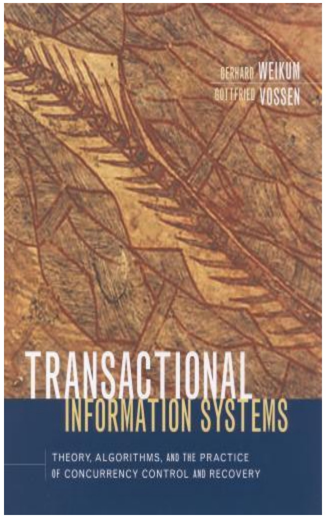
DIS
2019

2. Transactions
- Correctness -



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Basic Notion of Transactions
Histories and Schedules
Notions of Correctness
Serializability Classes

Preconsiderations Correctness (1)

- **Canonical Synchronization Problems**
  1. Dirty-Read

T1	T2
Read (A); A := A + 100; Write (A)	
	Read (A); Read (B); B := B + A; Write (B); Commit;
Abort;	

Time

Preconsiderations Correctness (2)

- **Canonical Synchronization Problems**

2. Lost Update

T1	T2
Read (A);	
	Read (A);
A := A - 1; Write (A);	
	A := A + 1; Write (A);

Time

Preconsiderations Correctness (3)

- **Canonical Synchronization Problems**

  3. Non-repeatable (inconsistent) Read

Read Transaction	Update Transaction	DB (PNR, Salary)
SELECT Salary INTO :salary FROM Pers WHERE Pnr = 2345; sum := sum + salary;		**2345 39.000 3456 48.000**
	UPDATE Pers SET Salary = Salary + 1000 WHERE Pnr = 2345;	**2345 40.000**
	UPDATE Pers SET Salary = Salary + 2000 WHERE Pnr = 3456;	**3456 50.000**
SELECT Sakary INTO :salary FROM Pers WHERE Pnr = 3456; sum := sum + salary;		

Time

Preconsiderations Correctness (4)

- **Canonical Synchronization Problems**

4. Phantom-Problem

Read Transaction	Update Transaction
SELECT SUM (Salary) INTO :sum FROM Pers WHERE Anr = 17;	
	INSERT INTO Pers (Pnr, Anr, Salary) VALUES (4567, 17, 55.000);
	UPDATE Abt SET SalarySum = SalarySum + 55.000 WHERE Anr = 17;
SELECT SalarySum INTO :salsum FROM Abt WHERE Anr = 17;	
IF salsum <> sum THEN <Error Handling>;	

Time

Preconsiderations Correctness (5)

- **Notion of Correctness: *Serializability***

  The concurrent execution of a set of transactions is considered to be correct, if there is a serial execution of the same set of transactions, leading

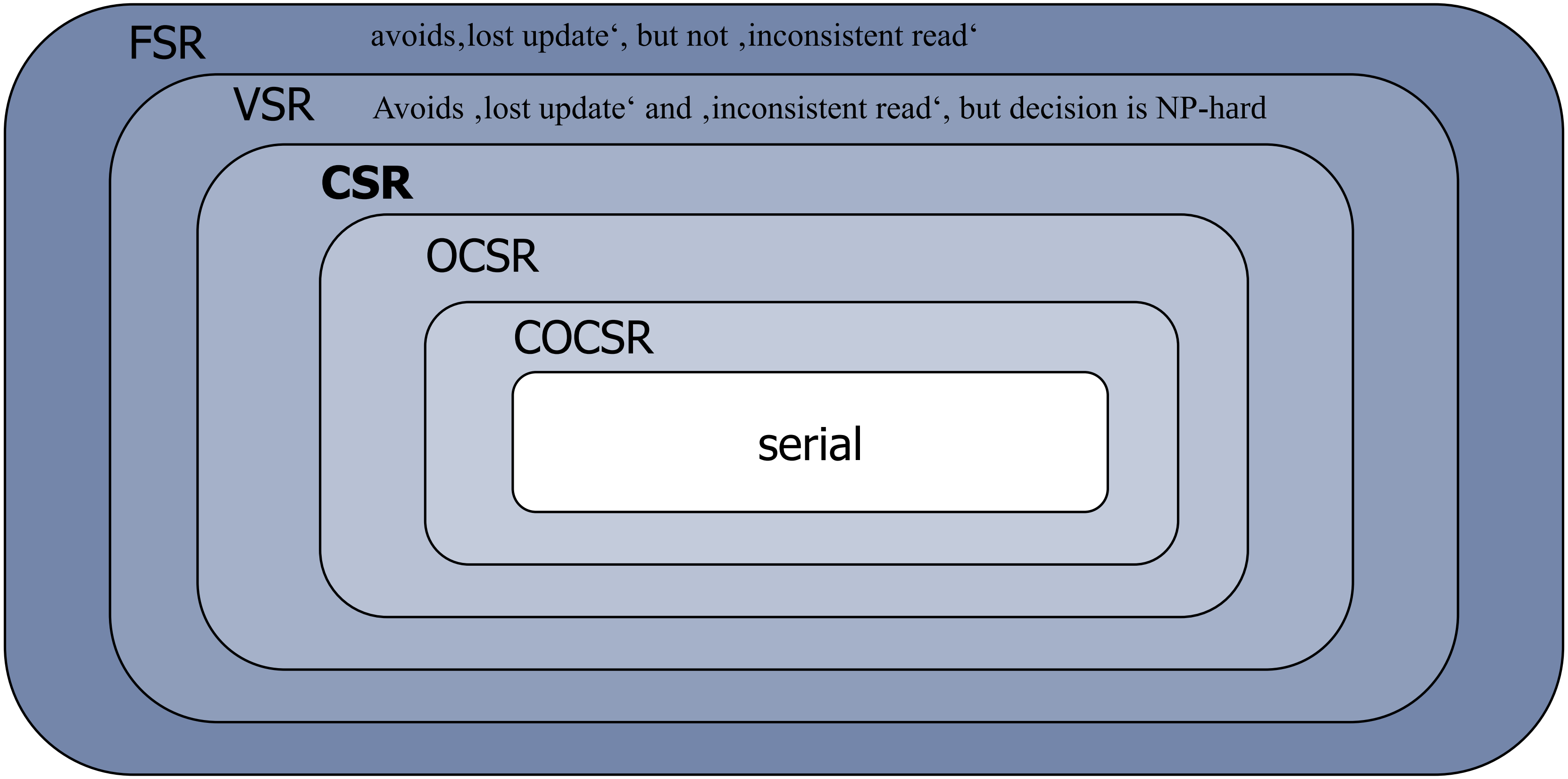
  to *the same resulting DB state*
  <u>as well as</u> *the same output values*

  as the original execution.

