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# Eckstein-Keane-Wolpin models

An invitation for transdisciplinary collaboration

The OSE team

November 13, 2020



Open Source  
Economics

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# Computational modeling in economics

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## Motivation

- Facilitate academic rigor
- Study mechanisms
- Predict public policies

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- Study mechanisms
- Predict public policies

## Transdisciplinary in nature

- Economic model
- Mathematical framework
- Computational implementation

# Eckstein–Keane–Wolpin models

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## Understanding individual decisions

- Human capital investment
- Consumption–savings decision

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- Welfare programs
- Tax schedules

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- Backward induction algorithm

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⇒ Transdisciplinary research on their **economics**, data, and computation

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# Partners

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Institute for  
Numerical Simulation



Statistisk sentralbyrå  
Statistics Norway



UNIL | Université de Lausanne

# Roadmap

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- Economic model
- Mathematical formulation
- Calibration

# Roadmap

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- Economic model
- Mathematical formulation
- Calibration
- Example
- Pipeline
- Projects

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## Economic model

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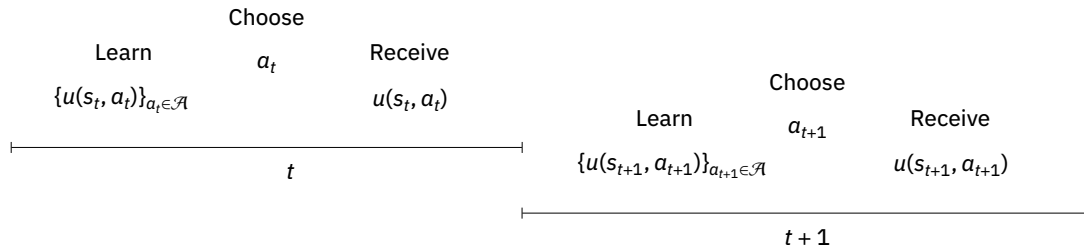
# Decision Problem

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$t = 1, \dots, T$	decision period
$s_t \in S$	state
$a_t \in A$	action
$a_t(s_t)$	decision rule
$u_t(s_t, a_t)$	immediate utility

## Timing of events

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$\pi = (a_1^\pi(s_1), \dots, a_T^\pi(s_T))$  policy

$\delta$  discount factor

$p_t(s_t, a_t)$  conditional distribution



$$\max_{\pi \in \Pi} E_{s_1}^{\pi} \left[ \sum_{t=1}^T \delta^{t-1} u_t(s_t, a_t^{\pi}(s_t)) \mid \mathcal{I}_1 \right]$$

### Core economics

- Rational expectations
- Exponential discounting
- Time-separability

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# Mathematical formulation

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$$v_t^\pi(s_t) = E_{s_t}^\pi \left[ \sum_{j=0}^{T-t} \delta^j u_{t+j}(s_{t+j}, a_{t+j}^\pi(s_{t+j})) \mid \mathcal{I}_t \right]$$

Inductive scheme

$$v_t^\pi(s_t) = u_t(s_t, a_t^\pi(s_t)) + \delta E_{s_t}^\pi [v_{t+1}^\pi(s_{t+1}) \mid \mathcal{I}_t]$$

## Optimality equations

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$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \left\{ u_t(s_t, a_t) + \delta \mathbb{E}_{S_t}^{\pi^*} \left[ v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \right] \right\}$$

## Backward induction algorithm

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**for**  $t = T, \dots, 1$  **do**

**if**  $t = T$  **then**

$$v_T^{\pi^*}(s_T) = \max_{a_T \in A} \left\{ u_T(s_T, a_T) \right\} \quad \forall s_T \in S$$

**else**

        Compute  $v_t^{\pi^*}(s_t)$  for each  $s_t \in S$  by

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \left\{ u_t(s_t, a_t) + \delta \mathbb{E}_{s_t}^{\pi} \left[ v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \right] \right\}$$

        and set

$$a_t^{\pi^*}(s_t) = \arg \max_{a_t \in A} \left\{ u_t(s_t, a_t) + \delta \mathbb{E}_{s_t}^{\pi} \left[ v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \right] \right\}$$

