Eckstein-Keane-Wolpin models

An invitation for transdisciplinary collaboration

Open Source Economics

Abstract

We present background material on a class of structural microeconometric models to facilitate transdisciplinary collaboration in their future development. We describe the economic framework, mathematical formulation, and calibration procedures for the so-called Eckstein-Keane-Wolpin (EKW) models. We provide an exemplifying analysis of the seminal model outlined in Keane & Wolpin (1997) and present our group's ensemble of research codes that allow for its specification, simulation, and calibration. We summarize our efforts drawing on research outside economics to address the computational challenges in applying EKW models and improve the reliability and interpretability of their results.

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Contents

1 Introduction

Economists use structural microeconometric models to study individual decision-making. These models specify the objective of individuals, their economic environment, and the institutional and informational constraints under which they operate. Calibration of the model to observed data on individual decisions and experiences allows quantifying the importance of competing economic mechanisms in determining economic outcomes and forecasting the effects of policy proposals (Wolpin, 2013).

We restrict our exposition to the class of Eckstein-Keane-Wolpin (EKW) models (Adda et al., 2017; Blundell et al., 2016; Keane & Wolpin, 1997). Labor economists use them to study human capital investment decisions. Human capital comprises the knowledge, skills, competencies, and attributes embodied in individuals facilitating the creation of personal, social, and economic well-being (Becker, 1964). Differences in human capital attainment lead to inequality in various life outcomes such as labor market success and health across and within countries (OECD, 2001).

In Bhuller et al. (2018), for example, we apply an EKW model to analyze the mechanisms determining schooling decisions in Norway. We calibrate the model using Norwegian population panel data with nearly career-long earnings histories. After validating our model using a mandatory schooling reform, we gain insights into the underlying economic mechanisms that generate the effects of the policy and forecast the impacts of several policy alternatives.

We offer this handout to facilitate transdisciplinary collaboration in the future development of EKW models. We first describe their economic framework, mathematical formulation, and calibration procedure. We then turn to the seminal model outlined in Keane & Wolpin (1997) as an example and present our group's ensemble of research codes that allow for its specification, simulation, and calibration. Finally, we summarize our efforts drawing on research outside economics to address the computational challenges in applying EKW models and improve the reliability and interpretability of their results.

Throughout, we only offer a limited number of seminal references and textbooks that invite further study. We introduce acronyms and symbols as needed, and our notation draws on the reviews by Aguirregabiria & Mira (2010), Arcidiacono & Ellickson (2011), and Puterman (1994).

2 Setup

We now present the basic setup of the EKW models. We first describe the economic framework, then turn to its mathematical formulation, and finally outline the calibration procedure.

2.1 Economic framework

EKW models describe sequential decision-making under uncertainty (Gilboa, 2009; Machina & Viscusi, 2014). At time $t = 1, \ldots, T$ each individual observes the state of the economic environment $s_t \in S$ and chooses an action a_t from the set of admissible actions $\mathcal{A}$. The decision has two consequences: an individual receives an immediate utility $u_t(s_t, a_t)$ and the economy evolves to a new state s_{t+1}. The transition from s_t to s_{t+1} is affected by the action but remains uncertain. Individuals are forward-looking. Thus they do not simply choose the alternative with the highest immediate utility. Instead, they take the future consequences of their current action into account.

A policy $\pi \equiv (a_1^\pi(s_1), \ldots, a_T^\pi(s_T))$ provides the individual with instructions for choosing an action in any possible future state. It is a sequence of decision rules $a_t^\pi(s_t)$ that specify the action at a particular time t for any possible state s_t under π. The implementation of a policy generates a sequence of utilities that depends on the objective transition probability distribution $p_t(s_t, a_t)$ for the evolution of state s_t to s_{t+1} induced by the model. Individuals have rational expectations (Muth, 1961) so their subjective beliefs about the future agree with the objective transition probabilities of the model.

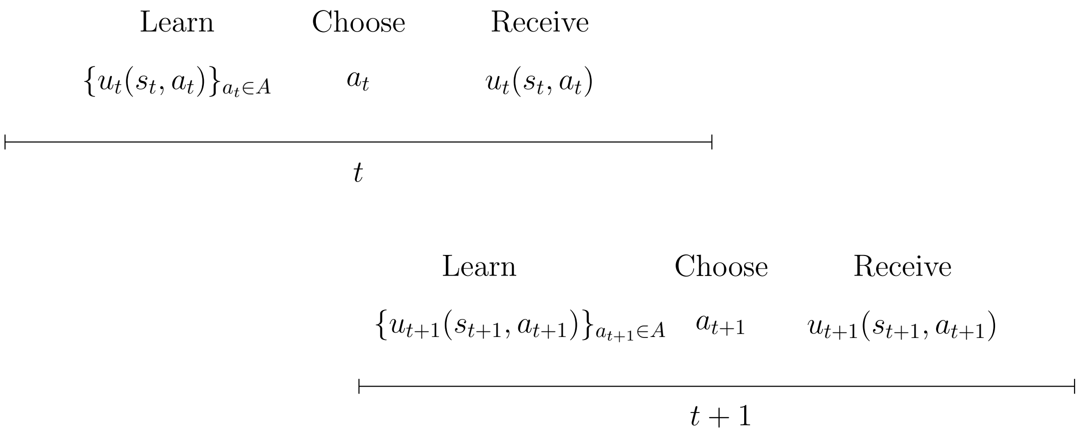
Figure 1 depicts the timing of events in the model for two generic periods. At the beginning of period t, an individual fully learns about the immediate utility of each alternative, chooses one of them, and receives its immediate utility. Then the state evolves from s_t to s_{t+1} and the process is repeated in $t + 1$. Individuals face uncertainty and they seek to maximize the expected total discounted utilities. An exponential discount factor $0 < \delta < 1$ parameterizes their time preference and captures a taste for immediate over future utilities.

