7.

(a) $m\dfrac{d^2x_2}{dt^2} + B\dfrac{dx_2}{dt} + k(x_2 - x_1) = 0$.

$$\frac{d^2y}{dt^2} + a_1\frac{dy}{dt} + a_2 y = b_0 u, \quad y = x_2,\ u = x_1,\ b_0 = \frac{k}{m},\ a_1 = \frac{B}{m},\ a_2 = \frac{k}{m}.$$

$$\Rightarrow \quad y(t) + a_1\int_0^{\tau_1} y(\tau)d\tau + a_2\int_0^{\tau_1}\int_0^{\tau} y(\tau)d\tau d\tau_1 = b_0\int_0^{\tau_1}\int_0^{\tau} u(\tau)d\tau d\tau_1,$$

$$y(t) = -a_1\int_0^{\tau} y(\tau)d\tau - a_2\int_0^{\tau_1}\int_0^{\tau} y(\tau)d\tau d\tau_1 + b_0\int_0^{\tau_1}\int_0^{\tau} u(\tau)d\tau d\tau_1,$$

$r(t) = y(t)$

$\phi(t) = [-\int_0^{\tau_1} y(\tau)d\tau,\ -\int_0^{\tau_1}\int_0^{\tau} y(\tau)d\tau d\tau_1,\ \int_0^{\tau_1}\int_0^{\tau} u(\tau)d\tau d\tau_1]^T$

$\theta(t) = [a_1,\ a_2,\ b_0]^T$

$\Rightarrow \ r(t) = \phi^T(t)\,\theta(t)$

given a sequence of N observations, we have

$$\Gamma = \begin{bmatrix} r_1(t) \\ r_2(t) \\ \vdots \\ r_N(t) \end{bmatrix}, \quad \Psi = \begin{bmatrix} \phi_1^T(t) \\ \phi_2^T(t) \\ \vdots \\ \phi_N^T(t) \end{bmatrix}, \quad \text{hence } \Gamma = \Psi\theta,\ \hat{\theta} = (\Psi^T\Psi)^{-1}\Psi^T\Gamma.$$

$$\begin{bmatrix} a_1 \\ a_2 \\ b_0 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}, \quad a_1 = \frac{B}{m} = \theta_1, \quad a_2 = \frac{k}{m} = \theta_2, \quad b_0 = \frac{k}{m} = \theta_3$$

(b) The laplace transform of the system signal is as follows:

$$s^2Y(s) + a_1 sY(s) + a_2 Y(s) = b_0 U(s),$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0}{s^2 + a_1 s + a_2}, \quad s^2 + a_1 s + a_2 = \frac{b_0}{G(s)}.$$

replace s by $j\omega_u$: $(j\omega_u)^2 + a_1(j\omega_u) + a_2 = \dfrac{b_0}{G(j\omega_u)} = \dfrac{4hb_0}{a\pi}$

$$-\omega_u^2 + a_2 + ja_1\omega_u = \frac{4hb_0}{a\pi}$$

equal the Re and Im parts, we have:

$$\begin{cases} a_2 - \omega_u^2 = \dfrac{4hb_0}{a\pi} \\ a_1\omega_u = 0 \end{cases}$$

2、(a)

$$K = G(0) = \begin{bmatrix} 0.261 & 0.261 \\ 0.247 & -0.247 \end{bmatrix}, \quad K^T = \begin{bmatrix} 0.261 & 0.247 \\ 0.261 & -0.247 \end{bmatrix}.$$

$$K^TK = \begin{bmatrix} 0.1291 & 0.0071 \\ 0.0071 & 0.1291 \end{bmatrix}, \quad |\lambda I - K^TK| = \begin{vmatrix} \lambda - 0.1291 & -0.0071 \\ -0.0071 & \lambda - 0.1291 \end{vmatrix}$$

$$= (\lambda - 0.1291)^2 - (0.0071)^2$$

$$\Rightarrow \lambda_1 = 0.1362, \quad \sigma_1 = \sqrt{\lambda_1} = 0.3691,$$

$$\lambda_2 = 0.122, \quad \sigma_2 = \sqrt{\lambda_2} = 0.3493,$$

condition number:

$$k = \frac{\sigma_{max}}{\sigma_{min}} = 1.0567.$$

(b) $$G(0) = \begin{bmatrix} 0.261 & 0.261 \\ 0.247 & -0.247 \end{bmatrix}, \quad G^{-1} = \begin{bmatrix} 1.9157 & 2.0243 \\ 1.9157 & -2.0243 \end{bmatrix}.$$

$$G_R = \begin{bmatrix} 0.261 & 0 \\ 0 & -0.247 \end{bmatrix}, \quad G_I = G^{-1}G_R = \begin{bmatrix} 0.5 & -0.5 \\ 0.5 & 0.5 \end{bmatrix},$$

$$\Rightarrow \begin{aligned} u_1 &= 0.5v_1 - 0.5v_2 = v_1 + g_{I1}u_2 \\ u_2 &= 0.5v_1 + 0.5v_2 = v_2 + g_{I2}u_1 \end{aligned}$$

$$u_1 = g_{I11}v_1 + g_{I12}v_2$$

$$u_2 = g_{I21}v_1 + g_{I22}v_2$$

$$g_{I1} = -\frac{g_{12}}{g_{11}} = -1, \quad g_{I2} = -\frac{g_{21}}{g_{22}} = 1$$

(c) $$K = \begin{bmatrix} 0.261 & 0.261 \\ 0.247 & -0.247 \end{bmatrix}, \quad K^{-T} = \begin{bmatrix} 1.9157 & 1.9157 \\ 2.0243 & -2.0243 \end{bmatrix}.$$

$$\Lambda = K \otimes K^{-T} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}.$$ the decoupling is feasible.

$$K_{new} = \begin{bmatrix} 0.261 & 0.261 \\ -0.0558 & -0.06175 \end{bmatrix}, \quad \det(K_{new}) = -0.00162,$$

$$K_{new}^{-T} = \begin{bmatrix} 39.7630 & -35.9316 \\ 168.0672 & -168.0672 \end{bmatrix}$$

$$\Lambda_{new} = K_{new} \otimes K_{new}^{-T} = \begin{bmatrix} 10.3781 & -9.3781 \\ -9.3781 & 10.3781 \end{bmatrix}.$$ hard to decouple.

The differen between the hot stream temperature (28.5°C) and cold stream temperature (28°C) is tiny, hence the effects of the two input cannot be distinguished well.

2、(a)

$$K = G(0) = \begin{bmatrix} 0.261 & 0.261 \\ 0.01235(T_H - T_S) & 0.01235(T_C - T_S) \end{bmatrix} = \begin{bmatrix} 0.261 & 0.261 \\ 0.247 & -0.247 \end{bmatrix}.$$

$$K^T = \begin{bmatrix} 0.261 & 0.247 \\ 0.261 & -0.247 \end{bmatrix}, \quad K^T K = \begin{bmatrix} 0.1291 & 0.0071 \\ 0.0071 & 0.1291 \end{bmatrix}.$$

$$|\lambda I - K^T K| = \left| \begin{bmatrix} \lambda - 0.1291 & -0.0071 \\ -0.0071 & \lambda - 0.1291 \end{bmatrix} \right| = (\lambda - 0.1291)^2 - (0.0071)^2 = \lambda^2 - 0.2582\lambda + 0.0166166$$

$\lambda_1 = 0.1362$, $\sigma_1 = \sqrt{\lambda_1} = 0.3691$

$\lambda_2 = 0.122$, $\sigma_2 = \sqrt{\lambda_2} = 0.3493$

condition number $k = \dfrac{\sigma_{max}}{\sigma_{min}} = 1.0567$.

