May 31, 2017

1 FORPI Correctness

The following are the definitions from the submitted version of the paper. Corrections are in red.

Definition 1. *The set of* safe literals *for a node η in a proof ψ with root clause Γ, denoted $\mathcal{S}(\eta)$, is such that $\ell \in \mathcal{S}(\eta)$ if and only if $\ell \in \Gamma$ or for all paths from η to the root of ψ there is an edge $v_1 \xrightarrow[\sigma]{\ell'} v_2$ with $\ell'\sigma = \ell$.*

Definition 2. *Let η be a node with safe literals $\mathcal{S}(\eta)$ and parents η_1 and η_2, assuming without loss of generality, $\eta_1 \xrightarrow[\sigma_1]{\{\ell_1\}} \eta$. The node η is said to be* pre-regularizable *in the proof ψ if $\ell_1\sigma_1$ is unifiable with a safe literal $\ell^* \in \mathcal{S}(\eta)$.*

Definition 3. *Let η be pre-regularizable, with safe literals $\mathcal{S}(\eta)$ and parents η_1 and η_2, with clauses Γ_1 and Γ_2 respectively, assuming without loss of generality that $\eta_1 \xrightarrow[\sigma_1]{\{\ell_1\}} \eta$ such that $\ell_1\sigma_1$ is unifiable with a safe literal $\ell^* \in \mathcal{S}(\eta)$. The node η is said to be* strongly regularizable *in ψ if $\Gamma_1\sigma_1 \subseteq \mathcal{S}(\eta)$.*

The notion of *pre-regulariziability* can be thought of as a *necessary* condition for recycling the node η, while the notion of *strongly regularizable* can be thought of as a *sufficient* condition. Note that these updated definitions more closely resemble their use, e.g. in Example 4.3 of the paper, when we say that η_3 is pre-regularizable, we only say that its pivot is unifiable with a safe literal; we don't look at η_2 at all, which would be required using the definition of pre-regularizable in the paper. Moreover, since η_1 replaces a strongly regularizable node η, η_1 remains in the proof - thus for any nodes η_2' used in the old definition of pre-regularizable, it shouldn't matter that $\mathcal{R}(\eta) \cup \{\overline{\ell_1}\}$ is unifiable - all of those nodes η_2' remain in the proof as well.

The following theorem is what the reviewer is looking for. We require the additional notion of subsumption. We will use $X \sqsubseteq Y$ to denote the following for clauses X and Y: there exists a substitution σ such that $X\sigma \subseteq Y$. We say that X *subsumes* Y.

Theorem 1. *Let ψ be a proof with root clause Γ, and $\eta \in \psi$ a node. Let $\psi' = \psi \setminus \{\eta\}$ and Γ' be the root of ψ'. If η is strongly regularizable, then $\Gamma' \sqsubseteq \Gamma$.*

Lemma 1. *Let η_1 be a node and $\rho(\eta_1)$ be a path from η_1 to the root of the proof. Suppose that $\eta \in \rho(\eta_1)$ is a node such that $\eta_1 \sqsubseteq \mathcal{S}(\eta)$. If η is replaced by η_1 in some proof ψ to obtain ψ', every literal $\ell_s \in \eta_1$ is either used as a pivot below η_1 in ψ' or is contained in the root clause $\Gamma(\psi')$.*

Proof. For a pair of nodes η_1, η that satisfy the conditions of the lemma, let σ_1 be the substitution such that $\eta_1\sigma_1 \subseteq \mathcal{S}(\eta)$. Assume that $\eta_1 \xrightarrow[\sigma]{\{\ell_1\}} \eta$ in ψ.

We proceed by induction $h(\eta)$, the height of η in ψ, which is the length of a longest path from the root to η. For the base case $h(\eta) = 0$, when deleting η, η is replaced by η_1 and by assumption there exists a σ_1 such that $\Gamma(\eta_1)\sigma_1 \subseteq \mathcal{S}(\eta) = \Gamma(\eta) \implies \Gamma(\eta_1) \sqsubseteq \Gamma(\eta)$. This concludes the base case; assume the result holds for any node η_I with height $h(\eta_I) > 0$ and consider a node η at height $h(\eta) = h(\eta_I) + 1$.

