NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Process Control: Part II- Model Predictive Control (EE6225, AY2019/20, S1)

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Q1 of Quiz

1. Answer the following questions about model predictive control (MPC)

(a) How to select the sampling time of MPC?

(1 Marks)

(b) How to select the prediction horizon of MPC;

(1 Marks)

(c) How to select the control horizon of MPC;

(1 Marks)

(d) How to select the constrains of MPC;

(1 Marks)

(e) How to select the weights of MPC;

(1 Marks)

Answer of Quiz's *Q*1

- **Sampling time:** Set the sampling time between 5% and 10% of the minimum desired closed-loop response time.

- **Prediction horizon:** If the setting time of the system is T_{set}, the prediction time should be larger than T_{set}.

- **Control horizon:** The control horizon m is selected between 0.1 P and 0.2 P, where P is the prediction horizon.

- **Constrains:** For the output variable, soft constrain is recommended, which allow to break the limits. In addition, hard constrains should be avoided to be given to the input variable and its variable rate at the same time.

- **Weights:** Larger weight for important cost item; less weight for unimportant cost item.

2. For a system, its CARIMA model is as follows

$$A(\mathrm{z})y_k = b(z)\Delta u_k \tag{1}$$

where

$$a(\mathrm{z}) = 1 - 0.5z^{-1} \tag{2}$$

$$b(\mathrm{z}) = 3z^{-1} + 2z^{-2} \tag{3}$$

The CARIMA model based MPC expression is as follows:

$$\underrightarrow{y}_{k+1} = C_A^{-1}C_b \cdot \Delta \underrightarrow{u}_k + \left(C_A^{-1}H_b \cdot \Delta \underleftarrow{u}_{k-1} - C_A^{-1}H_A \cdot \underleftarrow{y}_k \right) \tag{4}$$

The prediction horizon is 4, please answer the following questions:

Answer of Quiz's Q2

$$A(z)y_k = b(z)\Delta u_k \qquad a(z) = 1 - 0.5z^{-1} \qquad b(z) = 3z^{-1} + 2z^{-2}$$

- Using the definition of the prediction matrices and a horizon of 4.

$$\left.\begin{aligned} a(z) &= 1 - 0.5z^{-1} \\ \Delta &= 1 - Z^{-1} \end{aligned}\right\} \Rightarrow A(z) = a(z)\Delta = 1 \underbrace{-1.5}_{A_1} z^{-1} + \underbrace{0.5}_{A_2} z^{-2} + \underbrace{0}_{A_{3,4\cdots}=0}$$

$$C_A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ A_1 & 1 & 0 & 0 \\ A_2 & A_1 & 1 & 0 \\ A_3 & A_2 & A_1 & 1 \end{bmatrix} \Rightarrow C_A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1.5 & 1 & 0 & 0 \\ 0.5 & -1.5 & 1 & 0 \\ 0 & 0.5 & -1.5 & 1 \end{bmatrix}$$

$$H_A = \begin{bmatrix} A_1 & A_2 \\ A_2 & A_3 \\ A_3 & A_4 \\ A_4 & A_5 \end{bmatrix} \Rightarrow H_A = \begin{bmatrix} -1.5 & 0.5 \\ 0.5 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Answer of Quiz's Q2

$A(z)y_k = b(z)\Delta u_k$ $\quad a(z) = 1 - 0.5z^{-1}$ $\quad b(z) = 3z^{-1} + 2z^{-2}$

- Using the definition of the prediction matrices and a horizon of 4.

$$b(z) = 3z^{-1} + 2z^{-2} \Rightarrow b_1 = 3 \quad b_2 = 2 \quad b_{3,4\cdots} = 0$$

$$C_b = \begin{bmatrix} b_1 & 0 & 0 & 0 \\ b_2 & b_1 & 0 & 0 \\ b_3 & b_2 & b_1 & 0 \\ b_4 & b_3 & b_2 & b_1 \end{bmatrix} \Rightarrow C_b = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 0 & 2 & 3 \end{bmatrix}$$

$$H_b = \begin{bmatrix} b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix} \Rightarrow H_b = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Answer of Quiz's $Q2$

$$A(z)y_k = b(z)\Delta u_k \qquad a(z) = 1 - 0.5z^{-1} \qquad b(z) = 3z^{-1} + 2z^{-2}$$

- Using the definition of the prediction matrices and a horizon of 4.

$$\boxed{\underrightarrow{y}_{k+1} = H \cdot \Delta\underrightarrow{u}_k + \left(P \cdot \Delta\underleftarrow{u}_{k-1} - Q \cdot \underleftarrow{y}_k\right)}$$

$$H = C_A^{-1}C_b \qquad P = C_A^{-1}H_b \qquad Q = C_A^{-1}H_A$$

$$C_A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1.5 & 1 & 0 & 0 \\ 0.5 & -1.5 & 1 & 0 \\ 0 & 0.5 & -1.5 & 1 \end{bmatrix} \qquad C_b = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 0 & 2 & 3 \end{bmatrix}$$

$$H_A = \begin{bmatrix} -1.5 & 0.5 \\ 0.5 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad H_b = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

APPLICATION OF FINITE CONTROL SET MPC IN THREE PHASE INVERTER

[07/11/2019]

- Basic knowledge of power inverters
- Finite Control Set (FCS) MPC for power inverters
- Improved FCS-MPC for power inverters

Application of Finite Control Set MPC in Three Phase Inverter

[07/11/2019]

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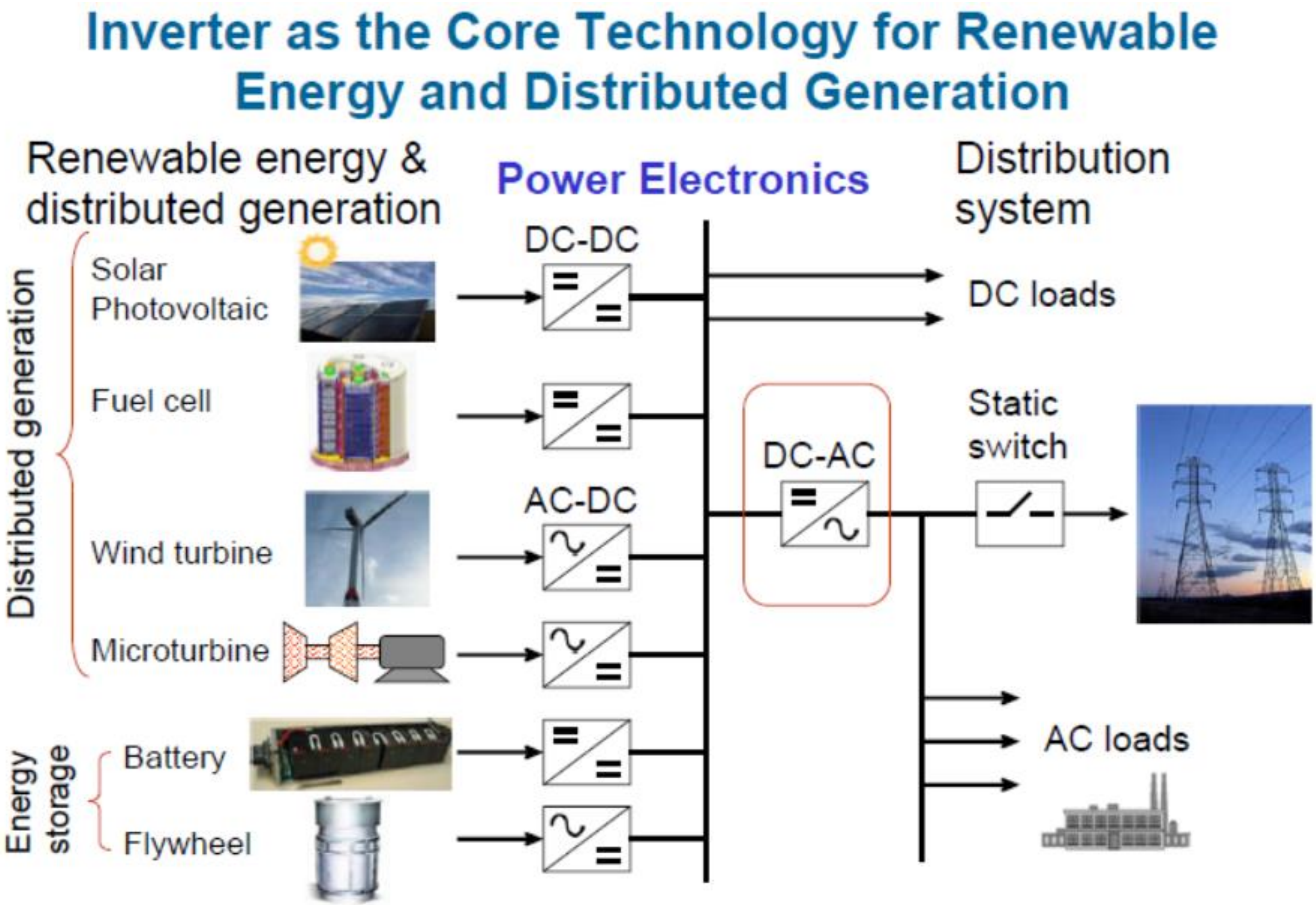
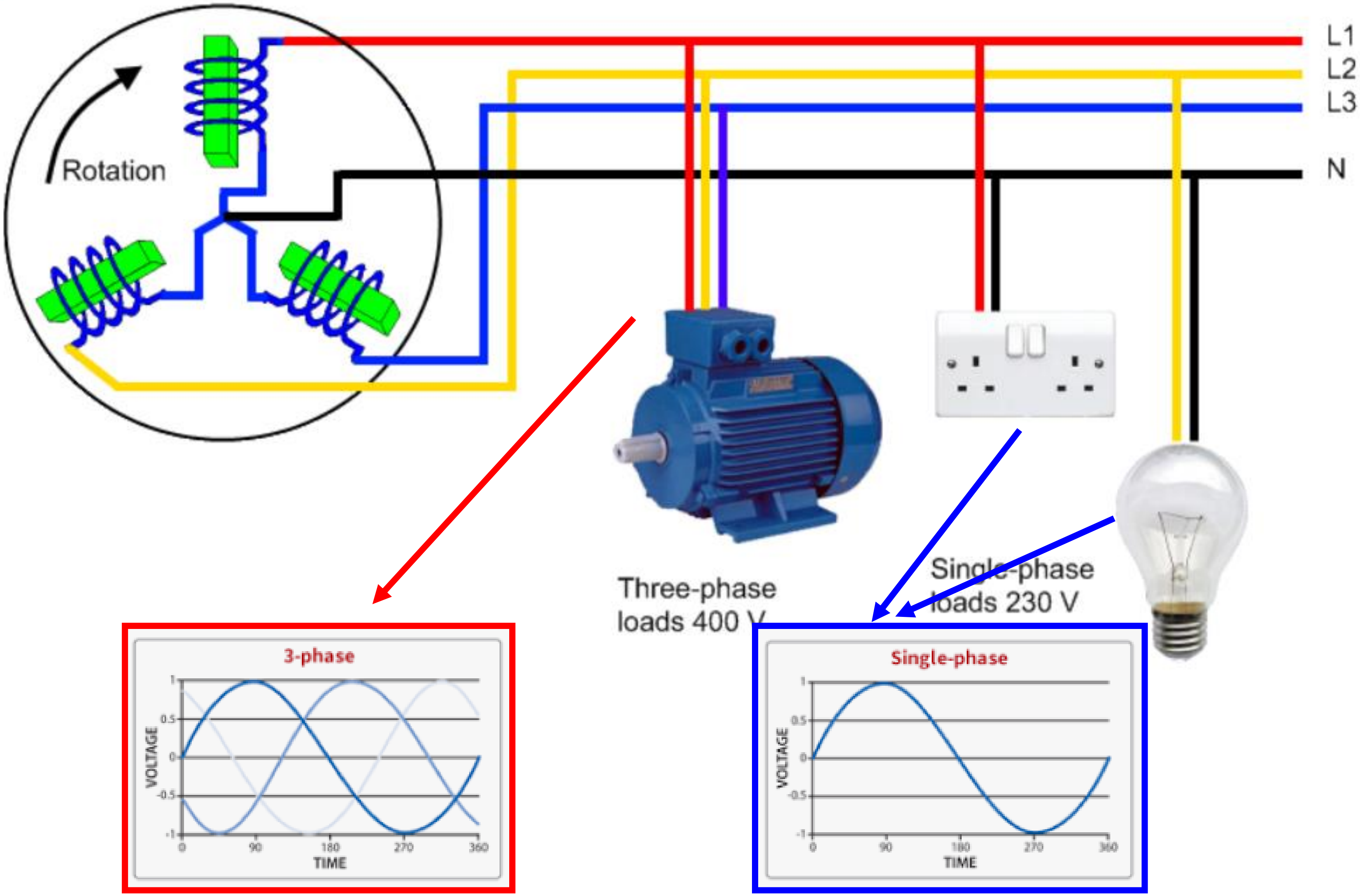
Inverter's applications

