„Small Data“: Estimation of Dynamic Models with Unobservable or Occasionally Observable States

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A Dynamic Model of Bus Engine Replacement

Rust, John (1987): Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher. *Econometrica*.

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General State

Mileage/Age

New Engine ($$$)

Maintenance ($-$$)

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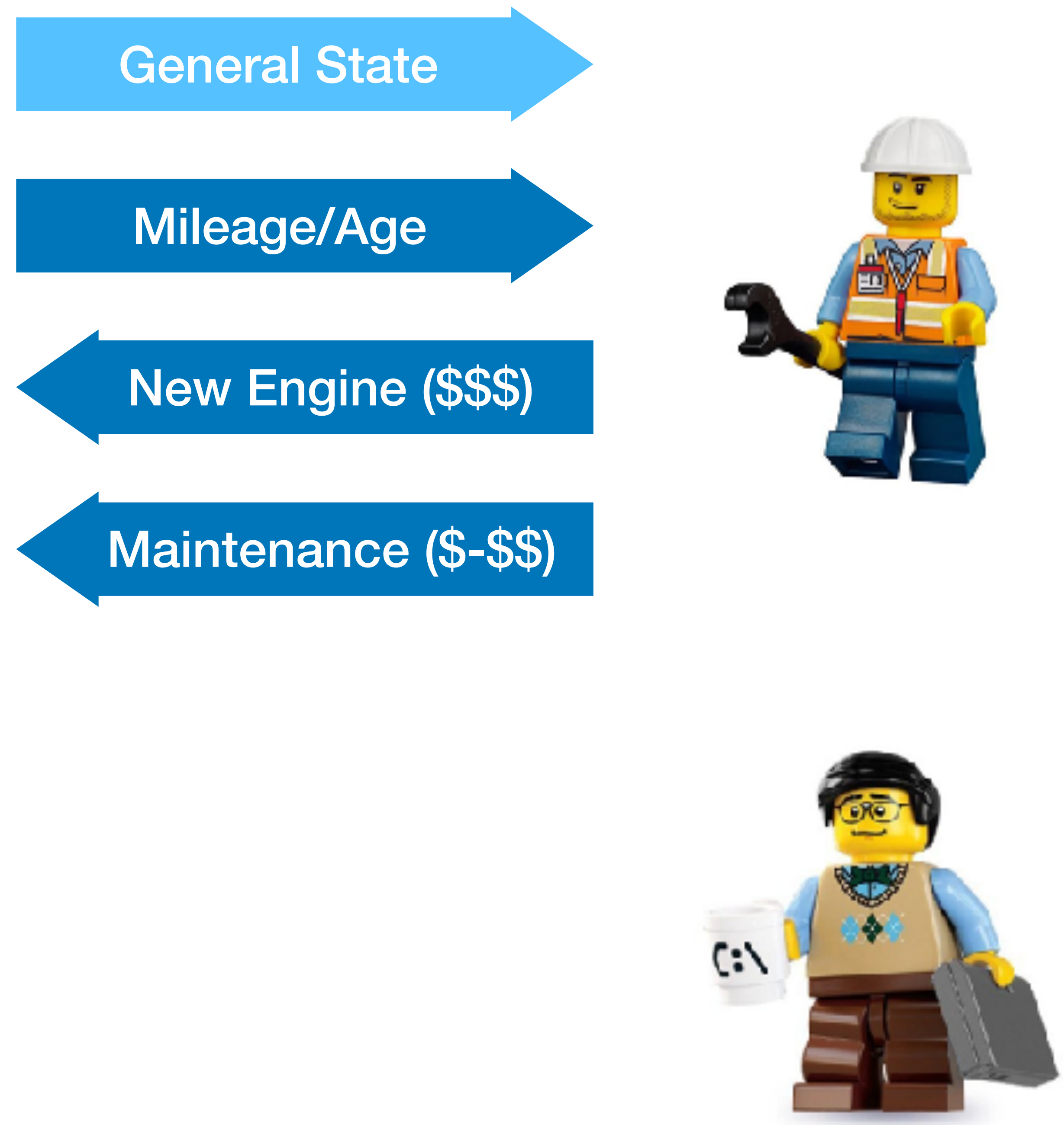

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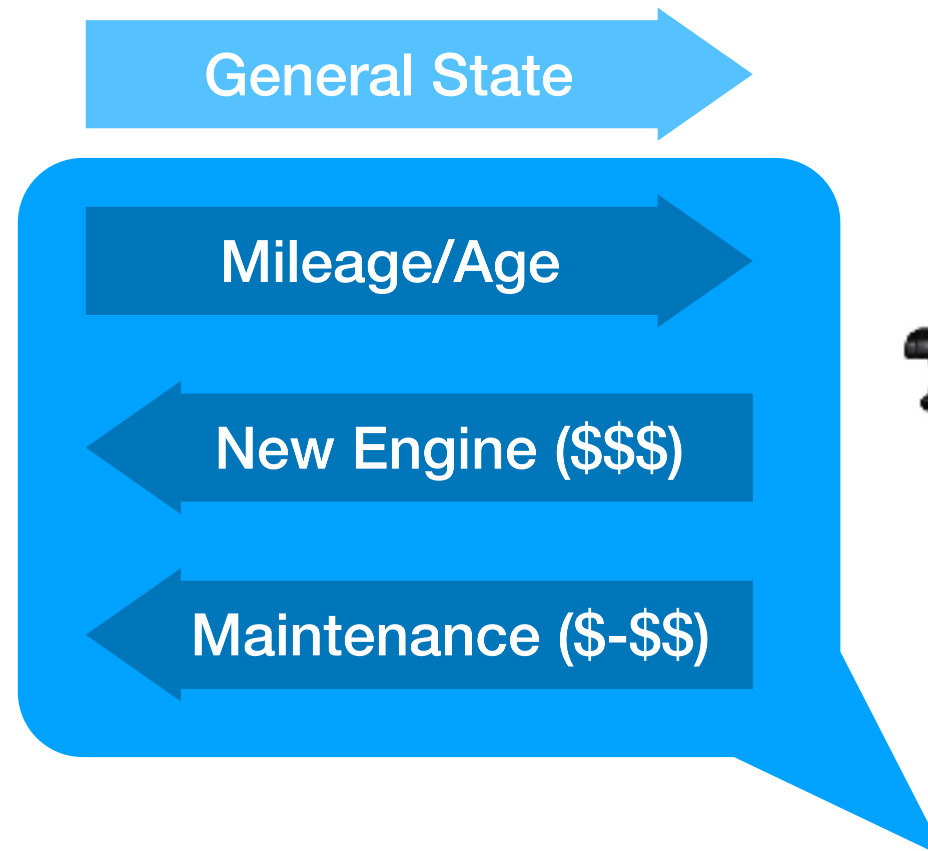

