

Eckstein-Keane-Wolpin models

An invitation for transdisciplinary collaboration



Open Source
Economics

Computational modeling in economics

- ▶ provide learning opportunities
- ▶ assess importance of competing mechanisms
- ▶ predict the effects of public policies

Eckstein-Keane-Wolpin (EKW) models

- ▶ understanding individual decisions
 - ▶ human capital investment
 - ▶ savings and retirement
- ▶ predicting effects of policies
 - ▶ welfare programs
 - ▶ tax schedules

Transdisciplinary components

- ▶ economic model
- ▶ mathematical formulation
- ▶ computational implementation

Cooperations



Institute for
Numerical Simulation

Roadmap

- ▶ Setup
- ▶ Example
- ▶ Pipeline
- ▶ Improvements
- ▶ Extensions

Setup

Components

- ▶ economic model
- ▶ mathematical formulation
- ▶ calibration procedure

Economic model

Decision problem

$t = 1, \dots, T$ decision period

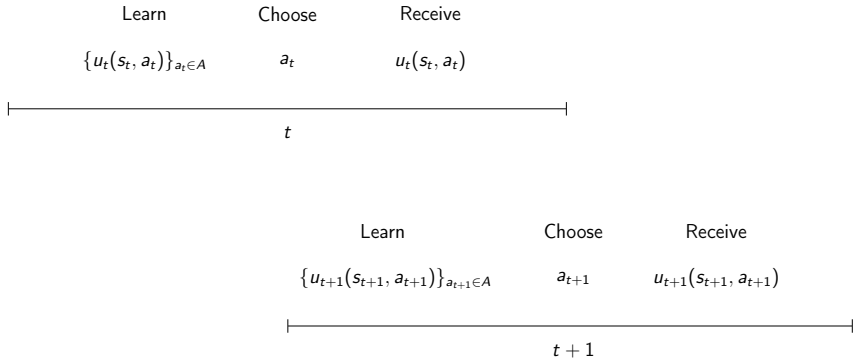
$s_t \in S$ state

$a_t \in A$ action

$a_t(s_t)$ decision rule

$u_t(s_t, a_t)$ immediate utility

Timing of events



$\pi = (a_1^\pi(s_1), \dots, a_T^\pi(s_T))$ policy

δ discount factor

$p_t(s_t, a_t)$ conditional distribution

Individual's objective

$$\max_{\pi \in \Pi} E_{s_1}^{\pi} \left[\sum_{t=1}^T \delta^{t-1} u_t(s_t, a_t^{\pi}(s_t)) \mid \mathcal{I}_1 \right]$$

Mathematical formulation

Policy evaluation

$$v_t^\pi(s_t) \equiv \mathbb{E}_{s_t}^\pi \left[\sum_{j=0}^{T-t} \delta^j u_{t+j}(s_{t+j}, a_{t+j}^\pi(s_{t+j})) \mid \mathcal{I}_t \right]$$

Inductive scheme

$$v_t^\pi(s_t) = u_t(s_t, a_t^\pi(s_t)) + \delta \mathbb{E}_{s_t}^\pi [v_{t+1}^\pi(s_{t+1}) \mid \mathcal{I}_t]$$

Optimality equations

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \left\{ u_t(s_t, a_t) + \delta \mathbb{E}_{s_t}^{\pi^*} \left[v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \right] \right\}$$

Backward induction algorithm

for $t = T, \dots, 1$ **do**

if $t == T$ **then**

$$v_T^{\pi^*}(s_T) = \max_{a_T \in A} \left\{ u_T(s_T, a_T) \right\} \quad \forall s_T \in S$$

else

 Compute $v_t^{\pi^*}(s_t)$ for each $s_t \in S$ by

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \left\{ u_t(s_t, a_t) + \delta \mathbb{E}_{s_t}^{\pi} \left[v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \right] \right\}$$

 and set

$$a_t^{\pi^*}(s_t) = \arg \max_{a_t \in A} \left\{ u_t(s_t, a_t) + \delta \mathbb{E}_{s_t}^{\pi} \left[v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \right] \right\}$$

end if

end for

Calibration procedure

Data

$$\mathcal{D} = \{a_{it}, \bar{s}_{it}, \bar{u}_{it} : i = 1, \dots, N; t = 1, \dots, T_i\}$$

