Application of AC-OPF with SOCP

Wind farm-related power distribution network access an ESS

Yue Zhao
Industrial Engineering
Ira A. Fulton Schools of Engineering

Motivation

- Understanding the AC-OPF and SOCP.
- Comparing Several Relaxation Methods with Julia and PowerModels.
- Finding the best bus to locate an ESS in a Wind farm-related power distribution network

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Optimal Power Flow(OPF)

- The goal of an optimal power flow (OPF) is to determine the “best” way to operate a power system. Usually “best” = minimizing operating cost.
    - Find a Power Flow distribution
    - Satisfy system operating constraints such as node voltage constraints, branch current/power constraints, generator power constraints, etc.
    - Find an optimal value for a certain aspect of the index, such as the lowest generation cost or the lowest system network loss

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AC Optimal Power Flow(OPF)

$sets$:

N- buses
R- reference buses
E, E^R- branches, forward and reverse orientation
G, G_i - generators and generators at bus i
L, L_i - loads and loads at bus i
S, S_i - shunts and shunts at bus I

$data$:

$S_k^{gl}, S_k^{gu} \forall k \in G$- generator complex power bounds
$c_{2k}, c_{1k}, c_{0k} \forall k \in G$- generator cost components
$v_i^l, v_i^u \forall i \in N$- voltage bounds
$S_k^d \ \forall k \in L$- load complex power consumption
$\mathrm{Y}_k^s \ \forall k \in S$- bus shunt admittance
$Y_{ij}, Y_{ij}^c, Y_{ji}^c \forall (i,j) \in E$- branch pi-section parameters
$T_{ij} \forall (i,j) \in E$- branch complex transformation ratio
$s_{ij}^u \ \forall (i,j) \in E$- branch apparent power limit
$i_{ij}^u \ \forall (i,j) \in E$- branch current limit
$\theta_{ij}^{\Delta l}, \theta_{ij}^{\Delta u} \ \forall (i,j) \in E$- branch voltage angle difference bounds

AC Optimal Power Flow(OPF)

$$variables:$$

$$S_k^g \; \forall k \in G$$

$$V_i \; \forall i \in N$$

$$\mathrm{S}_{ij} \; \forall (i,j) \in E \cup E^R$$

$$minimize:$$

$$\sum_{k \in G} c_{2k}(R(S_k^g))^2 + c_{1k}(R(S_k^g))^2 + c_{0k}$$

$$subject\ to:$$

$$\angle V_r = 0 \; \forall r \in R$$

$$S_k^{gl} \leq S_k^g \leq S_k^{gu} \; \forall k \in G$$

$$v_i^l \leq |V_i| \leq v_i^u \; \forall i \in N$$

$$\sum_{k \in G_i} S_k^g - \sum_{k \in L_i} S_k^g - \sum_{k \in S_i} (\mathrm{Y}_k^s)^* |V_i|^2 = \sum_{(i,j) \in E_i \cup E_i^R} \mathrm{S}_{ij} \; \forall i \in N$$

$$\mathrm{S}_{ij} = \left(Y_{ij} + \mathrm{Y}_{ij}^c\right)^* \frac{|V_i|^2}{\left|T_{ij}\right|^2} - Y_{ij}^* \frac{V_i V_j^*}{\left|T_{ij}\right|^2} \; \forall (i,j) \in E$$

$$\mathrm{S}_{ji} = \left(Y_{ij} + \mathrm{Y}_{ji}^c\right)^* \left|V_j\right|^2 - Y_{ij}^* \frac{V_i^* V_j}{T_{ij}^*} \; \forall (i,j) \in E$$

$$\left|\mathrm{S}_{ij}\right| \leq s_{ij}^u \; \forall (i,j) \in E \cup E^R$$

$$\left|I_{ij}\right| \leq i_{ij}^u \; \forall (i,j) \in E \cup E^R$$

$$\theta_{ij}^{\Delta l} \leq \angle (V_i V_j^*) \leq \theta_{ij}^{\Delta u} \; \forall (i,j) \in E$$

