7. (a) $g_p(s) = \frac{y(s)}{u(s)} = \frac{k e^{-sL}}{s^2+as+b}$, $y(s) = g_p(s)\, u(s)$.

$$= \frac{b_1 s + b_2}{s^2 + a_1 s + a_2} e^{-sL}, \text{ where } a_1 = a,\ a_2 = b,\ b_1 = 0,\ b_2 = k.$$

$$y(t) = -a_1 \int_0^t y(\tau)d\tau - a_2 \int_0^t \int_0^{\tau} y(\tau)d\tau d\tau_1 + \overset{h}{A} b_1 (t-L) + \frac{1}{2}\overset{h}{A} b_2 (t-L)^2$$

$$= -a_1 \int_0^t y(\tau)d\tau - a_2 \int_0^t \int_0^{\tau} y(\tau)d\tau d\tau_1 + b_1(t-L)A + \frac{1}{2} b_2 (t-L)^2 A$$

$$= -a_1 \int_0^t y(\tau)d\tau - a_2 \int_0^t \int_0^{\tau} y(\tau)d\tau d\tau_1 + (-b_1 L + \frac{1}{2} b_2 L^2)A + (b_1 - b_2 L) + A + \frac{1}{2} b_2 t^2 A$$

$$\begin{cases} \gamma(t) = y(t) \\ \phi(t) = [-\int_0^t y(\tau)d\tau \quad -\int_0^t \int_0^{\tau} y(\tau)d\tau d\tau_1 \quad A \quad tA \quad \frac{1}{2}t^2 A]^T \\ \theta(t) = [a_1 \quad a_2 \quad -b_1 L + \frac{1}{2} b_2 L^2 \quad b_1 - b_2 L \quad b_2]^T \end{cases}$$

$$\Rightarrow \gamma(t) = \phi^T(t)\theta(t), \quad \Gamma = \Psi\theta \Rightarrow \hat{\theta} = (\Psi^T \Psi)^{-1} \Psi^T \Gamma$$

$$\begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \\ L \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \beta \\ \theta_5 \\ \frac{\beta - \theta_4}{\theta_5} \end{bmatrix}, \quad \beta = \pm\sqrt{\theta_4^2 - 2\theta_5\theta_3}$$

$b_1 - b_2 L = \theta_4$

$L = \frac{\beta - \theta_4}{\theta_5}$

$-b_1 L + \frac{1}{2} b_2 L^2 = 0$

$\Delta = b_1^2$

$L = \pm$

(b) $g_p(s) = \frac{k}{s^2+as+b} e^{-Ls}$, $s^2 + as + b = \frac{k}{g_p(s)} e^{-Ls}$,

$(j\omega_n)^2 + (j\omega_n)a + b = \frac{k}{g_p(j\omega_n)} e^{-j\omega_n L}$, $g_p(j\omega_n) = \frac{a\pi}{4h}$,

$$\Rightarrow -\omega_n^2 + b + j\omega_n = \frac{4hk}{a\pi}(\cos(\omega_n L) - j\sin(\omega_n L))$$

$$\Rightarrow \begin{cases} -\omega_n^2 + b = \frac{4hk}{a\pi}\cos(\omega_n L) \\ \omega_n = -\frac{4hk}{a\pi}\sin(\omega_n L) \end{cases}$$

According to figure 1, $a = 0.5$, $h = 0.5$, $P_u = 10$, $\omega_n = \frac{2\pi}{P_u} = 0.2\pi$, $L = 5$.

$$\Rightarrow 0.2\pi = -\frac{2k}{0.5\pi}\sin(\pi).$$

$$-(0.2\pi)^2 + b = \frac{2k}{0.5\pi}\cos(\pi).$$

2. (a) (i) $K = G(0) = \begin{bmatrix} 22.89 & -11.64 \\ 4.689 & 5.8 \end{bmatrix}$, $K^{-T} = \begin{bmatrix} 0.0310 & 0250 \\ 0.0621 & 0.1222 \end{bmatrix}$

$\Delta = K \otimes K^{-T} = \begin{bmatrix} 0.7096 & 0.2904 \\ 0.2904 & 0.7096 \end{bmatrix}$. $NI = \dfrac{|G(0)|}{g_{11}(0) \cdot g_{22}(0)} = \dfrac{187.34196}{22.89 \times 5.8} > 0.$

the process is interactive, 1-1/2-2 pairing should be selected.

(ii) $|K| = 187.34196 \neq 0$. the inverse of the gain matrix exists.
hence the process is controllable at steady state.

