Combining RecyclePivotsWithIntersection and LowerUnits

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Overview

Benchmarks

Algorithms implemented in Scala for Skeptik

1000 proofs from VeriT

- 100 SAT proofs from the old external SAT solver
- 900 SMT proofs translated to propositional resolution

Propositional resolution calculus

Conventions

- Clauses are sets of literals.
- $\bar{a}$ is the dual of a.
- Let η_Γ be a proof of clause Γ and η_Δ a proof of Δ.
- If $\bar{a} \in \Gamma$ and $a \in \Delta$ then $\eta = \eta_\Gamma \odot_a \eta_\Delta$ is a proof of $(\Gamma \cup \Delta) \setminus \{a, \bar{a}\}$.
- a is the *pivot* of η.
- η_Γ and η_Δ are the *premises* of η.
- η is a *child* of both η_Γ and η_Δ.

Proof as DAG

- A node can have more than one child.

Regular proof

Definition (Regular proof)

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Theorem (Tseitin)

Given a set of axioms and a clause Γ*, the smallest regular proof of* Γ *might be exponentialy bigger than the smallest irregular proof of* Γ*.*

Extending Irregularity

Definition (Fully regular proof)

A proof is fully regular if for each variable there is at most one resolution node with this variable as pivot.

Conventions

- Usual irregularities are called *vertical irregularities*.
- Other irregularities are called *horizontal irregularities*.

RecyclePivotsWithIntersection (RPI)

Partial Vertical Regularization

- Delete a branch only if the pivot appears on **every** path from the root to the node.

Definition (Safe literal)

A literal is safe for a node η if it can be added to η's clause without changing the proof's conclusion (root's clause).

Two traversals

$\uparrow$ Collect safe literals and mark edges to delete.

$\downarrow$ Delete edges and fix the proof.

LowerUnits (LU)

Lowering

- Moving a node down the proof to resolve it only once.

Lowering Units

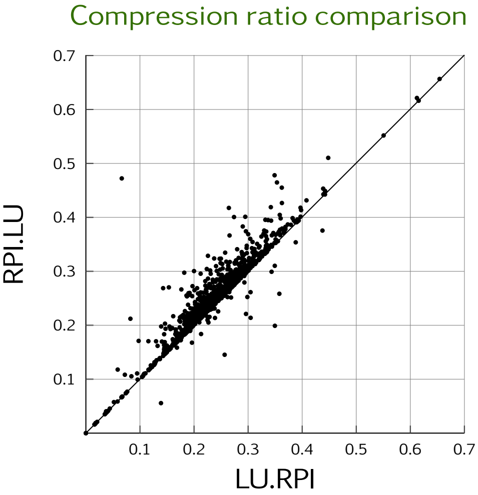
- Units can always be lowered.
- Reduces horizontal irregularities.

Two traversals

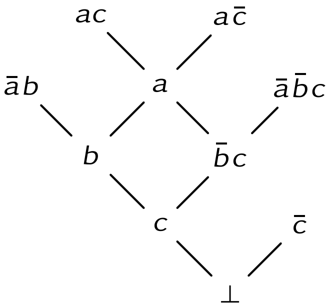
$\updownarrow$ Collect units with more than one child.

$\downarrow$ Delete units, fix the proof and then reintroduce the units at the bottom of the proof.

Sequential Composition



Irregular Units



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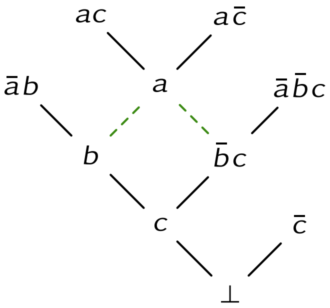

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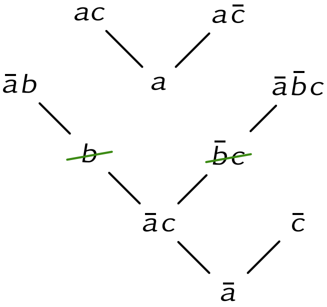

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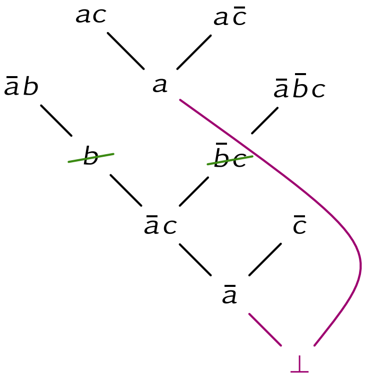

