

Eckstein-Keane-Wolpin models

An invitation for transdisciplinary collaboration



Open Source
Economics

Computational modeling in economics

- ▶ provide learning opportunities
- ▶ assess importance of competing mechanisms
- ▶ predict the effects of public policies

Eckstein-Keane-Wolpin (EKW) models

- ▶ understanding individual decisions
 - ▶ human capital investment
 - ▶ savings and retirement
- ▶ predicting effects of policies
 - ▶ welfare programs
 - ▶ tax schedules

Transdisciplinary components

- ▶ economic model
- ▶ mathematical formulation
- ▶ computational implementation

Cooperations



Institute for
Numerical Simulation

Roadmap

- ▶ Setup
- ▶ Example
- ▶ Improvements
- ▶ Extensions

Setup

Components

- ▶ economic model
- ▶ mathematical formulation
- ▶ calibration procedure

Economic model

Decision problem

$t = 1, \dots, T$ decision period

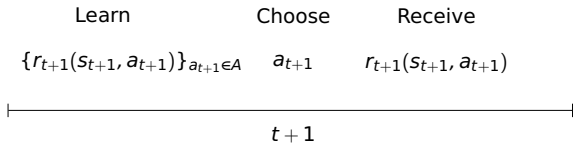
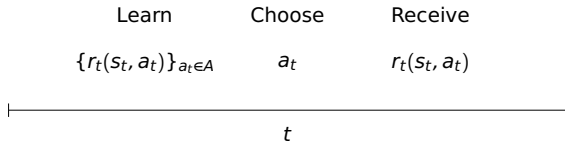
$s_t \in S$ state

$a_t \in A$ action

$a_t(s_t)$ decision rule

$r_t(s_t, a_t)$ immediate reward

Timing of events



$\pi = (a_1^\pi(s_1), \dots, a_T^\pi(s_T))$ policy

δ discount factor

$p_t(s_t, a_t)$ conditional distribution

Individual's objective

$$\max_{\pi \in \Pi} \mathbb{E}_{s_1}^{\pi} \left[\sum_{t=1}^T \delta^{t-1} r_t(s_t, a_t^{\pi}(s_t)) \mid \mathcal{I}_1 \right]$$

Mathematical formulation

Policy evaluation

$$v_t^\pi(s_t) \equiv \mathbb{E}_{s_t}^\pi \left[\sum_{j=0}^{T-t} \delta^j r_{t+j}(s_{t+j}, a_{t+j}^\pi(s_{t+j})) \mid \mathcal{I}_t \right]$$

Inductive scheme

$$v_t^\pi(s_t) = r_t(s_t, a_t^\pi(s_t)) + \delta \mathbb{E}_{s_t}^\pi \left[v_{t+1}^\pi(s_{t+1}) \mid \mathcal{I}_t \right]$$

Optimality equations

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \left\{ r_t(s_t, a_t) + \delta \mathbb{E}_{s_t}^{\pi^*} \left[v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \right] \right\}$$

Backward induction algorithm

for $t = T, \dots, 1$ **do**

if $t == T$ **then**

$$v_T^{\pi^*}(s_T) = \max_{a_T \in A} \left\{ r_T(s_T, a_T) \right\} \quad \forall s_T \in S$$

else

 Compute $v_t^{\pi^*}(s_t)$ for each $s_t \in S$ by

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \left\{ r_t(s_t, a_t) + \delta \mathbb{E}_{s_t}^{\pi} \left[v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \right] \right\}$$

 and set

$$a_t^{\pi^*}(s_t) = \arg \max_{a_t \in A} \left\{ r_t(s_t, a_t) + \delta \mathbb{E}_{s_t}^{\pi} \left[v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \right] \right\}$$

end if

end for

Calibration procedure

Data

$$\mathcal{D} = \{a_{it}, x_{it}, r_{it} : i = 1, \dots, N; t = 1, \dots, T_i\}$$

State variables

- ▶ $s_t = (x_t, \epsilon_t)$
 - ▶ x_t observed
 - ▶ ϵ_t unobserved

Procedures

- ▶ likelihood-based

$$\hat{\theta} \equiv \arg \max_{\theta \in \Theta} \prod_{i=1}^N \prod_{t=1}^{T_i} p_{it}(a_{it}, r_{it} \mid x_{it}, \theta)$$

