Efficient Solution, Filtering and Estimation of Models with OBCs

and Applications

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OSE *retreat* 2019

Note: All presented results are preliminary

Motivation

- **Zero-lower bound (ZLB) on nominal interest rates** binding eg. in US, Eurozone
- Conventional monetary policy **ineffective!**
- **Many questions:**
  - Effects of (unconventional monetary) policy at ZLB?
  - Economic costs of ZLB?
  - Structural changes in the last decade?

Contributions

- Robust & fast solution method for OBCs
- Robust & efficient nonlinear Bayesian filter
- **Robust** mode-search & sampling (stolen from astophysics)

- Set of **nice tools** for estimation of models with OBCs

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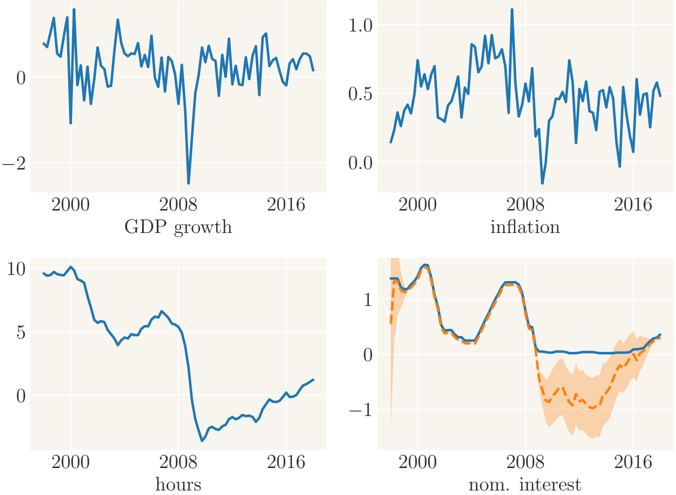
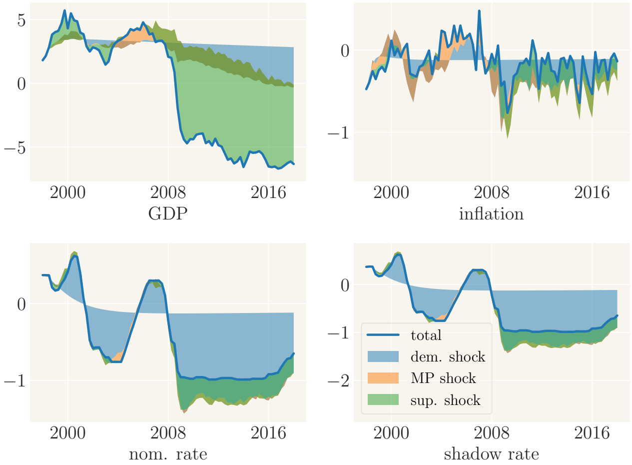
Motivation: US data



$$R_t = \max\{0, \rho \ln R_{t-1} + (1-\rho)(\phi_\pi \pi_t + \phi_y y_t + \phi_{\Delta y} \Delta y_t) + v_{r,t}\}$$

Decomposition: ZLB important?

- Model (estimated): small NK with habit, inflation indexation, hand-2-mouth & financial frictions
- Historical decomposition **without** ZLB:



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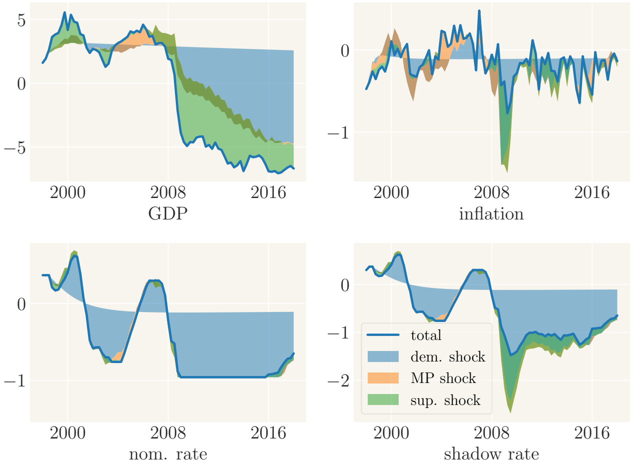

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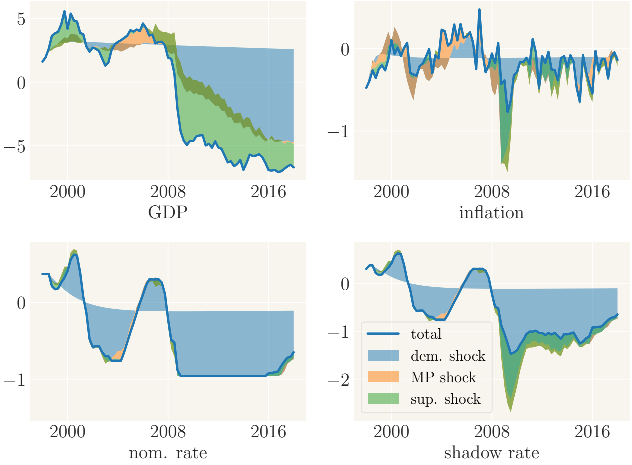

Motivation: QE (Boehl & Strobel, 2019)

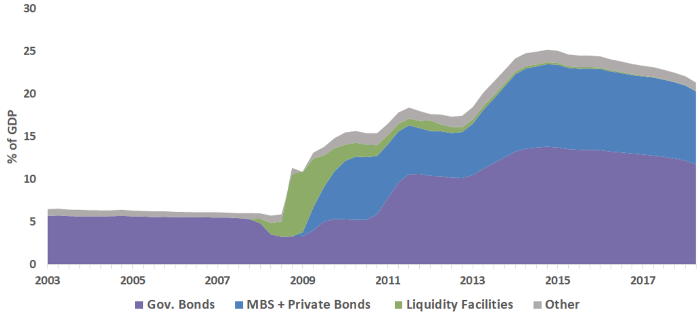

Figure: Fed Balance Sheet, Asset Side

- **What were the effects of unconventional monetary policy on the real economy?**

Motivation: QE (Boehl & Strobel, 2019)

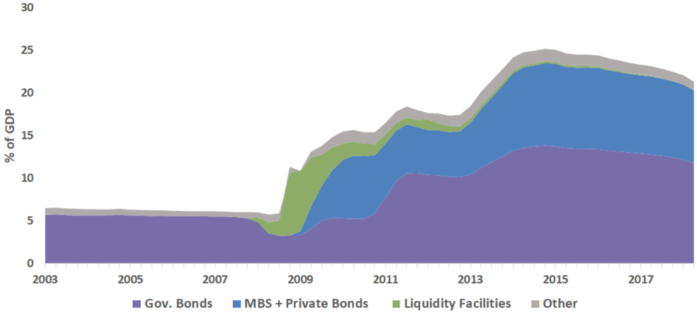

Figure: Fed Balance Sheet, Asset Side

- **What were the effects of unconventional monetary policy on the real economy?**

Example: Boehl & Strobel (2019)

- Use DSGE model to quantify QE effects:

Smets & Wouters (2007)	+	Gertler & Karadi (2013)
NK + bells + whistles	+	banks + QE

Issue: Implied effects of QE depend on parameter choice

▶ **Bayesian estimation necessary**

Problem: ZLB is a strong nonlinearity

▶ **New methodology**

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Outline

Literature (Methods)

- Solution Methods
  - `Occbin`: Guerrieri and Iacoviello (2015)
  - `RISE` (Regime switching): Binning & Maih (2016)
  - `DynareOBC`: Holden (2016)
  - Iterative Methods: Gust et al. (2017)
- Non-linear Filtering
  - **Inversion-Filter**: Fair & Taylor (1983), Guerrieri & Iacoviello (2017)
  - **Sigma-Point Filters**: Julier et al. (2000), Julier (2002), Binning & Maih (2015)
  - **Particle Filter**: An and Schorfheide (2007), Herbst and Schorfheide (2017)
- OBC Estimation
  - Linde et al. (2017), Kulish et al. (2017)
  - Richter & Throckmorton (2017,2019), Gust et al. (2017)
- **Problem:** speed and/or accuracy

Methodology

Method: Intro

General problem:

- requires recurrent solution & filtering of nonlinear model
- extremely large state space (42+)
- rugged likelihood landscape

Solution:

- Telescope-Method (Boehl, 2019)
- TEnKF & IPA-Smoother (Boehl, 2019)
- Nice tools from astrophysics (modefinding & sampling)

Method + Filter + Sampler = fast

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Solution Method

$$E_t \begin{bmatrix} \mathbf{x}_{t+1} \\ \mathbf{v}_t \end{bmatrix} = \mathbf{N} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_{t-1} \end{bmatrix} + \mathbf{c} \max \left\{ \mathbf{b} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_{t-1} \end{bmatrix}, \bar{r} \right\}$$

- Linear Problem:
  - $\mathbf{v}_{t-1}$ - vector of states
  - $\mathbf{x}_t$ - vector of forward looking variables
  - N - system matrix
- Solution $\mathbf{v}_t = \mathbf{\Omega} \mathbf{v}_{t-1}$ well known (Blanchard-Kahn, Schur-Decomposition,...)
- Nice:
  - (linear) Kalman Filter can be used
  - Everything is just linear algebra
  - (Sufficiently) fast, reference implemetations exist (`dynare`, ...)

Solution Method

$$E_t \begin{bmatrix} \mathbf{x}_{t+1} \\ \mathbf{v}_t \end{bmatrix} = \mathbf{N} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_{t-1} \end{bmatrix} + \mathbf{c} \max \left\{ \mathbf{b} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_{t-1} \end{bmatrix}, \bar{r} \right\}$$

- Nonlinear Problem:
  - $\mathbf{v}_t$ - vector of states
  - $\mathbf{x}_t$ - vector of forward looking variables
  - N - system matrix
  - b row vector
  - $\bar{r}$ - maximum value of constrained variable
    $r = \max \left\{ \mathbf{b} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_{t-1} \end{bmatrix}, \bar{r} \right\}$
  - c - vector of coefficients on constrained variables
- Duration of ZLB spell to be determined endogenously
- Piecewise linear representation of model
- Avoid simulations at runtime

Solution: Closed-Form Representation

- l: periods of transition toward ZLB
- k: periods at the ZLB

$$
\begin{aligned}
L_s(l,k,\mathbf{v}_{t-1}) = &\begin{bmatrix} \mathbf{x}_{t+1+s} \\ \mathbf{v}_{t+s}. \end{bmatrix} \\
= &\mathbf{N}^{\max\{s-l,0\}} \left(\mathbf{N}+\mathbf{cb}\right)^{\min\{l,s\}} S(l,k,\mathbf{v}_{t-1}) \\
&+ (\mathbf{I}-\mathbf{N})^{-1}(\mathbf{I}-\mathbf{N}^{\max\{s-l,0\}})\mathbf{c}\bar{r}
\end{aligned}
$$

- $S(l,k,\mathbf{v}_{t-1}) = \left\{ \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_{t-1} \end{bmatrix} : \mathbf{QN}^k \left(\mathbf{N}+\mathbf{cb}\right)^l \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_{t-1} \end{bmatrix} = -\mathbf{Q}(\mathbf{I}-\mathbf{N})^{-1}(\mathbf{I}-\mathbf{N}^k)\mathbf{c}\bar{r} \right\}$
- $\mathbf{Q} = \begin{bmatrix} \mathbf{I} & -\mathbf{\Omega} \end{bmatrix}$ ($\mathbf{x}_t = \mathbf{\Omega}\mathbf{v}_{t-1}$ is linear RE solution)
- Solution found by **iterating over** (l,k) until equilibrium conditions satisfied details

Solution: Implementation (`pydsge` package)

