Eckstein-Keane-Wolpin models

An invitation for transdisciplinary collaboration

The OSE team

November 10, 2020

Open Source
Economics

Computational modeling in economics

Motivation

- Facilitate academic rigor
- Study mechanisms
- Predict public policies

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Transdisciplinary in nature

- Economic model
- Mathematical framework
- Computational implementation

Understanding individual decisions

- Human capital investment
- Consumption–savings decision

Eckstein–Keane–Wolpin models

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Predicting effects of policies

- Welfare programs
- Tax schedules

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- Finite-horizon discrete Markov decision problem
- Backward induction algorithm

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Partners

Institute for
Numerical Simulation

DIW BERLIN

Statistisk sentralbyrå
Statistics Norway

UNIL | Université de Lausanne

Roadmap

- Economic model
- Mathematical formulation
- Calibration

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- Economic model
- Mathematical formulation
- Calibration
- Example
- Pipeline
- Projects

Economic model

Decision Problem

$t = 1, \ldots, T$	decision period
$s_t \in S$	state
$a_t \in A$	action
$a_t(s_t)$	decision rule
$u_t(s_t, a_t)$	immediate utility

Timing of events

Choose

Learn

a_t

Receive

$\{u(s_t, a_t)\}_{a_t \in \mathcal{A}}$

$u(s_t, a_t)$

t

Choose

Learn

a_{t+1}

Receive

$\{u(s_{t+1}, a_{t+1})\}_{a_{t+1} \in \mathcal{A}}$

$u(s_{t+1}, a_{t+1})$

$t + 1$

$\pi = (a_1^\pi(s_1), \ldots, a_T^\pi(s_T))$	policy
δ	discount factor
$p_t(s_t, a_t)$	conditional distribution

Individual's objective

$$\max_{\pi \in \Pi} \mathrm{E}_{s_1}^{\pi} \left[\sum_{t=1}^{T} \delta^{t-1} u_t(s_t, a_t^{\pi}(s_t)) \,\middle|\, \mathcal{I}_1 \right]$$

Core economics

- Rational expectations
- Exponential discounting
- Time-separability

Mathematical formulation

Policy evaluation

$$v_t^\pi(s_t) = \mathrm{E}_{s_t}^\pi\left[\sum_{j=0}^{T-t} \delta^j\, u_{t+j}(s_{t+j}, a_{t+j}^\pi(s_{t+j})) \,\middle|\, \mathcal{I}_t\right]$$

Inductive scheme

$$v_t^\pi(s_t) = u_t(s_t, a_t^\pi(s_t)) + \delta\, E_{s_t}^\pi\left[v_{t+1}^\pi(s_{t+1}) \mid \mathcal{I}_t\right]$$

Optimality equations

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \left\{ u_t(s_t, a_t) + \delta \, \mathrm{E}_{s_t}^{\pi^*} \left[v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \right] \right\}$$

Backward induction algorithm

for $t = T, \ldots, 1$ **do**

if t == T **then**

$$v_T^{\pi^*}(s_T) = \max_{a_T \in A} \left\{ u_T(s_T, a_T) \right\} \qquad \forall\, s_T \in S$$

else

Compute $v_t^{\pi^*}(s_t)$ for each $s_t \in S$ by

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \left\{ u_t(s_t, a_t) + \delta\, \mathrm{E}_{s_t}^{\pi} \left[v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \right] \right\}$$

and set

$$a_t^{\pi^*}(s_t) = \arg\max_{a_t \in A} \left\{ u_t(s_t, a_t) + \delta\, \mathrm{E}_{s_t}^{\pi} \left[v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \right] \right\}$$

end if

end for

Calibration procedure

Data

Dataset

$$\mathcal{D} = \{a_{it}, \bar{s}_{it}, \bar{u}_{it} : i = 1, \dots, N; t = 1, \dots, T_i\}$$

State variables

- $s_t = (\bar{s}_t, \varepsilon_t)$
  - $\bar{s}_t$ observed
  - ε_t unobserved

Procedures

- Likelihood-based

$$\hat{\vartheta} = \arg\max_{\vartheta \in \Theta} \prod_{i=1}^{N} \prod_{t=1}^{T_i} p_{it}(a_{it}, \bar{u}_{it} \mid \bar{s}_{it}, \vartheta)$$