**end if**

**end for**

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## Calibration procedure

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## Dataset

$$\mathcal{D} = \{a_{it}, \bar{s}_{it}, \bar{u}_{it} : i = 1, \dots, N; t = 1, \dots, T_i\}$$

## State variables

- $s_t = (\bar{s}_t, \varepsilon_t)$ 
  - $\bar{s}_t$  observed
  - $\varepsilon_t$  unobserved

- Likelihood-based

$$\hat{\vartheta} = \arg \max_{\vartheta \in \Theta} \prod_{i=1}^N \prod_{t=1}^{T_i} p_{it}(a_{it}, \bar{u}_{it} \mid \bar{s}_{it}, \vartheta)$$

- Simulation-based

$$\hat{\vartheta} = \arg \min_{\vartheta \in \Theta} (M_D - M_S(\vartheta))' W (M_D - M_S(\vartheta))$$



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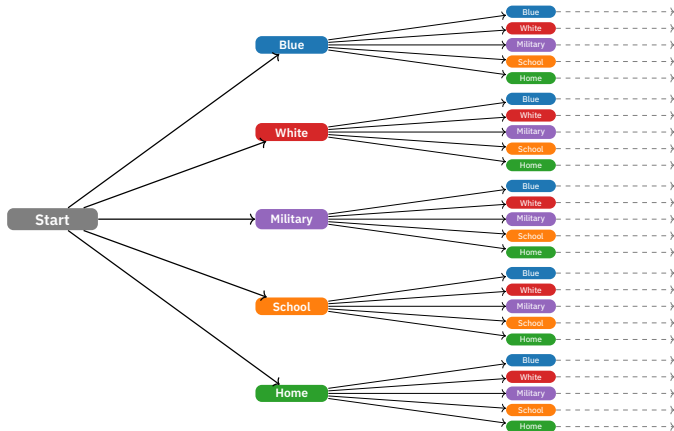
**Example**

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**Michael P Keane and Kenneth I Wolpin.** 1997. “The Career Decisions of Young Men.” *Journal of Political Economy* 105 (3): 473–522 .

- The study follows individuals over their working life from young adulthood at age 16 to retirement at age 65 where the decision period  $t = 16, \dots, 65$  is a school year.
- Individuals decide  $a \in \mathcal{A}$  whether to work in a blue-collar or white-collar occupation ( $a = 1, 2$ ), to serve in the military ( $a = 3$ ), to attend school ( $a = 4$ ), or to stay at home ( $a = 5$ ).

# Decision tree



### Informed by reduced-form evidence

$$u_t(s_t) = \begin{cases} \zeta_a(s_t) + w_a(s_t) & \text{if } a \in \{1, 2, 3\} \\ \zeta_a(s_t) & \text{if } a \in \{4, 5\} \end{cases}$$

- Mincer equation
- Sheepskin effects
- Skill depreciation
- Mobility and search costs
- Monetary and psychic cost of schooling

- Work experience  $k_t$  and years of completed schooling  $h_t$  evolve deterministically.

$$\begin{aligned}k_{a,t+1} &= k_{a,t} + 1[a_t = a] && \text{if } a \in \{1, 2, 3\} \\h_{t+1} &= h_t + 1[a_t = 4]\end{aligned}$$

- Productivity shocks  $\mathbf{e}_t$  are uncorrelated across time and follow a multivariate normal distribution with mean  $\mathbf{0}$  and covariance matrix  $\Sigma$ .
- Given the structure of the utility functions and the distribution of the shocks, the state at time  $t$  is  $s_t = \{k_t, h_t, t, a_{t-1}, \mathbf{e}, \boldsymbol{\varepsilon}_t\}$ .