- l: periods of transition toward ZLB
- k: periods at the ZLB
- $L_s(l, k, \mathbf{v}_{t-1}) = \begin{bmatrix} \mathbf{x}_{t+1+s} \\ \mathbf{v}_{t+s}. \end{bmatrix} =$ `LL_jit(l,k,s,v,*stuff)`

```
@njit(nogil=True, cache=True)
def bruite_wrapper(b, x_bar, v, mat, term):

    l_max = mat.shape[0] - 1
    k_max = mat.shape[1] - 1

    for l in range(l_max):
        for k in range(1, k_max):
            if l:
                if b @ LL_jit(l, k, 0, v, mat, term) - x_bar < 0:
                    continue
                if b @ LL_jit(l, k, l-1, v, mat, term) - x_bar < 0:
                    continue
            if b @ LL_jit(l, k, k+l, v, mat, term) - x_bar < 0:
                continue
            if b @ LL_jit(l, k, l, v, mat, term) - x_bar > 0:
                continue
            if b @ LL_jit(l, k, k+l-1, v, mat, term) - x_bar > 0:
                continue
            return l, k
    return 999, 999
```

- Five necessary **equilibrium conditions**

Solution: Implementation (`pydsge` package)

- l: periods of transition toward ZLB
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                continue
            return l, k
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```

- Efficient implementation (`numba`): ≈ 80.000 particles per second

Bayesian Filtering: Transposed-Ensemble KF

$$\begin{aligned} \mathbf{v}_t &= g(\mathbf{v}_{t-1}, \varepsilon_t), & \varepsilon_t &\sim \mathcal{N}(0, Q), \\ \mathbf{z}_t &= h(\mathbf{v}_t) + \nu_t, & \nu_t &\sim N(0, R) \end{aligned}$$

- **Problem**: nonlinear law-of-motion
- Normally: Particle Filter (PF)
- But: PF requires many (many [many]) particles ($> 10.000.000$)

Too expensive!

- **Solution:** Transposed-Ensemble Kalman Filter

Bayesian Filtering: Transposed-Ensemble KF

$$\begin{aligned}\mathbf{v}_t &= g(\mathbf{v}_{t-1}, \varepsilon_t), & \varepsilon_t &\sim \mathcal{N}(0, Q), \\ \mathbf{z}_t &= h(\mathbf{v}_t) + \nu_t, & \nu_t &\sim N(0, R)\end{aligned}$$

- Assume states are approx. Gaussian each t
- Ensemble (=small cloud of particles):

$$\mathbf{X}_t = [\mathbf{x}_t^1, \cdots, \mathbf{x}_t^N] \in \mathbb{R}^{n \times N}, \qquad \mathbf{X}_0 \overset{N}{\sim} \mathcal{N}(\bar{x}_0, P_0)$$

- **Predict** (like Particle Filter):

$$\begin{aligned}\mathbf{X}_{t|t-1} &= g(\mathbf{X}_{t-1|t-1}, \boldsymbol{\varepsilon}_t) \\ \mathbf{Z}_{t|t-1} &= h(\mathbf{X}_{t|t-1})\end{aligned}$$

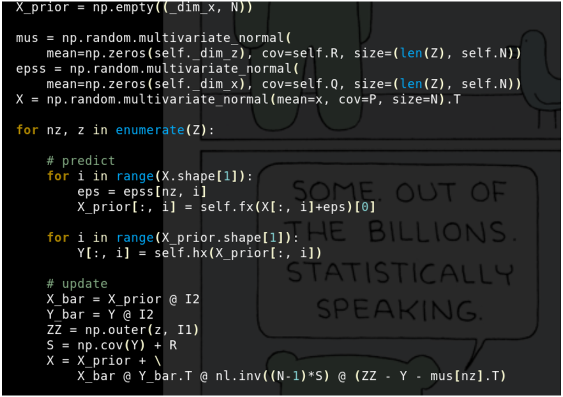
- **Update** (like Kalman Filter):

$$\mathbf{X}_{t|t} = \mathbf{X}_{t|t-1} + \bar{\mathbf{X}}_{t|t-1} \bar{\mathbf{Z}}_{t|t-1}^{\intercal} \left(\bar{\mathbf{Z}}_{t|t-1} \bar{\mathbf{Z}}_{t|t-1}^{\intercal} \right)^{-1} \left(z_t \mathbf{1}^{\intercal} - \mathbf{Z}_{t|t-1} \right).$$

(with *anomalies* $\bar{\mathbf{x}}_t = \mathbf{x}_t(\mathbf{I}_N - \mathbf{1}\mathbf{1}^{\intercal}/N)$)

Bayesian Filter: Implementation (`econsieve` package)

- For 42 states 300 particles seem sufficient (robustness)



```
X_prior = np.empty((_dim_x, N))

mus = np.random.multivariate_normal(
    mean=np.zeros(self._dim_z), cov=self.R, size=(len(Z), self.N))
epss = np.random.multivariate_normal(
    mean=np.zeros(self._dim_x), cov=self.Q, size=(len(Z), self.N))
X = np.random.multivariate_normal(mean=x, cov=P, size=N).T

for nz, z in enumerate(Z):

    # predict
    for i in range(X.shape[1]):
        eps = epss[nz, i]
        X_prior[:, i] = self.fx(X[:, i]+eps)[0]

    for i in range(X_prior.shape[1]):
        Y[:, i] = self.hx(X_prior[:, i])

    # update
    X_bar = X_prior @ I2
    Y_bar = Y @ I2
    ZZ = np.outer(z, I1)
    S = np.cov(Y) + R
    X = X_prior + \
        X_bar @ Y_bar.T @ nl.inv((N-1)*S) @ (ZZ - Y - mus[nz].T)
```


- (in-)Efficient implementation (not yet **numba**): $\approx$ 1 second per parameter draw

Bayesian Smoothing: IPA Smoother

- **Smoothing** (Transpose Ensemble Rauch-Tung-Striebel smoother):

$$\mathbf{X}_{t|T} = \mathbf{X}_{t|t} + \bar{\mathbf{X}}_{t|t}\bar{\mathbf{X}}_{t+1|t}^{+}\left[\mathbf{X}_{t+1|T} - \mathbf{X}_{t+1|t}\right] \tag{1}$$

- **Extraction/Decomposition** (Iterative path-adjustmend):
  - fully reflects the nonlinearity of the transition function
  - interested in shocks $\{\varepsilon_t\}_{t=0}^{T-1}$ that fully recover smoothened states (historical decomposition!)
  - Initiate $\hat{x}_0 = \mathrm{E}\mathbf{X}_{0|T}$, define $P_{t|T} = \mathrm{Cov}\{\mathbf{X}_{t|T}\}$.
  - For each t:

$$\hat{\varepsilon}_t = \arg\max_{\varepsilon}\left\{\log f_N\left(g(\hat{x}_{t-1},\varepsilon)|\bar{x}_{t|T},P_{t|T}\right)\right\}, \tag{2}$$

$$\hat{x}_t = g(\hat{x}_{t-1},\hat{\varepsilon}_t), \tag{3}$$

Sampling

Method + Filter + ? = fast

- Non-linear Filtering to obtain likelihood
  - Accurate results for small number of particles & many states
  - Preserves non-linearity of transition function
- **Benchmark:** $\approx 1s$ per draw (66 dimensions)
- **Problem:** likelihood function not very nice

Figure: Exclusive high-resolution footage of the likelihood function

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Estimation

- **Mode finding:** `PYGMO` (Biscani, Izzo & Yam, 2010)
  - Use heuristic global optimizers
  - Particle swarms, GA's, ants & bees
  - Massive parallelization and sulution candidate exchange

- **Posterior sampling:** Affine Invariant MCMC Ensemble sampler (Goodman & Weare, 2010)
  - Multiple self-tuning chains (parameter free)
  - Efficient parallelization

Implementation in `Python` (`pydsge` package on GitHub)

- **Benchmark:**
  - 42 dimensions take 2-3h
  - 40 cores with 3.1MHz, 32 GB RAM (not crucial), `Arch Linux`

Application: The Smets-Wouters Model™

Smets & Wouters (2007)

Model:

- The workhorse model in macro
- **Agents:** HH, unions, intermediate good producers, retailers, ...
- **Features:** closed economy, habit formation, variable capital utilization, Kimball, sticky prices & wages ...
- 7 exogenous disturbances

Our data:

- Full sample: 1966Q1 - 2018Q1 (incl ZLB period! SW: 2004Q1)
- 7 Observables:
  - growth rates of GDP, consumption, investment, real wages
  - GDP deflator
  - labor hours
  - effective federal fund rate

Full sample: posteriors modes (swarms)

```
loglike  -1015.872 -1015.872 -1021.104 -1021.104 -1024.509   -1024.509
ipython rsh/bs18/code/reprocess.py  7.73s user 0.20s system 99% cpu 7.956 total
gboehl@dyfp ~ % p rsh/bs18/code/reprocess.py
```

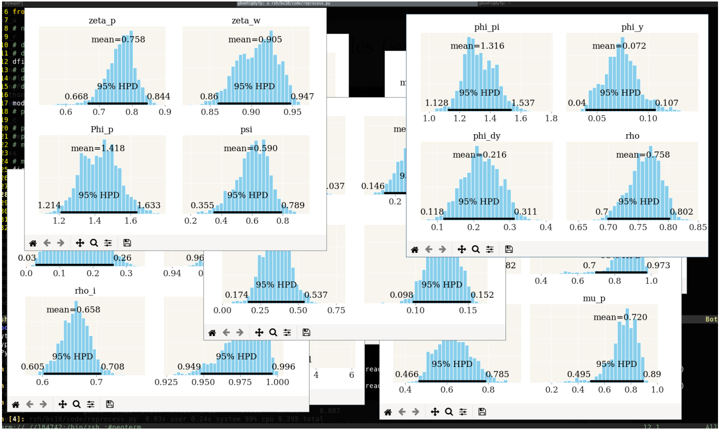
	PSO_0	GACO_1	ABC_2	GACO_3	GACO_4	sa-DE1220_5	ABC_6	sa-DE1220_7	GACO_8
sig_c	1.137	1.137	1.096	1.096	1.454	1.454	1.243	1.243	1.868
sig_l	2.933	2.933	3.440	3.440	2.830	2.830	2.143	2.143	2.909
tpr_beta	0.209	0.209	0.096	0.096	0.344	0.344	0.299	0.299	0.195
h	0.603	0.603	0.430	0.430	0.505	0.505	0.581	0.581	0.428
phiss	8.099	8.099	2.298	2.298	8.333	8.333	6.547	6.547	10.244
i_p	0.577	0.577	0.292	0.292	0.451	0.451	0.463	0.463	0.615
i_w	0.261	0.261	0.478	0.478	0.285	0.285	0.303	0.303	0.557
alpha	0.127	0.127	0.151	0.151	0.087	0.087	0.103	0.103	0.121
zeta_p	0.811	0.811	0.656	0.656	0.836	0.836	0.787	0.787	0.867
zeta_w	0.910	0.910	0.792	0.792	0.886	0.886	0.841	0.841	0.821
Phi_p	1.453	1.453	1.543	1.543	1.255	1.255	1.583	1.583	1.458
psi	0.593	0.593	0.673	0.673	0.345	0.345	0.624	0.624	0.525
phi_pi	1.380	1.380	1.533	1.533	1.328	1.328	1.303	1.303	1.271
phi_y	0.071	0.071	0.070	0.070	0.076	0.076	0.096	0.096	0.093
phi_dy	0.176	0.176	0.236	0.236	0.290	0.290	0.196	0.196	0.138
rho	0.770	0.770	0.786	0.786	0.742	0.742	0.770	0.770	0.757
rho_r	0.207	0.207	0.036	0.036	0.156	0.156	0.226	0.226	0.117
rho_g	0.988	0.988	0.987	0.987	0.991	0.991	0.975	0.975	0.991
rho_i	0.649	0.649	0.986	0.986	0.595	0.595	0.639	0.639	0.598
rho_z	0.977	0.977	0.985	0.985	0.990	0.990	0.976	0.976	0.982
rho_u	0.958	0.958	0.941	0.941	0.947	0.947	0.960	0.960	0.941
rho_p	0.575	0.575	0.858	0.858	0.593	0.593	0.613	0.613	0.441
rho_w	0.821	0.821	0.753	0.753	0.828	0.828	0.843	0.843	0.834
mu_p	0.385	0.385	0.675	0.675	0.436	0.436	0.365	0.365	0.278
mu_w	0.715	0.715	0.371	0.371	0.696	0.696	0.728	0.728	0.597
rho_gz	0.632	0.632	0.523	0.523	0.522	0.522	0.646	0.646	0.642
sig_g	2.574	2.574	0.216	0.216	2.566	2.566	2.221	2.221	0.103
sig_u	0.128	0.128	2.946	2.946	0.774	0.774	0.253	0.253	0.104
sig_z	0.087	0.087	0.143	0.143	0.172	0.172	0.066	0.066	0.403
sig_r	0.082	0.082	0.105	0.105	0.158	0.158	0.135	0.135	0.210
sig_i	0.085	0.085	0.493	0.493	0.353	0.353	0.258	0.258	0.066
sig_p	0.658	0.658	0.056	0.056	0.049	0.049	1.683	1.683	0.281
sig_w	0.120	0.120	0.360	0.360	0.092	0.092	0.063	0.063	0.494
trend	0.437	0.437	0.399	0.399	0.475	0.475	0.473	0.473	0.453
mean_l	5.585	5.585	2.489	2.489	6.922	6.922	5.242	5.242	6.598
mean_Pi	0.738	0.738	0.682	0.682	0.870	0.870	0.823	0.823	0.712
loglike	-1015.872	-1015.872	-1021.104	-1021.104	-1024.509	-1024.509	-1026.595	-1026.595	-1027.455

```
ipython rsh/bs18/code/reprocess.py  7.93s user 0.15s system 99% cpu 8.100 total
gboehl@dyfp ~ %
```