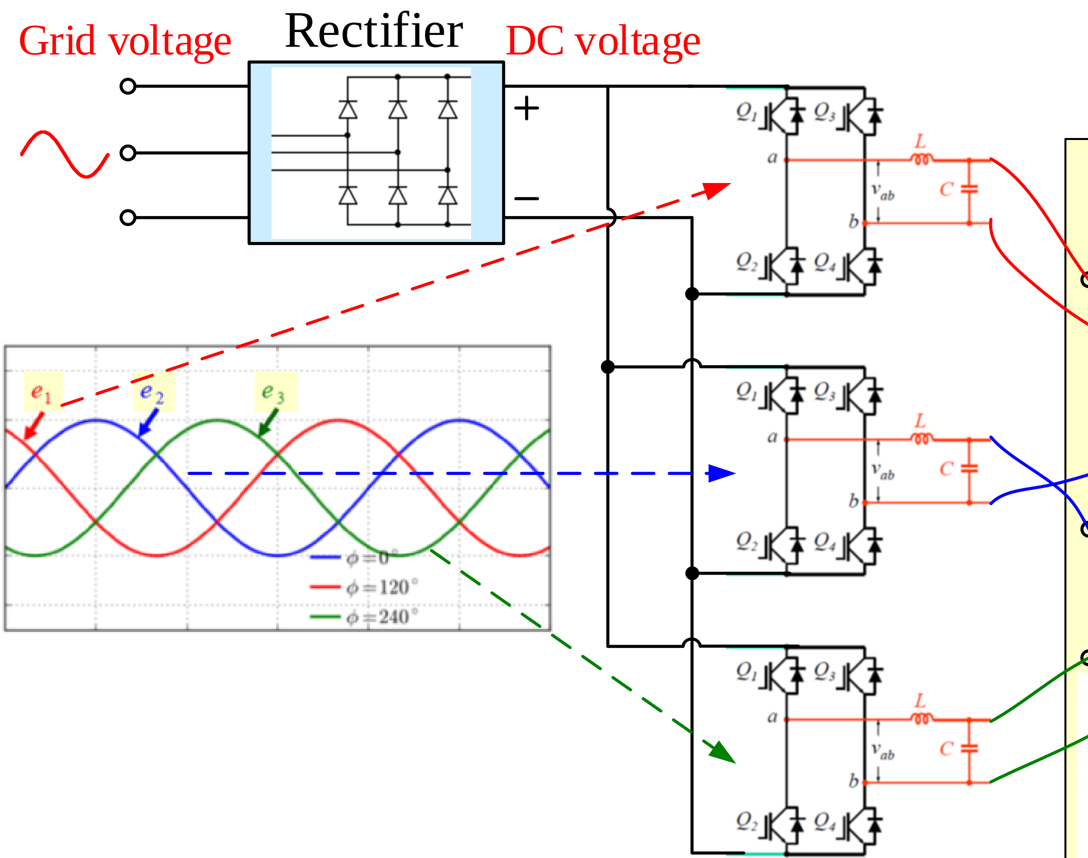


Single-phase inverter and three-phase inverter

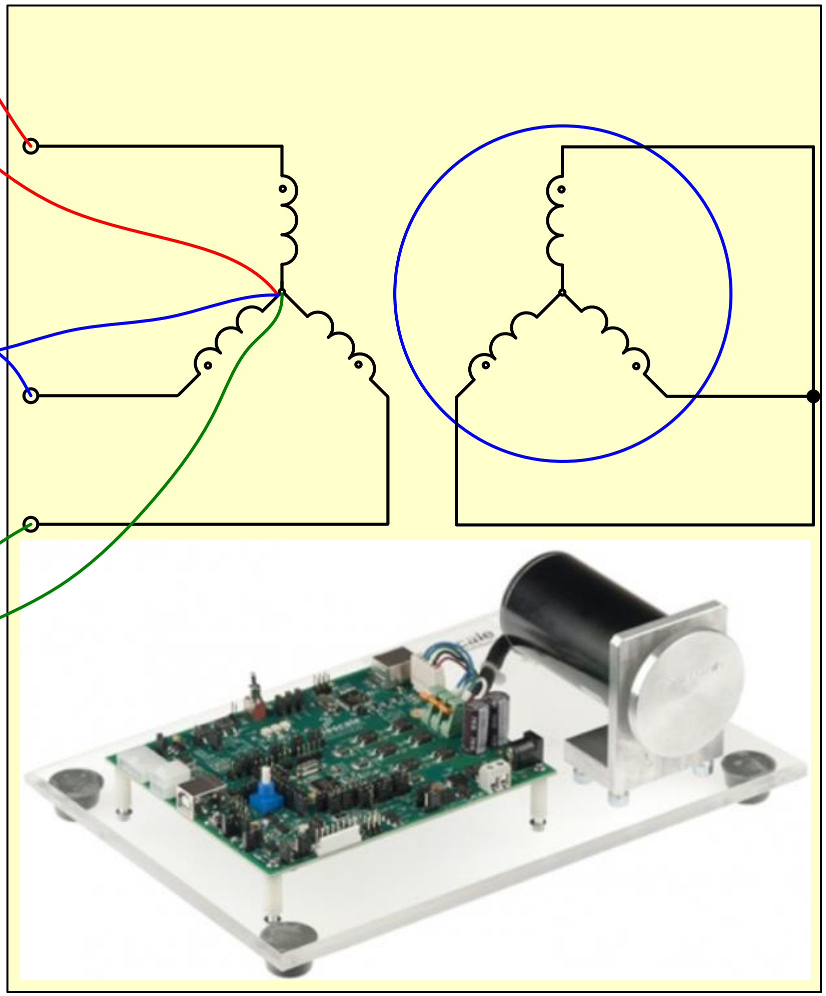


Single phase inverter's working principle

Three phase inverter = 3 single phase inverters



Three phase load

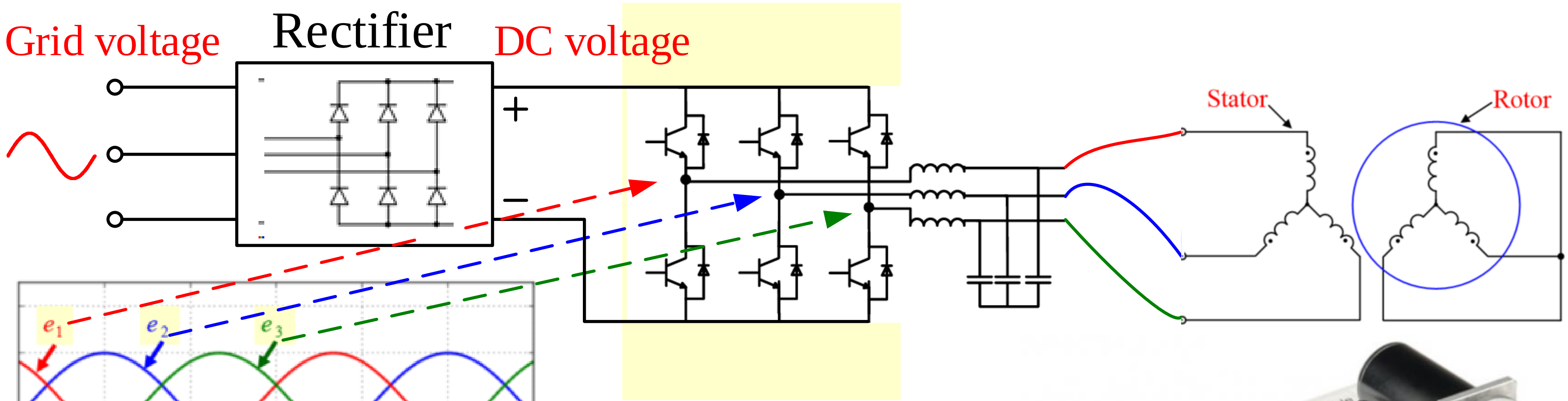


Three phase inverter

=

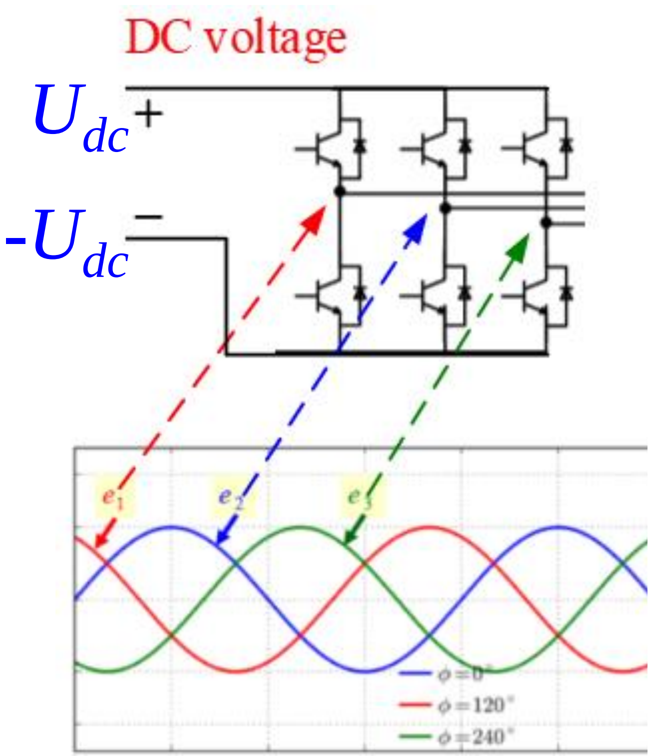
3 single phase inverters

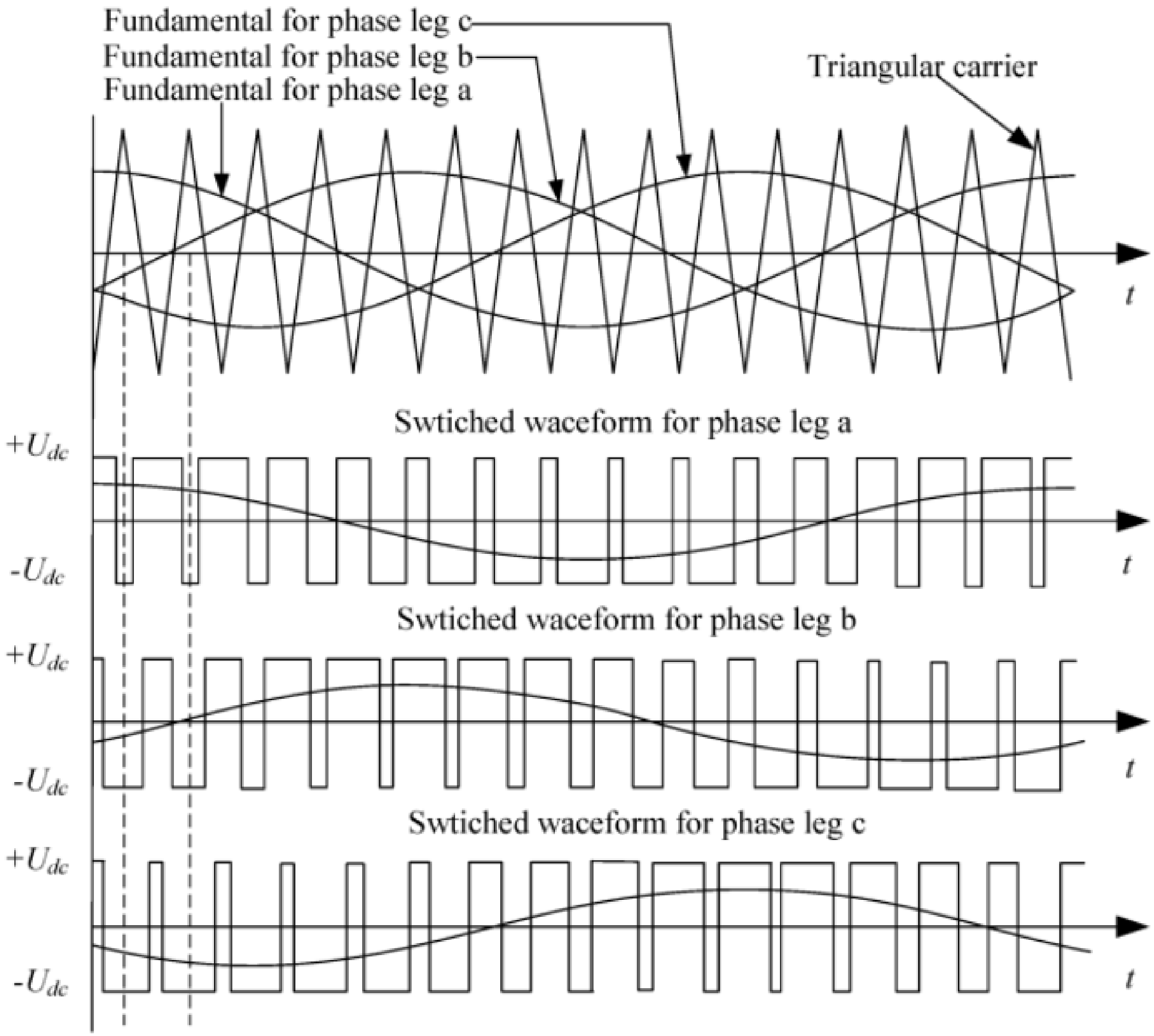
Topology of three phase inverter



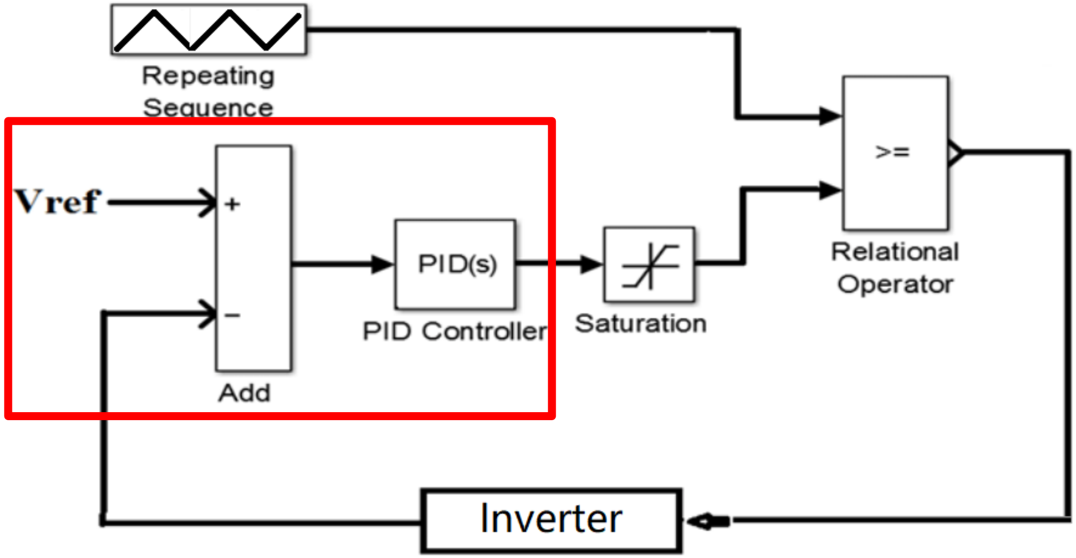
Topology of three phase inverter

Three phase inverter SPWM control

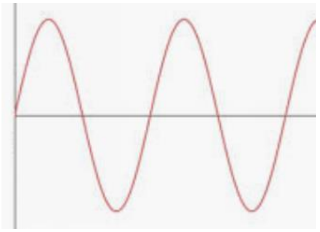


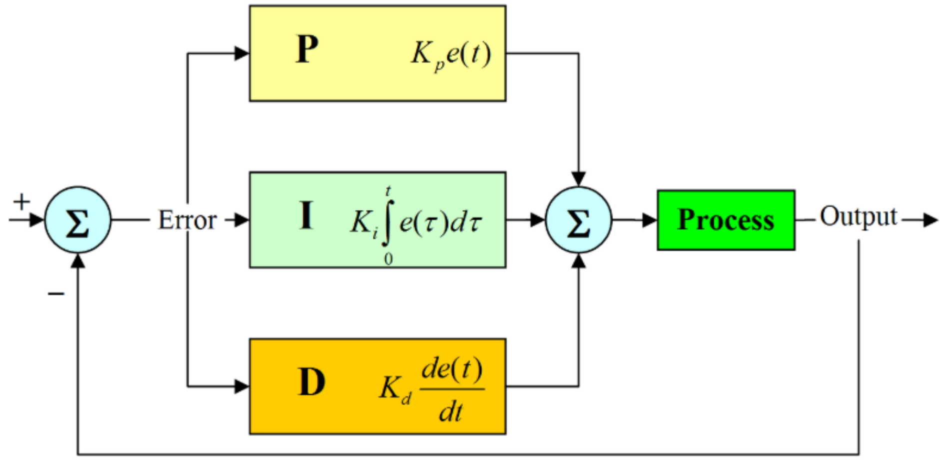


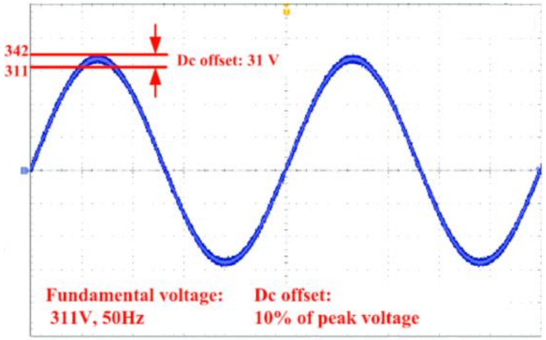
Why SPWM control needs further improvement?



PID control is optimal to DC, but not AC

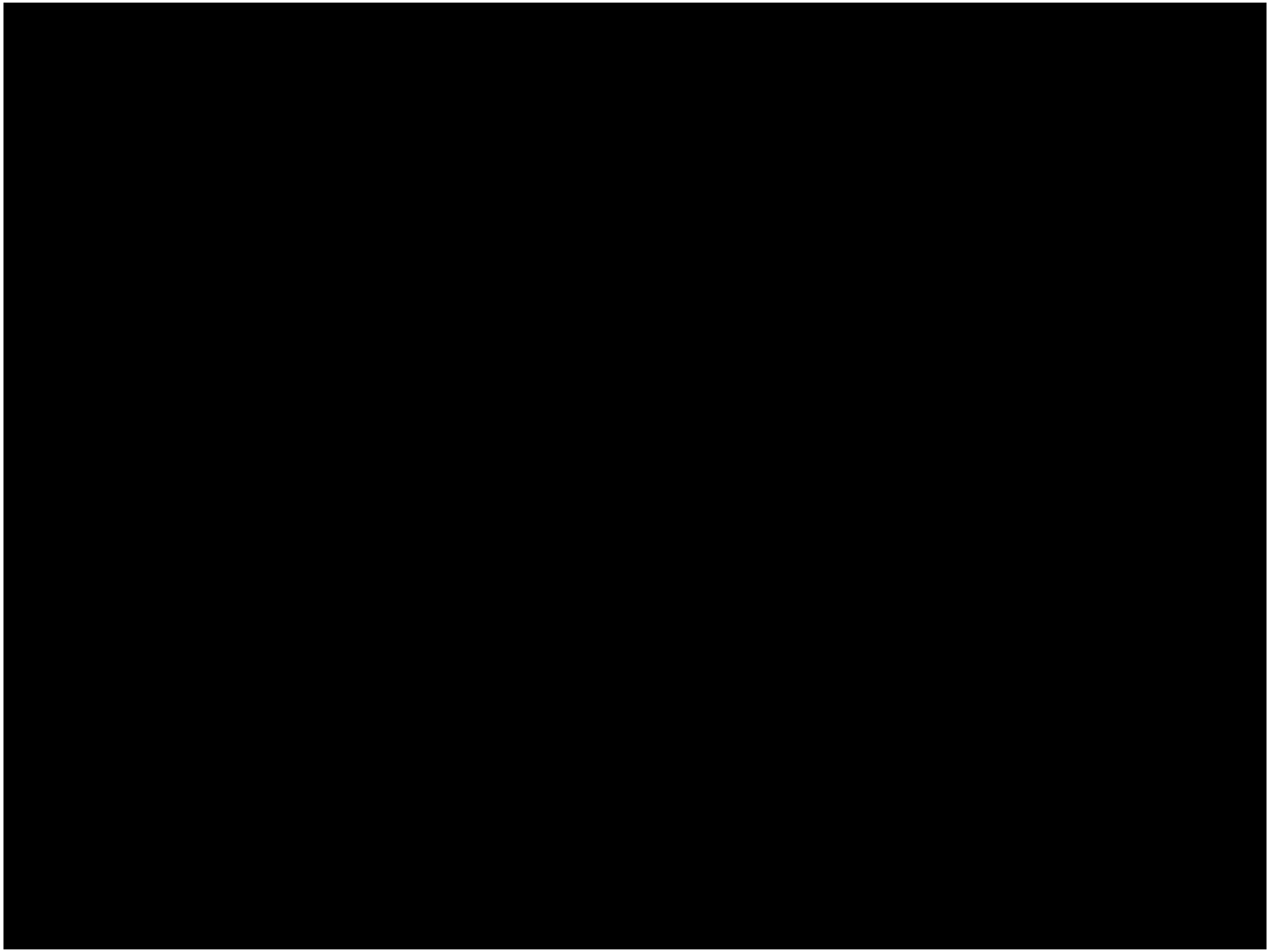






LET US TO TRY MPC CONTROLLER ☺

Application of the three-phase inverter



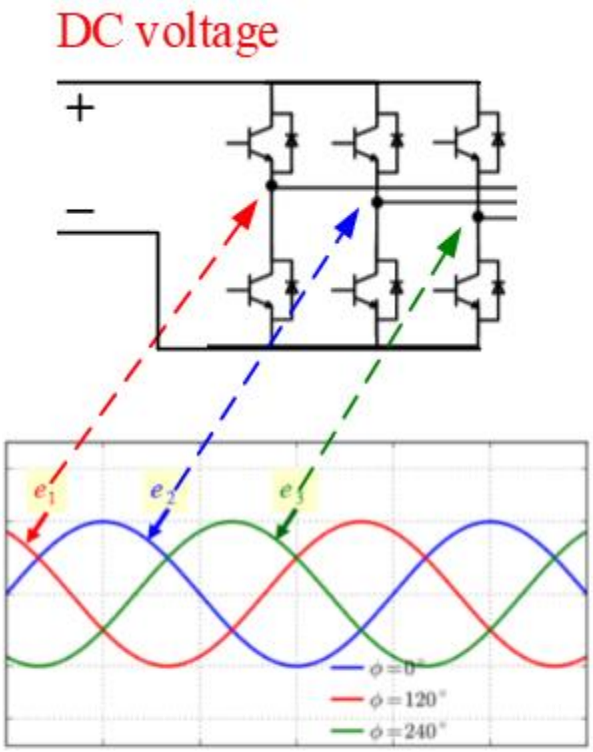
Application of Finite Control Set MPC in Three Phase Inverter

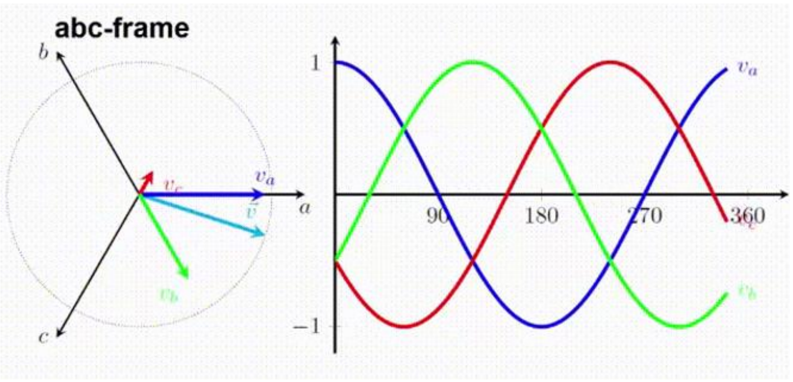
[07/11/2019]

- Basic knowledge of power inverters
- Finite Control Set (FCS) MPC for power inverters
- Improved FCS-MPC for power inverters

FCS-MPC for Power Inverters - Preliminary

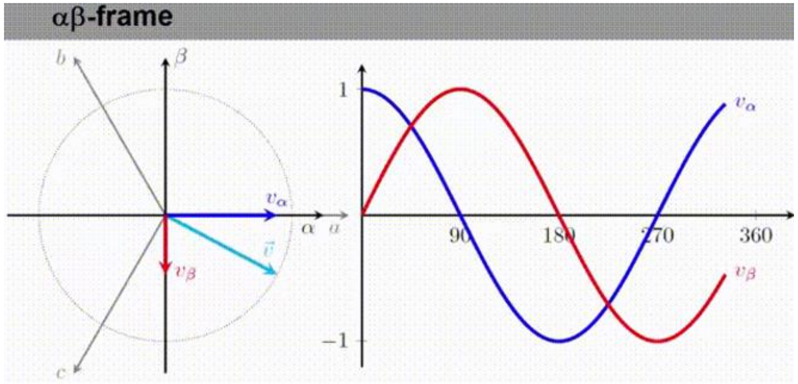
3 phase voltages can change to 2 phase voltages



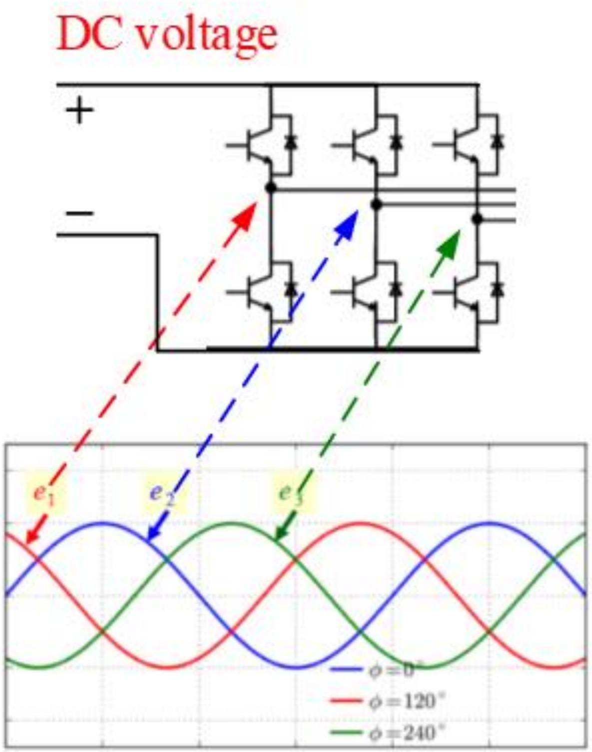


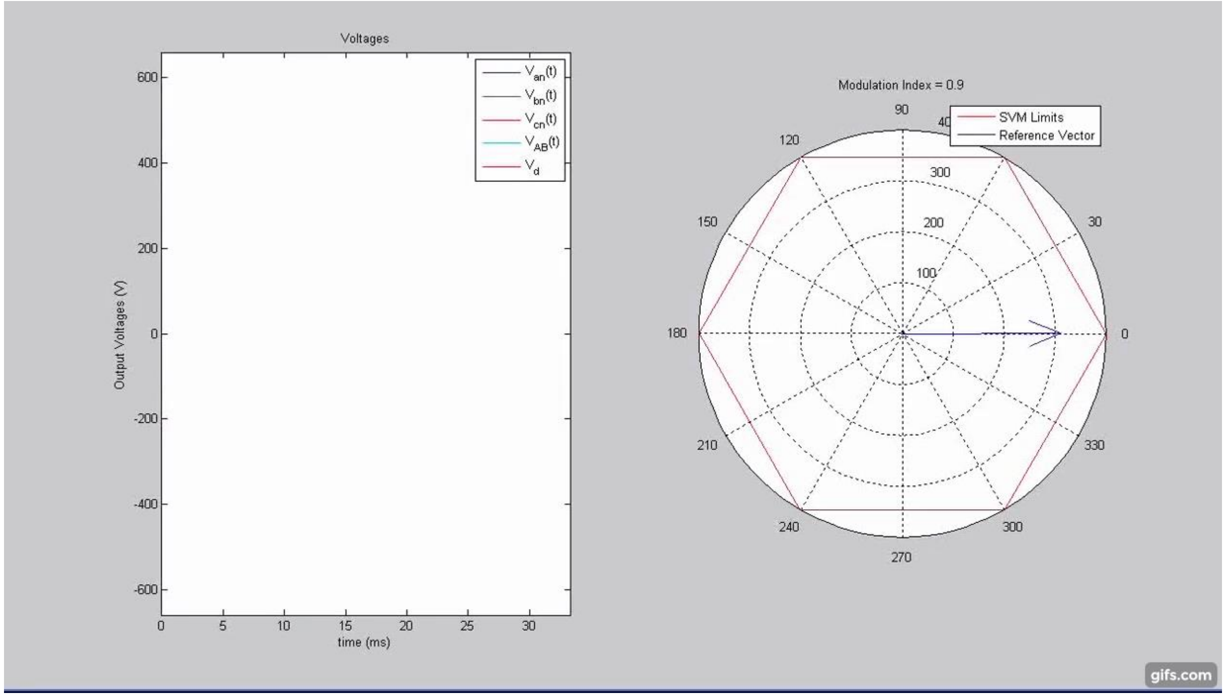
$$\begin{bmatrix}u_\alpha\\u_\beta\end{bmatrix}=\frac{2}{3}\begin{bmatrix}1&-\frac{1}{2}&-\frac{1}{2}\\0&\frac{\sqrt{3}}{2}&-\frac{\sqrt{3}}{2}\end{bmatrix}\begin{bmatrix}u_a\\u_b\\u_c\end{bmatrix}$$

$$\begin{bmatrix}u_a\\u_b\\u_c\end{bmatrix}=\begin{bmatrix}1&0\\-\frac{1}{2}&\frac{\sqrt{3}}{2}\\-\frac{1}{2}&-\frac{\sqrt{3}}{2}\end{bmatrix}\begin{bmatrix}u_\alpha\\u_\beta\end{bmatrix}$$



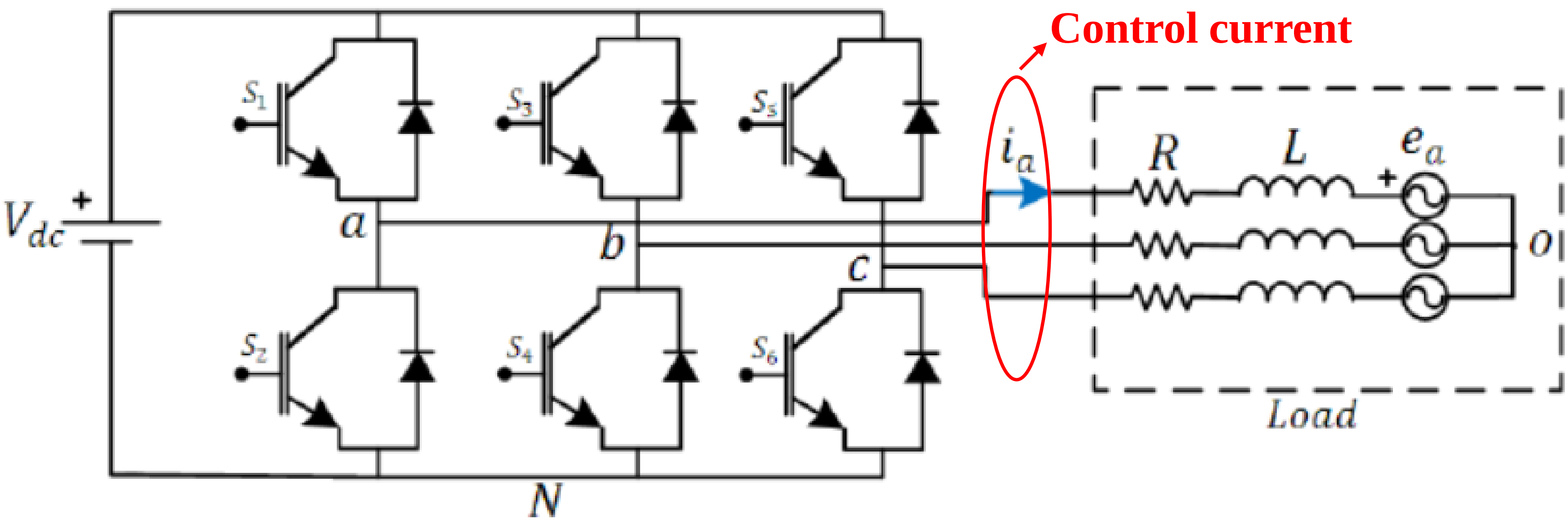
3 phase voltages can change to 1 voltage vector





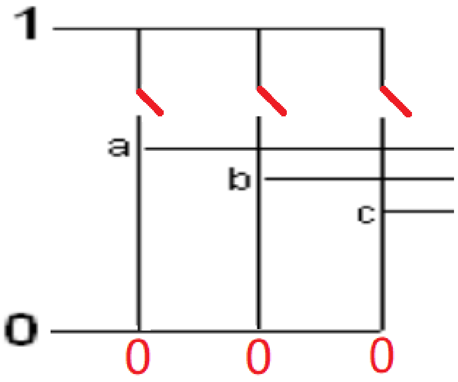
$$\vec{v} = \vec{u}_a + \vec{u}_b + \vec{u}_c = \vec{u}_\alpha + \vec{u}_\beta$$

Switches of power inverters

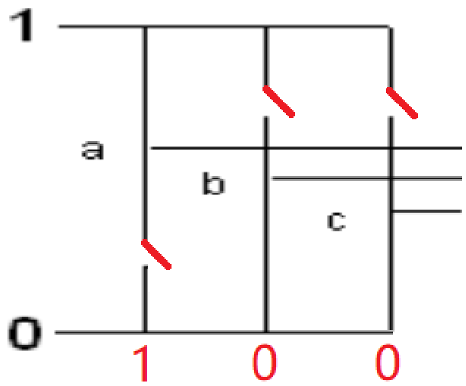


- S_1 & S_2, S_3 & S_4, S_5 & S_6
- $S_{1/2/3/4/5/6}$: 1 (Turn on); 0 (Turn off)
- Switching leg: 1 (upper switch turn on); 0 (Upper switch turn off)

