```
# 9.6.1 in the book
# An Introduction to Statistical Learning

# Construct a linearly seperable dataset on 2-D plane
set.seed (1)
x=matrix(rnorm(20*2), ncol=2)
y=c(rep(-1,10), rep(1,10))
x[y==1,]=x[y==1,] + 1
plot(x, col=(3-y))
dat=data.frame(x=x, y=as.factor(y))

# Load the libsvm R interface
# use LiblineaR for very large problem
library('e1071')

svmfit=svm(y ~ ., data=dat, kernel='linear', cost=10, scale=FALSE)
plot(svmfit,dat)
svmfit$index
summary(svmfit)

svmfit=svm(y ~ ., data=dat, kernel='linear', cost=0.1, scale=FALSE)
plot(svmfit,dat)
svmfit$index
summary(svmfit)

# Find optimal tuning parameter
set.seed (1)
tune.out=tune(svm,y ~ .,data=dat,kernel="linear",ranges=list(cost=c
    (0.001, 0.01, 0.1, 1,5,10,100)))
bestmod=tune.out$best.model
summary(bestmod)

# Construct the test data
xtest=matrix(rnorm(20*2), ncol=2)
ytest=sample(c(-1,1), 20, rep=TRUE)
xtest[ytest==1,]=xtest[ytest==1,] + 1
testdat=data.frame(x=xtest, y=as.factor(ytest))

ypred=predict(bestmod ,testdat)
# Confusion matrix
table(predict=ypred, truth=testdat$y)

# Change Cost parameter to be 0.01
svmfit=svm(y~., data=dat, kernel="linear", cost=.01, scale=FALSE)
ypred=predict(svmfit,testdat)
table(predict=ypred, truth=testdat$y)

x[y==1,]=x[y==1,]+0.5
plot(x, col=(y+5)/2, pch=19)

dat=data.frame(x=x,y=as.factor(y))
svmfit=svm(y~., data=dat, kernel="linear", cost=1e5)
summary(svmfit)
plot(svmfit , dat)

svmfit=svm(y~., data=dat, kernel="linear", cost=1)
summary(svmfit)
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plot(svmfit , dat)

# 9.6.2 Nonlinear kernel with radial basis
set.seed (1)
x=matrix(rnorm(200*2), ncol=2)
x[1:100,]=x[1:100,]+2
x[101:150,]=x[101:150,]-2
y=c(rep(1,150),rep(2,50))
dat=data.frame(x=x,y=as.factor(y))
plot(x, col=y)
train=sample(200,100)
svmfit=svm(y~., data=dat[train,], kernel="radial", gamma=1, cost
    =1)
plot(svmfit , dat[train ,])
summary(svmfit)
svmfit=svm(y~., data=dat[train,], kernel="radial",gamma=1, cost=1e5
    )
plot(svmfit ,dat[train ,])
tune.out=tune(svm, y~., data=dat[train,], kernel="radial", ranges=
    list(cost=c(0.1,1,10,100,1000), gamma=c(0.5,1,2,3,4) ))
table(true=dat[-train,"y"], pred=predict(tune.out$best.model, newx=
    dat[-train ,]))

# 9.6.3 ROC curves
library('ROCR')
rocplot=function(pred, truth, ...){
  predob = prediction(pred, truth)
  perf = performance(predob, "tpr", "fpr")
  plot(perf,...)}
svmfit.opt=svm(y~., data=dat[train,], kernel="radial", gamma=2,
    cost=1,decision.values=T)
fitted=attributes(predict(svmfit.opt,dat[train,],decision.values=
    TRUE))$decision.values
par(mfrow=c(1,2))
rocplot(fitted,dat[train,"y"],main="Training Data")
svmfit.flex=svm(y~., data=dat[train,], kernel="radial", gamma=50,
    cost=1, decision.values=T)
fitted=attributes(predict(svmfit.flex,dat[train,],decision.values=T
    ))$decision.values
rocplot(fitted ,dat[train ,"y"],add=T,col="red")
fitted=attributes(predict(svmfit.opt,dat[-train,],decision.values=T
    ))$decision.values
rocplot(fitted,dat[-train,"y"],main="Test Data")
fitted=attributes(predict(svmfit.flex,dat[-train,],decision.values=
    T))$decision.values
rocplot(fitted,dat[-train,"y"],add=T,col="red")

# 9.6.4 Multiclass SVM
set.seed (1)
x=rbind(x, matrix(rnorm(50*2), ncol=2))
y=c(y, rep(0,50))
x[y==0,2]=x[y==0,2]+2
dat=data.frame(x=x, y=as.factor(y))
par(mfrow=c(1,1))
plot(x,col=(y+1))
svmfit=svm(y~., data=dat, kernel="radial", cost=10, gamma=1)
plot(svmfit, dat)
```

```
# 9.6.5 Gene Expression data 'Khan'
library(ISLR)
names(Khan)
#[1] "xtrain" "xtest"  "ytrain" "ytest"
dim(Khan$xtrain )
#[1]   63 2308
dim(Khan$xtest )
#[1]   20 2308
length(Khan$ytrain )
#[1] 63
length(Khan$ytest )
#[1] 20
table(Khan$ytrain )
table(Khan$ytest )

#training error
dat=data.frame(x=Khan$xtrain, y=as.factor(Khan$ytrain))
out=svm(y~., data=dat, kernel="linear",cost=10)
summary(out)
table(out$fitted, dat$y)

# test error
dat.te=data.frame(x=Khan$xtest, y=as.factor(Khan$ytest ))
pred.te=predict(out, newdata=dat.te)
table(pred.te, dat.te$y)
```

svm_test.R

```
# Load Radon data
> radon <- read.csv('radon.csv')
> names(radon)
[1] "state"    "region"   "typebldg" "floor"    "room"      "wave"
[7] "rep"      "radon"    "county"
> dim(radon)
[1] 12777     9
> z <- lm(radon ~ county + floor + typebldg , data = radon)
> summary(z)

Call:
lm(formula = radon ~ county + floor + typebldg, data = radon)

Residuals:
    Min      1Q  Median      3Q     Max
-4.8853 -0.5818  0.0120  0.5980  4.9741

Coefficients:
                         Estimate Std. Error t value Pr(>|t|)
(Intercept)              1.555239   0.208025   7.476 8.16e-14 **
    *
countyADAIR             -0.701062   0.455642  -1.539 0.123922
countyADAMS             -0.053314   0.242150  -0.220 0.825743
countyAITKIN            -0.704999   0.538455  -1.309 0.190458
countyALLEGHENY         -0.477014   0.216331  -2.205 0.027471 *