Implementing AC-OPF with Julia and PowerModels

PowerModels.jl

Take MATPOWER's case30.m as an example for testing

```
network_data = PowerModels.parse_file("D:/asu/IEE 598/Project/case30.m");
```

```
ACP = instantiate_model(network_data, ACPPowerModel, PowerModels.build_opf);
result = optimize_model!(ACP, optimizer=CPLEX.Optimizer);
```

```
The solver does not support nonlinear problems (i.e., NLobjective and NLconstraint).
```

!! AC-OPF is Non-convex

(eg. $S_{ij} = V_{ij} I_{ij}$ is Non-convex)

Quick Comparison of Several Relaxation Methods with Julia and PowerModels

- Relaxation Methods
  - PowerModels.LPACCPowerModel - The LPAC Cold-Start AC Power Flow Approximation
  - PowerModels.SOCWRPowerModel – the Second-order cone relaxation
  - PowerModels. QCRMPowerModel – the Recursive McCormik relaxations
  - PowerModels. QCLSPowerModel - the "Quadratic-Convex" relaxation

- Comparison by solve time

```
solve time of LPACC is 0.054000139236450195
solve time of SOCWR is 0.03500008583068848
solve time of QCRM is 0.0820000171661377
solve time of QCLS is 0.07800006866455078
```

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Second-Order Cone Programming(SOCP)

- Second-order cone

$$C_k = \left\{ \begin{bmatrix} u \\ t \end{bmatrix} \middle| u \in R^{k-1}, t \in R, \|u\| \le t \right\}$$

- second-order cone constraint

$$\|Ax + b\| \le c^T x + d \Leftrightarrow \begin{bmatrix} A \\ c^T \end{bmatrix} x + \begin{bmatrix} b \\ d \end{bmatrix} \in C_k$$

- second-order cone constraint in AC-OPF

$$S_{ij} = V_{ij} I_{ij} \Leftrightarrow I_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{V_i} \Leftrightarrow \left\| \begin{matrix} P_{ij}^2 \\ Q_{ij}^2 \end{matrix} \right\|_2 \le V_{ij} I_{ij} \Leftrightarrow \left\| \begin{matrix} 2P_{ij} \\ 2Q_{ij} \\ \tilde{I}_{ij} - \tilde{V}_j \end{matrix} \right\|_2 \le \tilde{I}_{ij} + \tilde{V}_j, \forall ij \in E$$

AC-OPF with the Second-order cone relaxation

$$\min f(p, q, P, Q, \tilde{V}, \tilde{I})$$

$s.t.$

$$\begin{cases} p_j = \sum_{k \in \delta(j)} P_{jk} - \sum_{i \in \pi(j)} \left(P_{ij} - \tilde{I}_{ij} r_{ij}\right) + g_j \tilde{V}_j, \forall j \in B \\ q_j = \sum_{k \in \delta(j)} Q_{jk} - \sum_{i \in \pi(j)} \left(Q_{ij} - \tilde{I}_{ij} x_{ij}\right) + b_j \tilde{V}_j, \forall j \in B \end{cases}$$

$$\tilde{V}_j = \tilde{V}_i - 2\left(P_{ij} r_{ij} + Q_{ij} x_{ij}\right) + \tilde{I}_{ij}\left(r_{ij}^2 + x_{ij}^2\right), \forall ij \in E$$

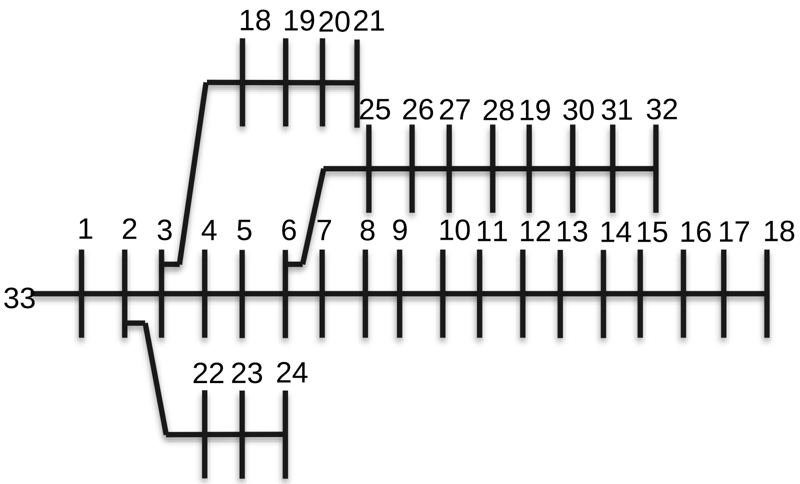
$$\left\| \begin{matrix} 2P_{ij} \\ 2Q_{ij} \\ \tilde{I}_{ij} - \tilde{V}_j \end{matrix} \right\|_2 \le \tilde{I}_{ij} + \tilde{V}_j, \forall ij \in E$$