State variables

- ▶ $s_t = (\bar{s}_t, \epsilon_t)$
 - ▶ \bar{s}_t observed
 - ▶ ϵ_t unobserved

Procedures

- ▶ likelihood-based

$$\hat{\theta} \equiv \arg \max_{\theta \in \Theta} \prod_{i=1}^N \prod_{t=1}^{T_i} p_{it}(a_{it}, \bar{u}_{it} \mid \bar{s}_{it}, \theta)$$

- ▶ simulation-based

$$\hat{\theta} \equiv \arg \min_{\theta \in \Theta} (M_D - M_S(\theta))' W (M_D - M_S(\theta))$$

Example

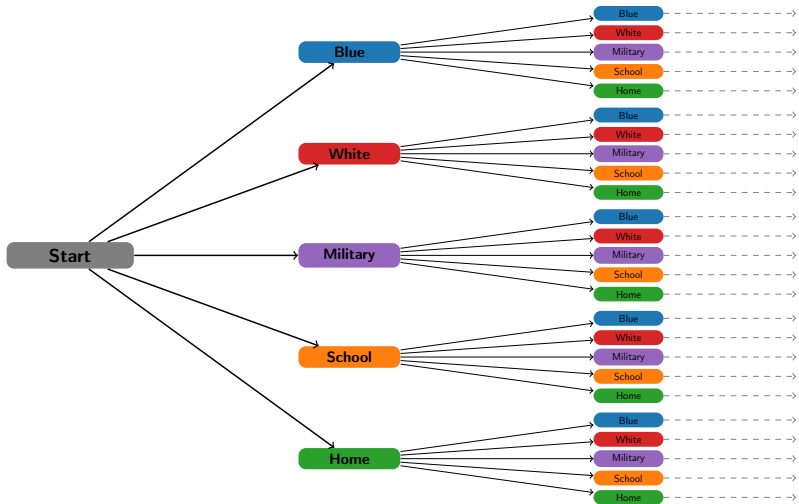
Seminal paper

- ▶ Keane, M. P. and Wolpin, K. I. (1997). The career decisions of young men. *Journal of Political Economy*, 105(3):473–522

Model of occupational choice

- ▶ life cycle histories
 - ▶ school attendance
 - ▶ occupation-specific work status
 - ▶ wages

Figure: Decision tree



Immediate utility

$$u(\cdot) = \begin{cases} \zeta_a(\cdot) + w_a(\cdot) & \text{if } a \in \{1, 2, 3\} \\ \zeta_a(\cdot) & \text{if } a \in \{4, 5\} \end{cases}$$

$\zeta_a(\cdot)$ non-pecuniary utility

$w_a(\cdot)$ wage component

Transitions

Work experience k_t and years of completed schooling h_t evolve deterministically.

$$\begin{aligned}k_{a,t+1} &= k_{a,t} + \mathbf{I}[a_t = a] && \text{if } a \in \{1, 2, 3\} \\h_{t+1} &= h_t + \mathbf{I}[a_t = 4]\end{aligned}$$

Productivity shocks \mathbf{e}_t are uncorrelated across time and follow a multivariate normal distribution with mean $\mathbf{0}$ and covariance matrix Σ .

Non-pecuniary utility of blue-collar occupation

$$\begin{aligned}\zeta_1(\cdot) = & \alpha_1 + c_{1,1} \cdot \mathbb{I}[a_{t-1} \neq 1] + c_{1,2} \cdot \mathbb{I}[k_{1,t} = 0] \\ & + \vartheta_1 \cdot \mathbb{I}[h_t \geq 12] + \vartheta_2 \cdot \mathbb{I}[h_t \geq 16] + \vartheta_3 \cdot \mathbb{I}[k_{3,t} = 1]\end{aligned}$$

Wage component

$$w_a(\cdot) = r_a x_a(\cdot)$$

with skill production function

$$x_1(\cdot) = \exp(\Gamma_1(\mathbf{k}_t, h_t, t, a_{t-1}, e_{j,1}) \cdot \epsilon_{1,t}).$$