Full sample: summary

```
ipython rsh/bs18/code/reprocess.py  7.98s user 0.16s system 99% cpu 8.166 total
gboehl@dyfp ~ % p rsh/bs18/code/reprocess.py
             distribution   mean  sd/df   mean     sd  mc_error  hpd_2.5  hpd_97.5
sig_c              normal  1.500  0.375  1.283  0.229     0.011    0.905     1.713
sig_l              normal  2.000  0.750  2.573  0.520     0.025    1.606     3.514
tpr_beta            gamma  0.250  0.100  0.251  0.098     0.005    0.061     0.439
h                    beta  0.700  0.100  0.518  0.063     0.003    0.404     0.633
phiss              normal  4.000  1.500  6.874  2.260     0.112    1.968     9.741
i_p                  beta  0.500  0.150  0.379  0.119     0.006    0.156     0.593
i_w                  beta  0.500  0.150  0.312  0.128     0.006    0.059     0.518
alpha              normal  0.300  0.050  0.126  0.018     0.001    0.091     0.157
zeta_p               beta  0.500  0.100  0.748  0.061     0.003    0.613     0.835
zeta_w               beta  0.500  0.100  0.775  0.086     0.004    0.616     0.911
Phi_p              normal  1.250  0.125  1.443  0.105     0.005    1.268     1.657
psi                  beta  0.500  0.150  0.593  0.140     0.007    0.375     0.903
phi_pi             normal  1.500  0.250  1.443  0.125     0.006    1.215     1.694
phi_y              normal  0.125  0.050  0.070  0.022     0.001    0.022     0.110
phi_dy             normal  0.125  0.050  0.235  0.059     0.003    0.118     0.351
rho                  beta  0.750  0.100  0.770  0.024     0.001    0.727     0.818
rho_r                beta  0.500  0.200  0.108  0.052     0.003    0.010     0.200
rho_g                beta  0.500  0.200  0.986  0.008     0.000    0.971     0.999
rho_i                beta  0.500  0.200  0.699  0.121     0.006    0.584     0.987
rho_z                beta  0.500  0.200  0.985  0.008     0.000    0.971     0.999
rho_u                beta  0.500  0.200  0.954  0.012     0.001    0.931     0.977
rho_p                beta  0.500  0.200  0.707  0.113     0.006    0.495     0.905
rho_w                beta  0.500  0.200  0.828  0.097     0.005    0.663     0.979
mu_p                 beta  0.500  0.200  0.478  0.130     0.006    0.248     0.710
mu_w                 beta  0.500  0.200  0.538  0.135     0.007    0.271     0.783
rho_gz             normal  0.500  0.250  0.523  0.073     0.004    0.379     0.653
sig_g    inv_gamma_dynare  0.100  2.000  0.990  1.086     0.053    0.025     3.203
sig_u    inv_gamma_dynare  0.100  2.000  0.761  1.107     0.055    0.022     3.194
sig_z    inv_gamma_dynare  0.100  2.000  0.504  0.649     0.032    0.021     1.940
sig_r    inv_gamma_dynare  0.100  2.000  0.355  0.430     0.021    0.020     1.234
sig_i    inv_gamma_dynare  0.100  2.000  0.470  0.640     0.032    0.020     2.126
sig_p    inv_gamma_dynare  0.100  2.000  0.453  0.667     0.033    0.022     1.938
sig_w    inv_gamma_dynare  0.100  2.000  0.461  0.499     0.025    0.021     1.397
trend              normal  0.400  0.100  0.446  0.025     0.001    0.388     0.487
mean_l             normal  4.000  2.000  5.017  1.207     0.058    2.854     7.685
mean_Pi             gamma  0.625  0.100  0.685  0.117     0.006    0.430     0.887
ipython rsh/bs18/code/reprocess.py  8.03s user 0.24s system 99% cpu 8.295 total
gboehl@dyfp ~ %
```

Full sample: posterior plots



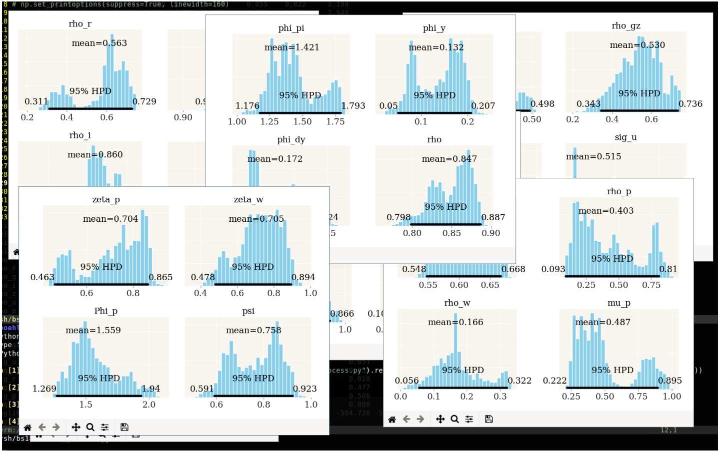
Short sample ('98-'18): posteriors modes (swarms)

```
gboehl@dyfp ~ % p rsh/bs18/code/reprocess.py
```

	PSO_0	GACO_1	ABC_2	GACO_3	GACO_4	sa-DE1220_5	ABC_6	sa-DE1220_7	GACO_8
sig_c	1.272	2.020	2.020	2.020	2.020	1.915	1.915	1.853	0.650
sig_l	0.428	1.289	1.289	1.289	1.289	0.863	0.863	0.738	1.100
tpr_beta	0.313	0.180	0.180	0.180	0.152	0.107	0.107	0.239	0.152
h	0.855	0.758	0.758	0.758	0.758	0.774	0.774	0.811	0.927
phiss	7.517	7.601	7.601	7.601	7.601	6.730	6.730	7.046	8.448
i_p	0.101	0.607	0.607	0.607	0.607	0.453	0.453	0.482	0.391
i_w	0.534	0.604	0.596	0.596	0.508	0.528	0.528	0.579	0.392
alpha	0.138	0.111	0.111	0.111	0.111	0.141	0.141	0.133	0.074
zeta_p	0.539	0.844	0.844	0.844	0.844	0.755	0.755	0.682	0.684
zeta_w	0.521	0.767	0.816	0.816	0.816	0.648	0.648	0.797	0.567
Phi_p	1.537	1.471	1.398	1.398	1.471	1.836	1.836	1.650	1.557
psi	0.675	0.850	0.850	0.850	0.850	0.846	0.846	0.633	0.836
phi_pi	1.700	1.341	1.341	1.341	1.341	1.248	1.248	1.444	1.499
phi_y	0.088	0.169	0.169	0.169	0.169	0.182	0.182	0.068	0.189
phi_dy	0.198	0.135	0.135	0.135	0.135	0.217	0.217	0.138	0.037
rho	0.829	0.860	0.860	0.860	0.860	0.878	0.878	0.856	0.798
rho_r	0.681	0.609	0.609	0.609	0.609	0.382	0.382	0.587	0.607
rho_g	0.986	0.984	0.986	0.986	0.988	0.996	0.996	0.982	0.992
rho_i	0.878	0.850	0.850	0.850	0.850	0.890	0.890	0.855	0.769
rho_z	0.925	0.916	0.916	0.916	0.916	0.885	0.885	0.908	0.896
rho_u	0.590	0.649	0.649	0.649	0.649	0.615	0.615	0.591	0.715
rho_p	0.740	0.166	0.166	0.166	0.166	0.454	0.454	0.225	0.319
rho_w	0.291	0.158	0.113	0.113	0.158	0.159	0.159	0.120	0.406
mu_p	0.503	0.349	0.349	0.349	0.349	0.814	0.814	0.248	0.418
mu_w	0.485	0.423	0.423	0.423	0.423	0.439	0.439	0.439	0.575
rho_gz	0.617	0.576	0.576	0.576	0.576	0.533	0.533	0.320	0.862
sig_g	0.010	0.010	0.057	0.057	0.010	0.010	0.010	0.010	1.500
sig_u	0.222	0.127	0.127	0.127	0.127	1.649	1.649	0.051	0.025
sig_z	0.088	0.282	0.282	0.282	0.282	0.212	0.212	0.139	0.229
sig_r	2.147	0.143	0.542	0.542	0.143	0.010	0.010	0.010	0.010
sig_i	1.330	0.625	0.030	0.030	0.625	1.556	1.556	0.477	1.960
sig_p	0.010	0.196	0.196	0.196	0.196	0.010	0.010	0.506	0.010
sig_w	0.173	0.010	0.010	0.010	0.010	0.175	0.175	0.080	0.010
loglike	-501.928	-502.213	-502.420	-502.420	-504.011	-504.626	-504.626	-504.726	-505.039

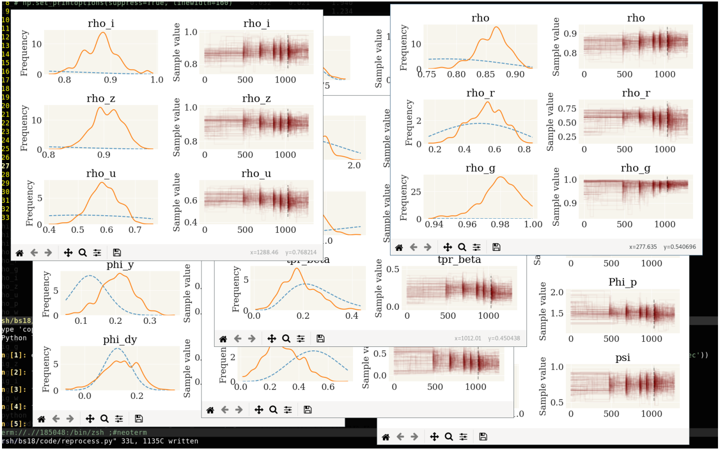
```
ipython rsh/bs18/code/reprocess.py 8.00s user 0.16s system 99% cpu 8.180 total
gboehl@dyfp ~ %
```