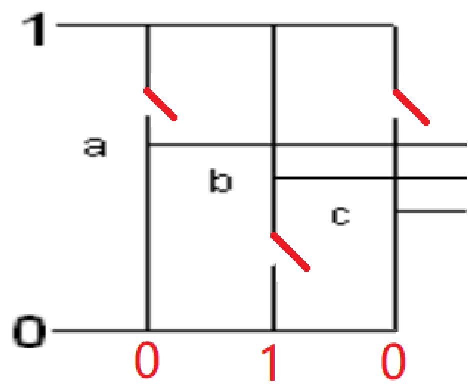
8 switching states of power inverters



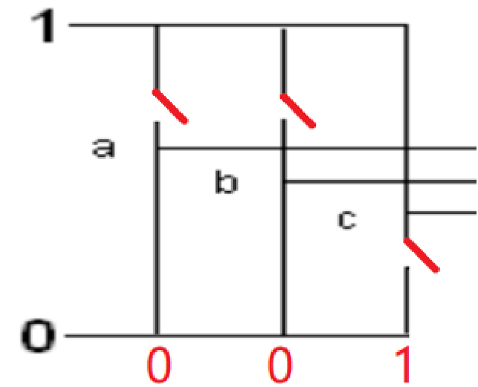
State 1



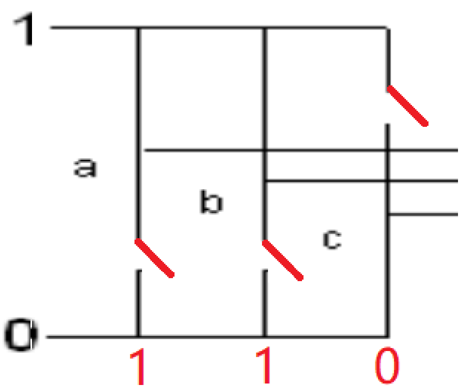
State 2



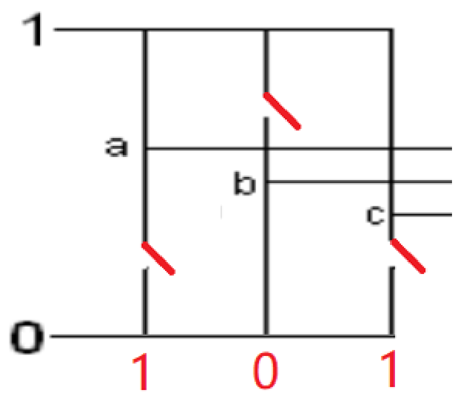
State 3



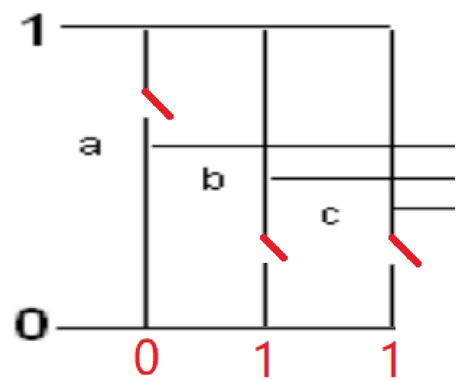
State 4



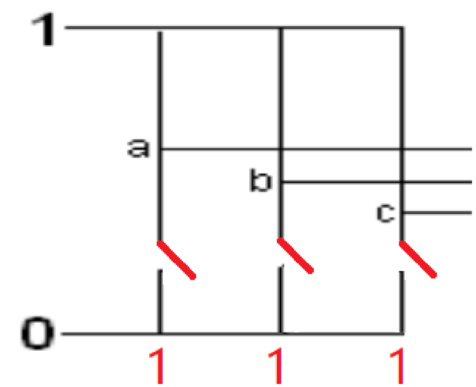
State 5



State 6



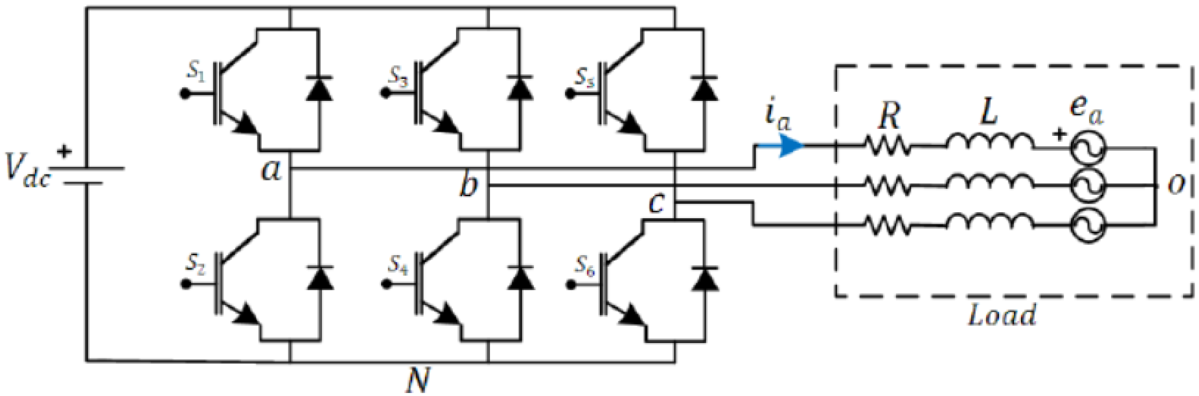
State 7

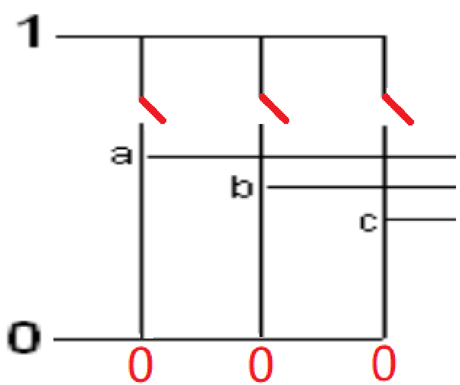


State 8

Switch state 1 of power inverters

S_1	S_2	S_3	S_4	S_5	S_6
0	1	0	1	0	1
1	0	0	1	0	1
0	1	1	0	0	1
0	1	0	1	1	0
1	0	1	0	0	1
1	0	0	1	1	0
0	1	1	0	1	0
1	0	1	0	1	0

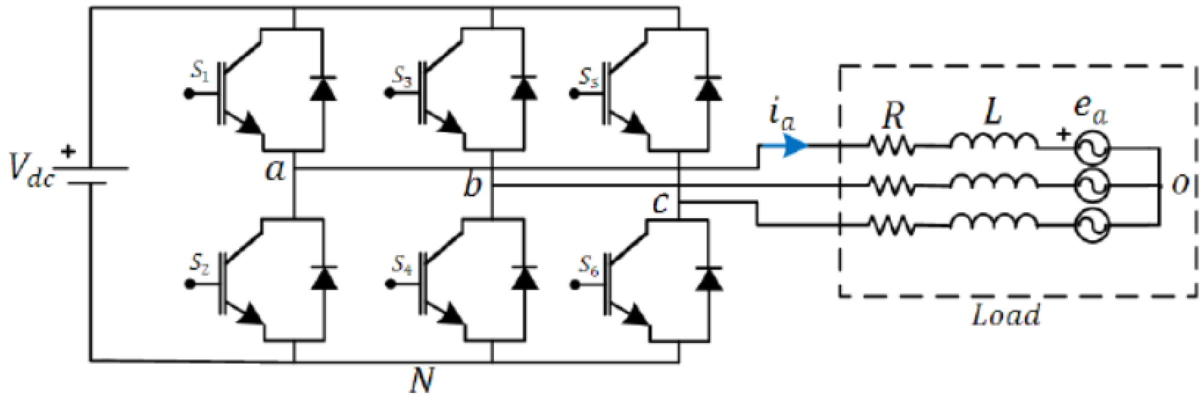


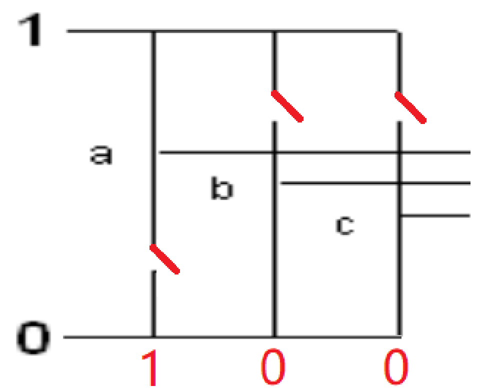


State 1

Switch state 2 of power inverters

S_1	S_2	S_3	S_4	S_5	S_6
0	1	0	1	0	1
1	0	0	1	0	1
0	1	1	0	0	1
0	1	0	1	1	0
1	0	1	0	0	1
1	0	0	1	1	0
0	1	1	0	1	0
1	0	1	0	1	0

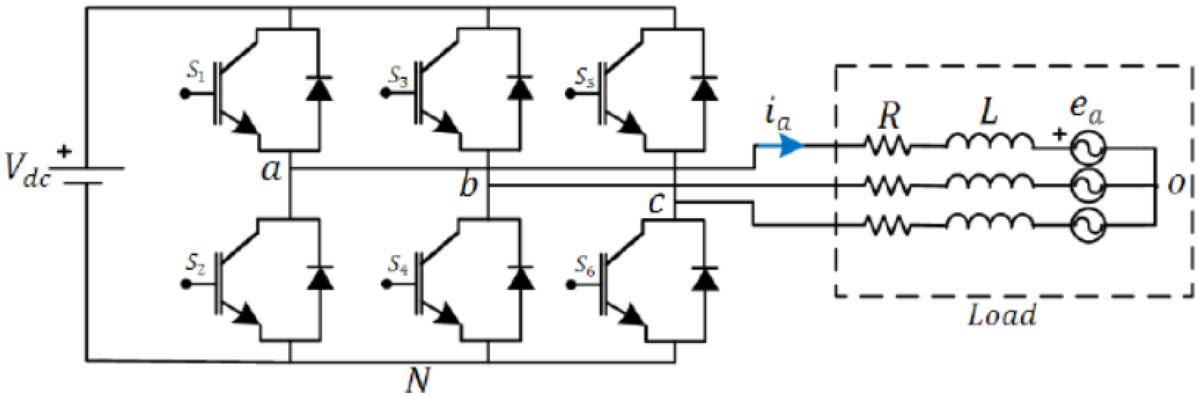


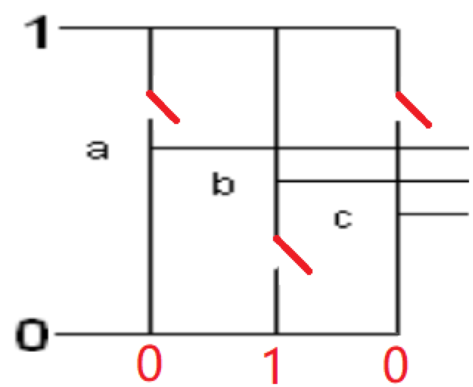


State 2

Switch state 3 of power inverters

S_1	S_2	S_3	S_4	S_5	S_6
0	1	0	1	0	1
1	0	0	1	0	1
0	1	1	0	0	1
0	1	0	1	1	0
1	0	1	0	0	1
1	0	0	1	1	0
0	1	1	0	1	0
1	0	1	0	1	0

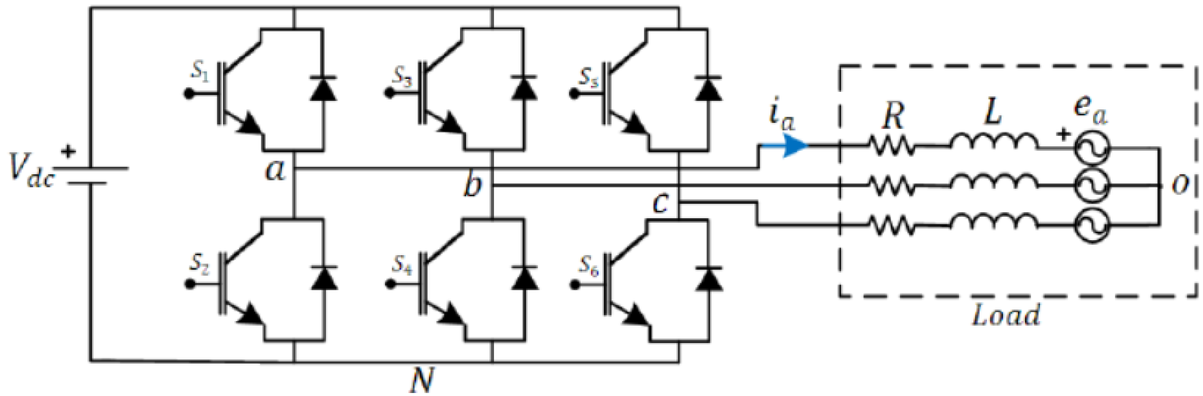


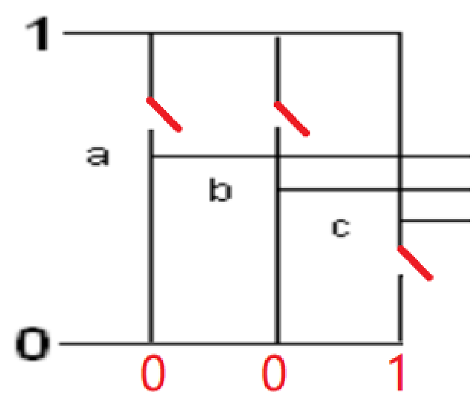


State 3

Switch state 4 of power inverters

S_1	S_2	S_3	S_4	S_5	S_6
0	1	0	1	0	1
1	0	0	1	0	1
0	1	1	0	0	1
0	1	0	1	1	0
1	0	1	0	0	1
1	0	0	1	1	0
0	1	1	0	1	0
1	0	1	0	1	0

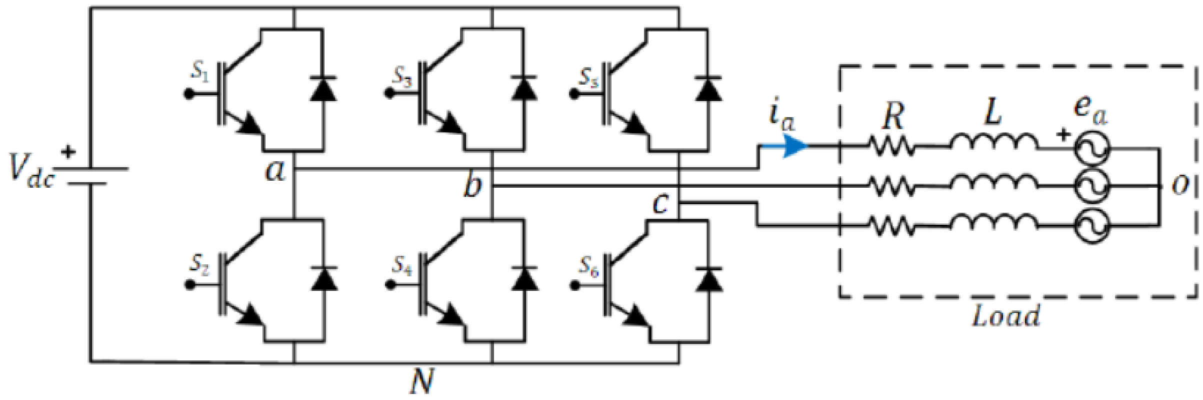


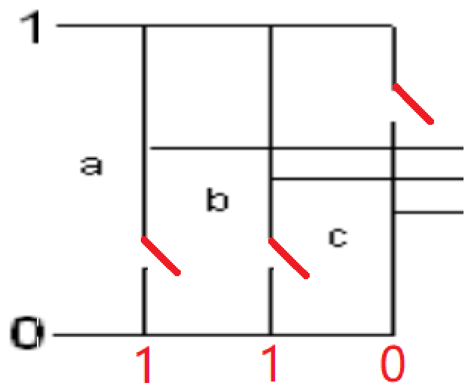


State 4

Switch state 5 of power inverters

S_1	S_2	S_3	S_4	S_5	S_6
0	1	0	1	0	1
1	0	0	1	0	1
0	1	1	0	0	1
0	1	0	1	1	0
1	0	1	0	0	1
1	0	0	1	1	0
0	1	1	0	1	0
1	0	1	0	1	0

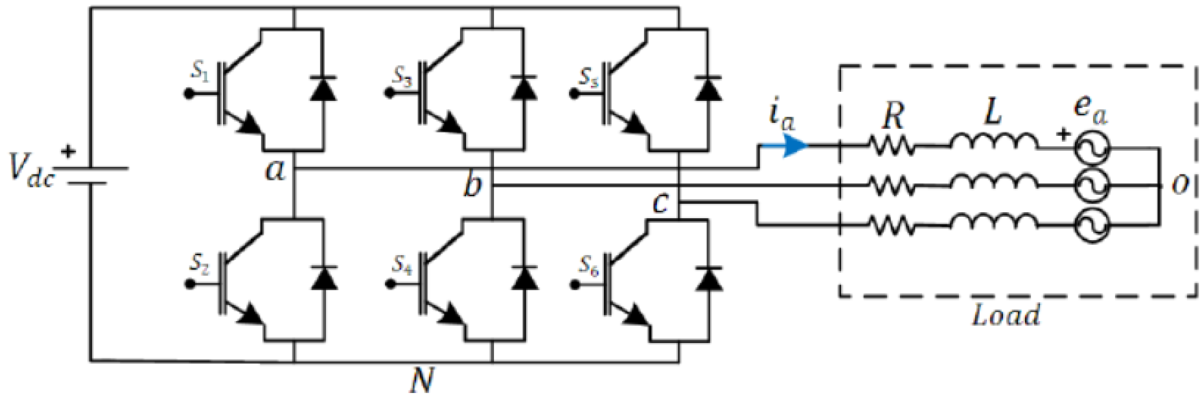


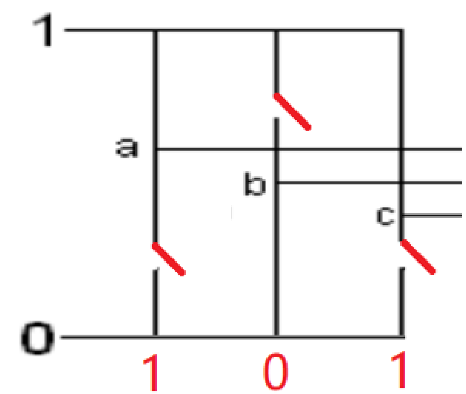


State 5

Switch state 6 of power inverters

S_1	S_2	S_3	S_4	S_5	S_6
0	1	0	1	0	1
1	0	0	1	0	1
0	1	1	0	0	1
0	1	0	1	1	0
1	0	1	0	0	1
1	0	0	1	1	0
0	1	1	0	1	0
1	0	1	0	1	0

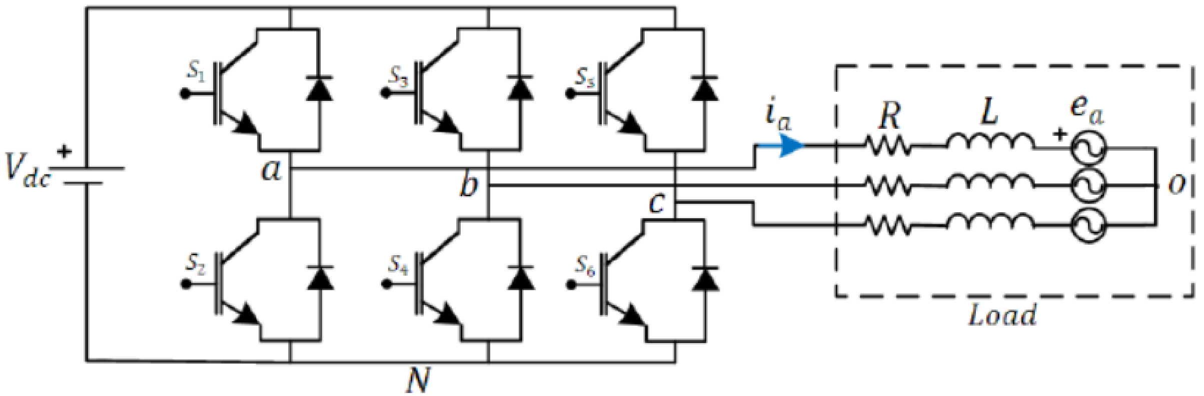


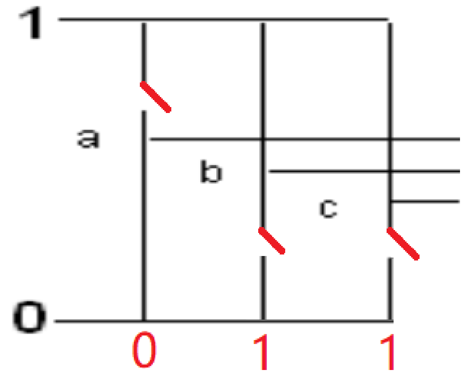


State 6

Switch state 7 of power inverters

S_1	S_2	S_3	S_4	S_5	S_6
0	1	0	1	0	1
1	0	0	1	0	1
0	1	1	0	0	1
0	1	0	1	1	0
1	0	1	0	0	1
1	0	0	1	1	0
0	1	1	0	1	0
1	0	1	0	1	0

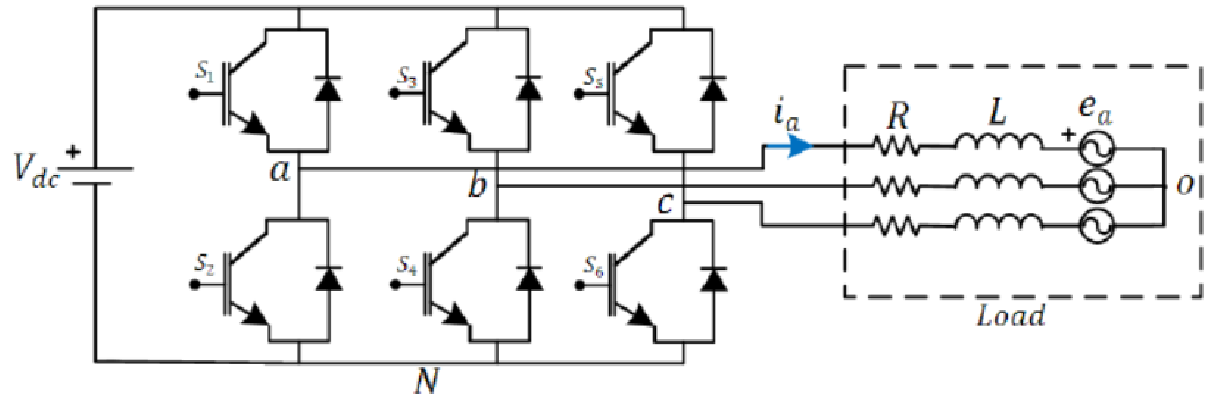


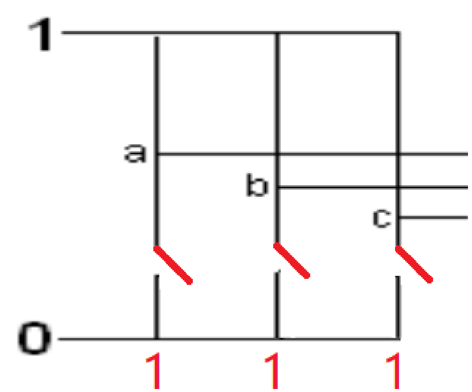


State 7

Switch state 8 of power inverters

S_1	S_2	S_3	S_4	S_5	S_6
0	1	0	1	0	1
1	0	0	1	0	1
0	1	1	0	0	1
0	1	0	1	1	0
1	0	1	0	0	1
1	0	0	1	1	0
0	1	1	0	1	0
1	0	1	0	1	0





State 8

Summary: 8 switch state of power inverters

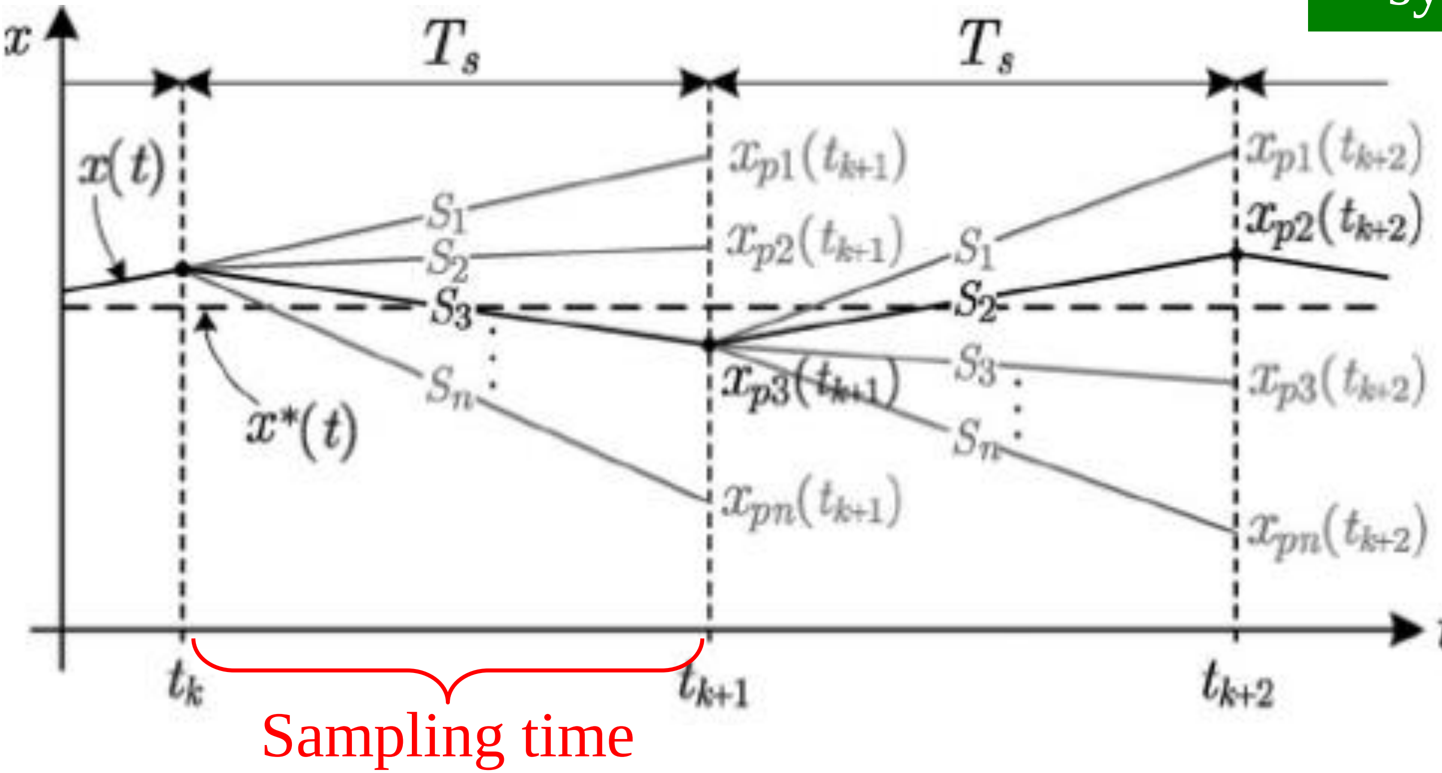
	S_1	S_2	S_3	S_4	S_5	S_6
State 1	0	1	0	1	0	1
State 2	1	0	0	1	0	1
State 3	0	1	1	0	0	1
State 4	0	1	0	1	1	0
State 5	1	0	1	0	0	1
State 6	1	0	0	1	1	0
State 7	0	1	1	0	1	0
State 8	1	0	1	0	1	0