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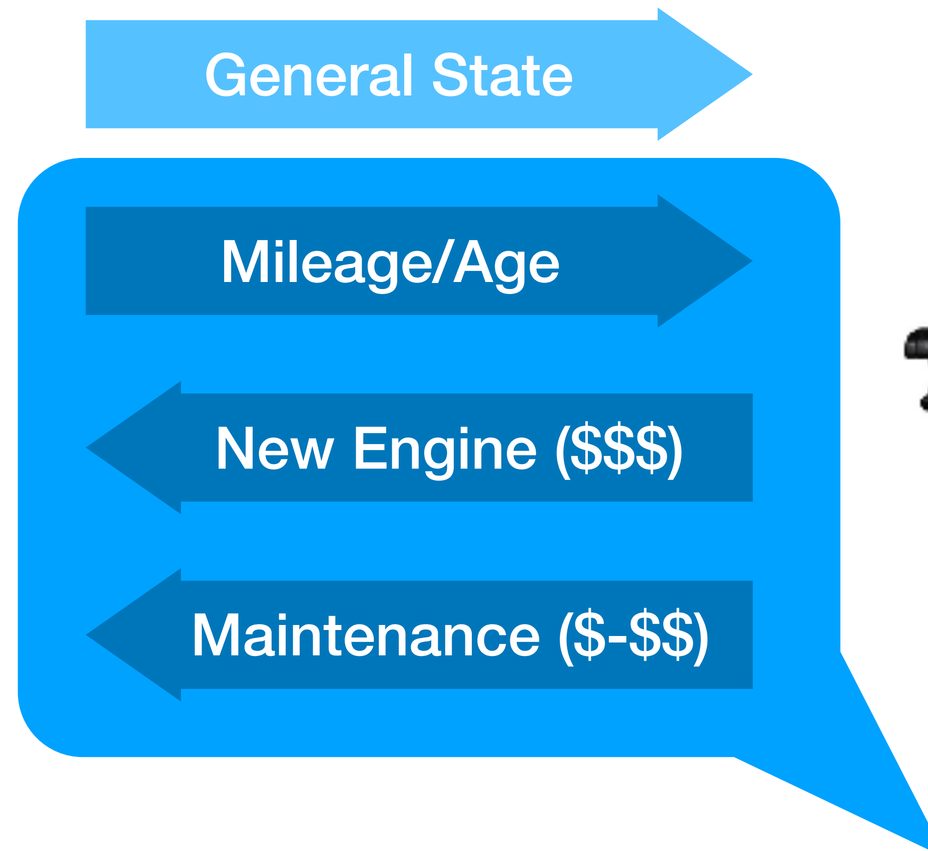

Goal: *Estimate* the dynamic cost tradeoff as a *regenerative optimal stopping problem*.

Rust (1987) with Serially Correlated EV1 Errors

- Reich (2018) estimates the models of Rust (1987) with

$$\varepsilon_{t+1} = \rho \varepsilon_t + \tilde{\varepsilon}_{t+1}$$

which nests the original model for for EV1 innovations and $\rho=0$

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	Bus groups 1–4		Bus group 4	
RC	9.7560	18.1927	10.0749	15.7403
	(0.898)	(6.724)	(1.351)	(9.452)
θ_1	2.6276	4.9894	2.2929	3.4330
	(0.469)	(2.424)	(0.554)	(2.610)
RC/θ_1	3.7128	3.6463	4.3935	4.5850
ρ	–	0.7396	–	0.7000
		(0.091)		(0.189)
L	-6,055.250	-6,053.340	-3,304.156	-3,303.912
p (LR)		0.0506		0.4848

$\beta = .9999$, p (LR) is p-value of likelihood ratio test $H_0 : \rho = 0$

Occasionally Observed States
(Work in Progress)

- So far, states were either observed or unobserved
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- RLI is still applicable
  - Straightforward for two-sided laws of motion
  - A little care needed for one-sided laws of motion (here)
  - Integration over a lower-dimensional submanifold in the presence of constraints on the law (e.g. aggregation)

- Literature: Hall & Rust (2019, *Journal of Econometrics*) use Method of Simulated Moments (MSM)