countyALLEN             -0.714868   0.220922  -3.236 0.001216 **
countyANDREW            -0.376171   0.455653  -0.826 0.409066
countyANOKA             -0.553931   0.248917  -2.225 0.026075 *
```

3

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countyAPACHE                 -1.279393    0.330213    -3.874 0.000107 **
      *
countyARMSTRONG              -0.036274    0.336932    -0.108 0.914266
countyATCHISON                0.745330    0.538465     1.384 0.166330
countyAUDRAIN                -1.848293    0.455731    -4.056 5.03e-05 **
      *
countyBAD RIVER             -1.515663    0.407976    -3.715 0.000204 **
      *
countyBARNES                  0.368073    0.262595     1.402 0.161036
countyBARNSTABLE             -0.993532    0.230519    -4.310 1.65e-05 **
      *
countyBARRY                  -0.946617    0.376491    -2.514 0.011939 *
countyBARTHOLOMEW            -0.061931    0.279706    -0.221 0.824774
countyBARTON                 -1.477536    0.376589    -3.923 8.78e-05 **
      *
countyBATES                  -0.662655    0.364368    -1.819 0.068990 .
countyBAY MILLS             -1.740983    0.317923    -5.476 4.43e-08 **
      *
countyBEAVER                  0.064046    0.226412     0.283 0.777278
countyBECKER                 -0.276791    0.610132    -0.454 0.650084
countyBEDFORD                 0.295805    0.336932     0.878 0.379996
countyBELTRAMI               -0.193349    0.429051    -0.451 0.652255
countyBENSON                  0.021397    0.407992     0.052 0.958175
countyBENTON                 -1.072567    0.344887    -3.110 0.001876 **
countyBERKS                   0.230512    0.260115     0.886 0.375533
countyBERKSHIRE              -0.742418    0.252943    -2.935 0.003340 **
countyBIG STONE              0.123439    0.610138     0.202 0.839675
countyBILLINGS                0.499747    0.390804     1.279 0.201004
countyBLACKFORD              -1.462718    0.538467    -2.716 0.006608 **
countyBLAIR                  -0.504725    0.271727    -1.857 0.063268 .
countyBLUE EARTH             0.545298    0.336935     1.618 0.105601
countyBOIS FORTE            -0.672850    0.296411    -2.270 0.023225 *
countyBOLLINGER             -1.127608    0.376492    -2.995 0.002750 **
countyBOONE                  -0.438888    0.296445    -1.481 0.138764
countyBOTTINEAU               0.043509    0.270009     0.161 0.871985
countyBOWMAN                  0.323702    0.273543     1.183 0.236685
countyBRADFORD                0.038304    0.260108     0.147 0.882929
countyBRISTOL                -0.817994    0.227200    -3.600 0.000319 **
      *
countyBROWN                   0.140396    0.429051     0.327 0.743504
countyBUCHANAN               -0.230862    0.261318    -0.883 0.377008
countyBUCKS                  -0.306658    0.253836    -1.208 0.227033
countyBURKE                  -0.505071    0.538463    -0.938 0.348269
countyBURLEIGH              -0.001913    0.229656    -0.008 0.993354
countyBUTLER                 -0.097793    0.228084    -0.429 0.668107
countyCALDWELL              -0.634332    0.610130    -1.040 0.298515
countyCALLAWAY              -0.999777    0.610138    -1.639 0.101320
countyCAMBRIA               -0.493426    0.257832    -1.914 0.055675 .
countyCAMDEN                -0.753941    0.290034    -2.599 0.009347 **
countyCAMERON               -0.555660    0.490451    -1.133 0.257254
countyCAPE GIRARDEAU       -0.512986    0.376494    -1.363 0.173055
countyCARBON                  0.210724    0.312860     0.674 0.500615
countyCARLTON               -0.448922    0.376498    -1.192 0.233143
countyCARROLL               -0.803816    0.364377    -2.206 0.027402 *
countyCARTER                -0.445359    0.732742    -0.608 0.543332
countyCARVER                -0.253127    0.455638    -0.556 0.578532
countyCASS                    0.032435    0.217008     0.149 0.881188
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countyCAVALIER               -0.861216    0.336949   -2.556 0.010602  *
countyCEDAR                  -1.497103    0.407979   -3.670 0.000244  **
                                                                      *
countyCENTRE                  0.779662    0.296424    2.630 0.008543  **
countyCHARITON               -1.057676    0.538458   -1.964 0.049521  *
countyCHESTER                -0.101145    0.269991   -0.375 0.707948
countyCHIPPEWA                0.340676    0.538463    0.633 0.526953
countyCHISAGO                -0.350850    0.455649   -0.770 0.441315
countyCHRISTIAN              -1.432323    0.429059   -3.338 0.000845  **
                                                                      *
countyCLARION                -0.426616    0.344896   -1.237 0.216133
countyCLARK                  -0.711148    0.230529   -3.085 0.002041  **
countyCLAY                   -0.409324    0.226872   -1.804 0.071223  .