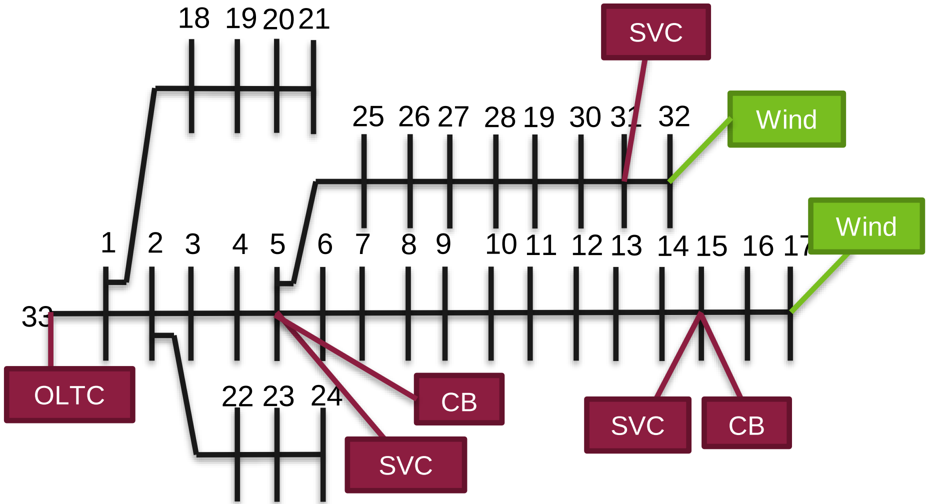
$$\underline{I}_{ij}^2 \le \tilde{I}_{ij} \le \tilde{I}_{ij}^2, \forall ij \in E$$

$$\underline{V}_j^2 \le \tilde{V}_j \le \tilde{V}_j^2, \forall ij \in B^+$$

Wind farm-related power distribution network with 33 buses

- Take data from MATPOWER's case33bw





Add OLTC and Constraints

- On-Load Tap Changers (OLTC) regulates the voltage value on the low voltage side
- With the addition of OLTC, the substation bus node is converted into an adjustable variable.
- The following substitutions can be made:

$$
\begin{cases}
\sigma_{j,1,t}^{OLTC,IN} \geq \sigma_{j,2,t}^{OLTC,} \leq \sigma_{j,SR_j,t}^{OLTC,IN}, \forall t, \forall j \in B^{OLTC} \\
\sigma_{j,t}^{OLTC,IN} + \delta_{j,t}^{OLTC,DE} \leq 1, \forall t, \forall j \in B^{OLTC} \\
\sum_{s} \sigma_{j,s,t}^{OLTC} - \sum_{s} \sigma_{j,s,t-1}^{OLTC} \geq \sigma_{j,t}^{OLTC,IN} - \delta_{j,t}^{OLTC,DE} SR_j, \forall t, \forall j \in B^{OLTC} \\
\sum_{s} \sigma_{j,s,t}^{OLTC} - \sum_{s} \sigma_{j,s,t-1}^{OLTC} \leq \delta_{j,t}^{OLTC,DE} SR_j - \delta_{j,t}^{OLTC,DE}, \forall t, \forall j \in B^{OLTC} \\
\sum_{s} (\sigma_{j,t}^{OLTC,IN} + \delta_{j,t}^{OLTC,DE}) \leq N_j^{OLTC,max}, \forall j \in B^{OLTC}
\end{cases}
$$

