Eckstein-Keane-Wolpin models

An invitation for transdisciplinary collaboration

Open Source Economics

Computational modeling in economics

- provide learning opportunities
- assess importance of competing mechanisms
- predict the effects of public policies

Eckstein-Keane-Wolpin (EKW) models

- understanding individual decisions
  - human capital investment
  - savings and retirement
- predicting effects of policies
  - welfare programs
  - tax schedules

Transdisciplinary components

- economic model
- mathematical formulation
- computational implementation

Cooperations

DIW BERLIN

Statistisk sentralbyrå
Statistics Norway

Institute for
Numerical Simulation

Roadmap

- Setup
- Example
- Pipeline
- Improvements
- Extensions

Setup

Components

- economic model
- mathematical formulation
- calibration procedure

Economic model

Decision problem

$t = 1, \ldots, T$	decision period
$s_t \in S$	state
$a_t \in A$	action
$a_t(s_t)$	decision rule
$u_t(s_t, a_t)$	immediate utility

Timing of events

Learn — $\{u_t(s_t, a_t)\}_{a_t \in A}$

Choose — a_t

Receive — $u_t(s_t, a_t)$

t

Learn — $\{u_{t+1}(s_{t+1}, a_{t+1})\}_{a_{t+1} \in A}$

Choose — a_{t+1}

Receive — $u_{t+1}(s_{t+1}, a_{t+1})$

$t+1$

$\pi = (a_1^\pi(s_1), \ldots, a_T^\pi(s_T))$	policy
δ	discount factor
$p_t(s_t, a_t)$	conditional distribution

Individual's objective

$$\max_{\pi \in \Pi} \mathrm{E}_{s_1}^{\pi} \left[\sum_{t=1}^{T} \delta^{t-1} u_t(s_t, a_t^{\pi}(s_t)) \,\middle|\, \mathcal{I}_1 \right]$$

Mathematical formulation

Policy evaluation

$$v_t^\pi(s_t) \equiv \mathrm{E}_{s_t}^\pi \left[\sum_{j=0}^{T-t} \delta^j \, u_{t+j}(s_{t+j}, a_{t+j}^\pi(s_{t+j})) \,\middle|\, \mathcal{I}_t \right]$$

Inductive scheme

$$v_t^\pi(s_t) = u_t(s_t, a_t^\pi(s_t)) + \delta \, \mathrm{E}_{s_t}^\pi \left[v_{t+1}^\pi(s_{t+1}) \,\middle|\, \mathcal{I}_t \right]$$

Optimality equations

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \left\{ u_t(s_t, a_t) + \delta \, \mathrm{E}_{s_t}^{\pi^*} \left[v_{t+1}^{\pi^*}(s_{t+1}) \,\middle|\, \mathcal{I}_t \right] \right\}$$

Backward induction algorithm

for $t = T, \ldots, 1$ **do**

  if t == T **then**

$$v_T^{\pi^*}(s_T) = \max_{a_T \in A} \Big\{ u_T(s_T, a_T) \Big\} \qquad \forall\, s_T \in S$$

  else

    Compute $v_t^{\pi^*}(s_t)$ for each $s_t \in S$ by

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \Big\{ u_t(s_t, a_t) + \delta\, \mathrm{E}_{s_t}^{\pi} \left[v_{t+1}^{\pi^*}(s_{t+1}) \middle| \, \mathcal{I}_t \right] \Big\}$$

    and set

$$a_t^{\pi^*}(s_t) = \arg\max_{a_t \in A} \Big\{ u_t(s_t, a_t) + \delta\, \mathrm{E}_{s_t}^{\pi} \left[v_{t+1}^{\pi^*}(s_{t+1}) \middle| \, \mathcal{I}_t \right] \Big\}$$

  end if

end for

Calibration procedure

Data

$$\mathcal{D} = \{a_{it}, \bar{s}_{it}, \bar{u}_{it} : i = 1, \ldots, N; t = 1, \ldots, T_i\}$$