Skill production for blue-collar occupation

$$\begin{aligned}\Gamma_1(\cdot) = & e_{j,1} + \beta_{1,1} \cdot h_t + \beta_{1,2} \cdot \mathbf{I}[h_t \geq 12] + \beta_{1,3} \cdot \mathbf{I}[h_t \geq 16] \\ & + \gamma_{1,1} \cdot k_{1,t} + \gamma_{1,2} \cdot (k_{1,t})^2 + \gamma_{1,3} \cdot \mathbf{I}[k_{1,t} > 0] \\ & + \gamma_{1,4} \cdot t + \gamma_{1,5} \cdot \mathbf{I}[t < 18] \\ & + \gamma_{1,6} \cdot \mathbf{I}[a_{t-1} = 1] + \gamma_{1,7} \cdot k_{2,t} + \gamma_{1,8} \cdot k_{3,t}\end{aligned}$$

Empirical data

National Longitudinal Survey of Youth 1979

- ▶ 1,373 individuals starting at age 16
- ▶ life cycle histories
 - ▶ school attendance
 - ▶ occupation-specific work status
 - ▶ wages

Figure: Choices

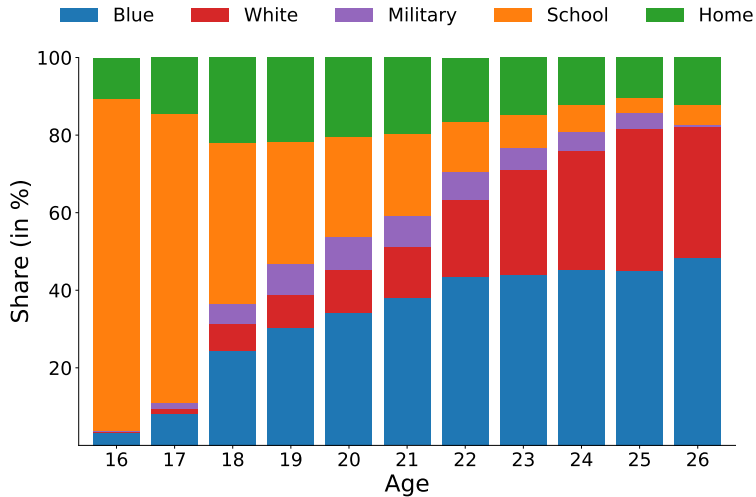
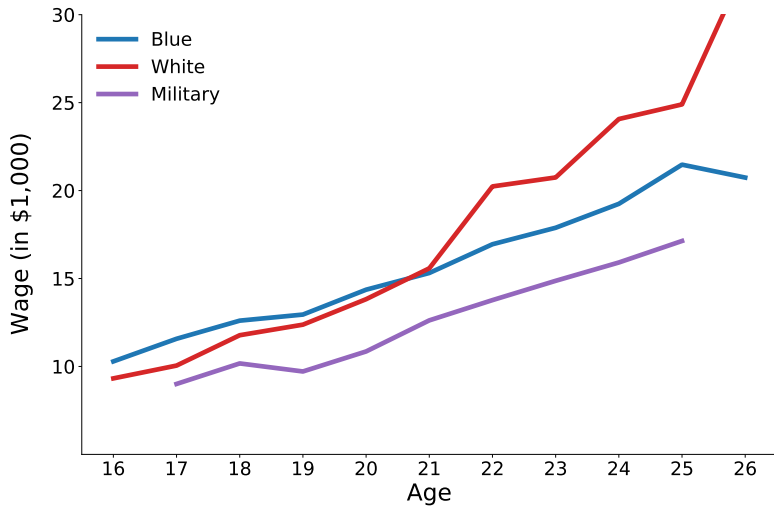
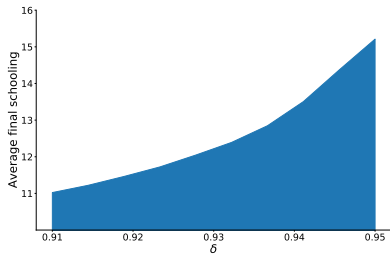