Add Reactive Power Regulator and Constraints

- Capacitor Banks(CB)

$$\begin{cases} \sum_{s} \delta_{j,t}^{CB} \leq N_j^{CB,max} \\ -\delta_{j,t}^{CB} Y_j^{CB,max} \leq Y_{j,t}^{CB} \leq \delta_{j,t}^{CB} Y_j^{CB,max} \end{cases}, \forall t, \forall j \in B^{CB}$$

- Static VAR compensation(SVC)

$$Q_j^{SVC,min} \leq Q_{j,t}^{SVC} \leq Q_j^{SVC,max}, \forall t, \forall j \in B^{SVC}$$

Select which bus to add an ESS?

- Place the ESS on each bus in turn
- Add ESS constraints to the power system model
- The objective values are obtained by solving with the CPLEX solver.
- Find the minimum objective value and index its bus
- Get the best position for ESS

Select which bus to add an ESS?

- Charging and discharging mode limitation

$$u_{j,t}^{discharge} + u_{j,t}^{charge} \leq 1, \forall j \in B^{ESS}, \forall t$$

- Power Limit

$$\begin{cases} u_{j,t}^{discharge} P_j^{discharge,min} \leq P_{j,t}^{discharge} \leq u_{j,t}^{discharge} P_j^{discharge,max} \\ u_{j,t}^{charge} P_j^{charge,min} \leq P_{j,t}^{charge} \leq u_{j,t}^{charge} P_j^{charge,max} \end{cases}$$

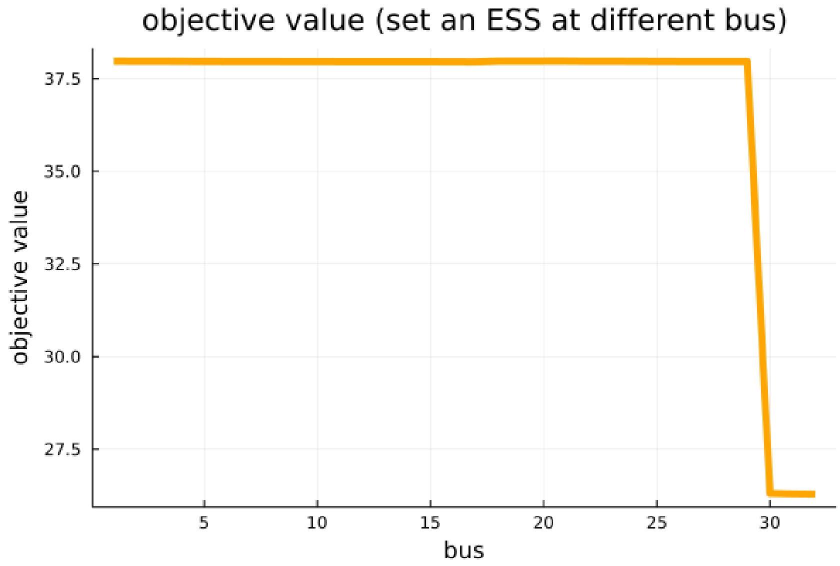
$$\forall t, \forall j \in B^{ESS}$$

- Storage Limit

$$\begin{cases} E_{j+1}^{ESS} = E_{j,t}^{ESS} + \alpha_j^{charge} P_{j,t}^{charge} - \alpha_j^{discharge} P_{j,t}^{discharge} \\ E_j^{ESS,min} \leq E_{j,t}^{ESS} \leq E_j^{ESS,max} \end{cases}$$