Short sample ('98-'18): posterior plots (MCMC)



Problem: Chains not designed for bimodal distributions

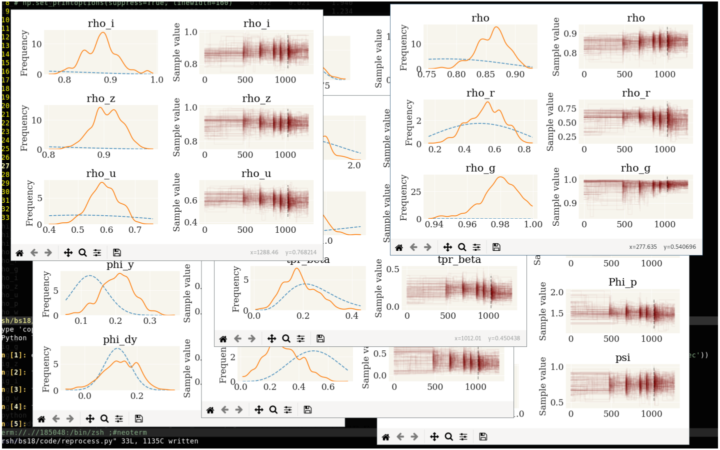
Short sample ('98-'18): trace plots (alternative sampler)



kombine **sampler:**

multi-chains with KDE based proposal distribution

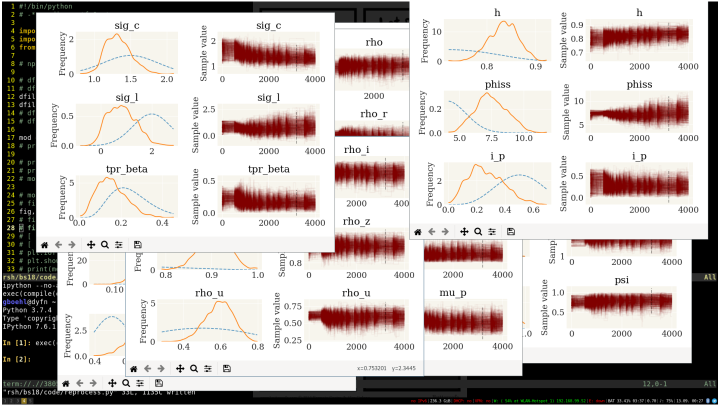
Short sample ('98-'18): trace plots (alternative sampler)



Problem:

System probably too high-dimensional $\rightsquigarrow$ try more walkers!

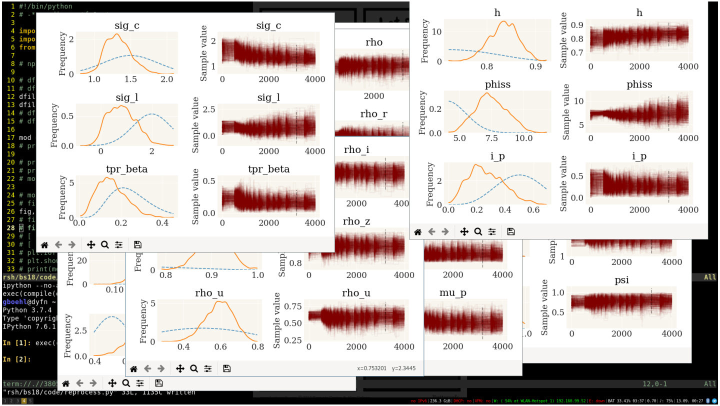
Short sample ('98-'18): trace plots (400 chains)



"A lot helps a lot."

— Confucius

Short sample ('98-'18): trace plots (400 chains)



"A lot helps a lot."?!

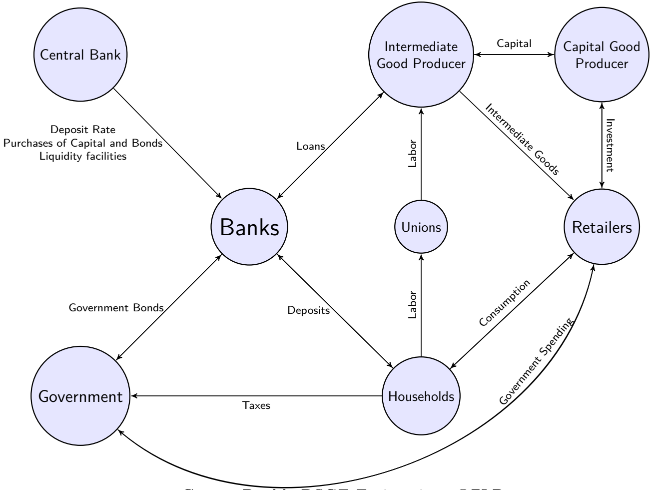
— Confucius

Application: the Real Effects of QE

–Model–

Model Overview details

Smets & Wouters (2007)	+	Gertler and Karadi (2013)	=
NK + bells + whistles	+	banks + QE	



Monetary Policy: Liquidity Facilities

- Liquidity injections relax the incentive constraint of banks (Gertler and Kiyotaki, 2010)

$$V_t \geq \lambda Q_t K_{b,t} + \lambda_b Q_t^b B_{b,t} - \lambda_{cbl} CBL_t,$$

- CBL_t - central bank liquidity injections
- λ_{cbl} - degree to which the liquidity injections relax the constraint.
- Central bank lends out liquidity at zero interest
- Model: exogenous AR(1) process (but fed with time series)

Monetary Policy: LSAP & FG

LSAP

$$X_{K,t} = \frac{K_t - K_{b,t}}{K_t}, \qquad X_{B,t} = \frac{B_t - B_{b,t}}{B_t}.$$

- $X_{K,t}$ and $X_{B,t}$ - share of central bank holdings in economy-wide capital assets and gov. bonds
- Model: exogenous AR(1) process (fed with time series)

Taylor Rule / Forward Guidance

$$\begin{aligned} R_t =& \max\{0, \rho \ln R_{t-1} + (1-\rho)(\phi_\pi \pi_t + \phi_y y_t + \phi_{\Delta y} \Delta y_t) + v_{r,t}\} \\ v_{r,t} =& \rho v_{r,t-1} + \epsilon_t \end{aligned}$$

- negative v_t also lowers *expected* interest rate
- @ZLB constitutes (positive) forward guidance shock

Application: the Real Effects of QE

– Quantitative Results –

Data

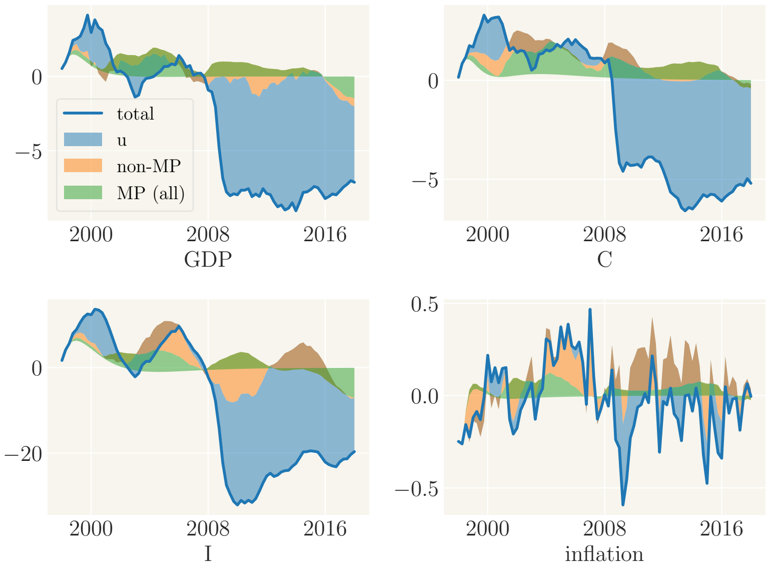
- Sample: 1998Q1 - 2018Q1 (USA)
  - short sample to adequately capture properties of recent decades
- 10 Observables:
  - as in Smets and Wouters (2007): growth rates of GDP, consumption, investment, real wages, GDP deflator, labor hours, effective federal fund rate
  - CB holdings of private assets (MBS + corporate bonds)
  - CB holdings of Treasuries (all maturities)
  - Liquidity facilities (TAF + CB Liquidity Swaps + ...)

Filtered Series

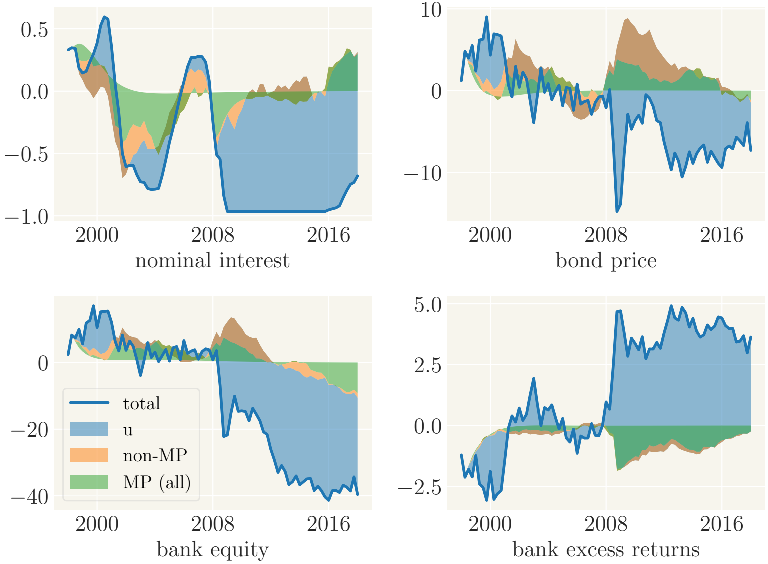
Some key parameter estimates estimates no banks

	distribution	mean	sd	mean	sd	hpd_2.5	hpd_97.5
$\imath_p$	beta	0.500	0.15	0.340	0.084	0.174	0.492
$\imath_w$	beta	0.500	0.15	0.445	0.142	0.167	0.733
ζ_p	beta	0.500	0.10	0.805	0.044	0.727	0.887
ζ_w	beta	0.500	0.10	0.680	0.058	0.555	0.783
Φ_p	normal	1.250	0.12	1.305	0.119	1.062	1.516
LEV	normal	3.000	1.00	1.802	0.457	1.131	2.616
θ	beta	0.950	0.05	0.908	0.074	0.762	0.994
λ_{cbl}	uniform	0.000	10.00	2.694	0.842	0.995	4.134
ρ	beta	0.700	0.20	0.784	0.040	0.697	0.849
ρ_u	beta	0.500	0.20	0.766	0.051	0.667	0.862
ρ_r	beta	0.700	0.20	0.488	0.092	0.332	0.683
ρ_g	beta	0.500	0.20	0.838	0.101	0.661	0.983
ρ_i	beta	0.500	0.20	0.816	0.069	0.692	0.944
ρ_z	beta	0.500	0.20	0.583	0.193	0.214	0.888
ρ_p	beta	0.700	0.20	0.260	0.053	0.158	0.363
ρ_w	beta	0.700	0.20	0.455	0.094	0.314	0.660
ρ_{cbl}	beta	0.500	0.20	0.555	0.064	0.439	0.670
ρ_{qe_b}	beta	0.500	0.20	0.863	0.041	0.778	0.945
ρ_{qe_k}	beta	0.500	0.20	0.921	0.033	0.857	0.980
...	...	...	...	...	...	...	...

