Eckstein-Keane-Wolpin models

OpenSourceEconomics*

Abstract

We present background material for a particular class structural economic models to facilitate transdiciplinary collaboration in their future development. We describe the economic setup, mathematical formulation, and calibration procedures for so-called Eckstein-Keane-Wolpin (EKW) models. We provide an example application using our group's research code `respy`. We draw on research outside economics to identify model components ripe for improvement and explore possible extensions.

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Contents

1 Introduction

Focus of handout We present background material for a particular class structural economic models to facilitate transdiciplinary collaboration in their future development. We describe the economic setup, mathematical formulation, and calibration procedures for the so-called Eckstein-Keane-Wolpin (EKW) models (**?**). We provide an example application using our group's research code `respy` (**?**). We draw on research outside economics to identify model components ripe for improvement and explore possible extensions.

Structural models Structural economic models clearly specify an individual's objective and the constraints of their economic environment under which they operate. They are used to quantify the importance of competing economic mechanisms in determining economic outcomes and evaluate the effects of alternative policies before their implementation (**?**).

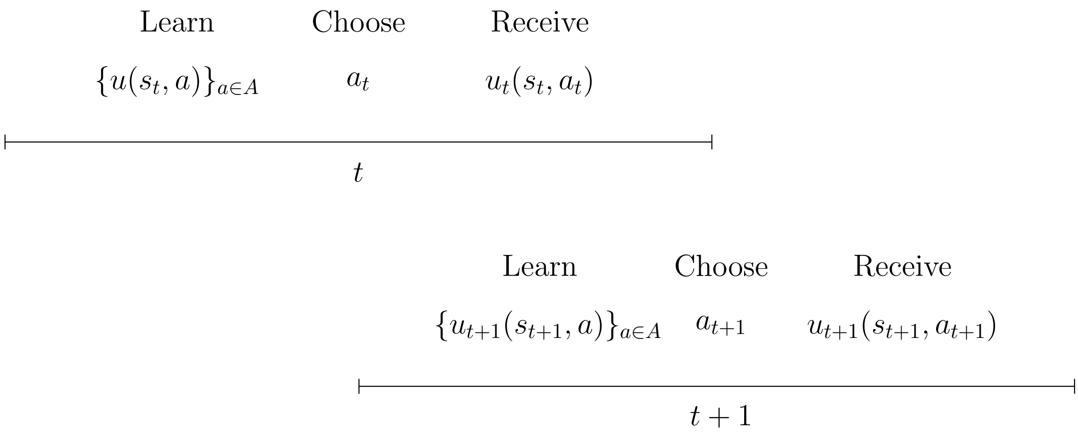
EKW models We restrict us to the class of Eckstein-Keane-Wolpin (EKW) models. Labor economists often apply these models for the analysis of human capital investment decisions. Differences in human capital attainment are a major determinant of inequality in a variety of life outcomes such as labor market success and educational attainment across and within countries.

?, for example, build a model to analyze the mechanisms determining schooling decisions in Norway. They find an important role for the option value of schooling, which measures the value of the information generated by each additional year of schooling. After validating their model using an increase in mandatory schooling, they then use the model to study the underlying mechanisms that generate the increase of average years of schooling and evaluate the effects of several policy alternatives.

Notation Throughout, we only provide a very limited number of references and only point to related textbooks instead. We will introduce acronyms and symbols as needed, but a full list of both is provided in the Appendix. The notation draws form the related work by **?**, **?**, and **?**.

Stucture This handout is structured as follows. We first present the basic setup and introduce the economic framework, its mathematical formalization, and the calibration procedure. We then offer one example. Finally, we outline possible improvements and extensions.

Figure 1: Timing of events



2 Setup

2.1 Economic framework

Basic setup EKW models describe sequential decision-making under risk (**??**). At time $t = 1, \ldots, T$ each individual observes the state of the economic environment $s_t \in S$ and chooses an action a_t from the set of admissible actions $\mathcal{A}$. The decision has two consequences: an individual receives an immediate reward $r(s_t, a_t)$ and the economy evolves to a new state s_{t+1}. The transition from s_t to s_{t+1} is affected by the action. Individuals are forward-looking, thus they do not simply choose the alternative with the highest immediate reward. Instead, they take the future consequences of their current action into account.

Decision rule A policy $\pi \equiv (a_1^\pi(s), \ldots, a_T^\pi(s))$ provides the individual with a prescription for choosing an action in any possible future state, where $a_t^\pi(s)$ specifies the action at a particular time t for any possible state under π. It is a sequence of decision rules and its implementation generates a sequence of rewards. The evolution of states over time is at least partly unknown as future rewards depend on, for example, shocks to preferences. Individuals use models about their economic environment to inform their beliefs about the future. For a given model, individuals thus face risk as each induces a unique objective transition probability distribution $p_t(s_t, a_t)$ for the evolution of state s_t to s_{t+1} that depends on the action a_t.

Timing of events Figure 1 depicts the timing of events in the model for two generic time periods. At the beginning of time t an individual fully learns about the immediate reward of each alternative, chooses one of them, and receives its immediate reward. Then the state evolves from s_t to s_{t+1} and the process is repeated in $t+1$.