FCS-MPC for Power Inverters - Overview

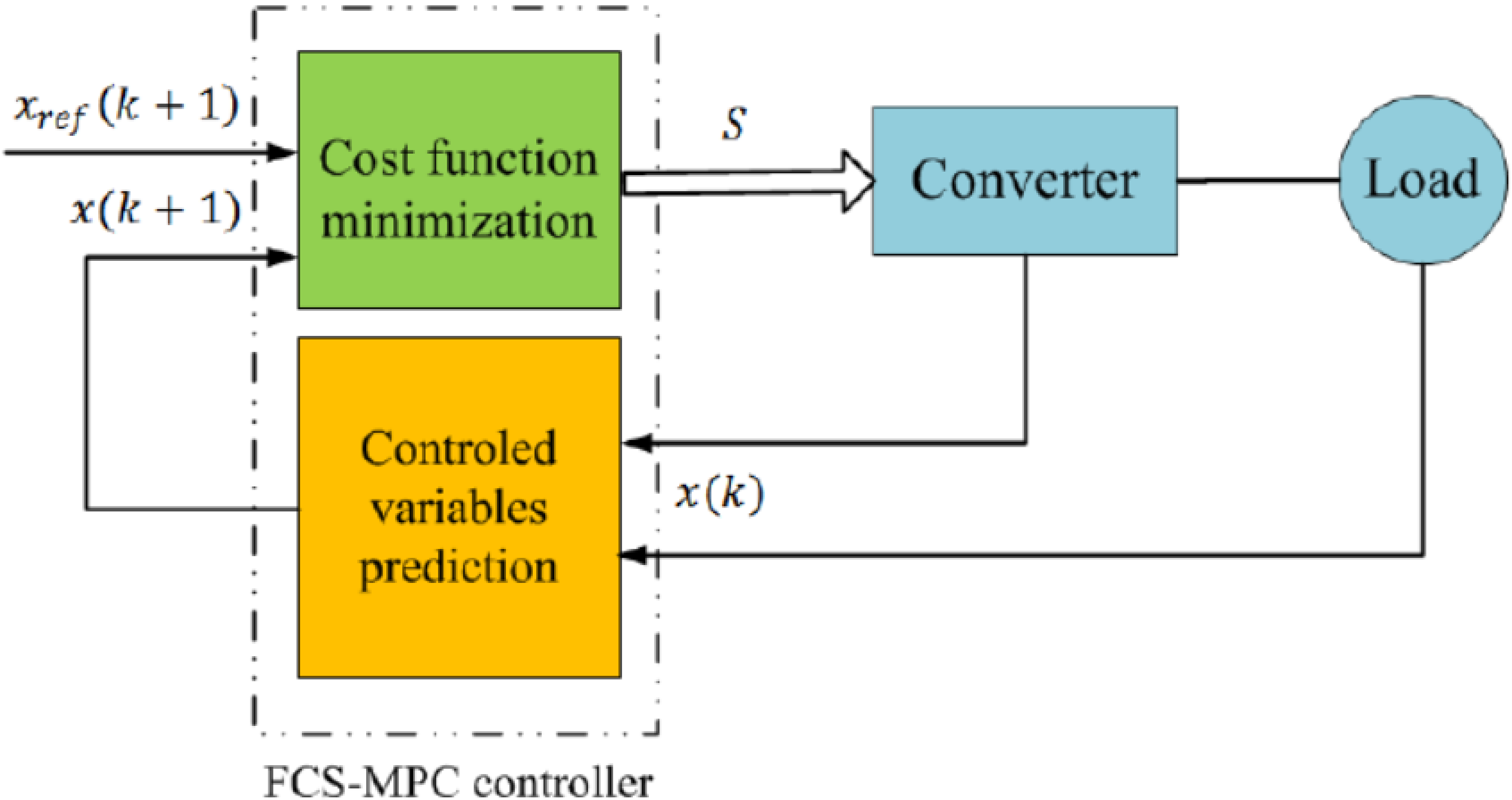
- Finite Control Set (FCS) MPC can reduce processing time.

- Finite number of switching states in a inverter
  - Prediction limited only to these states.
  - Select state via cost function minimization.

Suitable for discrete system

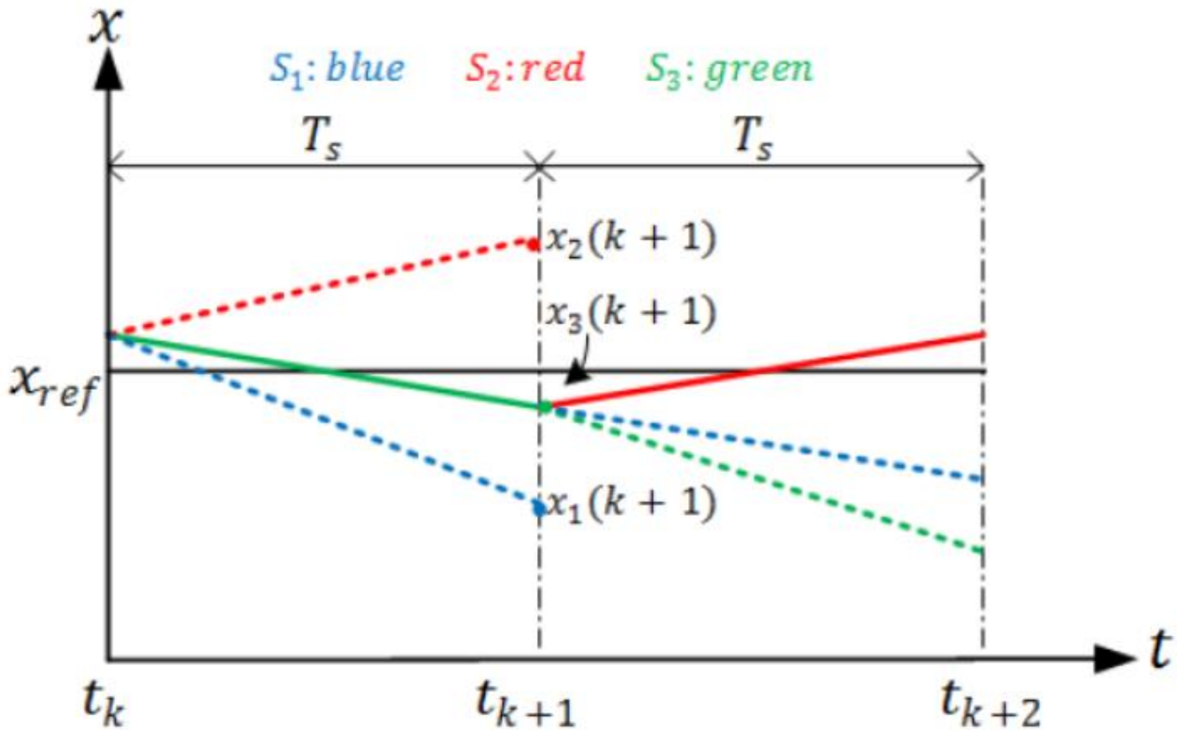


Control blocks of the FCS-MPC



- ***x*(*k*):** Controlled variable in current state;
- ***x*(*k*+1):** Controlled variable in next sampling time state.

FCS-MPC operation principle



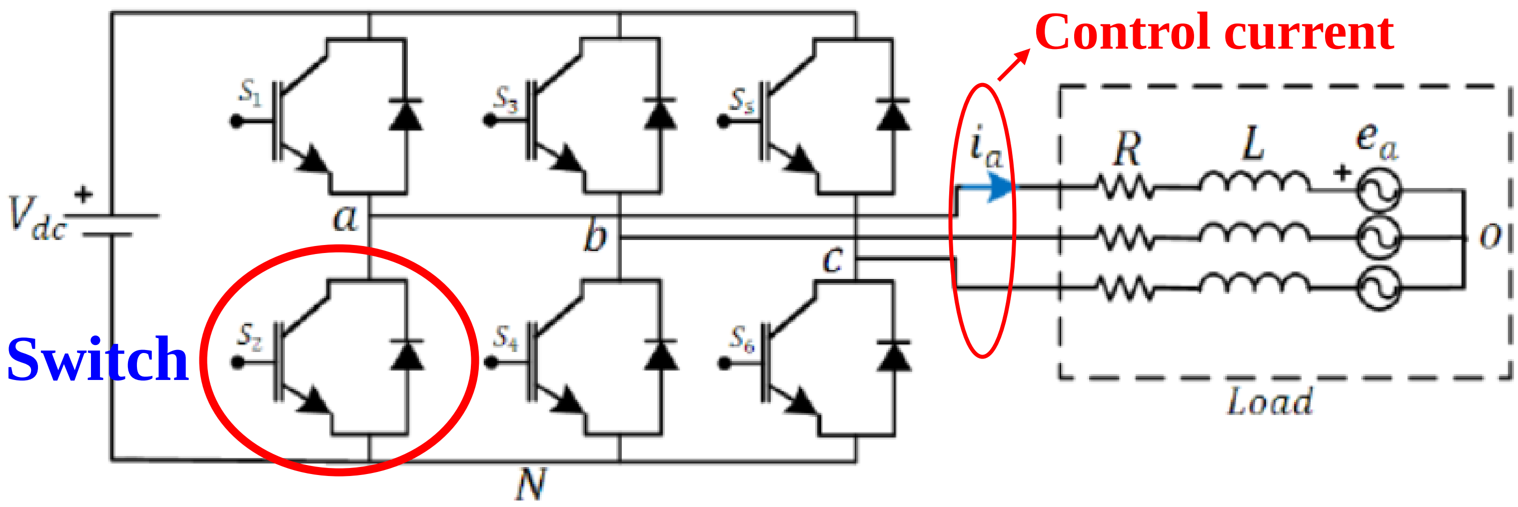
Primary: Identifying all the possible switching states.

Step 1: Obtaining the discrete model

Step 2: Defining a cost function.

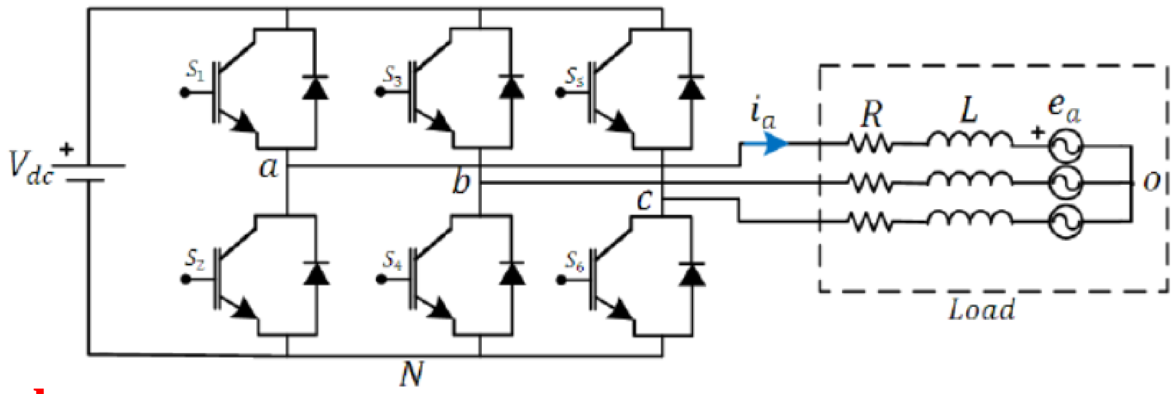
Step 3: Select the optimal switching states

Inverter topology and assumptions



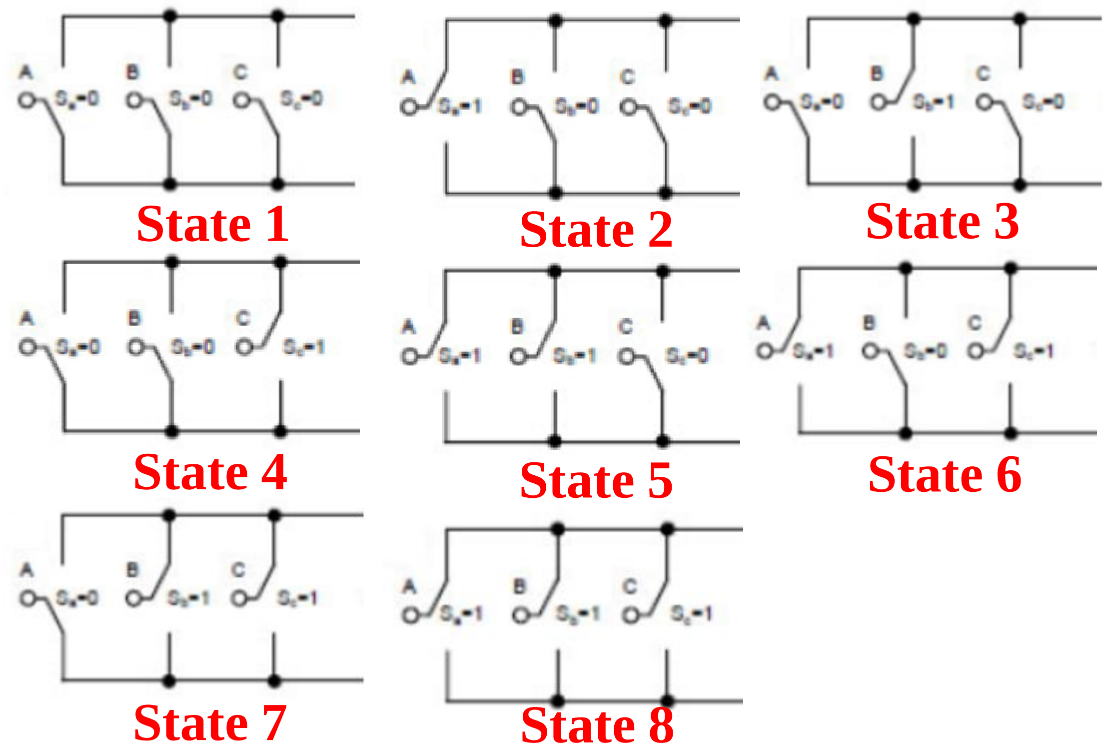
- Only two possible states for Switch: ON or OFF.
- These switching states are not acceptable:
  - Both up & down switches are ON (**short circuit**).
  - Both switches in each phase are OFF (**no power transfer**).

Review of 8 switch states



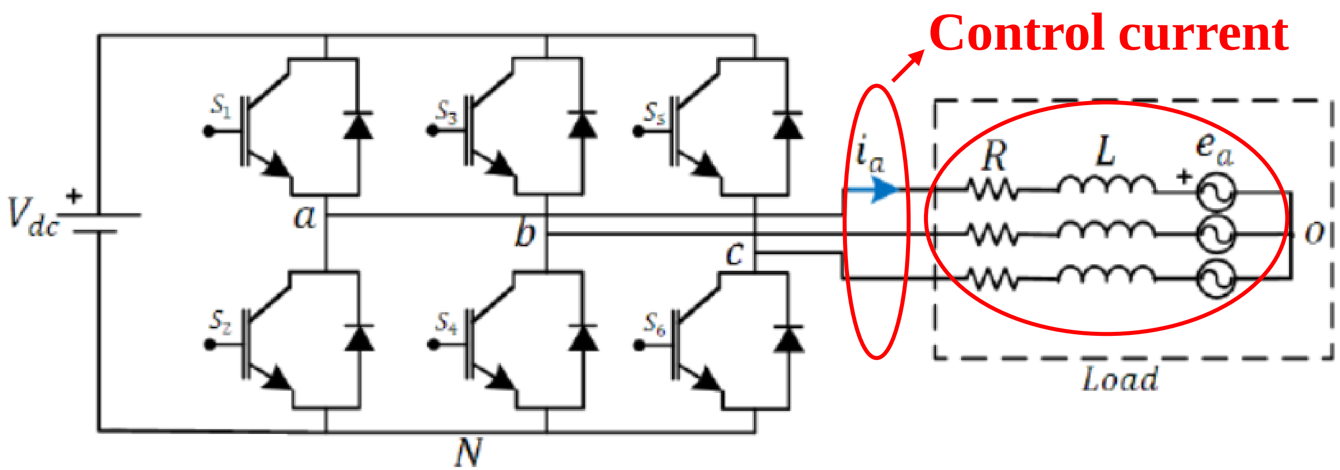
8 switch states

	S_1	S_2	S_3	S_4	S_5	S_6
State 1	0	1	0	1	0	1
State 2	1	0	0	1	0	1
State 3	0	1	1	0	0	1
State 4	0	1	0	1	1	0
State 5	1	0	1	0	0	1
State 6	1	0	0	1	1	0
State 7	0	1	1	0	1	0
State 8	1	0	1	0	1	0



FCS-MPC for Power Inverters - Step 1: Model

Inverter's continues model

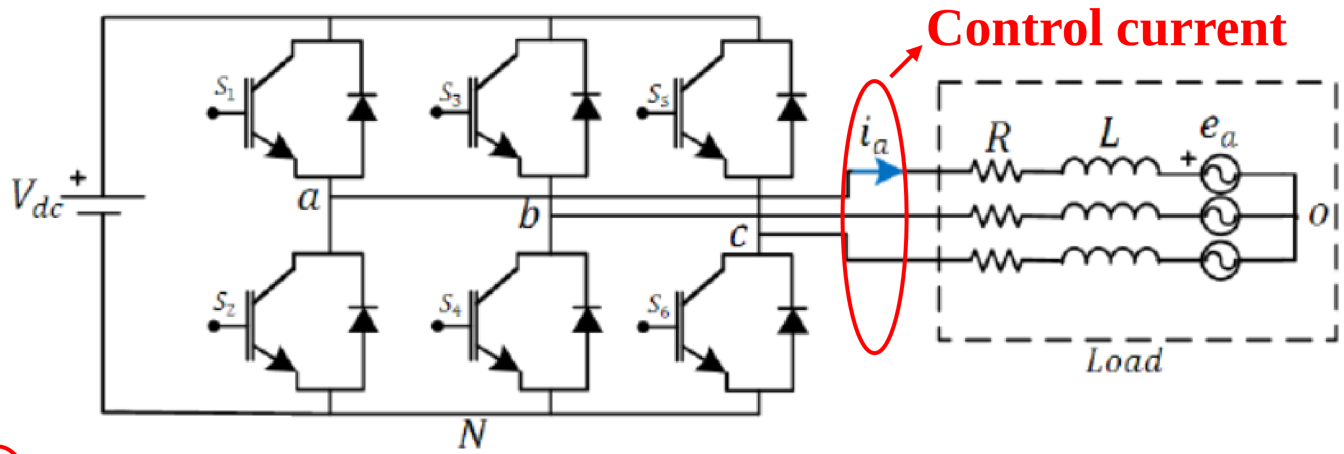


$$i = \frac{2}{3}(i_a + a i_b + a^2 i_c)$$

$$v = L\frac{di}{dt} + Ri + e \qquad e = \frac{2}{3}(e_a + a e_b + a^2 e_c) \qquad a = e^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$v = \frac{2}{3}(v_{aN} + a v_{bN} + a^2 v_{cN})$$

Inverter discrete model1: Forward Euler method



$$\frac{dx}{dt} = \frac{x(k+1) - x(k)}{T_s} \Rightarrow \underline{x(k+1) = x(k) + T_s\, f\big(x(k), u(k)\big)}$$

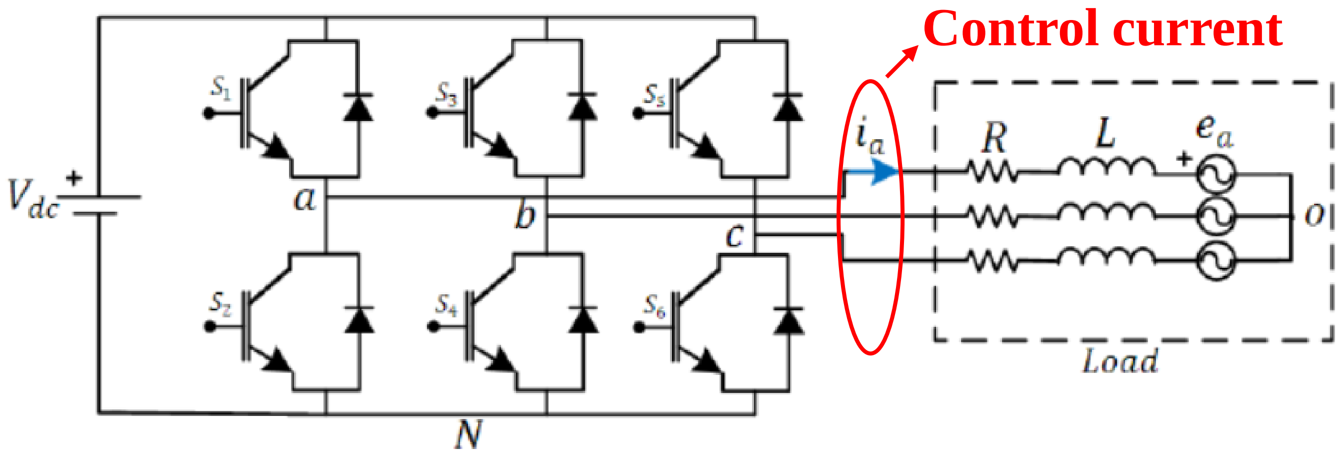
Forward Euler method

$$v = L\frac{di}{dt} + Ri + e$$

Forward Euler method

$$i(k+1) = \left(1 - \frac{T_s R}{L}\right) i(k) + \frac{T_s}{L}\,(v(k) - e(k))$$

Inverter discrete model2: Backward Euler method



$$\frac{dx}{dt} = \frac{x(k+1) - x(k)}{T_s} \Longrightarrow x(k+1) = x(k) + T_s\, f(x(k+1), u(k+1))$$