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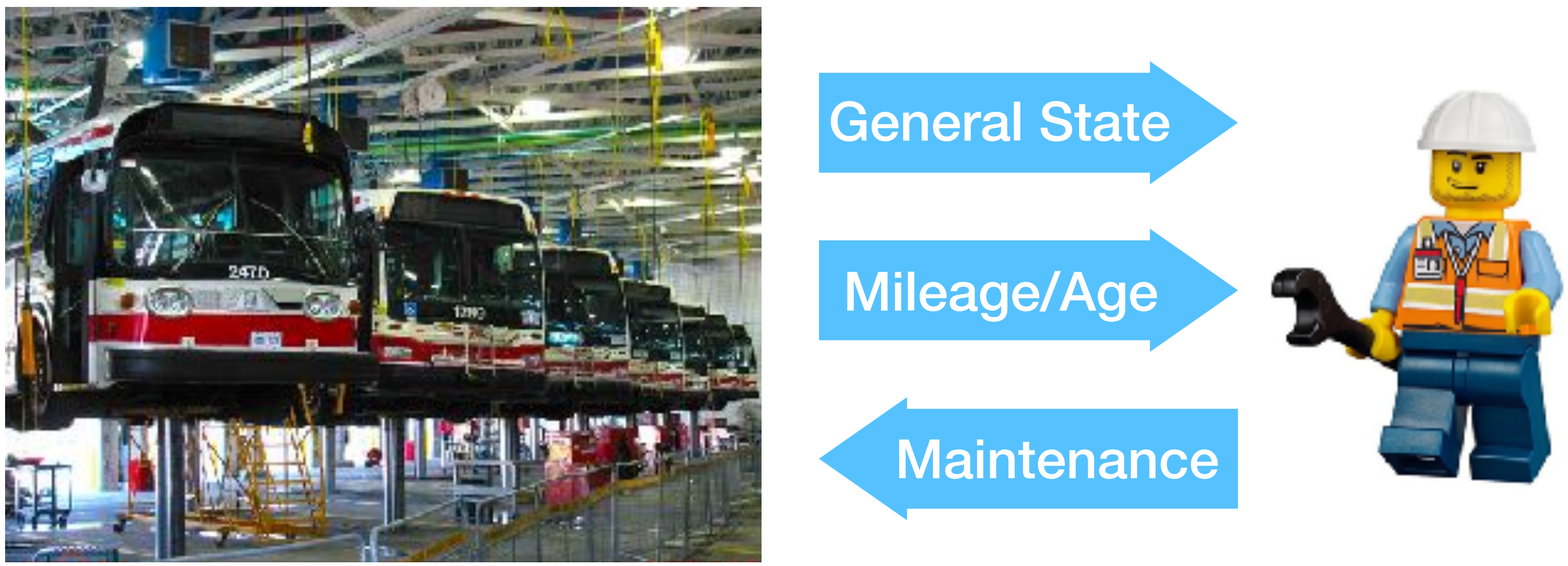

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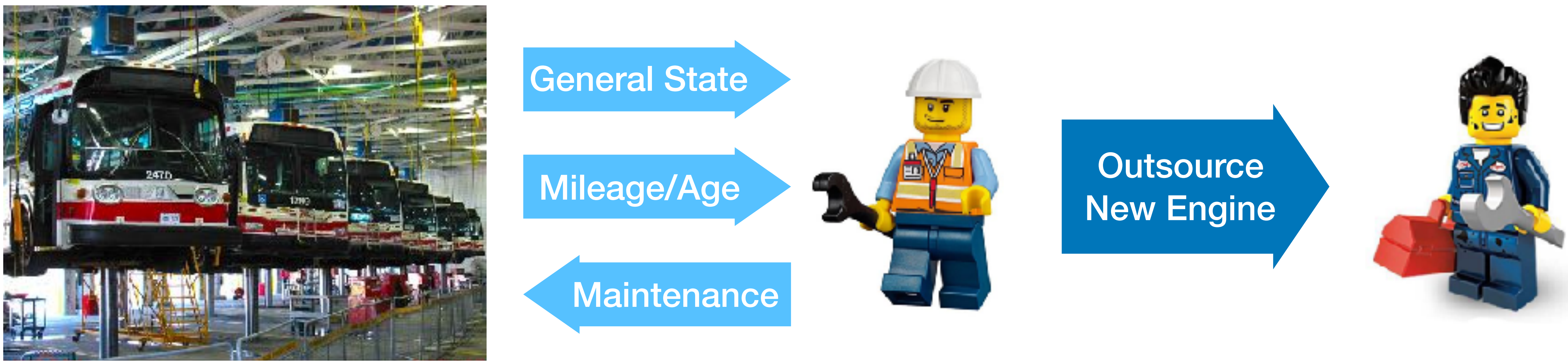

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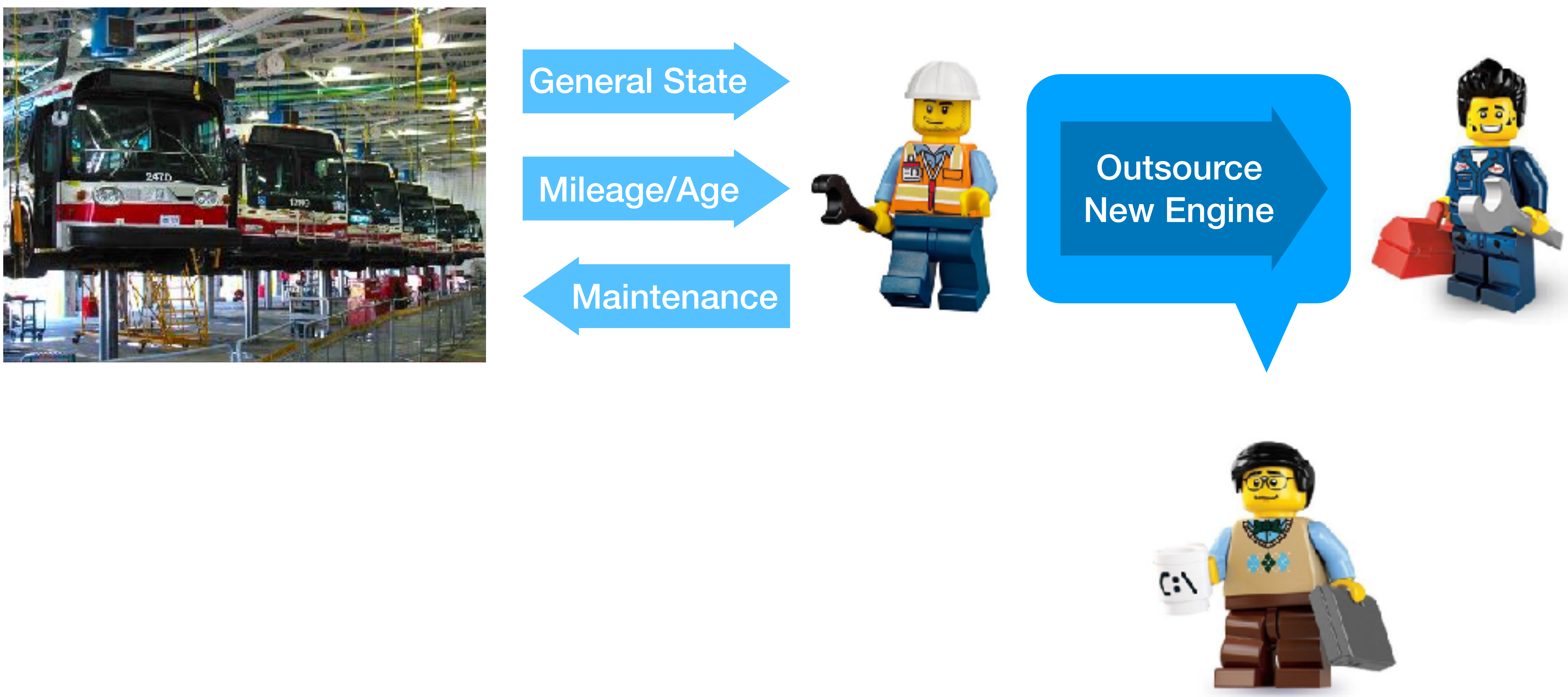