Historical decomposition I. more ZLB effects no banks

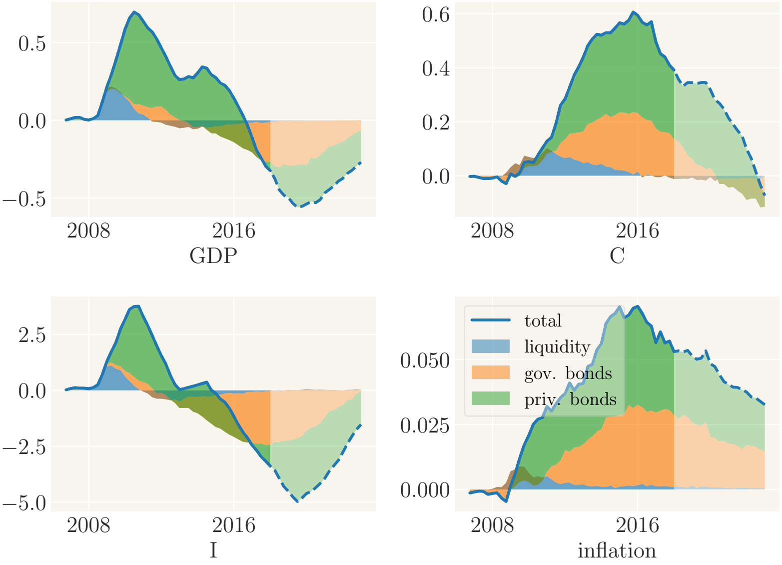


Historical decomposition II. more ZLB effects no banks



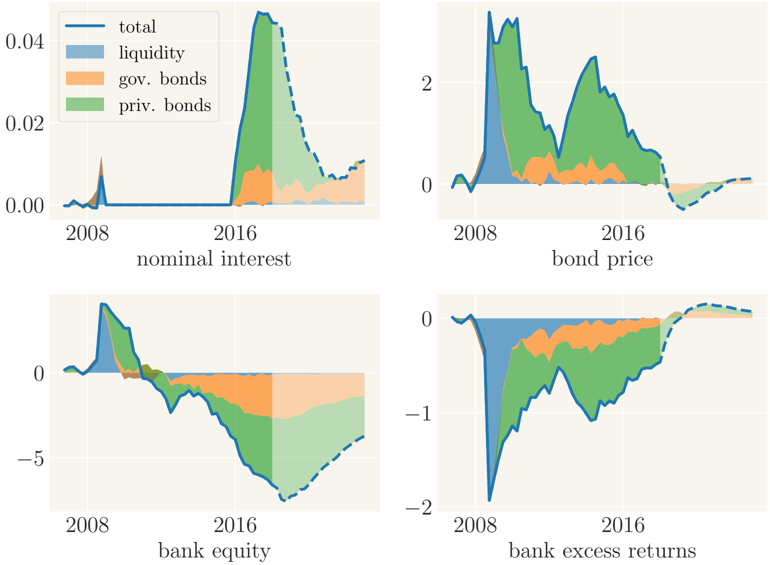
The net effects of QE Measures I

IRFs QE vs. FG



The net effects of QE Measures II

IRFs QE vs. FG



Conclusion

- Tools for:
  - Solution
  - Filtering
  - Estimation of DSGE with OBCs
- Estimate model with banks & FF at the ZLB
- Investigate the effects of QE in US

Todo:

- Some robustness
- Some more robustness
- Robustness
- # comment code
- Provide `documentation` & examples

