Eckstein-Keane-Wolpin models

An invitation for transdisciplinary collaboration

Open Source Economics Bonn

Computational modeling in economics

- provide learning opportunities
- assess importance of competing mechanisms
- predict the effects of public policies

Eckstein-Keane-Wolpin (EKW) models

- understanding individual decisions
  - human capital investment
  - savings and retirement
- predicting effects of policies
  - welfare programs
  - tax schedules

Transdisciplinary components

- economic model
- mathematical formulation
- computational implementation

Cooperations

DIW BERLIN

Statistisk sentralbyrå
Statistics Norway

Institute for
Numerical Simulation

Roadmap

- Setup
- Example
- Improvements
- Extensions

Setup

Components

- economic model
- mathematical formulation
- calibration procedure

Economic model

Decision problem

$t = 1, \ldots, T$	decision period
$s_t \in S$	state
$a_t \in A$	action
$a_t(s_t)$	decision rule
$r_t(s_t, a_t)$	immediate reward

Timing of events

Learn	Choose	Receive
$\{r_t(s_t, a_t)\}_{a_t \in A}$	a_t	$r_t(s_t, a_t)$

t

Learn	Choose	Receive
$\{r_{t+1}(s_{t+1}, a_{t+1})\}_{a_{t+1} \in A}$	a_{t+1}	$r_{t+1}(s_{t+1}, a_{t+1})$

$t+1$

$\pi = (a_1^\pi(s_1), \ldots, a_T^\pi(s_T))$	policy
δ	discount factor
$p_t(s_t, a_t)$	conditional distribution

Individual's objective

$$\max_{\pi \in \Pi} \mathrm{E}_{s_1}^{\pi} \left[\sum_{t=1}^{T} \delta^{t-1} r_t(s_t, a_t^{\pi}(s_t)) \,\middle|\, \mathcal{I}_1 \right]$$

Mathematical formulation

Policy evaluation

$$v_t^\pi(s_t) \equiv \mathrm{E}_{s_t}^\pi \left[\sum_{j=0}^{T-t} \delta^j \, r_{t+j}(s_{t+j}, a_{t+j}^\pi(s_{t+j})) \,\middle|\, \mathcal{I}_t \right]$$

Inductive scheme

$$v_t^\pi(s_t) = r_t(s_t, a_t^\pi(s_t)) + \delta \, \mathrm{E}_{s_t}^\pi \left[v_{t+1}^\pi(s_{t+1}) \,\middle|\, \mathcal{I}_t \right]$$

Optimality equations

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \left\{ r_t(s_t, a_t) + \delta \, \mathrm{E}_{s_t}^{\pi^*} \left[v_{t+1}^{\pi^*}(s_{t+1}) \,\middle|\, \mathcal{I}_t \right] \right\}$$

Backward induction algorithm

for $t = T, \ldots, 1$ **do**

if t == T **then**

$$v_T^{\pi^*}(s_T) = \max_{a_T \in A} \Big\{ r_T(s_T, a_T) \Big\} \qquad \forall\, s_T \in S$$

else

Compute $v_t^{\pi^*}(s_t)$ for each $s_t \in S$ by

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \Big\{ r_t(s_t, a_t) + \delta\, \mathrm{E}_{s_t}^{\pi} \Big[v_{t+1}^{\pi^*}(s_{t+1}) \Big|\, \mathcal{I}_t \Big] \Big\}$$

and set

$$a_t^{\pi^*}(s_t) = \arg\max_{a_t \in A} \Big\{ r_t(s_t, a_t) + \delta\, \mathrm{E}_{s_t}^{\pi} \Big[v_{t+1}^{\pi^*}(s_{t+1}) \Big|\, \mathcal{I}_t \Big] \Big\}$$

end if

end for

Calibration procedure

Data

$$\mathcal{D} = \{a_{it}, x_{it}, r_{it} : i = 1, \ldots, N; t = 1, \ldots, T_i\}$$

State variables

- $s_t = (x_t, \epsilon_t)$
  - x_t observed
  - ϵ_t unobserved

Procedures

- likelihood-based

$$\hat{\theta} \equiv \arg\max_{\theta \in \Theta} \prod_{i=1}^{N} \prod_{t=1}^{T_i} p_{it}(a_{it}, r_{it} \mid x_{it}, \theta)$$