Equation (2.1) provides the formal representation of the individual's objective. Given an initial state s_1, individuals implement the policy π from the set of all possible policies Π that maximizes the expected total discounted utilities over all T decision periods given the information $\mathcal{I}_1$ available in the first period.

$$\max_{\pi \in \Pi} \mathrm{E}_{s_1}^{\pi} \left[\sum_{t=1}^{T} \delta^{t-1} u_t(s_t, a_t^\pi(s_t)) \,\middle|\, \mathcal{I}_1 \right] \tag{2.1}$$

The superscript of the expectation emphasizes that each policy π induces a different probability

Figure 1: Timing of events



distribution over the sequences of utilities.

2.2 Mathematical formulation

EKW models are set up as a standard Markov decision process (MDP) (Puterman, 1994; White, 1993). When making sequential decisions under uncertainty, the task is to determine the optimal policy π^* with the largest expected total discounted utilities $v_1^{\pi^*}(s_1)$ as formalized in equation (2.1). In principle, this requires evaluating the performance of all policies based on all possible sequences of utilities, each weighted by the probability with which they occur. Fortunately, however, the multistage problem can be solved by a sequence of simpler inductively defined single-stage problems.[1]

The value function $v_t^\pi(s_t)$ captures the expected total discounted utilities under policy π from period t onwards for an individual experiencing state s_t:

$$v_t^\pi(s_t) \equiv \mathrm{E}_{s_t}^\pi \left[\sum_{j=0}^{T-t} \delta^j \, u_{t+j}(s_{t+j}, a_{t+j}^\pi(s_{t+j})) \,\middle|\, \mathcal{I}_t \right].$$

Then we can determine $v_1^\pi(s_1)$ for any policy by recursively evaluating equation (2.2):

$$v_t^\pi(s_t) = u_t(s_t, a_t^\pi(s_t)) + \delta \, \mathrm{E}_{s_t}^\pi \left[v_{t+1}^\pi(s_{t+1}) \,\middle|\, \mathcal{I}_t \right]. \tag{2.2}$$

Equation (2.2) expresses the total value $v_t^\pi(s_t)$ of adopting policy π going forward as the sum

[1] Optimal decisions in an MDP are a deterministic function of the current state s only, i.e., an optimal decision rule is always deterministic and Markovian. We restrict our notation to this special case right from the beginning.

of its immediate utility and all expected discounted future utilities.

The principle of optimality (Bellman, 1954) allows to construct π^* by solving the optimality equations (2.3) for all s and t recursively:

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \left\{ u_t(s_t, a_t) + \delta \, \mathrm{E}_{s_t}^{\pi^*} \left[v_{t+1}^{\pi^*}(s_{t+1}) \,\middle|\, \mathcal{I}_t \right] \right\}. \tag{2.3}$$

The optimal value function $v_t^{\pi^*}$ is the sum of the expected discounted utilities in t over the remaining time horizon assuming the optimal policy is implemented going forward. The optimal action is choosing the alternative with the highest total value:

$$a_t^{\pi^*}(s_t) = \arg\max_{a_t \in A} \left\{ u_t(s_t, a_t) + \delta \, \mathrm{E}_{s_t}^{\pi^*} \left[v_{t+1}^{\pi^*}(s_{t+1}) \middle| \, \mathcal{I}_t \right] \right\}.$$

Algorithm 1 allows to solve the MDP by a simple backward induction procedure. In the final period T, there is no future to take into account, and the optimal action is choosing the alternative with the highest immediate utilities in each state. With the decision rule for the final period at hand, the other optimal decisions can be determined recursively following equation (2.3) as the calculation of their expected future utilities is straightforward given the relevant transition probabilities.

Algorithm 1 Backward induction procedure

for $t = T, \ldots, 1$ **do**

if t == T **then**

$$v_T^{\pi^*}(s_T) = \max_{a_T \in A} \left\{ u_T(s_T, a_T) \right\} \qquad \forall \, s_T \in S$$

else

Compute $v_t^{\pi^*}(s_t)$ for each $s_t \in S$ by

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \left\{ u_t(s_t, a_t) + \delta \, \mathrm{E}_{s_t}^{\pi} \left[v_{t+1}^{\pi^*}(s_{t+1}) \middle| \, \mathcal{I}_t \right] \right\}$$

and set

$$a_t^{\pi^*}(s_t) = \arg\max_{a_t \in A} \left\{ u_t(s_t, a_t) + \delta \, \mathrm{E}_{s_t}^{\pi} \left[v_{t+1}^{\pi^*}(s_{t+1}) \middle| \, \mathcal{I}_t \right] \right\}.$$

end if

end for

2.3 Calibration procedure

EKW models are calibrated to data on observed individual decisions and experiences under the hypothesis that the individual behaves according to the model. The goal is to back out information on utility functions, preference parameters, and transition probabilities. This requires the full parameterization θ of the model.