- Simulation-based

$$\hat{\vartheta} = \arg\min_{\vartheta \in \Theta} \big(M_D - M_S(\vartheta)\big)' W \big(M_D - M_S(\vartheta)\big)$$

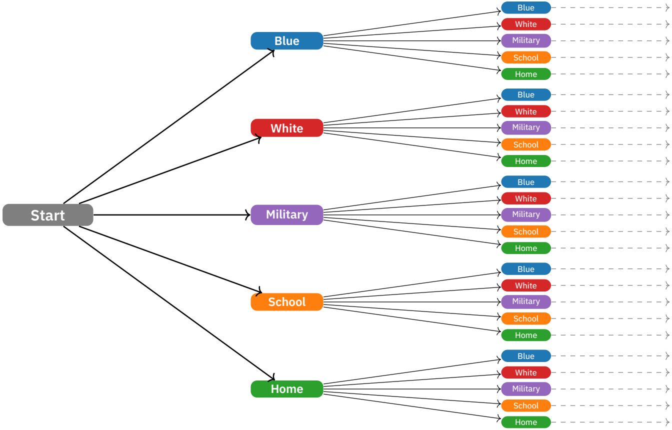
Example

Seminal paper

Michael P Keane and Kenneth I Wolpin. 1997. “The Career Decisions of Young Men.” *Journal of Political Economy* 105 (3): 473–522 .

- They follow individuals over their working life from young adulthood at age 16 to retirement at age 65 where the decision period $t = 16, \ldots, 65$ is a school year. Individuals decide $a \in \mathcal{A}$ whether to work in a blue-collar or white-collar occupation ($a = 1, 2$), to serve in the military ($a = 3$), to attend school ($a = 4$), or to stay at home ($a = 5$).

Decision tree



Immediate utility

$$u_t(s_t) = \begin{cases} \zeta_a(s_t) + w_a(s_t) & \text{if } a \in \{1, 2, 3\} \\ \zeta_a(s_t) & \text{if } a \in \{4, 5\} \end{cases}$$

Informed by reduced-form evidence

- Mincer equation
- Sheepskin effects
- Skill depreciation
- Mobility and search costs
- Monetary and psychic cost of schooling

Transitions

Work experience $\boldsymbol{k}_t$ and years of completed schooling h_t evolve deterministically.

$$\begin{aligned} k_{a,t+1} &= k_{a,t} + \mathbb{1}[a_t = a] \qquad \text{if } a \in \{1,2,3\} \\ h_{t+1} &= h_t \quad + \mathbb{1}[a_t = 4] \end{aligned}$$

Productivity shocks $\boldsymbol{e}_t$ are uncorrelated across time and follow a multivariate normal distribution with mean $\mathbf{0}$ and covariance matrix $\boldsymbol{\Sigma}$.

Given the structure of the utility functions and the distribution of the shocks, the state at time t is $s_t = \{\boldsymbol{k}_t, h_t, t, a_{t-1}, \boldsymbol{e}, \boldsymbol{\varepsilon}_t\}$.

Utility of blue-collar occupation

- Non-pecuniary

$$\zeta_1(\cdot) = \alpha_1 + c_{1,1} \cdot \mathbb{1}[a_{t-1} \neq 1] + c_{1,2} \cdot \mathbb{1}[k_{1,t} = 0] \\ + \vartheta_1 \cdot \mathbb{1}[h_t \geq 12] + \vartheta_2 \cdot \mathbb{1}[h_t \geq 16] + \vartheta_3 \cdot \mathbb{1}[k_{3,t} = 1]$$

- Wage component

$$w_1(\cdot) = r_1\, x_1(\cdot),$$

where $x_1(\cdot)$ is the occupation-specific skill level.