(iii) poles: $P_1 = -\frac{1}{4.572} < 0$, $P_2 = -\frac{1}{1.8000} < 0$, $P_3 = -\frac{1}{2.174} < 0$,

$P_4 = -\frac{1}{1.8000} < 0$, since all the poles lie on the left-hand-plane
the system is open loop stable.

(iv)

$$|G(s)| = \frac{132.762}{(4.572s+1)(1.8s+1)} + \frac{54.57996}{(2.174s+1)(1.8s+1)}$$

$$= \frac{132.762(2.174s+1) + 54.57996(4.572s+1)}{(4.572s+1)(1.8s+1)(2.174s+1)} = 0$$

(v)

$$\Rightarrow 288.624s + 132.762 + 249.5396s + 54.57996 = 0$$

$$538.1636s + 187.34196 = 0.$$

$\Rightarrow$ zero: $s = -0.3481$

(b). $K = \begin{bmatrix} 22.89 & -11.64 \\ 4.689 & 5.8 \end{bmatrix}$, $K^T = \begin{bmatrix} 22.89 & 4.689 \\ -11.64 & 5.8 \end{bmatrix}$.

$$K^T K = \begin{bmatrix} 545.9388 & -239.2434 \\ -239.2434 & 169.1296 \end{bmatrix}.$$

$$|(\lambda I - K^T K)| = \begin{vmatrix} \lambda - 545.9388 & 239.2434 \\ 239.2434 & \lambda - 169.1296 \end{vmatrix}$$

$$= (\lambda - 545.9388)(\lambda - 169.1296) - (239.2434)^2$$

$$= \lambda^2 - 715.0684\lambda - 35097.0064.$$

$\Rightarrow \lambda_1 = 761.1772$, $\lambda_2 = 46.1088$,

$\sigma_1 = (\lambda_1)^{\frac{1}{2}} = 27.5894$, $\sigma_2 = (\lambda_2)^{\frac{1}{2}} = 6.7903$,

$\Rightarrow$ condition number $k = \dfrac{\sigma_{max}}{\sigma_{min}} = 4.0631$, well-conditioned.

3. (a) $K = G(0) = \begin{bmatrix} -2.2 & 1.5 \\ -2.8 & 4.3 \end{bmatrix}$, $K^{-T} = \begin{bmatrix} -0.1500 & [illegible] \\ 0.2234 & 0.3780 \end{bmatrix}$

$\Lambda = K \otimes K^{-T} = \begin{bmatrix} 1.6254 & -0.6254 \\ -0.6254 & 1.6254 \end{bmatrix}$, $\hat{K} = K \odot \Lambda = \begin{bmatrix} -1.3535 & -2.0787 \\ 4.4771 & 2.6455 \end{bmatrix}$

$K_N = \begin{bmatrix} -0.275 & 0.1781 \\ -0.2478 & 0.4503 \end{bmatrix}$, $K_N^{-T} = \begin{bmatrix} -5.6499 & -3.1092 \\ 2.2346 & 3.4505 \end{bmatrix}$,

$\Lambda_N = K_N \otimes K_N^{-T} = \begin{bmatrix} 1.5537 & -0.5537 \\ -0.5537 & 1.5537 \end{bmatrix}$, $\Gamma = \Lambda_N \odot \Lambda = \begin{bmatrix} 0.9559 & 0.8853 \\ 0.8853 & 0.9559 \end{bmatrix}$

$\hat{T} = T \otimes \Gamma = \begin{bmatrix} 6.6913 & 6.1971 \\ 8.4103 & 8.7943 \end{bmatrix}$, $\hat{L} = L \otimes \Gamma = \begin{bmatrix} 0.9559 & 0.2656 \\ 1.5935 & 0.3346 \end{bmatrix}$

$$\hat{G}(s) = \begin{bmatrix} \dfrac{-1.3535e^{-0.9559}}{6.6913s+1} & \dfrac{-2.0787e^{-0.2656}}{6.1971s+1} \\ \dfrac{4.4771e^{-1.5935}}{8.4103s+1} & \dfrac{2.6455e^{-0.3346}}{8.7943s+1} \end{bmatrix}.$$

with consideration of integrity:

$$\hat{G}(s) = \begin{bmatrix} \dfrac{-1.3535e^{-s}}{7s+1} & \dfrac{1.3e^{-0.3s}}{7s+1} \\ \dfrac{4.4771e^{-1.8s}}{9.5s+1} & \dfrac{4.3e^{-0.35s}}{9.2s+1} \end{bmatrix}.$$

(b) $\tilde{g}_{11}(s) = \dfrac{-2.2e^{-s}}{7s+1}$, $\tilde{g}_{+}(s) = e^{-s}$, $\tilde{g}_{-}(s) = \dfrac{-2.2}{7s+1}$, $\tilde{q}(s) = \tilde{g}_{-}^{-1}(s) = \dfrac{7s+1}{-2.2}$