Economists have access to information for $i = 1, \ldots, N$ individuals in each time period t. For every observation (i, t) in the data, we observe action a_{it}, some components $\bar{u}_{it}$ of the utility, and a subset $\bar{s}_{it}$ of the state s_{it}. Therefore, from an economist's point of view, we need to distinguish between two types of state variables $s_{it} = (\bar{s}_{it}, \epsilon_{it})$. At time t, the economist and individual both observe $\bar{s}_{it}$ while ϵ_{it} is only observed by the individual. In summary, the data $\mathcal{D}$ has the following structure:

$$\mathcal{D} = \{a_{it}, \bar{s}_{it}, \bar{u}_{it} : i = 1, \ldots, N; t = 1, \ldots, T_i\},$$

where T_i is the number of observations for which we observe individual i.

Numerous calibration procedures for different settings exist (Davidson & MacKinnon, 2003; Gourieroux & Monfort, 1996). We briefly outline likelihood-based and simulation-based calibration. Independent of the calibration criterion, it is necessary to solve for the optimal policy π^* at each candidate parameterization of the model.

Likelihood-based calibration seeks to find the parameterization $\hat{\theta}$ that maximizes the likelihood function $\mathcal{L}(\theta \mid \mathcal{D})$, i.e. the probability of observing the given data as a function of θ. As we only observe a subset $\bar{s}_t$ of the state, we can determine the probability $p_{it}(a_{it}, \bar{u}_{it} \mid \bar{s}_{it}, \theta)$ of individual i at time t in $\bar{s}_{it}$ choosing a_{it} and receiving u_{it} given parametric assumptions about the distribution of ϵ_{it}. The objective function takes the following form:

$$\hat{\theta} \equiv \underset{\theta \in \Theta}{\arg\max} \underbrace{\prod_{i=1}^{N} \prod_{t=1}^{T_i} p_{it}(a_{it}, \bar{u}_{it} \mid \bar{s}_{it}, \theta)}_{\mathcal{L}(\theta \mid \mathcal{D})}.$$

In simulation-based calibration, our goal is to find the parameterization $\hat{\theta}$ that yields a simulated data set from the model that closest resembles the observed data. More precisely, the goal is often to minimize the weighted squared distance between a set of moments M_D computed on the observed data and the same set of moments computed on the simulated data $M_S(\theta)$. The objective function takes the following form:

$$\hat{\theta} \equiv \underset{\theta \in \Theta}{\arg\min} (M_D - M_S(\theta))' W (M_D - M_S(\theta)).$$

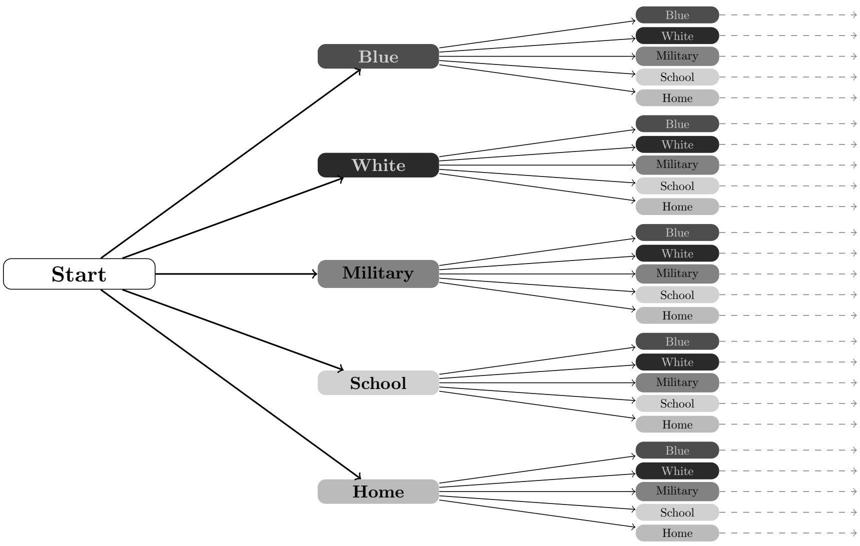
3 Example

We now present an exemplifying analysis of a canonical EKW model on human capital investment. The model was initially studied in Keane & Wolpin (1997) to explore the career decisions of young men about their schooling, work, and occupational choice. We first outline the basic setup of the model, provide some descriptive statistics of the empirical data used for its calibration, and then explore selected economic insights.

3.1 Basic setup

We follow individuals over their working life from young adulthood at age 16 to retirement at age 65 where the decision period $t = 16, \ldots, 65$ is a school year. Figure 2 illustrates the initial decision problem as individuals decide $a \in \mathcal{A}$ whether to work in a blue-collar or white-collar occupation ($a = 1, 2$), to serve in the military ($a = 3$), to attend school ($a = 4$), or to stay at home ($a = 5$).

