dfrac{4}{10s+1}e^{-s} & \dfrac{3}{15s+1}e^{-2s}\end{bmatrix},$$

draw a block diagram for the inverted decoupling control scheme for the process and find the parameters of the decoupler transfer functions. Are the decoupler transfer functions realizable? If necessary, suggest appropriate modifications to make the decoupler transfer functions realizable.

(8 Marks)

5. Consider the transfer function matrix:

$$G(s)=\begin{bmatrix}\dfrac{-0.3505}{2.9727s+1}e^{-0.24s} & \dfrac{0.5068}{18.9322s+1}e^{-4.47s}\\ \dfrac{-0.2556}{29.6560s+1}e^{-8.01s} & \dfrac{0.4565}{14.6071s+1}e^{-2.65s}\end{bmatrix}$$

(a) Recommend an appropriate loop pairing using Relative Normalized Gain Array (RNGA) based method. Justify your recommendations.

(5 Marks)

(b) Find the equivalent transfer function matrix based on the results obtained in part 5(a).

(10 Marks)

(c) Design appropriate decentralized controllers by using the gain and phase margin method with gain margin of 3 for each loop.

(5 Marks)

END OF PAPER