countyCLEARFIELD             -0.285285    0.279706   -1.020 0.307775
countyCLEARWATER             -0.280703    0.538538   -0.521 0.602214
countyCLINTON                -0.596376    0.312797   -1.907 0.056597  .
countyCOCHISE                -1.277813    0.261611   -4.884 1.05e-06  **
                                                                      *
countyCOCONINO               -1.302659    0.232651   -5.599 2.20e-08  **
                                                                      *
countyCOLE                   -0.896952    0.268358   -3.342 0.000833  **
                                                                      *
countyCOLUMBIA                0.666516    0.376498    1.770 0.076701  .
countyCOOK                   -0.724709    0.732749   -0.989 0.322669
countyCOOPER                 -0.628988    0.538481   -1.168 0.242798
countyCOTTONWOOD             -0.755967    0.538458   -1.404 0.160359
countyCRAWFORD               -0.900787    0.275476   -3.270 0.001079  **
countyCROW WING              -0.423992    0.353951   -1.198 0.230986
countyCUMBERLAND              1.009348    0.254768    3.962 7.48e-05  **
                                                                      *
countyDADE                   -0.299655    1.015324   -0.295 0.767897
countyDAKOTA                 -0.080356    0.242164   -0.332 0.740028
countyDALLAS                 -1.317474    0.538458   -2.447 0.014429  *
countyDAUPHIN                 0.660092    0.251273    2.627 0.008625  **
countyDAVIESS                -0.816877    0.353949   -2.308 0.021021  *
countyDE KALB                -0.289177    0.287176   -1.007 0.313969
countyDEARBORN               -0.777071    0.455639   -1.705 0.088135  .
countyDECATUR                -0.245462    0.490451   -0.500 0.616745
countyDELAWARE               -0.930740    0.247489   -3.761 0.000170  **
                                                                      *
countyDENT                   -2.069074    0.455640   -4.541 5.65e-06  **
                                                                      *
countyDICKEY                  0.219670    0.364387    0.603 0.546621
countyDIVIDE                  0.631258    0.538463    1.172 0.241085
countyDODGE                   0.410744    0.610138    0.673 0.500833
countyDOUGLAS                -0.324153    0.317908   -1.020 0.307918
countyDUBOIS                 -1.224868    0.490448   -2.497 0.012522  *
countyDUKES                  -0.869228    0.456004   -1.906 0.056649  .
countyDUNKLIN                -1.199518    0.353964   -3.389 0.000704  **
                                                                      *
countyDUNN                    0.479302    0.279720    1.714 0.086645  .
countyEDDY                   -0.545780    0.538455   -1.014 0.310792
countyELK                    -0.489694    0.317923   -1.540 0.123516
countyELKHART                -0.273907    0.236542   -1.158 0.246902
countyEMMONS                  0.181538    0.329881    0.550 0.582114
countyERIE                   -0.764648    0.238896   -3.201 0.001374  **
countyESSEX                  -0.441725    0.218806   -2.019 0.043530  *
```

```
countyFARIBAULT            -0.856759   0.455642   -1.880 0.060086  .
countyFAYETTE              -0.328041   0.268353   -1.222 0.221571
countyFILLMORE             -0.301518   0.732742   -0.411 0.680718
countyFLOYD                -0.627099   0.271712   -2.308 0.021018  *
countyFOND DU LAC          -0.078822   0.336943   -0.234 0.815041
countyFOREST               -0.930372   0.610130   -1.525 0.127316
countyFOREST COUNTY        -1.660929   0.455638   -3.645 0.000268  **
    *
countyFOSTER               -0.307912   0.429054   -0.718 0.472984
countyFOUNTAIN              0.135948   0.344887    0.394 0.693453
countyFRANKLIN             -0.615382   0.231992   -2.653 0.007998  **
countyFREEBORN             0.591820    0.390806    1.514 0.129960
countyFULTON               -0.006939   0.353953   -0.020 0.984359
countyGASCONADE            -2.244026   0.429050   -5.230 1.72e-07  **
    *
countyGENTRY               -0.864099   0.490445   -1.762 0.078117  .
countyGIBSON               -0.740024   0.323571   -2.287 0.022209  *
countyGILA                 -1.483245   0.344903   -4.300 1.72e-05  **
    *
countyGOLDEN VALLEY        -0.121061   0.429062   -0.282 0.777832
countyGOODHUE              0.472027    0.336932    1.401 0.161253
countyGRAHAM               -1.637953   0.277678   -5.899 3.76e-09  **
    *
countyGRAND FORKS          0.852564    0.220686    3.863 0.000112  **
    *
countyGRAND PORTAGE        -0.508475   0.287177   -1.771 0.076652  .
countyGRAND TRAVERSE       -1.569784   0.329907   -4.758 1.97e-06  **
    *
countyGRANT                0.259673    0.265326    0.979 0.327751
countyGREENE               -0.804843   0.241148   -3.338 0.000848  **
    *
countyGREENLEE             -1.384914   0.407992   -3.394 0.000690  **
    *
countyGRIGGS               -0.322103   0.376492   -0.856 0.392269
countyGRUNDY               -1.185179   0.538465   -2.201 0.027752  *
countyHAMILTON             -0.491359   0.293102   -1.676 0.093683  .