Figure 2: Decision tree



Individuals are already heterogeneous when entering the model. They differ with respect to their level of completed schooling h_{16} and have one of four different $\mathcal{J} = \{1, \ldots, 4\}$ alternative-specific skill endowments $\boldsymbol{e} = (e_{j,a})_{\mathcal{J} \times \mathcal{A}}$.

The immediate utility $u(\cdot)$ of each alternative consists of a non-pecuniary utility $\zeta_a(\cdot)$ and, at least for the working alternatives, an additional wage component $w_a(\cdot)$. Both depend on the level of human capital as measured by their alternative-specific skill endowment $\boldsymbol{e}$, years of completed schooling h_t, and occupation-specific work experience $\boldsymbol{k_t} = (k_{a,t})_{a \in \{1,2,3\}}$. The immediate utilities are influenced by last-period choices a_{t-1} and alternative-specific productivity shocks $\boldsymbol{\epsilon_t} = (\epsilon_{a,t})_{a \in \mathcal{A}}$ as well. Their general form is given by:

$$u(\cdot) = \begin{cases} \zeta_a(\boldsymbol{k_t}, h_t, t, a_{t-1}) + w_a(\boldsymbol{k_t}, h_t, t, a_{t-1}, e_{j,a}, \epsilon_{a,t}) & \text{if } a \in \{1,2,3\} \\ \zeta_a(\boldsymbol{k_t}, h_t, t, a_{t-1}, e_{j,a}, \epsilon_{a,t}) & \text{if } a \in \{4,5\}. \end{cases}$$

Work experience $\boldsymbol{k_t}$ and years of completed schooling h_t evolve deterministically.

$$\begin{aligned} k_{a,t+1} &= k_{a,t} + \mathbf{I}[a_t = a] \qquad \text{if } a \in \{1,2,3\} \\ h_{t+1} &= h_t \;\; + \mathbf{I}[a_t = 4] \end{aligned}$$

The productivity shocks $\boldsymbol{\epsilon_t}$ are uncorrelated across time and follow a multivariate normal distribution with mean $\mathbf{0}$ and covariance matrix $\mathbf{\Sigma}$. Given the structure of the utility functions and the distribution of the shocks, the state at time t is $s_t = \{\boldsymbol{k_t}, h_t, t, a_{t-1}, \boldsymbol{e}, \boldsymbol{\epsilon_t}\}$.

Theoretical and empirical research from specialized disciplines within economics informs the specification of each $u_a(\cdot)$ and we discuss the exact functional form of the immediate utility function in the blue-collar occupation as an example.[2]

Equation (3.1) shows the parameterization of the non-pecuniary utility from working in a blue-collar occupation.

$$\begin{aligned} \zeta_1(\boldsymbol{k_t}, h_t, a_{t-1}) = \alpha_1 &+ c_{1,1} \cdot \mathbf{I}[a_{t-1} \neq 1] + c_{1,2} \cdot \mathbf{I}[k_{1,t} = 0] \\ &+ \vartheta_1 \cdot \mathbf{I}[h_t \geq 12] + \vartheta_2 \cdot \mathbf{I}[h_t \geq 16] + \vartheta_3 \cdot \mathbf{I}[k_{3,t} = 1] \end{aligned} \tag{3.1}$$

It includes job amenities α_1 and mobility and search costs $(c_{1,1}, c_{1,2})$ that capture the extra effort for individuals who only recently started working in a white-collar occupation. Additional components depend on whether an individual has a high school ϑ_1 or college ϑ_2 degree. There is a detrimental impact of leaving the military after a single year ϑ_3.

The wage component $w_1(\cdot)$ is given by the product of the market-equilibrium rental price r_1 and an occupation-specific skill level $x_1(\cdot)$. The latter is determined by the overall level of

[2] All additional details are available in Appendix **??**.

human capital. This specification leads to a standard logarithmic wage equation in which the constant term is the skill rental price $\ln(r_1)$ and wages follow a log-normal distribution.

The occupation-specific skill level $x_1(\cdot)$ is determined by a skill production function, which includes a deterministic component $\Gamma_1(\cdot)$ and a multiplicative stochastic productivity shock $\epsilon_{1,t}$.

$$x_1(\boldsymbol{k_t}, h_t, t, a_{t-1}, e_{j,1}, \epsilon_{1,t}) = \exp\left(\Gamma_1(\boldsymbol{k_t}, h_t, t, a_{t-1}, e_{j,1}) \cdot \epsilon_{1,t}\right)$$

Equation (3.2) shows the parameterization of the deterministic component of the skill production function.

