Encoding of SAT into congruence closure with full propagation.

Let $\mathcal{C}$ be a set of clause. We introduce a constant v_i for each propositional variable p_i, and a function f_i for each clause c_i in $\mathcal{C}$, the arity of f_i being the number of literals in c_i. We furthermore introduce the constants $\top$ and $\bot$.

For every clause $c_i = \{\ell_1, \ldots \ell_n\} \in \mathcal{C}$, we build a literal $f_i(v_1, \ldots v_n) \neq f_i(\mathrm{pol}(\ell_1), \ldots \mathrm{pol}(\ell_n))$, where $\mathrm{pol}(\ell)$ is $\bot$ if ℓ is v, $\top$ otherwise. We furthermore add (might be not necessary) $\top \neq \bot$.

Then $\mathcal{C}$ is unsatisfiable if and only if $v \neq \top$ and $v \neq \bot$ are both propagated, for some v.

We'd have to precisely define the problem, but I think it is trivially encoded into propositional unsatisfiability, thus trivially in Co-NP.