Backward Euler method

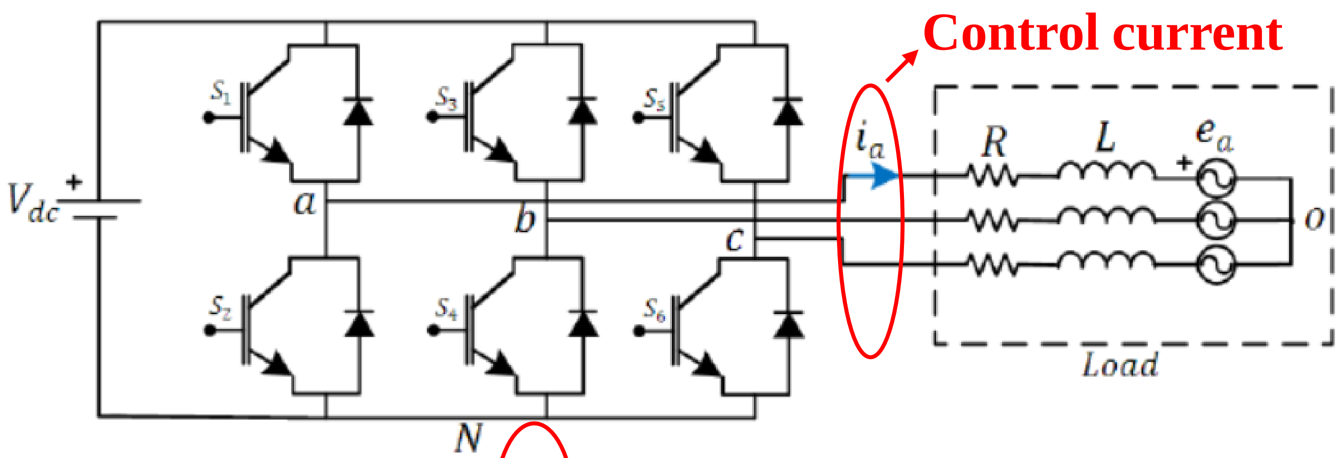
Sampling time

$$v = L\frac{di}{dt} + Ri + e$$

Backward Euler method

$$i(k+1) = \frac{T_s}{L + RT_s}[v(k+1) - e(k+1)] + \frac{L}{L + RT_s}i(k)$$

Inverter discrete model3: Midpoint Euler method



$$\frac{dx}{dt} = \frac{x(k+1) - x(k)}{T_s}$$

Midpoint Euler method

$$x(k+1) = x(k) + \frac{T_s}{2}\left[f\big(x(k), u(k)\big) + f\big(x(k+1), u(k+1)\big) \right]$$

$$v = L\frac{di}{dt} + Ri + e$$

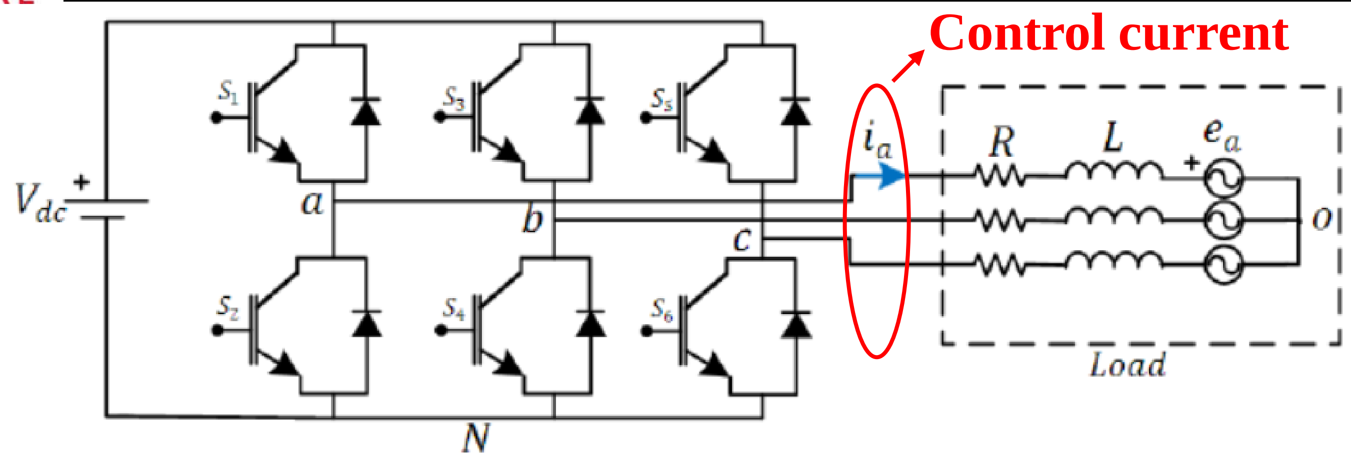
Midpoint Euler method

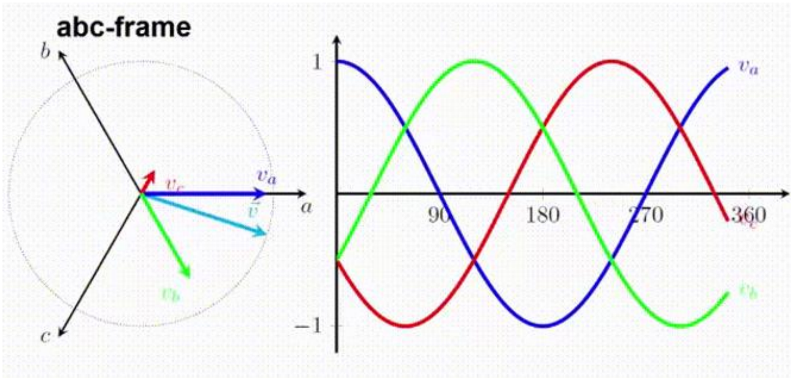
$$i(k+1) = \frac{2 - RT_s}{2 + RT_s}\, i(k) + \frac{T_s}{2 + RT_s}\left[v(k) + v(k+1) - e(k) - e(k+1)\right]$$

FCS-MPC for Power Inverters

- Step 2: Cost Function

Basic cost function

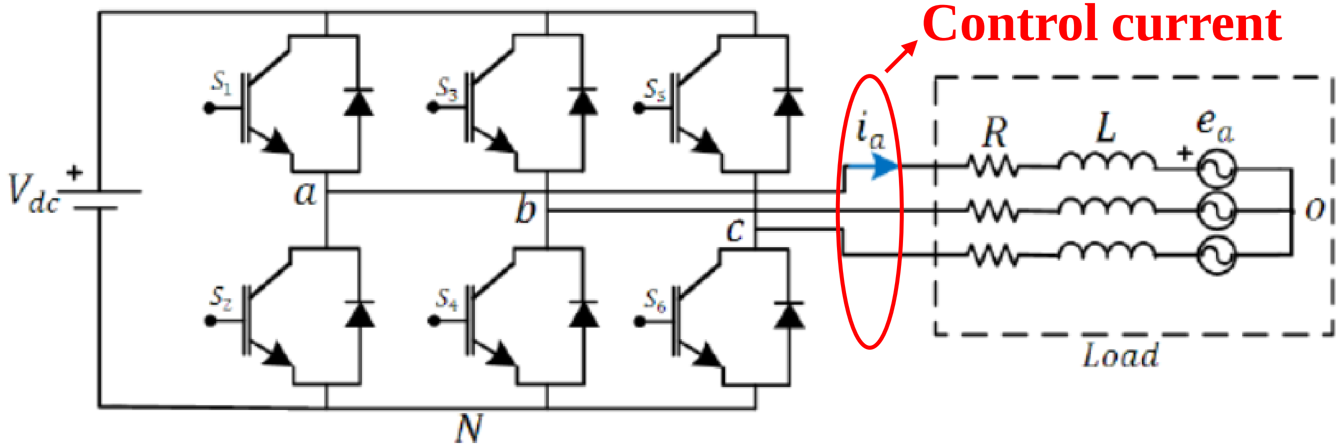


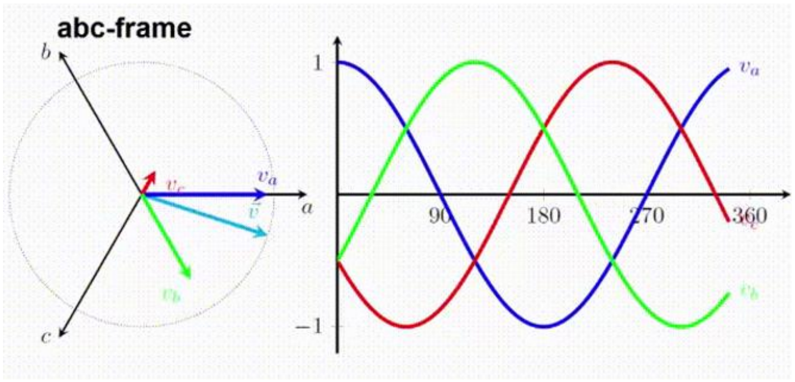


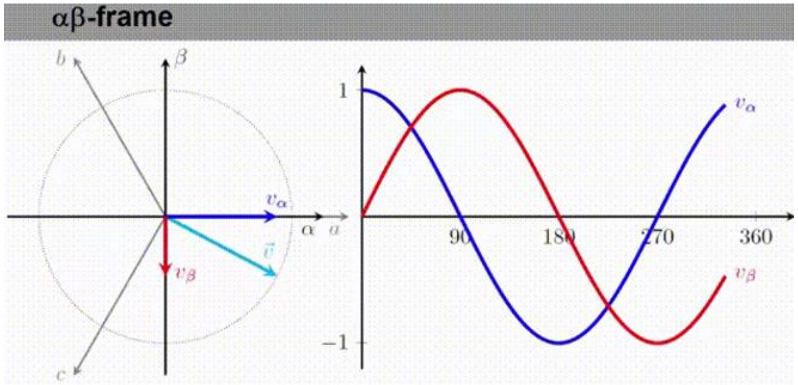
- Basic cost function: $i_{a/b/c}$ is the real current, $i^*_{a/b/c}$ is the reference

$$J = \left| i_a^*(k+1) - i_a(k+1) \right| + \left| i_b^*(k+1) - i_b(k+1) \right| + \left| i_c^*(k+1) - i_c(k+1) \right|$$

Improved cost function: reduce items





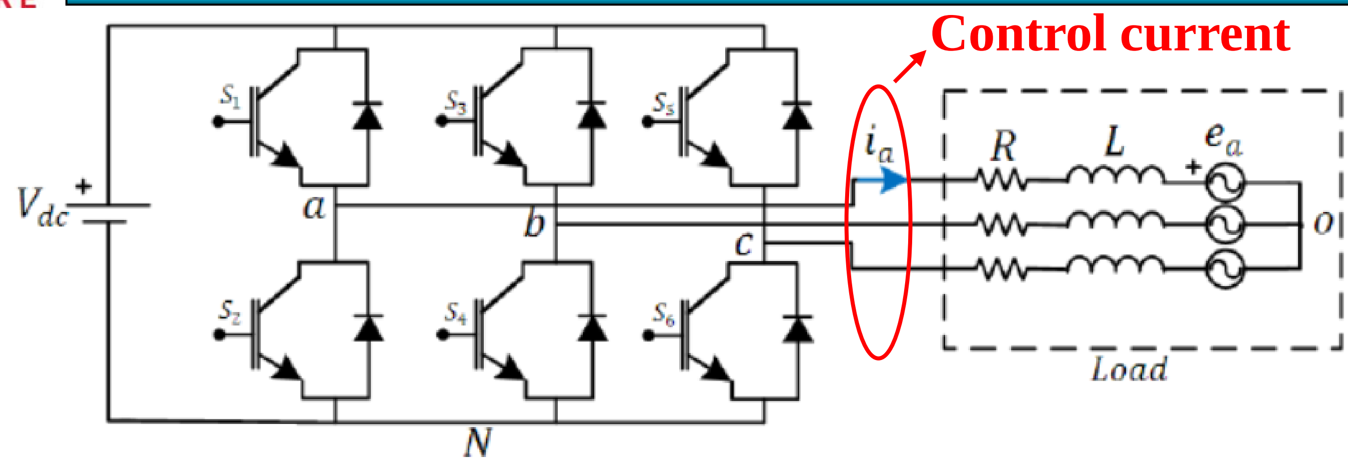


- Improved cost function:

Reduce from $i_{a/b/c}$ to $i_{\alpha/\beta}$

$$J = |i_\alpha^*(k+1) - i_\alpha(k+1)| + |i_\beta^*(k+1) - i_\beta(k+1)|$$

Practical cost function



- Ideal cost function:

$$J = |i_\alpha^*(k+1) - i_\alpha(k+1)| + |i_\beta^*(k+1) - i_\beta(k+1)|$$

- If high sampling frequency
  - Current is approximated to be constant in one step time,
  - Practical cost function:

$$J = |i_\alpha^*(k) - i_\alpha(k+1)| + |i_\beta^*(k) - i_\beta(k+1)|$$

* ADD CONSTRAINS TO THE COST FUNCTION OF THREE PHASE INVERTERS

Constraints to cost function

- ✓ Switching frequency minimization
- ✓ Voltage and current ripple minimization
- ✓ Defining maximum allowed current and voltage

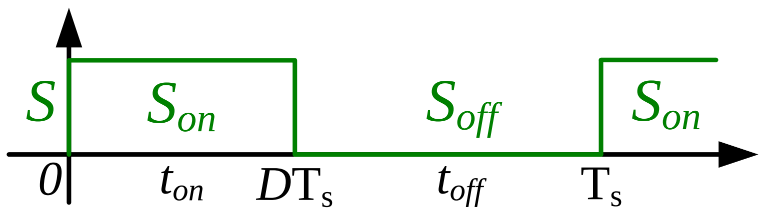
Constraints I: Switching frequency minimization

➢ Switch states amount changed at each sampling time:

Weight factor: (0~1)

$$J=\left|i_{\alpha}^{*}(\mathrm{k}+1)-i_{\alpha}(\mathrm{k}+1)\right|+\left|i_{\beta}^{*}(\mathrm{k}+1)-i\beta(\mathrm{k}+1)\right|+\boxed{\lambda\cdot n}$$

Switching frequency



$$n=\sum_{i=1}^{N}|S_i(k+1)-S_i(k)|$$

- $S_i(k)$: switch state i at the current state;
- $S_i(k+1)$: switch state i at the next sampling period;

Constraints II: Voltage & current ripple minimization

- General form of adding voltage ripple constrain:

$$J = \left\| x_{ref} - x_{prediction} \right\| + \boxed{\lambda \cdot \left\| v(k+1) - v(k) \right\|}$$

⇩

Voltage ripple constrain

Weight factor: (0~1)

- General form of adding current ripple constrain:

$$J = \left\| x_{ref} - x_{prediction} \right\| + \boxed{\lambda \cdot \left\| i(k+1) - i(k) \right\|}$$

⇩

Current ripple constrain

Constraints III: Defining allowed maximum current & voltage

- Cost function considering allowed maximum current:

$$J = \left\| x_{ref} - x_{prediction} \right\| + \boxed{f_{lim}(i_{prediction})}$$

Allowed max. I

Weight factor: (0~1)

$$f_{lim}(i_{prediction}) = \begin{cases} \infty & if \ \left| i_{prediction} \right| > I_{\max} \\ 0 & if \ \left| i_{prediction} \right| < I_{\max} \end{cases}$$

- Cost function considering allowed maximum voltage:

$$J = \left\| x_{ref} - x_{prediction} \right\| + \boxed{f_{lim}(v_{prediction})}$$

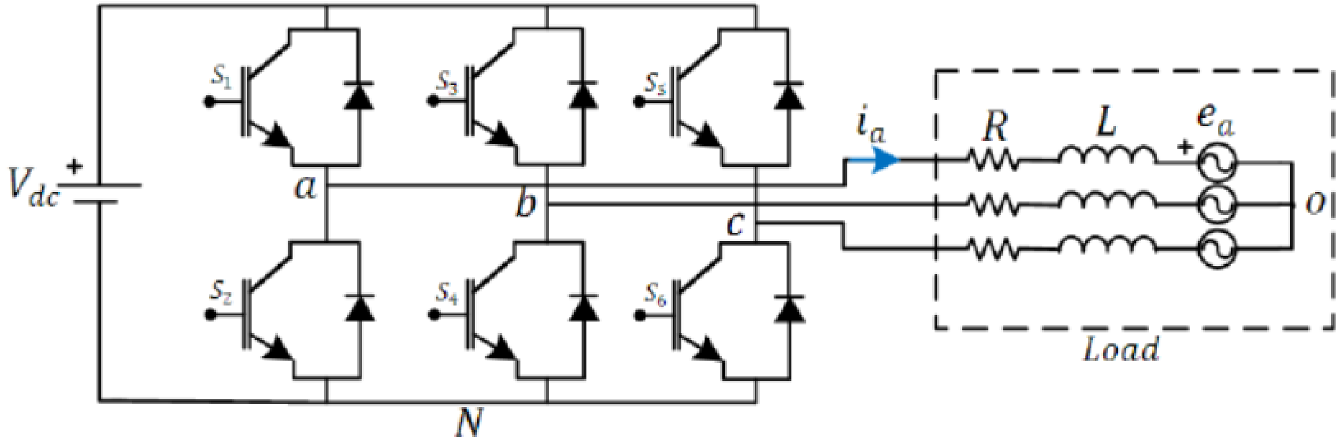
Allowed max. V

$$f_{lim}(v_{prediction}) = \begin{cases} \infty & if \ \left| v_{prediction} \right| > V_{\max} \\ 0 & if \ \left| v_{prediction} \right| < V_{\max} \end{cases}$$

FCS-MPC FOR POWER INVERTERS

- STEP 3: SELECT THE OPTIMAL SWITCH STATES

Select the suitable switching state



S_1	S_2	S_3	S_4	S_5	S_6
0	1	0	1	0	1
1	0	0	1	0	1
0	1	1	0	0	1
0	1	0	1	1	0
1	0	1	0	0	1
1	0	0	1	1	0
0	1	1	0	1	0
1	0	1	0	1	0

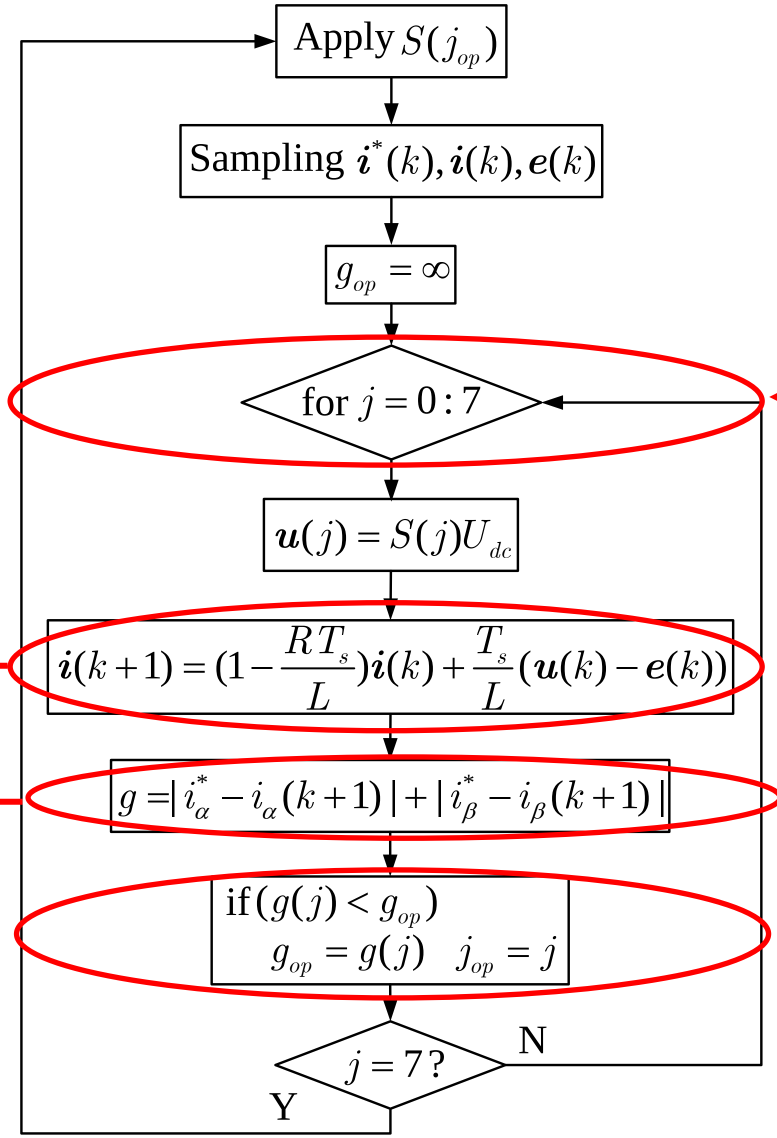
- Practical cost function:

$$J = |i_\alpha^*(k) - i_\alpha(k+1)| + |i_\beta^*(k) - i_\beta(k+1)|$$

$\Downarrow$

Select the suitable switching state to get the minimal value

MPC flowchart



8 states

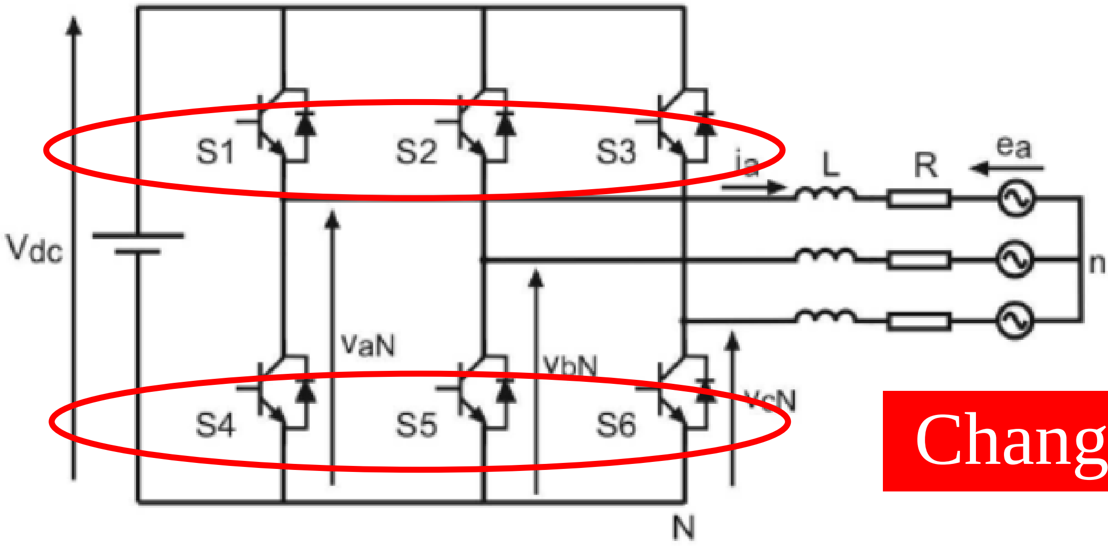
Forward Euler method based model

Cost function

Minimal cost

FCS-MPC for Power Inverters - Example

Parameters of the three-phase inverter

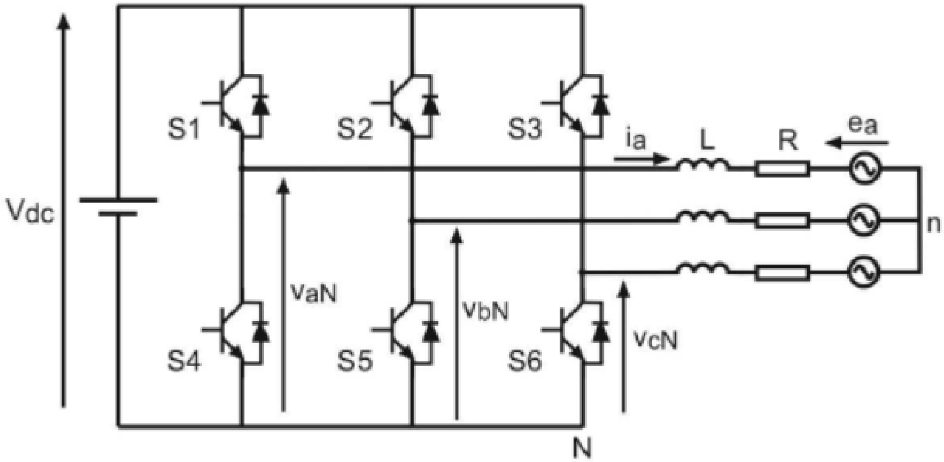


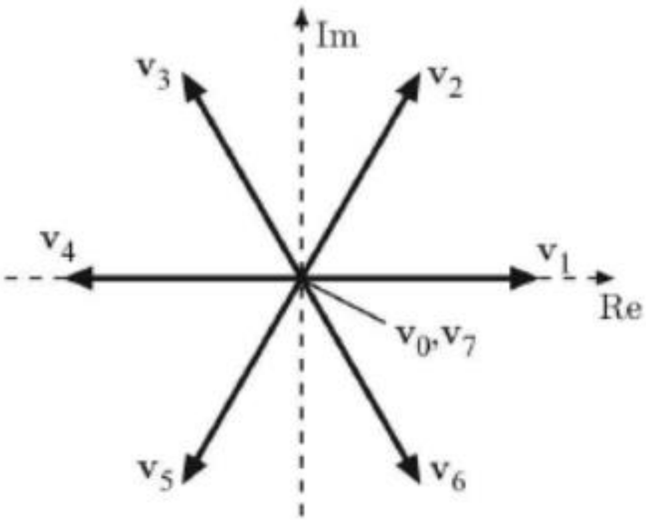
Change orders

V_{dc}	6.6 kV	R	0.3 Ω
$e_{line\text{-}line}$	3.3 kV (rms)	L	2.5 mH
F	50 Hz	T_S	100 µs
$I_{nominal}$	3.5 kA (peak)	f_s	10 kHz

