OpenSourceEconomics

Presentation: Numerical Toolbox

Integration

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September 13, 2019

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Keane and Wolpin in 1994

"However, for problems of the size we would like to consider, a [...] simulation strategy is not computationally practicable."

Keane and Wolpin (1994): Model Outline

Setting: In each of T periods an individual chooses between $K = 4$ mutually exclusive alternatives: occupation one / two, schooling, and home.

Each of the alternatives k is associated with a *per-period reward function*

$$R_k(t, s_t, x_{1t}, x_{2t}; \theta, \epsilon_{kt}) \quad \text{for } k = 1, \dots, 4.$$

Keane and Wolpin (1994): Model Outline

With $\epsilon_t \sim \mathcal{N}_4(\mu, \Sigma)$, ϵ_{kt} are alternative-specific shocks to

- skill level ($k = 1, 2$)
- consumption value of schooling ($k = 3$)
- value of non-market time ($k = 4$),

Cross-correlations admissible, but not serial correlation.

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Cross-correlations admissible, but not serial correlation.

The vector θ summarizes parameters for wage, schooling, and non-market time.

Question of interest: Does a school subsidy change the average years of schooling in the sample?

Keane and Wolpin (1994): Solution

Individual's objective function, alternative-specific value function at $t = 0, \ldots, T$:

$$V_k(S(t), t) = \mathbb{E}\left[\sum_{\tau=t}^{T} \delta^{\tau-t} \sum_{k}^{K} R_k(\tau) d_k(\tau) \;\middle|\; S(t)\right] \quad , T < \infty.$$

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State Space S(t): All factors, known to the individual, that affect *current* rewards or the probability distribution of any of the *future* rewards.

Keane and Wolpin (1994): Solution

Maximizing Choice: Optimal sequence of control variables $\{d_k(t)\}_{k \in K}$ for $t = 0, \ldots, T$

Value function for period t:

$$V\left(S(t), t\right) = \max_{\{d_k(t)\}_{k \in K}} \left\{V_k\left(S(t), t\right)\right\},$$

Keane and Wolpin (1994): Solution

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Value function for period t:

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where *alternative-specific value functions* $V_k\left(S(t), t\right)$

$$V_k\left(S(t), t\right) = R_k\left(S(t), t\right) + \delta \mathbb{E}\Big[V\left(S(t+1), t+1\right)\Big] \;, t \leq T - 1$$
$$V_k\left(S(T), T\right) = R_k\left(S(T), T\right)$$

obey the Bellman equation (Bellman, 1957).

Keane and Wolpin (1994): Solution

Individual at $(T-1)$: For each k, calculate

$$\begin{aligned}&\mathrm{Emax}\Big(R_1(T), R_2(T), R_3(T), R_4(T) \;\Big|\; \bar{S}(T-1), d_k(T-1)\Big) \\ &\quad = \int_{\epsilon_T} \max\Big(R(T) \;\Big|\; \bar{S}(T-1), d_k(T-1)\Big) f(\epsilon_T) \mathrm{d}\epsilon_T\end{aligned}$$

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$\Rightarrow V_k\left(S(T-1), T-1\right)$ are known

Keane and Wolpin (1994): Solution

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$\Rightarrow V_k\,(S(T-1), T-1)$ are known

$\rightarrow$ Receive random draw of $\epsilon_{T-1} = \big(\epsilon_{1,T-1}, \ldots, \epsilon_{4,T-1}\big)$
$\rightarrow$ Choose alternative k with highest value

Keane and Wolpin (1994): Solution

Individual at $t < (T-1)$: At every t, calculate

$$\mathrm{Emax}\Big[V_1\left(S(t+1), t+1\right), \ldots, V_4\left(S(t+1), t+1\right) \;\Big|\; \bar{S}(t), d_k(t)\Big]$$

Keane and Wolpin (1994): Solution

Individual at $t < (T-1)$: At every t, calculate

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Requirement: Alternative-specific value functions at t must have been calculated for *all* of the possible state space values at $t+1$

$\Rightarrow$ Need to calculate the $V_k(\cdot)$ at each future dates,
and at all feasible state points.

