Recommender System Using SVD

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***Abstract*—Recommender System are simple algorithms which aim to provide the most relevant and accurate items to the user by filtering useful stuff from of a huge pool of information base. Recommendation engines discovers data patterns in the data set by learning consumers choices and produces the outcomes that co-relates to their needs and interests. A recommender system allows you to provide personalized recommendations to users. With this SVD algorithm, you can train a model based on past interaction data and use that model to make recommendations. Collaborative Filtering algorithm considers "User Behavior" for recommending items. They exploit behavior of other users and items in terms of transaction history, ratings, selection and purchase information. Other user's behavior and preferences over the items are used to recommend items to the new users. In this case, features of the items are not known.**

***Index Terms*—Recommender System, SVD**

I. Introduction

The first question which arises in mind is basically is" ***What is SVD***" In linear algebra, the singular-value decomposition (SVD) is a factorization of a real or complex matrix. Some definitions: Let A be an m by n matrix. Then the SVD of A is A = UΣVT where U is m by m, V is n by n, and Σ is an m by n diagonal matrix where the diagonal entries Σii = σi are nonnegative, and are arranged in non-increasing order. The positive diagonal entries are called the singular values of A. The columns of U are called the left singular vectors for A, and the columns of V are called the right singular values for A

II. Applications of SVD

It should be noted that part of what makes the SVD so useful are the efficient and accurate algorithms for computing it. Computing the EVD of the matrix product A T A is sometimes of interest, but can be prone to a loss of accuracy. However, the SVD can be computed reliably directly from A, and the right singular values of A are the eigenvectors of A T A and the squares of the singular values of A are the eigenvalues of A T A. Thus the SVD can be used to accurately determine the EVD of the matrix product A T A. Effective rank estimation is another application of the SVD. The SVD of a matrix can be used to numerically estimate the effective rank of a matrix. In the case that the columns of a matrix A represent data the measurement error from obtaining the data can make a matrix seem to be of greater rank than it really should be. By only looking at the singular values of A greater than the measurement error we can determine the effective rank of A. The third application is just briefly mentioned, and is that the SVD can be used in computing what is called the generalized inverse of a matrix.

III. How to Analytically Calculate a SVD

If A is an m by n matrix, then it can be expressed as UΣVT . V is the matrix such that A T A = VDVT with the diagonal entries of D arranged in non-increasing order. The nonzero entries of Σ are equal to the square roots of the corresponding entries of D. U is made up of the non-vanishing normalized columns of AV extended to an orthonormal basis in R m . This results in V and U being orthogonal matrices. Proof: First off, we will find an orthonormal basis for R n such that its image under A is orthogonal. Let A be of rank k such that k is less than n. Let A T A = VDVT be the EVD of A T A with the diagonal entries λi of D be arranged in non-increasing order, and the columns of V be the orthonormal basis {v1, v2, …, vn} since symmetric matrices have orthogonal eigenvectors. Then ( ) ( ) ( ) so the we clearly have an orthonormal basis for R n with an orthogonal image. Also, the nonzero vectors in this image set form a basis for the range of A. We have V, so next we will normalize the vectors Avi to form U. If i = j in the equation above, we get |Avi| 2 = λi . Then λi ≥ 0 so, since they were assumed to be in non-increasing order, λ1 ≥ λ2 ≥ … ≥ λk ≥ 0 and λi is zero for all i greater than k because k is the rank of A. With this we have an orthonormal basis for the range of A defined as

$$u_i = \frac{Av_i}{|Av_i|} = \frac{1}{\sqrt{\lambda_i}} Av_i \text{ for } 1 \leq i \leq k$$

Extend this to an orthonormal basis in R m if k < m. Now all that is left is to determine Σ, and then show that we have the SVD of A. Set $\sigma_i = \sqrt{\lambda_i}$. Then by the equation above, we have for all i ≤ k. Putting the vi together as the columns of a matrix V and the ui together as the columns of a matrix U we get , where Σ has the same size as A, its diagonal entries are σi and the rest are zero. Multiplying both sides of by V T gives $A = U\Sigma V^T$ which is the SVD of A.

IV. References

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