ABSTRACT

This experiment was deigned to analysis of motion of a symmetrical rolling object. It was considered a symmetrical object about its axis and it was rolling along the plane because of the frictional force between the plane and itself. Both transitional and rotational motions of the object was analyzed using Newton's laws of motion. Note that the booth inertia and the rotational inertia will be determined its eventual motion.

A wooden board as an inclined plane, a glue tape as the object symmetrical about the axis to rolling, measuring tape were used to prepared the basic setup. The mobile phone was taken as the video camera to recorded the videos, tracker software was done the duty of taking necessary time and displacement measurements and the Matlab software was helped to plotting all the graphs and calculations including the error calculations as well. Different videos were taken for different height values (y values) as *figure 2*. There was a known value for the length of the wooden board and hence calculated the correspondent sin and cos values.

Relationships between the graphs were derived using the Newton's laws of motion, motion equations, equation of friction. $a = \left(\frac{g}{1+k}\right) sin\beta$ was derived as the equation of the 'graph of the acceleration vs sin(β)' and $a = \left(\frac{\mu g}{k}\right) cos\beta$ was the equation of the 'graph of the acceleration vs cos(β)'.

By equaling the gradient values,

- Then coefficient of the moment of inertia (k) is (0.77 ± 0.04)
- Then coefficient of the kinetic friction (μ) is (1.46 ± 0.09)
  Were found as the final results.

Air drag force can be neglected, the object was symmetrical, its shape doesn't change during the experiment, density of the object was constant and the frictional force is a constant everywhere between the two surfaces were some of general assumptions considered in this practical. But those can have some errors to affected on the final results.

Another assumption was made to be easier to the calculations was initial velocity of the object was zero. But in the tracker software, it wasn't easy to find an initial velocity was zero moment. This may be hardly affected with the final results. This error can be reduce using a camera with a higher framerate or much small angle values instead of these angles.

It wasn't easy to find the center of the mass every time. This might be another huge error maker to the results. Using a symmetrical object with much small radius this error can be reduced and observed much accurate results.

Resulted coefficient of the moment of inertia (k) is (0.77 ± 0.04) but the expected value was 1 for the hollow cylinder shapes. Resulted value doesn't lie within the range of the expected

values but the it can can be said as reasonable since it was near with the real value and with considering above facts as well.

Since μ lies between 0 and 1, $\mu = 1.46 \pm 0.09$ is an impossible observation.

Other major assumption was the object was travelled downwards along the plane only by rolling because of the friction but there might be slipping too. So, this might be a huge fact for the impossible results for μ. So, this method is not good method to evaluate the coefficient of the kinetic friction.

- I was unable to find a protractor so, I used lengths to calculate angles. So, this may can occur some errors. By using a protractor, it can be prevented from those errors.
- Calculating accelerations using velocity-time graphs instead of the equation will produce much perfect accelerations and hence much perfect final results as well.

METHODOLOGY

- A wooden board as an inclined plane, a glue tape as the object symmetrical about the axis to rolling, measuring tape, mobile phone as the video camera, tracker software and the Matlab software were the apparatus and accessories used in this practical.
- Using above, experimental setup was prepared as the *figure 1*. Camera was well fixed to take videos from a clear angle.
- It was unable to found a protractor so, then used lengths to calculate angles as *figure 2* shown. Length of the wooden board (l =55.2 cm) was known and marked on the wall 5cm by 5cm (y values) from bellow point. Using Pythagoras equation, x values were calculated. Hence, it was calculated the angles.

- Implicit plane was sat for the lowest y point marked (5 cm from the below point), switched on the camera and then kept the object on the top of plane and release the object by rest.
- This procedure was repeated by changing the angle (marked y values) hence there had been six videos for different angles.

- First video was inserted into the tracker software and tried so hard to tracked the initial moment by sliding the frames. Initial moment was taken approximately then tracked the object in a below point as some good difference with the initial point taken.
- This procedure was repeated to the all videos.
- Correspondent time values and the distance values for the all different angles were measured and recorded in the *table 1* to further analysis.