$$\begin{aligned}\Gamma_1(\boldsymbol{k_t}, h_t, t, a_{t-1}, e_{j,1}) = e_{j,1} &+ \beta_{1,1} \cdot h_t + \beta_{1,2} \cdot \mathbf{I}[h_t \geq 12] + \beta_{1,3} \cdot \mathbf{I}[h_t \geq 16] \\ &+ \gamma_{1,1} \cdot k_{1,t} + \gamma_{1,2} \cdot (k_{1,t})^2 + \gamma_{1,3} \cdot \mathbf{I}[k_{1,t} > 0] \\ &+ \gamma_{1,4} \cdot t + \gamma_{1,5} \cdot \mathbf{I}[t < 18] \\ &+ \gamma_{1,6} \cdot \mathbf{I}[a_{t-1} = 1] + \gamma_{1,7} \cdot k_{2,t} + \gamma_{1,8} \cdot k_{3,t}\end{aligned} \tag{3.2}$$

There are several notable features. Skills increase with schooling $\beta_{1,1}$ and white-collar work experience $(\gamma_{1,1}, \gamma_{1,2})$. There are so-called sheep-skin effects associated with completing a high school $\beta_{1,2}$ and graduate $\beta_{1,3}$ education that capture the impact of completing a degree beyond just the associated years of schooling. Also, skills depreciate when working in a different occupation $\gamma_{1,6}$ but other work experience $(\gamma_{1,7}, \gamma_{1,8})$ is transferable.

3.2 Empirical data

We analyze the original dataset used by Keane & Wolpin (1997) and thus only provide a brief description here.[3] The authors construct their sample based on the National Longitudinal Survey of Youth 1979 (NLSY79). The NLSY79 is a nationally representative sample of young men and women living in the United States in 1979 and born between 1957 and 1964. Individuals were followed from 1979 onwards and repeatedly interviewed about their schooling decisions and labor market experiences. Based on this information, individuals are assigned to either working in one of the three occupations, attending school, or simply staying at home.

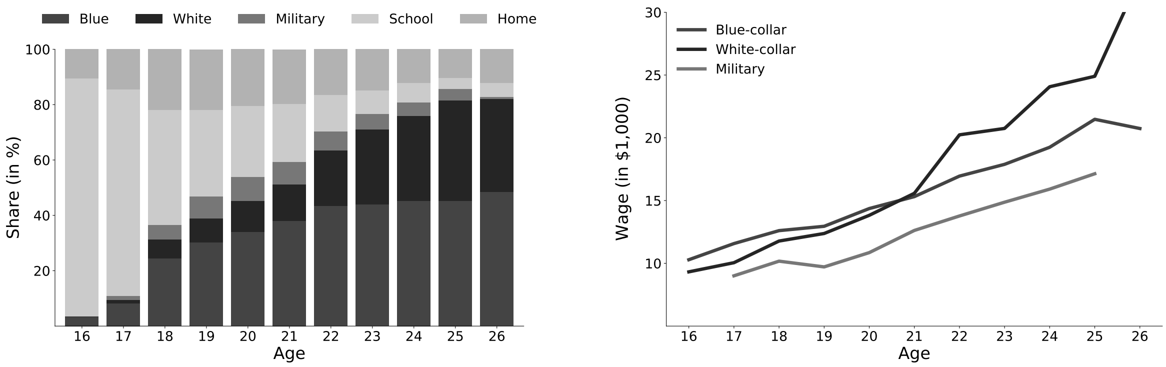
Keane & Wolpin (1997) restrict attention to white males that turn 16 between 1977 and 1981 and exploit the information collected between 1979 and 1987. Thus individuals in the sample are all between 16 and 26 years old. While the sample initially consists of 1,373 individuals

[3] We provide additional details in Appendix ??.

at age 16, this number drops to 256 at the age of 26 due to sample attrition, missing data, and the short observation period. Overall, the final sample consists of 12,359 person-period observations.

Figure 3 summarizes our information about choices and wages by age. We show the distribution of choices on the left, and report average wages on the right. Initially, roughly 86% of individuals enroll in school, but this share steadily declines with age. Nevertheless, about 39% obtain more than a high school degree and continue their schooling for more than twelve years. As individuals leave school, most of them initially pursue a blue-collar occupation. But the relative share of the white-collar occupation increases as individuals entering the labor market later have higher levels of schooling. At age 26, about 48% work in a white-collar occupation and 34% in a blue-collar occupation. The share of individuals in the military peaks around age 20 when it amounts to 8%. At its maximum around age 18, approximately 20% of individuals stay at home.

Figure 3: Data overview



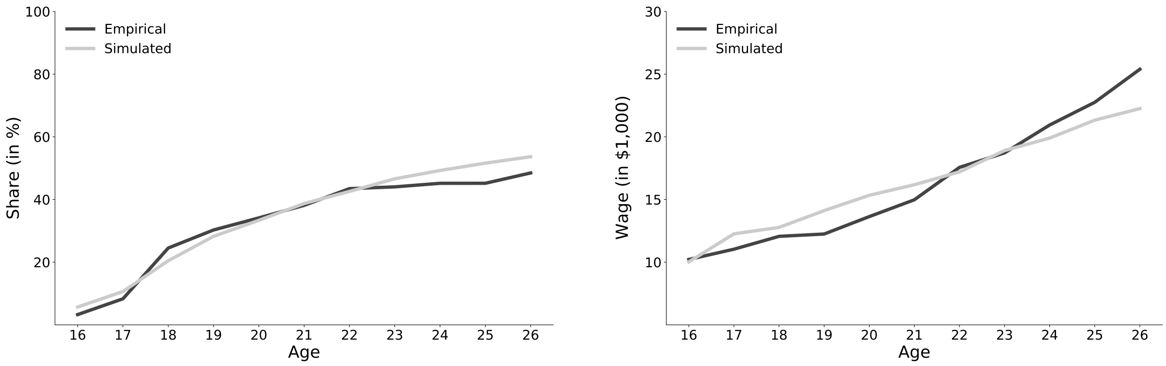
(a) Choices **(b)** Average wage

Notes: The wage is a full-time equivalent deflated by the gross national product deflator, with 1987 as the base year. We do not report the wage if less than ten observations are available.