Figure: Average wage

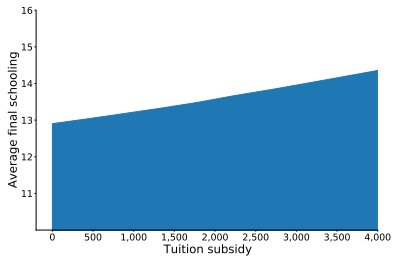


Economic insights

Figure: Economic mechanism and policy forecast



Time preference



Tuition subsidy

Pipeline

respy

GitHub [OpenSourceEconomics/respy](https://github.com/OpenSourceEconomics/respy)

Docs respy.readthedocs.io

estimagic

GitHub [OpenSourceEconomics/estimagic](https://github.com/OpenSourceEconomics/estimagic)

Docs estimagic.readthedocs.io

Figure: Typical workflow

```
import respy as rp
from estimagic import maximize

# obtain model input
params, options, df = rp.get_example_model("kw_97_extended_respy")

# process model specification
log_like = rp.get_log_like_func(params, options, df)
simulate = rp.get_simulate_func(params, options)

# perform calibration
results, params_rslt = maximize(log_like, params, "nlopt_bobyqa")

# conduct analysis
df_rslt = simulate(params_rslt)
```

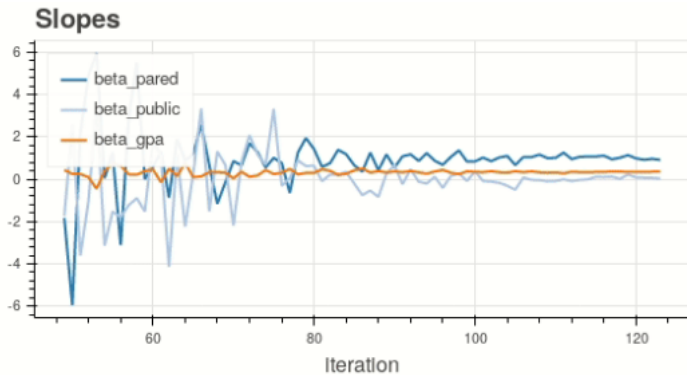
Figure: Model parameterization

		value	name
category	name		
delta	delta	9.370735e-01	delta_delta
wage_white-collar	constant	8.741888e+00	wage_white-collar_constant
	exp_school	6.548940e-02	wage_white-collar_exp_school
	exp_white-collar	1.763655e-02	wage_white-collar_exp_white-collar
	exp_white-collar_square	-4.215936e-02	wage_white-collar_exp_white-collar_square
	exp_blue-collar	3.431936e-02	wage_white-collar_exp_blue-collar
	exp_military	1.406945e-02	wage_white-collar_exp_military
	hs_graduate	-3.599855e-03	wage_white-collar_hs_graduate
	co_graduate	2.301313e-03	wage_white-collar_co_graduate
	period	9.577717e-03	wage_white-collar_period
	is_minor	-1.509984e-01	wage_white-collar_is_minor

Figure: Model options

	value
estimation_draws	200
estimation_seed	500
estimation_tau	500
interpolation_points	-1
n_periods	50
simulation_agents	5000
simulation_seed	132
solution_draws	500
solution_seed	456
monte_carlo_sequence	random
covariates	{'hs_graduate': 'exp_school >= 12', 'co_gradua...

Figure: Dashboard



Roadmap

Improvements

- ▶ numerical integration
- ▶ global optimization
- ▶ function approximation
- ▶ high-performance computing

Extensions

- ▶ robust decision-making
- ▶ uncertainty quantification
- ▶ model validation
- ▶ non-standard expectations

Join us!

GitHub <http://bit.ly/ose-github>

Meetup <http://bit.ly/ose-meetup>

Chat <http://bit.ly/ose-zulip>

Appendix

Content

- ▶ Contact
- ▶ References

Contact

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References

Keane, M. P. and Wolpin, K. I. (1997). The career decisions of young men.
Journal of Political Economy, 105(3):473–522.