countyHAMPDEN              -1.027208   0.225540   -4.554 5.30e-06  **
    *
countyHAMPSHIRE            -0.731660   0.247516   -2.956 0.003122  **
countyHANCOCK              -0.603954   0.407979   -1.480 0.138803
countyHANNAHVILLE          -1.770139   0.407974   -4.339 1.44e-05  **
    *
countyHARRISON             -0.223198   0.287184   -0.777 0.437059
countyHENDRICKS            -1.026022   0.296416   -3.461 0.000539  **
    *
countyHENNEPIN             -0.072259   0.228853   -0.316 0.752201
countyHENRY                -0.954269   0.261316   -3.652 0.000262  **
    *
countyHETTINGER            0.293444    0.273551    1.073 0.283418
countyHICKORY              -1.611889   0.733620   -2.197 0.028027  *
countyHOLT                 -1.014928   0.733620   -1.383 0.166551
countyHOUSTON              0.175829    0.455639    0.386 0.699580
countyHOWARD               -0.591851   0.271777   -2.178 0.029447  *
countyHOWELL               -0.487028   0.329876   -1.476 0.139863
countyHUBBARD              -0.522307   0.490446   -1.065 0.286914
countyHUNTINGDON           -0.147732   0.429064   -0.344 0.730618
countyHUNTINGTON           -0.470831   0.344889   -1.365 0.172225
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countyINDIANA               -0.340146    0.317921   -1.070 0.284682
countyIRON                  -1.788043    0.429076   -4.167 3.10e-05 **
      *
countyISANTI                -0.333570    0.610138   -0.547 0.584586
countyITASCA                -0.463812    0.364382   -1.273 0.203086
countyJACKSON               -0.388107    0.215513   -1.801 0.071750 .
countyJASPER                -1.284880    0.249655   -5.147 2.69e-07 **
      *
countyJAY                   -0.556679    0.490446   -1.135 0.256378
countyJEFFERSON             -0.664443    0.235498   -2.821 0.004789 **
countyJENNINGS              -0.939868    0.308229   -3.049 0.002299 **
countyJOHNSON               -0.833897    0.240670   -3.465 0.000532 **
      *
countyJUNIATA               -0.557725    0.490445   -1.137 0.255485
countyKANABEC               -0.153281    0.538463   -0.285 0.775907
countyKANDIYOHI              0.672303    0.538463    1.249 0.211850
countyKEWEENAW BAY          -1.570799    0.243767   -6.444 1.21e-10 **
      *
countyKIDDER                -0.160587    0.407984   -0.394 0.693875
countyKITTSON               -0.215049    0.610132   -0.352 0.724497
countyKNOX                  -0.809466    0.353987   -2.287 0.022230 *
countyKOOCHICHING           -0.926873    0.429051   -2.160 0.030770 *
countyKOSCIUSKO             -0.022114    0.275471   -0.080 0.936017
countyLA MOURE               0.215658    0.490455    0.440 0.660154
countyLA PAZ                -3.137347    0.732752   -4.282 1.87e-05 **
      *
countyLA PORTE              -0.533011    0.240675   -2.215 0.026802 *
countyLAC COURTE OREILLES   -0.822248    0.244937   -3.357 0.000790 **
      *
countyLAC DU FLAMBEAU       -0.369957    0.265321   -1.394 0.163229
countyLAC QUI PARLE          1.257457    0.732742    1.716 0.086169 .
countyLAC VIEUX DESERT      -1.165350    0.353955   -3.292 0.000996 **
      *
countyLACKAWANNA           -0.779350    0.235510   -3.309 0.000938 **
      *
countyLACLEDE              -1.417951    0.353955   -4.006 6.21e-05 **
      *
countyLAFAYETTE            -0.347114    0.268348   -1.294 0.195856
countyLAGRANGE              0.434975    0.390936    1.113 0.265880
countyLAKE                 -1.366951    0.224336   -6.093 1.14e-09 **
      *
countyLAKE OF THE WOODS     0.166207    0.538454    0.309 0.757575
countyLANCASTER             0.877442    0.239327    3.666 0.000247 **
      *
countyLAWRENCE            -0.626976    0.230983   -2.714 0.006649 **
countyLE SUEUR             0.233720    0.490448    0.477 0.633696
countyLEBANON              1.181597    0.300027    3.938 8.25e-05 **
      *
countyLEECH LAKE          -0.925415    0.234557   -3.945 8.01e-05 **
      *
countyLEHIGH               0.830494    0.293102    2.833 0.004612 **
countyLEWIS               -2.305862    1.015322   -2.271 0.023160 *
countyLINCOLN             -0.596905    0.376492   -1.585 0.112892
countyLINN                -0.645043    0.490462   -1.315 0.188476
countyLIVINGSTON          -0.281057    0.732749   -0.384 0.701307
countyLOGAN                0.223527    0.344897    0.648 0.516933
countyLOWER SIOUX         -0.762045    0.490535   -1.553 0.120330
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countyLUZERNE              -0.487915    0.226574   -2.153 0.031303  *
countyLYCOMING              0.022862    0.279720    0.082 0.934861
countyLYON                  0.499597    0.407980    1.225 0.220764
countyMACON                -1.368647    0.490453   -2.791 0.005269  **
countyMADISON              -0.520410    0.268368   -1.939 0.052504  .
