Homework I - Advanced Game Theory

Andrea Titton

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(1) Directed Search

(a)

Given pairwise matching, within a submarket the number of firms matching with a worker needs to be equal to the number of workers matching with a firm. Using this we can rewrite,

$$\begin{aligned}\alpha_w \cdot n_w &= \alpha_v \cdot n_v \\ \alpha_w &= \alpha_v \cdot \frac{n_v}{n_w} \\ &= \alpha(n) \cdot \frac{1}{n}\end{aligned} \tag{1}$$

(b)

First notice that the expected value that a worker receives by matching is the extra value, on top of their outside option, they obtain by accepting a job, weighted by the matching probability, namely

$$V_w = \alpha_w \cdot (w - b) = \frac{\alpha(n)}{n} \cdot (w - b). \tag{2}$$

When offering a wage, w, thereby attracting a certain amount of workers that generate a queue, n, the firm needs to take the workers expected value from matching in account as a participation constraint. On the other hand, the firm picks a wage and queue that maximizes its own expected value from matching, namely,

$$V_v = \max_{n,w} \left\{ \underbrace{\alpha(n)}_{\text{match probability}} \cdot \underbrace{(y - w)}_{\text{surplus of match}} \right\} \tag{3}$$

(c)

Using V_w we can rewrite the wage as,

$$w = \frac{n \cdot V_w}{\alpha(n)} + b. \tag{4}$$

By plugging in the wage in Equation (3), te optimization problem of the firm reduces to,

$$V_v = \max_n \{\alpha(n) \cdot (y - b) - n \cdot V_w\} \tag{5}$$

Taking the first order condition with respect to n to solve the maximization problem yields,

$$\alpha'(n) \cdot (y - b) - V_w = 0 \tag{6}$$

(d)

Using (6) in the wage definition we can rewrite,

$$w = \frac{n \cdot \alpha'(n) \cdot (y - b)}{\alpha_n} + b \tag{7}$$

Now let $\varepsilon := \frac{n \cdot \alpha \prime(n)}{\alpha(n)}$ be the elasticity of $\alpha(n)$ with respect to tightness. Then the wage can be written as,

$$\begin{aligned} w &= \varepsilon \cdot (y - b) + b \\ &= \varepsilon \cdot y - (1 - \varepsilon) \cdot b \end{aligned} \tag{8}$$

(2) Albrecht Gautier and Vroman (2016 RED)