Thank you for your attention

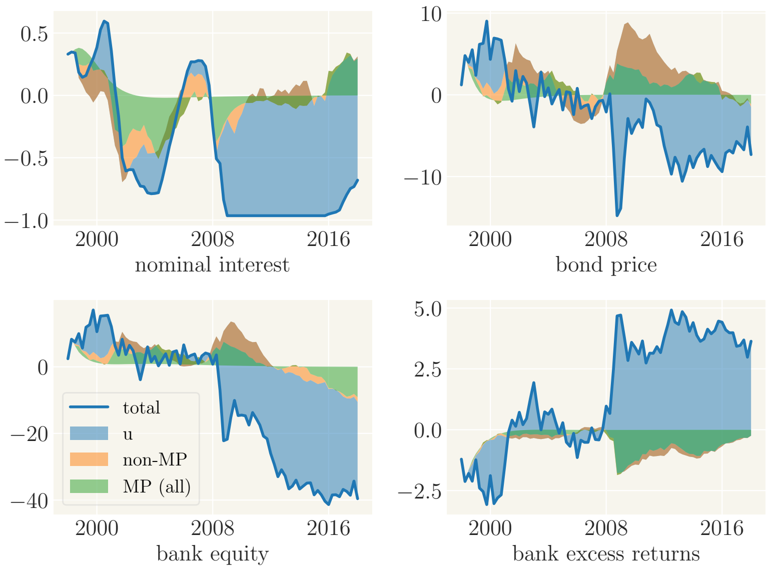
x x x x x x Advertisement x x x x x x

https://github.com/gboehl/macro_puzzles

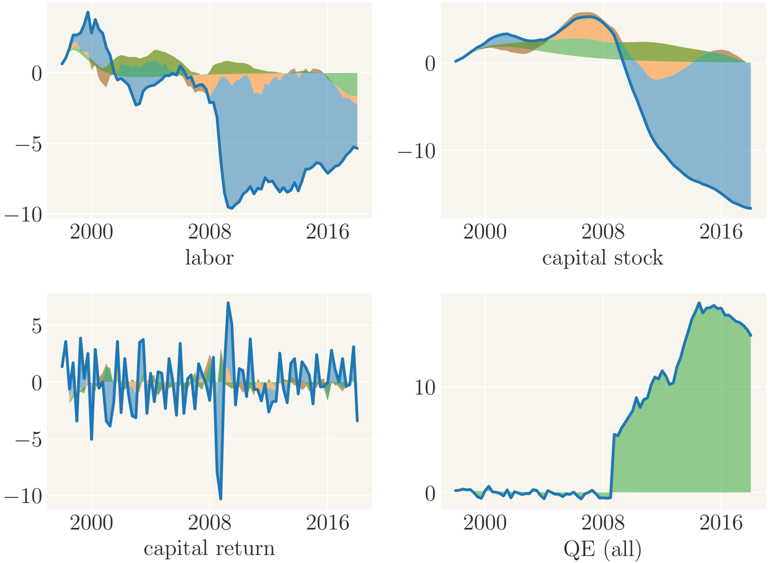
or just

https://gregorboehl.com

Historical decomposition back

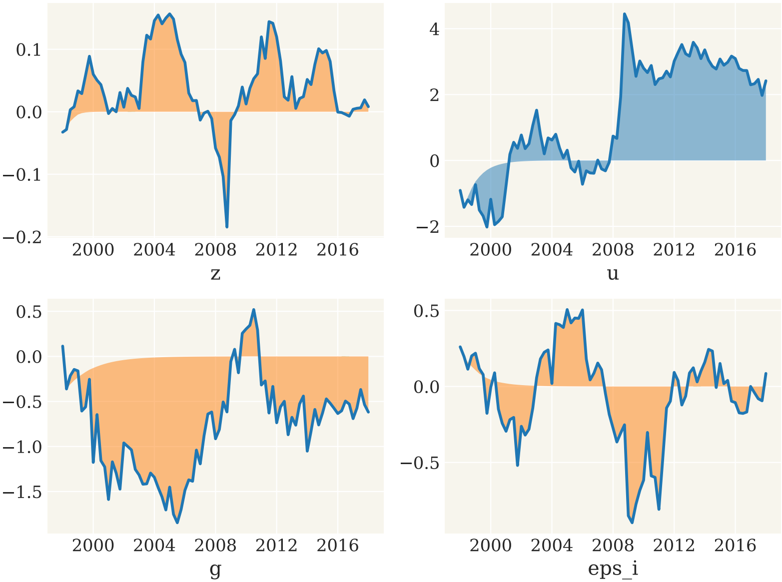


Historical decomposition back

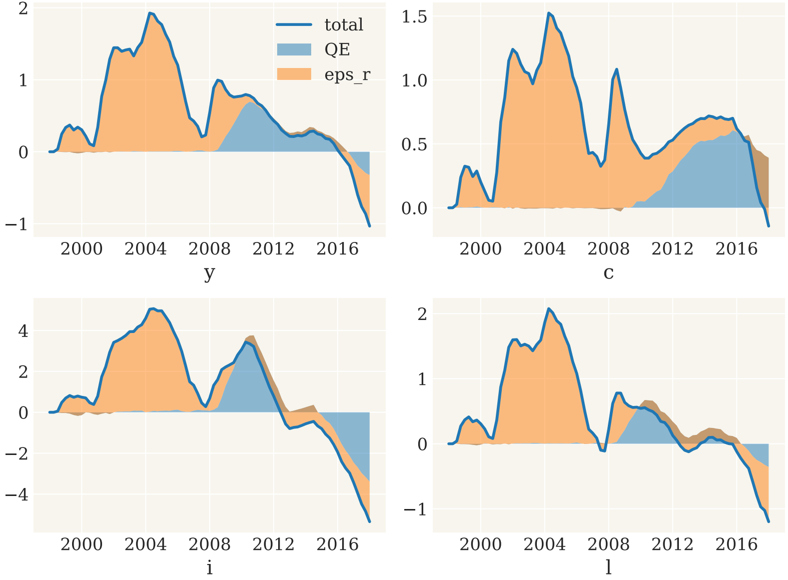


Historical decomposition

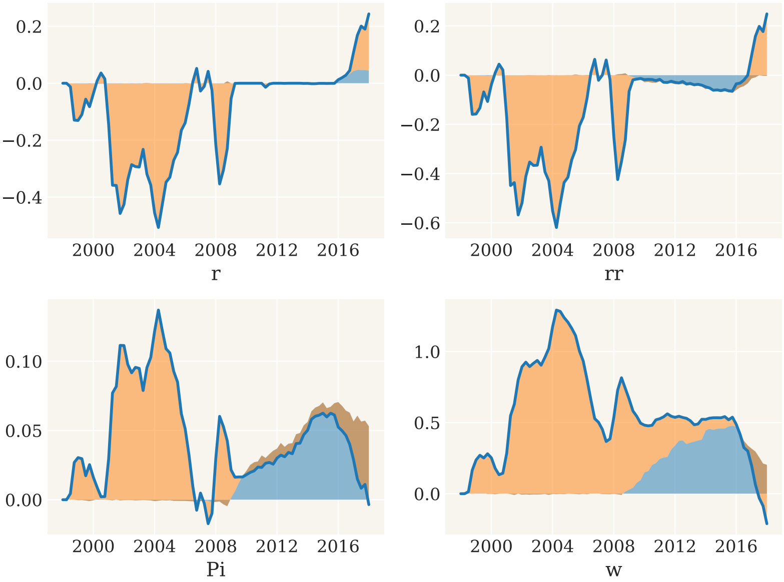
back



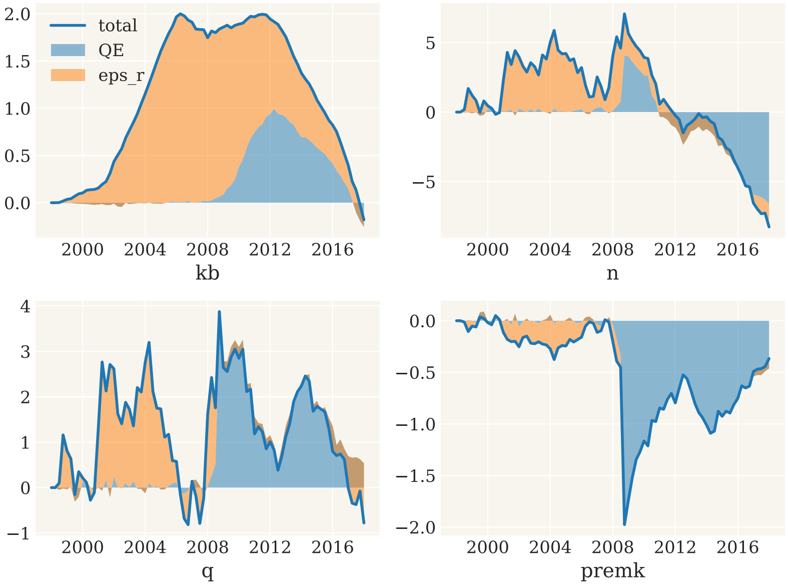
QE vs. FG I back



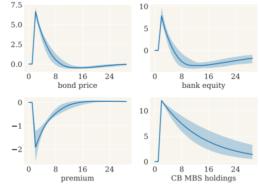
QE vs. FG II back

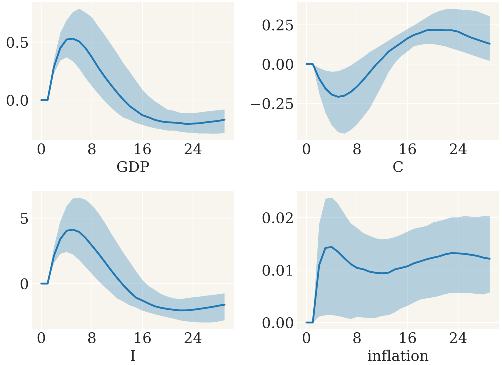


QE vs. FG III back

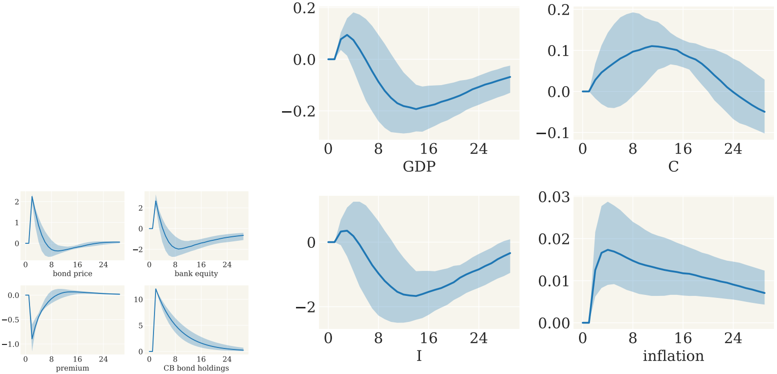


IRF to a shock to CB capital purchases back





IRF to a shock to CB gov. bond purchases back

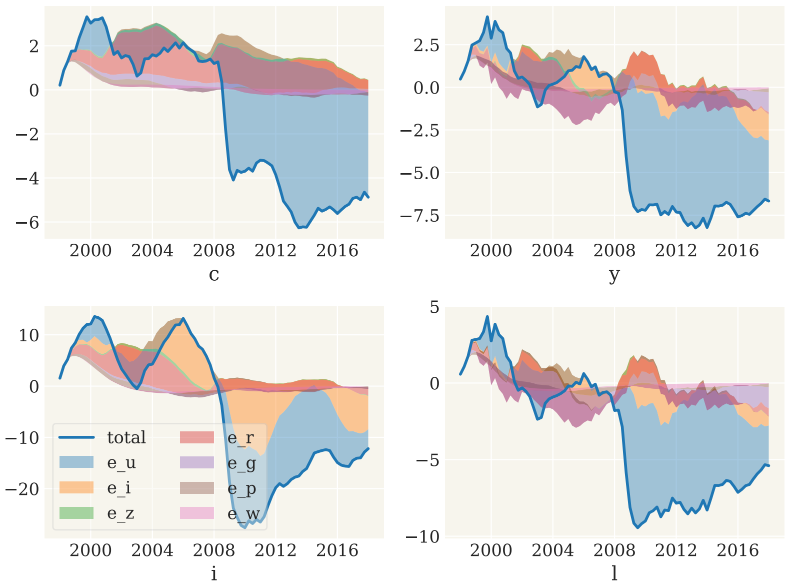


Parameter estimates SW07 model

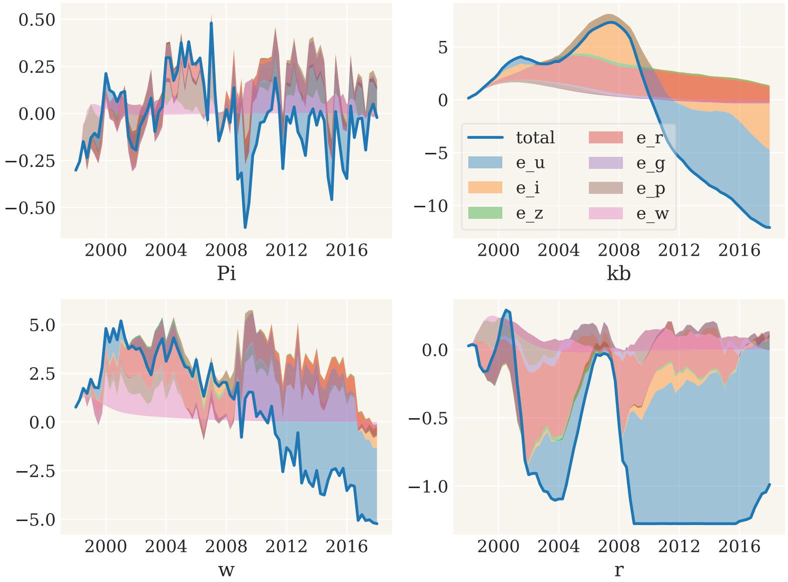
go back

	Prior		Posterior			
	mean	sd	mean	sd	hpd_2.5	hpd_97.5
ζ_p	0.500	0.10	0.758	0.044	0.664	0.835
ζ_w	0.500	0.10	0.630	0.052	0.533	0.732
Φ	1.250	0.12	1.803	0.081	1.658	1.973
ϕ_π	1.700	0.25	1.517	0.257	1.033	2.018
ϕ_y	0.125	0.05	0.199	0.032	0.141	0.263
ϕ_{dy}	0.125	0.05	0.181	0.044	0.092	0.265
ρ	0.700	0.20	0.790	0.047	0.699	0.879
ρ_r	0.700	0.20	0.688	0.114	0.518	0.897
ρ_i	0.500	0.20	0.782	0.139	0.413	0.964
ρ_z	0.500	0.20	0.722	0.165	0.345	0.934
ρ_u	0.500	0.20	0.761	0.044	0.653	0.839
ρ_p	0.700	0.20	0.341	0.091	0.181	0.515
ρ_w	0.700	0.20	0.300	0.056	0.192	0.405
σ_u	0.100	2.00	1.769	0.429	0.925	2.620
σ_z	0.100	2.00	0.214	0.131	0.058	0.505
σ_r	0.100	2.00	0.150	0.078	0.076	0.245
σ_i	0.100	2.00	0.323	0.300	0.119	1.039
σ_p	0.100	2.00	0.284	0.104	0.112	0.496
σ_w	0.100	2.00	1.482	0.313	0.848	2.059