countyMAHNOMEN             -0.028594    1.015322   -0.028 0.977533
countyMARICOPA             -1.129202    0.210504   -5.364 8.27e-08  **
    *
countyMARIES               -2.699613    0.732752   -3.684 0.000230  **
    *
countyMARION               -0.302916    0.225803   -1.342 0.179780
countyMARSHALL             -0.406955    0.354018   -1.150 0.250360
countyMARTIN               -0.526669    0.353987   -1.488 0.136824
countyMCDONALD             -2.246716    0.490451   -4.581 4.67e-06  **
    *
countyMCHENRY              -0.304204    0.275493   -1.104 0.269520
countyMCINTOSH             -0.277287    0.390796   -0.710 0.478000
countyMCKEAN               -0.954304    0.329879   -2.893 0.003824  **
countyMCKENZIE             -0.244516    0.455651   -0.537 0.591533
countyMCLEAN                0.135445    0.317916    0.426 0.670087
countyMCLEOD               -0.326694    0.344893   -0.947 0.343538
countyMEEKER               -0.208094    0.490469   -0.424 0.671372
countyMENOMINEE             0.063287    0.219903    0.288 0.773509
countyMERCER               -0.120359    0.230523   -0.522 0.601600
countyMIAMI                 0.005350    0.279726    0.019 0.984742
countyMIDDLESEX            -0.498561    0.213272   -2.338 0.019420  *
countyMIFFLIN              -0.343434    0.376507   -0.912 0.361703
countyMILLE LACS           -0.817580    0.732742   -1.116 0.264538
countyMILLER               -1.426228    0.376493   -3.788 0.000152  **
    *
countyMILLIE LACS          -1.261750    0.344908   -3.658 0.000255  **
    *
countyMISSISSIPPI          -2.440247    0.610142   -3.999 6.39e-05  **
    *
countyMOHAVE               -1.736913    0.230175   -7.546 4.80e-14  **
    *
countyMONITEAU             -1.813527    0.538454   -3.368 0.000759  **
    *
countyMONROE               -0.222523    0.231714   -0.960 0.336904
countyMONTGOMERY           -0.378298    0.235501   -1.606 0.108222
countyMONTOUR               0.442817    0.455649    0.972 0.331151
countyMORGAN               -1.117911    0.329880   -3.389 0.000704  **
    *
countyMORRISON            -0.310771    0.390804   -0.795 0.426507
countyMORTON               0.082952    0.230099    0.361 0.718476
countyMOUNTRAIL            0.258335    0.303904    0.850 0.395312
countyMOWER                0.203813    0.344895    0.591 0.554569
countyMURRAY               1.103634    1.015322    1.087 0.277067
countyNAVAJO              -1.138828    0.245628   -4.636 3.58e-06  **
    *
countyNELSON               0.139625    0.284527    0.491 0.623630
countyNEW MADRID          -1.543013    0.429068   -3.596 0.000324  **
    *
countyNEWTON              -0.664282    0.293101   -2.266 0.023445  *
countyNICOLLET             0.775468    0.538463    1.440 0.149850
countyNOBLE               -0.339941    0.303917   -1.119 0.263362
countyNOBLES               0.538120    0.610138    0.882 0.377813
```

```
countyNODAWAY              -0.265541   0.407983   -0.651 0.515148
countyNORFOLK              -0.698656   0.221500   -3.154 0.001613 **
countyNORMAN               -0.346726   0.610130   -0.568 0.569854
countyNORTHAMPTON           0.729678   0.284562    2.564 0.010353 *
countyNORTHUMBERLAND        0.363910   0.330089    1.102 0.270284
countyOHIO                 -0.631026   0.538458   -1.172 0.241254
countyOLIVER                0.384836   0.308148    1.249 0.211737
countyOLMSTED              -0.099364   0.293190   -0.339 0.734685
countyONEIDA               -0.951849   0.235176   -4.047 5.21e-05 **
    *
countyORANGE               -0.393420   0.364374   -1.080 0.280291
countyOREGON               -0.093011   0.610132   -0.152 0.878839
countyOSAGE                -1.499105   0.455638   -3.290 0.001004 **
countyOTTER TAIL           -0.005530   0.407974   -0.014 0.989186
countyOWEN                 -1.602658   0.490451   -3.268 0.001087 **
countyOZARK                -0.727211   0.490452   -1.483 0.138170
countyPARKE                -0.771695   0.429054   -1.799 0.072107 .
countyPEMBINA               0.499995   0.244337    2.046 0.040745 *
countyPEMISCOT             -0.299655   1.015324   -0.295 0.767897
countyPENNINGTON           -0.704389   0.610132   -1.154 0.248323
countyPERRY                -0.488839   0.344895   -1.417 0.156404
countyPETTIS               -0.867354   0.312791   -2.773 0.005563 **
countyPHELPS               -1.576189   0.336925   -4.678 2.93e-06 **
    *
countyPHILADELPHIA         -1.040991   0.225534   -4.616 3.96e-06 **
    *
countyPIERCE               -0.169446   0.317926   -0.533 0.594061
countyPIKE                 -0.788542   0.287178   -2.746 0.006045 **
countyPIMA                 -1.345018   0.216384   -6.216 5.27e-10 **
    *
countyPINAL                -1.192385   0.270360   -4.410 1.04e-05 **
    *
countyPINE                 -0.731366   0.455642   -1.605 0.108490
countyPIPESTONE             0.398219   0.538532    0.739 0.459647
countyPLATTE               -0.094518   0.287183   -0.329 0.742071
countyPLYMOUTH             -1.070867   0.223524   -4.791 1.68e-06 **
    *
countyPOLK                 -0.836962   0.308144   -2.716 0.006614 **
countyPOPE                 -0.110183   0.732749   -0.150 0.880476
countyPORTER               -0.829522   0.233910   -3.546 0.000392 **
    *
countyPOSEY                -0.535115   0.455639   -1.174 0.240246
countyPOTTER                0.278560   0.353961    0.787 0.431308
countyPRAIRIE ISLAND       -0.255881   0.490445   -0.522 0.601867
countyPULASKI              -1.582897   0.317971   -4.978 6.51e-07 **
    *
countyPUTNAM               -1.466333   0.390803   -3.752 0.000176 **
    *
countyRALLS                -1.398268   1.015324   -1.377 0.168487
countyRAMSEY               -0.078078   0.250440   -0.312 0.755226
countyRANDOLPH             -0.799986   0.329874   -2.425 0.015317 *
countyRANSOM                0.422048   0.429050    0.984 0.325292
countyRAY                  -0.702784   0.317923   -2.211 0.027085 *
countyRED CLIFF            -1.762588   0.455653   -3.868 0.000110 **
    *
countyRED LAKE             -0.774998   0.241657   -3.207 0.001345 **
countyREDWOOD               0.468666   0.490448    0.956 0.339300
```

```
countyRENVILLE                0.148403   0.353958    0.419 0.675028
countyREYNOLDS               -0.396319   1.015322   -0.390 0.696293
countyRICE                    0.401549   0.364377    1.102 0.270476
countyRICHLAND                0.134789   0.253838    0.531 0.595424
countyRIPLEY                 -0.880107   0.317909   -2.768 0.005641 **
countyROCK                   -0.090453   0.732749   -0.123 0.901758
countyROLETTE                -0.056172   0.303908   -0.185 0.853364
countyROSEAU                 -0.080330   0.336926   -0.238 0.811560
countyRUSH                   -1.803733   1.015324   -1.777 0.075674 .