$$\forall j \in B^{ESS}, \forall t$$

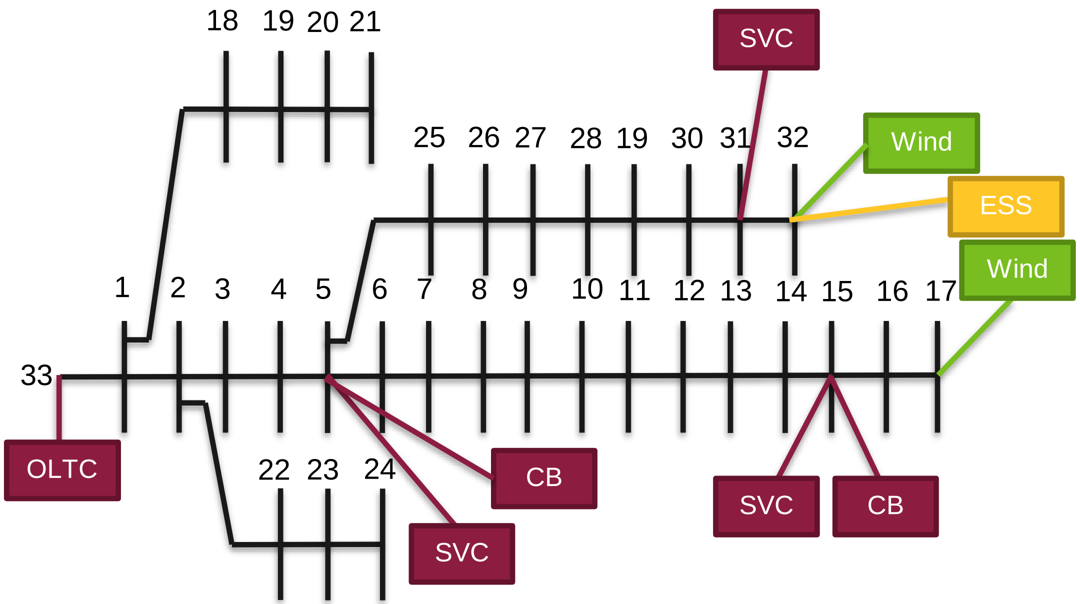
Result



optimalvalue	
⋮	
21.0	37.9755
22.0	37.9708
23.0	37.9716
24.0	37.9693
25.0	37.9668
26.0	37.9649
27.0	37.9645
28.0	37.9635
29.0	37.9633
30.0	26.3077
31.0	26.2949
32.0	26.2882

- choose 32 bus to set an ESS

Result



18
19
20
21
SVC
25
26
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28
19
30
31
32
Wind
ESS
Wind
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7
8
9
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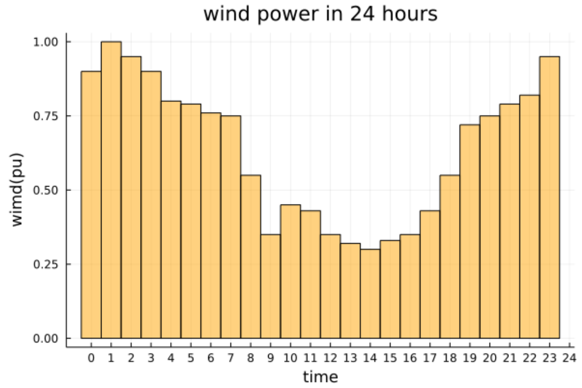
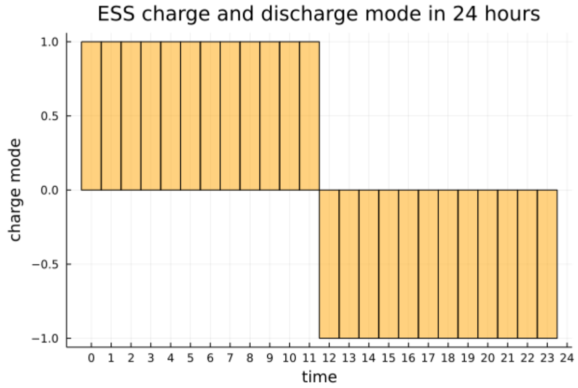
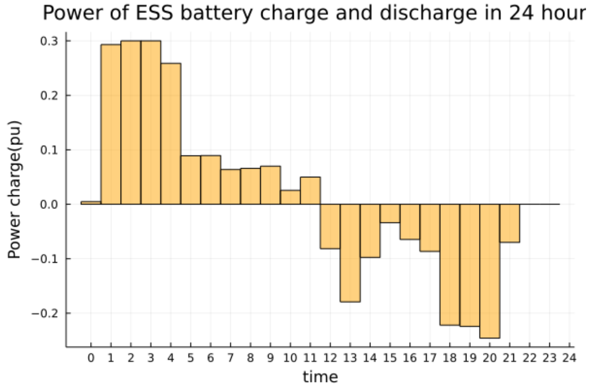
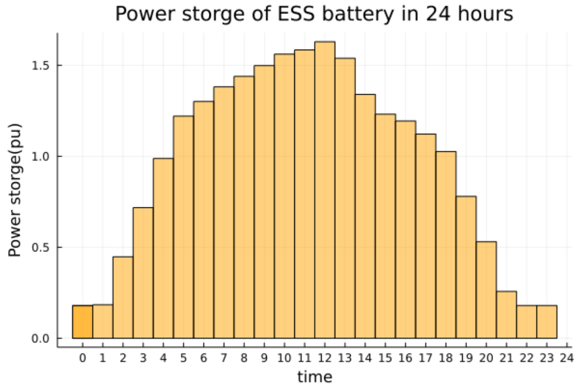
12
13
14
15
16
17
33
OLTC
22
23
24
CB
SVC
SVC
CB