- It was taken the time difference and the distance difference and hence calculated correspondent accelerations using $s = ut + \frac{1}{2}at^2$.
- All necessary data to plotting graphs were recorded in *table 2*.
- Then, graphs of graph of the ‘acceleration vs sin(β)’ and ‘acceleration vs cos(β) then taken its gradient and intersections.
- Using Newton’s methods of motion, it was derivatived (*) and (**) equation as the equations of the graph.
- Then the coefficient of the moment of inertia (k) and coefficient of the kinetic friction (μ) were evaluated using above relationships.
- Plotting graphs and all the calculations including error calculations done by Matlab software.
- Finally, after all the calculations were done, they were compared with the real values and checked there reasonability.

FIGURE PANEL

Figure 1: Experimental setup

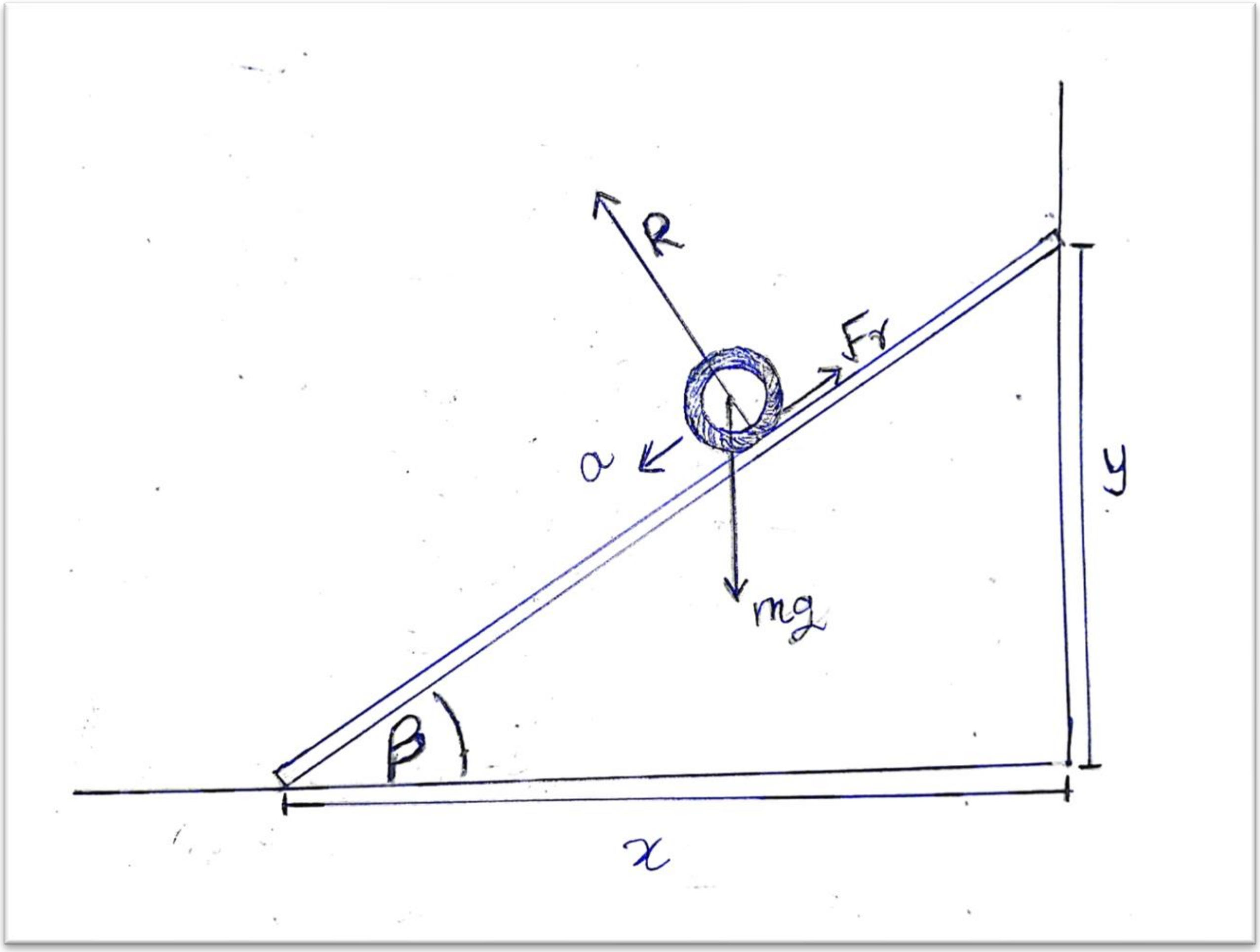

Figure 2: Cross section diagram and the force diagram

Table 1: Measurements

$(y \pm 0.05)$ cm	$(t_1 \pm 0.001)$ s	$(t_2 \pm 0.001)$ s	$(l_1 \pm 0.01)$ cm	$(l_2 \pm 0.01)$ cm