countySAGINAW CHIPPEWA       -1.178781   0.265344   -4.442 8.97e-06 **
   *
countySALINE                 -0.139429   0.390797   -0.357 0.721262
countySANTA CRUZ             -0.942364   0.344969   -2.732 0.006309 **
countySARGENT                 0.210727   0.376498    0.560 0.575693
countySAULT ST. MARIE        -2.217608   0.246179   -9.008  < 2e-16 **
   *
countySCHUYLER               -0.556662   1.015322   -0.548 0.583522
countySCHUYLKILL              0.243291   0.271716    0.895 0.370598
countySCOTLAND               -1.652904   0.538465   -3.070 0.002148 **
countySCOTT                  -0.960119   0.252940   -3.796 0.000148 **
   *
countySHAKOPEE-MDEWAKANTO    -0.643123   0.538454   -1.194 0.232349
countySHANNON                -0.368983   0.455640   -0.810 0.418064
countySHELBY                 -0.175713   0.376567   -0.467 0.640782
countySHERBURNE              -0.299549   0.407986   -0.734 0.462833
countySHERIDAN                0.201907   0.490448    0.412 0.680582
countySIBLEY                 -0.147118   0.538463   -0.273 0.784690
countySIOUX                  -0.437033   0.732785   -0.596 0.550919
countySLOPE                   0.099478   0.429052    0.232 0.816655
countySNYDER                  0.451619   0.455649    0.991 0.321629
countySOKAOGAN CHIPPEWA      -0.610601   0.538480   -1.134 0.256844
countySOMERSET               -0.650907   0.296428   -2.196 0.028122 *
countySPENCER                -0.882444   0.364368   -2.422 0.015456 *
countyST CHARLES             -0.849048   0.250443   -3.390 0.000701 **
   *
countyST CLAIR               -0.417023   0.610132   -0.683 0.494306
countyST FRANCOIS            -0.887646   0.252064   -3.522 0.000431 **
   *
countyST JOSEPH              -0.444090   0.227198   -1.955 0.050648 .
countyST LOUIS               -0.717824   0.218472   -3.286 0.001020 **
countyST LOUIS CITY          -0.870979   0.218472   -3.987 6.74e-05 **
   *
countyST. CROIX              -0.933138   0.344904   -2.705 0.006830 **
countySTARK                   0.261600   0.225963    1.158 0.247004
countySTARKE                 -1.254362   0.407976   -3.075 0.002112 **
countySTE GENEVIEVE          -1.521231   0.364368   -4.175 3.00e-05 **
   *
countySTEARNS                 0.002452   0.287183    0.009 0.993187
countySTEELE                  0.088999   0.317921    0.280 0.779528
countySTEUBEN                 0.170333   0.344910    0.494 0.621423
countySTEVENS                 0.402188   0.732749    0.549 0.583100
countySTOCKBRIDGE-MUNSEE      0.296578   0.252937    1.173 0.241003
countySTODDARD               -1.335445   0.390851   -3.417 0.000636 **
   *
countySTONE                  -0.940992   0.390797   -2.408 0.016060 *
countySTUTSMAN                0.045857   0.260108    0.176 0.860062
countySUFFOLK                -1.021282   0.236966   -4.310 1.65e-05 **
```

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```
              *
countySULLIVAN                -1.356866   0.329880  -4.113 3.93e-05 **
              *
countySUSQUEHANNA             -0.330537   0.303898  -1.088 0.276767
countySWIFT                   -0.402531   0.538463  -0.748 0.454743
countySWITZERLAND             -0.686937   0.732752  -0.937 0.348533
countyTANEY                   -0.873530   0.376554  -2.320 0.020368 *
countyTEXAS                   -1.434406   0.407976  -3.516 0.000440 **
              *
countyTIOGA                   -0.425572   0.277534  -1.533 0.125202
countyTIPPECANOE              -0.029220   0.261313  -0.112 0.910969
countyTIPTON                  -0.951878   0.490451  -1.941 0.052303 .
countyTODD                     0.126190   0.610130   0.207 0.836150
countyTOWNER                   0.258824   0.376502   0.687 0.491815
countyTRAILL                   0.122467   0.284525   0.430 0.666893
countyTRAVERSE                 0.462899   0.538455   0.860 0.389982
countyUNION                    0.767014   0.407980   1.880 0.060128 .