Overall, average wages start at about \$10,000 at age 16 but increase considerably up to about \$25,000 at age 26. While wages in the blue-collar occupation are initially highest with about \$10,286, wages in the white-collar occupation and military start around \$9,000. However, wages in the white-collar occupation increase steeper over time and overtake blue-collar wages around age 21. At the end of the observation period, wages in the white-collar occupation are about 50% higher than blue-collar wages with \$32,756 as opposed to only \$20,739. Military wages remain lowest throughout.

We fit the model to the empirical data using maximum likelihood calibration. Figure 4 shows the overall agreement between the empirical data and a dataset simulated using the calibrated parameters within the support of the data. On the left, we show the choice probability of working in a blue-collar occupation, while we plot the average wage across all occupations on the right.

Figure 4: Model fit



(a) Blue-collar **(b)** Average wage

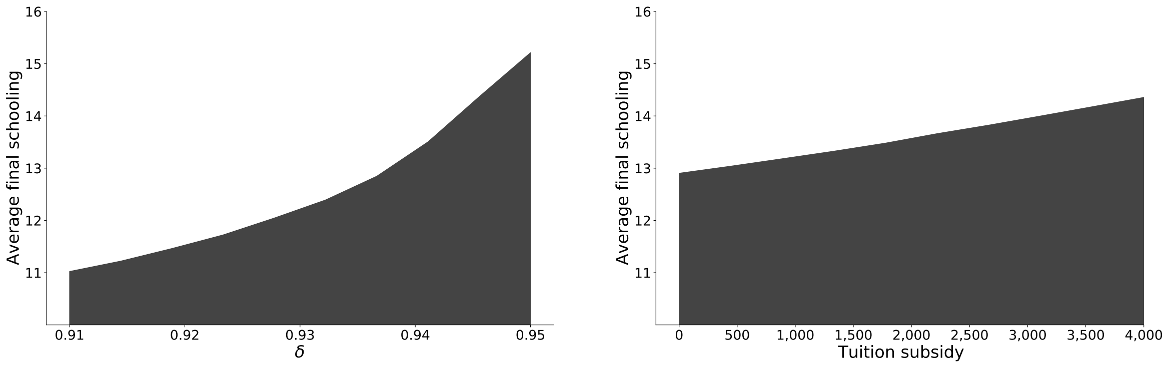
Notes: We simulate a sample of 1,000 individuals using the calibrated model.

Overall, the values of the calibrated parameters of the model are in broad agreement with the relevant literature. For example, individuals discount future utilities by 6% per year, and wages increase by about 7% with each additional year of schooling.

3.3 Economic insights

Figure 5 illustrates the ability of the model to quantify the impact of economic mechanisms and to forecast the effect of public policies. On the left, we vary the discount factor capturing time preferences between 0.91 and 0.95 while we introduce a tuition subsidy of up to $\$4,000$ on the right. In both cases, we are interested in the changes to average final schooling.

Figure 5: Economic mechanism and policy forecast



(a) Time preference **(b)** Tuition subsidy

Notes: We simulate a sample of 1,000 individuals using the calibrated model.

Increases in the discount factor and the tuition subsidy both result in higher average final schooling. However, they do so for very different reasons. While individuals emphasize the future benefits of their schooling investment in the former, they react to a reduction of its immediate cost in the latter.

4 Pipeline

We are actively developing an ensemble of research codes that provide an analysis pipeline for EKW models. Among them are `respy` and `estimagic`. The former allows for the flexible specification and simulation of EKW models while the latter provides the means for their calibration. We briefly showcase the typical workflow of using both packages in our research.

Figure 6 illustrates a typical workflow. Initially, the user provides the empirical data, the parameterization of the model, and other options to `respy`. All together define the structure of the model, and we can construct the functionality for the simulation of data and the evaluation of the criterion function. `estimagic` allows calibrating the model to the empirical data. The results from the calibration steps are used to, for example, analyze the economic mechanisms underlying the observed behaviors.

Figure 6: Typical workflow

```
import respy as rp
from estimagic import maximize

# obtain model input
params, options, df = rp.get_example_model("kw_97_extended_respy")

# process model specification
log_like = rp.get_log_like_func(params, options, df)
simulate = rp.get_simulate_func(params, options)

# perform calibration
results, params_rslt = maximize(log_like, params, "nlopt_bobyqa")

# conduct analysis
df_rslt = simulate(params_rslt)
```

Figure 7 shows the model specification files for Keane & Wolpin (1997). The file on the left sets the parameter values for the utility functions and the distribution of the unobservable state variables. On the right, we provide details on the construction of the observed state variables and numerous tuning parameters for the numerical solution of the model.

