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Modeling and Control of Mechatronic Aeropendulum

Mila Mary Job
PG Scholar
Dept. Electronics and Instrumentation
Karunya University
Coimbatore, India
milamaryjob@gmail.com

P.Subha Hency Jose
Assistant Professor
Dept. Electronics and Instrumentation
Karunya University
Coimbatore, India
hency20002000@karunya.edu

Abstract- **Modeling and controller design for an aeropendulum system is important because it enables to control the pendulum action by controlling the voltage given such as the stability, dead time,rise time, overshoot etc. This concept of pendulum system is very useful and can be applied in day to day life. There are different kinds of applications of pendulum systems such as measurement applications, Schuler tuning methods, coupled pendulum applications, entertainment purposes, etc. By equation of motion, the open loop transfer function can be obtained and simulation is done accordingly.Aeropendulum is a suspended pendulum, which has a propeller at the end of the stick which is motorized,It can be controlled by controlling the voltage given to dc electric motor.Designing of Proportional Integral Derivative (PID),Linear Quadratic Regulator (LQR),PID based LQR controllers are used for nonlinear pendulum dynamic system in this project. The promising performance of the proposed controllers investigates in simulation.The effectiveness and the comparisons of the controller methods for Nonlinear Aeropendulum Dynamical System are delivered in this project.**

Keywords- Aeropendulum, PID, Linear Quadratic Regulator(LQR)

I. INTRODUCTION

Pendulum is a classical control problem that has been used till today in day to day life which is easiest and stabilizes the machine base by the pendulum system. By mathematical modeling and suitable controller design, the pendulum characteristic can be easily understood, viewed and designed. In this paper the properties of an aeropendulum will be studied.A simple pendulum consists of a small body of mass called a bob which is attached to the end point of a string.The length of the string is large compared to the dimensions of the bob.The mass of the string is negligible compared with that of the bob mass.Due to these conditions the mass of the bob is concentrated at its center of mass, and the length of the pendulum is the distance to this center of mass from the axis of suspension or from pivot point.An aeropendulum has a motorized propeller at the end of the pendulum so it can lift the pendulum for the given voltage.A motorized propeller is rotated by a positioning servo around an axis parallel to the pendulums axis of rotation[1].The direction and the magnitude of the thrust is varied to hold the pendulum at any desired position. The main objective of this paper is modeling and control of an aeropendulum.The first objective is derivation of mathematical modeling of an aeropendulum system such as transfer function and state apace equation. Second objective is to design a suitable controller for the aeropendulum system, where for this paper PID controller,LQR controller, PID based LQR controller is used then the comparison between controllers is obtained. Next objective is to simulate the mathematical models with controller based on input by using simulation in MATLAB.

II. PROCESS DESCRIPTION

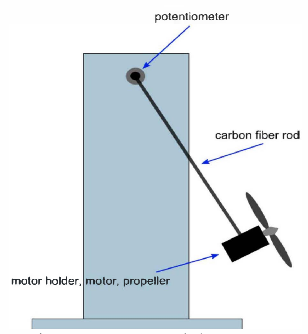


Figure.1.Aeropendulum system

The setup consists of a small dc electric motor driven by a 5-V pulse-width modulated signal.The motor is attached to the free end of a light carbon rod, while the other end of the rod is connected to the shaft of a low-friction potentiometer. The potentiometer is fixed on a plastic stand at a height where the pendulum can swing freely .A 2-in propeller is attached to the motor shaft to produce a thrust force in order to control the angular position of the pendulum. An auto calibrating step during the initialization allows the system to automatically find the vertical position .There is a DC motor with a propeller on the lead of a suspended stick. After applied voltage, the propeller spins and generates torque T to pull up the pendulum. It is the most benefits of driven pendulum that enables us controlling its behavior with regulating the applied voltage. Therefore, the control variable for this system is the angle of the pendulum settled and the manipulated variable is the voltage fed to the motorized-propeller[4].

III.MATHEMATICAL MODELING

An aeropendulum system is shown in figure 2. After applied voltage, the propeller spins and generates torque T to pull up the pendulum. The aim is to command the pendulum to a specified angle. The advantage of an aeropendulum is that it controls its behavior by adjusting the applied voltage. Therefore, the controlled variable for this system is the angle of the pendulum and the manipulated variable is the voltage given to the motorized-propeller[2].