  - Background:
    - Serial processing is correct
    - Each schedule having the same effect as an (arbitrary) serial one is considered to be correct

Schedule Classes (1)

- **Overview (simplified)**

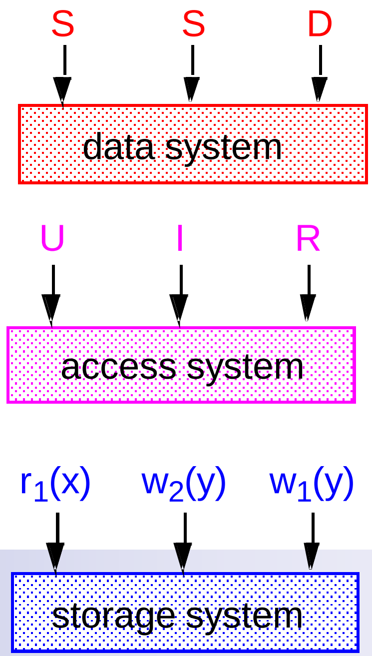


Schedule Classes (2)

- **Requirements for an acceptable class of schedules**
  - At least *lost update* and *inconsistent read* are avoided
  - Decision (of membership) can be taken efficiently
  - In presence of failures (Aborts) also *dirty read* is avoided
- **Focus: Conflict Serializability (CSR)**
  - Most important for practical application

Page Model (1)

- **Modeling**
  - Page Model (Foundation)
    - Abstract model, not necessarily restricted to the notion of database pages
    - However, page-oriented Synchronization and Recovery (in the DBS storage system) are the major application areas of the page model
- **Basics**
  - Set of *atomic, uninterpreted data objects* (pages)
    - D = {x, y, z, ...}
    - with *atomic read and write operations*



Page Model (2)

- **Basics (contd.)**
  - A *Transaction t* is a finite sequence of steps/actions of the form r(x) or w(x):
    - $t = p_1 \ldots p_{n,}$ with $n < \infty$, $p_i \in \{r(x), w(x)\}$ for $1 \leq i \leq n$, $x \in D$;
    - r stands for read, w for write
  - Different transactions do not have steps in common; steps can be identified uniquely:
    - p_{ij} describes the j^{th} step of Transaction i
      (Transaction index can be omitted, if context clear)

Page Model (3)

- **Interpretation of a Transaction (Semantics)**
  - $p_j = r(x)$
    - the j^{th} step of the transaction is a read operation assigning the current value of x to the local variable v_j
    - $v_j := x$
  - $p_j = w(x)$
    - the j^{th} step of the transaction is a write operation assigning a computed value to x
    - each value written by a transaction potentially depends on all data objects previously read by this transaction
    - $x := f_j (v_{j1}, \ldots, v_{jk})$
    - x is the return value of an arbitrary, unknown function f_j with $\{j_1, \ldots j_k\} = \{j_r \mid p_{jr} \text{ is a read operation} \wedge j_r < j\}$

Page Model (4)

- **So far assumption of total ordering of transaction steps**
  - Not necessary, as far as ACID is ensured
  - Not reasonable, e.g., in case of parallelized transactions on multi processor system
- **Definition *(Strict) Partial Order***
  A arbitrary set. R $\subseteq$ A $\times$ A is (Strict) Partial Order on A, if for elements a, b, c $\in$ A holds:
  - $(a, a) \notin R$ (Irreflexivity)
  - $(a, b) \in R \Rightarrow (b, a) \notin R$ (Anti-Symmetry)
  - $(a, b) \in R \wedge (b, c) \in R \Rightarrow (a, c) \in R$ (Transitivity)

  Note: each R can be represented as directed graph.

Page Model (5)

- **Definition *Transaction***
  - A *Transaction t* is a partial order of steps of the form r(x) or w(x) with $x \in D$ and read and write operations as well as multiple write operations on the same object are ordered.
  - Formal: $t = (op, <)$
    - op is finite set of steps r(x) or w(x), $x \in D$
    - $< \subseteq op \times op$ is partial order over op with:

      if $\{p, q\} \subseteq op$ and p and q access the same data object and at least one of the two is a write operation, then:

      $p < q \vee q < p$.

Page Model (6)

- **Ordering ensures unambiguous interpretation**
  - for example, in case of unordered read and write operations on the same object
    - the read value would be ambiguous
    - it could be the value before or after the write operation
- **Further assumptions**
  - in each TA each data object is only read or written once
  - no data object will be read again, after it has been written (does not exclude blind writes)

Histories and Schedules (1)

- **Goal**
  - Correctness notion for parallel TA executions
  - The scheduler, which is the core component of concurrency control needs correctness criteria that can be applied efficiently

- **(additional) Termination Operations**
  - c_i: successful completion of TA t_i, Commit
  - a_i: non-successful completion of TA t_i, Abort

Histories and Schedules (2)