Switching states of the example inverter

- Model of a three-phase inverter
  - Only 8 possible switching states
  - 7 different voltage vectors





$$S_a = \begin{cases} 1 & \text{if } S_1 \text{ on and } S_4 \text{ off} \\ 0 & \text{if } S_1 \text{ off and } S_4 \text{ on} \end{cases}$$

$$S_b = \begin{cases} 1 & \text{if } S_2 \text{ on and } S_5 \text{ off} \\ 0 & \text{if } S_2 \text{ off and } S_5 \text{ on} \end{cases}$$

$$S_c = \begin{cases} 1 & \text{if } S_3 \text{ on and } S_6 \text{ off} \\ 0 & \text{if } S_3 \text{ off and } S_6 \text{ on} \end{cases}$$

- Space vectors

$$\mathbf{S} = \frac{2}{3}(S_a + \mathbf{a}S_b + \mathbf{a}^2 S_c)$$

$$\mathbf{v} = \frac{2}{3}(v_{aN} + \mathbf{a}v_{bN} + \mathbf{a}^2 v_{cN})$$

$$\mathbf{v} = V_{dc}\mathbf{S}$$

Detailed switching states of example inverter

$$\mathbf{v} = \frac{2}{3}(v_{aN} + \mathbf{a}v_{bN} + \mathbf{a}^2 v_{cN}) \qquad a = e^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

S_1	S_2	S_3	S_4	S_5	S_6	Inverter terminal voltage space vector v
0	1	0	1	0	1	$v_0 = 0$
1	0	0	1	0	1	$v_1 = {}^2/_3\, V_{dc}$
0	1	1	0	0	1	$v_2 = {}^1/_3\,(-1 + j\sqrt{3})V_{dc}$
0	1	0	1	1	0	$v_3 = {}^1/_3\,(-1 - j\sqrt{3})V_{dc}$
1	0	1	0	0	1	$v_4 = {}^1/_3\,(1 + j\sqrt{3})V_{dc}$
1	0	0	1	1	0	$v_5 = {}^1/_3\,(1 - j\sqrt{3})V_{dc}$
0	1	1	0	1	0	$v_6 = -{}^2/_3\, V_{dc}$
1	0	1	0	1	0	$v_7 = 0$

Model of the example inverter

- Load model
  - Vector equation for the load current dynamics

$$\mathbf{v} = R\mathbf{i} + L\frac{d\mathbf{i}}{dt} + \mathbf{e}$$

where

$$\mathbf{v} = \frac{2}{3}(v_{aN} + \mathbf{a}v_{bN} + \mathbf{a}^2 v_{cN})$$

$$\mathbf{i} = \frac{2}{3}(i_a + \mathbf{a}i_b + \mathbf{a}^2 i_c)$$

$$\mathbf{e} = \frac{2}{3}(e_a + \mathbf{a}e_b + \mathbf{a}^2 e_c)$$

  - Discrete-time equations

$$\hat{\mathbf{i}}(k+1) = \left(1 - \frac{RT_s}{L}\right)\mathbf{i}(k) + \frac{T_s}{L}\left(\mathbf{v}(k) - \hat{\mathbf{e}}(k)\right)$$

$$\hat{\mathbf{e}}(k-1) = \mathbf{v}(k-1) - \frac{L}{T_s}\mathbf{i}(k) - \left(R - \frac{L}{T_s}\right)\mathbf{i}(k-1)$$

$$\frac{d\mathbf{i}}{dt} \approx \frac{\mathbf{i}(k+1) - \mathbf{i}(k)}{T_s}$$

Forward Euler method

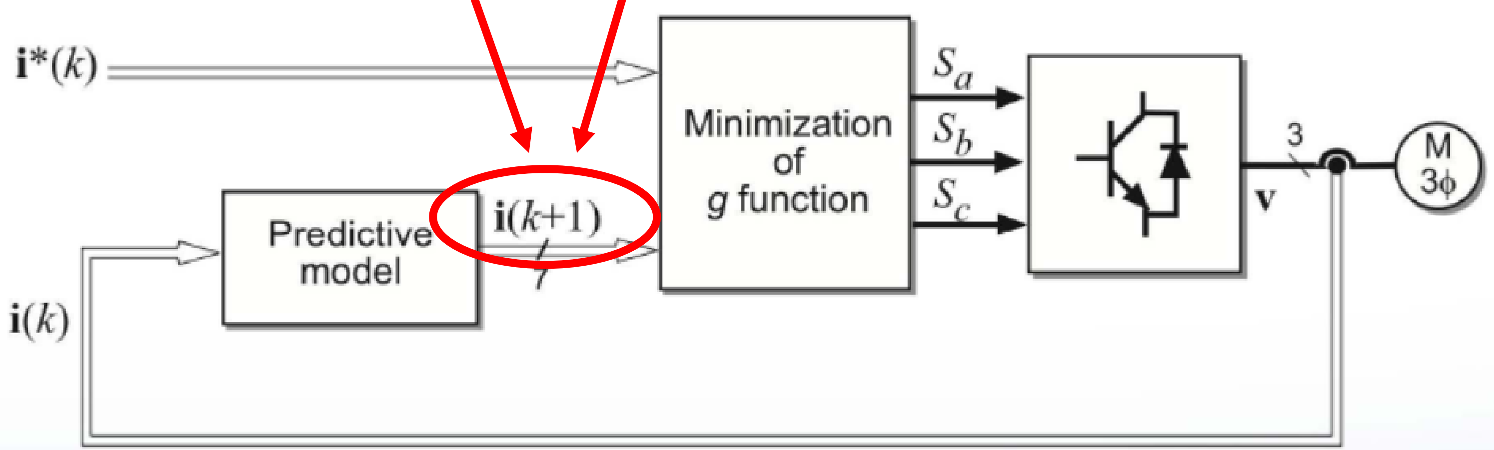
Cost function of the example inverter

- Cost function:

$$g = |i_\alpha^* - i_\alpha^p| + |i_\beta^* - i_\beta^p|$$

i_α^*, i_β^* : reference values

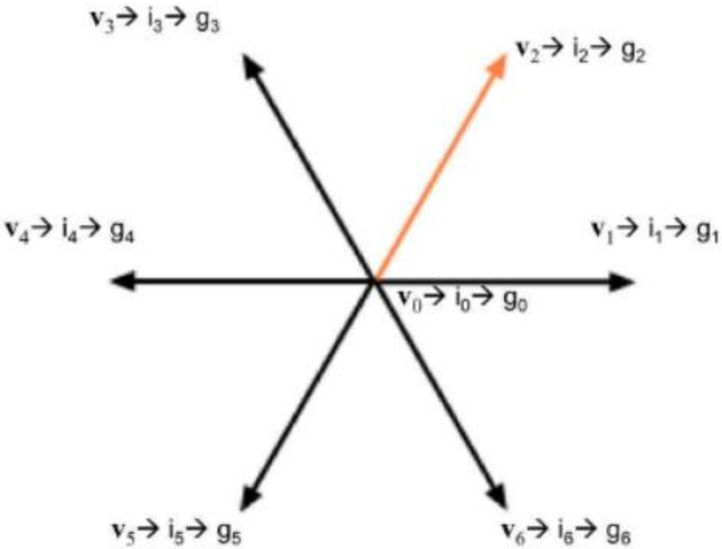
i_α^p, i_β^p : predicted values



- No need for linear controllers !!
- No need for modulator (PWM or SVM) !!

Select the optimal state for the example inverter

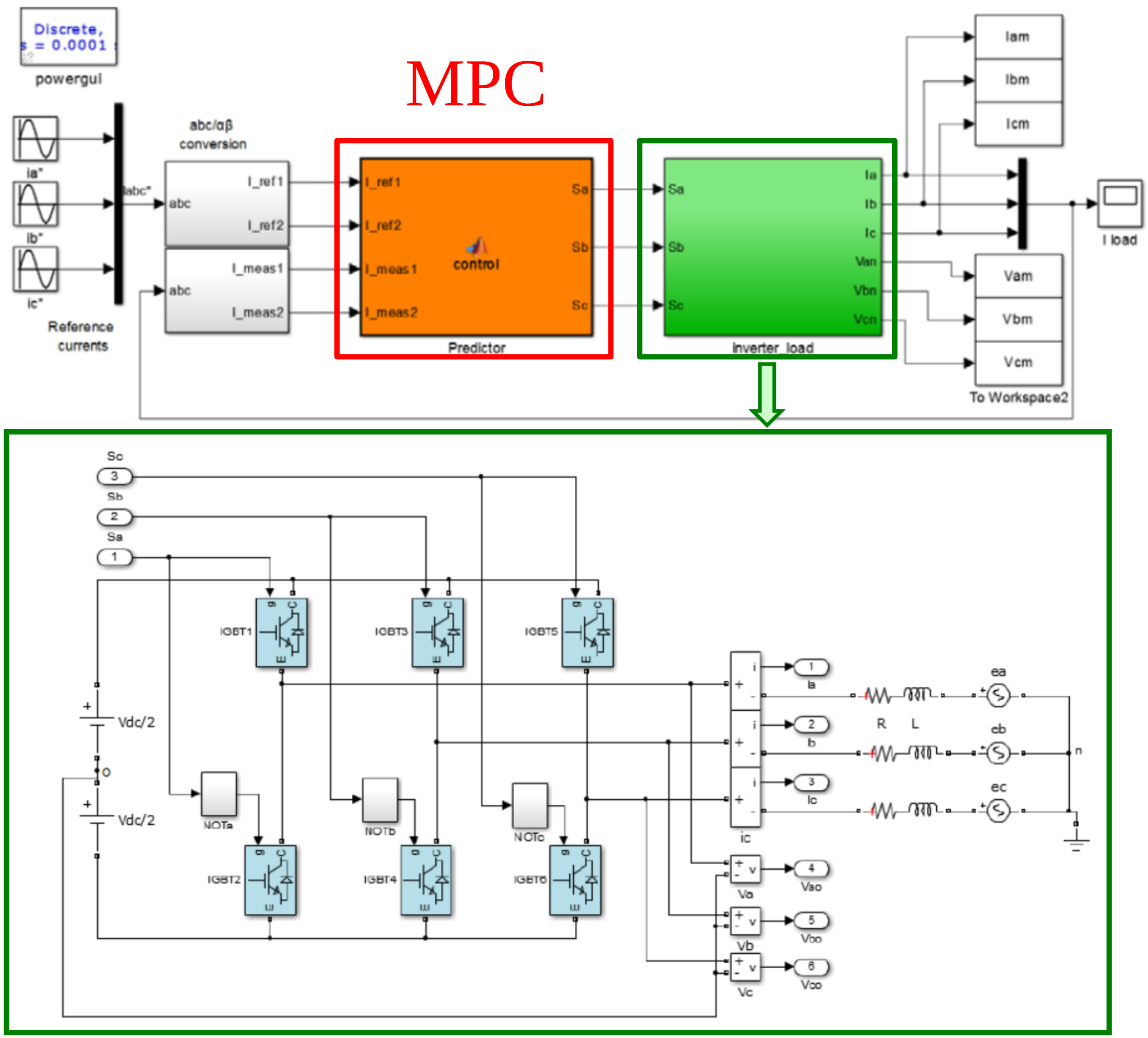
Cost function minimization



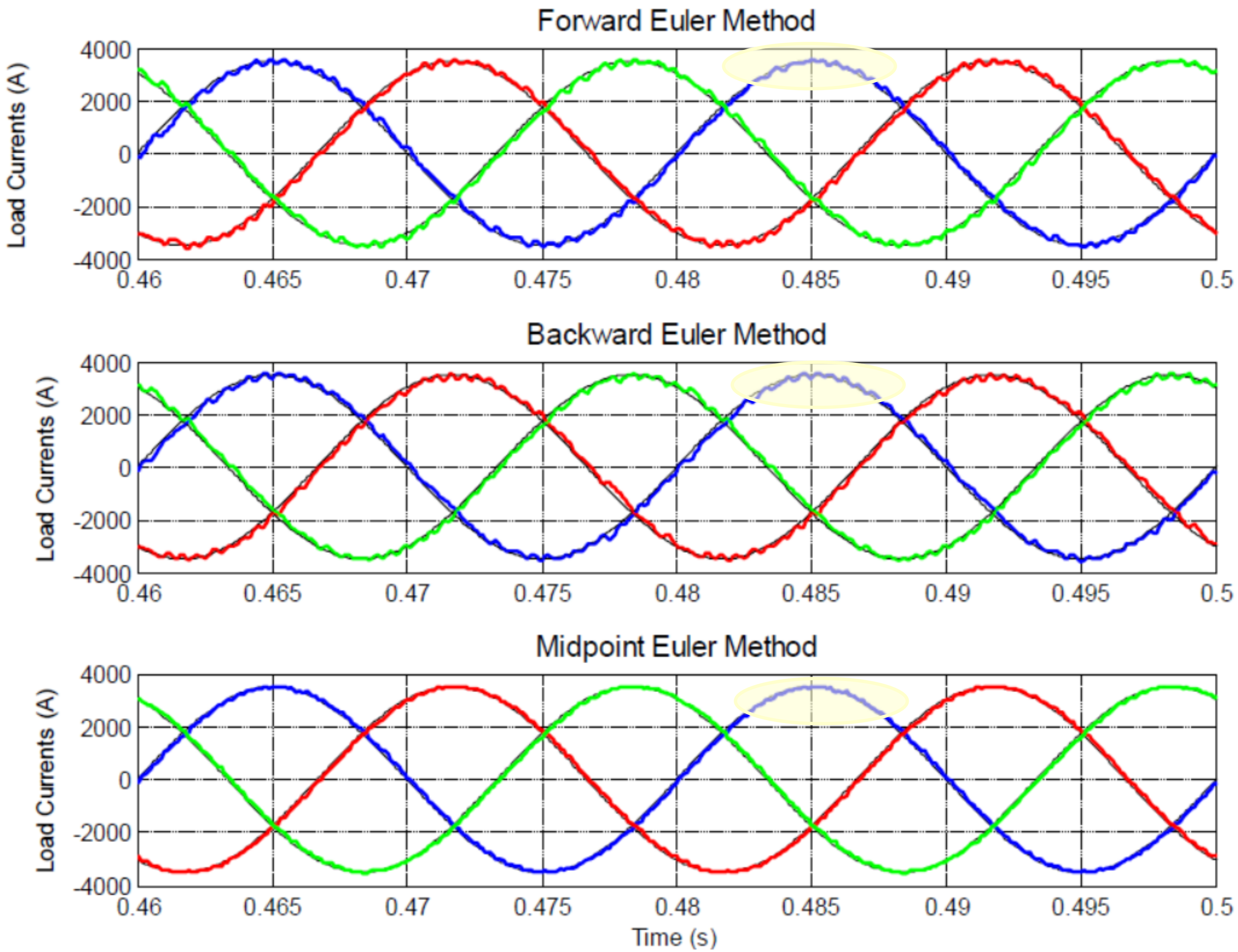
$\mathbf{v}_0$	g_0	0.60	
$\mathbf{v}_1$	g_1	0.82	
$\mathbf{v}_2$	g_2	0.24	← g_{min}
$\mathbf{v}_3$	g_3	0.42	
$\mathbf{v}_4$	g_4	0.96	
$\mathbf{v}_5$	g_5	1.24	
$\mathbf{v}_6$	g_6	1.19	

- Voltage vector $\mathbf{v}_0$ is used to predict i_0 and to calculate cost function (error) g_0.
- Voltage vector $\mathbf{v}_1$ is used to predict i_1 and to calculate cost function (error) g_1.
- …
- $g_{min} = g_2$.
- Voltage vector $\mathbf{v}_2$ is selected and will be applied during the next sampling interval.

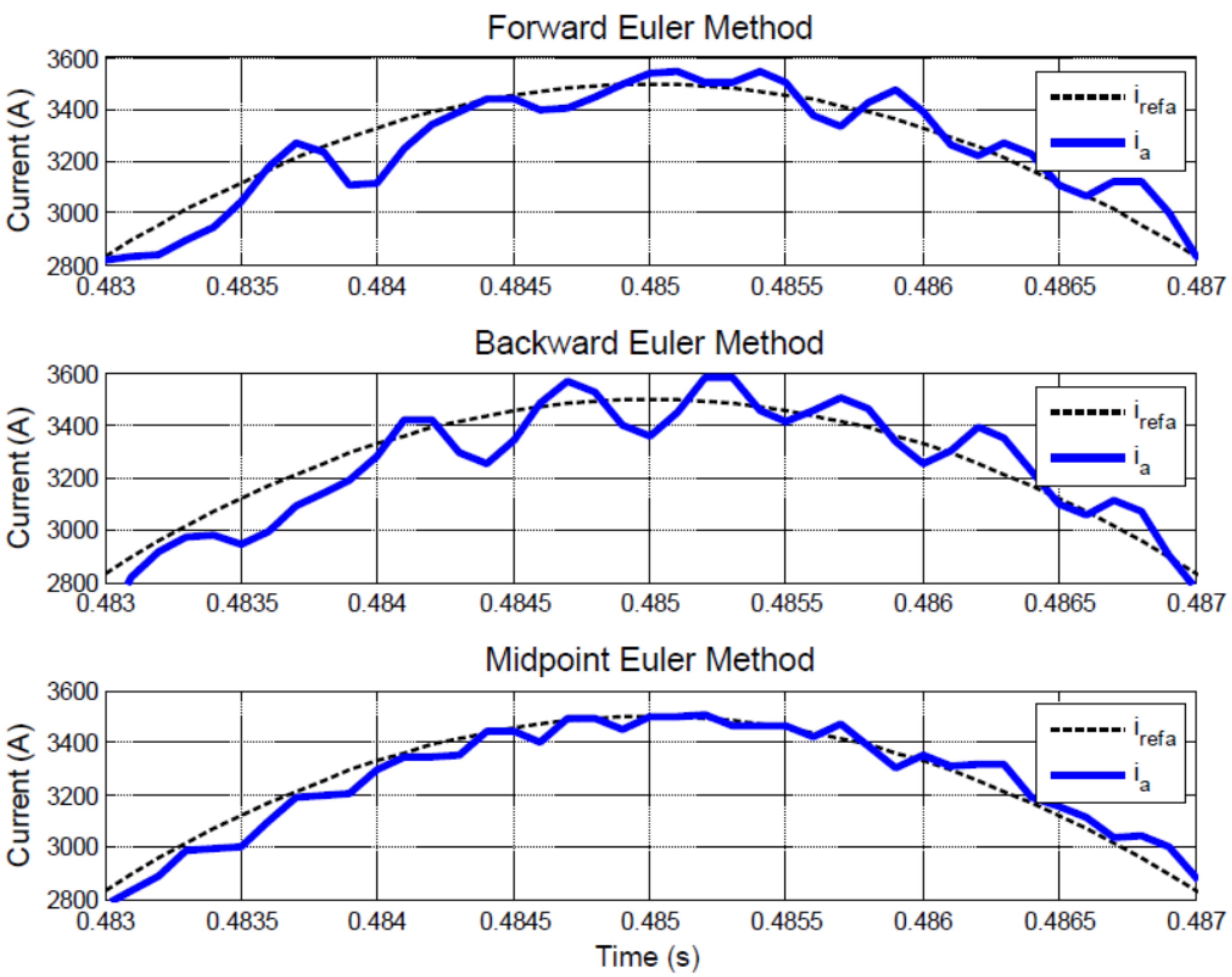
Simulation of the example inverter



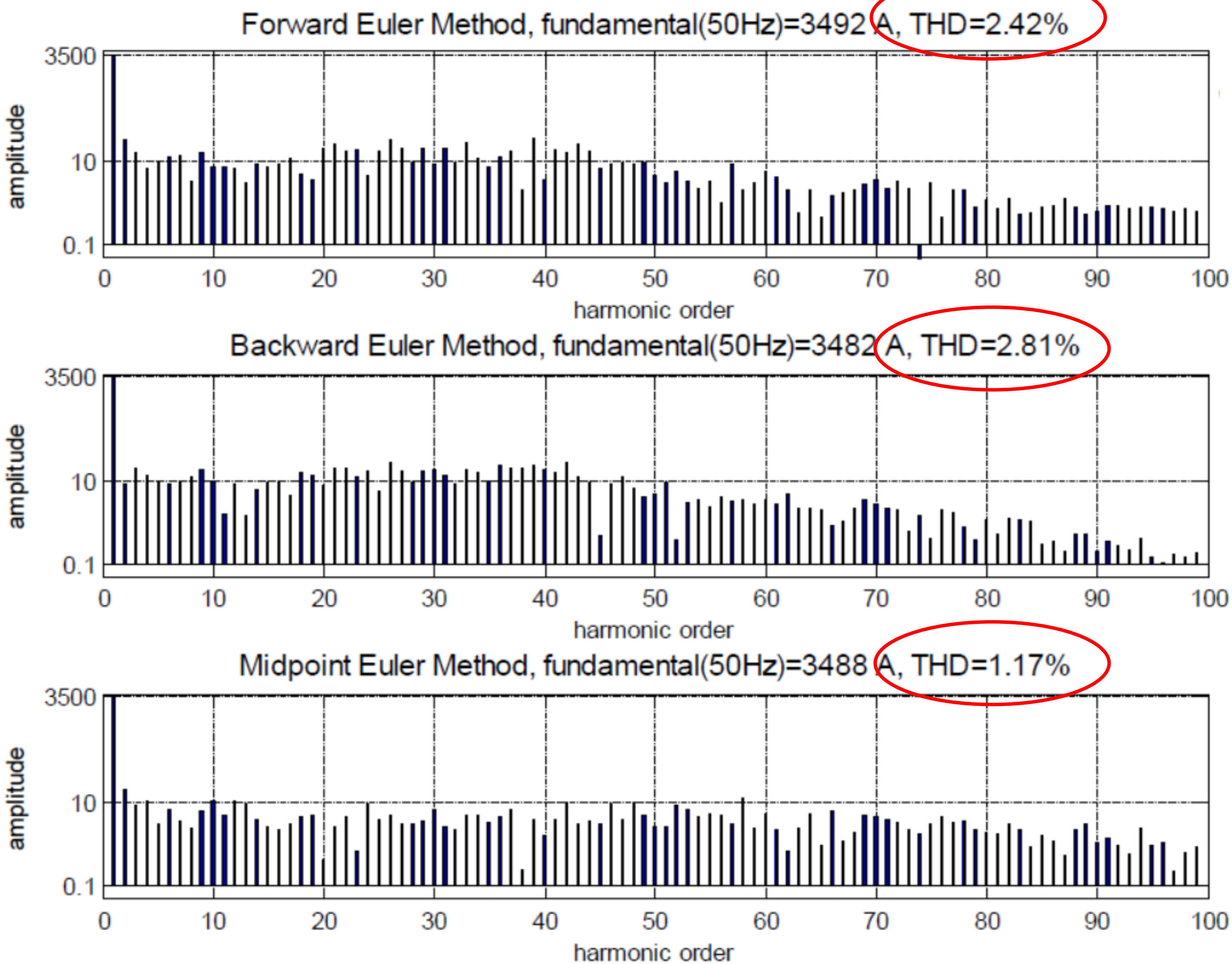
Simulation results with different discrete models



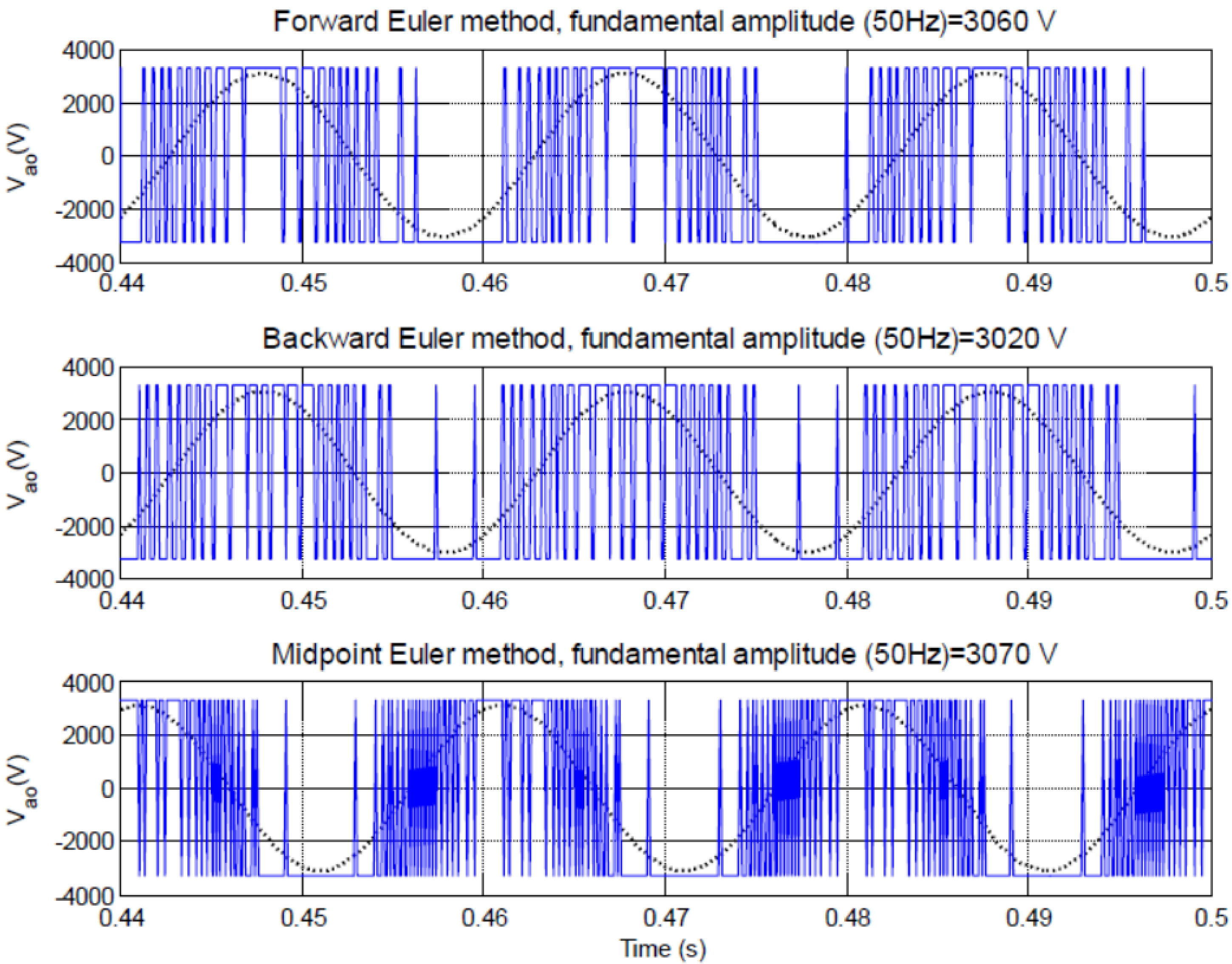
Simulation : ripple waveforms (comparisons)



Simulation: THD (comparisons)



Simulation: switching waveforms (comparisons)



Comparisons results

- The backward & forward Euler based models give smaller switching frequency.