(b) X steady-state decoupler

$$G_R = \begin{bmatrix} \dfrac{0.261}{43.5s+1} & 0 \\ 0 & \dfrac{-0.247}{21.7s+1} \end{bmatrix},$$

$$G_I = G^{-1} G_R = \hat{G}^T G_R, \quad G^{-1} = \frac{1}{\Delta} \begin{bmatrix} \dfrac{-0.247}{21.7s+1} & \dfrac{-0.261}{43.5s+1} \\ \dfrac{-0.247}{21.7s+1} & \dfrac{0.261}{43.5s+1} \end{bmatrix}.$$

$$\Delta = \frac{-0.064467}{(21.7s+1)(43.5s+1)} - \frac{0.064467}{(21.7s+1)(43.5s+1)} = \frac{-0.128934}{(21.7s+1)(43.5s+1)},$$

$$\Rightarrow G^{-1} = \begin{bmatrix} 1.9157(43.5s+1) & 2.0243(21.7s+1) \\ 1.9157(43.5s+1) & -2.0243(21.7s+1) \end{bmatrix}$$

$$\Rightarrow G_I = G^{-1} G_R = \begin{bmatrix} 0.4999 & -0.5000 \\ 0.4999 & 0.5000 \end{bmatrix}$$

(c)

$$K' = \begin{bmatrix} 0.261 & 0.261 \\ -0.055575 & -0.06175 \end{bmatrix}, \quad |K'| = -1.6117 \times 10^{-3}$$

3. (a) $K = G(0) = \begin{bmatrix} -2.2 & 1.3 \\ -2.8 & 4.3 \end{bmatrix}$. $K^{-1} = \dfrac{\begin{bmatrix} 4.3 & -1.3 \\ 2.8 & -2.2 \end{bmatrix}}{-5.82} = \begin{bmatrix} -0.7388 & 0.2234 \\ -0.4811 & 0.3780 \end{bmatrix}$

$\Lambda = K \otimes K^{-T} = \begin{bmatrix} 1.6254 & -0.6254 \\ -0.6254 & 1.6254 \end{bmatrix}$,

$\Rightarrow$ select 1-1/2-2 pairing. $NI = \dfrac{|G(0)|}{g_{11}(0)\cdot g_{22}(0)} = \dfrac{-5.82}{-2.2\times 4.3} = 0.6152 > 0$

(b) $K_N = \begin{bmatrix} \dfrac{-2.2}{7+1} & \dfrac{1.3}{7+0.3} \\ \dfrac{-2.8}{9.5+1.8} & \dfrac{4.3}{9.2+0.35} \end{bmatrix} = \begin{bmatrix} -0.275 & 0.1781 \\ -0.2478 & 0.4503 \end{bmatrix}$.

$K_N^{-1} = \dfrac{\begin{bmatrix} 0.4503 & -0.1781 \\ 0.2478 & -0.275 \end{bmatrix}}{-0.07969} = \begin{bmatrix} -5.6506 & 2.2349 \\ -3.1095 & 3.4509 \end{bmatrix}$.

$\Lambda_N = K_N \otimes K_N^{-T} = \begin{bmatrix} 1.5539 & -0.5539 \\ -0.5539 & 1.5539 \end{bmatrix}$, $\Rightarrow$ select 1-1/2-2 pairing.

$NI = \dfrac{-0.0797}{-0.275\times 0.4503} = 0.6436 > 0$

(c) $\hat{K} = K \odot \Lambda = \begin{bmatrix} -1.3535 & -2.0787 \\ 4.4771 & 2.6455 \end{bmatrix}$,

$\Gamma = \Lambda_N \odot \Lambda = \begin{bmatrix} 0.9560 & 0.8857 \\ 0.8857 & 0.9560 \end{bmatrix}$, $\hat{T} = T \otimes \Gamma = \begin{bmatrix} 6.6920 & 6.1999 \\ 8.4141 & 8.7952 \end{bmatrix}$

