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Series DC Motor Modeling and Identification

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***Abstract*—The modeling and identification of a series DC motor without saturation is presented. The identification is performed using the Strejc method. The procedure requires only the measurement of the current consumption and the motor rotors speed. The resulting model is validated comparing the responses of the estimated model against real time responses of an actual motor.**

Keywords—Electric Machine; Identification; Series dc motor; Linear systems

I. Introduction

Series DC motors, as well as series universal motors, are a kind electric motors with one voltage supply and the field winding connected in series with the rotor winding. This series connection results in a motor with very high starting torque. However, torque decreases as the speed builds up due to an increment of the back or counter electromotive force EMF. This is why series DC motors have poor speed regulation. That is, increasing the motors load tends to slow its speed which in turns reduces the back EMF and increases the torque to accommodate the load. A limitation of these motors is that the sense of rotation is fixed for most of their applications. In order to change the direction of torque and rotation, it is necessary to change the polarity of the current flow.

Despite the fact series DC motors generate high torques with very low current consumption and small dimensions they are commonly used open loop for short periods of time. This is mainly, as mention above, because they have poor speed regulation. Nonetheless, this kind of motors can be fully exploited if good closed loop controllers are designed. However, in general many control strategies to this kind of motors are based or depend on dynamic cancellations [1]-[5] requiring good models.

Models, in particular mathematical models, are a corner stone for many control strategies. These are normally obtained by analyzing the physical properties of the phenomena. However, a second crucial problem is to estimate the values of the different physical parameters involved in the model. In the case of linear system and some nonlinear models the algorithm of Least Square is one of the most popular methodologies to estimate or identify these parameters. Nonetheless, a necessary condition to apply this algorithm is that the model must be linear at its parameters, [6]. Another identification approach is the Strecj algorithm, successfully applied in [7] for the identification of the asynchronous machine which is based on the transient response analysis to a step input of the process. In the case of linear first and second order systems this procedure renders good models by estimating its poles and steady state gain. In some cases, it is also possible to estimate the physical parameters of the process.

Therefore, the main objective of the paper is to present the modeling, without magnetic saturation, and identification of series DC motor. A second objective is to present a simple procedure to estimate the parameters of the model using only the measurement of the current consumption and the speed of the motor rotor. This methodology is based on the transient and steady state responses of the mechanical and electrical subsystems of the motor known as the Strecj algorithm. In order to assess the estimated model comparisons between the time responses of the model and an actual motor are presented

II. Series DC Motor Model

Series DC motors similar to shunt wound DC motors or compound wound DC motors are self-excited DC motors. They get their name because the field winding is connected internally in series to the armature winding as shown in figure 1. They are also considered self-excited motors because instead of two separate voltage sources -one for the armature and one for the field winding- they required only one voltage source.

The electric diagram of a series DC motor is shown in figure 2. Based on the electric diagram of figure 2 the differential equations comprising the mechanical and electrical subsystems of the series DC motor are given by:

$$V(t) = R_a i_a(t) + R_f i_f(t) + L_a \frac{d}{dt} i_a(t) + L_f \frac{d}{dt} i_f(t) + E_a \qquad (1)$$

$$T_e(t) = T_L(t) + b\omega(t) + J \frac{d}{dt} \omega(t) \qquad (2)$$

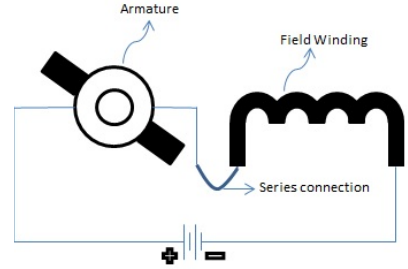

Fig. 1. Series connection of a DC motor

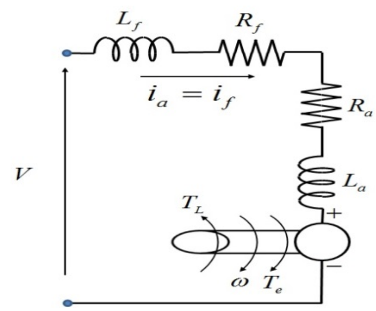

Fig. 2. Electric diagram of a series DC motor

As $i_a(t) = i_f(t)$ equations (1) and (2) reduce to:

$$V(t) = \left(R_a + R_f\right)i(t) + \left(L_a + L_f\right)\frac{d}{dt}i(t) + E_a \tag{3}$$

$$T_e(t) = T_L(t) + b\omega(t) + J\frac{d}{dt}\omega(t) \tag{4}$$

Where, $\omega(t)$ is the rotors speed, E_a represents the counter electromotive force *EMF*, $T_L(t)$ is the load torque, $i(t) = i_a(t) = i_f(t)$ is the current, b is the friction coefficient, J is the rotor's inertia and $T_e(t)$ is the electromagnetic torque produced by the motor.