- **Definition *Histories and Schedules***
  - Let $T = \{t_1, ..., t_n\}$ be a (finite) set of TA, each $t_i \in T$ be of the form $t_i = \{op_i, <_i)$, op_i is the set of operations of t_i and $<_i$ the corresponding ordering ($1 \leq i \leq n$).
  - A *History for* T is a pair $s = (op(s), <_s)$, with:
    a) $op(s) \subseteq \bigcup_{i=1}^{n} op_i \cup \bigcup_{i=1}^{n} \{a_i, c_i\}$ **and** $\bigcup_{i=1}^{n} op_i \subseteq op(s)$
    b) $(\forall\ i, 1 \leq i \leq n)\ c_i \in op(s) \Leftrightarrow a_i \notin op(s)$
    c) $\bigcup_{i=1}^{n} <_i \ \subseteq\ <_s$
    d) $(\forall\ i, 1 \leq i \leq n)\ (\forall\ p \in op_i\ )\ p <_s a_i$ or $p <_s c_i$
    e) Each pair of operations $p, q \in op(s)$ of different TAs, which access the same data object and at least one of which is a write operation is ordered, so that $p <_s q$ or $q <_s p$.
  - A *Schedule* is a prefix of a History

Histories and Schedules (3)

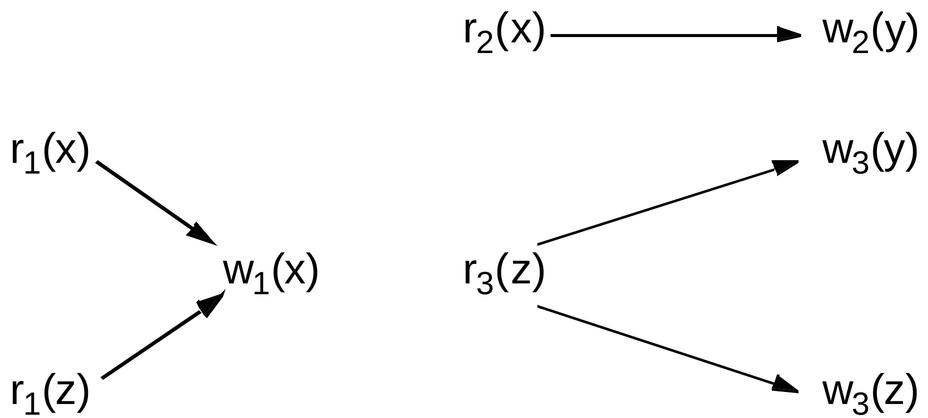
- **Explanations:**
  - A History (for partially ordered TA)
    - a) Contains all operations of all TA
    - b) Requires a single termination operation for each TA
    - c) Retains orderings within TA
    - d) Contains the termination operation of each TA as the last operation of this TA
    - e) Orders conflicting operations
  - Because of (a) and (b) a History is also called a complete Schedule.

Histories and Schedules (4)

- **Comment**
  - A prefix of a history can be the history itself
  - Histories can be considered to be special cases of Schedules. Thus, it is (mostly) sufficient, to deal with schedules.
- **Definition *Serial History***
  - A History s is *serial*, if for any two TA t_i and t_j ($i \neq j$) all operations of t_i occur in s before all operations of t_j or vice versa.

Histories and Schedules (5)

- **Example**
  - 3 TA as DAG (directed acyclic graph)

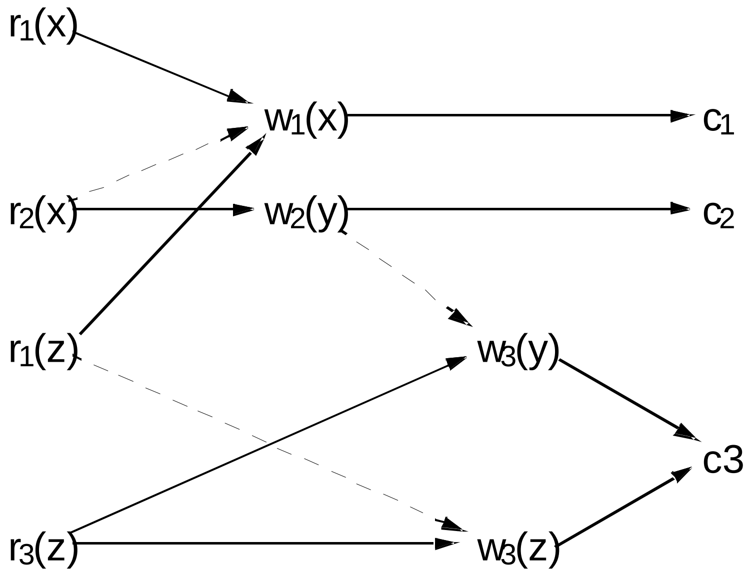


  - Example of a completely ordered history of these 3 TA

$$r_1(x) \quad r_2(x) \quad r_1(z) \quad w_1(x) \quad w_2(y) \quad r_3(z) \quad w_3(y) \quad c_1 \quad c_2 \quad w_3(z) \quad c_3$$

Histories and Schedules (6)

- **Example (contd.)**
  - Example of a partially ordered history of these 3 TA



$r_1(x)\quad r_2(x)\quad r_1(z)\quad w_1(x)\quad w_2(y)\quad r_3(z)\quad w_3(y)\quad c_1\quad c_2\quad w_3(z)\quad c_3$

  - Partial orderings can always be extended to a variety of complete orderings (as special cases)

Histories and Schedules (7)

- ***Prefix of a partial ordering***
  - Omitting parts at the end of the *„accessibility chain"*
  - If $s = (op(s), <_s)$, then a *Prefix* of s has the form $s' = (op_{s'}, <_{s'})$, with:
    - $op_{s'} \subseteq op(s)$
    - $<_{s'} \subseteq <_s$
    - $(\forall p \in op_{s'}) (\forall q \in op(s))\ q <_s p \Rightarrow q \in op_{s'}$
    - $(\forall p, q \in op_{s'})\ p <_s q \Rightarrow p <_{s'} q$

Histories and Schedules (8)