Skill production for blue-collar occupation

$$x_1(\cdot) = \exp\left(\Gamma_1(\boldsymbol{k}_t, h_t, t, a_{t-1}, e_{j,1}) \cdot \varepsilon_{1,t}\right)$$

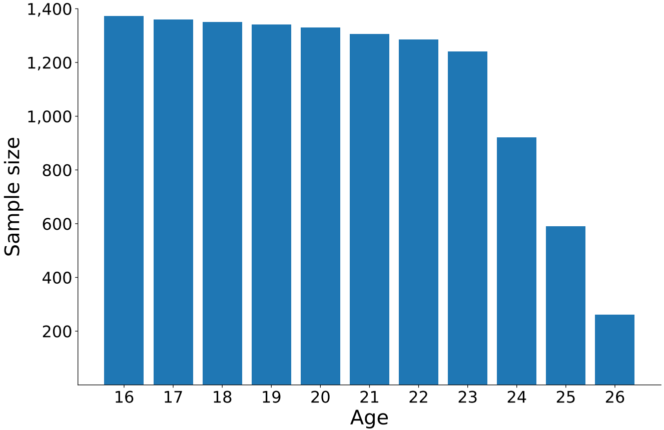
Parameterization of the deterministic component of the skill production function:

$$\begin{aligned}\Gamma_1(\cdot) = e_{j,1} &+ \beta_{1,1} \cdot h_t + \beta_{1,2} \cdot 1[h_t \geq 12] + \beta_{1,3} \cdot 1[h_t \geq 16] \\ &+ \gamma_{1,1} \cdot k_{1,t} + \gamma_{1,2} \cdot (k_{1,t})^2 + \gamma_{1,3} \cdot 1[k_{1,t} > 0] \\ &+ \gamma_{1,4} \cdot t + \gamma_{1,5} \cdot 1[t < 18] \\ &+ \gamma_{1,6} \cdot 1[a_{t-1} = 1] + \gamma_{1,7} \cdot k_{2,t} + \gamma_{1,8} \cdot k_{3,t}\end{aligned}$$

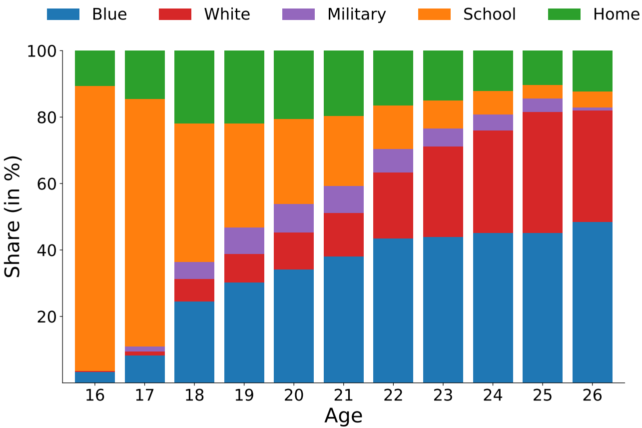
National Longitudinal Survey of Youth 1979

- 1,373 individuals starting at age 16
- Life cycle histories
  - School attendance
  - Occupation-specific work status
  - Wages

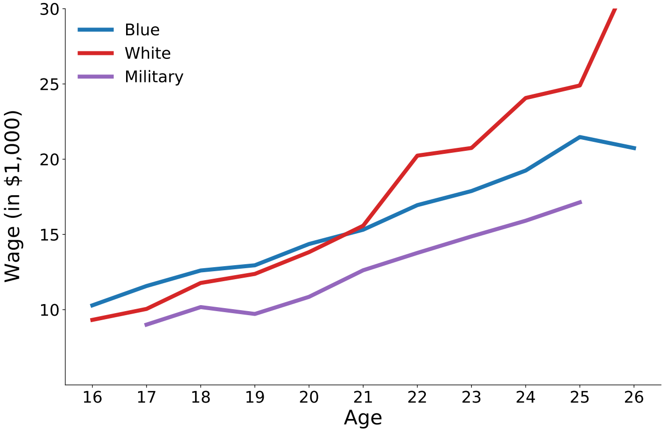
Sample size


1,400
1,200
1,000
800
600
400
200
Sample size
16
17
18
19
20
21
22
23
24
25
26
Age

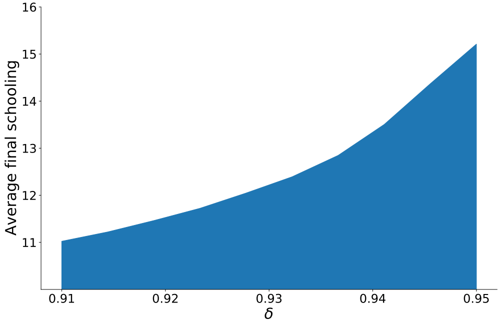
Choices


Blue
White
Military
School
Home
Share (in %)
100
80
60
40
20
16
17
18
19
20
21
22
23
24
25
26
Age