$q(s) = \tilde{q}(s)f(s) = \dfrac{-(7s+1)}{2.2(1+\lambda s)} = -\dfrac{7s+1}{2.2(1+0.8s)}$, $1-\tilde{g}(s)q(s) = 1-\dfrac{e^{-s}}{1+0.8s} = 1-\dfrac{1-s}{1+0.8s} = \dfrac{1+0.8s-(1-s)}{1+0.8s}$

$g_c(s) = \dfrac{q(s)}{1-\tilde{g}(s)q(s)} = \dfrac{-(7s+1)}{2.2\times1.8s} = -\dfrac{7s+1}{3.96s} = K_c^{*}(1+\dfrac{1}{\tau_I s})$,

$K_c^{*} = -1.7677$

$K_c = |0.8+\sqrt{0.8^2-0.8}|\,K_c^{*} = -2.1212.$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2013-2014

EE6225 – Process Control

November/December 2013 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains FIVE (5) questions and comprises FIVE (5) pages.
2. Answer ALL questions.
3. All questions carry equal marks.

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1. Consider a process described by a second order transfer function

$$g_p(s)=\frac{y(s)}{u(s)}=\frac{ke^{-sL}}{s^2+as+b} \qquad \text{------- Equation (I)}$$

where a, b, k and L are the process parameters, u and y are process input and output, respectively.

(a) Given a sequence of N observations (y,u) under step test with amplitude of h, and assuming that L is determined through observation of the response curve, write the mathematical equations using Least Squares method to estimate the process parameters a, b and k in Equation (I).

(10 marks)

(b) The process input and output response curves in a relay plus step test are as shown in Figure 1 on page 2. Describe the procedures for determining the process parameters a, b, k and L in Equation (I).

Note: Question No. 1 continues on page 2

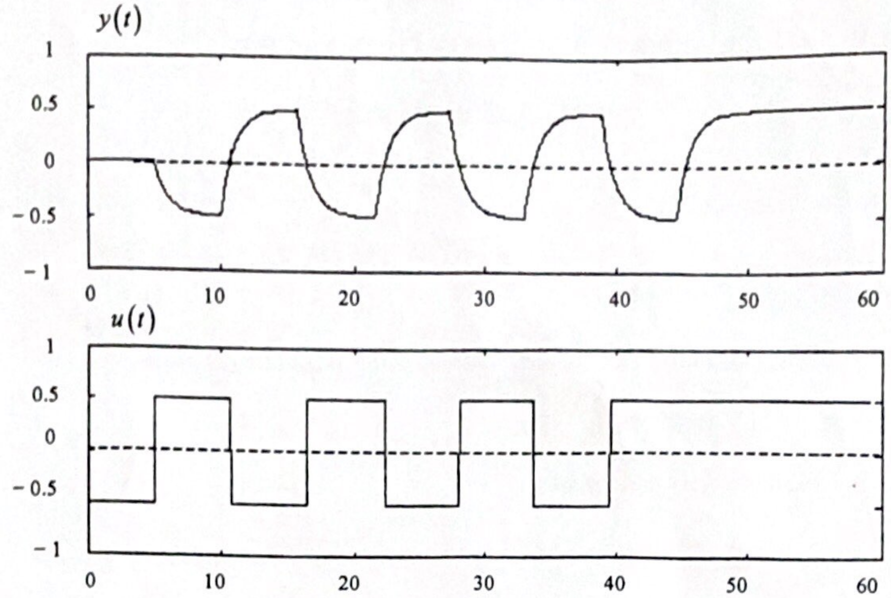

Figure 1: Process Input $u(t)$ and Output $y(t)$ in a Relay Test

(10 marks)

2. The transfer function matrix of a Two-Input Two-Output (TITO) is given as:

$$G(s) = \begin{bmatrix} \dfrac{22.89}{4.572s+1} & \dfrac{-11.64}{1.800s+1} \\ \dfrac{4.689}{2.174s+1} & \dfrac{5.8}{1.800s+1} \end{bmatrix} \quad \text{------- Equation (II)}$$

(a) Answer the following questions with your justifications

(i) Is the process interactive?
(ii) Is the process controllable at steady state?
(iii) Is the process open loop stable?
(iv) What are the system poles?
(v) Find the system zeros.

(10 marks)

(b) For the transfer function matrix given in Equation (II), determine the singular value and condition number of the process gain matrix. Is the system well-conditioned?

