Last Time:

- Hybrid methods for contact

Today:

- Reasoning about friction
- Iterative Learning Control

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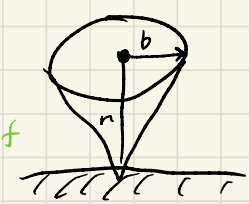
* Friction in Trajectory Optimization

- In our hybrid TrajOpt setup we assumed sticking (no slip).
- Equivalent to infinite friction
- We can enforce additional friction cone constraints for each contact point to make sure the robot won't slip:

$$\|b\|_2 \leq \mu n$$

(b: friction force $\in \mathbb{R}^2$; μ: friction coefficient; n: normal force $\in \mathbb{R}$)



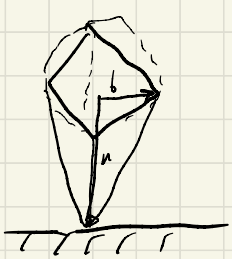
- This constraint is often linearized:

$$e^T d \leq \mu n, \quad d \in \mathbb{R}^4, \quad e = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$d \geq 0$$

$$b = \underbrace{[I}_{2\times 2} \ \underbrace{-I]}_{2\times 2} d$$

$= \sum \text{abs}(b) = \|b\|_1$



- The "friction pyramid" constraint is easy to enforce in QP-based MPC to avoid foot slip.

- Hybrid methods generally try to avoid. If you want to model stick-slip behavior, need to add more modes.

- Friction is hard to model. Coulomb is only an approximation. Typically use a conservatively small value of μ.

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* What Happens when our model has errors?

  - Models are always approximations

  - Simpler models are often preferred even if they're less accurate.

  - Feedback (e.g. LQR or MPC) can often compensate for model error.

  - Sometimes that isn't good enough (e.g. very tight constraints, performance requirements)

  - Several Options!

    1) Parameter Estimation: Classical "grey-box" system identification. Fit e.g. masses in your model using data from the real system.
       + Very sample efficient
       + Generalizes
       - Assumes model structure

    - Improve the model

2) Learn Model: Fit a generic function approximator to full dynamics or residual. Classical "black-box" system identification.
+ Doesn't assume model structure
+ Generalizes
- Not sample efficient. Requires lots of data.

3) Learn a Policy: Standard reinforcement learning approach: Optimize a function approximator for feedback policy.
+ Makes few assumptions
- Doesn't generalize
- Not sample efficient. Requires many "rollouts"

4) Improve a Trajectory: Assuming we have a reference trajectory computed with nominal model, Improve it with data from the real system.
+ Makes few assumptions
- Assumes you have a decent model
- Doesn't generalize
+ Very sample efficient

Improve the policy

* Iterative Learning Control

- Can think of this as a very specialized policy gradient method for the policy class

$$u_n = \bar{u}_n - K_n(x_n - \bar{x}_n)$$

$\bar{u}_n$: reference control input

$K_n(x_n - \bar{x}_n)$: can be any tracking controller

where we are improving the reference $\bar{u}$

- Can also think of this as SQP method where we get the "right-hand side" vector from a rollout on the actual system.

- Assume we have a reference trajectory $\bar{x}, \bar{u}$ that we want to track:

$$\min_{x_{1:N},\, u_{1:N-1}} \quad J = \sum_{n=1}^{N-1} \frac{1}{2}(x_n - \bar{x}_n)^T Q (x_n - \bar{x}_n) + \frac{1}{2}(u_n - \bar{u}_n)^T R (u_n - \bar{u}_n)$$

(can be any cost function) $+ \frac{1}{2}(x_N - \bar{x}_N)^T Q_N (x_N - \bar{x}_N)$

$$\text{s.t.} \quad x_{n+1} = f(x_n, u_n)$$

- As we've seen before, the KKT system for this problem looks like:

$$\begin{bmatrix} H & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \Delta z \\ \lambda \end{bmatrix} = \begin{bmatrix} -\nabla J \\ -c(z) \end{bmatrix}$$

where $z = \begin{bmatrix} x_1 \\ u_1 \\ \vdots \\ x_N \end{bmatrix}$, $c(z) = \begin{bmatrix} \vdots \\ f(x_n, u_n) - x_{n+1} \\ \vdots \end{bmatrix}$, $C = \frac{\partial c}{\partial z}$,

$H = \begin{bmatrix} Q & & & \\ & R & & \\ & & Q & \\ & & & Q_N \end{bmatrix}$ ← Gauss-Newton Hessian

- Note that we have ∇J in the RHS instead of ∇L and λ instead of $\nabla\lambda$ in the LHS. We can do this because the Lagrangian is linear in λ. We normally prefer solving for $\nabla\lambda$ because that lets us regularize.

- Two important observations:

  1) If we do a rollout on the real system, $c(z) = 0$ always (for the true dynamics).

  2) Since we know J, given x_n, u_n from a rollout, we can compute ∇J

- Now we have the RHS vector for the KKT system. We also know H from the cost function.

- We can compute $C = \frac{\partial c}{\partial z}$ using x_n, u_n and the nominal model.

- However, since our nominal model is approximate and assuming x_n, u_n is already close to $\bar{x}, \bar{u}$, we can just use $C = \frac{\partial c}{\partial z}\Big|_{\bar{x},\bar{u}}$, which can be pre-computed once.

- Now just solve the KKT system for δz, update $\bar{u} \leftarrow \bar{u} + \delta u$, and repeate.

- Can easily add inequality constraints (e.g. torque limits) and solve a QP.

* Why Should ILC Converge?

- We've already seen approximations in Newton's method: Gauss-Newton, regularization.

- In general, these are called "inexact" and "quasi-Newton" methods. Many variants (e.g. BFGS, Broyden), with well-developed convergence theory.

- For a generic root-finding problem:

$$f(x+\Delta x) \approx \underbrace{f(x) + \frac{\partial f}{\partial x}\Delta x = 0}_{\text{Exact Newton step}}$$

- As long as Δx satisfies $\| f(x) + \frac{\partial f}{\partial x}\Delta x \| \leq \eta \| f(x) \|$ for some $\eta < 1$, an inexact Newton method will converge.

- Convergence is slower than exact Newton

- This means we can use $J \approx \frac{\partial f}{\partial x}$ to compute Δx

* ILC Algorithm:

do:

$x_n, u_n \leftarrow rollout(\bar{x}, \bar{u})$ (on real system) — whole trajectory from real system

$\delta x, \delta u \leftarrow \underset{}{\operatorname{argmin}} \; J(\delta x, \delta u)$

$s.t. \quad \delta x_{n+1} = A_n \delta x_n + B_n \delta u_n$

$u_{min} \leq u_n \leq u_{max}$

This is a QP

$\bar{u} \leftarrow \bar{u} + \delta u$

while $\| x_n - \bar{x}_n \| \geq tol$