5	0.314	1.601	0.07	37.49
10	0.330	1.271	0.01	39.32
15	0.297	0.990	0.06	36.74
20	0.380	1.057	0.09	44.38
25	0.314	0.892	0.08	41.65
30	0.281	0.809	0.12	40.99

RESULTS AND ANLYSIS

- Length of the inclined plane (l) = 55.2 cm

- By *figure 2*, x values can be find using Pythagoras equation.

$$x = \sqrt{l^2 - y^2}$$

So,

$$\sin\beta = {}^{y}/_{55.2}$$

$$\cos\beta = {}^{X}/_{55.2}$$

- $t = (t_2 - t_1)$ and $\Delta l = (l_2 - l_1)$

And correspondent accelerations can be determined using the equation $s = ut + \frac{1}{2}at^2$ (approximately assuming initial velocity u=0)

$$a = \frac{2(\Delta l)}{t^2}$$

Table 2 : Prepared data table for analysis

(y ± 0.05) cm	(t ± 0.002) s	(Δl ± 0.0002) m	a (ms^{-2})	$\sin\beta$	$\cos\beta$
5	1.287	0.3742	0.45 ± 0.02	0.0906 ± 0.0009	0.996 ± 0.001
10	0.941	0.3931	0.89 ± 0.03	0.1812 ± 0.0009	0.983 ± 0.001
15	0.693	0.3668	1.53 ± 0.06	0.272 ± 0.001	0.962 ± 0.001
20	0.677	0.4429	1.93 ± 0.06	0.362 ± 0.001	0.932 ± 0.001
25	0.578	0.4157	2.49 ± 0.08	0.453 ± 0.001	0.892 ± 0.001
30	0.528	0.4087	2.9 ± 0.1	0.543 ± 0.001	0.839 ± 0.001

