3. $P_u = 10$, $w_u = \frac{2\pi}{P_u} = 0.2\pi$, $h = 0.5$, $a = 0.5$, $L = 5$

$K_u = \frac{4h}{a\pi} = \frac{4}{\pi}$,

$K = \frac{y(\infty)}{u(\infty)} = \frac{0.5}{0.5} = 1$

gain condition: $K_u |G_p(jw_u)| = 1$;

phase condition: $\arg[G_p(jw_u)] = -\pi$;

$$g_p(s) = \frac{Ke^{-sL}}{s^2+as+b} = \frac{1}{a_1 s^2 + b_1 s + c_1} e^{-Ls}, \quad a_1 = \frac{1}{K},\ b_1 = \frac{a}{K},\ c_1 = \frac{b}{K}$$

$$a_1 s^2 + b_1 s + c = \frac{e^{-Ls}}{g_p(s)}, \quad (jw_u)^2 a_1 + jw_u b_1 + c = \frac{e^{-jw_u L}}{g_p(jw_u)}$$

$$c_1 - w_u^2 a_1 + jw_u b_1 = \frac{e^{-jw_u L}}{g_p(jw_u)}$$

$$\Rightarrow \begin{cases} w_u^2 a_1 = c - Re\left\{\frac{e^{-jw_u L}}{g_p(jw_u)}\right\}, \\ w_u b_1 = Im\left\{\frac{e^{-jw_u L}}{g_p(jw_u)}\right\}, \end{cases} \quad g_p(jw_u) = \frac{1}{K_u} = \frac{\pi a}{4h}$$

$$\Rightarrow \begin{cases} a_1 = \frac{1}{w_u^2}\left[c_1 - \frac{4h}{\pi a}\cos(w_u L)\right] = \frac{1}{(0.2\pi)^2}\left(\frac{b}{K} - \frac{4}{\pi}\cos(0.2\pi\times 5)\right) \\ b_1 = \frac{1}{w_u}\left[-\frac{4h}{\pi a}\sin(w_u L)\right] = \frac{1}{0.2\pi}\left(-\frac{4}{\pi}\sin(0.2\pi\times 5)\right) \\ \quad = 0 \end{cases}$$

$$g_p(s) = \frac{K}{s^2+as+b} e^{-sL}, \quad s^2+as+b = \frac{K}{g_p(s)} e^{-Ls},$$

$$(jw_u)^2 + a(jw_u) + b = \frac{K}{g_p(jw_u)} e^{-jw_u L}. \quad g_p(s) = \frac{1}{K_u} = \frac{\pi a}{4h}$$

$$-w_u^2 + b + jw_u a = \frac{4hK}{\pi a}(\cos(w_u L) - j\sin(w_u L))$$

$$\Rightarrow \begin{cases} b - w_u^2 = \frac{4hK}{\pi a}\cos(w_u L) \\ w_u a = -\frac{4hK}{\pi a}\sin(w_u L) \end{cases} \Rightarrow \begin{cases} b - (0.2\pi)^2 = \frac{4K}{\pi}\times\cos\pi = \frac{4K}{\pi}, \\ 0.2\pi a = -\frac{4K}{\pi}\sin\pi = 0 \end{cases}$$

$$\Rightarrow a = 0, \quad b = \frac{4K}{\pi} + (0.2\pi)^2$$

5. (a)

$$K = G(0) = \begin{bmatrix} 22.89 & -11.64 \\ 4.689 & 5.8 \end{bmatrix}, \quad K^{-1} = \begin{bmatrix} 0.0310 & 0.0621 \\ -0.0250 & 0.1222 \end{bmatrix}$$

$$\Lambda = K \otimes K^{-T} = \begin{bmatrix} 0.7090 & 0.2910 \\ 0.2910 & 0.7090 \end{bmatrix}, \quad \hat{K} = K \odot \Lambda = \begin{bmatrix} 32.2849 & -40 \\ 16.1134 & 8.1805 \end{bmatrix}$$

$$K_N = \begin{bmatrix} 4.7967 & -5.2741 \\ 1.9751 & 2.6352 \end{bmatrix}, \quad K_N^{-1} = \begin{bmatrix} 0.1143 & 0.2287 \\ -0.0857 & 0.2080 \end{bmatrix},$$