- ***Shuffle Product***
  - Be $T = \{t_1, ..., t_n\}$ a set of completely ordered TA
  - shuffle(T) denotes the *Shuffle Product*, i.e., the set of all operation sequences, in which the sequence $t_i \in T$ occurs as partial sequence and contains no other operations
- **Completely ordered Histories and Schedules**
  - a History s for T is derived from sequence $s' \in$ shuffle(T), whereat c_i or a_i for each $t_i \in T$ is added (Rules b) and d) in definition on slide 16).
  - As before, a Schedule is a prefix of a history.
  - A history s is serial, if $s = t_{i_1}, ..., t_{i_n}$ with $i_1, ..., i_n$ permutation of 1, ..., n

Histories and Schedules (9)

- **Example (continuing slide 19)**
  - Completely ordered TA:
    $t_1 = r_1(x)\ r_1(z)\ w_1(x)$
    $t_2 = r_2(x)\ w_2(y)$
    $t_3 = r_3(z)\ w_3(y)\ w_3(z)$
  - The History

    $r_1(x)\ r_2(x)\ r_1(z)\ w_1(x)\ w_2(y)\ r_3(z)\ w_3(y)\ c_1\ c_2\ w_3(z)\ c_3$

    is completely ordered and has (among others)

    $r_1(x)\ r_2(x)\ r_1(z)\ w_1(x)\ w_2(y)\ r_3(z)\ w_3(y)$,

    $r_1(x)\ r_2(x)\ r_1(z)\ w_1(x)\ w_2(y)$, and

    $r_1(x)\ r_2(x)\ r_1(z)$

    as Prefixes

Histories and Schedules (10)

- **(New) Example**
  - $T = \{t_1, t_2, t_3\}$ with

    $t_1 = r_1(x)\ w_1(x)\ r_1(y)\ w_1(y)$
    $t_2 = r_2(z)\ w_2(x)\ w_2(z)$
    $t_3 = r_3(x)\ r_3(y)\ w_3(z)$

    $s_1 = r_1(x)\ r_2(z)\ r_3(x)\ w_2(x)\ w_1(x)\ r_3(y)\ r_1(y)\ w_1(y)\ w_2(z)\ w_3(z)$
    $\in$ shuffle(T);

    $s_2 = s_1\ c_1\ c_2\ a_3$ is a History, in which s_1 ($\in$ shuffle(T)) has been amended by termination steps;

    $s_3 = r_1(x)\ r_2(z)\ r_3(x)$ is a Schedule;

    $s_4 = s_1\ c_1$ is another Schedule;

    $s_5 = t_1\ c_1\ t_3\ a_3\ t_2\ c_2$ is a serial History.

Histories and Schedules (11)

- **Remark**
  - The statements given here hold for complete as well as partial orderings.
  - Mostly it is easier to show them for complete orderings.
- ***TA-Sets of Schedules***
  - trans(s) $:= \{t_i \mid$ s contains steps of $t_i\}$
  - commit(s) $:= \{t_i \in \text{trans}(s) \mid c_i \in s\}$
  - abort(s) $:= \{t_i \in \text{trans}(s) \mid a_i \in s\}$
  - active(s) $:= \text{trans}(s) - (\text{commit}(s) \cup \text{abort}(s))$

Histories and Schedules (12)

- **Example (continuing slide 24)**
  - $s_2 = r_1(x)\ r_2(z)\ r_3(x)\ w_2(x)\ w_1(x)\ r_3(y)\ r_1(y)\ w_1(y)\ w_2(z)\ w_3(z)\ c_1\ c_2\ a_3$

    $trans(s_2) = \{t_1, t_2, t_3\}$
    $commit(s_2) = \{t_1, t_2\}$
    $abort(s_2) = \{t_3\}$
    $active(s_2) = \emptyset$

  - $s_4 = r_1(x)\ r_2(z)\ r_3(x)\ w_2(x)\ w_1(x)\ r_3(y)\ r_1(y)\ w_1(y)\ w_2(z)\ w_3(z)\ c_1$

    $trans(s_4) = \{t_1, t_2, t_3\}$
    $commit(s_4) = \{t_1\}$
    $abort(s_4) = \emptyset$
    $active(s_4) = \{t_2, t_3\}$

Histories and Schedules (13)

- **For each History s the following is true:**
  - trans(s) = commit(s) $\cup$ abort(s)
  - active(s) = Ø

Histories and Schedules (14)

- **Definition *Monotonic Classes of Histories***
  - A class E of Histories is monotonic, if the following holds:
    - If s in E, then $\Pi_T(s)$, the Projection of s on T, is in E for each $T \subseteq$ trans(s)
    - In other words: E is closed under arbitrary projections
- **Monotonicity**
  - Monotonicity is a wanted property of a history class, since it preserves E under arbitrary projections
  - VSR is not monotonic

Correctness (1)

- **A correctness criterion can formally be considered to be a mapping**
  - $\sigma : S \rightarrow \{0, 1\}$ with S set of all Schedules.
  - $correct(S) := \{s \in S \mid \sigma(s)=1 \}$
- **A concrete correctness criterion at least must fulfill the following requirements**
  1. $correct(S) \neq \emptyset$
  2. „$s \in correct(S)$" can be decided efficiently
  3. correct(S) is „sufficiently large",
     - so that the scheduler has many possibilities to derive correct schedules
     - the bigger the set of allowed (correct) schedules, the higher concurrency and efficiency

Correctness (2)

- **Basic Idea of Serializability**
  - Single TA is correct, since it leaves the database in consistent state
  - Consequence: serial histories are correct!
  - However, serial histories should ‚only' be used as correctness measures via appropriate equivalence relations
- **Approach**
  1. Definition of an equivalence relation ‚$\approx$' on S (set of all schedules) with
     - $[S]_{\approx} = \{[s]_{\approx} \mid s \in S\}$ set of equivalence classes
  2. Consideration of those classes having serial schedules as representatives

CSR (1)