Result

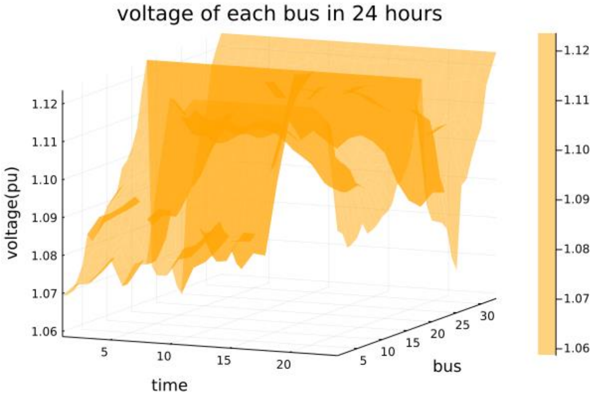

wind power in 24 hours
wimd(pu)
time

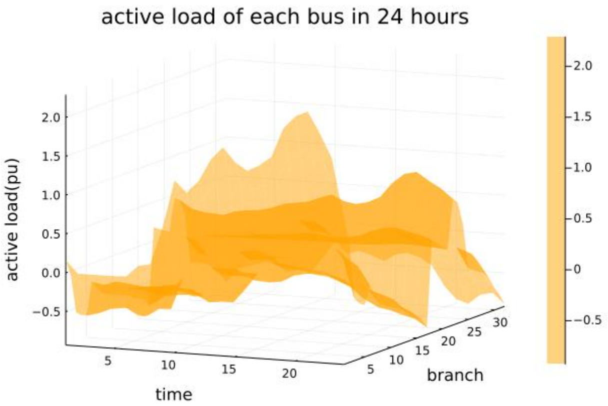

ESS charge and discharge mode in 24 hours
charge mode
time

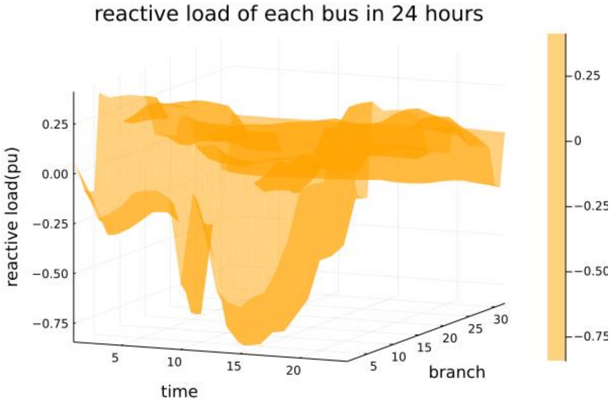

Power of ESS battery charge and discharge in 24 hour
Power charge(pu)
time


Power storge of ESS battery in 24 hours
Power storge(pu)
time

Result


voltage of each bus in 24 hours
voltage(pu)
time
bus


active load of each bus in 24 hours
active load(pu)
time
branch


reactive load of each bus in 24 hours
reactive load(pu)
time
branch

Future Research

- Considering a large scale grid
- Involving some geographical factors related to a real world case
- Adding more constraints to the model, like factors related to the electricity market

References

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