For the inductive step, consider any path $\rho(\eta')$ from η' to the root of the proof, and let η'' be the node which is resolved against η in ψ. The deletion of η from ψ attempts to replace the resolution $\eta' = \eta \odot \eta''$ with $\eta' = \eta_1 \odot \eta''$. For each path $\rho(\eta')$, there are two cases: either there exists an $\ell_1'' \in \eta_1$ such that $\ell_1''\sigma_1$ can be used as the instantiated resolved literal between η_1 and η''', or no such ℓ_1'' exists.

Case 1: $\eta_1 \xrightarrow[\sigma_1''=\sigma_1]{\{\ell_1''\}} \eta'$ and $\eta'' \xrightarrow[\sigma_2'']{\{\ell_2''\}} \eta'$ for some ℓ_1'', ℓ_2'', and σ_2''. Since all instantiated literals of $\eta_1\sigma_1$ are safe, for each of the remaining literals $\ell_s\sigma_1 \in \Gamma(\eta_1)\sigma_1 \cap \Gamma(\eta')$ such that $\ell_s \neq \ell_1''$, there is a node $\eta_{\ell_s} \in \rho(\eta')$ that uses $\ell_s\sigma_1$ as a resolved literal or $\ell_s\sigma_1$ is contained in the root clause Γ; i.e. every remaining literal $\ell \in \eta_1$ that is not contained in Γ will eventually be used as a resolved literal. The nodes using $\ell_{\eta''}\sigma_2'' \in (\Gamma(\eta'')\sigma_2'' \cap \Gamma(\eta')) \setminus (\Gamma(\eta_1)\sigma_1)$ are unchanged, so these literals will still be used as a resolved literal for some node below η'. It remains to be shown that ℓ_1 is still used as a resolved literal. To see this, recall that clauses are sets and that $\ell_1\sigma_1$ is safe. Therefore the resolution on $\rho(\eta')$ which uses $\ell_1\sigma_1$ as a resolved literal removes all copies[1] of $\ell_1\sigma_1$.

Case 2: σ_1 cannot be used as a unifier for literals of η_1 and η''; i.e. resolution between η_1 and η'' is not possible for any $\ell_1'' \in \eta_1$ with the instantiated resolved literal $\ell_1''\sigma_1$. In this case, replace η' by η_1; since $\ell_1''\sigma_1'' \notin \Gamma(\eta_1)\sigma_1$, every $\ell_s\sigma_1 \in \Gamma(\eta_1)\sigma_1$ must still be used as a resolved literal below η', i.e. $\eta_1\sigma_1 \subseteq \mathcal{S}(\eta') \implies \eta_1 \sqsubseteq \mathcal{S}(\eta')$. Since $h(\eta') < h(\eta) = h(\eta_I) + 1$, we are done by the induction hypothesis.

■

Proof (of Theorem 1). Let ψ be a proof with root clause Γ, and let $\eta_S \in \psi$ be a strongly regularizable node. Let $\psi' = \psi \setminus \{\eta_S\}$ with root clause Γ'. To prove the theorem, it suffices to observe that any strongly regularizable node η_S satisfies Lemma 1's hypothesis for some $\rho(\eta_1)$.

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[1] Note that the desired result can be obtained by inserting a contraction before performing resolution with η' if clauses are defined as multi-sets.

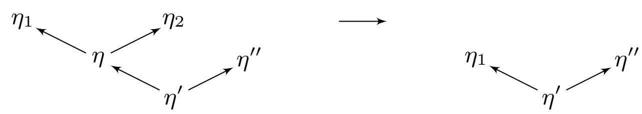

Fig. 1: The a layout of η_1 and η in proofs ψ (left) and $\psi \setminus \{\eta\}$ (right), as used in the proof of Lemma 1.

2 Other Corrections

The set to which σ is applied in Example 4.4 of the paper is wrong; it should read "$\{\neg p(X), \neg q(X), \neg r(X)\}$".

When the definitions are final, we'll need to check the pseudo-code in Algorithm 2 again. Note that the correction already discussed in the previous email also needs to be reflected in this algorithm.