- simulation-based

$$\hat{\theta} \equiv \arg\min_{\theta \in \Theta} (M_D - M_S(\theta))' W (M_D - M_S(\theta))$$

Example

Seminal paper

- Keane, M. P. and Wolpin, K. I. (1994). The solution and estimation of discrete choice dynamic programming models by simulation and interpolation: Monte Carlo evidence. *Review of Economics and Statistics*, 76(4):648–672

Model of occupational choice

- 1,000 individuals starting at age 16
- life cycle histories
  - school attendance
  - occupation-specific work status
  - wages

Labor market

$$r_t(s_t, 1) = w_{1t} = \exp\{ \underbrace{\alpha_{10}}_{\text{endowment}} + \underbrace{\alpha_{11} g_t}_{\text{schooling}} + \underbrace{\alpha_{12} e_{1t} + \alpha_{13} e_{1t}^2}_{\text{own experience}}$$
$$+ \underbrace{\alpha_{14} e_{2t} + \alpha_{15} e_{2t}^2}_{\text{other experience}} + \underbrace{\epsilon_{1t}}_{\text{shock}} \}$$

Schooling

$$r_t(s_t, 3) = \underbrace{\beta_0}_{\text{taste}} - \underbrace{\beta_1 \mathbb{I}[g_t \geq 12]}_{\text{direct cost}} - \underbrace{\beta_2 \mathbb{I}[a_{t-1} \neq 3]}_{\text{reenrollment effort}} + \underbrace{\epsilon_{3t}}_{\text{shock}}$$

Home

$$r_t(s_t, 4) = \underbrace{\gamma_0}_{\text{taste}} + \underbrace{\epsilon_{4t}}_{\text{shock}}$$

State space

$$s_t = \{g_t, e_{1t}, e_{2t}, a_{t-1}, \epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t}, \epsilon_{4t}\}$$

Transitions

- observed state variables

$$
\begin{aligned}
e_{1,t+1} &= e_{1t} + \mathbb{I}[a_t = 1] \\
e_{2,t+1} &= e_{2t} + \mathbb{I}[a_t = 2] \\
g_{t+1} &= g_t \ \ + \mathbb{I}[a_t = 3]
\end{aligned}
$$

- unobserved state variables

$$\{\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t}, \epsilon_{4t}\} \sim N(0, \Sigma)$$

Figure: Choices over the life cycle

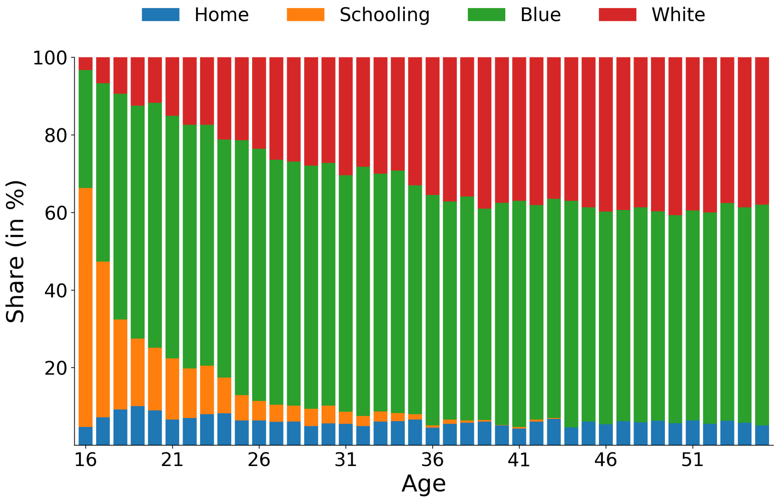
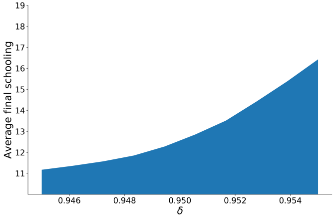
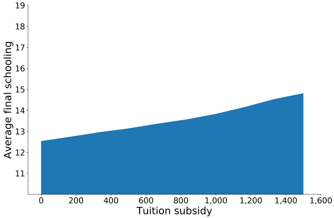