Decision theory Individuals make their decisions facing risk and have rational expectations as their model about the future also turns out to be true. In this case, there is a consensus that rational choices are expressed by the maximization of their expected total discounted rewards. A constant discount factor ensures dynamic consistency of preferences as the individual's future actions agree with the planned-for contingencies. Beliefs are updated according to Bayes's rule.

Equation (1) provides the formal representation of the individual's objective. Given an initial state s_1, individuals seek to implement the policy π from the set of all possible policies Π that maximizes the expected total discounted rewards over all T decision periods given the information $\mathcal{I}_1$ available.

$$\max_{\pi \in \Pi} \mathrm{E}_{s_1}^{\pi} \left[\sum_{t=1}^{T} \delta^{t-1} r(s_t, a_t^{\pi}(s_t)) \,\middle|\, \mathcal{I}_1 \right] \tag{1}$$

The exponential discount factor $0 < \delta < 1$ captures a preference for immediate over future rewards. The superscript of the expectation emphasizes that each policy π induces a different unique probability distribution over the sequences of rewards.

2.2 Mathematical formulation

EKW models are set up as a standard Markov decision processes (MDP) (**??**). When making sequential decisions, the task is to determine the optimal policy π^* with the largest expected total discounted rewards $v_1^{\pi^*}$ as formalized in equation (1). In principle, this requires to evaluate the performance of all policies based on all possible sequences of rewards and the probability that each occurs. Fortunately, however, the multistage problem can be solved by a sequence of simpler inductively defined single-stage problems. Note that in slight abuse of notation s_{t+1} is a random variable given the information available at $\mathcal{I}_t$.

Let $v_t^{\pi}(s)$ denote the expected total discounted rewards under π from period t onwards:

$$v_t^{\pi}(s_t) \equiv \mathrm{E}_{s_t}^{\pi} \left[\sum_{j=0}^{T-t} \delta^{j}\, r(s_{t+j}, a_{t+j}^{\pi}(s_{t+j})) \,\middle|\, \mathcal{I}_t \right]$$

Then $v_1^{\pi}(s_t)$ can be determined for any policy by recursively evaluating equation (2):

$$v_t^{\pi}(s) = r(s_t, a_t^{\pi}(s_t)) + \delta\, \mathrm{E}_{s_t}^{\pi} \left[v_{t+1}^{\pi}(s_{t+1}) \,\middle|\, \mathcal{I}_t \right]. \tag{2}$$

Equation (2) expresses the rewards $v_t^{\pi}(s_t)$ of adopting policy π going forward as the sum of its immediate rewards and all expected discounted future rewards.

The principle of optimality allows to construct the optimal policy π^* by solving the optimality equations for all s and t in equation (3) recursively:

$$v_t^{\pi^*}(s_t) = \max_{a\in A}\left\{ r(s_t,a) + \delta\, \mathrm{E}_{s_t}^{\pi}\left[v_{t+1}^{\pi^*}(s_{t+1}) \,\middle|\, \mathcal{I}_t \right] \right\}. \tag{3}$$

The value function $v_t^{\pi^*}$ is the expected discounted rewards in t over the remaining time horizon assuming the optimal policy is implemented going forward.

The optimal decision is simply the alternative with the highest value:

$$a_t^{\pi^*}(s_t) \equiv \arg\max_{a\in A}\left\{ r(s_t,a) + \delta\, \mathrm{E}_{s_t}^{\pi}\left[v_{t+1}^{\pi^*}(s_{t+1}) \,\middle|\, \mathcal{I}_t \right] \right\}$$

Solution approach Algorithm (1) allows to solve the MDP by a simple backward induction procedure. In the final period T, there is no future to take into account and so the optimal decision is simply to choose the alternative with the highest immediate rewards in each state. With the results for the final period at hand, the other optimal decisions can be determined recursively as the calculation of their expected future rewards is straightforward given the relevant transition probabilities.

Algorithm 1 Backward induction procedure

for $t = T, \ldots, 1$ **do**
  if t == T **then**
    $$v_T^{\pi^*}(s_T) = \max_{a\in A}\left\{ r(s_T,a) \right\} \qquad \forall\, s_T \in S$$
  else
    Compute $v_t^{\pi^*}(s_t)$ for each $s_t \in S$ by
    $$v_t^{\pi^*}(s_t) = \max_{a\in A}\left\{ r(s_t,a) + \delta\, \mathrm{E}_{s_t}^{\pi}\left[v_{t+1}^{\pi^*}(s_{t+1}) \,\middle|\, \mathcal{I}_t \right] \right\}$$
    and set
    $$a_t^{\pi^*}(s_t) = \arg\max_{a\in A}\left\{ r(s_t,a) + \delta\, \mathrm{E}_{s_t}^{\pi}\left[v_{t+1}^{\pi^*}(s_{t+1}) \,\middle|\, \mathcal{I}_t \right] \right\}.$$
  end if
end for

2.3 Calibration procedure

EKW models are calibrated to data to obtain information on preference parameters and transition probabilities (??). Given this information, the quantitative importance of competing economic mechanisms can be assessed and the effects of public policies predicted. This requires the parameterization of all elements of the model which we collect in θ.

