Eckstein-Keane-Wolpin models

An invitation for transdisciplinary collaboration

The OSE team

November 10, 2020

Open Source Economics

Computational modeling in economics

Motivation

- Facilitate academic rigor
- Study mechanisms
- Predict public policies

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Transdisciplinary in nature

- Economic model
- Mathematical framework
- Computational implementation

Eckstein–Keane–Wolpin models

Understanding individual decisions

- Human capital investment
- Consumption–savings decision

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- Backward induction algorithm

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Partners

Institute for
Numerical Simulation

Statistisk sentralbyrå
Statistics Norway

DIW BERLIN

Unil
UNIL | Université de Lausanne

Roadmap

- Economic model
- Mathematical formulation
- Calibration

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- Economic model
- Mathematical formulation
- Calibration
- Example
- Pipeline
- Projects

Economic model

Decision Problem

$t = 1, \ldots, T$	decision period
$s_t \in S$	state
$a_t \in A$	action
$a_t(s_t)$	decision rule
$u_t(s_t, a_t)$	immediate utility

Timing of events

Choose

Learn a_t Receive

$\{u(s_t, a_t)\}_{a_t \in \mathcal{A}}$ $u(s_t, a_t)$

t

Choose

Learn a_{t+1} Receive

$\{u(s_{t+1}, a_{t+1})\}_{a_{t+1} \in \mathcal{A}}$ $u(s_{t+1}, a_{t+1})$

$t+1$

$\pi = (a_1^\pi(s_1), \dots, a_T^\pi(s_T))$	policy
δ	discount factor
$p_t(s_t, a_t)$	conditional distribution

Individual's objective

$$\max_{\pi \in \Pi} \mathrm{E}_{s_1}^{\pi} \left[\sum_{t=1}^{T} \delta^{t-1} u_t(s_t, a_t^{\pi}(s_t)) \,\middle|\, \mathcal{I}_1 \right]$$

Core economics

- Rational expectations
- Exponential discounting
- Time-separability

Mathematical formulation

Policy evaluation

$$v_t^{\pi}(s_t) = \mathrm{E}_{s_t}^{\pi}\left[\sum_{j=0}^{T-t} \delta^j \, u_{t+j}(s_{t+j}, a_{t+j}^{\pi}(s_{t+j})) \;\middle|\; \mathcal{I}_t\right]$$

Inductive scheme

$$v_t^{\pi}(s_t) = u_t(s_t, a_t^{\pi}(s_t)) + \delta \, E_{s_t}^{\pi}\left[v_{t+1}^{\pi}(s_{t+1}) \mid \mathcal{I}_t\right]$$

Optimality equations

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \left\{ u_t(s_t, a_t) + \delta \, \mathrm{E}_{s_t}^{\pi^*} \left[v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \right] \right\}$$

Backward induction algorithm

for $t = T, \ldots, 1$ **do**

if t == T **then**

$$v_T^{\pi^*}(s_T) = \max_{a_T \in A} \Big\{ u_T(s_T, a_T) \Big\} \qquad \forall\, s_T \in S$$

else

Compute $v_t^{\pi^*}(s_t)$ for each $s_t \in S$ by

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \Big\{ u_t(s_t, a_t) + \delta\, \mathrm{E}_{s_t}^{\pi} \Big[v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \Big] \Big\}$$

and set

$$a_t^{\pi^*}(s_t) = \arg\max_{a_t \in A} \Big\{ u_t(s_t, a_t) + \delta\, \mathrm{E}_{s_t}^{\pi} \Big[v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \Big] \Big\}$$

end if

end for

Calibration procedure

Data

Dataset

$$\mathcal{D} = \{a_{it}, \bar{s}_{it}, \bar{u}_{it} : i = 1, \dots, N; t = 1, \dots, T_i\}$$

State variables

- $s_t = (\bar{s}_t, \varepsilon_t)$
  - $\bar{s}_t$ observed
  - ε_t unobserved

Procedures

- Likelihood-based

$$\hat{\vartheta} = \arg\max_{\vartheta \in \Theta} \prod_{i=1}^{N} \prod_{t=1}^{T_i} p_{it}(a_{it}, \bar{u}_{it} \mid \bar{s}_{it}, \vartheta)$$