Historical decomposition (no Banks) I back

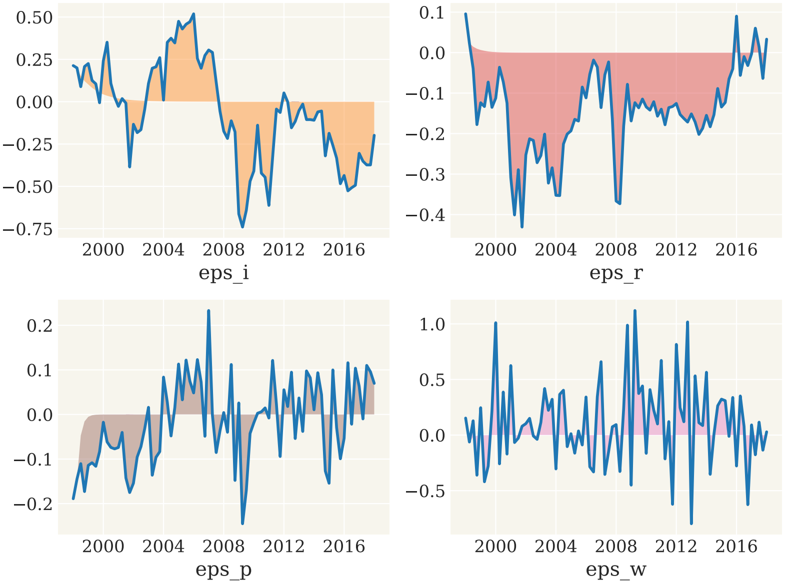


Historical decomposition (no Banks) II back

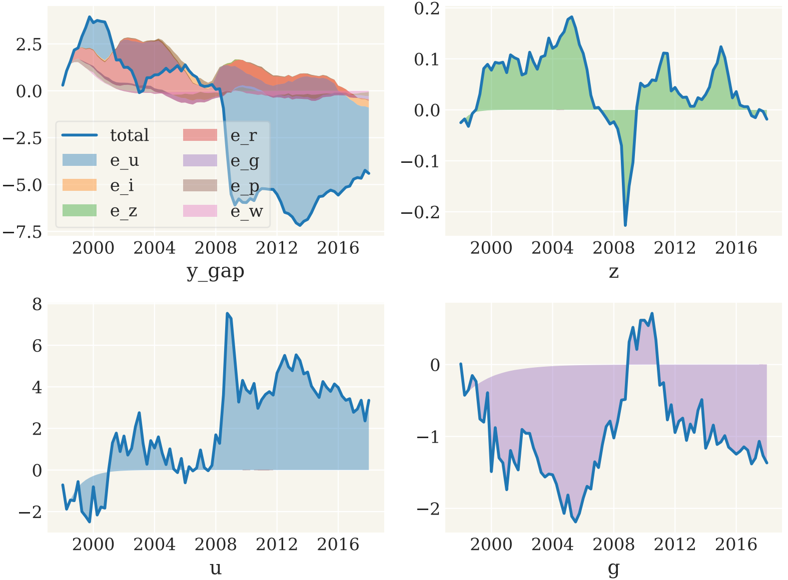


Historical decomposition (no Banks) III

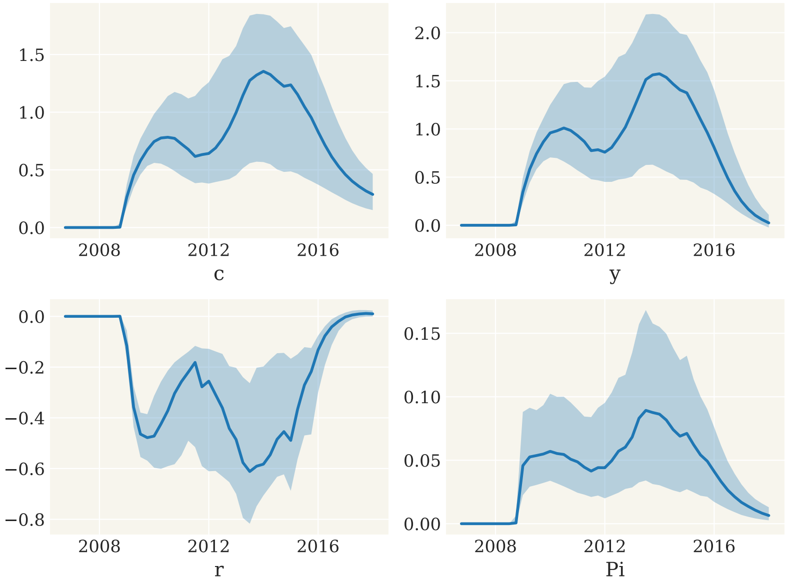
back



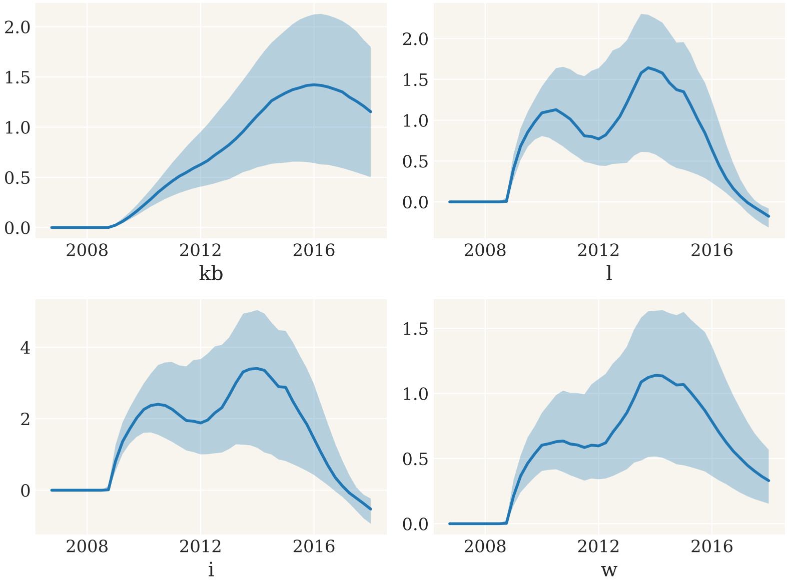
Historical decomposition (no Banks) IV back



(Counterfactual) Effects of hitting the ZLB I back

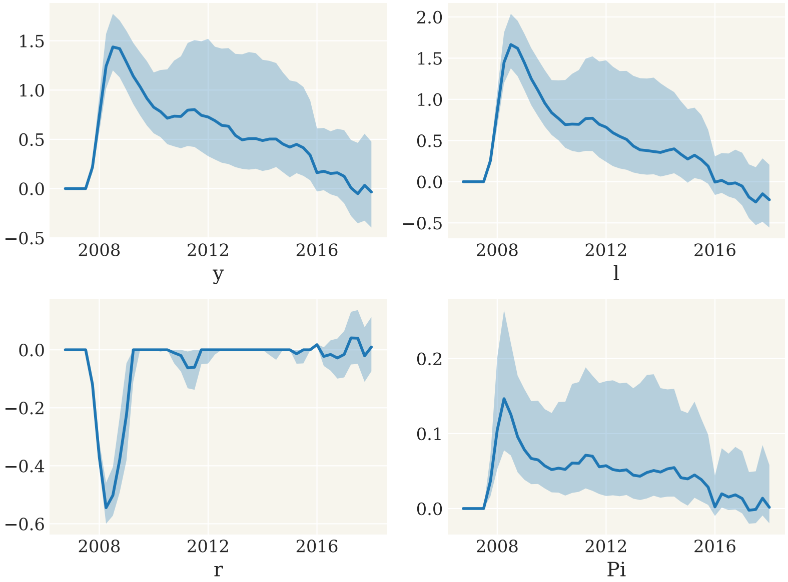


(Counterfactual) Effects of hitting the ZLB II back



(Counterfactual) Effects of MP / Forward Guidance

back



Smoothing & Path-adjustment (Boehl, 2019) go back

- **Smoothing** (Transposed-Ensemble Rauch-Tung-Striebel smoother):

$$\mathbf{X}_{t|T} = \mathbf{X}_{t|t} + \bar{\mathbf{X}}_{t|t}\bar{\mathbf{X}}^{+}_{t+1|t}\left[\mathbf{X}_{t+1|T} - \mathbf{X}_{t+1|t}\right] \tag{4}$$

- **Extraction** (Iterative path-adjustmend):
  - Fully reflects the nonlinearity of the transition function
  - Interested in shocks $\{\varepsilon_t\}_{t=0}^{T-1}$ that fully recover smoothened states (historical decomposition!)
  - Initialize $\hat{x}_0 = \mathrm{E}\mathbf{X}_{0|T}$, define $P_{t|T} = \mathrm{Cov}\{\mathbf{X}_{t|T}\}$.
  - For each t:

$$\hat{\varepsilon}_t = \arg\max_{\varepsilon} \left\{ \log f_N \left(g(\hat{x}_{t-1}, \varepsilon) | \bar{x}_{t|T}, P_{t|T} \right) \right\}, \tag{5}$$

$$\hat{x}_t = g(\hat{x}_{t-1}, \hat{\varepsilon}_t), \tag{6}$$

The (flat) Phillips Curve

	Prior		Posterior			
	mean	sd	mean	sd	hpd_2.5	hpd_97.5
$\imath_p$	0.500	0.15	0.247	0.094	0.072	0.419
$\imath_w$	0.500	0.15	0.452	0.143	0.196	0.723
ζ_p	0.500	0.10	0.758	0.044	0.664	0.835
Φ	1.250	0.12	1.803	0.081	1.658	1.973

$$\pi_t = \frac{\overline{\beta}}{1+\imath_p\overline{\beta}} E_t\pi_{t+1} + \kappa\hat{x}_t + \frac{\imath_p}{1+\imath_p\overline{\beta}}\pi_{t-1}$$

$$\kappa = \frac{(1-\zeta_p\overline{\beta})(1-\zeta_p)}{(1+\overline{\beta}\imath_p)\zeta_p(\epsilon_p(\Phi-1)+1)} \qquad \text{(slope of PC)}$$

$$\hat{x}_t = w_t - z_t + \alpha(l_t - k_t) \qquad \text{(marginal costs)}$$

- SW07: $\kappa \approx 0.02$
- here: $\kappa \approx 0.007$!
- Key-incredient of NK model?

Equilibrium conditions

Definition (transition equilibrium)

A rational expectation solution $S(l^*, k^*)$ is a rational expectations *equilibrium* iff

$$\mathbf{b}L_s(l^*, k^*) \geq \bar{r} \quad \forall s < l^* \wedge s \geq k^* + l^* \tag{7}$$

and

$$\mathbf{b}L_s(l^*, k^*) < \bar{r} \quad \forall l^* \leq s < k^* + l^*. \tag{8}$$

go back

Linearized equilibrium back

$$
\begin{aligned}
c_t =& \frac{1/\gamma}{(1+1/\gamma)} c_{t-1} + \frac{1}{1+1/\gamma} E_t[c_{t+1}] \\
& - \frac{(1-1/\gamma)}{(1+1/\gamma)\sigma_c} (r_t - E_t[\pi_{t+1}] + v_{d,t}) \\
i_t =& \frac{1}{1+\overline{\beta}} [i_{t-1}] + \frac{\overline{\beta}}{1+\overline{\beta}} E_t[i_{t+1}] + \frac{1}{(1+\overline{\beta})\gamma^2 S''} q_t^k \\
\overline{k}_t =& (1-\delta)/\gamma \overline{k}_{t-1} + (1-(1-\delta)/\gamma)\hat{i}_t \\
& + (1-(1-\delta)/\gamma)(1+\overline{\beta})\gamma^2 S'' v_{i,t} \\
R_t - E_t[\pi_{t+1}] + v_{d,t} =& \frac{R^k}{R^k + (1-\delta)} E_t[r_{t+1}^k] + \frac{(1-\delta)}{R^k + (1-\delta)} E_t[q_{t+1}^k] - q_t^k \\
k_t =& \frac{1-\psi}{\psi} r_t^k + \overline{k}_{t-1} \\
k_t =& w_t - r_t^k + l_t \\
y_t =& \Phi(\alpha k_t + (1-\alpha) l_t + z_t)
\end{aligned}
$$

Linearized equilibrium II back

$$
\begin{aligned}
y_t =& \frac{G}{Y} g_t + \frac{C}{Y} c_t + \frac{I}{Y} i_t + \frac{R^k K}{Y} \frac{1-\psi}{\psi} r_t^k \\
\pi_t =& \frac{\overline{\beta}}{1+\imath_p \overline{\beta}} E_t \pi_{t+1} + \frac{\imath_p}{1+\imath_p \overline{\beta}} \pi_{t-1} \\
& + \frac{(1-\zeta_p \overline{\beta})(1-\zeta_p)}{(1+\overline{\beta}\imath_p)\zeta_p((\Phi-1)\epsilon_p+1)} (w_t - z_t + \alpha l_t - \alpha k_t) \\
w_t =& \frac{1}{1+\overline{\beta}\gamma} (w_{t-1} + \imath_w \pi_{t-1}) + \frac{\overline{\beta}\gamma}{1+\overline{\beta}\gamma} E_t [w_{t+1} + \pi_{t+1}] \\
& - \frac{1+\imath_w \overline{\beta}\gamma}{1+\overline{\beta}\gamma} \pi_t + \frac{(1-\zeta_w \overline{\beta}\gamma)(1-\zeta_w)}{(1+\overline{\beta}\gamma)\zeta_w((\lambda_w-1)\epsilon_w+1)} (w_t^h - w_t) \\
w_t^h =& \frac{\sigma_c}{(1-h)} (c_t - h c_{t-1}) + \frac{L}{1-L} l_t \\
r_t =& max\{0, \rho r_{t-1} + (1-\rho)(\phi_\pi \pi_t + \phi_y \widetilde{y}_t + \phi_{dy}(\widetilde{y}_t - \widetilde{y}_{t-1})) + v_{r_t}\}
\end{aligned}
$$

Linearized equilibrium III back

$$
\begin{aligned}
v_{d,t} =& \rho_d v_{d,t-1} + \epsilon_t^d, \\
z_t =& \rho_z z_{t-1} + \epsilon_t^z, \\
g_t =& \rho_g g_{t-1} + \epsilon_t^g, \\
v_{r,t} =& \rho_r v_{r,t-1} + \epsilon_t^r, \\
v_{i,t} =& \rho_i v_{i,t-1} + \epsilon_t^i, \\
v_{p,t} =& \rho_p v_{p,t-1} + \epsilon_t^p, \\
v_{w,t} =& \rho_w v_{w,t-1} + \epsilon_t^w,
\end{aligned}
$$

Households

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - hC_{t-1})^{1-\sigma_c} - 1}{1-\sigma_c} - \nu \, log(1 - L_t) \right)$$

$$P_t C_t + \frac{D_t}{v_{d,t} R_t} = D_{t-1} + W_t L_t - T_t + \mathfrak{P}_t.$$

- C_t - consumption
- L_t - labor
- W_t - real wage
- D_t - bank deposits
- $v_{d,t}$ - risk premium shock
- R_t - real rate on deposits
- T_t - lump sum taxes
- $\mathfrak{P}_t$- profits from firms and banks

back

Unions

$$\max_{W_t(i)} E_t \sum_{s=0}^{\infty} (\beta\zeta_w)^s \frac{\Lambda_{t,t+s}}{\Pi_{t,t+s}} \left[W_t(i)\Pi_{l=1}^{s}(\Pi_{t+l-1}^{\iota_w}\Pi^{1-\iota_w}) - MRS_{t+s}\right] L_{t+s}(i)$$

$$s.t. \frac{L_{t+s}(i)}{L_{t+s}} = G_w^{'-1}\left(\frac{W_t(i)\Pi_{l=1}^{s}(\Pi_{t+l-1}^{\iota_w}\Pi^{1-\iota_w})}{W_{t+s}}\tau_{t+s}^{w}\right).$$