State variables

- $s_t = (\bar{s}_t, \epsilon_t)$
  - $\bar{s}_t$ observed
  - ϵ_t unobserved

Procedures

- likelihood-based

$$\hat{\theta} \equiv \arg\max_{\theta \in \Theta} \prod_{i=1}^{N} \prod_{t=1}^{T_i} p_{it}(a_{it}, \bar{u}_{it} \mid \bar{s}_{it}, \theta)$$

- simulation-based

$$\hat{\theta} \equiv \arg\min_{\theta \in \Theta} (M_D - M_S(\theta))' W (M_D - M_S(\theta))$$

Example

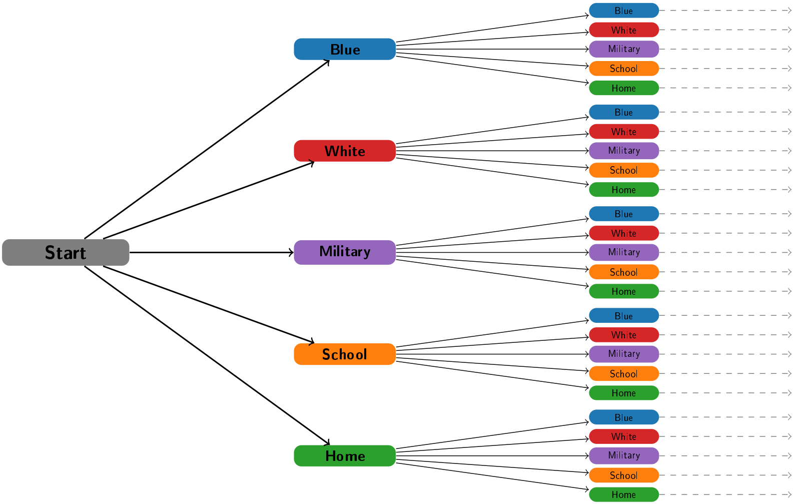
Seminal paper

- Keane, M. P. and Wolpin, K. I. (1997). The career decisions of young men. *Journal of Political Economy*, 105(3):473–522

Model of occupational choice

- life cycle histories
  - school attendance
  - occupation-specific work status
  - wages

Figure: Decision tree



Immediate utility

$$u(\cdot) = \begin{cases} \zeta_a(\cdot) + w_a(\cdot) & \text{if } a \in \{1, 2, 3\} \\ \zeta_a(\cdot) & \text{if } a \in \{4, 5\} \end{cases}$$

$\zeta_a(\cdot)$	non-pecuniary utility
$w_a(\cdot)$	wage component

Transitions

Work experience $\boldsymbol{k_t}$ and years of completed schooling h_t evolve deterministically.

$$
\begin{aligned}
k_{a,t+1} &= k_{a,t} + \mathbf{I}[a_t = a] \qquad \text{if } a \in \{1,2,3\} \\
h_{t+1} &= h_t \quad + \mathbf{I}[a_t = 4]
\end{aligned}
$$

Productivity shocks $\boldsymbol{e_t}$ are uncorrelated across time and follow a multivariate normal distribution with mean $\mathbf{0}$ and covariance matrix $\boldsymbol{\Sigma}$.

Non-pecuniary utility of blue-collar occupation

$$\begin{aligned}\zeta_1(\cdot) = \alpha_1 &+ c_{1,1} \cdot \mathbf{I}[a_{t-1} \neq 1] + c_{1,2} \cdot \mathbf{I}[k_{1,t} = 0] \\ &+ \vartheta_1 \cdot \mathbf{I}[h_t \geq 12] + \vartheta_2 \cdot \mathbf{I}[h_t \geq 16] + \vartheta_3 \cdot \mathbf{I}[k_{3,t} = 1]\end{aligned}$$

Wage component

$$w_a(\cdot) = r_a\, x_a(\cdot)$$

with skill production function

$$x_1(\cdot) = \exp\left(\Gamma_1(\boldsymbol{k_t}, h_t, t, a_{t-1}, e_{j,1}) \cdot \epsilon_{1,t}\right).$$