$\Rightarrow$ Computational complexity

Keane and Wolpin (1994): Estimation

Exact solution requires to solve multiple (aka millions) K-dimensional integrals (aka $\text{Emax}(\cdot)$).

A priori unknown: extent of finite sample bias as accuracy of $\text{Emax}(\cdot)$ varies.

Keane and Wolpin (1994): Estimation

Exact solution requires to solve multiple (aka millions) K-dimensional integrals (aka Emax($\cdot$)).

A priori unknown: extent of finite sample bias as accuracy of Emax($\cdot$) varies.

Our goal: Use quasi-Monte Carlo approach to lower required points for given accuracy
$\Rightarrow$ Use respy for a simulation study

Tool of choice: respy

respy 1.2.1 documentation »

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Welcome to respy's documentation!

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`respy` is an open-source Python package for the simulation and estimation of a prototypical finite-horizon discrete choice dynamic programming model. We build on the baseline model presented in:

> Keane, M. P. and Wolpin, K. I. (1994). The Solution and Estimation of Discrete Choice Dynamic Programming Models by Simulation and Interpolation: Monte Carlo Evidence. *The Review of Economics and Statistics*, 76(4): 648-672.

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New option in respy

Implementation of *low-discrepancy sequences for quasi-Monte Carlo simulation*:

- Halton ((co)prime numbers)
- Sobol (primitive polynomial)
- R_d-sequence (additive recurrence method based on irrational numbers)

Halton and Sobol sequences rely on implementation from chaospy.

R_d implemented based on a blog post by M. Roberts.

Background $\mathbf{R_d}$ sequence

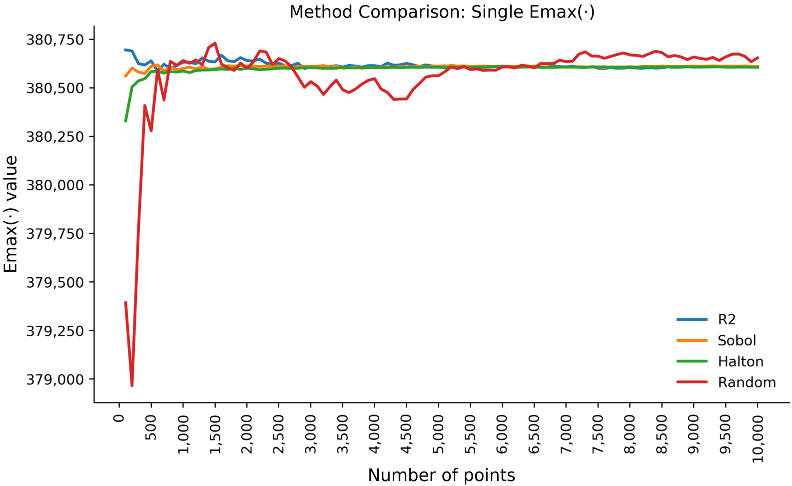
Intuition: Connection of generalized golden ratio (Marohnic and Strmecki (2012)) with construction of higher-dimensional low discrepancy sequences

Construction method: The parameter-free d-dimensional infinite sequence $R_d(\phi_d)$ is constructed as follows:

$$t_n = \{n\alpha\}; \quad \alpha = \left(\frac{1}{\phi_d}, \frac{1}{\phi_d^2}, \ldots, \frac{1}{\phi_d^d}\right); \quad n = 1, 2, \ldots,$$