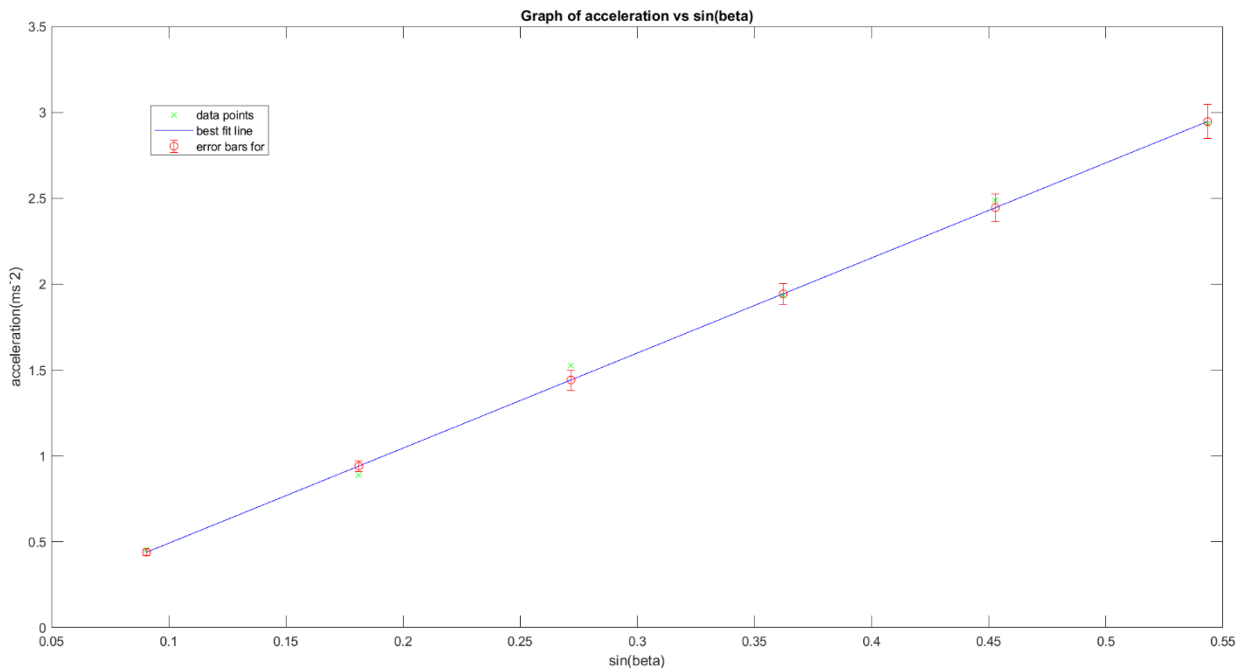

Figure 3: Graph of the acceleration vs sin(beta)

Considering the figure 2,

Putting f = ma downwards along the inclined plane we have,

$$\text{mgsin}\,\beta - f_r = ma\text{--------(1)}$$

m = mass of the object $\quad$ f_r = frictional force

Considering the rotational motion of the object (anti-clock vise),

$$\tau = I\alpha\text{---------(2)}$$

$\alpha = {}^{a}/_{r}$ $\quad$ $I = kmr^2$ $\quad$ and $\quad$ $\tau = f_r$r

Where k is the coefficient of the moment inertia I and r is the radius of the object and α is the angular acceleration of the object.

Then we have $\quad$ $f_r = kma$--------(3)

From equation (1),

$$\text{a} = \left(\frac{g}{1+k}\right)\sin\beta\text{-----------(*)}$$

This is the equation of the graph of acceleration vs sin β.

```
>> gpl
error of the gradient is;
(0.134363)
error of the interction is;
(0.025496)
The gradient is;
(5.536987)
The intersection is;
(-0.062632)
Coefficient of the moment of innertia is;
(0.771722)
fx >>
```

Figure 4: Details of graph of acceleration vs sin(beta)

Considering the gradient,

$$\left(\frac{g}{1+k}\right) = (5.537 \pm 0.003)\ \mathrm{ms}^{-2}$$

(taken g = 9.81 ms^{-2})

Then coefficient of the moment of inertia is (0.77 ± 0.04)

Coefficient of the moment of inertia of the object was used (hollow cylinder) in the experiment was 1.

It doesn’t lie within the range of the expected values but the resulted value can be said as reasonable since it was near with the real value.