A Modern Tale of Bus Engine Replacement

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Can we still consistently estimate…

- the cost trade-off? YES
- the law of motion of mileage? NO

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    - Full information (positive benchmark)
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  - *Asymptotic distribution*: Normal

Nature and Law of the Mileage State

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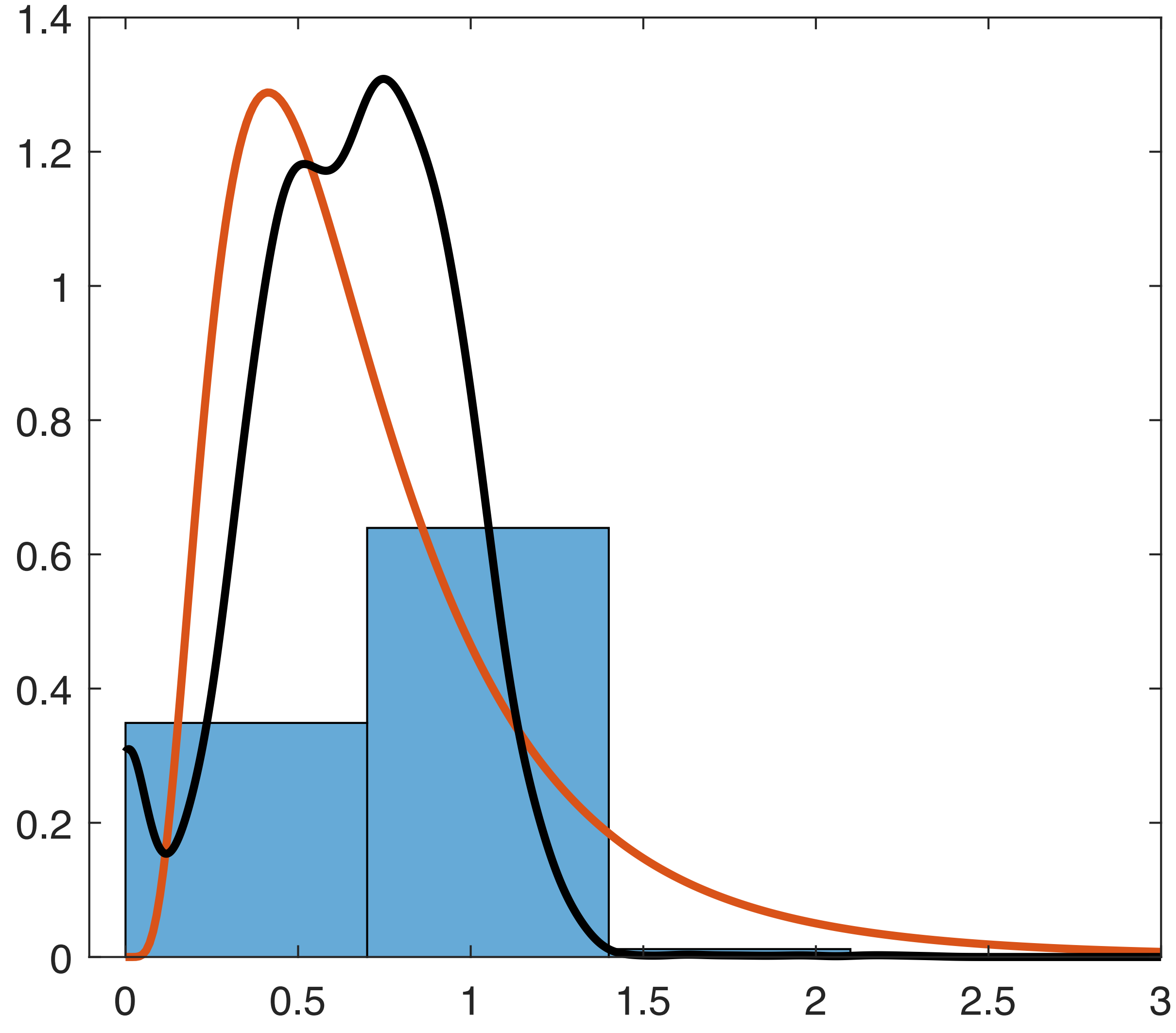
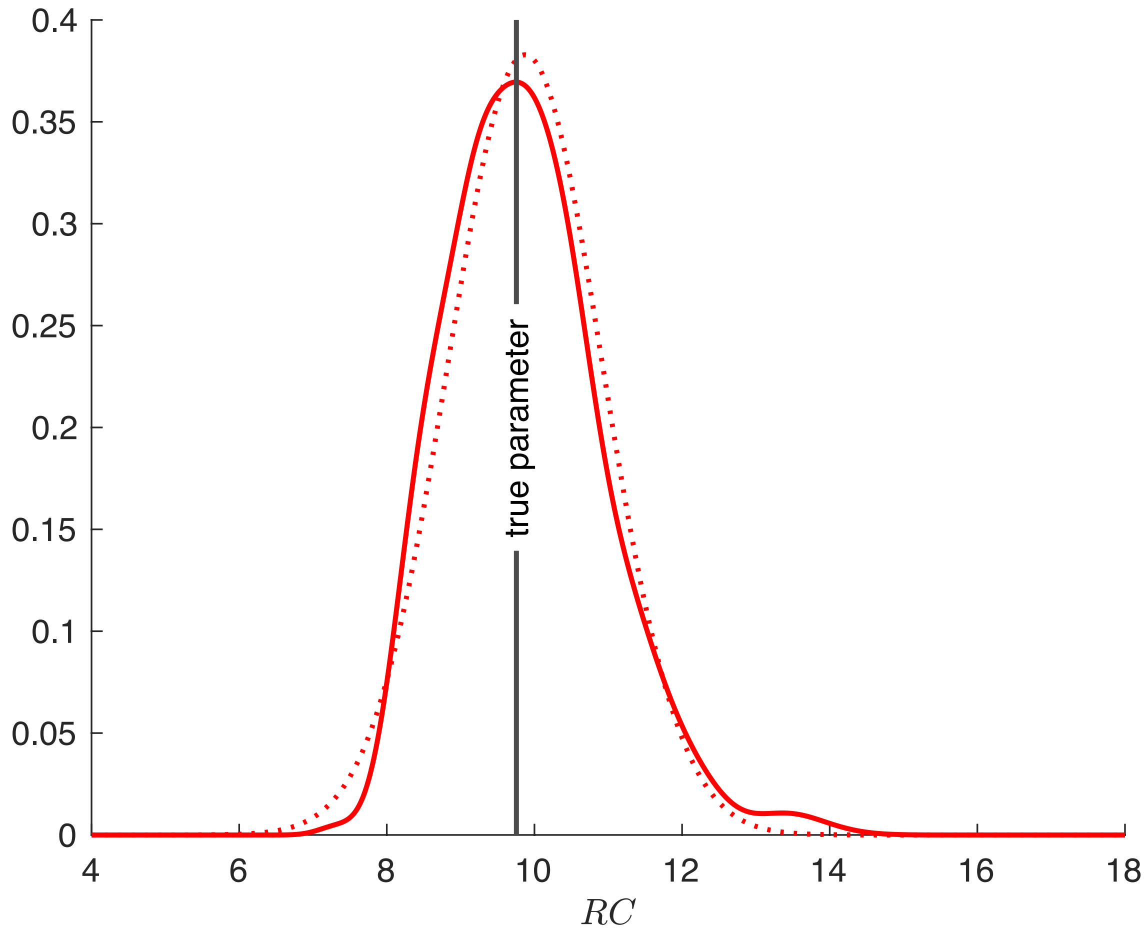
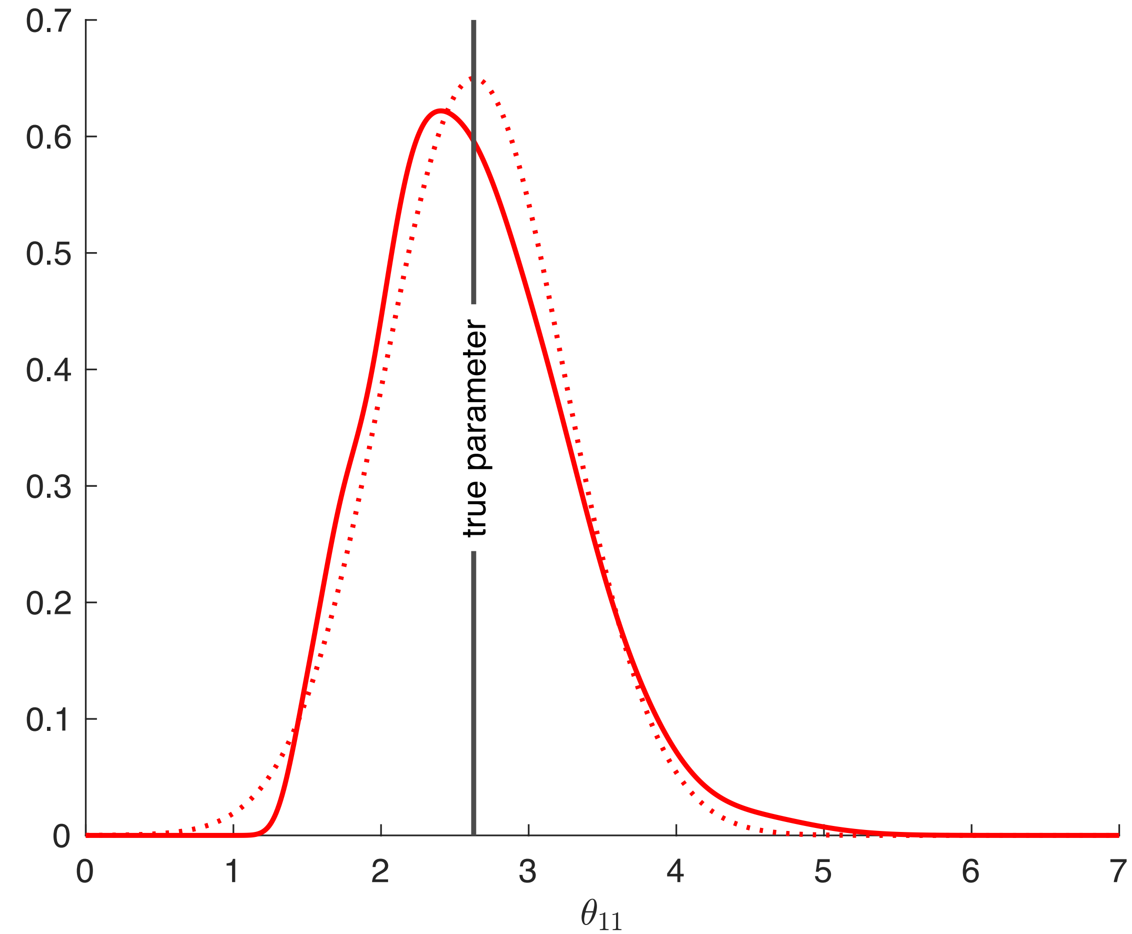