Figure: Economic mechanism and policy forecast



Time preference



Tuition subsidy

Research codes

respy

GitHub	OpenSourceEconomics/respy
Docs	respy.readthedocs.io

estimagic

GitHub	OpenSourceEconomics/estimagic
Docs	estimagic.readthedocs.io

Figure: Typical workflow

```
from estimagic.optimization.optimize import maximize
import respy as rp

# obtain model input
params, options, df = rp.get_example_model("kw_94_two")

# process model specification
crit_func = rp.get_crit_func(params, options, df)
simulate = rp.get_simulate_func(params, options)

# perform calibration
results, params_rslt = maximize(crit_func, params, "nlopt_bobyqa")

# conduct analysis
df_rslt = simulate(params_rslt)
```

Figure: Model specification

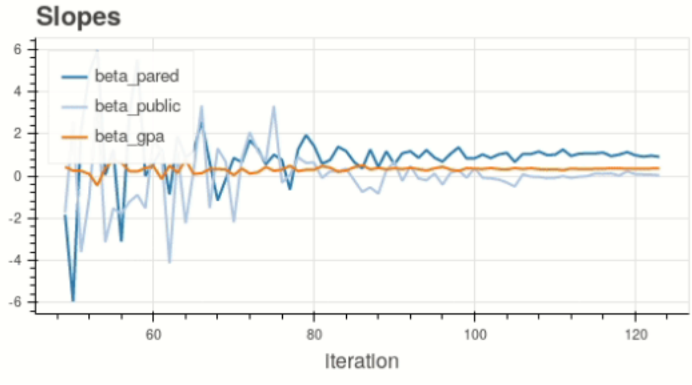
category	name	value	comment
delta	delta	0.9500	discount factor
wage_a	constant	9.2100	log of rental price
wage_a	exp_edu	0.0400	return to an additional year of schooling
wage_a	exp_a	0.0330	return to same sector experience
wage_a	exp_a_square	-0.0005	return to same sector, quadratic experience
wage_a	exp_b	0.0000	return to other sector experience
wage_a	exp_b_square	0.0000	return to other sector, quadratic experience
wage_b	constant	8.2000	log of rental price
wage_b	exp_edu	0.0800	return to an additional year of schooling
wage_b	exp_b	0.0670	return to same sector experience
wage_b	exp_b_square	-0.0010	return to same sector, quadratic experience
wage_b	exp_a	0.0220	return to other sector experience
wage_b	exp_a_square	-0.0005	return to other sector, quadratic experience
nonpec_edu	constant	5000.0000	constant reward for choosing education
nonpec_edu	at_least_twelve_exp_edu	-5000.0000	reward for going to college (tuition, etc.)

Parameterization

estimation_draws	200
estimation_seed	500
estimation_tau	500
interpolation_points	-1
n_periods	40
simulation_agents	1000
simulation_seed	132
solution_draws	500
solution_seed	456
monte_carlo_sequence	random
core_state_space_filters	[period > 0 and exp_{i} == period and lagged_c...
covariates.constant	1
covariates.exp_a_square	exp_a ** 2
covariates.exp_b_square	exp_b ** 2
covariates.at_least_twelve_exp_edu	exp_edu >= 12
covariates.not_edu_last_period	lagged_choice_1 != 'edu'

Options

Figure: Dashboard



Roadmap

Improvements

- numerical integration
- global optimization
- function approximation
- high-performance computing

Extensions

- robust decision-making
- uncertainty quantification
- model validation
- nonstandard expectations

Join us!

GitHub	`http://bit.ly/ose-github`
Meetup	`http://bit.ly/ose-meetup`
Chat	`http://bit.ly/ose-zulip`

Appendix

Content

Contact

Philipp Eisenhauer

Mail	peisenha@uni-bonn.de
Web	`http://eisenhauer.io`
GitHub	`https://github.com/peisenha`

References

Keane, M. P. and Wolpin, K. I. (1994). The solution and estimation of discrete choice dynamic programming models by simulation and interpolation: Monte Carlo evidence. *Review of Economics and Statistics*, 76(4):648–672.