Skill production for blue-collar occupation

$$
\begin{aligned}
\Gamma_1(\cdot) = e_{j,1} &+ \beta_{1,1} \cdot h_t + \beta_{1,2} \cdot \mathbf{I}[h_t \geq 12] + \beta_{1,3} \cdot \mathbf{I}[h_t \geq 16] \\
&+ \gamma_{1,1} \cdot k_{1,t} + \gamma_{1,2} \cdot (k_{1,t})^2 + \gamma_{1,3} \cdot \mathbf{I}[k_{1,t} > 0] \\
&+ \gamma_{1,4} \cdot t + \gamma_{1,5} \cdot \mathbf{I}[t < 18] \\
&+ \gamma_{1,6} \cdot \mathbf{I}[a_{t-1} = 1] + \gamma_{1,7} \cdot k_{2,t} + \gamma_{1,8} \cdot k_{3,t}
\end{aligned}
$$

Empirical data

National Longitudinal Survey of Youth 1979

- 1,373 individuals starting at age 16
- life cycle histories
  - school attendance
  - occupation-specific work status
  - wages

Figure: Choices

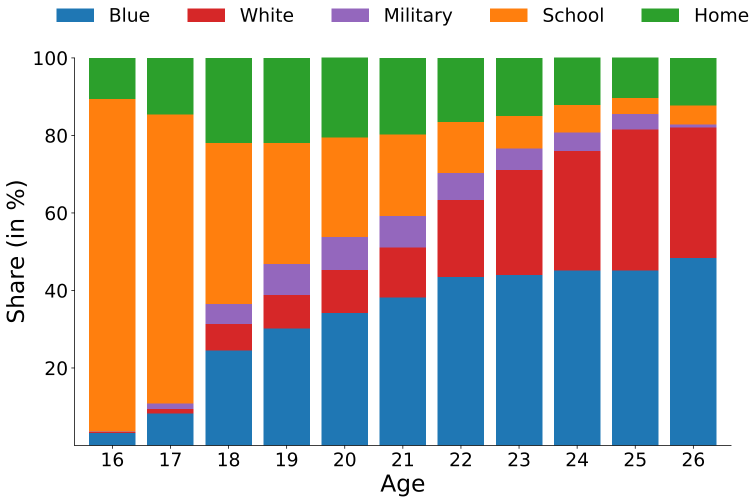
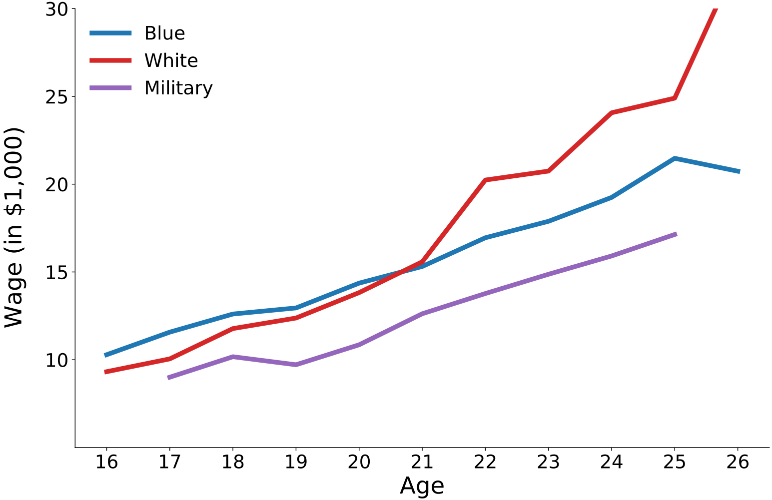
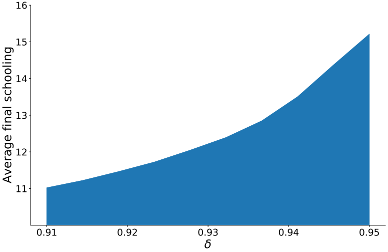