Distribution of RLI Estimator for OOS (1)

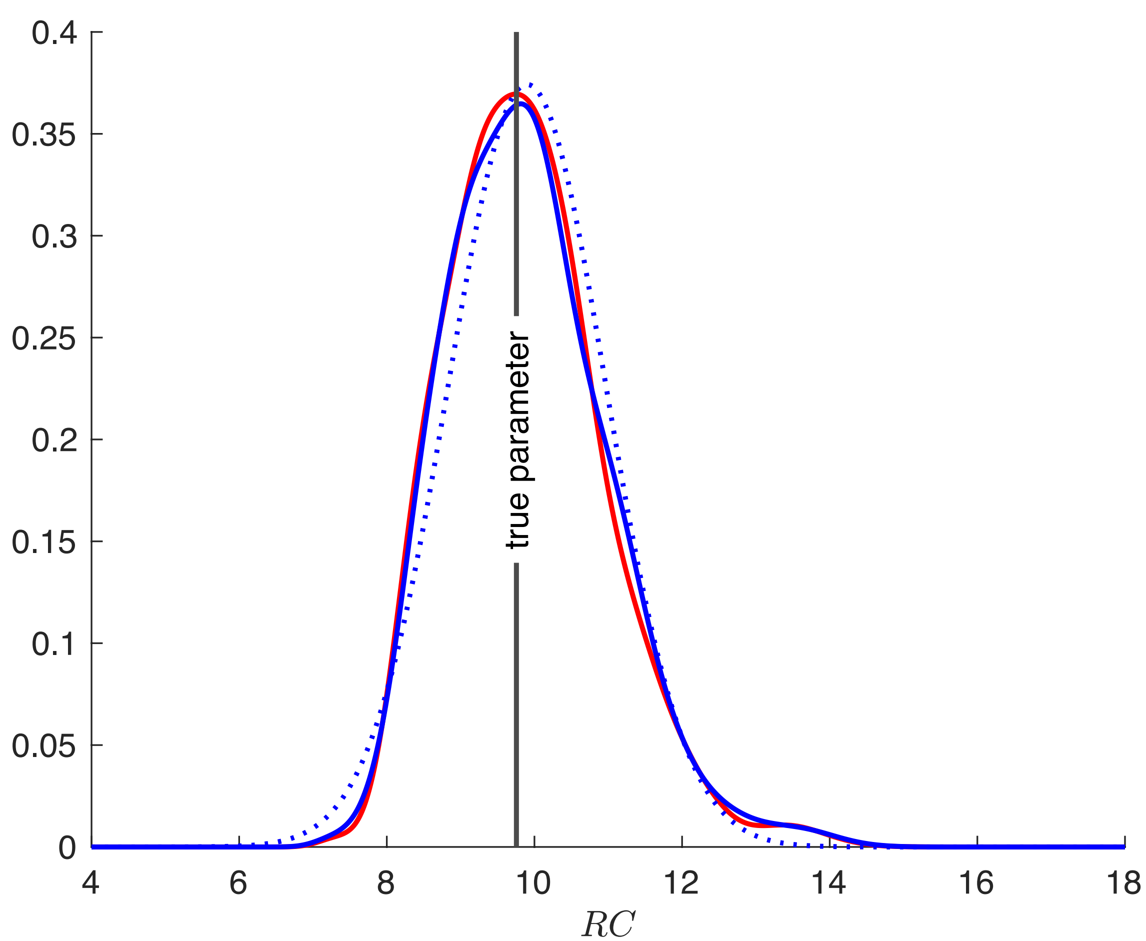
- We compare the distributions of various observation frequencies
  - full information (8'112 state observations)