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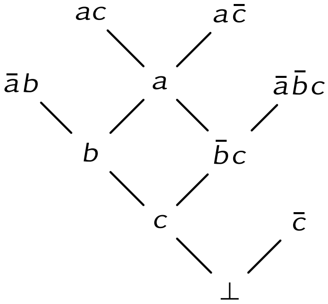

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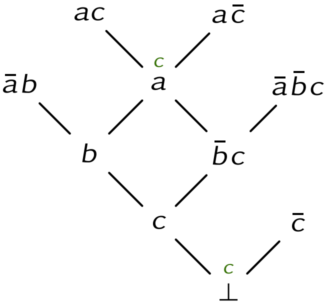

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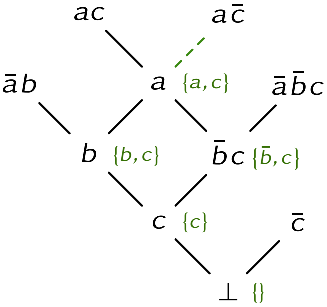
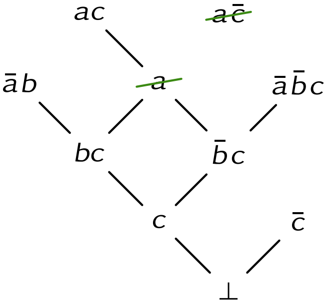


Irregular Units



Root and Units Safe Literals

In RPI.LU, after LU :

- the proof is of the form $\eta \odot_{a_0} \eta_0 \odot_{a_1} \cdots \odot_{a_n} \eta_n$;
- $\{\bar{a}_i \mid i \leq n\}$ is the safe literals for η : *the root's safe literals* ;
- $\forall i < n, \left\{\bar{a}_j \mid i < j \leq n\right\}$ is the safe literals for η_i.

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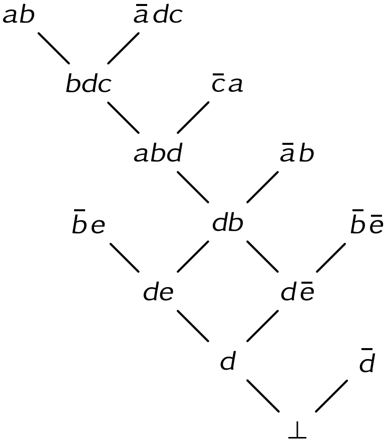
Units introduced by RPI



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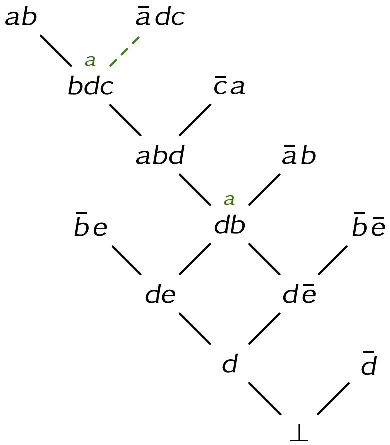

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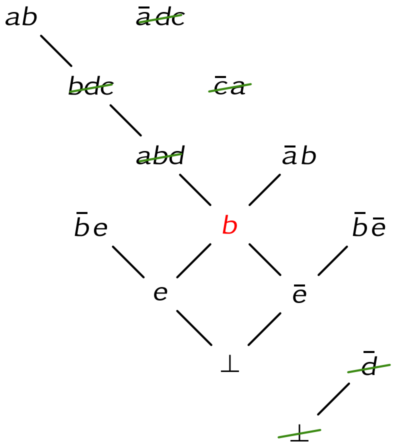

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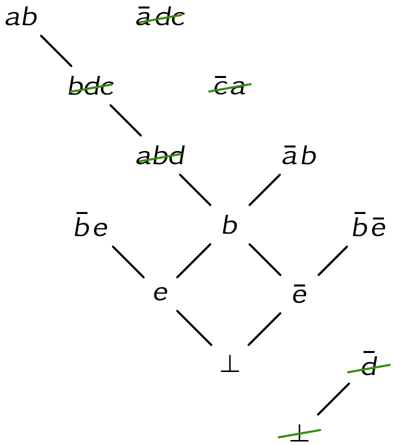


For a combined algorithm to be always at least as good as LU.RPI it has to be able to lower units introduced by RPI.