Figure: Average wage

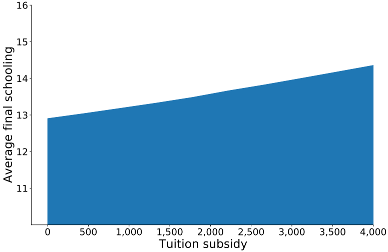


Economic insights

Figure: Economic mechanism and policy forecast



Time preference



Tuition subsidy

Pipeline

respy

GitHub	`OpenSourceEconomics/respy`
Docs	`respy.readthedocs.io`

estimagic

GitHub	`OpenSourceEconomics/estimagic`
Docs	`estimagic.readthedocs.io`

Figure: Typical workflow

```python
import respy as rp
from estimagic import maximize

# obtain model input
params, options, df = rp.get_example_model("kw_97_extended_respy")

# process model specification
log_like = rp.get_log_like_func(params, options, df)
simulate = rp.get_simulate_func(params, options)

# perform calibration
results, params_rslt = maximize(log_like, params, "nlopt_bobyqa")

# conduct analysis
df_rslt = simulate(params_rslt)
```

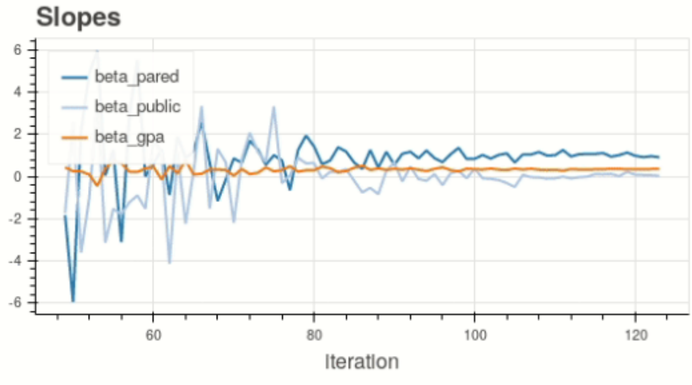
Figure: Model parameterization

category	name	value	name
delta	**delta**	9.370735e-01	delta_delta
wage_white_collar	**constant**	8.741888e+00	wage_white_collar_constant
	exp_school	6.548940e-02	wage_white_collar_exp_school
	exp_white_collar	1.763655e-02	wage_white_collar_exp_white_collar
	exp_white_collar_square	-4.215936e-02	wage_white_collar_exp_white_collar_square
	exp_blue_collar	3.431936e-02	wage_white_collar_exp_blue_collar
	exp_military	1.406945e-02	wage_white_collar_exp_military
	hs_graduate	-3.599855e-03	wage_white_collar_hs_graduate
	co_graduate	2.301313e-03	wage_white_collar_co_graduate
	period	9.577717e-03	wage_white_collar_period
	is_minor	-1.509984e-01	wage_white_collar_is_minor

Figure: Model options

	value
estimation_draws	200
estimation_seed	500
estimation_tau	500
interpolation_points	-1
n_periods	50
simulation_agents	5000
simulation_seed	132
solution_draws	500
solution_seed	456
monte_carlo_sequence	random
covariates	{'hs_graduate': 'exp_school >= 12', 'co_gradua...

Figure: Dashboard



Roadmap

Improvements

- numerical integration
- global optimization
- function approximation
- high-performance computing

Extensions

- robust decision-making
- uncertainty quantification
- model validation
- non-standard expectations

Join us!

GitHub	http://bit.ly/ose-github
Meetup	http://bit.ly/ose-meetup
Chat	http://bit.ly/ose-zulip

Appendix

Content

Contact

Philipp Eisenhauer

Mail	peisenha@uni-bonn.de
Web	http://peisenha.github.io
GitHub	https://github.com/peisenha

References

Keane, M. P. and Wolpin, K. I. (1997). The career decisions of young men. *Journal of Political Economy*, 105(3):473–522.