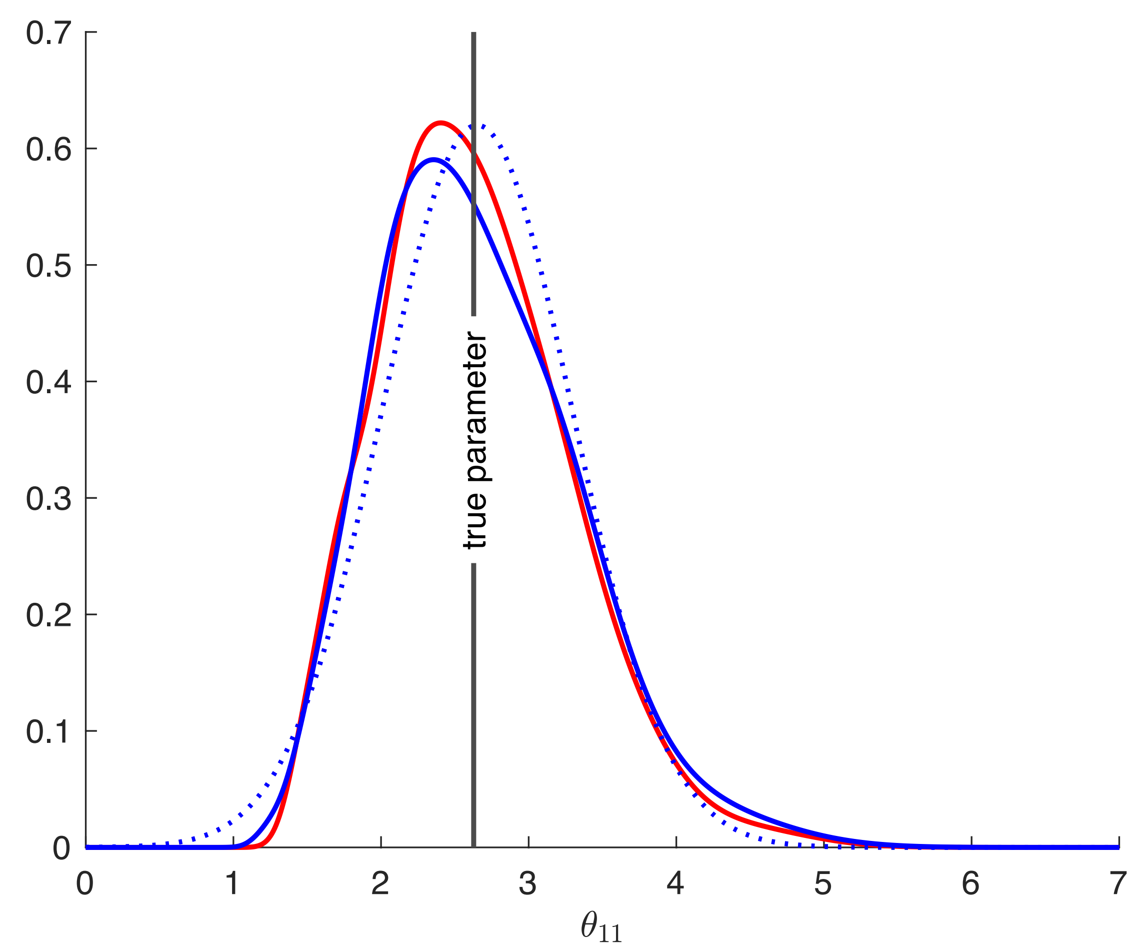




Distribution of RLI Estimator for OOS (1)

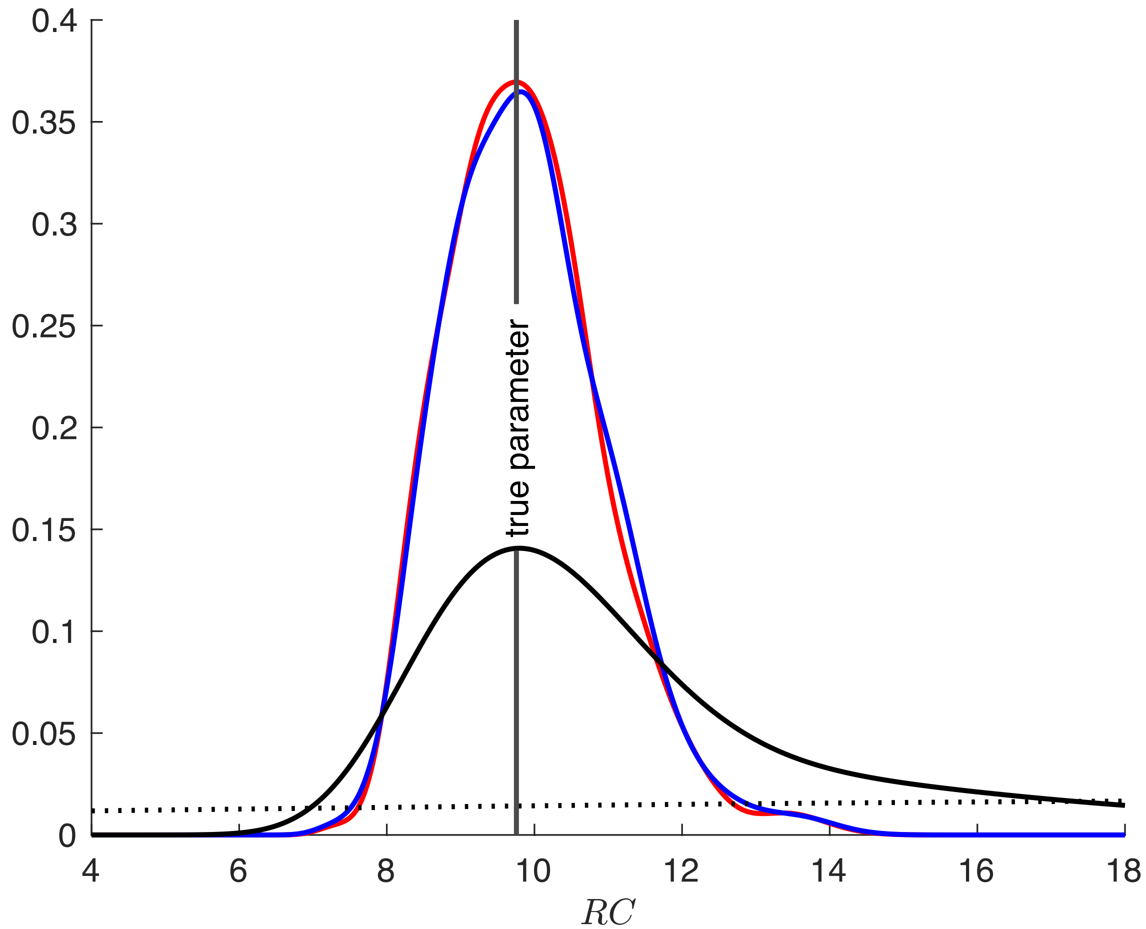
- We compare the distributions of various observation frequencies
  - full information (8’112 state observations)
  - at replacement (~164 state observations)

