

Pure Mathematics Core

Attempt **all** the questions. The allocation of marks is shown in brackets; Section A is worth 60 marks in total, and Section B is worth 30 marks.

A1 Convert the function $(x^2 + x + 1)/(x + 1)^2$ to partial fraction form, and thus find $\int \frac{x^2 + x + 1}{(x + 1)^2} dx$. (6 marks)

A2 Let $f: (0, \infty) \rightarrow (1, \infty)$ be given by $f(x) = 1/(1 - e^{-x})$. Find a formula for $f^{-1}(x)$. (4 marks)

A3 If $f(x) = 2x + 2$, what is $(\exp \circ f \circ \log)(x)$? Simplify your answer as much as possible. (4 marks)

A4 Find $\log_{16}(1/2)$. (2 marks)

A5 Find $\sin(-7\pi/3)$. (You should give an exact answer, not a decimal approximation.) (2 marks)

A6 Show that $\frac{1 + \tanh(x)^2}{1 - \tanh(x)^2} = \cosh(2x)$. (6 marks)

A7 Let a , n and m be constants. Find $f'(x)$, where $f(x) = (x^n + a)^m$. (2 marks)

A8 Find $\frac{d}{dx} \log(1 + x + x^2 + x^3)$. (2 marks)

A9 Find $\frac{d}{dx} \left(\frac{x}{\log(x)} \right)$. (3 marks)

A10 Find $\frac{d}{dx} \left(\frac{3x + 2}{4x + 3} \right)$. (2 marks)

A11 Let a , b and ω be constants. Find $f'(x)$, where $f(x) = e^{-(x-a)^2/b} \sin(\omega x)$.
(4 marks)

A12 Find $\int x^2 e^x dx$ (5 marks)

A13 Find $\int e^{3x} \sin(4x) dx$ (5 marks)

A14 Find the general solution of the following system of equations:

$$\begin{aligned} w + x + y + z &= 0 \\ w + x - y - z &= 0 \\ w - x + y - z &= 0. \end{aligned}$$

(7 marks)

A15 Find the inverse of the matrix

$$\begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}.$$

(6 marks)

B1 Define $f: (-1, 1) \rightarrow \mathbb{R}$ by $f(x) = x^2 + 2x + 3$. Find the range of f .
(4 marks)

B2 Find $\int \sin(x)^2 \cos(x)^2 dx$ (7 marks)

B3 By making a suitable substitution, find $\int \sin(x) \log(\cos(x)) dx$.
(6 marks)

B4 You may assume that $\int x^2 \log(x)^2 dx = x^3(a \log(x)^2 + b \log(x) + c)$ for some constants a , b and c . Find these constants, and thus evaluate $\int_1^e x^2 \log(x)^2 dx$.
(7 marks)

B5 Find the determinant of the following matrix:

$$\begin{bmatrix} 1 & a & 0 & 0 \\ a & 1 & b & 0 \\ 0 & b & 1 & c \\ 0 & 0 & c & 1 \end{bmatrix}.$$

(6 marks)

End of Question Paper