$\hat{L} = L \otimes \Gamma = \begin{bmatrix} 0.9560 & 0.2657 \\ 1.5943 & 0.3346 \end{bmatrix}$,

$$\Rightarrow \hat{G}(s) = \begin{bmatrix} \dfrac{-1.3535e^{-0.9560s}}{6.6920s+1} & \dfrac{-2.0787e^{-0.2657s}}{6.1999s+1} \\ \dfrac{4.4771e^{-1.5943s}}{8.4141s+1} & \dfrac{2.6455e^{-0.3346s}}{8.7952s+1} \end{bmatrix}$$

with consideration of integrity:

$$\hat{G}(s) = \begin{bmatrix} \dfrac{-2.2e^{-s}}{7s+1} & \dfrac{1.3e^{-0.3s}}{7s+1} \\ \dfrac{-2.8e^{-1.8s}}{9.5s+1} & \dfrac{4.3e^{-0.35s}}{9.2s+1} \end{bmatrix}.$$

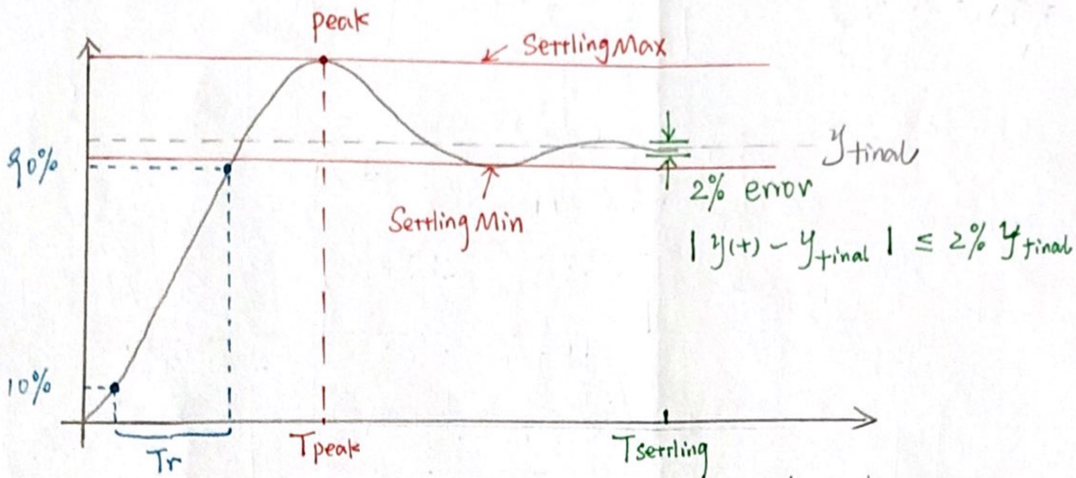
4. (a) sampling time: $\frac{1}{20} T_r \le T_s \le \frac{1}{10} T_r$

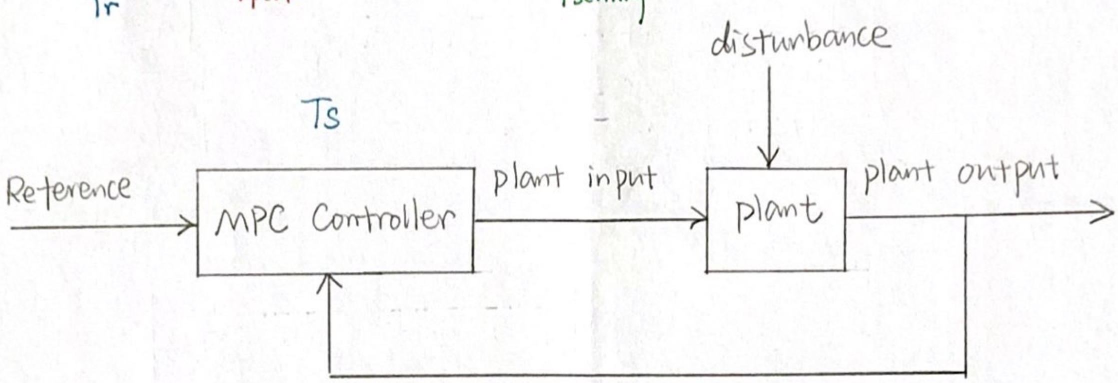
(b) prediction horizon: $p \cdot T_s \ge T_{settling}$

(c) control horizon: $0.1p \le m \le 0.2p$

(d) constraints: the system constraint and the control constraint cannot be both hard at the same time. (1 hard + 1 soft or 2 soft is ok)

(e) weight: make a tradeoff between multiple options





Constraint: set output constraint as soft constraint and avoid having both hard constraints on input constraint and input rate constraint.

weight: (the weight of) the output which desires more attentation should have larger weight than those desire less attentations.

