Last Time:

- Tips + Tricks
- Control History

Today:

- How to land a Space Ship

---

* The Rocket Soft-Landing Problem

  - Go from an initial state x_0 to some final position r_f with $z_f = 0$ and $V_f = 0$

    ("Landing" $\Rightarrow z_f = 0$, "soft" $\Rightarrow V_f = 0$)

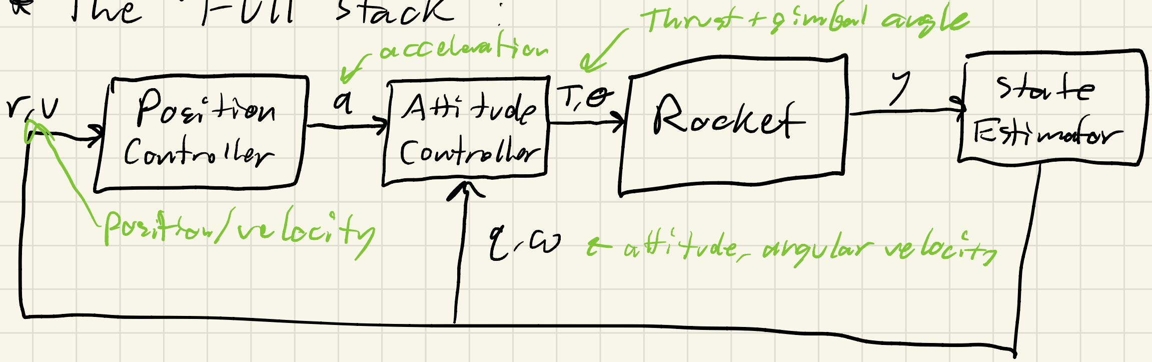
- Minimize some combination of fuel consumption and landing position error $\|r_f - r_g\|$

- Respect thrust limits + safety constraints

* Examples:

  - NASA Curiosity "Sky Crane" (2012)
  - SpaceX Falcon 9 (2018)
  - NASA Perseverance TRN (2021)
  - Lots of tricks to make these work in practice.

* The "Full Stack":



- State Estimation:

SpaceX: GPS + IMU, with good filtering accurate to < 1 m position, < 1 cm/sec velocity, < 1 deg attitude. Enables high-precision landing.

Mars: No GPS. IMU + Radar Altimiter + Vision. ~ 30 meter accuracy. Avoid boulders.

- Decoupled Control Loop:

High-Level Position Controller: Uses a point-mass model. Reasoning about safety, thrust, and fuel constraints. Generates acceleration/thrust commands. Runs at ~ Hz

Low-Level Attitude Controller: Reasons about attitude, flexible modes, fluid slosh. Generates thrust and gimble commands to track desired acceleration. Runs at ~10s of Hz

⋆ Rocket Dynamics:

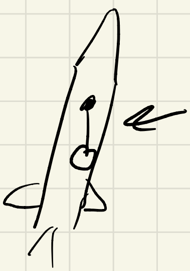
- Rigid Body Model:

Position Controller (slow) $\left\{ \begin{array}{l} \dot{v} = -g + \frac{T}{m} \quad \leftarrow \text{point mass} \\ \dot{m} = -\alpha T \quad \leftarrow \text{fuel burn} \end{array} \right.$

attitude controller (fast) $\left\{ J\dot{\omega} + \omega \times J\omega = l \times T \quad \leftarrow \text{attitude} \right.$

- Fuel can be 80% + of initial vehicle mass. Total mass can change 2-5x. Have to account for this.

- Fluid Slosh: Highly nonlinear, hard to model. Standard model:



pendulum with l_p - M_p fit to data.

- Flexible Modes: Rockets are built to be very light ⇒ not very stiff ⇒ low-frequency bending modes. First bending mode might be ~Hz. Dealt with by adding a notch filter to the attitude controller at the bending frequency to avoid exciting it.

- Aerodynamic Forces: Mostly ignored. Velocity constraints in the position controller can make sure these aren't too big.

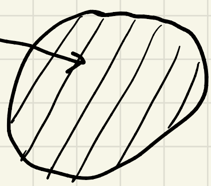
- Lots of model uncertainty! Linear robust control ideas (e.g. H_∞ loop shaping) are used in the attitude controller.

---

* Background: Convex Relaxation

- Sometimes we can have a non-convex constraint that can be expressed as the boundary of a larger convex set e.g.:

Enlarged set
$S_2 = \{x \mid \|x\| \leq 1\}$



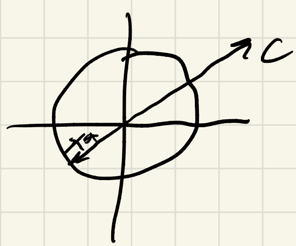
Original set S_1
$S_1 = \{x \mid \|x\| = 1\}$

$$S_1 = \partial S_2$$

- Replacing the original constraint with the larger convex one is called a "convex relaxation"

- Sometimes if the cost is particularly nice, we can still get the answer to the original problem by solving the relaxed version e.g.:

$$\min \; c^T x$$
$$\text{s.t.} \; \|x\| = 1$$
$$\Downarrow$$
$$\|x\| \leq 1$$



- When this happens we call it a "tight relaxation"

* Convex Relaxation of Thrust Constraints:

- Maximum thrust constraint: (convex):

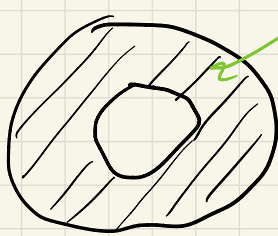
$$T \in \mathbb{R}^3, \quad \|T\| \leq T_{max}$$

- Thrust angle constraint (convex):

$$n^T \frac{T}{\|T\|} \leq \cos(\theta_{max})$$

- Rocket engine also has minimum thrust constraint once turned on:

$$T_{min} \leq \|T\| \leq T_{max}$$



Feasible set is spherical shell.

- Let's add a new "slack variable" $\Gamma \in \mathbb{R}$ that equals the thrust magnitude:

(1) $\|T\| = \Gamma$ ← boundary of a convex set (sphere)

(2) $T_{min} \leq \Gamma \leq T_{max}$

(3) $n^T T \leq \Gamma \cos(\theta_{max})$

} – convex (linear)

- Now we can convexify the constraints by relaxing (1):

(1') $\|T\| \leq \Gamma$

(2) $T_{min} \leq \Gamma \leq T_{max}$

(3) $n^T T \leq \Gamma \cos(\theta_{max})$

} All convex!

- The paper proves that this relaxation is tight using Pontryagin's minimum principle.