Figure 7: Model specification

category	name	value	name
delta	delta	9.370735e-01	delta_delta
wage_white_collar	constant	8.741888e+00	wage_white_collar_constant
	exp_school	6.548940e-02	wage_white_collar_exp_school
	exp_white_collar	1.763655e-02	wage_white_collar_exp_white_collar
	exp_white_collar_square	-4.215936e-02	wage_white_collar_exp_white_collar_square
	exp_blue_collar	3.431936e-02	wage_white_collar_exp_blue_collar
	exp_military	1.406945e-02	wage_white_collar_exp_military
	hs_graduate	-3.599855e-03	wage_white_collar_hs_graduate
	co_graduate	2.301313e-03	wage_white_collar_co_graduate
	period	9.577717e-03	wage_white_collar_period
	is_minor	-1.509984e-01	wage_white_collar_is_minor

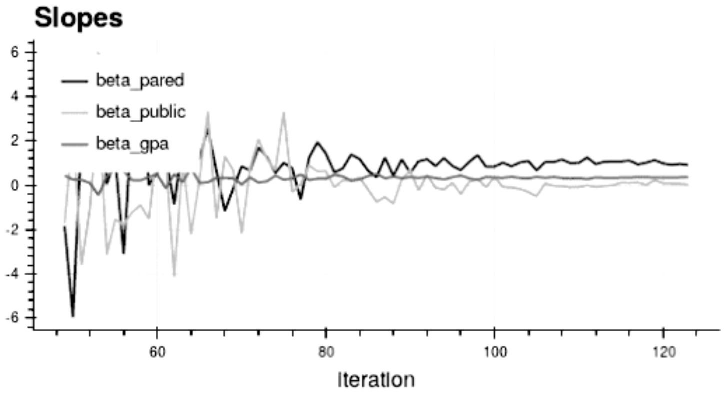
(a) Parameterization

	value
estimation_draws	200
estimation_seed	500
estimation_tau	500
interpolation_points	-1
n_periods	50
simulation_agents	5000
simulation_seed	132
solution_draws	500
solution_seed	456
monte_carlo_sequence	random
covariates	{'hs_graduate': 'exp_school >= 12', 'co_gradua...

(b) Options

Figure 8 depicts the dashboard provided by `estimagic` to monitor the progress and parameter values of the calibration in real-time. This allows us to detect problems during calibration right away and facilitates the debugging process.

Figure 8: Dashboard



We adopt a modern software engineering workflow in the development of both packages and tutorials, source code, testing harness, as well as implementation details are available in their respective online documentations at `https://respy.rtfd.io` and `https://estimagic.rtfd.io`.

5 Improvements

The implementation of EKW models poses several computational challenges. Among them are numerical integration, global optimization, function approximation, and efficient parallelization. We now describe some of our efforts to align `respy` and `estimagic` with the state-of-the-art in computational methods. We have concluded our preparatory work and actively seek input from domain experts for further improvements and joint publication.

5.1 Numerical integration

The solution of EKW models requires the evaluation of millions of integrals to determine the future value of each action in each state. In Eisenhauer, Gabler, & Suchy (2020), we draw on the extensive literature on numerical integration (Davis & Rabinowitz, 2007; Gerstner & Griebel, 1998) to improve the precision and reliability of their solution. The current practice in economics is to implement a random Monte Carlo integration which introduces considerable numerical error and computational instabilities (Judd & Skrainka, 2011).

We consider the optimality equation in a generic time period t to clarify the structure of the integral. Let $v_t^\pi(s_t, a_t)$ denote the action-specific value function of choosing action a_t in state

s_t while continuing with the optimal policy going forward.

$$\begin{aligned}
v_t^{\pi}(s_t, a_t) &= u_t(s_t, a_t) + \delta\, \mathrm{E}_{s_t}\left[v_{t+1}^{\pi^*}(s_{t+1}) \,\middle|\, \mathcal{I}_t \right] \\
&= u_t(s_t, a_t) + \delta \int_S v_{t+1}^{\pi^*}(s_{t+1})\ \mathrm{d}p_t(a_t, s_t) \\
&= u_t(s_t, a_t) + \delta \int_S \max_{a_{t+1} \in A} \left\{ v_{t+1}^{\pi^*}(s_{t+1}, a_{t+1}) \right\} \mathrm{d}p_t(a_t, s_t).
\end{aligned}$$

Let's consider an atemporal version of the typical integral from Keane & Wolpin (1994) as an example. As outlined earlier, individuals can choose among four alternatives. Each of the alternative-specific utilities is, in part, determined by a stochastic continuous state variable ϵ. The transition of all other state variables x is deterministic. This results in a four-dimensional integral of the following form:

$$\int_{\epsilon} \max_{a \in A} \left\{ v^{\pi^*}(x, \epsilon, a) \right\} \phi_{\mu,\Sigma}(\epsilon)\, \mathrm{d}\epsilon \quad \forall\, x \in X,$$

where ϵ follows a multivariate normal distribution with mean μ, covariance matrix Σ, and probability density function $\phi_{\mu,\Sigma}$.