  - Less accurate

  - Less Power loss

- The Midpoint Euler based models give bigger switching frequency.

  - More accurate

  - More Power loss

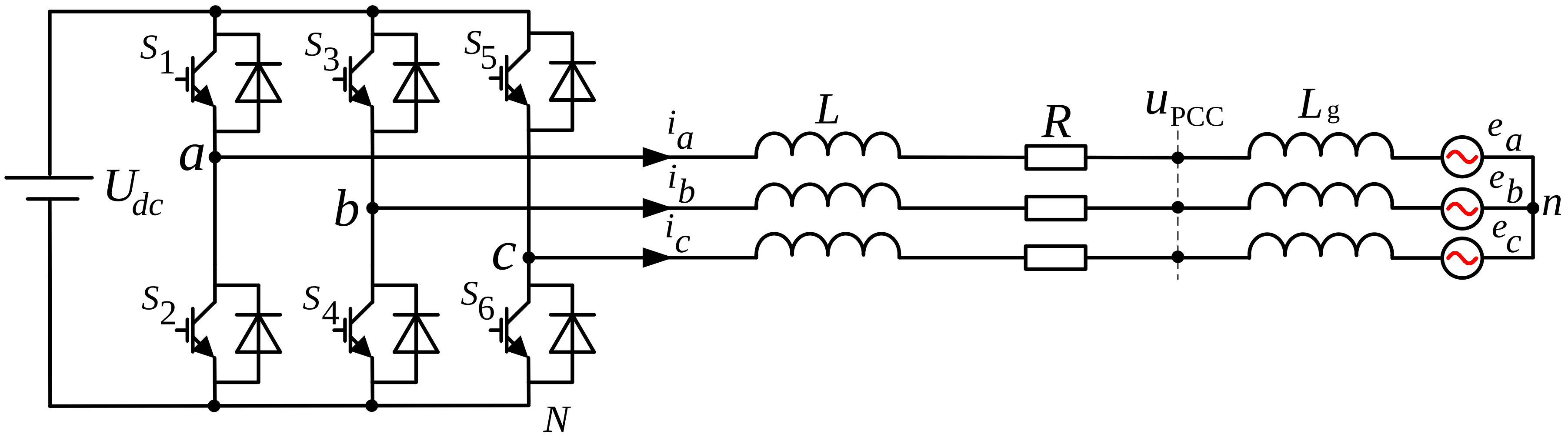
APPLICATION OF FINITE CONTROL SET MPC IN THREE PHASE INVERTER

[07/11/2019]

- Basic knowledge of power inverters
- Finite Control Set (FCS) MPC for power inverters
- Improved FCS-MPC for power inverters

Review of traditional FCS-MPC: definition

- 2 level 3 phase grid-connected inverter
- Control in αβ coordinates
- Differential equation of the load: $\underline{v} = R \cdot \underline{i} + L \frac{d\underline{i}}{dt} + \underline{e}$
- v: Applied voltage vector



Review of traditional FCS-MPC

- Time-discrete load model:

  Euler-forwards approximation:

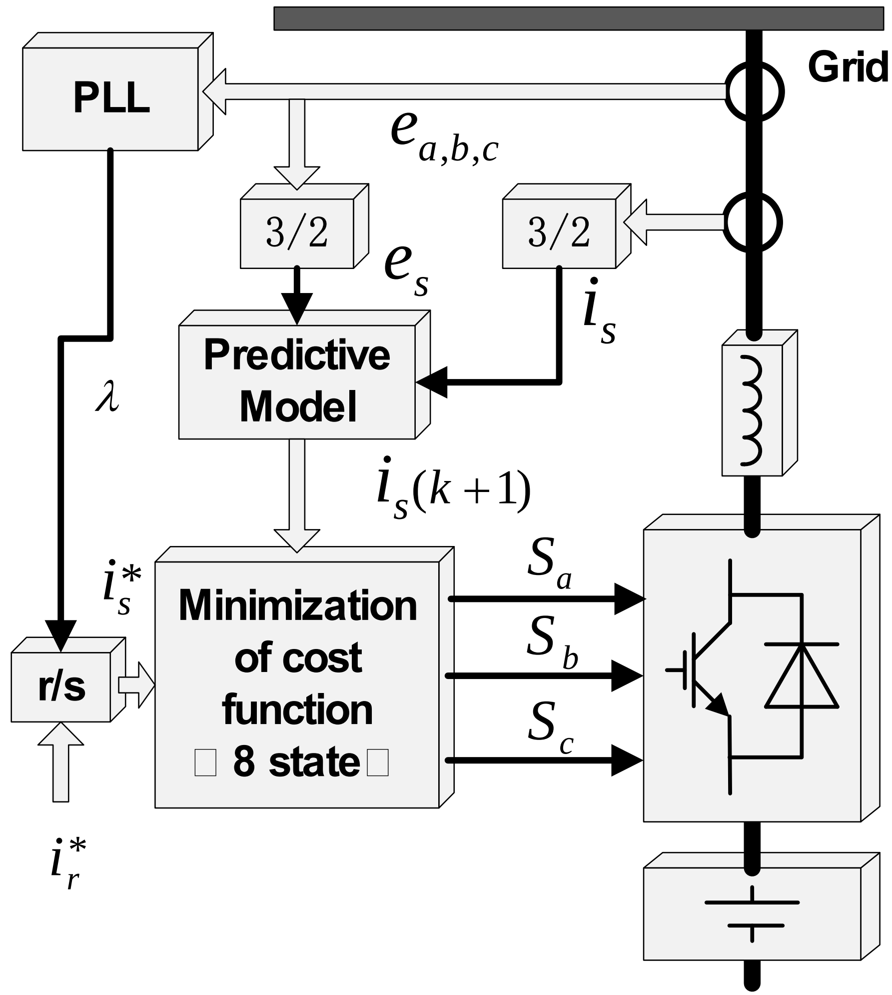
$$\frac{d\underline{i}}{dt} \approx \frac{\underline{i}(k+1) - \underline{i}(k)}{T_s}$$

- Current prediction:

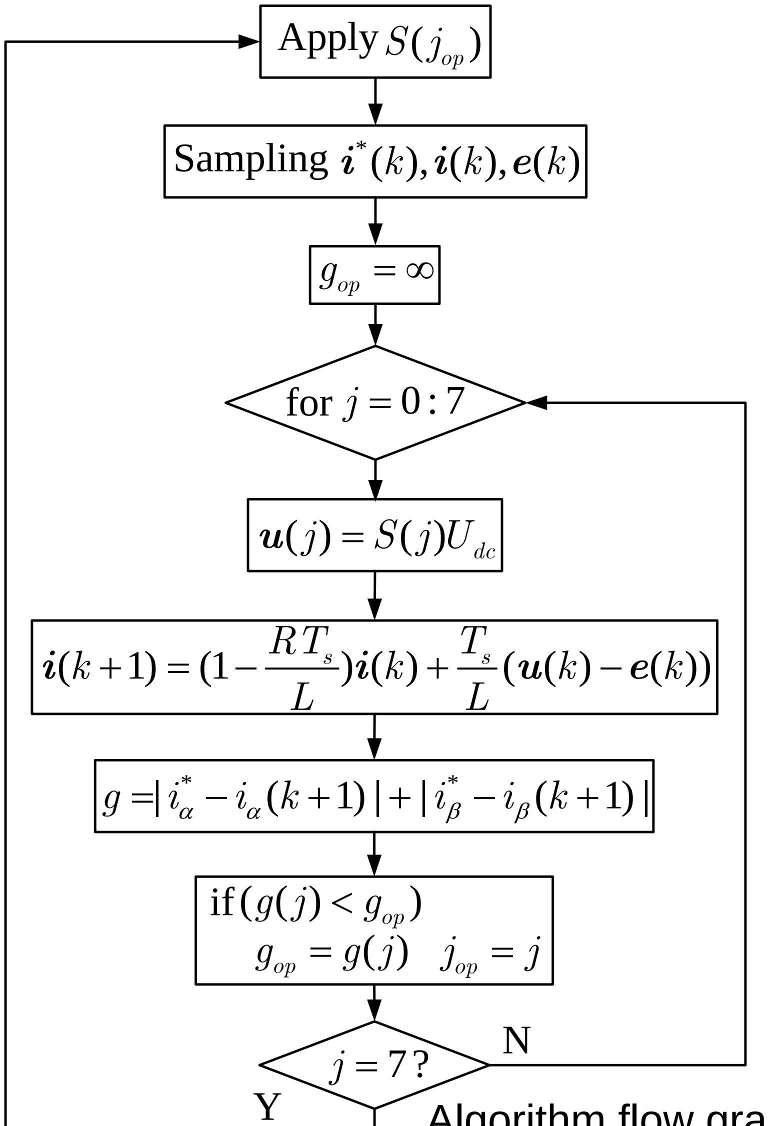
$$\underline{i}(k+1) = \left(1 - \frac{RT_s}{L}\right)\underline{i}(k) + \frac{T_s}{L}\left(\underline{v}(k) - \underline{e}(k)\right)$$

- Cost function:

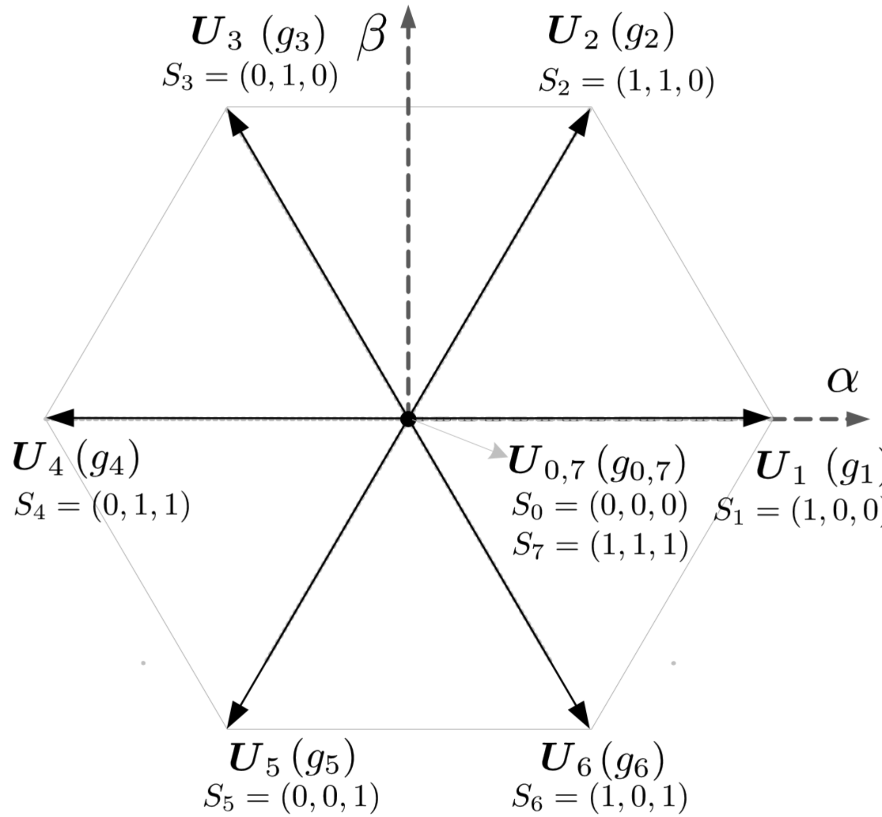
$$G = \left|i_\alpha^* - i_\alpha(k+1)\right| + \left|i_\beta^* - i_\beta(k+1)\right|$$



Review of traditional FCS-MPC: flowchart



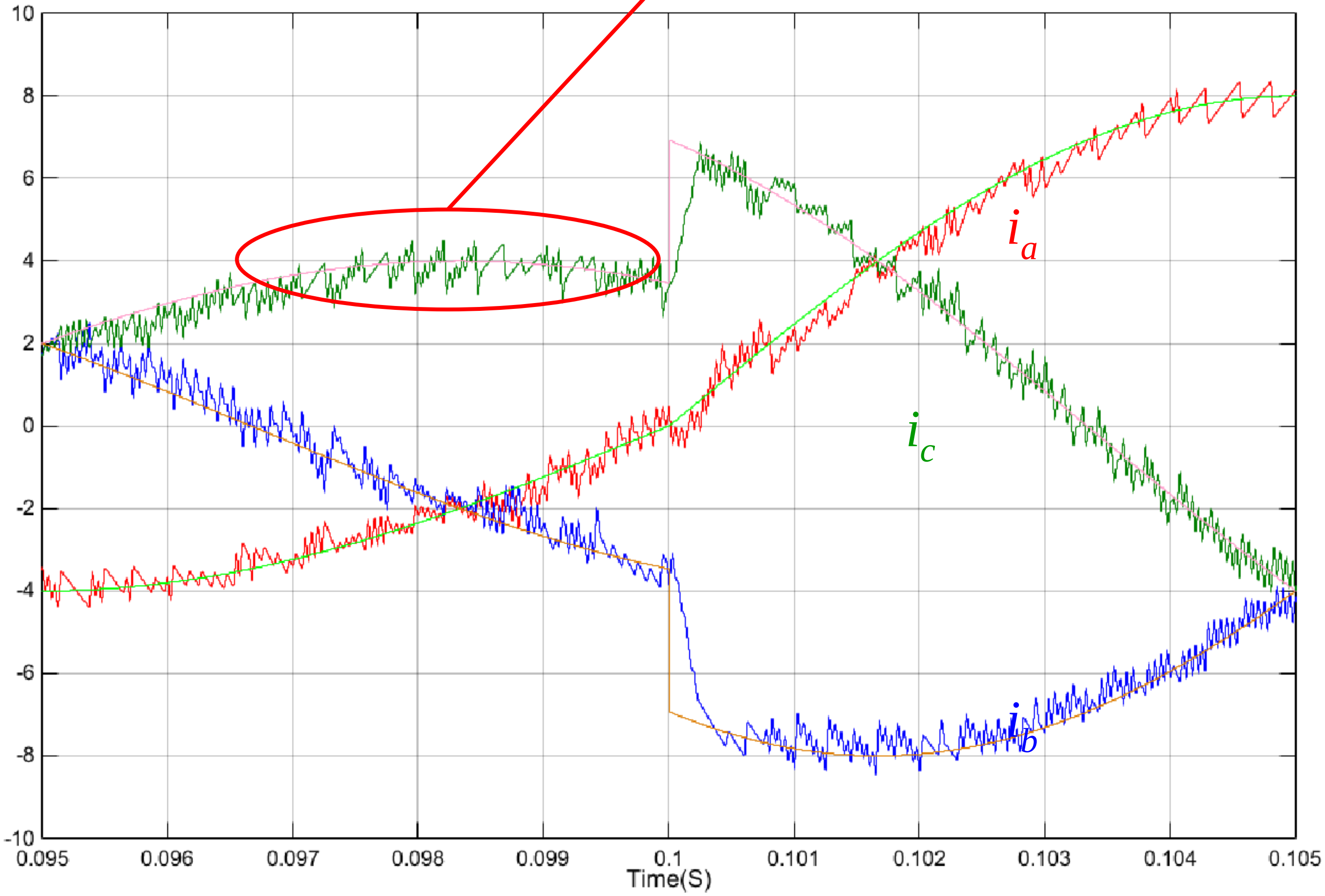
Algorithm flow graph



Eight voltage vectors

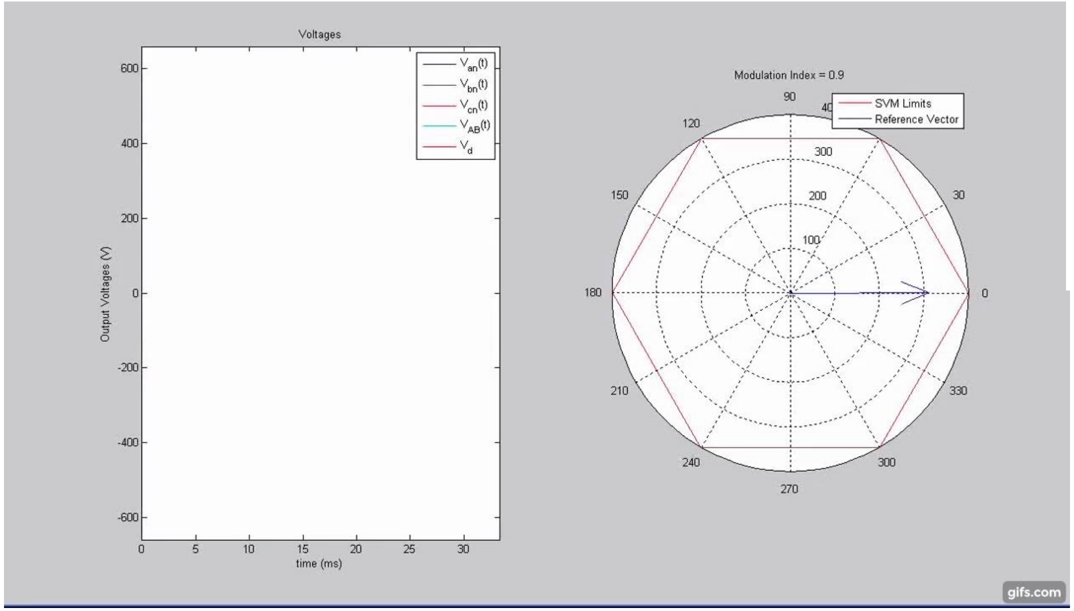
Problem: variable switching frequency

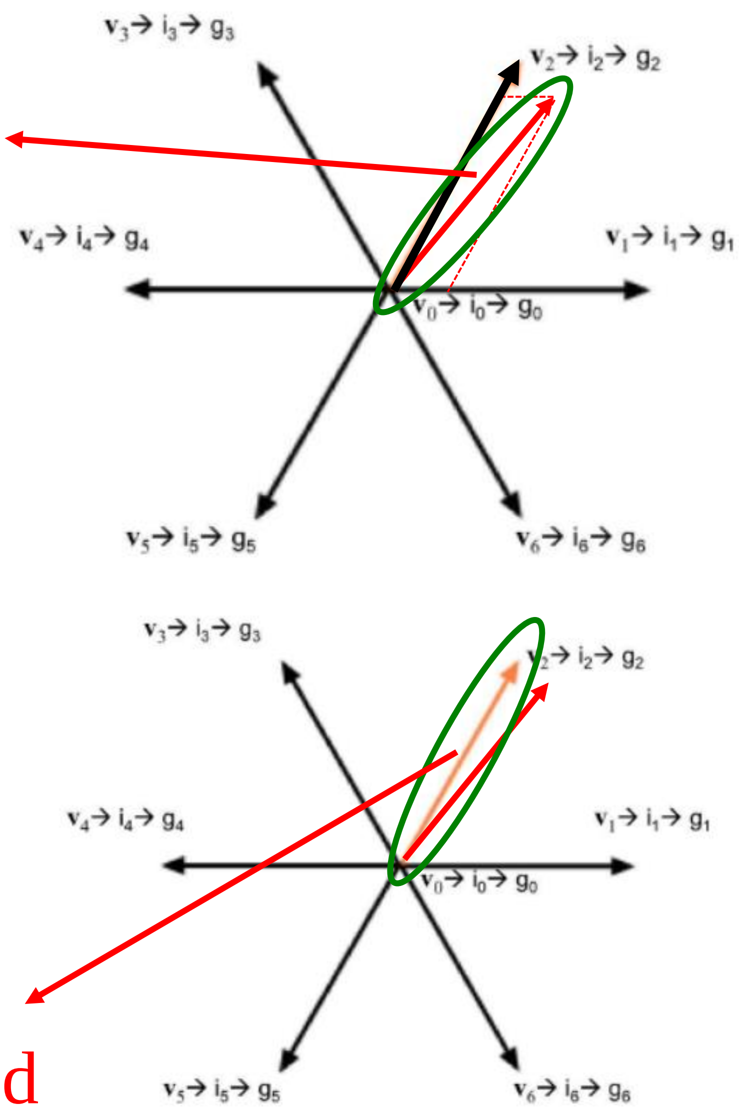
Variable switching frequency



Reason

Best vector

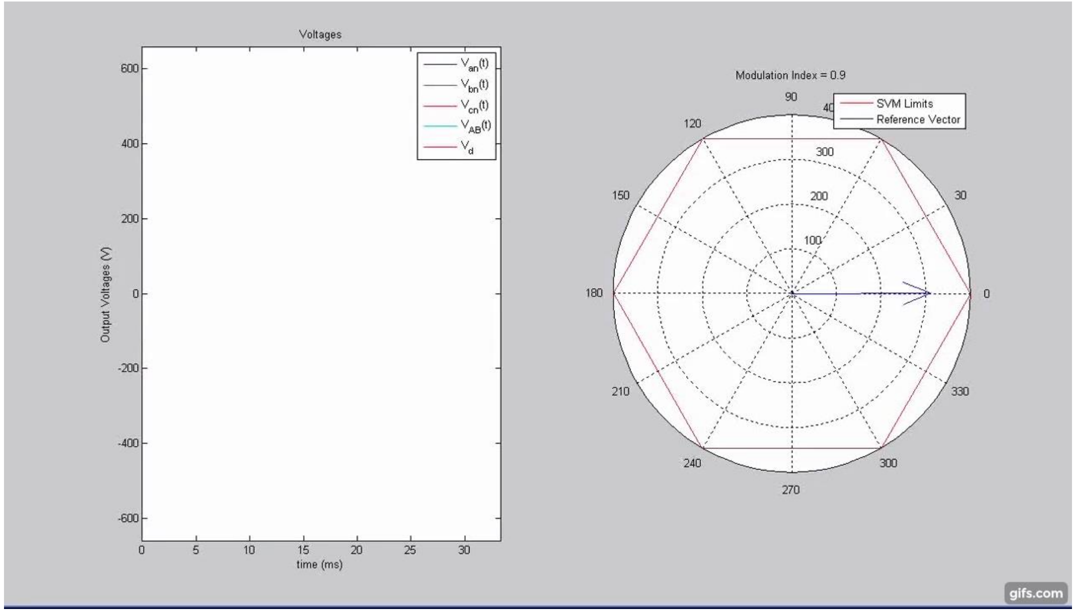


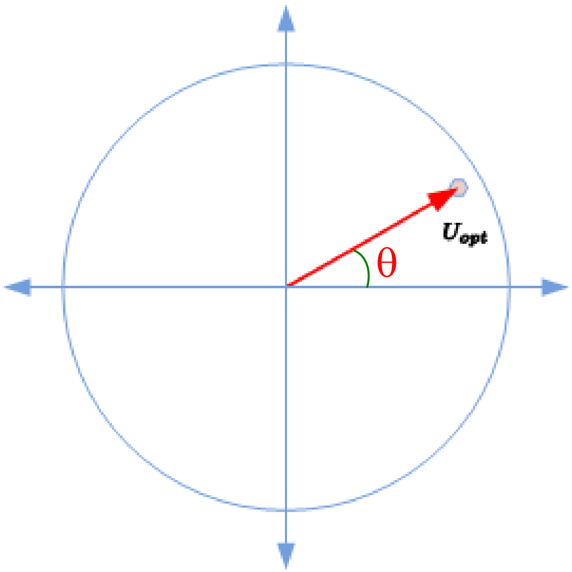


Not the targeted vector,
But we select it via FCS MPC method

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Improved direction: Directly determine the amplitude and angle