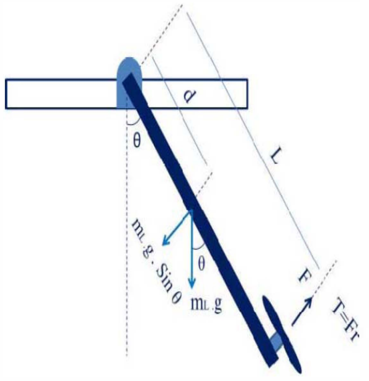


Figure.2.Schematic diagram of an aeropendulum

According to Newton's laws and angular momentum, the motion equation of aeropendulum is derived

$$J\ddot{\theta} + c\dot{\theta} + m.l.g.d.\sin\theta = T \qquad (1)$$

m = weight of the pendulum (kg)
l = length of pendulum (m)
d = distance from pivot point to center of mass (m)
θ = angular position (degrees)
c = viscous damping coefficient (Nms/rad)
J = inertia moment (kgm^2)
g = acceleration of gravity (m/s^2)

The equation between voltage applied to dc motor and thrust T is,

$$T(s) = K_m V(s) \qquad (2)$$

Where, T = torque provided by dc motor(Nm)
V = voltage to dc motor (volts)
K_m = motorized propeller gain

To linearize the aeropendulum system around equilibrium point consider
$\sin\theta \approx \theta$,
Motion equation can be written as,

$$J\ddot{\theta} + c\dot{\theta} + m.l.g.d = T \qquad (3)$$

TABLE I. SYMBOLIC DESCRIPTIONS AND UNITS

SYMBOLS	DESCRIPTION	UNIT	VALUE
d	Distance between centre of mass and extended point	m	.03m
ml	Weight of the pendulum	kg	.36 kg
g	Acceleration due to gravity	ms^{-1}	$9.8ms^{-1}$
J	Moment of inertia	Kgm^2	0.0106Kgm^2
c	Viscous damping coefficient	Nms/rad	0.0076 Nms/rad
K_m	Gain of motorized propeller	Nm/volts	0.0296 Nm/volts
θ	Angular position	°	Given angle in degrees
V	Voltage input to dc motor	v	Given voltage
T	Thrust	N	

Equation gives the transfer function aeropendulum,

$$\frac{\theta(s)}{T(S)} = \frac{1}{J\ddot{\theta} + c\dot{\theta} + m.l.g.d} \qquad (4)$$

And the standard representation is,

$$\frac{\theta(s)}{T(S)} = \frac{1/J}{S^2 + \frac{c}{J}S + \frac{m.l.g.d}{J}} \qquad (5)$$

The generated trust T in above equations is not the manipulated variable for control system since the aeropendulum is controlled by applied voltage.Transfer function of motorized propeller can be represented as a block diagram[5].

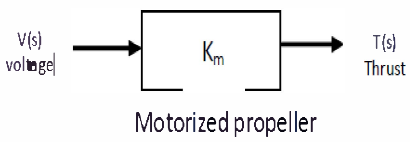


Figure .3.Block diagram of Motorized Propeller

Where,

$$K_m = \frac{m.l.g.d.\theta}{V} \qquad (6)$$

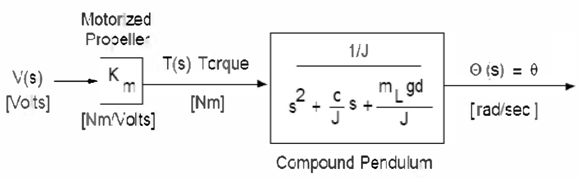


Figure .4.Block diagram of an aeropendulum

$$\frac{\theta(s)}{V(S)} = \frac{Km/J}{S^2 + \frac{c}{J}S + \frac{m.l.g.d}{J}} \qquad (7)$$

For nonlinear model of an aeropendulum system it is required to represent it in state space form and it is given of the form,

$$\frac{d}{dt}\mathrm{x} = \mathrm{f}(\mathrm{x}, \mathrm{u}, \mathrm{t}) \tag{8}$$

Consider state variables as,

$$X_1 = \theta; \qquad X_2 = \dot{\theta}; \qquad X_3 = \ddot{\theta}; \tag{9}$$

Final state space equations for driven pendulum is,

$$\frac{d}{dt}\mathrm{X} = \frac{d}{dt}\begin{bmatrix} x1 \\ x2 \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} f1 \\ f2 \end{bmatrix} \tag{10}$$

Where,

$$f_1 = X_2 ; \tag{11}$$

$$f_2 = \frac{\mathrm{Km.V} - \mathrm{c}\dot{\theta} - \mathrm{m.l.g.d.}\sin\theta}{\mathrm{J}} \tag{12}$$

The pendulum angle θ is the variable of interest, and then output equation can be written as,

$$\mathrm{Y} = [\theta] = \mathrm{CX} = [1 \ \ 0]\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \tag{13}$$

To linearize the system around equilibrium point, consider $\sin\theta \approx \theta$.