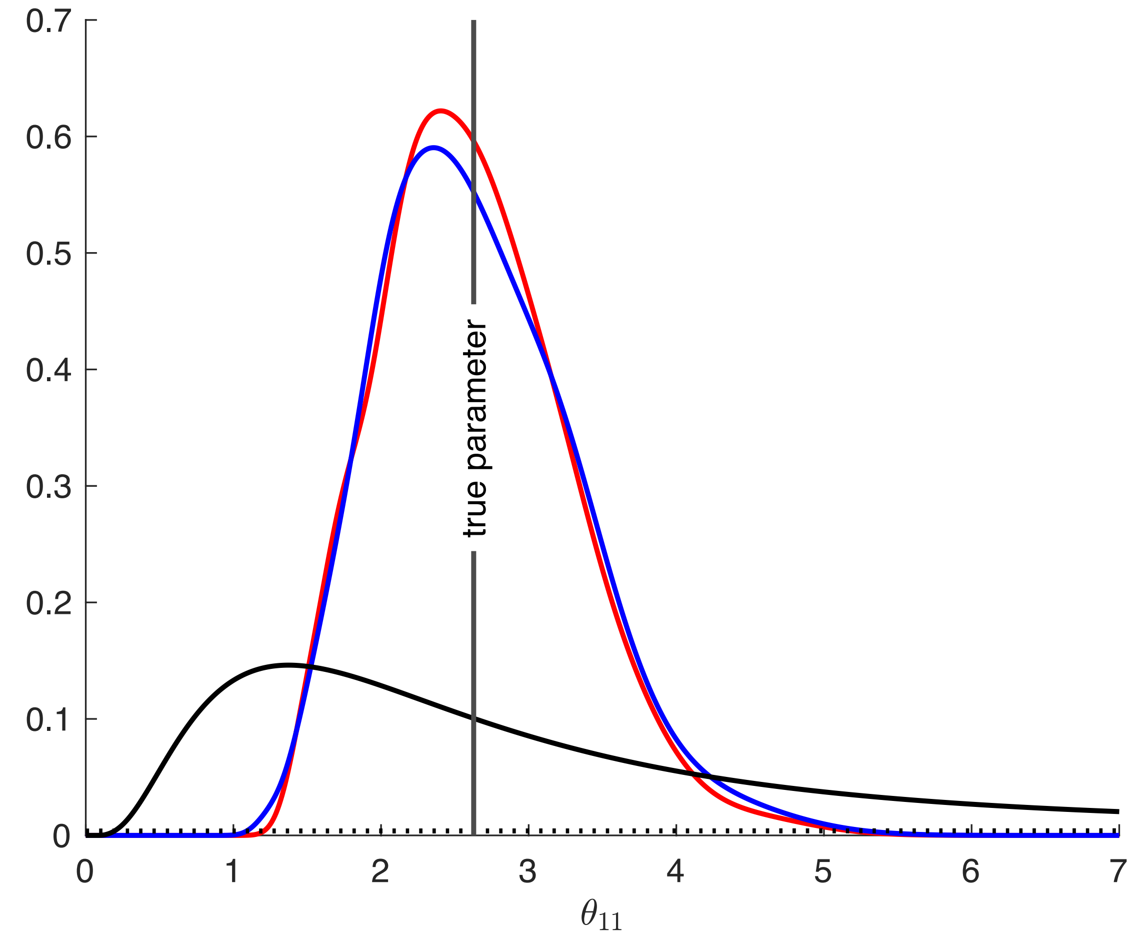




Distribution of RLI Estimator for OOS (1)

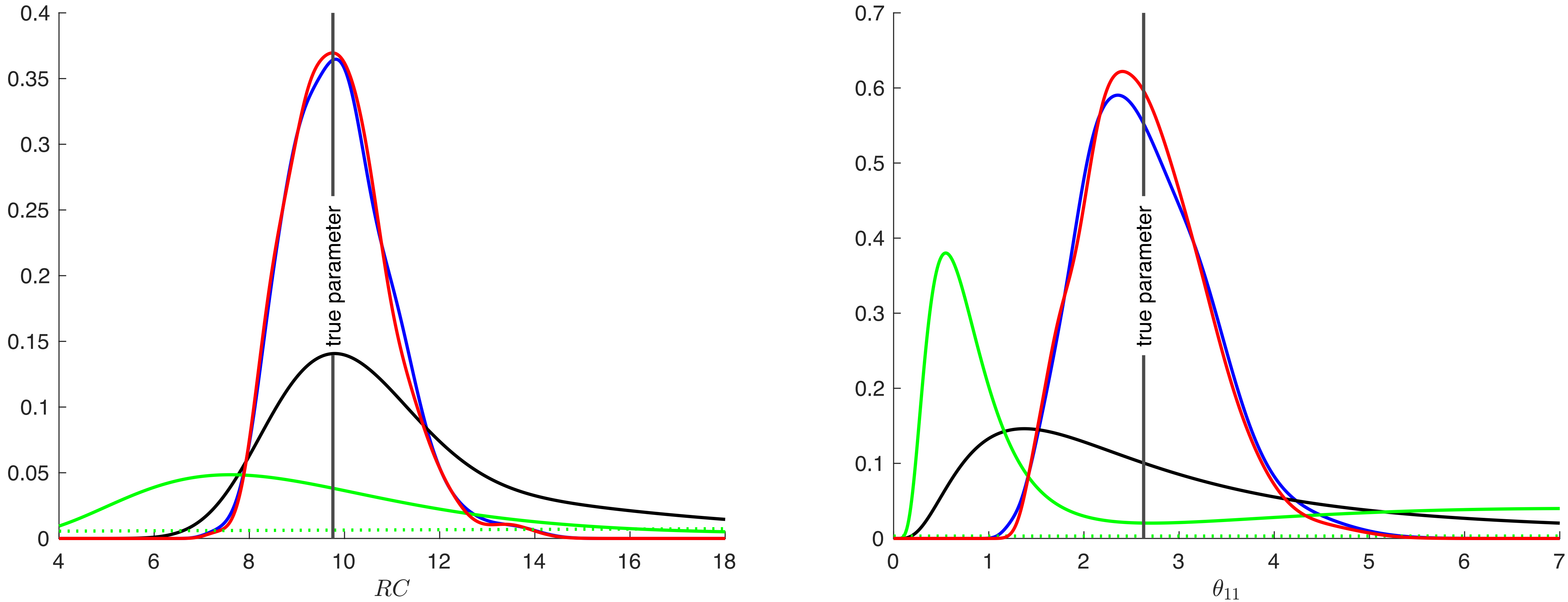
- We compare the distributions of various observation frequencies
  - full information (8'112 state observations)
  - at replacement (~164 state observations)
  - new bus (104 state observations)





Distribution of RLI Estimator for OOS (1)

- We compare the distributions of various observation frequencies
  - full information (8’112 state observations)
  - at replacement (~164 state observations)
  - new bus (104 state observations)
  - never (0 state observations)



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