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2018-2019

EE6225 – PROCESS CONTROL

November/December 2018 | Time Allowed: 3 hours

INSTRUCTIONS

1. This Paper contains 5 questions and comprises 5 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

B A

1. Consider a physical spring-damper system as shown in Figure 1, where x_2 and x_1 are position B (output variable) and position A (input variable), respectively.

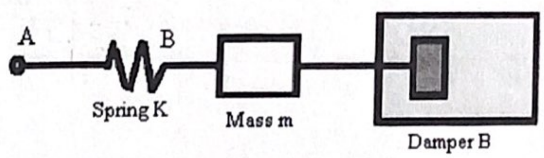


Figure 1

The differential equation of the process is given as

$$m\frac{d^2x_2}{dt^2}+B\frac{dx_2}{dt}+k(x_2-x_1)=0$$

which can be further simplified to

$\frac{k}{m}x_2$

$$\frac{d^2y}{dt^2}+a_1\frac{dy}{dt}+a_2y=b_0u$$

$\frac{k}{m}x_1$

where $y\equiv x_2$, $u\equiv x_1$, $a_2=k/m$, $a_1=B/m$, and $b_0=k/m$.

Note: Question No. 1 continues on page 2

(a) Given a sequence of N observations (y, u) obtained using the step test, formulate the Least Squares method to estimate the system parameters.

(8 Marks)

(b) If the test signal is relay plus step, write the necessary equations for estimating the system parameters.

(7 Marks)

(c) Can you use a relay feedback test to identify the system parameters? Justify your answer.

(5 Marks)

2. The transfer function of a stirred mixing tank expanded with an additional heater is given as

$$G(s) = \begin{bmatrix} \dfrac{0.261}{43.5s+1} & \dfrac{0.261}{43.5s+1} \\ \dfrac{0.01235(T_H - T_s)}{21.7s+1} & \dfrac{0.01235(T_C - T_s)}{21.7s+1} \end{bmatrix}$$

where: Hot Stream Temperature: $T_H = 53^{\circ}$C,
Cold Stream Temperature: $T_c = 13^{\circ}$C and
Operating steady-state tank temperature: $T_s = 33^{\circ}$C.

(a) Determine the singular value and conditioning number of the system's gain matrix.

$G_I = G^{-1} G_R \quad u_1 = g_{I11} v_1 + g_{I12} v_2 , \quad u_2 = g_{I21} v_1 + g_{I22} v_2$

(10 Marks)

(b) Find the generalized steady-state decoupler, assuming that the diagonal elements of steady state gain matrix K are retained as the elements of the decoupled gain matrix K_R.

(6 Marks)

(c) If the operating conditions of the system are changed to

Hot Stream Temperature: $T_H = 28.5^{\circ}$C,
Cold Stream Temperature: $T_c = 28^{\circ}$C and
Operating steady-state tank temperature: $T_s = 33^{\circ}$C,

is decoupling still practically possible? Explain your results from both mathematical and physical viewpoints.

(4 Marks)

3. The transfer function matrix of a multivariable process is described as

$$G(s) = \begin{bmatrix} \dfrac{-2.2}{7s+1}e^{-s} & \dfrac{1.3}{7s+1}e^{-0.3s} \\ \dfrac{-2.8}{9.5s+1}e^{-1.8s} & \dfrac{4.3}{9.2s+1}e^{-0.35s} \end{bmatrix}$$

(a) Determine the loop pairing for the process using Relative Gain Array (RGA) method.

(6 Marks)

(b) Determine the loop pairing for the process using Relative Normalized Gain Array (RNGA) method.