The *EMF* E_a and $T_e(t)$ depend both on the air-gap flux Φ, that is:

$$E_a(t) = \omega(t)\,\Phi(i) \tag{5}$$

$$T_e(t) = i(t)\,\Phi(i) \tag{6}$$

The flux $\Phi(i)$ is a function of the current $i(t)$ so equations (1)-(4) are non-linear. Also, it is common practice to approximate the flux $\Phi(i)$ by a linear relation when the magnetic saturation is neglected, that is:

$$\Phi(i) = k_0 i(t) \tag{7}$$

where k_0 is the mutual inductance between the armature and field coils.

Finally, the nonlinear model of a series DC motor without saturation is given by:

$$V(t) = Ri(t) + L\frac{d}{dt}i(t) + \omega(t)i(t)k_0 \tag{8}$$

$$i^2(t)k_0 = T_L(t) + b\omega(t) + J\frac{d}{dt}\omega(t) \tag{9}$$

Where $R = R_a + R_f$ and $L = L_a + L_f$

III. Model Identification

The identification of the monophasic universal-motor Koblenz model HC8825M110 with nominal maximum speed and power of 24000 RPM and 0.815HP is presented.

In particular, these machines must operate loaded in order to avoid damage. Hence, a steel disc load was added as shown in figure 3. This extra load is considered as a part of the rotors inertia. From equations (8) and (9) it is clear that it is possible to estimate the motors parameters by analyzing the responses of the current and angular velocity of the rotor to step input voltages. This is performed using the set up experiment depicted in figure 4.

The experimental setup consists of a voltage source with a maximum voltage of 50 volts and a maximum current of 3 Amp, the series DC motor, and a power driver with a USB communication. The power driver was designed considering the electric demand of the motor. Hence, the power driver is based on the MOSFET 12N65, which operates with a maximum current of 12Amp and a maximum input voltage of 650 Volts. An equally important element is the ACS711LC circuit used to monitor the power consumed by the motor.

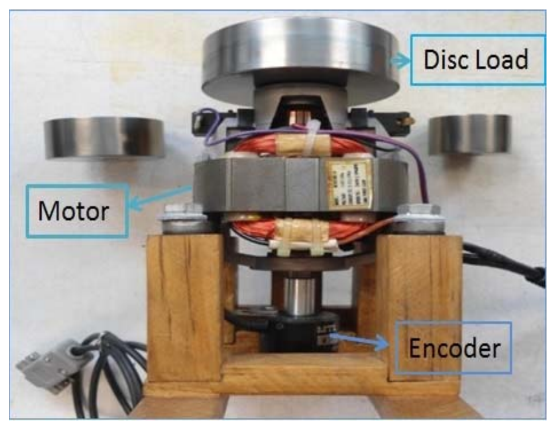

Fig. 3. Series DC motor

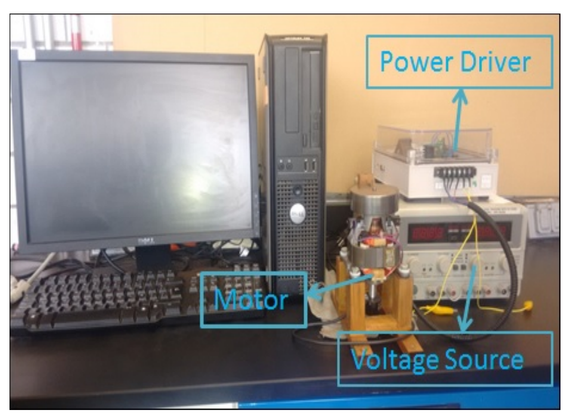

Fig. 4. Experimental Set Up

Inductive loads together with pulsed excitation signals generate reverse currents that may damage switching elements such as MOSFETs. Although the MOSFETs used in the implementation have an internal protection diode, two transistors FR307 of rapid recovery were added in order for additional protection, as shown in figure 5.