- ▶ simulation-based

$$\hat{\theta} \equiv \arg \min_{\theta \in \Theta} (M_D - M_S(\theta))' W (M_D - M_S(\theta))$$

Example

Seminal paper

- ▶ Keane, M. P. and Wolpin, K. I. (1994). The solution and estimation of discrete choice dynamic programming models by simulation and interpolation: Monte Carlo evidence. *Review of Economics and Statistics*, 76(4):648–672

Model of occupational choice

- ▶ 1,000 individuals starting at age 16
- ▶ life cycle histories
 - ▶ school attendance
 - ▶ occupation-specific work status
 - ▶ wages

Labor market

$$r_t(s_t, 1) = w_{1t} = \exp\left\{ \underbrace{\alpha_{10}}_{\text{endowment}} + \underbrace{\alpha_{11}g_t}_{\text{schooling}} + \underbrace{\alpha_{12}e_{1t} + \alpha_{13}e_{1t}^2}_{\text{own experience}} \right. \\ \left. + \underbrace{\alpha_{14}e_{2t} + \alpha_{15}e_{2t}^2}_{\text{other experience}} + \underbrace{\epsilon_{1t}}_{\text{shock}} \right\}$$

Schooling

$$r_t(s_t, 3) = \underbrace{\beta_0}_{\text{taste}} - \underbrace{\beta_1 \mathbb{I}[g_t \geq 12]}_{\text{direct cost}} - \underbrace{\beta_2 \mathbb{I}[a_{t-1} \neq 3]}_{\text{reenrollment effort}} + \underbrace{\epsilon_{3t}}_{\text{shock}}$$

Home

$$r_t(s_t, 4) = \underbrace{\gamma_0}_{\text{taste}} + \underbrace{\epsilon_{4t}}_{\text{shock}}$$

State space

$$s_t = \{g_t, e_{1t}, e_{2t}, a_{t-1}, \epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t}, \epsilon_{4t}\}$$

Transitions

- ▶ observed state variables

$$e_{1,t+1} = e_{1t} + \mathbb{I}[a_t = 1]$$

$$e_{2,t+1} = e_{2t} + \mathbb{I}[a_t = 2]$$

$$g_{t+1} = g_t + \mathbb{I}[a_t = 3]$$

- ▶ unobserved state variables

$$\{\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t}, \epsilon_{4t}\} \sim N(0, \Sigma)$$

Figure: Choices over the life cycle

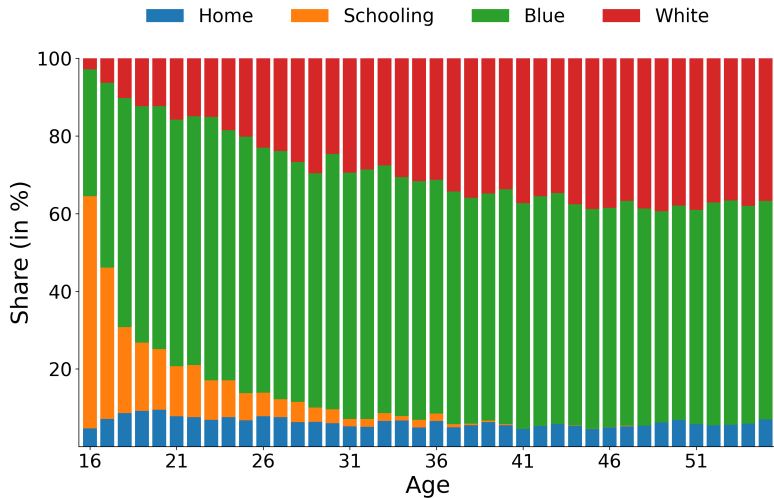
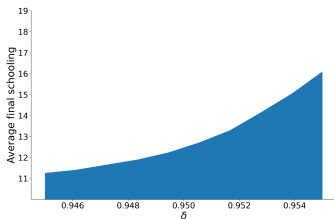
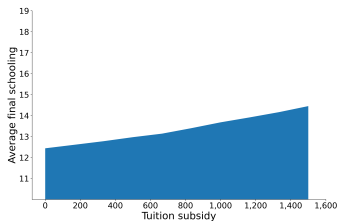