(6 Marks)

(c) Using the information obtained in Part 3(b), determine the Equivalent Transfer Functions (ETFs) for the paired loops of the process.

(8 Marks)

4. Answer the following questions related to Model Predictive Control (MPC).

(a) What are the selection criteria for the sampling time?

(4 Marks)

(b) What are the selection criteria for the prediction horizon?

(4 Marks)

(c) What are the selection criteria for the control horizon?

(4 Marks)

(d) What are the selection criteria for the constraints of the MPC?

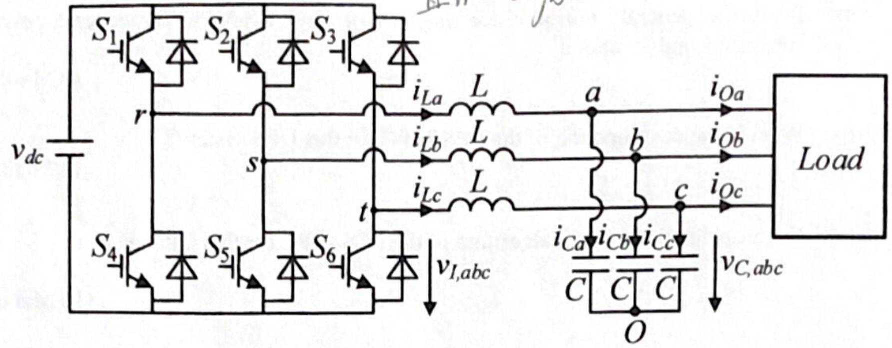
(4 Marks)

(e) What are the selection criteria for the weights of the MPC?

(4 Marks)

5. The voltage source inverter (VSI) of an uninterruptible power supply (UPS) is a system to obtain a high-quality output sinusoidal voltage. Figure 2 presents a typical UPS system with its system variables and parameters defined in Table 1.

直流电源



交流负载

Figure 2

Table 1

Variable	Description
$v_{C,abc} = \{v_{aO}\ v_{bO}\ v_{cO}\}^T$	Output filter capacitor voltage vector
$i_{L,abc} = \{i_{La}\ i_{Lb}\ i_{Lc}\}^T$	Output inductor current vector
$v_{I,abc} = \{v_{rO}\ v_{sO}\ v_{tO}\}^T$	VSI output voltage vector
$S_{abc} = \{S_a\ S_b\ S_c\}^T$	Switching vector
$S_{P\vert p} = \{-1,\ 1\}^T$	Switching functions
L	Output filter inductance
C	Output filter capacitance
v_{dc}	dc-link voltage

The discrete model of the above UPS system is as follows:

$$x_{abc}(k+1) = A_q^{abc} x_{abc}(k) + B_q^{abc}\overline{v}_{I,abc}(k) + B_{dq}^{abc}\overline{i}_{O,abc}(k)$$

where A_q^{abc}, B_q^{abc} and B_{dq}^{abc} are default to be known in this question. x_{abc}, $\overline{v}_{I,abc}$ and $\overline{i}_{O,abc}$ are expressed as

$$x_{abc} = \{i_{L,abc}\ v_{C,abc}\}^T$$

$$\overline{v}_{I,abc} = \{v_{I,abc}\ 0\ 0\ 0\}^T$$

$$\overline{i}_{O,abc} = \{0\ 0\ 0\ i_{O,abc}\}^T$$

Note: Question No. 5 continues on page 5

If the system is required to control its output filter capacitor voltage vector ($v_{C,abc} = \{v_{aO}\ v_{bO}\ v_{cO}\}^T$) to the reference voltage vector ($v^*_{C,abc} = \{v^*_{aO}\ v^*_{bO}\ v^*_{cO}\}^T$) via Finite Control Set Model Predictive Control (FCS-MPC), answer the following questions:

(a) Draw the general control block diagram of the FCS-MPC for general power converters and explain it.

(6 Marks)

(b) What is the cost function of the FCS-MPC for this UPS system?

(4 Marks)

(c) Develop detailed control algorithm of the FCS-MPC for this UPS system.

(10 Marks)

END OF PAPER