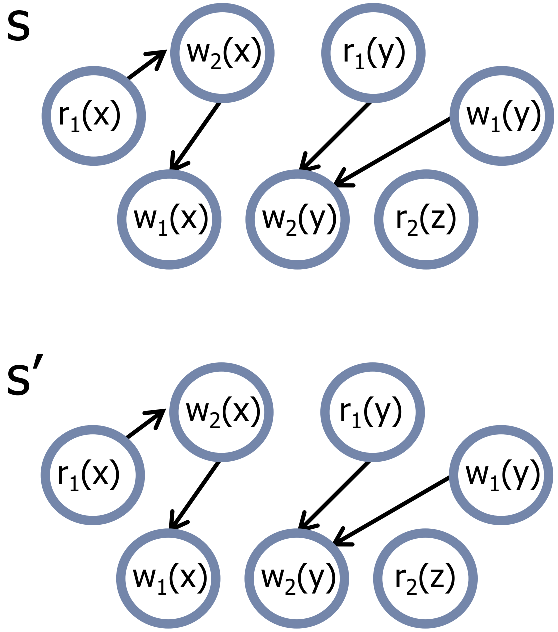
- **Conflict Serializability**
  - Most important serializability class w.r.t. practical use
- **Goal**
  - Further reduction in comparison to VSR
    (VSR is not monotonic and membership test is NP-hard)
  - Concept that is easy to test and, thus, is feasible for being applied in schedulers
- **Definitions *Conflict and Conflict Relation***
  - s Schedule; t, t' $\in$ trans(s), t $\neq$ t':
    - Two operations p $\in$ t and q $\in$ t' are in *Conflict* in s, if they access the same data object and at least one of them is a write operation
    - conf(s) := {(p, q) | p, q are in *Conflict* in s and p $<_s$ q} is called *Conflict Relation* of s

CSR (2)

- **Remark**
  - Conflicts only occur between data operations, independently from the termination state of a TA; operations of aborted TAs can be ignored
- **Example**
  - $s = w_1(x)\ r_2(x)\ w_2(y)\ r_1(y)\ w_1(y)\ w_3(x)\ w_3(y)\ c_1\ a_2$
  - $conf(s) = \{(w_1(x), w_3(x)), (r_1(y), w_3(y)), (w_1(y), w_3(y))\}$
- **Definition *Conflict Equivalence***
  - Schedules s and s' are conflict equivalent, denoted as $s \approx_c s'$, if
    - $op(s) = op(s')$
    - $conf(s) = conf(s')$

CSR (3)

- **Example ($s \approx_c s'$)**
  - $s = r_1(x)\ r_1(y)\ w_2(x)\ w_1(y)\ r_2(z)\ w_1(x)\ w_2(y)$
  - $s' = r_1(y)\ r_1(x)\ w_1(y)\ w_2(x)\ w_1(x)\ r_2(z)\ w_2(y)$

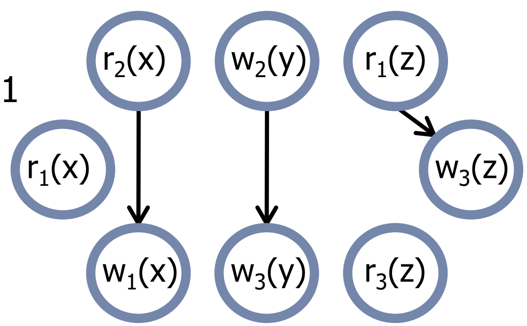


- **Conflicting-Step-Graph $D_2(s)$**
  - Conflict equivalence can be illustrated as graph $D_2(s) := (V, E)$ with $V = op(s)$ and $E = conf(s)$
  - $D_2(s)$ is called Conflicting-Step-Graph
  - $s \approx_c s' \Leftrightarrow D_2(s) = D_2(s')$
- **Definition *Conflict Serializability***
  - A History s is conflict serializable, if there is a serial History s′ with $s \approx_c s'$
  - CSR denotes the class of all conflict serializable Histories

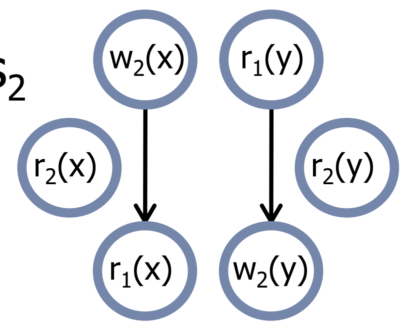
CSR (4)

- **Examples**
  - $s_1 = r_1(x)\ r_2(x)\ r_1(z)\ w_1(x)\ w_2(y)\ r_3(z)\ w_3(y)\ c_1\ c_2\ w_3(z)\ c_3$
    $s_1 \in CSR$
  - $s_2 = r_2(x)\ w_2(x)\ r_1(x)\ r_1(y)\ r_2(y)\ w_2(y)\ c_1\ c_2$
    $s_2 \notin CSR$

s_1



s_2



$s_1^{serial} = r_2(x)\ w_2(y)\ c_2\ r_1(x)\ r_1(z)\ w_1(x)\ c_1\ r_3(z)\ w_3(y)\ w_3(z)\ c_3 = t_2\ t_1\ t_3$

CSR (5)

- **Lost Update**
  - $L = r_1(x)\ r_2(x)\ w_1(x)\ w_2(x)\ c_1\ c_2$
  - $conf(L) = \{(r_1(x), w_2(x)), (r_2(x), w_1(x)), (w_1(x), w_2(x))\}$
  - $L \not\approx_c t_1\ t_2$ and $L \not\approx_c t_2\ t_1$
- **Inconsistent Read**
  - $I = r_2(x)\ w_2(x)\ r_1(x)\ r_1(y)\ r_2(y)\ w_2(y)\ c_1\ c_2$
  - $conf(I) = \{(w_2(x), r_1(x)), (r_1(y), w_2(y))\}$
  - $I \not\approx_c t_1\ t_2$ and $I \not\approx_c t_2\ t_1$

CSR (6)

- **CSR $\subset$ VSR $\subset$ FSR**

- **Example**
  - s = $w_1(x)\ w_2(x)\ w_2(y)\ c_2\ w_1(y)\ c_1\ w_3(x)\ w_3(y)\ c_3$
  - $s \not\approx_c t_1\ t_2\ t_3$ and $s \notin$ CSR, but
    $s \approx_v t_1\ t_2\ t_3$ and thus $s \in$ VSR