- Simulation-based

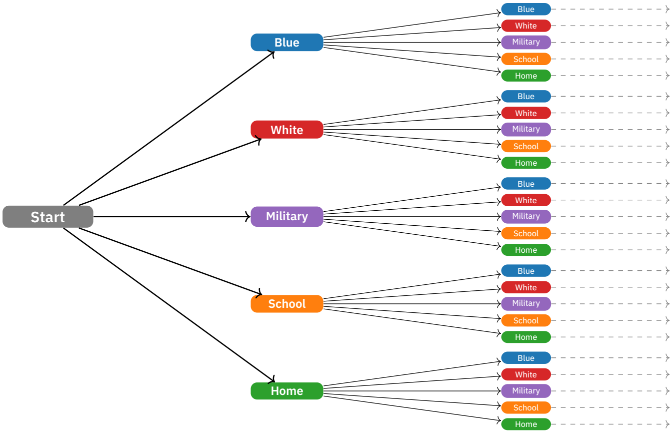
$$\hat{\vartheta} = \arg\min_{\vartheta \in \Theta} \left(M_D - M_S(\vartheta)\right)' W \left(M_D - M_S(\vartheta)\right)$$

Example

Seminal paper

- **Michael P Keane and Kenneth I Wolpin.** 1997. “The Career Decisions of Young Men.” *Journal of Political Economy* 105 (3): 473–522 .

Decision tree



Immediate utility

$$u_t(s_t) = \begin{cases} \zeta_a(s_t) + w_a(s_t) & \text{if } a \in \{B, W, M\} \\ \zeta_a(s_t) & \text{if } a \in \{S, H\} \end{cases}$$

Informed by reduced-form evidence

- Mincer equation
- Sheepskin effects
- Skill depreciation
- Mobility and search costs
- Monetary and psychic cost of schooling

Transitions

Work experience $\boldsymbol{k}_t$ and years of completed schooling h_t evolve deterministically.

$$\begin{aligned} k_{a,t+1} &= k_{a,t} + \mathbb{1}[a_t = a] \qquad \text{if } a \in \{B, W, M\} \\ h_{t+1} &= h_t \quad + \mathbb{1}[a_t = 4] \end{aligned}$$

Productivity shocks $\boldsymbol{e}_t$ are uncorrelated across time and follow a multivariate normal distribution with mean $\mathbf{0}$ and covariance matrix $\boldsymbol{\Sigma}$.

Given the structure of the utility functions and the distribution of the shocks, the state at time t is $s_t = \{\boldsymbol{k}_t, h_t, t, a_{t-1}, \boldsymbol{e}, \boldsymbol{\varepsilon}_t\}$.

Utility of blue-collar occupation

- Non-pecuniary

$$\begin{aligned}\zeta_1(\cdot) = \alpha_1 + c_{1,1} \cdot \mathbb{1}[a_{t-1} \neq 1] + c_{1,2} \cdot \mathbb{1}[k_{1,t} = 0] \\ + \vartheta_1 \cdot \mathbb{1}[h_t \geq 12] + \vartheta_2 \cdot \mathbb{1}[h_t \geq 16] + \vartheta_3 \cdot \mathbb{1}[k_{3,t} = 1]\end{aligned}$$

- Wage component

$$w_a(\cdot) = r_a \, x_a(\cdot),$$

where $x_a(\cdot)$ is the occupation-specific skill level.

Skill production for blue-collar occupation

$$x_1(\cdot) = \exp\left(\Gamma_1(\boldsymbol{k}_t, h_t, t, a_{t-1}, e_{j,1}) \cdot \varepsilon_{1,t}\right)$$

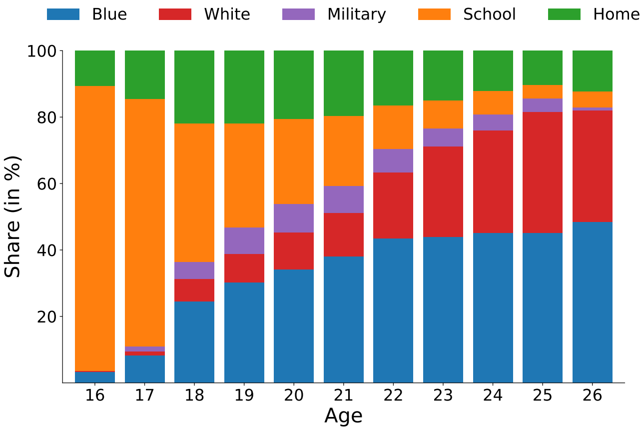
Parameterization of the deterministic component of the skill production function:

$$\begin{aligned}
\Gamma_1(\cdot) = e_{j,1} &+ \beta_{1,1} \cdot h_t + \beta_{1,2} \cdot \mathbb{1}[h_t \geq 12] + \beta_{1,3} \cdot \mathbb{1}[h_t \geq 16] \\
&+ \gamma_{1,1} \cdot k_{1,t} + \gamma_{1,2} \cdot (k_{1,t})^2 + \gamma_{1,3} \cdot \mathbb{1}[k_{1,t} > 0] \\
&+ \gamma_{1,4} \cdot t + \gamma_{1,5} \cdot \mathbb{1}[t < 18] \\
&+ \gamma_{1,6} \cdot \mathbb{1}[a_{t-1} = 1] + \gamma_{1,7} \cdot k_{2,t} + \gamma_{1,8} \cdot k_{3,t}
\end{aligned}$$

