Last Time

- How to drive a car

Today:

- How to walk

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* History:

- First legged robots built 1960s
- Serious research began in 1980s
- Different approaches:

  Honda/Waseda: Humanoids, Control based on manipulator ideas.

  Raibert/CMU: Hoppers. Control focused on floating-base dynamics

- Past 10~15: lots of work on mechanical design (series elasticity, direct drive) and MPC for quadrupeds. (ANYmal, Cheetah, Spot)

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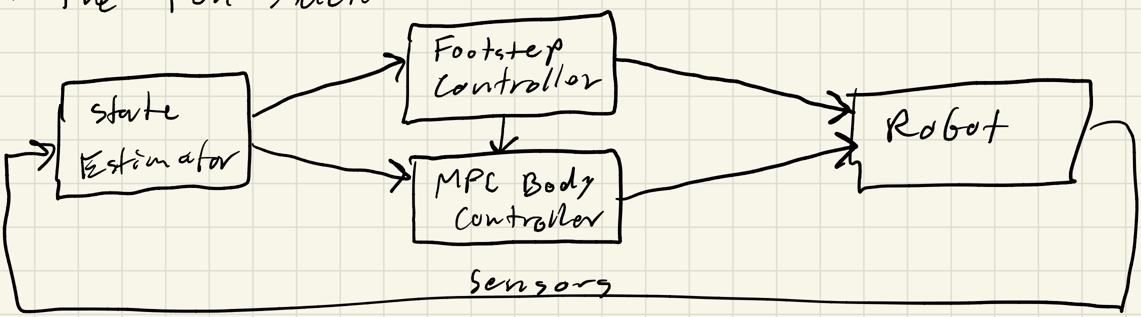
* The "Full Stack":



- Sensors: joint encoders, IMU, contact/force sensors

- State Estimator: some kind of EKF with extra tricks for reasoning about contact + foot slip

- Footstep Planner/Controller: plans + tracks foot locations based on pre specified gait and desired body velocity.

- MPC Controller: Treats the robot as a single rigid body and assumes given foot positions to compute contact forces.

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* Legged Robot Dynamics:

$$x = \begin{bmatrix} q \\ v \end{bmatrix}, \quad q = \begin{bmatrix} \text{body position} \\ \text{body attitude} \\ \text{joint angles} \end{bmatrix}, \quad v = \dot{q}$$

$$\underbrace{M(q)}_{\text{Mass Matrix}}\dot{v} + \underbrace{C(q,v)}_{\text{Coriolis}} = \underbrace{B(q)}_{\text{Input Jacobian}}u + \underbrace{J(q)^T}_{\text{Contact Jacobian}} f$$

f ← contact forces

$$\underbrace{\phi(q)}_{\text{Signed Distance Function}} \geq 0$$

← Interpenetration Constraint

$$\| f^{2:3} \| \leq \mu f^1$$

← Friction Cone

$f^{2:3}$: friction force

μ: Friction Coefficient

f^1: normal force

- Very messy for online control
  - State dimension is large (~36). TrajOpt scales like n^3

* Centroidal Dynamics

- Common Assumptions:
  1) Leg mass/inertia << body mass/inertia (~10%)
  2) Leg actuators are very fast compared to body motion

⇒ Use a lumped single rigid body model for the whole robot!

total mass of robot

$$m\dot{v} = mg \sum_i f_i$$

← sum of all contact forces

$$J\dot{\omega} + \underbrace{\omega \times J\omega}_{\text{Coriolis}} = \overbrace{\sum_i r_i \times f_i}^{\text{total torque}}$$

Total inertia in some reference pose

foot positions

$$\underbrace{\dot{r} = v, \qquad \dot{q} = \frac{1}{2} q * \omega}_{\text{Kinematics}}$$

- With a few more assumptions we can get a linear model:

3) Body angular velocities are small

4) Body pitch and roll are small

5) Foot positions track reference (almost) exactly

$$\Downarrow$$

$$m\dot{v} = -mg + \sum_i f_i$$

$$J\dot{\omega} = \sum_i \underbrace{R_i}_{\hat{r}_i} f_i$$

$\hat{r}_i$ cross-product matrix using reference from footstep planner

$$\dot{r} = v$$

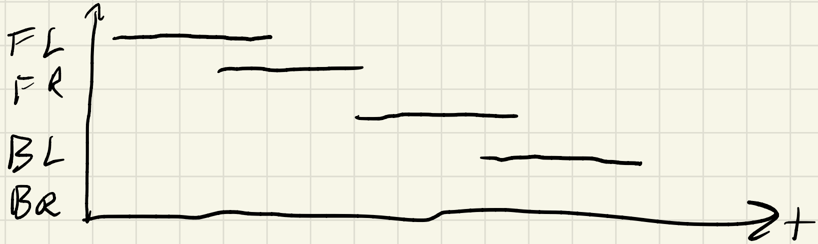
$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

roll, pitch, yaw Euler angles

$$\underbrace{\begin{bmatrix} \dot{r} \\ \dot{\theta} \\ \dot{v} \\ \dot{\omega} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} r \\ \theta \\ v \\ \omega \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} & 0 & & \\ I & I & I & I \\ R_1 & R_2 & R_3 & R_4 \end{bmatrix}}_{B} \underbrace{f}_{u}$$

* Footstep Planner / Controller

- Specify a gait sequence (foot order/timing):



- Says when to put the feet down.
- Where to put the feet down?
- Standard technique: "Raibert Heuristic"

gait period

$$r_{foot} = r_{hip} + \frac{\Delta t}{2} \underbrace{V_{body}}_{\text{desired forward velocity}}$$

positions projected on the ground

- How to swing the leg between stance phases?
- Hand-designed spline curve tracked with joint space PD control:

$$\tau = J_{swing}^T \left(K_p (r - r_{ref}) + K_d (\dot{r} - \dot{r}_{ref}) \right)$$

- We assume this tracking is very good in the centroidal dynamics / MPC controller.

* MPC Controller

- Try to track desired body motion (velocity + heading)
- Horizon is typically 1~2 gait periods
- With linearized centroidal dynamics, this is a convex optimization problem:

$$\min_{x_{1:N}\, u_{1:N-1}} \sum_{n=1}^{N-1} (x_n - \bar{x}_n)^T Q (x_n - \bar{x}_n) + (u_n - \bar{u}_n) R (u_n - \bar{u}_n)$$

$$\text{s.t.} \quad x_{n+1} = A x_n + B_n u_n$$

$$\| u_n^{2:3} \|_2 \leq \mu u_n^1 \quad (\| u_n^{2:3} \|_1 \leq \mu u_n^1)$$

- Friction cone is typically linearized to make this a QP
- Difficult to enforce torque limits
- Output is desired foot forces $u = f$
- Joint torques can then be calculated using joint Jacobian:

$$\tau = \underbrace{J(q)}_{\text{stance feet}}^T f$$

- Many extra hacks/extensions to handle unexpected contact events, foot slip, etc.