160102 Test 2 Niamh Ferns

1)

a) $\begin{bmatrix} 1 & 2 & 1 & -3 & 7 \\ 3 & 0 & 3 & 3 & 15 \\ 0 & 0 & 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 3 & 0 & 3 & 3 & 15 \\ 1 & 2 & 1 & -3 & 7 \\ 0 & 0 & 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 & 5 \\ 1 & 2 & 1 & -3 & 7 \\ 0 & 0 & 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 & 5 \\ 0 & 2 & 0 & -4 & 2 \\ 0 & 0 & 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 & 5 \\ 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$

Free variable $= x_4 = s$

$x_2 - 2s = 1$ $\qquad x_3 + x_4 = 2$

$x_2 = 1 + 2s$ $\qquad x_3 = 2 - s$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 2 \\ -1 \\ 1 \end{bmatrix}$$

b) $\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 5 & 1 & 2 & 0 & 1 & 0 \\ 3 & 2 & -2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -3 & -5 & 1 & 0 \\ 0 & 2 & \cancel{4} \; -5 & -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -3 & -5 & 1 & 0 \\ 0 & 0 & 1 & 7 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -6 & 2 & -1 \\ 0 & 1 & 0 & 16 & -5 & 3 \\ 0 & 0 & 1 & 7 & -2 & 1 \end{bmatrix}$ $\quad B^{-1} = \begin{bmatrix} -6 & 2 & -1 \\ 16 & -5 & 3 \\ 7 & -2 & 1 \end{bmatrix}$

2)

a) B cant be added to P as $B_{ij} = 2 \times 3$ and $P_{ij} = 3 \times 2$

b) AC is undefined as $A = 3 \times 3$ and $C = 2 \times 2$

c) $\begin{bmatrix} Y & 1 & 0 \\ 0 & Y & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} Y1 + 1(0) + 0(3) & Y1 + 1(2) + 0(4) \\ 0(1) + Y(0) + 2(3) & 0(1) + Y2 + 2(4) \end{bmatrix} = \begin{bmatrix} Y & Y+2 \\ 6 & 2Y+8 \end{bmatrix}$

d) $C^{-1} = \frac{1}{2} C = \frac{1}{2} \begin{bmatrix} 2 & z \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{z}{2} \\ 0 & \frac{1}{2} \end{bmatrix}$

e) $\begin{bmatrix} Y & 1 & 0 \\ 0 & Y & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 1+Y & 1 & 1 \\ 1 & Y-1 & 4 \end{bmatrix}$

f) $B^T = \begin{bmatrix} Y & 0 \\ 1 & Y \\ 0 & 2 \end{bmatrix}$ $\qquad AB^T = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & x \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} Y & 0 \\ 1 & Y \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0(Y) + 1(1) + 2(0) & 0(0) + 1(Y) + 2(2) \\ 1(Y) + 0(1) + x(0) & 1(0) + 0(Y) + x(2) \\ 1(Y) + 1(1) + 0(0) & 1(0) + 1(Y) + 0(2) \end{bmatrix}$

$$= \begin{bmatrix} 1 & 4+Y \\ Y & 2x \\ 1+Y & Y \end{bmatrix}$$

g) This operation is not defined because C^{-1} is 2×2 ad B is 3×2.

3)

b) As per rref(A) basis $\mathrm{col}(A) = \left\{ \begin{bmatrix} 1 \\ 2 \\ 5 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \\ 0 \\ 2 \end{bmatrix} \right\}$

c) As per rref(A) basis $\mathrm{Row}(A) = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \\ 10 \end{bmatrix} \right\}$

e) Rank (A) is given by the dimension of basis Row (A) and basic Col(A). These are spanned by 2 vectors each, therefor dim = 2, therefor Rank (A) = 2

g) No, they are not the rows of a are linearly dependand as some (Row 3, 4, and 5) can be constructed from Rows 1 and 2.

d) $\cancel{x_1 + 2x_3 = 5}$ $\cancel{x_2 - x_3 = 4}$ $\cancel{\text{Let } x_3 = t}$

$\cancel{x_1 = 5 - 2x_3}$ $\cancel{x_2 = 4 + x_3}$

$x_1 + 2x_3 + 3x_4 = 0$ $\quad$ $x_2 - x_3 + 4x_4 = 0$ $\quad$ Let $x_3 = s$, $x_4 = t$

$x_1 = -2x_3 - 3x_4$ $\quad$ $x_2 = x_3 - 4x_4$

$$x = s\begin{bmatrix} -2 \\ +1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3 \\ -4 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{basis Null}(A) = \left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -4 \\ 0 \\ 1 \end{bmatrix} \right\}$$

f) Nullity of A is given by the dimension of the basis nullspace of A. Therefor, the nullity of A is 2.

We can check because nullity + Rank = # cols. $2 + 2 = 4$ = nuber of cols in A.

a)
$$\begin{bmatrix} 1 & 1 & 1 & 7 \\ 2 & 1 & 3 & 10 \\ 5 & 3 & 7 & 27 \\ 1 & 0 & 2 & 3 \\ 3 & 2 & 4 & 17 \end{bmatrix} \times \begin{bmatrix} 1 \\ 5 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1(1) + 1(5) + 1(1) + 7(-1) \\ 2(1) + 1(5) + 3(1) + 10(-1) \\ 5(1) + 3(5) + 7(1) + 27(-1) \\ 1(1) + 0(5) + 2(1) + 3(-1) \\ 3(1) + 2(5) + 4(1) + 17(-1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$A\underline{u} = \underline{0}$, therefor $\underline{u}$ is in the nullspace of A.