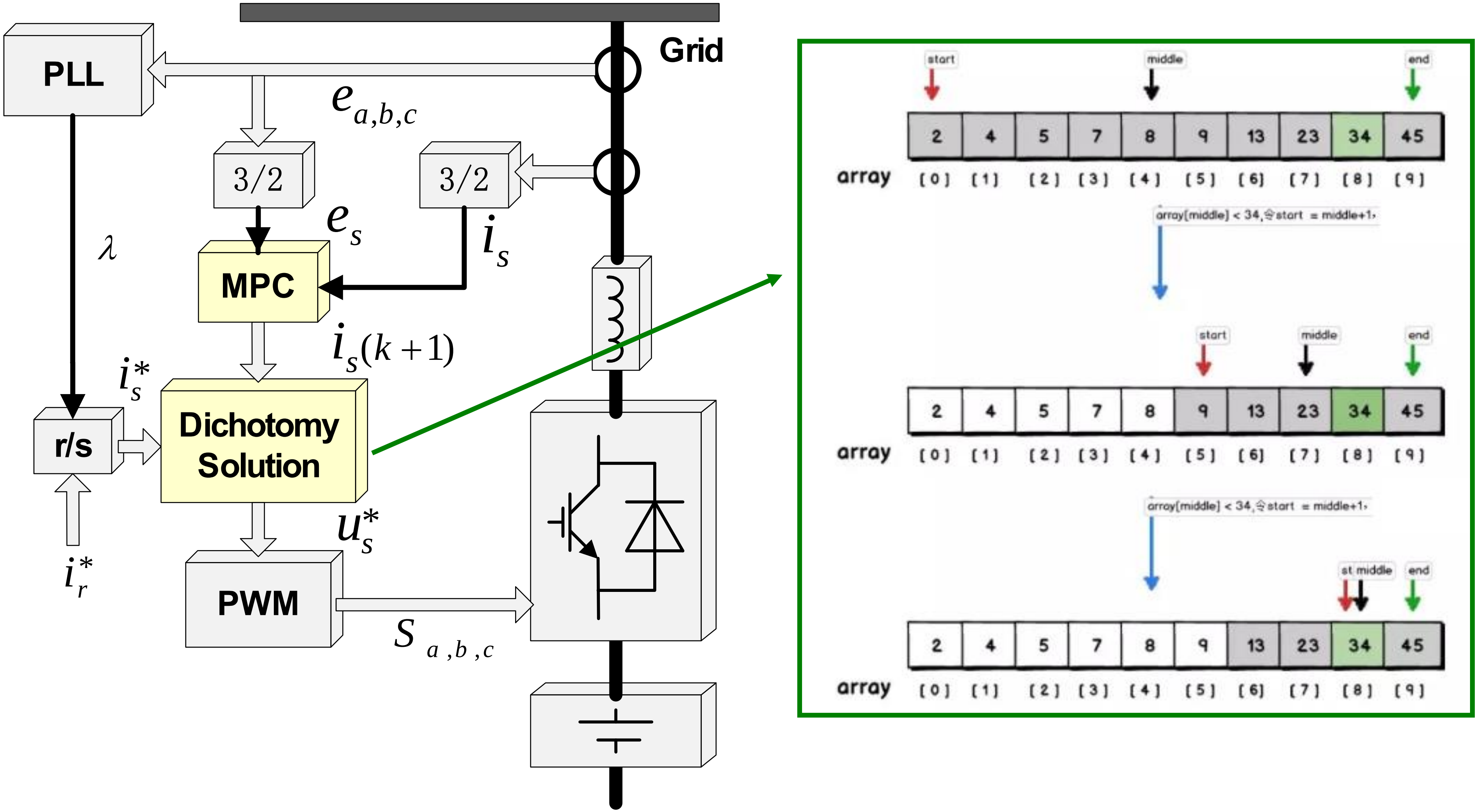


- Need to confirm the amplitude of the phasor
- Need to confirm the angle of the phasor

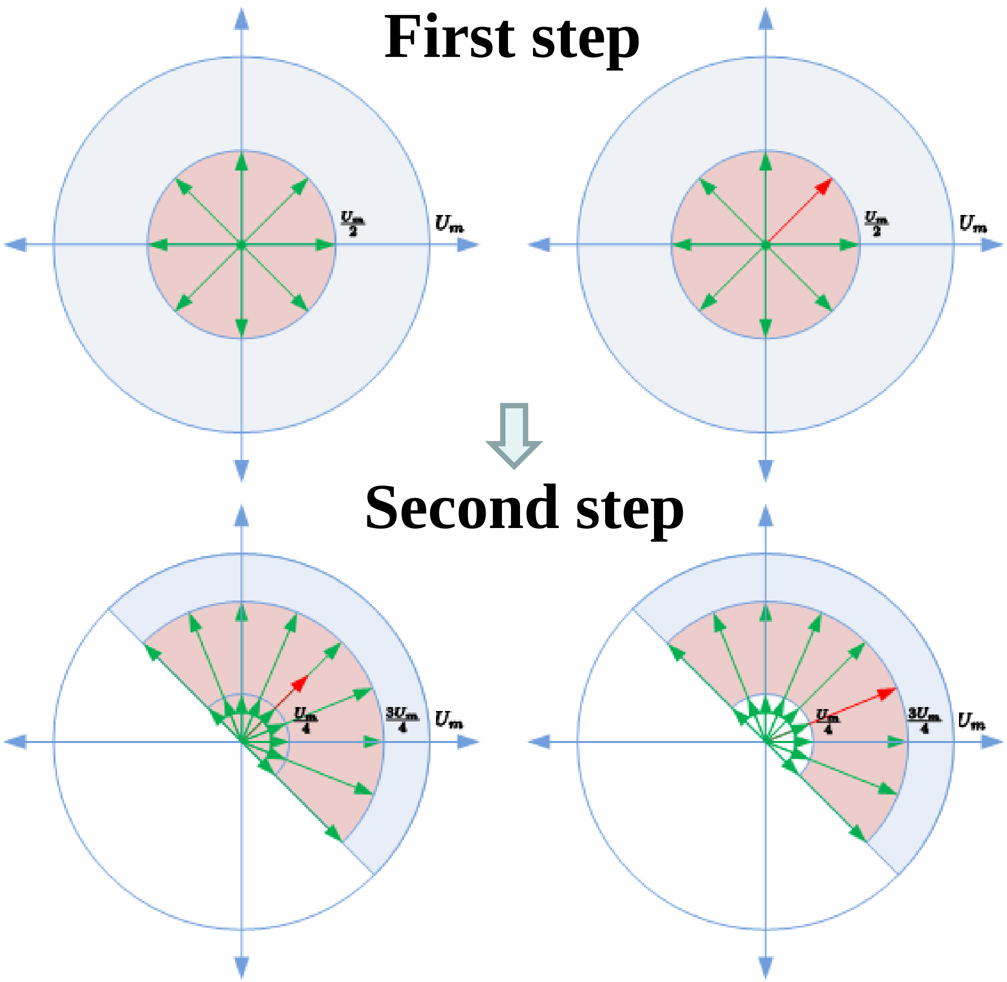
Constant switching frequency (CSF)

Dichotomy solution to find the amplitude & angle

Proposed CSF-MPC

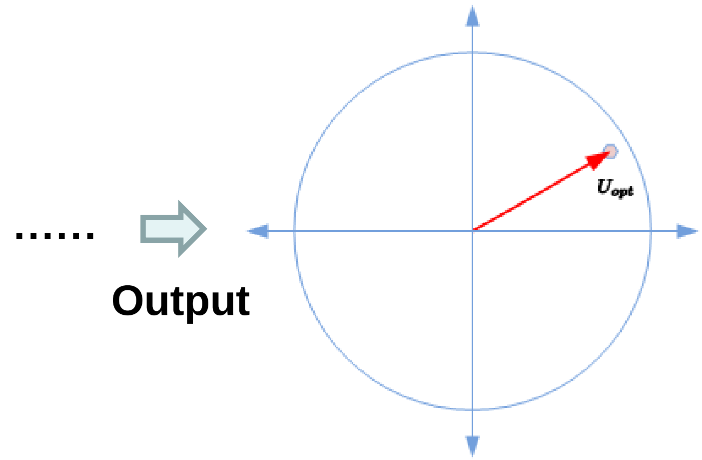


Improved MPC: Schematic diagram



Key idea: **Dichotomy**

- voltage vector search area decrease to half of the former steps.
- The convergence of voltage vector selection is fast.



Improved MPC with constant switching frequency (CSF)

Flow chart:

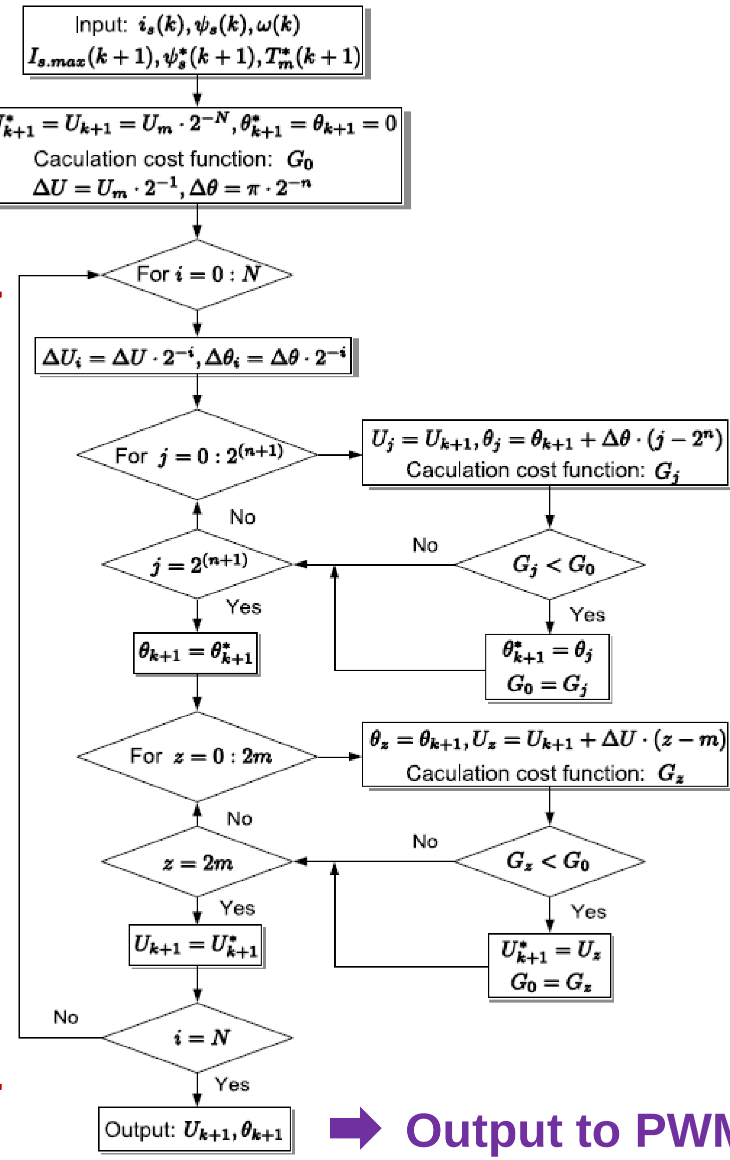
N: Resolution of the output voltage vector

n: **Phase** search resolution

2m: **Amplitude** search resolution

Initialization

Search Resolution



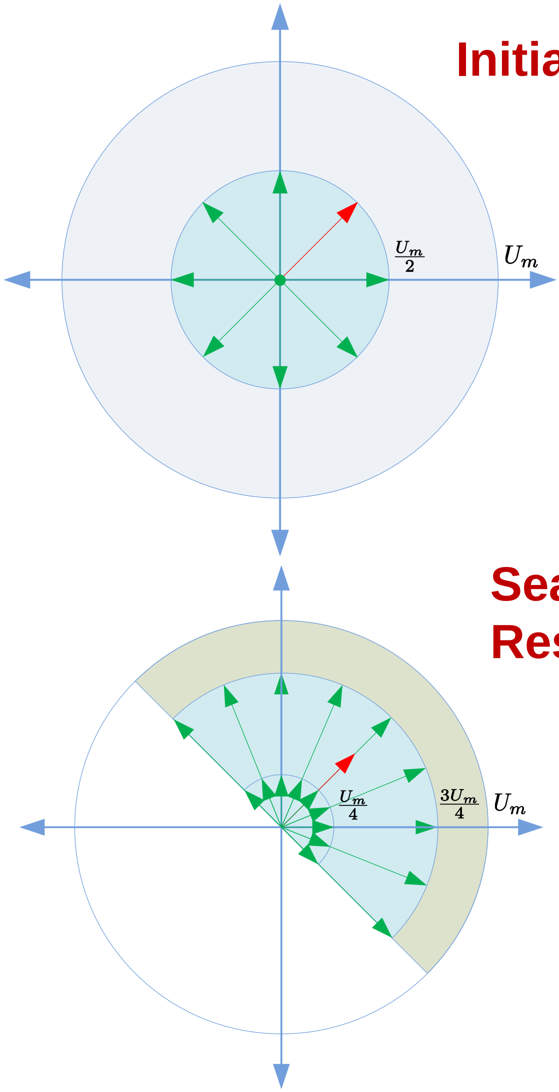
Phase Calculation Loop

Amplitude Calculation Loop

➡ Output to PWM modulator

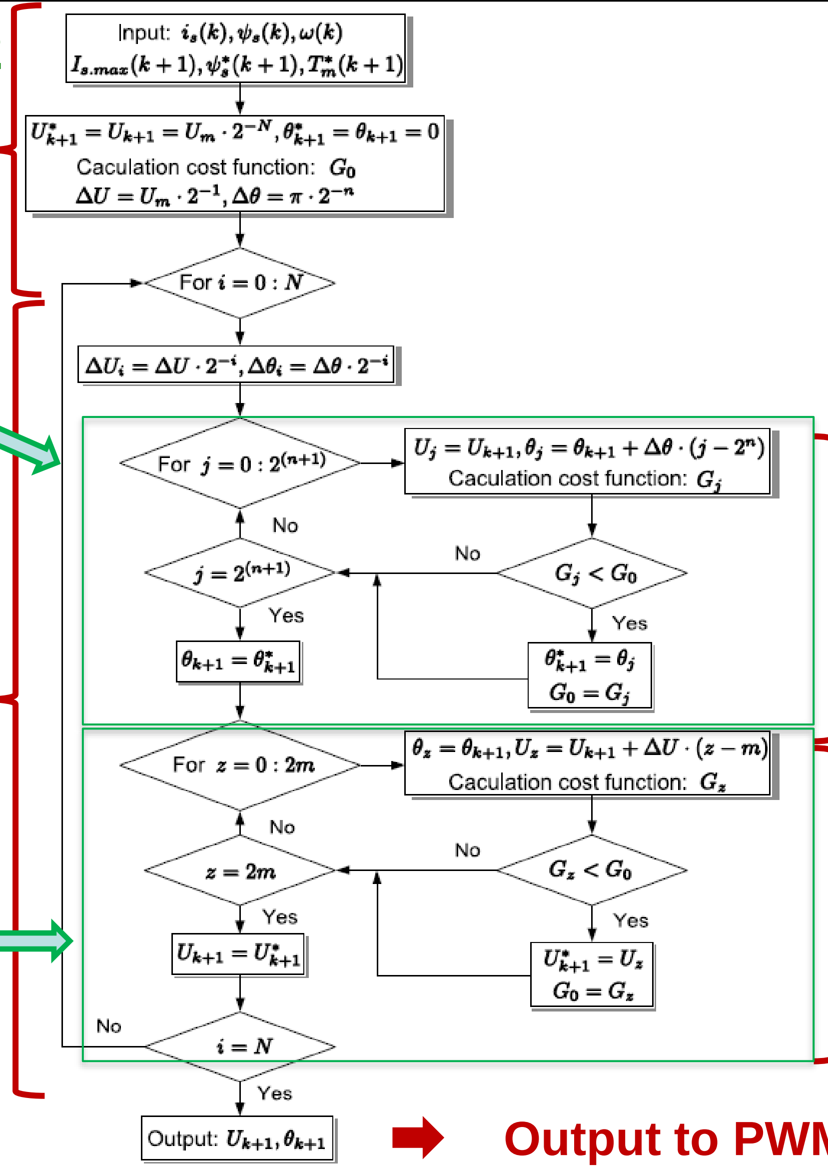
Explain of the flowchart

One example: N=14, n=2, m=2



Initialization

Search Resolution

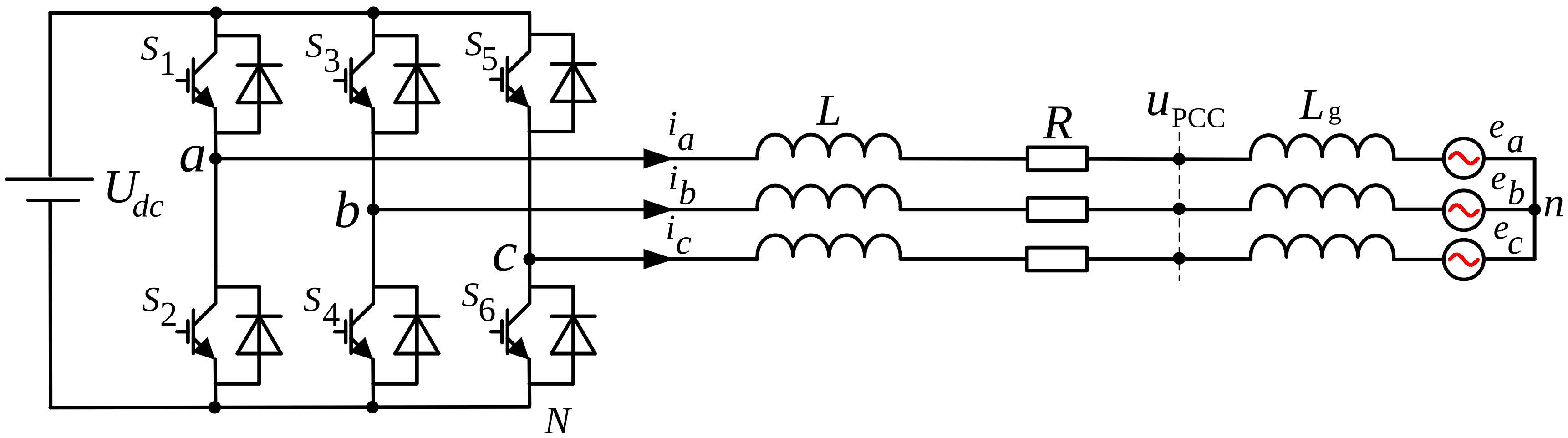


Phase Calculation Loop

Amplitude Calculation Loop

Output to PWM modulator

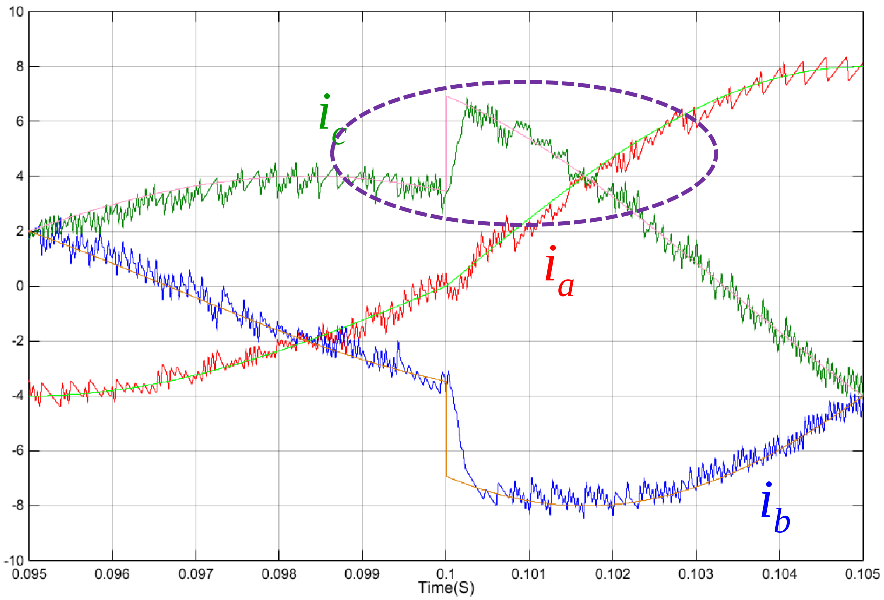
Example of the improved FCS-MPC method



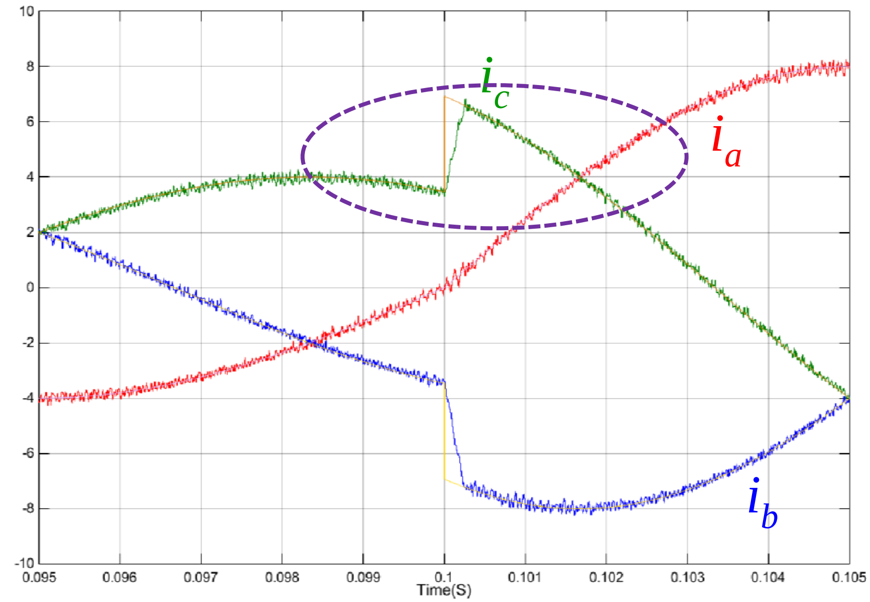
PARAMETERS	VALUE
RATED POWER P_N	2 kW
DC BUS VOLTAGE U_{DC}	300 V
AC BUS VOLTAGE	115 V (RMS)
FILTER INDUCTANCE L	1 mH
GRID FUNDAMENTAL FREQUENCY	50 Hz
LINE RESISTANCE R	0.01 Ω

Dynamic process of the active current

Conventional FCS-MPC



Improved CSF-MPC



Constant switching frequency

Review of today's topic

- Understand the basic concept of the FCS-MPC in power converters
- Know the tips of the FCS-MPC for a power inverter, i.e., model, cost function, select methods.
- Know the improved FCS-MPC method for a power inverter.

Thank you!

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