Then state space takes the form,

$$\begin{bmatrix} \dot{X}1 \\ \dot{X}2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\mathrm{m.l.g.d}/\mathrm{J} & -c/J \end{bmatrix}\begin{bmatrix} X1 \\ X2 \end{bmatrix} + \begin{bmatrix} 0 \\ Km/J \end{bmatrix}.\mathrm{u} \tag{14}$$

$$\mathrm{Y} = [1 \ \ 0]\begin{bmatrix} X1 \\ X2 \end{bmatrix} \tag{15}$$

IV.CONTROLLER DESIGN

Optimal Control Method Using Linear Quadratic Regulator (LQR)

The linear quadratic regulator plays a key role in many control design method applications.Linear quadratic regulator design applications involve the determination of an input signal that will take a linear system from a given initial state X (t_0) to a final state X (t_f) while minimizing a quadratic form of performance index[3]. The quadratic performance index in question is the time integral of a quadratic form in the state vector X and the input vector u such as

$$(X^T Q X) + (U^T R U) \tag{16}$$

where Q and R is positive definite matrices which is found out by iterative method and given as,

$$Q = \begin{bmatrix} 28.77 & 0 \\ 0 & 0 \end{bmatrix} \qquad R = 1 \tag{17}$$

With this basic definition in place, various forms of the quadratic linear regulator design problem can be stopped for example: finite horizon (t_f finite), infinite horizon (t_f infinite), time-varying (the system, R and Q matrices themselves) etc. The final state itself may or may not contribute to the performance index as separate term.LQR is an optimal control technique which is based on closed loop optimal control with the linear state feedback method. The proposed cost function in LQR method is as,

$$\int_0^\infty((X^TQX) + (U^TRU))dt = \int_0^\infty(X^TQX)t + \int_0^\infty(U^TRU)dt \tag{18}$$

The selection weight matrices Q and R should be symmetric and positive matrices.The weight matrices affect the control behaviour.Determining matrix K of the optimal control

$$U = -(KX) \tag{19}$$

Where,

$$K = R^{-1}B^TP \tag{20}$$

P is defined by Reccati's equation,

$$A^TP + PA - PBR^{-1}B^T P + Q = 0 \tag{21}$$

After finding the weight matrices Q and R, the controller gain is obtained by the above Reccati's equation which is found by the Matlab program.

V.SIMULATION RESULTS

A. Closed loop response using PID controller

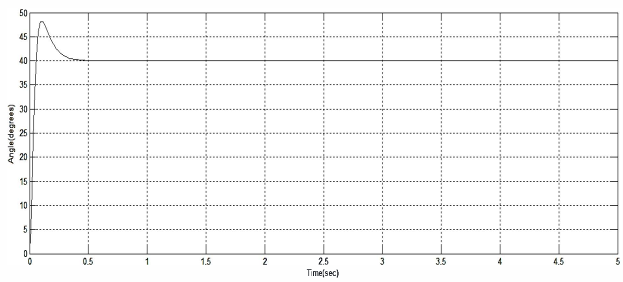


Figure.5.closed loop response using PID controller

From Fig 5 it is clear that the settling time of the aerpendulum when controlled by PID controller is very high and also the overshoot and peak time is high for this controller and rise time is less.

B .Closed loop response with LQR controller

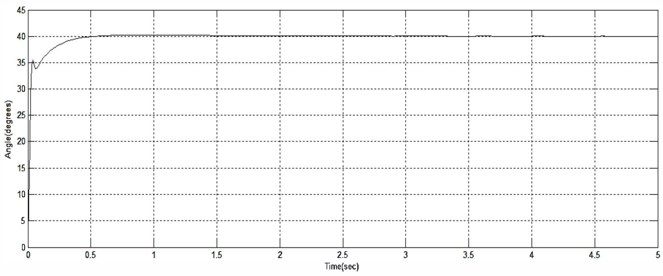


Figure .6.Closed loop response of LQR controller in the system

From Fig 6 it is clear that the settling time of the aeropendulum when controlled with an LQR controller is very less and the rise time is also very less compared to PID controller, where as overshoot and peak time is zero.