Data The econometrician has access to panel data for N individuals. For every observation (i, t) in the panel data set, the researcher observes action a_{it} and a subvector x_{it} of the state vector. Therefore, from an econometricians point of view, we need to distinguish between two types of state variables $s_{it} = (x_{it}, \epsilon_{it})$. Variables x_{it} that are observed by the econometrician and the individual i at time t and those that are only observed by the individual ϵ_{it}. In addition, also some realizations of the rewards $r_{it} = r(x_{it}, \epsilon_{it}, a_{it})$.

$$\mathcal{D} = \{a_{it}, x_{it}, r_{it} : i = 1, 2, \ldots, N; t = 1, \ldots, T_i\},$$

where T_i is the number of observations over which we observe individual i.

Procedures We briefly outline maximum likelihood estimation and the method of simulated moments. Whatever the estimation criterion, in order to evaluate it for a particular value of θ it is necessary to construct the optimal policy π^*. Therefore at each trial value of θ the whole model needs to solved by the backward induction algorithm.

Likelihood-based The individual chooses the alternative with the highest total value $a_t^{\pi^*}(s_t)$ which is determined by the complete state and rewards are also determined by s. However, the econometrician only observes the subset x. Given parametric assumptions about the distribution of ϵ, we can determine the probability $p_{it}(a_{it}, r_{it} \mid x_{it}, \theta)$ of individual i at time t choosing d_{it} and receiving r_{it} given x_{it}.

$$\mathcal{L}(\theta \mid \mathcal{D}) \equiv \prod_{i=1}^{N} \prod_{t=1}^{T_i} p_{it}(a_{it}, r_{it} \mid x_{it}, \theta)$$

The goal of likelihood-based estimation is to find the value of the model parameters θ that maximize the likelihood function:

$$\hat{\theta} \equiv \operatorname*{arg\,max}_{\theta \in \Theta} \mathcal{L}(\theta \mid \mathcal{D})$$

Simulation-based ... to be written

3 Example

We now provide the seminal implementation of **?**. Individuals live for a total of T periods, are risk neutral, and make a decision about their human capital investment each period. They choose to either work in one of two occupations (a = 1, 2), attend school (a = 3), or stay at home (a = 4). The immediate utility from each alternative is the following:

$$u_t(s,a) = \begin{cases} w_{1t} = \exp\{\alpha_{10} + \alpha_{11} g_t + \alpha_{12} x_{1t} + \alpha_{13} x_{1t}^2 + \alpha_{14} x_{2t} + \alpha_{15} x_{2t}^2 + \epsilon_{1t}\} & \text{if } a = 1 \\ w_{2t} = \exp\{\alpha_{20} + \alpha_{21} g_t + \alpha_{22} x_{1t} + \alpha_{23} x_{1t}^2 + \alpha_{24} x_{2t} + \alpha_{25} x_{2t}^2 + \epsilon_{2t}\} & \text{if } a = 2 \\ \beta_0 - \beta_1 \mathbb{I}\,[\, g_t \geq 12\,] - \beta_2 (1 - \mathbb{I}\,[\, a_{t-1} = 3\,]) + \epsilon_{3t} & \text{if } a = 3 \\ \gamma_0 + \epsilon_{4t} & \text{if } a = 4. \end{cases}$$

g_t is the number of periods of schooling obtained by the beginning of period t, x_{1t} and x_{2t} are the number of periods that the individual worked in the two occupations respectively. The utility for each labor market alternative corresponds to its wage (w_{1t}, w_{2t}) and α_1 and α_2 are thus parameters associated with the wage functions. They capture the returns to schooling and occupation-specific human capital. β_0 is the consumption utility of schooling, β_1 is the post-secondary cost of schooling, and β_2 is an adjustment cost associated with returning to school. The mean utility of the home alternative is denoted γ_0. The ϵ_{at}'s are alternative-specific shocks to occupational productivity, the consumption utility of schooling, and the utility of home time. They are serially uncorrelated.

Given the structure of the utility functions and the lack of serial correlation, the state at time t is:

$$s_t = \{g_t, x_{1t}, x_{2t}, a_{t-1}, \epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t}, \epsilon_{4t}\}.$$

The observable components of s_t evolve according to the following rules:

$$\begin{aligned} x_{1,t+1} &= x_{1t} + \mathbb{I}\,[\, a_t = 1\,] \\ x_{2,t+1} &= x_{2t} + \mathbb{I}\,[\, a_t = 2\,] \\ g_{t+1} &= g_t \;\; + \mathbb{I}\,[\, a_t = 3\,]. \end{aligned}$$

The transitions of all observable components of s_t are deterministic. However, there is uncertainty about the realization of its unobservable components. All unobservable components are jointly normally distributed with mean zero and covariance matrix $\mathbf{\Sigma}$.

$$[\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t}, \epsilon_{4t}]^T \sim \mathcal{N}_0(\mathbf{0}, \mathbf{\Sigma})$$