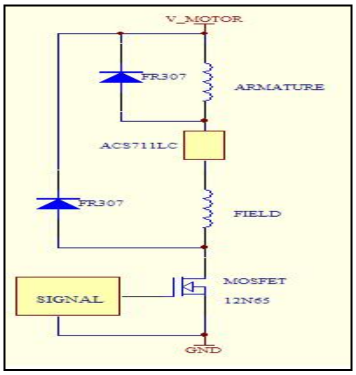

Fig. 5. Power driver

A. Electric Subsytem Identification

From equation (8), it is clear that if the rotor shaft is fully locked, then $\omega(t) = 0 \;\forall t$, reducing the electric subsystem to a simple RL circuit with a differential equation given by:

$$V(t) = Ri(t) + L\frac{d}{dt}i(t) \tag{10}$$

with a transfer function $G_E(s)$ given by:

$$G_E(s) = \frac{I(s)}{V(s)} = \frac{1}{Ls + R} \tag{11}$$

The steady state gain and the steady state time of $G_E(s)$ are $K_{G_E} = 1/R$ and, $t_{ss_E} = 4L/R$ respectively. This is in fact the Strecj method for the identification of first order systems without delay.

With the rotor shaft locked, the current response $i(t)$, obtained by feeding a step input voltage $V(t)$ from 0 to 25 volts is shown in figure 6. The current variation is approximately 1.2 Amp so the steady state gain is $K_{G_E} = 0.048$, hence $R = 20.833\Omega$. Also, from Figure 6, the steady state time of the current response is $t_{ss_E} = 0.03\,\text{sec}$. Thus, the inductance is $L = 156.24mH$.

The mutual inductance k_0 can be estimated measuring the steady state responses of the current $i(t)$ and the speed of the rotor $\omega(t)$ to a step input voltage $V(t)$.

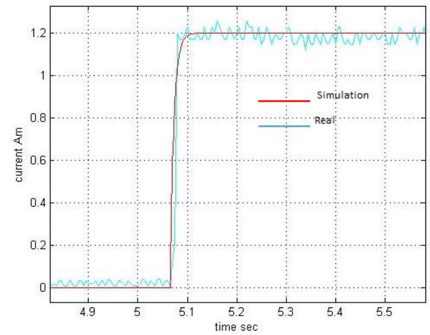

Fig. 6. Current step response (rotor shaft locked)

It should be noted that under this condition $\frac{d}{dt}i(t) \approx 0$.

Rearranging equation (8):

$$k_0 = \frac{V(t) - Ri(t)}{\omega(t)i(t)} \tag{12}$$

In figures 7 and 8, the current $i(t)$ and rotor speed, $\omega(t)$, responses to a step input voltage from 0 to 25 volts are shown. From the steady state responses, the previously estimated resistance, R, and equation (12), the mutual inductance value is: $k_0 = 0.17554$ N-m/Wb-A.

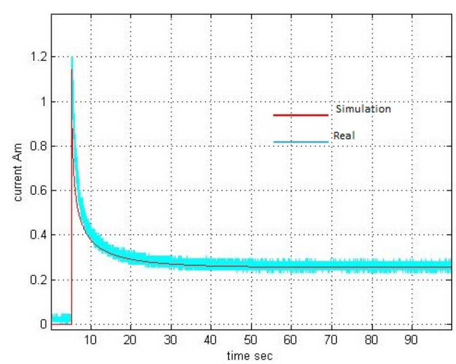

Fig. 7. Current step response

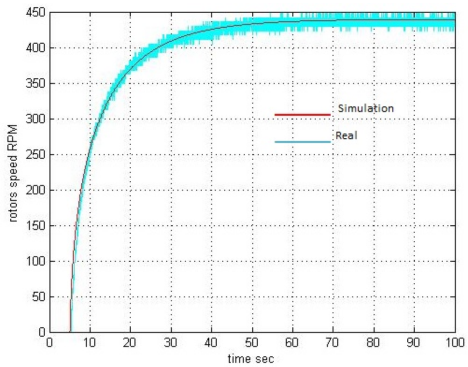

Fig. 8. Rotors speed step response

B. Mechanical Subsystem Identification

From equation (4), the mechanical subsystem without torque load, $T_L(t) = 0$, is described by:

$$T_e(t) = b\omega(t) + J\frac{d}{dt}\omega(t) \tag{13}$$

With a transfer function

$$G_M(s) = \frac{\omega(s)}{T_e(s)} = \frac{1}{Js + b} \tag{14}$$

Similar to the electrical subsystem identification, the steady state gain and the steady state time for the mechanical subsystem are $K_{G_M} = 1/b$ and $t_{ss_M} = 4J/b$, respectively. It should be noted that the electromagnetic input torque $T_e(t) = k_0 i^2(t)$ can be obtained by measuring the current $i(t)$ together with the estimated mutual inductance $k_0 = 0.17554$.