countyUPPER SIOUX              0.255736   0.376494   0.679 0.496989
countyVANDERBURGH             -0.919414   0.271708  -3.384 0.000717 **
              *
countyVENANGO                 -0.187665   0.284580  -0.659 0.509621
countyVERMILLION              -0.396170   0.407979  -0.971 0.331540
countyVERNON                  -1.410995   0.344895  -4.091 4.32e-05 **
              *
countyVIGO                    -0.227942   0.268348  -0.849 0.395661
countyWABASH                  -0.027189   0.329870  -0.082 0.934311
countyWABASHA                  0.319318   0.429062   0.744 0.456755
countyWADENA                  -0.353433   0.490445  -0.721 0.471147
countyWALSH                    0.590069   0.251241   2.349 0.018859 *
countyWARD                    -0.183495   0.240678  -0.762 0.445830
countyWARREN                  -0.707522   0.287180  -2.464 0.013765 *
countyWARRICK                 -1.384988   0.300009  -4.616 3.94e-06 **
              *
countyWASECA                  -0.930657   0.538455  -1.728 0.083944 .
countyWASHINGTON              -0.411924   0.228839  -1.800 0.071875 .
countyWATONWAN                 0.904046   0.610132   1.482 0.138440
countyWAYNE                   -0.564261   0.245537  -2.298 0.021575 *
countyWEBSTER                 -1.320250   0.329871  -4.002 6.31e-05 **
              *
countyWELLS                   -0.709963   0.312790  -2.270 0.023238 *
countyWESTMORELAND            -0.380048   0.239321  -1.588 0.112306
countyWHITE                   -1.171433   0.323607  -3.620 0.000296 **
              *
countyWHITE EARTH             -0.038566   0.287199  -0.134 0.893181
countyWHITLEY                 -0.255376   0.293098  -0.871 0.383609
countyWILKIN                   0.840443   1.015322   0.828 0.407822
countyWILLIAMS                -0.011229   0.293109  -0.038 0.969442
countyWINONA                   0.102124   0.344894   0.296 0.767157
countyWISCONSIN WINNEBAGO     -1.203646   0.303890  -3.961 7.51e-05 **
              *
countyWORCESTER              -0.306700   0.217877  -1.408 0.159253
countyWORTH                   -1.442251   0.732749  -1.968 0.049059 *
countyWRIGHT                  -0.369518   0.303893  -1.216 0.224028
countyWYOMING                 -0.553656   0.353963  -1.564 0.117804
countyYAVAPAI                 -1.486729   0.249728  -5.953 2.70e-09 **
              *
countyYELLOW MEDICINE         -0.203049   0.732749  -0.277 0.781703
```

```
countyYORK                      0.638449    0.240683    2.653 0.007996 **
countyYUMA                     -1.824017    0.268522   -6.793 1.15e-11 **
        *
floor                          -0.096664    0.007033 -13.745  < 2e-16 **
        *
typebldg                       -0.165668    0.017538   -9.446  < 2e-16 **
        *
---
Signif. codes:  0    ***    0.001    **    0.01    *    0.05    .
        0.1         1


Residual standard error: 0.9939 on 12389 degrees of freedom
Multiple R-squared:  0.2907,  Adjusted R-squared:  0.2685
F-statistic: 13.12 on 387 and 12389 DF,  p-value: < 2.2e-16

> z <- lm(radon ~ floor + typebldg, data = radon)
> summary(z)

Call:
lm(formula = radon ~ floor + typebldg, data = radon)

Residuals:
    Min     1Q  Median     3Q     Max
-4.1502 -0.7300 -0.0024  0.7300  4.7933

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.154429   0.023758   48.59   <2e-16 ***
floor       -0.188644   0.007512  -25.11   <2e-16 ***
typebldg    -0.235770   0.019562  -12.05   <2e-16 ***
---
Signif. codes:  0    ***    0.001    **    0.01    *    0.05    .
        0.1         1

Residual standard error: 1.133 on 12774 degrees of freedom
Multiple R-squared:  0.04999, Adjusted R-squared:  0.04985
F-statistic: 336.1 on 2 and 12774 DF,  p-value: < 2.2e-16

> library(lme4)
Loading required package: Matrix
> zc <- lmer(radon ~ floor + typebldg + (1|county), data = radon)
> summary(zc)
Linear mixed model fit by REML ['lmerMod']
Formula: radon ~ floor + typebldg + (1 | county)
   Data: radon

REML criterion at convergence: 36855.6

Scaled residuals:
    Min      1Q  Median      3Q     Max
-4.8701 -0.5880  0.0128  0.6086  4.9877

Random effects:
 Groups    Name        Variance Std.Dev.
 county   (Intercept) 0.3396   0.5828
 Residual             0.9881   0.9940
Number of obs: 12777, groups:  county, 386
```

```
Fixed effects:
              Estimate Std. Error t value
(Intercept)   1.078113   0.038677  27.875
floor        -0.102853   0.007005 -14.683
typebldg     -0.169967   0.017485  -9.721

Correlation of Fixed Effects:
         (Intr) floor
floor    -0.203
typebldg -0.485  0.255
> zcw <- lmer(radon ~ floor + typebldg + (1|county) + (1|wave),
    data = radon)
> summary(zcw)
Linear mixed model fit by REML ['lmerMod']
Formula: radon ~ floor + typebldg + (1 | county) + (1 | wave)
   Data: radon

REML criterion at convergence: 36829.9

Scaled residuals:
    Min     1Q  Median      3Q     Max
-4.7942 -0.5904  0.0113  0.6028  5.0171

Random effects:
 Groups   Name        Variance Std.Dev.
