UNIVERSITY OF SHEFFIELD PMA101

PURE MATHEMATICS

Autumn Semester 2003–2004   2 hours

Pure Mathematics Core — Mock exam

*Attempt **all** the questions. The allocation of marks is shown in brackets; Section A is worth 60 marks in total, and Section B is worth 30 marks.*

A1   Convert the function $f(x) = \dfrac{4}{(x+1)(x^2-1)}$ to partial fraction form, and thus find $\displaystyle\int f(x)\,dx$. *(6 marks)*

A2   Let $f\colon(-1,1) \to (-1,1)$ be given by $f(x) = (5x+4)/(4x+5)$. Find a formula for $f^{-1}(x)$. (You need not check anything about the domain or range of f or f^{-1}.) *(4 marks)*

A3   Let a and b be constants, and define $f, g\colon \mathbb{R} \to \mathbb{R}$ by $f(x) = a + x$ and $g(x) = b - x$. Find $f^{-1}(x)$, $g^{-1}(x)$ and $(g \circ f \circ g)(x)$. *(6 marks)*

A4   Simplify the expression $\log\left(\dfrac{e^a e^b}{(e^c)^d}\right)$. *(2 marks)*

A5   Sketch the graphs of the functions $\tan(\theta)$ and $\sin(\theta)$. Find an angle θ such that $\tan(\theta) = 1$ and $\sin(\theta) < 0$. *(4 marks)*

A6   Show that $8\cosh(x)^4 - 8\cosh(x)^2 + 1 = \cosh(4x)$. *(5 marks)*

A7   Let a, b and n be constants. Find $f'(x)$, where $f(x) = \left(\dfrac{x-a}{x-b}\right)^n$. *(3 marks)*

A8   Find $\dfrac{d}{dx}\cos\left(\left(\dfrac{x+1}{2}\right)^2\right)$. *(2 marks)*

A9   Find $\dfrac{d}{dx}\log(\cos(x))$. *(2 marks)*

A10 Let a, b, c and d be constants. Find $\dfrac{d}{dx}\left(\dfrac{ax+bx^{-1}}{cx+dx^{-1}}\right)$. *(4 marks)*

A11 If $y=\sqrt{2\pi}x^{x-1/2}e^{-x}$, show that $y'/y=\log(x)-1/(2x)$. *(4 marks)*

A12 By putting $u=\log(x)$, find $\displaystyle\int\frac{(1+\log(x))^2}{x}\,dx$. *(4 marks)*

A13 Find $\displaystyle\int(4x^2+2x+1)e^{2x}\,dx$. *(4 marks)*

A14 Find the general solution of the following system of equations:
$$\begin{aligned}
y+2z &= 1\\
-x+3z &= 2\\
-2x-3y &= 1
\end{aligned}$$
(6 marks)

A15 Find the determinant of the following matrix, simplifying your answer as much as possible.
$$\begin{bmatrix} t & a & b \\ a & t & c \\ b & c & t \end{bmatrix}$$
(4 marks)

B1 Let $g\colon[0,\infty)\to\mathbb{R}$ be given by $g(x)=\sin(\pi e^{-x}/2)$. Find the range of g. *(4 marks)*

B2 Find $\displaystyle\int 8x\sin(x)\cos(x)\,dx$ *(7 marks)*

B3 By substituting $u=x^n$, find $\displaystyle\int\frac{dx}{x\sqrt{x^{-2n}-1}}$. *(7 marks)*

B4 Find $\displaystyle\int\frac{x^5-1}{x^2(x-1)}\,dx$. *(6 marks)*

B5 Find the inverse of the following matrix:
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$
(6 marks)

End of Question Paper