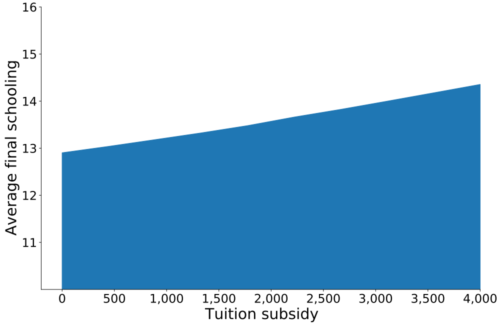
Average wages



Economic mechanism and policy forecast



(a) Time preference



(b) Tuition subsidy

Pipeline

Tooling

respy

GitHub	OpenSourceEconomics/respy
Docs	respy.readthedocs.io

estimagic

GitHub	OpenSourceEconomics/estimagic
Docs	estimagic.readthedocs.io

Workflow

```
import respy as rp
from estimagic import maximize

# obtain model input
params, options, df = rp.get_example_model("kw_97_extended_respy")

# process model specification
log_like = rp.get_log_like_func(params, options, df)
simulate = rp.get_simulate_func(params, options)

# perform calibration
results, params_rslt = maximize(log_like, params, "nlopt_bobyqa")

# conduct analysis
df_rslt = simulate(params_rslt)
```

Model parameterization

category	name	value	name
delta	delta	9.370735e-01	delta_delta
wage_white_collar	constant	8.741888e+00	wage_white_collar_constant
	exp_school	6.548940e-02	wage_white_collar_exp_school
	exp_white_collar	1.763655e-02	wage_white_collar_exp_white_collar
	exp_white_collar_square	-4.215936e-02	wage_white_collar_exp_white_collar_square
	exp_blue_collar	3.431936e-02	wage_white_collar_exp_blue_collar
	exp_military	1.406945e-02	wage_white_collar_exp_military
	hs_graduate	-3.599855e-03	wage_white_collar_hs_graduate
	co_graduate	2.301313e-03	wage_white_collar_co_graduate
	period	9.577717e-03	wage_white_collar_period
	is_minor	-1.509984e-01	wage_white_collar_is_minor

Model options

	value
estimation_draws	200
estimation_seed	500
estimation_tau	500
interpolation_points	-1
n_periods	50
simulation_agents	5000
simulation_seed	132
solution_draws	500
solution_seed	456
monte_carlo_sequence	random
covariates	{'hs_graduate': 'exp_school >= 12', 'co_gradua...

Projects

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Improvements

- Numerical integration
- Global optimization
- Function approximation
- High-performance computing

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Extensions

- Robust decision making
- Uncertainty quantification
- Model validation
- Non-standard expectations

Conclusion

Join us!

http://bit.ly/ose-github

ZULIP http://bit.ly/ose-zulip

https://twitter.com/open_econ

http:// https://open-econ.org

Open Source Economics

respy

econsa

References

References

Aguirregabiria, Victor, and Pedro Mira. 2010. “Dynamic Discrete Choice Structural Models: A survey.” *Journal of Econometrics* 156 (1): 38–67.

Becker, Gary S. 1964. *Human Capital*. New York City, NY: Columbia University Press.

Bellman, Richard E. 1954. “The theory of dynamic programming.” *Bulletin of the American Mathematical Society* 60 (6): 503–15.

Keane, Michael P, and Kenneth I Wolpin. 1997. “The Career Decisions of Young Men.” *Journal of Political Economy* 105 (3): 473–522. [PDF p. 26]

Keane, Micheal P, and Kenneth I Wolpin. 1994. “The Solution and Estimation of Discrete Choice Dynamic Programming Models by Simulation and Interpolation: Monte Carlo Evidence.” *Review of Economics and Statistics* 76 (4): 648–72.

Puterman, Martin L. 1994. *Markov Decision Processes: Discrete Stochastic Dynamic Programming*. New York City, NY: John Wiley & Sons.

White, D. J. 1993. *Markov Decision Processes*. New York City, NY: John Wiley & Sons.

Wolpin, Kenneth I. 2013. *The Limits to Inference without Theory*. Cambridge, MA: MIT University Press.