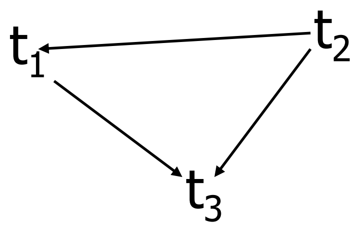
- **Theorem**
  - CSR is monotonic
  - $s \in \text{CSR} \Leftrightarrow \Pi_T(s) \in \text{CSR}$ for all $T \subseteq \text{trans}(s)$
    (i.e., CSR is the largest monotonic subset of VSR)

CSR (7)

- **Definition *Conflict Graph (Serialization Graph)***
  - Let s be a Schedule. The Conflict Graph G(s) = (V, E) is a directed graph with
    - V = commit(s)
    - $(t, t') \in E \Leftrightarrow t \neq t' \wedge (\exists\, p \in t)\ (\exists\, q \in t')\ (p, q) \in conf(s)$
- **Remark**
  - The Conflict Graph abstracts from individual conflicts between pairs of TA (conf(s)) and represents multiple conflicts between the same (terminated) TA by a single edge.

CSR (8)

- **Example**
  - $s = r_1(x)\ r_2(x)\ w_1(x)\ r_3(x)\ w_3(x)\ w_2(y)\ c_3\ c_2\ w_1(y)\ c_1$
  - G(s) =

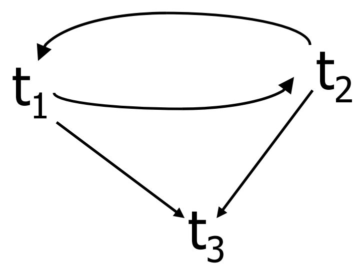


- ***Serialization Theorem***
  - Let s be a History; then s $\in$ CSR if and only if G(s) acyclic
- **Problem**
  - Find a serial History, which is consistent to all edges in G(s)

CSR (9)

- **Example**
  - $s = r_1(y)\ r_3(w)\ r_2(y)\ w_1(y)\ w_1(x)\ w_2(x)\ w_2(z)\ w_3(x)\ c_1\ c_3\ c_2$

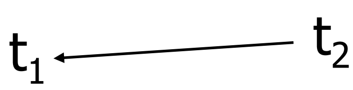
    $G(s) =$

    $s \notin CSR$

  - $s' = r_1(x)\ r_2(x)\ w_2(y)\ w_1(x)\ c_2\ c_1$

    $G(s') =$

    $s' \in CSR$

CSR (10)

- **Corollary**
  - Membership in CSR can be tested in polynomial time w.r.t the set of TA contributing the considered schedule
- **Blind Write**
  - A blind Write (of a data object x) is given, if a TA performs a Write(x) without preceding Read(x)
  - If blind writes are prohibited, the TA definition is intensified as follows:
    - If $w_i(x) \in T_i$, then $r_i(x) \in T_i$ and $r_i(x) < w_i(x)$
  - Then it is true: a history is view-serializable (element of VSR) if and only if it is conflict-serializable (element of CSR)!

CSR (11)

- **Conflicts and Commutativity**
  - So far Conflict Serializability has been shown by use of the Conflict Graph
  - (New) Testing approach
    - s is supposed to be stepwise transformed by the help of commutativity rules
    - Find a sequence of steps that transforms s into a serial History

CSR (12)

- **Commutativity Rules**
  - $\sim$ means that ordered pairs of actions can be replaced by each other
    - C1: $r_i(x)\ r_j(y) \sim r_j(y)\ r_i(x)$ if $i \neq j$
    - C2: $r_i(x)\ w_j(y) \sim w_j(y)\ r_i(x)$ if $i \neq j,\ x \neq y$
    - C3: $w_i(x)\ w_j(y) \sim w_j(y)\ w_i(x)$ if $i \neq j,\ x \neq y$
  - Ordering rule for partially ordered schedules
    - C4: $o_i(x), p_j(y)$ unordered $\Rightarrow o_i(x)\ p_j(y)$
      if $x \neq y \vee (o = r \wedge p = r)$
    - says that unordered operations can be ordered arbitrarily if they are not in conflict

CSR (13)

- **Example**

$$s = \quad w_1(x)\ \underbrace{r_2(x)\ w_1(y)}\ w_1(z)\ \underbrace{r_3(z)\ w_2(y)}\ w_3(y)\ w_3(z)$$

$$\rightarrow(C2) \quad w_1(x)\ w_1(y)\ \underbrace{r_2(x)\ w_1(z)}\ w_2(y)\ r_3(z)\ w_3(y)\ w_3(z)$$

$$\rightarrow(C2) \quad w_1(x)\ w_1(y)\ w_1(z)\ r_2(x)\ w_2(y)\ r_3(z)\ w_3(y)\ w_3(z)$$

$$= \quad t_1\ t_2\ t_3$$

- **Definition *Commutativity-based Equivalence***
  - Two Schedules s and s’ with op(s) = op(s’) are *commutativity-based equivalent*, denoted as s ~* s’, if s can be transformed to s’ by a finite sequence of steps following the rules C1, C2, C3 and C4.

CSR (14)

- **Theorem**
  - Let s and s′ be Schedules with op(s) = op(s′)
  - Then: s $\approx_c$ s′ if and only if s ~* s′
- **Definition *Commutativity-based Reducibility***
  - History s is commutativity-based reducible, if there is a serial History s′ with s ~* s′
- **Corollary**
  - A History s is commutativity-based reducible
    if and only if s $\in$ CSR

CSR (15)

- **Generalization of the Conflict Notion**
  - Scheduler does not have to ‘know’ the operations in detail, but only which of them are in conflict
  - Example
    - $s = p_1\ q_1\ p_2\ o_1\ p_3\ q_2\ o_2\ o_3\ p_4\ o_4\ q_3$ with the conflicts (q_1, p_2), (p_2, o_1), (q_1, o_2) and (o_4, q_3)
  - Applicable for ‘semantic’ concurrency control
    - Specification of a Commutativity- or Conflict Table for ‚new‘ (possibly application-specific) Operations and
    - Derivation of Conflict Serializability from this Table
  - Examples for operations
    - increment/decrement
    - enqueue/dequeue
    - ...