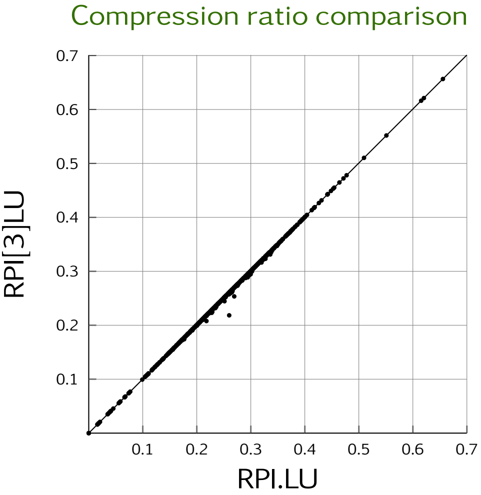
RPI[3]LU

For a combined algorithm to be always at least as good as RPI.LU it has to compute root and units safe literals.

Three traversals

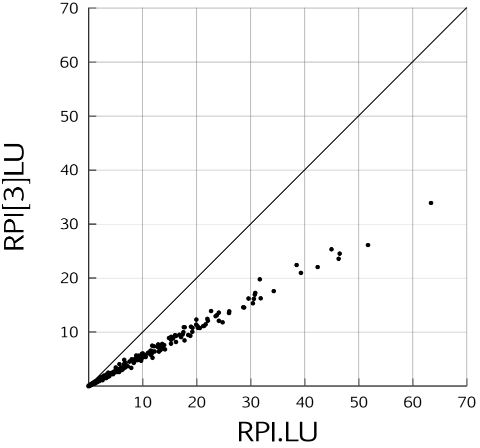
- ↓ collect units and compute root and units safe literals ;
- ↑ compute safe literals and mark edges to be deleted ;
- ↓ fix the proof and reintroduce units.

RPI[3]LU vs RPI.LU



RPI[3]LU vs RPI.LU

Compression time comparison (in seconds)



LUnivRPI

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- Collect units during fixing (top-down traversal).

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- If a unit $\{a\}$ depends on a unit $\{b\}$ it'll be seen as $\{a, \bar{b}\}$.
- What to do if we find $\{a, \bar{b}\}$ after $\{b\}$?
- A new algorithm extending LU is needed.

Lowering a node

The (generalized) problem

- Given $\psi[\eta] \odot_{a_0} \eta_0 \odot_{a_1} \cdots \odot_{a_{n-1}} \eta_{n-1}$
- is $\mathsf{Fix}(\psi[]) \odot_a \eta \odot_{a_0} \eta_0 \odot_{a_1} \cdots \odot_{a_{n-1}} \eta_{n-1}$ equivalent ?

Two steps

- Deleting the node : $\mathsf{Fix}(\psi[])$;
- Reintroducing it : $\odot_a \eta$.

Beware of introduced literals

- $\Delta = \{\bar{a}_i \mid i < n\}$ is the safe literals of $\mathsf{Fix}(\psi[]) \odot_a \eta$.

Conditions

Literals introduced by reintroducing the node

- Let Γ_+ be η's clause,
- $\Gamma \setminus \Delta = \{a\}$.

Definition (Active literal)

Let's consider a node η with clause Γ_+. A literal a from Γ_+ is said to be an active literal of η iff a is the pivot of one of η's child.

Literals introduced by deleting the node

- Let Γ_- be the set of the duals of η's active literals,
- $\Gamma_- \setminus \Delta = \{\bar{a}\}$.

Partial regularization

Deletable node

- If $\Gamma_{-} \setminus \Delta = \varnothing$ then delete η.

Partial regularization

Deletable node

- If $\Gamma_{-} \setminus \Delta = \varnothing$ then delete η.

Partial regularization

- If the dual of any of η's active literal belongs to Δ then delete the edge.

Partial regularization

Deletable node (implemented)

- If $\Gamma_{-} \setminus \Delta = \varnothing$ then delete η.