Figure 7 shows that the steady state variation of the current $i(t)$ is 0.255. Thus, from equations (6) and (7) the variation of the electromagnetic torque $T_E(t)$ is 0.01141. On the other hand, from Figure 8, the steady state variation of the speed of the rotor is 439.82*RPM*. Hence, the gain of the mechanical subsystem is $K_{G_M} = 1/b = 38531.73$, resulting in a friction coefficient $b = 0.000026$ N-m/Wb-A .

Estimating the rotor inertia J requires measuring the steady state time t_{ss_M} of the velocity $\omega(t)$ to a step input. However, Figure 6 clearly indicates that the magnetic torque input cannot be assumed as a step input. Nevertheless, the inertia J was initially estimated using the relation $J = t_{ss_M} b/4$. Further adjustments via trial and error were carried out finding that $J = 0.0006206$ Kg-m^2 was closer to the actual response of the system.

The résumé of the estimated parameters of the series DC motors is shown in Table I.

TABLE I. ESTIMATED PARAMETERS

$k_0 = 0.17554$ N-m/Wb-A
$R = 20.833\Omega$
$L = 156.24mH$
$b = 0.000026$ N-m/Wb-A
$J = 0.0006206$ Kg-m^2

The Simulink program of the Series DC motor non-linear model without magnetic saturation is shown in figure 9.

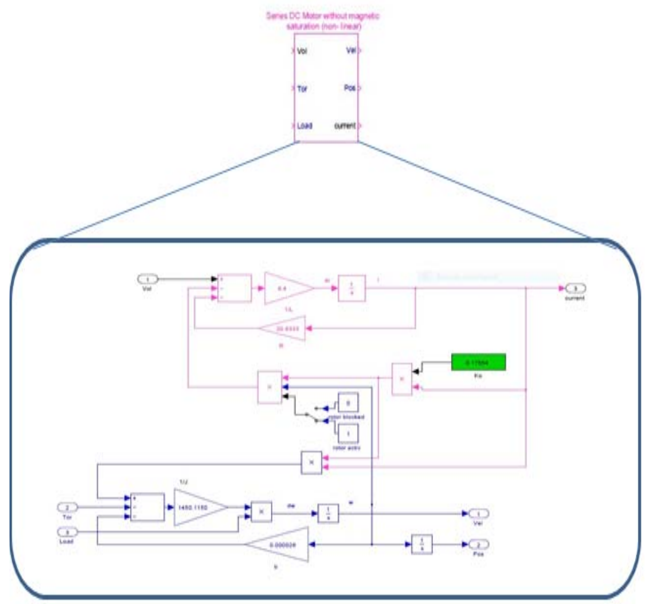

Fig. 9. Simulink program Series DC motor

C. Model Linearization

A linear approximation of equations (8) and (9) around any equilibrium point may represent a better model for certain control objectives. In this sense, equations (8) and (9) are rearranged as follows

$$\frac{d}{dt}i(t) = -\frac{R}{L}i(t) - \frac{k_0}{L}\omega(t)i(t) + \frac{1}{L}V(t) \tag{15}$$

$$\frac{d}{dt}\omega(t) = -\frac{b}{J}\omega(t) - \frac{1}{J}T_L(t) + \frac{k_0}{J}i^2(t) \tag{16}$$

defining

$$\begin{aligned} a_1 &=: \frac{k_0}{J}; & b_1 &=: \frac{R}{L} & & \\ a_2 &=: \frac{b}{J}; & b_2 &=: \frac{k_0}{L} & \text{and} \quad & \begin{aligned} x_1 &=: \omega \\ x_2 &=: i \end{aligned} \\ a_3 &=: \frac{1}{J}; & b_3 &=: \frac{1}{L} & & \end{aligned} \tag{17}$$

The nonlinear state space representation of the series DC motor is given by:

$$\begin{aligned}\dot{x}_1 &= a_1 x_2^2 - a_2 x_1 - a_3 T_L \\ \dot{x}_2 &= -b_1 x_2 - b_2 x_1 x_2 + b_3 V\end{aligned} \tag{18}$$

$$\dot{x} = \begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} a_1 x_2^2 - a_2 x_1 - a_3 T_L \\ -b_1 x_2 - b_2 x_1 x_2 + b_3 V\end{bmatrix} = f(x,u) \tag{19}$$

The equilibrium point $\left(x_1^0, x_2^0\right)$ of equation (18) is given by:

$$x_2^0 = \sqrt{\frac{a_2 x_1^0 + a_3 T_L}{a_1}}; \qquad V = \frac{x_2^0\left(b_1 + b_2 x_1^0\right)}{b_3} \tag{20}$$