Figure: Economic mechanism and policy forecast



Time preference



Tuition subsidy

Research codes

respy

GitHub [OpenSourceEconomics/respy](https://github.com/OpenSourceEconomics/respy)

Docs respy.readthedocs.io

estimagic

GitHub [OpenSourceEconomics/estimagic](https://github.com/OpenSourceEconomics/estimagic)

Docs estimagic.readthedocs.io

Figure: Typical workflow

```
from estimagic.optimization.optimize import maximize
import respy as rp

# obtain model input
params, options, df = rp.get_example_model("kw_94_two")

# process model specification
crit_func = rp.get_crit_func(params, options, df)
simulate = rp.get_simulate_func(params, options)

# perform calibration
results, params_rslt = maximize(crit_func, params, "nlopt_bobyqa")

# conduct analysis
df_rslt = simulate(params_rslt)
```


Figure: Model specification

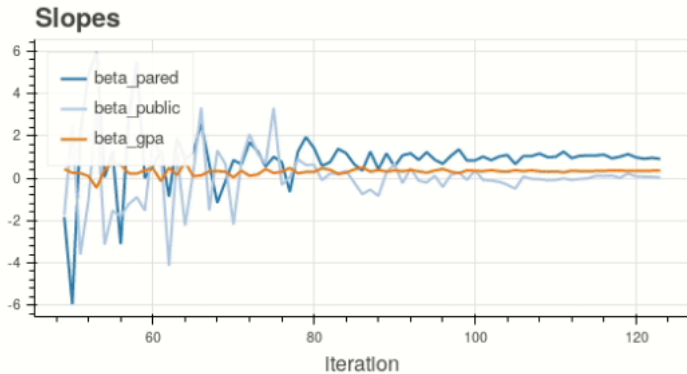
category	name	value	comment
delta	delta	0.9500	discount factor
wage_a	constant	9.2100	log of rental price
wage_a	exp_edu	0.0400	return to an additional year of schooling
wage_a	exp_a	0.0330	return to same sector experience
wage_a	exp_a_square	-0.0005	return to same sector, quadratic experience
wage_a	exp_b	0.0000	return to other sector experience
wage_a	exp_b_square	0.0000	return to other sector, quadratic experience
wage_b	constant	8.2000	log of rental price
wage_b	exp_edu	0.0800	return to an additional year of schooling
wage_b	exp_b	0.0670	return to same sector experience
wage_b	exp_b_square	-0.0010	return to same sector, quadratic experience
wage_b	exp_a	0.0220	return to other sector experience
wage_b	exp_a_square	-0.0005	return to other sector, quadratic experience
nonpec_edu	constant	5000.0000	constant reward for choosing education
nonpec_edu	at_least_twelve_exp_edu	-5000.0000	reward for going to college (tuition, etc.)

Parameterization

estimation_draws	200
estimation_seed	500
estimation_tau	500
interpolation_points	-1
n_periods	40
simulation_agents	1000
simulation_seed	132
solution_draws	500
solution_seed	456
monte_carlo_sequence	random
core_state_space_filters	[period > 0 and exp_{i}] == period and lagged_c...
covariates.constant	1
covariates.exp_a_square	exp_a ** 2
covariates.exp_b_square	exp_b ** 2
covariates.at_least_twelve_exp_edu	exp_edu >= 12
covariates.not_edu_last_period	lagged_choice_1 != 'edu'

Options

Figure: Dashboard



Roadmap

Improvements

- ▶ numerical integration
- ▶ global optimization
- ▶ function approximation
- ▶ high-performance computing

Extensions

- ▶ robust decision-making
- ▶ uncertainty quantification
- ▶ model validation
- ▶ nonstandard expectations

Join us!

GitHub <http://bit.ly/ose-github>

Meetup <http://bit.ly/ose-meetup>

Chat <http://bit.ly/ose-zulip>

Appendix

Content

- ▶ Contact
- ▶ References

Contact

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GitHub <https://github.com/peisenha>

References

Keane, M. P. and Wolpin, K. I. (1994). The solution and estimation of discrete choice dynamic programming models by simulation and interpolation: Monte Carlo evidence. *Review of Economics and Statistics*, 76(4):648–672.