 county   (Intercept) 0.325611 0.57062
 wave     (Intercept) 0.009425 0.09708
 Residual             0.980985 0.99045
Number of obs: 12777, groups:  county, 386; wave, 135

Fixed effects:
              Estimate Std. Error t value
(Intercept)   1.072987   0.039626  27.078
floor        -0.103019   0.007011 -14.694
typebldg     -0.170289   0.017471  -9.747

Correlation of Fixed Effects:
         (Intr) floor
floor    -0.197
typebldg -0.472  0.254

> zcwSlope <- lmer(radon ~ floor + typebldg + (1|county) + (floor|
    wave), data = radon)
> summary(zcwSlope)
Linear mixed model fit by REML ['lmerMod']
Formula: radon ~ floor + typebldg + (1 | county) + (floor | wave)
   Data: radon

REML criterion at convergence: 36766.8

Scaled residuals:
    Min     1Q  Median      3Q     Max
-4.8636 -0.5927  0.0105  0.6056  5.0344

Random effects:
 Groups   Name        Variance Std.Dev. Corr
```

```
 county    (Intercept) 0.321956 0.56741
 wave      (Intercept) 0.005531 0.07437
           floor       0.010106 0.10053  0.25
 Residual              0.968386 0.98407
Number of obs: 12777, groups:  county, 386; wave, 135

Fixed effects:
            Estimate Std. Error t value
(Intercept)  1.07908    0.03887   27.76
floor       -0.13066    0.01245  -10.49
typebldg    -0.16346    0.01743   -9.38

Correlation of Fixed Effects:
         (Intr) floor
floor    -0.087
typebldg -0.478  0.134

> anova(zcwSlope, zcw)
refitting model(s) with ML (instead of REML)
Data: radon
Models:
zcw: radon ~ floor + typebldg + (1 | county) + (1 | wave)
zcwSlope: radon ~ floor + typebldg + (1 | county) + (floor | wave)
         Df    AIC    BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
zcw       6 36823 36867 -18405    36811
zcwSlope  8 36765 36824 -18374    36749 61.983      2  3.472e-14 **
    *
---
Signif. codes:  0    ***    0.001    **    0.01    *    0.05    .
        0.1         1

> zcw <- lmer(radon ~ floor + typebldg + (1|county) + (wave|county)
    , data = radon)
Error: number of observations (=12777) <= number of random effects
    (=52110) for term (wave | county); the random-effects
    parameters and the residual variance (or scale parameter) are
    probably unidentifiable

# Produce new responses for the fitted model zcw
> zcwYSim <- simulate(zcw, nsim = 100)
# Fit the same model to the new responses.
# Put each new model in a list.
> nsim=100
> zcwSim <- vector('list', length = 100)
>  for (i in 1:nsim) {
+  zcwSim[[i]] <- refit(zcw, newresp =
+  zcwYSim[,i])
+  }

# Exam data analysis
> library(mlmRev)
> names(Exam)
 [1] "school"   "normexam" "schgend"  "schavg"   "vr"        "intake
     "
 [7] "standLRT" "sex"      "type"     "student"
# Null model
> lmer(normexam ~ 1 + (1 | school), data=Exam)
```

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```
Linear mixed model fit by REML ['lmerMod']
Formula: normexam ~ 1 + (1 | school)
   Data: Exam
REML criterion at convergence: 11014.65
Random effects:
 Groups   Name        Std.Dev.
 school   (Intercept) 0.4142
 Residual             0.9207
Number of obs: 4059, groups:  school, 65
Fixed Effects:
(Intercept)
   -0.01325

> lmer(normexam ~ standLRT + (1 | school), data=Exam)
Linear mixed model fit by REML ['lmerMod']
Formula: normexam ~ standLRT + (1 | school)
   Data: Exam
REML criterion at convergence: 9368.765
Random effects:
 Groups   Name        Std.Dev.
 school   (Intercept) 0.3063
 Residual             0.7522
Number of obs: 4059, groups:  school, 65
Fixed Effects:
(Intercept)     standLRT
   0.002323     0.563307

 > lmer(normexam ~ standLRT + (standLRT | school), data=Exam,
     method="ML")
Linear mixed model fit by REML ['lmerMod']
Formula: normexam ~ standLRT + (standLRT | school)
   Data: Exam
REML criterion at convergence: 9327.6
Random effects:
 Groups   Name        Std.Dev. Corr
 school   (Intercept) 0.3035
          standLRT    0.1223   0.49
 Residual             0.7441
Number of obs: 4059, groups:  school, 65
Fixed Effects:
(Intercept)     standLRT
   -0.01165        0.55653
Warning message:
Argument   m e t h o d   is deprecated. Use the REML argument to
    specify ML or REML estimation.

> lmer(normexam ~ standLRT + schavg + (1 + standLRT | school), data
    =Exam)
Linear mixed model fit by REML ['lmerMod']
Formula: normexam ~ standLRT + schavg + (1 + standLRT | school)
   Data: Exam
REML criterion at convergence: 9323.885
Random effects:
 Groups   Name        Std.Dev. Corr
 school   (Intercept) 0.2778
          standLRT    0.1238   0.37
 Residual             0.7440
```

```
Number of obs: 4059, groups:  school, 65
Fixed Effects:
(Intercept)      standLRT         schavg
  -0.001423      0.552242       0.294731

> lmer(normexam ~ standLRT * schavg + (1 + standLRT | school), data
    =Exam)
Linear mixed model fit by REML ['lmerMod']
Formula: normexam ~ standLRT * schavg + (1 + standLRT | school)
   Data: Exam
REML criterion at convergence: 9320.387
Random effects:
 Groups    Name         Std.Dev. Corr
 school    (Intercept) 0.2763
           standLRT     0.1106    0.36
 Residual               0.7442
Number of obs: 4059, groups:  school, 65
Fixed Effects:
    (Intercept)            standLRT             schavg   standLRT:schavg
      -0.007092            0.557943           0.373398          0.161829
```

multilevel.R