## Utility of blue-collar occupation

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- Non-pecuniary

$$\begin{aligned}\zeta_1(\cdot) = & \alpha_1 + c_{1,1} \cdot 1[a_{t-1} \neq 1] + c_{1,2} \cdot 1[k_{1,t} = 0] \\ & + \vartheta_1 \cdot 1[h_t \geq 12] + \vartheta_2 \cdot 1[h_t \geq 16] + \vartheta_3 \cdot 1[k_{3,t} = 1]\end{aligned}$$

- Wage component

$$w_1(\cdot) = r_1 x_1(\cdot),$$

where  $x_1(\cdot)$  is the occupation-specific skill level.

$$x_1(\cdot) = \exp \left( \Gamma_1(\mathbf{k}_t, h_t, t, a_{t-1}, e_{j,1}) \cdot \varepsilon_{1,t} \right)$$

- Parameterization of the deterministic component of the skill production function:

$$\begin{aligned} \Gamma_1(\cdot) = & e_{j,1} + \beta_{1,1} \cdot h_t + \beta_{1,2} \cdot 1[h_t \geq 12] + \beta_{1,3} \cdot 1[h_t \geq 16] \\ & + \gamma_{1,1} \cdot k_{1,t} + \gamma_{1,2} \cdot (k_{1,t})^2 + \gamma_{1,3} \cdot 1[k_{1,t} > 0] \\ & + \gamma_{1,4} \cdot t + \gamma_{1,5} \cdot 1[t < 18] \\ & + \gamma_{1,6} \cdot 1[a_{t-1} = 1] + \gamma_{1,7} \cdot k_{2,t} + \gamma_{1,8} \cdot k_{3,t} \end{aligned}$$

# National Longitudinal Survey of Youth 1979

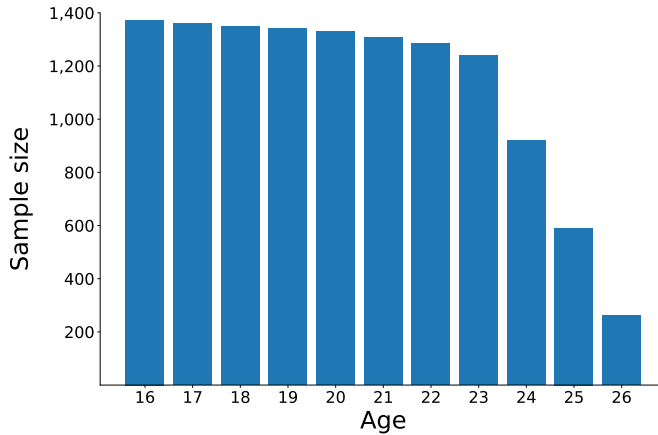
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- 1,373 individuals starting at age 16
- Life cycle histories
  - School attendance
  - Occupation-specific work status
  - Wages



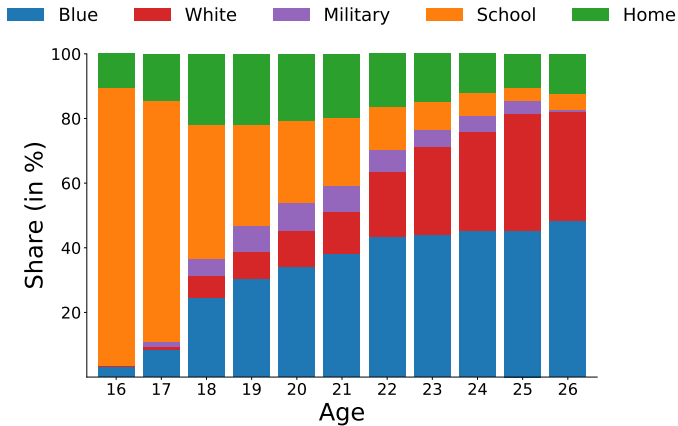
## Sample size

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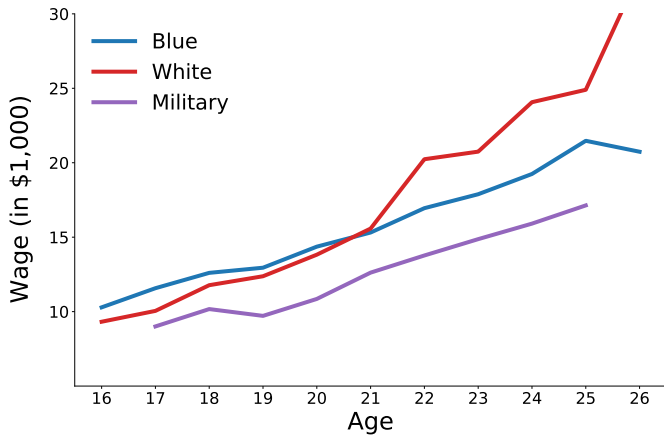
# Choices