(10 marks)

3. The transfer function matrix of a Two-Input Two-Output (TITO) distillation process is given as

$$G(s)=\begin{bmatrix} \dfrac{-2.2e^{-s}}{7s+1} & \dfrac{1.3e^{-0.3s}}{7s+1} \\ \dfrac{-2.8e^{-1.8s}}{9.5s+1} & \dfrac{4.3e^{-0.35s}}{9.2s+1} \end{bmatrix}$$

(a) Calculate the Relative Gain Array (RGA), Λ; Relative Normalized Gain Array (RNGA), Λ_N; and Relative Average Residence Time Array (RARTA); Γ. Determine the Equivalent Transfer Function Matrix for the closed loop system.

(10 marks)

(b) One of the classical methods for designing MIMO decentralized control systems is the detuning factor method, in which PID controllers are designed for the paired open loops, and then the designed controller gains are detuned by a detuning factor

$$K_{ci}=\begin{cases} \left(\lambda-\sqrt{\lambda^2-\lambda}\right)K_{ci}^{\bullet} & \lambda>1.0 \\ \left|\lambda+\sqrt{\lambda^2-\lambda}\right|K_{ci}^{\bullet} & \lambda<1.0 \end{cases}$$

where $K_{ci}^{\bullet}$ is the proportional gain of i loop controllers when other loop open. Design decentralized PI controllers for the system by detuning method using the IMC technique with $\lambda=0.8\times L$ and Taylor approximation for the time delay. Do you think that the performance of the decentralized controller will be satisfactory? Justify your answer based on interaction analysis.

(5 marks)

(c) By selecting the maximum steady state gain, maximum time constant and maximum time delay on each column of Equivalent Transfer Function Matrix in part 3(a) as the parameters of the decoupled process transfer functions, find the parameters of the decoupling transfer function matrix.

(5 marks)

4. Consider the state space model of a system with time delay d

$$x(k+1) = Ax(k) + B\Delta u(k-d)$$
$$y(k) = Cx(k)$$

where y, u, and x are the output, input and state vectors of the system, respectively; A, B, and C are known matrices of appropriate dimensions and $\Delta u(k) = u(k) - u(k-1)$ is the control increment at time k.

(a) Given a constant set-point w and the cost function

$$J = \sum_{j=1}^{N} \left[w - y(k+d+j) \right]^2 + \lambda \Delta u(k)^2$$

where N is the prediction horizon, λ is a tuning parameter, and $\Delta u(k+j) = 0$ for $j = 1, 2, \ldots$.

Derive the MPC law and write it in the form

$$\Delta u(k) = K_1 w + K_2 x(k) + K_3 f.$$

Show clearly how the gains K_1, K_2 and K_3 are obtained, and which variables are contained in the vector f.

(12 marks)

(b) If $N = 3$, $d = 2$, $A = \begin{bmatrix} 1 & 0.8 \\ 0 & 0.8 \end{bmatrix}$, $B = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, obtain an expression for the gain K_1 in part 4(a) in terms of the tuning parameter λ.

(8 marks)

5. A two-zone temperature control system is represented by the following discrete-time state space model:

$$x(k+1) = Ax(k) + B\Delta u(k),\ \ y_1(k) = C_1 x(k),\ \ y_2(k) = C_2 x(k)$$

where y_1 and y_2 are the temperatures for zone 1 and zone 2, respectively; $\Delta u(k) = u(k) - u(k-1)$ is the incremental control input at time k; x is the state vector; the matrices A, B, C_1 and C_2 have dimensions $n \times n$, $n \times 2$, $1 \times n$ and $1 \times n$, respectively.

It is desirable to implement an MPC which minimizes the following cost function:

$$\min J = \sum_{j=1}^{3} \left[w - y_1(k+j) \right]^2 + \sum_{j=1}^{3} \left[w - y_2(k+j) \right]^2$$

where w is the common constant set-point for the two zones and $\Delta u(k+j) = 0$ for $j = 1, 2, \ldots$.

In addition, the control specification states that the temperature difference between zone 1 and zone 2 should not exceed 1 degree to ensure product quality.

Assume that the function $u = QP(M, N, H, g)$ which solves the Quadratic Program

$$\min_{z} z^T M z - 2z^T N, \quad s.t. \quad Hz \le g$$

is available.

(a) Show clearly how you derive the quantities M, N, H and g so that the function $u = QP(M, N, H, g)$ can be used to implement the constrained MPC. What are the dimensions of M, N, H and g? What is z and what are its dimensions?

(12 marks)

(b) Using pseudo code, illustrate how you would implement a constrained MPC for the two-zone temperature control problem. In your pseudo code, show clearly what quantities can be pre-computed and what quantities need to be updated at every sampling instance. State any other assumptions that you may need.

(8 marks)

End of Paper