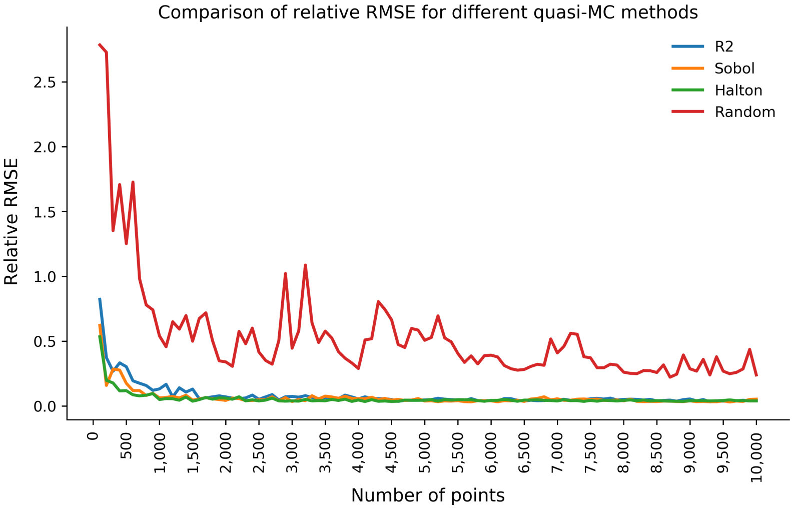
and ϕ_d is the unique positive root of $x^{d+1} = x + 1$.

Simulation Study



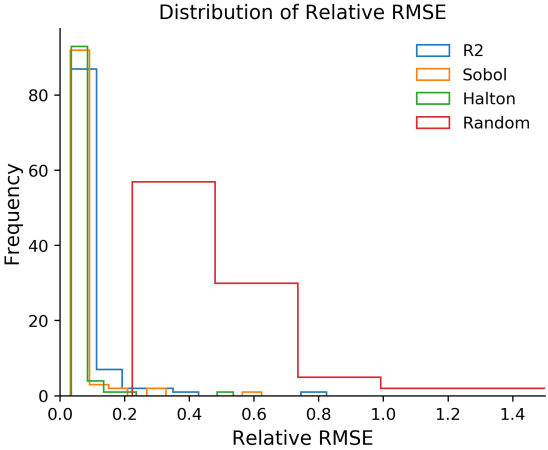
Take Emax(·) for one state, iterate over points

Simulation Study



Compute relative RMSE by aggregating over all state spaces

Simulation Study



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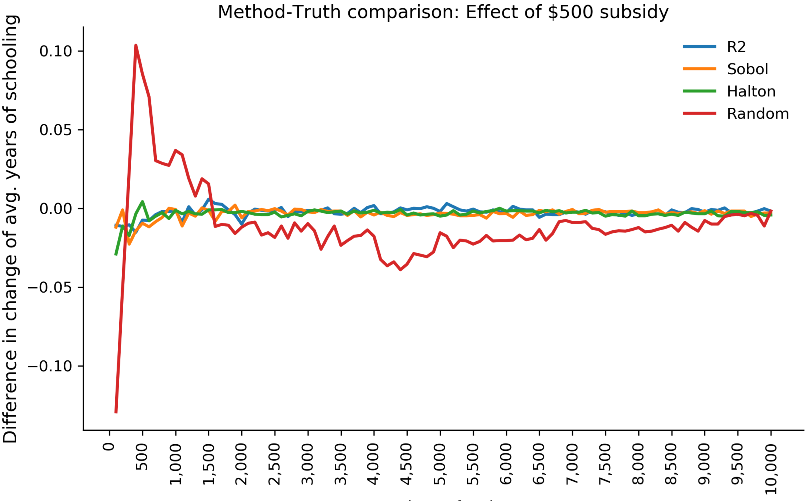

Compute differences in policy effect dependent on number of points

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Thank you for your attention

and enjoy the workshop!

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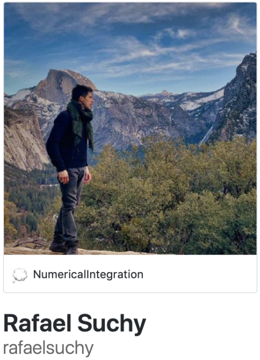

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Appendix