OCSR (1)

- **Restrictions**
  - Histories/Schedules of VSR and FSR cannot be used in practice!
  - Further restrictions of CSR, on the other hand, are beneficial for certain practical applications!
- **Example**
  - $s = w_1(x)\ r_2(x)\ c_2\ w_3(y)\ c_3\ w_1(y)\ c_1$
  - $G(s) =$

$$t_3 \longrightarrow t_1 \longrightarrow t_2$$

- **Contrast between serialization and actual processing order possibly unwanted**
- **Can be avoided by order preservation!**

OCSR (2)

- **Definition *Order Preserving Conflict Serializability***
  - A History s is called order preserving conflict serializable, if
    - s conflict serializable, i.e., there is s', with op(s) = op(s') and s $\approx_c$ s', and
    - additionally the following holds for all t_i, t_j $\in$ trans(s): If t_i completely before t_j in s, then the same holds for s'
- **Theorem**
  - Let OCSR be the class of all order preserving conflict serializable histories: OCSR $\subset$ CSR
- **Idea of prove**
  - From Definition: OCSR $\subseteq$ CSR
  - s (previous) shows that inclusion is strict: s $\in$ CSR - OCSR

COCSR (1)

- **Further Restriction of CSR**
  - beneficial for distributed and possibly heterogeneous applications
  - Observation: for conflict serializability it is sufficient that transactions which are in conflict, perform there commits in conflict order
- **Definition *Preservation of Commit Order***
  - A History s is called *commit order-preserving conflict serializable*, if the following holds:
    - For all t_i, $t_j \in$ commit(s), $i \neq j$:

      If $(p, q) \in$ conf(s) for $p \in t_i$, $q \in t_j$, then $c_i < c_j$ in s
- **Order of conflicting operations determines the order of the corresponding commit operations**

COCSR (2)

- **Theorem**
  - Let COCSR be the class of all commit order-preserving conflict serializable histories; then
    - $COCSR \subset CSR$
- **Sketch of proof**
  - $s = r_1(x)\ w_2(x)\ c_2\ c_1$
  - $s \in CSR - COCSR$ (Inclusion is strict)
- **Theorem**
  - Let s be a History: $s \in COCSR$ if and only if:
    - $s \in CSR$ and
    - there is a serial History s' so that $s' \approx_c s$ and for all $t_i, t_j \in trans(s)$, $t_i <_{s'} t_j \Rightarrow c_{t_i} <_s c_{t_j}$
- **Theorem: $COCSR \subset OCSR$**

Commit Serializability (1)

- **Assumption so far,**
  - Every transaction terminates.
- **Requirements w.r.t. possible failure cases**
  1. A correctness notion should only take successfully completed TA into account
  2. For each correct schedule all its prefixes should be correct, too
- **Definition *Closure Properties***
  - Let E be Class of Schedules
    1. E is *prefix-closed*, if for every Schedule s in E all the prefixes of s are in E, too
    2. E is *commit-closed*, if for every Schedule s in E, CP(s) also in E, with CP(s) = $\Pi_{commit(s)}$ (s)

Commit Serializability (2)

- ***Prefix-Commit-Closed***
  - Both, previously mentioned closure properties
  - If class E prefix-commit-closed, then for each schedule s in E it is true that CP(s') in E for each prefix s' of s
- **FSR is not prefix-commit-closed**
  - $s = w_1(x)\ w_2(x)\ w_2(y)\ c_2\ w_1(y)\ c_1\ w_3(x)\ w_3(y)\ c_3$
  - $s \approx_v t_1\ t_2\ t_3$, that means $s \in$ VSR, that means $s \in$ FSR
  - $s' = w_1(x)\ w_2(x)\ w_2(y)\ c_2\ w_1(y)\ c_1$ is Prefix of s
  - $CP(s') = s'$
  - $s' \not\approx_f t_1\ t_2$ and $s' \not\approx_f t_2\ t_1$, that means $s' \notin$ FSR
- **VSR is not prefix-commit-closed**, **since VSR not monotonic**

Commit Serializability (3)

- **Theorem**
  - CSR is prefix-commit-closed
  - Proof
    - s $\in$ CSR, then G(s) acyclic
    - For each partial sequence s′ of s, G(s′) is acyclic, too
    - Especially G(CP(s′)) is acyclic
    - Thus: CP(s′) $\in$ CSR
- **Definition *Commit-Serializability***
  - A Schedule s is called *commit-serializable*, if for every Prefix s′ CP(s′) serializable.
- **Classes of commit-serializable Schedules**
  - CMFSR
  - CMVSR
  - CMCSR