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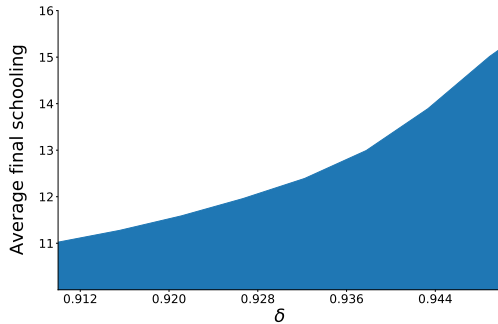


## Average wages

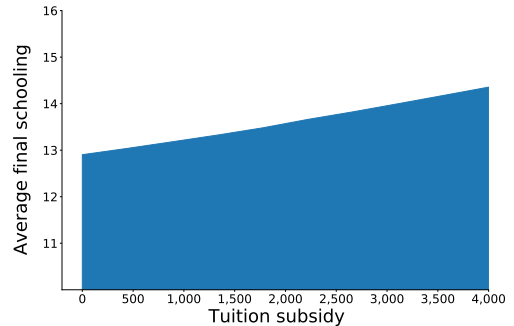
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## Economic mechanism and policy forecast



(a) Time preference



(b) Tuition subsidy

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# Pipeline

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### **respy**

GitHub [OpenSourceEconomics/respy](https://github.com/OpenSourceEconomics/respy)

Docs [respy.readthedocs.io](https://respy.readthedocs.io)

### **estimagic**

GitHub [OpenSourceEconomics/estimagic](https://github.com/OpenSourceEconomics/estimagic)

Docs [estimagic.readthedocs.io](https://estimagic.readthedocs.io)

## Workflow

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```
import respy as rp
from estimagic import maximize

# obtain model input
params, options, df = rp.get_example_model("kw_97_extended_respy")

# process model specification
log_like = rp.get_log_like_func(params, options, df)
simulate = rp.get_simulate_func(params, options)

# perform calibration
results, params_rslt = maximize(log_like, params, "nlopt_bobyqa")

# conduct analysis
df_rslt = simulate(params_rslt)
```

# Model parameterization

		value	name
category	name		
delta	delta	9.370735e-01	delta_delta
wage_white-collar	constant	8.741888e+00	wage_white-collar_constant
	exp_school	6.548940e-02	wage_white-collar_exp_school
	exp_white-collar	1.763655e-02	wage_white-collar_exp_white-collar
	exp_white-collar_square	-4.215936e-02	wage_white-collar_exp_white-collar_square
	exp_blue-collar	3.431936e-02	wage_white-collar_exp_blue-collar
	exp_military	1.406945e-02	wage_white-collar_exp_military
	hs_graduate	-3.599855e-03	wage_white-collar_hs_graduate
	co_graduate	2.301313e-03	wage_white-collar_co_graduate
	period	9.577717e-03	wage_white-collar_period
	is_minor	-1.509984e-01	wage_white-collar_is_minor



## Model options

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	value
estimation_draws	200
estimation_seed	500
estimation_tau	500
interpolation_points	-1
n_periods	50
simulation_agents	5000
simulation_seed	132
solution_draws	500
solution_seed	456
monte_carlo_sequence	random
covariates	{'hs_graduate': 'exp_school >= 12', 'co_gradua...

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# Projects

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## Improvements

- Numerical integration
- Global optimization
- Function approximation
- High-performance computing

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- Numerical integration
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## Extensions

- Robust decision making
- Uncertainty quantification
- Model validation
- Non-standard expectations

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## Conclusion

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## Join us!

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<http://bit.ly/ose-github>



<http://bit.ly/ose-zulip>



[https://twitter.com/open\\_econ](https://twitter.com/open_econ)



<https://open-econ.org>



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**respy**



**econsa**

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## References

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## References

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