$W_t(i)$ - wage set by union i; ζ_w - Calvo parameter; $\Lambda_{t,t+s}$ - SDF; ι_w - the degree of wage indexation; MRS_t - mraginal rate of substitution; G_w - Kimball aggregator.

$$W_t = [(1-\zeta_w)(W_t^*)G_w^{'-1}\left[\frac{W_t^*\tau_t^w}{W_t}\right] +$$

$$\zeta_w \Pi_{t-1}^{\iota_w}\Pi^{(1-\iota_w)})W_{t-1}G_w^{'-1}\left[\frac{\Pi_{t-1}^{\iota_w}\Pi^{(1-\iota_w)})W_{t-1}\tau_t^w}{W_t}\right],$$

W_t^* - optimal wage

back

Firms

- Intermediate good producers details
  - Cobb Douglas production function; employ labor and capital
  - perfect competition
  - buy and re-sell entire capital stock each period
  - capital purchases are financed with bank loans
- Capital good producers details
  - perfect competition
  - buys and re-sells capital to intermediate good producer
  - repairs used capital, invests in new capital
  - subject to investment adjustment costs.
- Retailers details
  - monopolistic competition, Calvo pricing

Intermediate goods producers

$$\max_{K_t, L_t, U_t} E_t[\beta \Lambda_{t,t+1}(-R_{k,t+1} Q_t \overline{K}_t(i) + P_{m,t+1}(i) Y_{m,t+1}(i) - W_{t+1} L_{t+1}(i) ...$$

$$... - a(U_t) K_t(i) + (1-\delta) Q_{t+1} \overline{K}_t(i))]$$

$$s.t. \qquad Y_{m,t}(i) = e^{z_t} K_t(i)^\alpha \left(\gamma^t L_t(i)\right)^{1-\alpha} - \gamma^t \Phi, \tag{9}$$

Y_{mt} - intermediate good; P_{mt} - price of intermediate good; z_t - technology shock; δ_t - depreciation rate; $\overline{K}_t$ - physical capital stock; K_t - effective capital; U_t - utilization rate; Q_t - price of capital; $R_{k,t+1}$ - real return of capital; Φ - fixed cost; γ - growth trend;

back

Capital Goods Producers

Capital accumulation

$$\overline{K}_t = (1-\delta)\overline{K}_{t-1} + v_{i,t}\left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t,$$

Objective of Capital Good producer

$$\max_{I_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_{0,t} \left\{ Q_t \left(1 - S\left(\frac{I_t(k)}{I_{t-1}(k)}\right)\right) v_{i,t} - 1 \right\} I_t.$$

First-order condition for optimal investment:

$$\begin{aligned} 1 \quad = \quad & Q_t v_{i,t} \left(1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right) \\ & + E_t \left\{ Q_{t+1} v_{i,t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \right\} \end{aligned}$$

- I_t - Investment; $v_{i,t}$ - investment specific technology shock

Retailers

$$\max_{P_t(i)} E_t \sum_{s=0}^{\infty} (\beta\zeta_p)^s \frac{\Lambda_{t,t+s}}{\Pi_{t,t+s}} \left[P_t(i)\Pi_{l=1}^{s}(\Pi_{t+l-1}^{\imath_p}\Pi^{1-\imath_p}) - MC_{t+s}\right] Y_{t+s}(i)$$

$$s.t. \quad \frac{Y_{t+s}(i)}{Y_{t+s}} = G^{'-1}\left(\frac{P_t(i)\Pi_{l=1}^{s}(\Pi_{t+l-1}^{\imath_p}\Pi^{1-\imath_p})}{P_{t+s}}\tau_{t+s}\right).$$

Aggregate price index

$$P_t = [(1-\zeta_p)(P_t^*)G^{'-1}\left[\frac{P_t^*\tau_t}{P_t}\right] + \zeta_p\Pi_{t-1}^{\imath_p}\Pi^{(1-\imath_p)}P_{t-1}G^{'-1}\left[\frac{\Pi_{t-1}^{\imath_p}\Pi^{(1-\imath_p)})P_{t-1}\tau_t}{P_t}\right]$$

$P_t(i)$ - price set by firm i; $\Pi_{t,t+s}$ - gross inflation, $\Lambda_{t,t+s}$ -SDF, Y_t - demand for intermediate goods; MC_t - marginal cost; $\imath_p$ - degree of price indexation; G - Kimball aggregator; P_t^* - optimal price;

back

Banks

Banks' balance sheet

$$Q_t K_{b,t} + Q_t^b B_{b,t} = D_t + N_t$$

Law of motion of net worth

$$N_t = R_{k,t} Q_{t-1} K_{b,t-1} + R_{b,t} Q_{t-1}^b B_{b,t-1} - v_{d,t-1} R_{t-1} D_{t-1}$$

- Q_t, Q_t^b - prices of capital asset and government bonds
- $K_{b,t}$ - claims on capital stock held by banks
- $B_{b,t}$ - government bond held by banks
- D_t deposits
- N_t - net worth
- $R_{k,t}, R_{b,t}, R_t$ - interest rates on capital, bonds and deposits
- $v_{d,t}$ - risk premium shock (AR(1)-process)

back

Banks

Each period a fraction of bankers, (1-θ), exits the business with a fixed probability. When they exit they consume their accumulated net worth. Hence, bankers maximize the terminal value of their net worth

$$V_t = \max_{\{K_{b,t}\},\{B_{b,t}\},\{D_t\}} \quad E_t \Lambda_{t,t+1}[(1-\theta)N_{t+1} + \theta V_{t+1}],$$

subject to an incentive constraint.

$$V_t \geq \lambda Q_t K_{b,t} + \lambda_b Q_t^b B_{b,t}.$$

- λ, λ_b - fraction of assets that banker can divert

Assumption: Incentive constraint is always binding

back

Banks - First Order Conditions

Guess: value function is linear in loans, government bonds and net worth:

$$V_t = \nu_{kt} Q_t K_t + \nu_{bt} Q_t^b B_t + \nu_{nt} N_t$$

FOC for K_t, B_t. μ_t:

$$\nu_{kt} = \lambda \frac{\mu_t}{1+\mu_t} \tag{10}$$

$$\nu_{bt} = \lambda_b \frac{\mu_t}{1+\mu_t} \tag{11}$$

$$Q_t K_t = \frac{\nu_{bt} - \lambda_b}{(\lambda - \nu_{kt})} Q_t^b B_t + \frac{\nu_{nt}}{\lambda - \nu_{kt}} N_t \tag{12}$$

- μ_t - Lagrange Multiplier of the incentive constraint

back

Solution to the Bank's Problem

Guess: value function is linear in loans, government bonds and net worth:

$$V_t = \nu_{kt} Q_t K_t + \nu_{bt} Q_t^b B_t + \nu_{nt} N_t$$

solution for the coefficients:

$$\begin{aligned} \nu_{k,t} &= \beta E_t \Omega_{t+1} (R_{k,t+1} - v_{d,t} R_t), & (13) \\ \nu_{b,t} &= \beta E_t \Omega_{t+1} (R_{b,t+1} - v_{d,t} R_t), & (14) \\ \nu_{n,t} &= \beta E_t \Omega_{t+1} v_{d,t} R_t. & (15) \end{aligned}$$

where the stochastic discount factor of the banker is defined as:

$$\Omega_t \equiv \Lambda_{t-1,t} [(1-\theta) + \theta\left(\nu_{nt}(1+\mu_t)\right)] \quad (16)$$

with $\Lambda_{t-1,t}$ being the SDF of the household

back

Banks - Aggregation

Each period a fraction of bankers, (1-θ), exits the business with a fixed probability, and is replaced by new bankers, which are given a fraction ω of the total assets.
net worth by existing and new bankers:

$$N_t = N_{nt} + N_{et} \tag{17}$$

$$N_{et} = \theta[(R_{kt}Q_{t-1}K_{t-1} + R_{bt}Q^b_{t-1}B_{t-1} - v_{d,t-1}R_{t-1}D_{t-1}] \tag{18}$$

$$N_{nt} = \omega[(R_{kt}Q_{t-1}K_{t-1} + R_{bt}Q^b_{t-1}B_{t-1}] \tag{19}$$

aggregate balance sheet

$$Q_tK_t + Q^b_tB_t = D_t + N_t \tag{20}$$

leverage ratio:

$$\phi_t = (Q_tK_t + Q^b_tB_t)/N_t \tag{21}$$

back

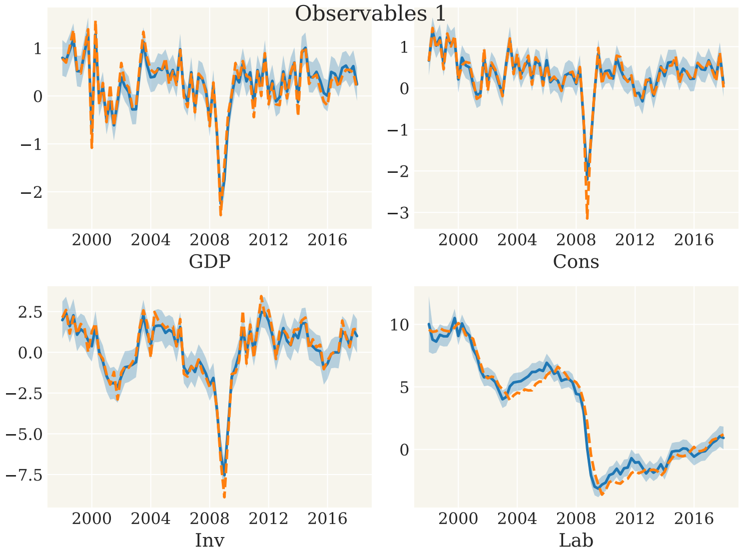
Fiscal Policy

budget constraint	$G_t + R_{b,t} Q^b_{t-1} B_{t-1} = Q^b_t B_t + T_t$
government spending	$G_t = G \cdot e^{g_t}$
government spending	$g_t = \rho_g g_{t-1} + \varepsilon^g_t$
tax revenues	$T_t = T + \kappa_b (B_{t-1} - B)$
return on bonds	$R_{bt} = \frac{r_c + \rho_c Q^b_t}{Q^b_{t-1}}$

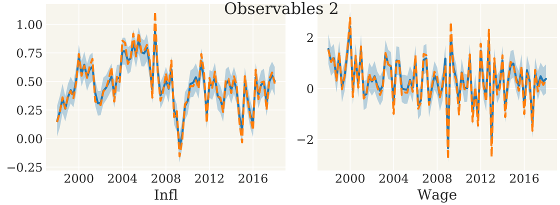
G_t - government spending;
T_t - tax revenues;
g_t - government spending shock;
r_c - coupon on bond;
ρ_c - decay rate of consol;

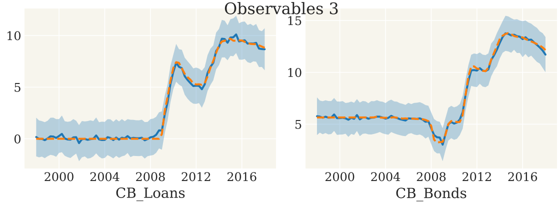
back

Data and Filtered series I go back



Data and Filtered series II go back





Calibrated parameter

$trend$	0.344	pre-crisis average
$mean_L$	6.5415	pre-crisis average
$mean_\Pi$	0.5	2% inflation target
$mean_{LSAP,B}$	5.65	pre-crisis average
$mean_{LSAP,K}$	0	pre-crisis average
$mean_{CBL}$	0	pre-crisis average
λ_w	1.1	10% markup in labor market
ϵ	10	as in SW (2007)
h	0.72	as in SW (2007)
α	0.19	as in SW (2007)
ψ	0.79	mean value for trial estimations

go back