5.2 Global optimization

The calibration of EKW models is challenging due to a large number of parameters and multiplicity of local minima. In Eisenhauer, Gabler, & Röhl (2020), we draw on the literature on global optimization to assess and improve the reliability of the calibrations (Locatelli & Schoen, 2013; Nocedal & Wright, 2006).

We conduct a benchmarking exercise using Keane & Wolpin (1994, 1997) as a well-known and empirically-grounded test case. Depending on the calibration procedure, particular challenges arise. For example, while likelihood-based calibration requires smoothing of the choice probabilities, simulation-based calibration involves the optimization of a noisy function. We provide guidelines for selecting the appropriate algorithm in each setting and showcase diagnostics to assess the reliability of the calibration results.

6 Extensions

We are actively pursuing several extensions to the standard analysis of EKW models. For example, we draw on the methodological literature on robust-decision making and uncertainty quantification to account for the uncertainties within and outside the model (Hansen, 2015). We also work with the German Institute for Economic Research and Statistics Norway to improve the available data for the calibration of the models. Again, we have concluded our preparatory work and actively seek input from domain experts for further improvements and joint publication.

6.1 Robust decision-making

The uncertainties involved in human capital investments are ubiquitous (Becker, 1964). Individuals usually make investments early in life when they are still uncertain about their abilities and tastes. Their returns also depend on demographic, economic, and technological trends that only start to unfold years from now. However, the treatment of uncertainty in EKW models of human capital investment is very narrow. A model provides individuals with a formalized view of their economic environment and implies unique probabilities for all possible future events. Individuals have no fear of model misspecification.

In Eisenhauer & Suchy (2020), we address this shortcoming by formulating, implementing, and exploring an EKW model of robust human capital investment where individuals face risk within a model and ambiguity about the model (Arrow, 1951). Ambiguity arises as individuals do not know the true model and consider a whole set of models as reasonable descriptions of their economic environment. Individuals fear model misspecification and thus seek robust decisions, i.e., decisions that perform well over the whole range of models.

We incorporate methods from robust optimization (Ben-Tal et al., 2009; Rahimian & Mehrotra, 2019; Wiesemann et al., 2014) and robust Markov decision processes (Iyengar, 2005; Nilim & El Ghaoui, 2005) that allow to construct decision rules that explicitly take potential model misspecification into account.

6.2 Uncertainty quantification

There are numerous sources of uncertainty in the policy forecasts produced by a calibrated EKW model. The model is subject to misspecification, its numerical implementation introduces approximation error, the data is subject to measurement error, and the calibrated parameters remain partly uncertain. However, economists display incredible certitude as they disregard all uncertainty (Manski, 2013) in their forecasts.

In Eisenhauer, Gabler, Janys, & Mensinger (2020), we draw on a rich literature in other disciplines where a proper accounting of the uncertainty in forecasts from complex computational models is mandatory (Saltelli et al., 2004, 2008; Smith, 2014). However, uncertainty quantification for EKW models poses several unique challenges. They usually have a large number of uncertain and correlated parameters, and the quantity of interest is time-consuming to compute and a complex function of the model parameters. Using machine learning methods, we set up an emulator that approximates the full model but is fast to evaluate. We revisit the analysis of Keane & Wolpin (1994, 1997) to showcase our approach in a well-known and empirically-motivated setting and characterize the uncertainty in their key findings.

We construct our approximating emulator using recent advances in surrogate modeling (Forrester et al., 2008) and machine learning (Hastie et al., 2008; Murphy, 2012).

6.3 Model validation

The validation of computational models is a prerequisite for their use in other disciplines (Adams, 2012; Oberkampf & Roy, 2010). However, it is extremely rare in economics as drastic regime shifts are seldom available in observational data and costly to implement in large-scale experiments.

In Bhuller et al. (2018), we calibrate an EKW model on Norwegian population panel data with nearly career-long earnings histories. Due to the richness of the data, we can validate the model using a mandatory schooling reform. Our data includes substantial geographic variation in compulsory schooling across Norway between 1960 and 1975 as mandatory schooling increased from seven to nine years at different points in time across municipalities. We split our data into a calibration and validation sample. We only use pre-reform data in our calibration, forecast the effect of increasing mandatory schooling by two years, and compare our forecast with the post-reform outcome. Doing so allows us to assess our model's ability to extrapolate individual responses outside the support of our calibration data. We use the validated model to gain insights into the underlying economic mechanisms that generate the effects of the policy and forecast the effects of several policy alternatives.

6.4 Nonstandard expectations

When economists analyze individual decision-making through the lens of an EKW model, they impose rational expectations. The subjective beliefs about the future correspond to the objective transition probabilities induced by the model.

In Eisenhauer, Haan, et al. (2020), we relax this assumption. We analyze and quantify the effect of biased expectations about wage growth in part-time employment on life cycle wage profiles of female workers. We design specific survey questions for the German Socio-Economic Panel (Goebel et al., 2019) and elicit the expected wage trajectories for full-time and part-time employment directly. Thus, we can incorporate the belief elicitation directly in our life cycle model.

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