The linear approximation of (18) around the equilibrium point (19) is given by:

$$\dot{x} = Ax + Bu;\ y = Cx \tag{21}$$

where

$$\begin{aligned} A &= \left.\frac{\delta f(x,u)}{\delta x}\right|_{x_1^0, x_2^0} = \begin{bmatrix} -a_2 & 2a_1 x_2^0 \\ -b_2 x_2^0 & -\left(b_1 + b_2 x_1^0\right)\end{bmatrix}; \\ B &= \left.\frac{\delta f(x,u)}{\delta u}\right|_{x_1^0, x_2^0} = \begin{bmatrix} -a_3 & 0 \\ 0 & b_3\end{bmatrix}; \\ C &= [1, 0]\end{aligned} \tag{22}$$

with

$$u = \begin{bmatrix} T_L & V\end{bmatrix}^T, \quad y = \omega(t) \tag{23}$$

If the load torque is assumed zero $T_L = 0$

$$x_2^0 = \sqrt{\frac{a_2 x_1^0}{a_1}}; \qquad V = \frac{x_2^0\left(b_1 + b_2 x_1^0\right)}{b_3}; \quad B = \begin{bmatrix} 0 \\ b_3\end{bmatrix} \tag{24}$$

These calculations lead to a model, which can be considered as a system with one input, voltage $V(t)$, subjected to a torque perturbation.

The transfer function $G(s)$ associated to the state space representation (21) around the equilibrium point (20) with $x_1^0 = 439.82\ RPM$ is:

$$G(s) = \frac{\omega(s)}{V(s)} = C\left(sI - A\right)^{-1} B = \frac{923.3}{s^2 + 626.6s + 67.58} \tag{25}$$

The poles of the transfer function (25) are $\{-3256.2, -0.1\}$ so the series DC motor is stable and over damped. It is also possible to distinguish the two typical modes of a DC motor from the poles: $(s + 0.1)$ representing the slow dynamic of the mechanical subsystem and $(s + 3256.2)$ the fast dynamic of the electrical subsystem.

In [8], was shown that linear models may be suitable for regulation and tracking control objectives.

IV. Real Time Model Validation

To validate the nonlinear model the responses of the actual series DC motor and the identified nonlinear model to different input voltages were compared.

In figures (6) and (7) the step responses of the current and rotors speed to a step input voltage from 0 to 25 volts are shown. From these figures it is clear that the model matched the actual motors response.

To verify the validity of the model in a range of input variation and frequency a sinusoidal input voltage was applied.

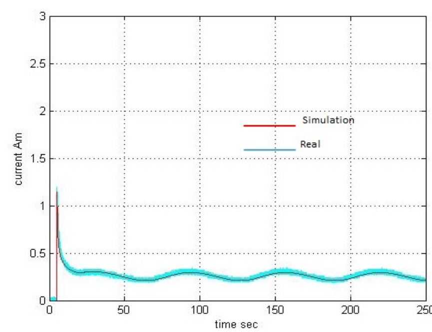

Fig. 10. Rotors speed response to a sinusoidal input

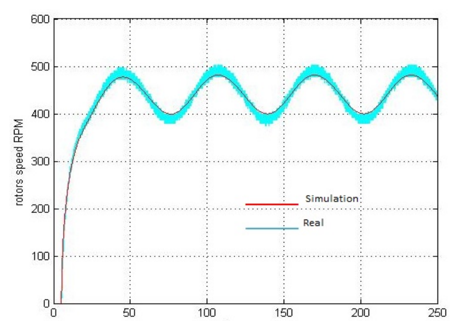

Fig. 11. Rotors speed response to a sinusoidal input

The input voltage is given by:

$$V(t) = 25 + 5\sin(0.05t) \qquad (26)$$

The frequency of the signal takes into account the slow mode of the mechanical subsystem. The responses of the current and rotors speed to the sinusoidal input voltage of equation (26) are shown in figures (10) and (11). From these plots it is clear that the estimated model responses are comparable to the actual motor responses.

V. CONCLUSIONS

The modeling, without magnetic saturation, and estimation of a series DC motor was obtained. The estimation is based only on the transient response of the current and rotor speed resulting in a simple and easy to implement procedure. The resulting model matched the actual response of a real series DC motor. Also, a linearization of the nonlinear model in an equilibrium point is presented. This linear model which clearly identifies the slow mode of the mechanical subsystem and the fast mode of the Electrical subsystem may be adequate to regulation purposes where classical linear controllers can be implemented.

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