National Longitudinal Survey of Youth 1979

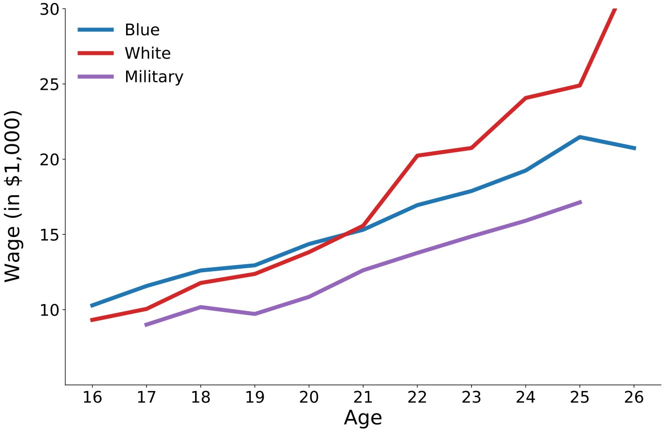
- 1,373 individuals starting at age 16
- Life cycle histories
  - School attendance
  - Occupation-specific work status
  - Wages

Choices

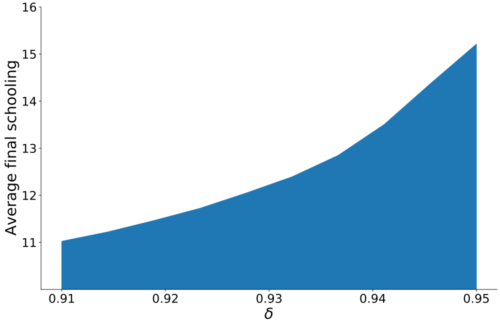


Blue
White
Military
School
Home
100
80
60
40
20
Share (in %)
16
17
18
19
20
21
22
23
24
25
26
Age

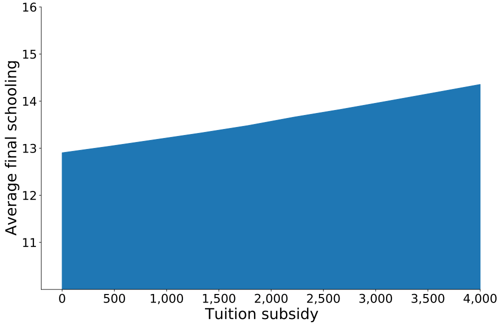
Average wages



Economic mechanism and policy forecast



(a) Time preference



(b) Tuition subsidy

Pipeline

Tooling

respy

GitHub	OpenSourceEconomics/respy
Docs	respy.readthedocs.io

estimagic

GitHub	OpenSourceEconomics/estimagic
Docs	estimagic.readthedocs.io

Workflow

```python
import respy as rp
from estimagic import maximize

# obtain model input
params, options, df = rp.get_example_model("kw_97_extended_respy")

# process model specification
log_like = rp.get_log_like_func(params, options, df)
simulate = rp.get_simulate_func(params, options)

# perform calibration
results, params_rslt = maximize(log_like, params, "nlopt_bobyqa")

# conduct analysis
df_rslt = simulate(params_rslt)
```

Model parameterization

category	name	value	name
delta	delta	9.370735e-01	delta_delta
wage_white_collar	constant	8.741888e+00	wage_white_collar_constant
	exp_school	6.548940e-02	wage_white_collar_exp_school
	exp_white_collar	1.763655e-02	wage_white_collar_exp_white_collar
	exp_white_collar_square	-4.215936e-02	wage_white_collar_exp_white_collar_square
	exp_blue_collar	3.431936e-02	wage_white_collar_exp_blue_collar
	exp_military	1.406945e-02	wage_white_collar_exp_military
	hs_graduate	-3.599855e-03	wage_white_collar_hs_graduate
	co_graduate	2.301313e-03	wage_white_collar_co_graduate
	period	9.577717e-03	wage_white_collar_period
	is_minor	-1.509984e-01	wage_white_collar_is_minor

Model options

	value
estimation_draws	200
estimation_seed	500
estimation_tau	500
interpolation_points	-1
n_periods	50
simulation_agents	5000
simulation_seed	132
solution_draws	500
solution_seed	456
monte_carlo_sequence	random
covariates	{'hs_graduate': 'exp_school >= 12', 'co_gradua...

Projects

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Improvements

- Numerical integration
- Global optimization
- Function approximation
- High-performance computing

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Extensions

- Robust decision making
- Uncertainty quantification
- Model validation
- Non-standard expectations

Conclusion

Join us!

http://bit.ly/ose-github

http://bit.ly/ose-zulip

https://twitter.com/open_econ

https://open-econ.org

Open Source Economics

respy

econsa

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