$$\Lambda_N = K_N \otimes K_N^{-1} = \begin{bmatrix} 0.5482 & 0.4518 \\ 0.4518 & 0.5482 \end{bmatrix}.$$

$$\Gamma = \Lambda_N \odot \Lambda = \begin{bmatrix} 0.7732 & 1.5526 \\ 1.5526 & 0.7732 \end{bmatrix}, \quad \hat{T} = T \otimes \Gamma = \begin{bmatrix} 3.5351 & 2.8055 \\ 3.3754 & 1.3925 \end{bmatrix}$$

$$\hat{L} = L \otimes \Gamma = \begin{bmatrix} 0.1546 & 0.6210 \\ 0.3105 & 0.3093 \end{bmatrix}.$$

$$\hat{G}(s) = \begin{bmatrix} \dfrac{32.2849 e^{-0.1546s}}{3.5351s+1} & \dfrac{-40 e^{-0.6210s}}{2.8055s+1} \\ \dfrac{16.1134 e^{-0.3105s}}{3.3754s+1} & \dfrac{8.1805 e^{-0.3093s}}{1.3925s+1} \end{bmatrix}$$

(b)

$$\hat{G}^{T}(s) = \begin{bmatrix} \dfrac{3.5351s+1}{32.2849} e^{0.1546s} & \dfrac{3.3754s+1}{16.1134} e^{0.3105s} \\ \dfrac{2.8055s+1}{-40} e^{0.6210s} & \dfrac{1.3925s+1}{8.1805} e^{0.3093s} \end{bmatrix}$$

$$G_R(s) = \begin{bmatrix} \dfrac{40 e^{-0.6210s}}{3.5351s+1} & 0 \\ 0 & \dfrac{16.1134 e^{-0.3105s}}{3.3754s+1} \end{bmatrix}$$

$$G_I(s) = \hat{G}^{T}(s)\, G_R(s) = \begin{bmatrix} 1.239 e^{-0.4664s} & 1 \\ (-1)\dfrac{2.8055s+1}{3.5351s+1} & 1.97 \dfrac{1.3925s+1}{3.3754s+1} e^{0.0012s} \end{bmatrix}$$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2015-2016

EE6225 – PROCESS CONTROL

November/December 2015 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 5 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

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1. (a) "Model Predictive Control (MPC) is an open-loop control strategy". Do you agree with this statement? Justify your answer by describing how MPC works.

(8 Marks)

(b) Consider the following single-input/single-output discrete-time system:

$$y(k+1) = 1.6y(k) - 0.8y(k-1) + 0.2u(k)$$

where $u(k)$ is the input, $y(k)$ is the output.

(i) Calculate the range within which the control $u(k)$ should fall so that the following constraint is satisfied:

$-3 \leq y(k+2) \leq 3$

You may assume that

$u(k) = u(k+1)$, and $y(k) = y(k-1) = 1.2$

(8 Marks)

Note: Question No. 1 continues on page 2

(ii) If $u(k) \neq u(k+1)$, explain how you would determine the range of $u(k)$ and $u(k+1)$ to satisfy the same constraints on $y(k+2)$.

(4 Marks)

2. Consider the following single-input/single-output discrete-time system:

$$y(k+1) = a_1 y(k) + a_2 y(k-1) + bu(k)$$

where $u(k)$ is the input, $y(k)$ is the output and a_1, a_2 and b are constants.

(a) Obtain an incremental model of the form

$$\xi(k+1) = A\xi(k) + B\Delta u(k), \qquad y(k) = C\xi(k)$$

where $\Delta u(k) = u(k) - u(k-1)$ and $\xi(k)$ is the state vector. State clearly the matrices A, B and C in terms of the constant parameters a_1, a_2 and b. What is your state vector $\xi(k)$ and why did you choose it this way?

(8 Marks)

(b) With the state space model obtained in part 2(a), design an MPC law which minimises the following cost function:

$$J = \xi(k+1)^T P\xi(k+1) + y(k+1)^T(k+1) + y(k+2)^T(k+2)$$

where P is a matrix of appropriate dimensions and the control increment $\Delta u(k)$ is subject to the constraints $\Delta u(k+j) = 0, j > 0$. Write the MPC law in the form

$$\Delta u(k) = K\,\xi(k)\,.$$

Show clearly how you obtain the gain K. You may present your answer in terms of the A, B and C matrices you have obtained in part 2(a). What are the dimensions of the gain K?