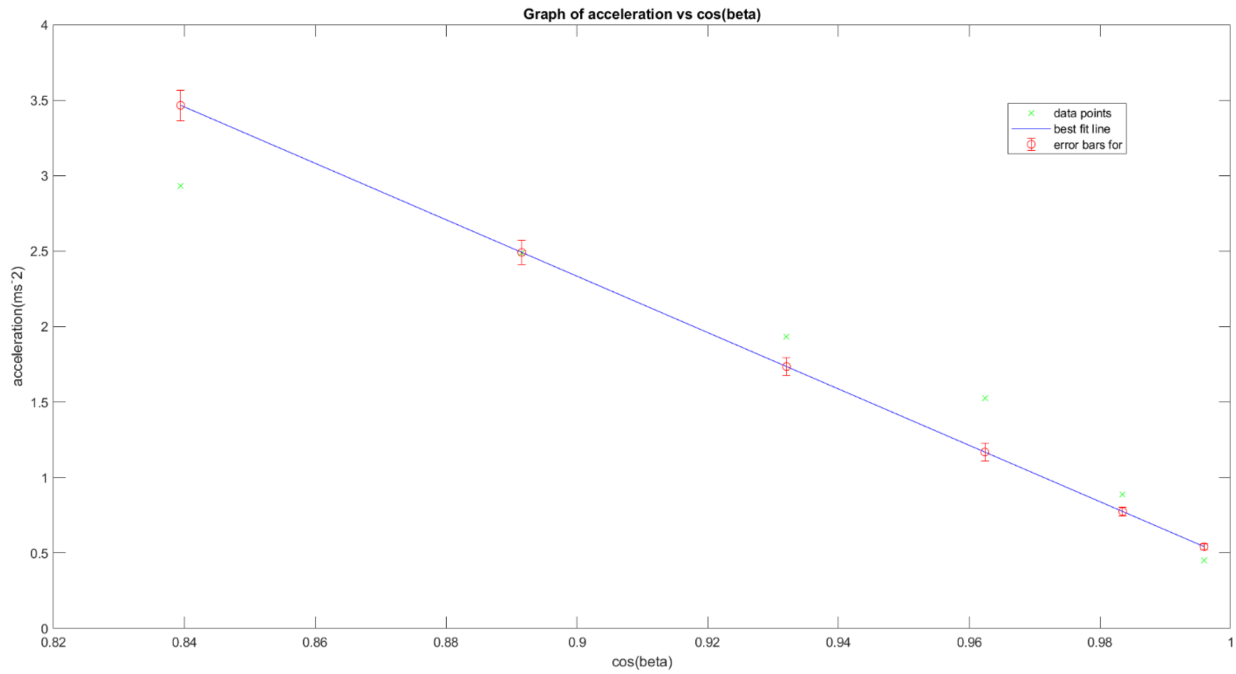

Figure 5: Graph of the acceleration vs cos(beta)

Considering the static friction $f_r = \mu$R--------(5)

where μ is the coefficient of the kinetic friction.

Putting f=ma to the object perpendicular to the inclined plane upwards,

$$R = mg\cos\beta \text{---------(6)}$$

By (3),(5) and (6) we have,

$$\mu mg\cos\beta = kma \text{--------(7)}$$

Then, $$a = \left(\frac{\mu g}{k}\right)\cos\beta \text{-----------(**)}$$

This is the equation of the graph of acceleration vs cos β.

```
>> Untitled
error of the gradient is;
(0.467983)
error of the interction is;
(0.025496)
The gradient is;
(-18.685770)
The intersection is;
(19.151288)
Coefficient of the coefficient of fraction is;
(1.469951)
fx >>
```

Figure 6: Details of graph of acceleration vs sin(beta)

Considering the gradient,

$$\left(\frac{\mu g}{k}\right) = (-18.7 \pm 0.5)\ ms^{-2}$$

(taken $g = 9.81\ ms^{-2}$)

Then coefficient of the kinetic friction is (1.46 ± 0.09)

Since μ lies between 0 and 1, this is an impossible observation.

- It was assumed that the air drag force can be neglected.
- The object was symmetrical, its shape doesn't change during the experiment, density of the object was constant and the frictional force is a constant everywhere between the two surfaces were the other major assumptions.

Those can have some errors to affected on the final results.

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