C .Closed loop response of PID controller based LQR

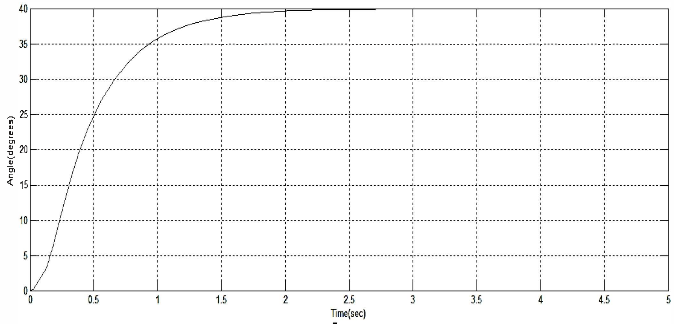


Figure.7.Closed loop response of PID based LQR

From Fig 7 it is clear that the settling time of the aeropendulum when controlled with a PID based LQR controller is higher than that of with LQR alone. Also the rise time is more. But the overshoot and peak time remains zero.

D. Comparison of PID, LQR, PID based LQR controllers in aeropendulum system

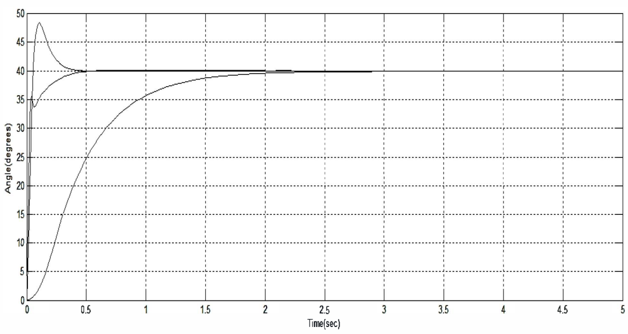


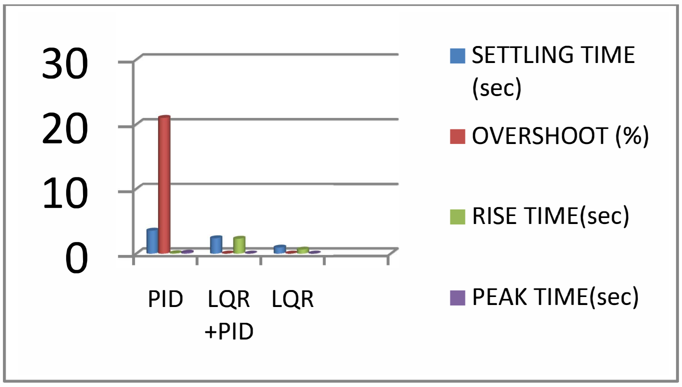
Figure 8.Closed loop Responses of comparison between PID, LQR, PID based LQR

E .Comparison

Controller	Settling time (sec)	Overshoot (%)	Rise time(sec)	Peak overshoot (%)
PID	3.538	21	.0576	.1346
PID + LQR	2.36	0	2.288	0
LQR	.9230	0	.61538	0

From the table, it is found that the settling time of PID control is 3.538 seconds but when using PID based LQR the settling time reduces to 2.36 and when LQR alone is used the settling time is again reduced to .9230 seconds. Further rise time when using PID is .0576 seconds but it is reduced to .61538 seconds when using LQR also increased to 2.288 when PID based LQR is used.

F.Bar diagram of compared response parameter



VI.CONCLUSION AND FUTURE WORKS

Nonlinear dynamical equations of aeropendulum are derived. The objectives of this project have been completed. In order to improve response parameters such as oscillations, rise time, settling time, overshoot, a conventional PID controller, LQR controller, PID based LQR controller has been applied to the aeropendulum system. Based on simulation results, the system responses indicate that the performance of an aeropendulum has improved using LQR controller. There are several improvements that can be added in this project which is using other controller such as Fuzzy Logic controller and Lead Lag controller, Sliding mode controller. Then comparison of this controller can be made.

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