(12 Marks)

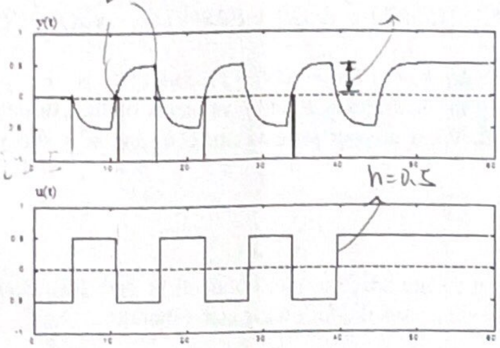
3. (a) Consider a process described by the second order transfer function

$$g_p(s) = \frac{y(s)}{u(s)} = \frac{ke^{-sL}}{s^2 + as + b}$$

where a, b, k and L are the process parameters, u and y are process input and output, respectively.

The process input and output response curves in a relay plus step test are shown as in Figure 1. Determine the process parameters a, b, k and L.

$L=5$ $a=0.5$



$P_u = 10$

$\omega_u = \frac{2\pi}{P_u} = 0.2\pi$

Figure 1. Process input and output in the relay plus step test

(10 Marks)

(b) For a given open-loop unstable process with transfer function

$$G(s) = \frac{(3s+1)e^{-s}}{(3s-1)(10s+1)}$$

it is argued that a stable limit cycle may not be guaranteed under relay feedback test. Please confirm this argument with your justifications.

(5 Marks)

(c) For the transfer function given in part 3(b), propose a simple scheme to guarantee a stable limit cycle under relay feedback test.

(5 Marks)

4. (a) The following incomplete Relative Gain Array (RGA, denoted by Λ) was determined experimentally for a 4 x 4 system with outputs y_1, y_2, y_3 and y_4 to be paired with the inputs m_1, m_2, m_3 and m_4, where ? represents an unknown value. Is there enough information to determine an input/output pairing? If so, complete the RGA matrix and determine the loop pairing based on the RGA information.

$$\Lambda = \begin{bmatrix} 0.64 & 0.72 & ? & -0.20 \\ 0.87 & ? & -0.35 & ? \\ ? & -0.21 & 0.70 & ? \\ -0.36 & ? & ? & 0.88 \end{bmatrix}$$

$\sum row = \sum col = 1$

(5 Marks)

(b) A binary distillation column has three temperature measurements (17th, 24th and 30th trays) that can be used as possible controlled variables for composition. Step testing gives the following steady state input-output relationships:

$$T_{17} = 1.5u_1 + 0.5u_2$$
$$T_{24} = 2.0u_1 + 1.7u_2$$
$$T_{30} = 3.4u_1 + 2.9u_2$$

where u_1 is the steam pressure in reboiler, u_2 is the reflux ratio and all variables are deviation variables. Select a 2×2 control configuration that has the most desirable interactions, as determined by the RGA.

(10 Marks)

(c) Is the information provided in part 4(b) enough to guarantee the best loop pairing decision? Justify your answer.

(5 Marks)

5. The transfer function matrix of a Two-Input Two-Output (TITO) Industrial-scale Polymerization Reactor (IPR) is given as:

$$G(s) = \begin{bmatrix} \frac{22.89}{4.572s+1}e^{-0.2s} & \frac{-11.64}{1.807s+1}e^{-0.4s} \\ \frac{4.689}{2.174s+1}e^{-0.2s} & \frac{5.8}{1.801s+1}e^{-0.4s} \end{bmatrix}$$

Note: Question No. 5 continues on page 5

(a) Determine the Equivalent Transfer Function Matrix for the closed loop system based on Relative Gain Array (RGA), Λ, Relative Normalized Gain Array (RNGA), Φ and Relative Average Residence Time Array (RARTA), Γ.

(10 Marks)

(b) Determine the Normalized Decoupling Matrix based on the Equivalent Transfer Function Matrix obtained in part 5(a) by selecting the decoupler parameters as the maximum values of each column transfer function in the transpose of Equivalent Transfer Function Matrix.

(5 Marks)

(c) Will the decoupling control scheme result in a better performance than the decentralized control structure? Justify your answer based on the interaction analysis.

(5 Marks)

END OF PAPER