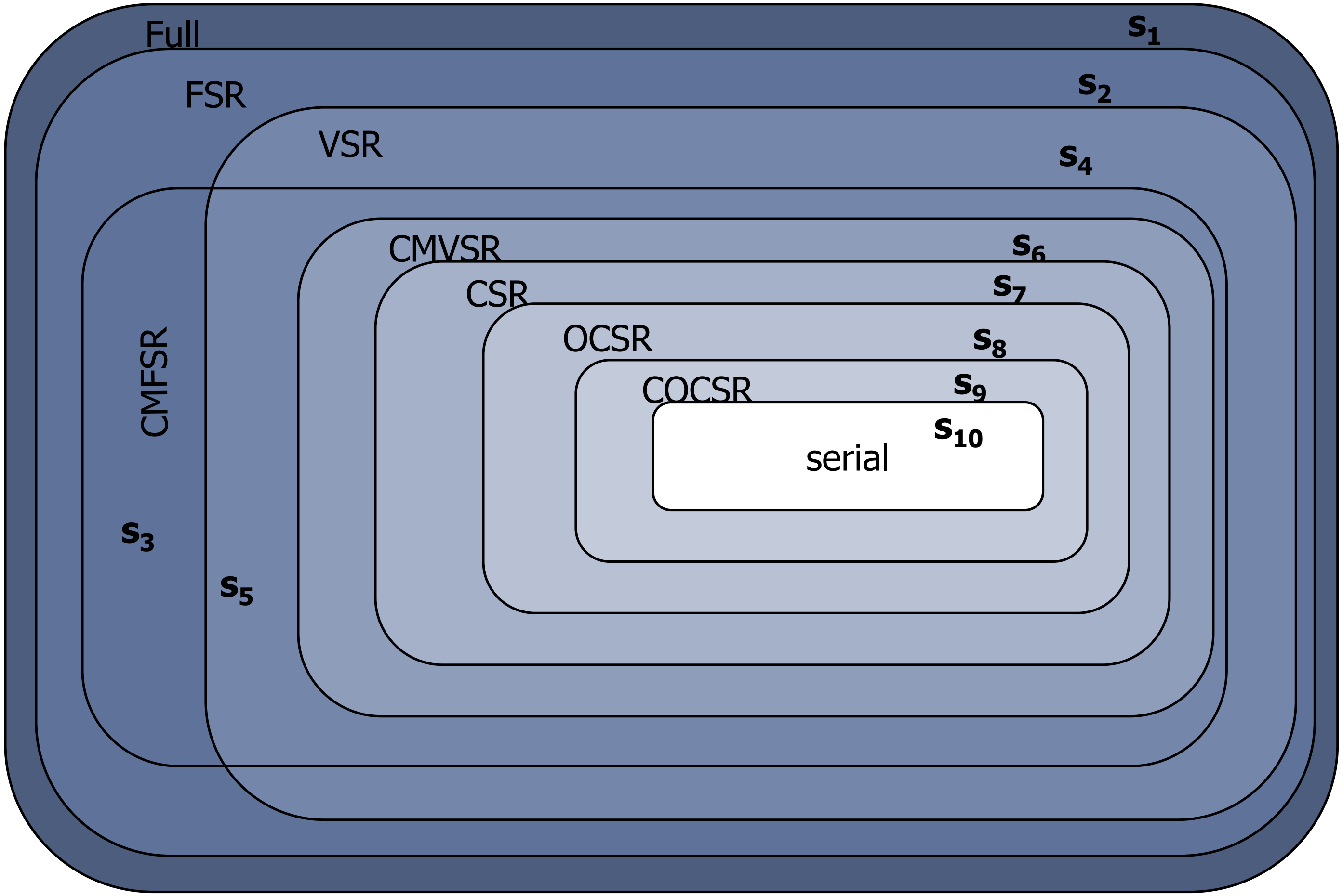
Commit Serializability (4)

- **Theorem**
  1. CMFSR, CMVSR, CMCSR are commit-closed
  2. CMCSR $\subset$ CMVSR $\subset$ CMFSR
  3. CMFSR $\subset$ FSR
  4. CMVSR $\subset$ VSR
  5. **CMCSR = CSR**

Overview (1)

- **Historien**
  - $s_1 = w_1(x)\ w_2(x)\ w_2(y)\ c_2\ w_1(y)\ c_1$
  - $s_2 = w_1(x)\ r_2(x)\ w_2(y)\ c_2\ r_1(y)\ w_1(y)\ c_1\ w_3(x)\ w_3(y)\ c_3$
  - $s_3 = w_1(x)\ r_2(x)\ w_2(y)\ w_1(y)\ c_1\ c_2$
  - $s_4 = w_1(x)\ w_2(x)\ w_2(y)\ c_2\ w_1(y)\ c_1\ w_3(x)\ w_3(y)\ c_3$
  - $s_5 = w_1(x)\ r_2(x)\ w_2(y)\ w_1(y)\ c_1\ c_2\ w_3(x)\ w_3(y)\ c_3$
  - $s_6 = w_1(x)\ w_2(x)\ w_2(y)\ c_2\ w_1(y)\ w_3(x)\ w_3(y)\ c_3\ w_1(z)\ c_1$
  - $\mathbf{s_7 = w_1(x)\ w_2(x)\ w_2(y)\ c_2\ w_1(z)\ c_1}$
  - $\mathbf{s_8 = w_3(y)\ c_3\ w_1(x)\ r_2(x)\ c_2\ w_1(y)\ c_1}$
  - $\mathbf{s_9 = w_3(y)\ c_3\ w_1(x)\ r_2(x)\ w_1(y)\ c_1\ c_2}$
  - $\mathbf{s_{10} = w_1(x)\ w_1(y)\ c_1\ w_2(x)\ w_2(y)\ c_2}$

Overview (2)


Full
s_1
FSR
s_2
VSR
s_4
CMFSR
s_3
s_5
CMVSR
s_6
CSR
s_7
OCSR
s_8
COCSR
s_9
serial
s_{10}

Conclusion (1)

- **Basic Correctness Notion:**
  - (Conflict-) Serializability
- **Theory of Serializablility**
  - Simple Read/Write-Model
  - Conflict Operations: order-depending operations of different transactions on the same data objects
  - Conflict-Serializability
    - relevant for practical applications
      (in contrast to Final-State- and View-Serializability)
    - can be checked efficiently
    - CSR $\subset$ VSR $\subset$ FSR
  - Serialization Theorem: A History s is conflict serializable if and only if the corresponding G(s) is acyclic

Conclusion (2)

- **Theory of Serializablility (contd.)**
  - CSR, albeit less general than VSR, is best suited
    - for complexity reasons
    - because of its monotonicity
    - because of its generalizabilty to semantically richer operations
  - OCSR and COCSR have further beneficial properties
  - Commit-Serializability also takes possible failures into account
- **Serializable Processes**
  - Ensure correctness of multi user processing automatically
  - Number of possible schedules determines maximal degree of concurrency (parallelism)