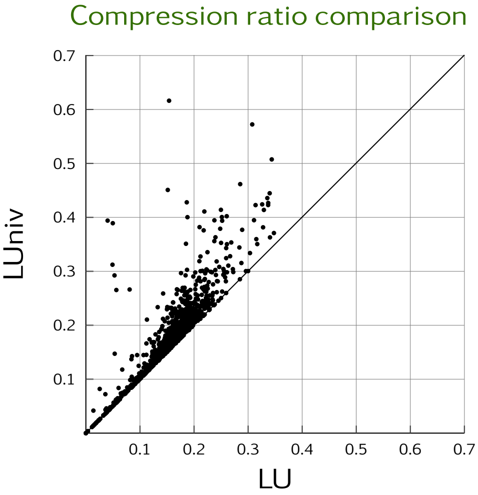
Partial regularization (not implemented)

- If the dual of any of η's active literal belongs to Δ then delete the edge.

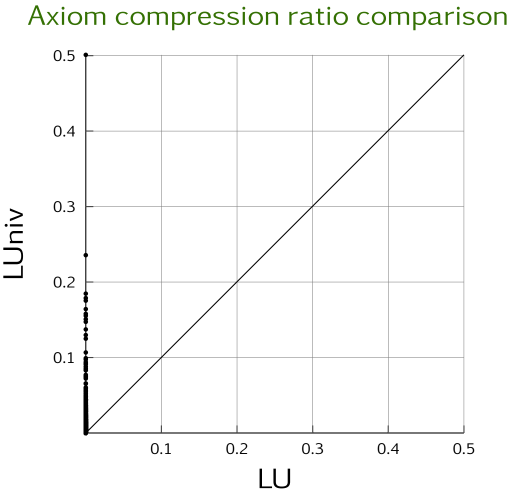
Algorithm

$\Delta \leftarrow \emptyset$;
for *every node* η *in a top-down traversal* **do**
    Fix η ;
    Compute $\Gamma_{-} \setminus \Delta$;
    if $\Gamma_{-} \setminus \Delta = \emptyset$ **then**
        Delete η ;
    else if $\Gamma_{-} \setminus \Delta = \{\bar{a}\}$ **and** $\Gamma_{+} \setminus \Delta = \{a\}$ **then**
        Lower η ;
        $\Delta \leftarrow \Delta \cup \{a\}$;

Comparison with LU

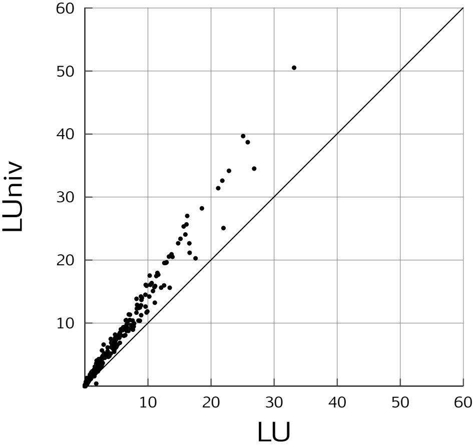


Comparison with LU



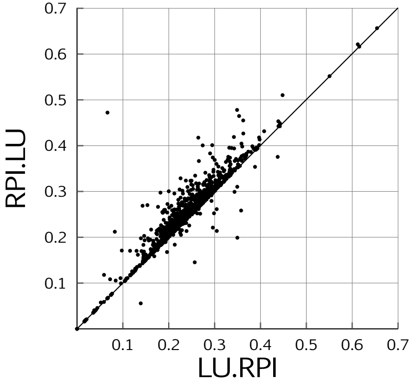
Comparison with LU

Compression time comparison (in seconds)



Conclusion

Compression ratio comparison



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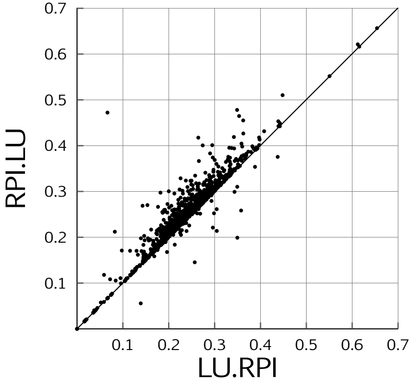

References

Skeptik

- http://github.com/Paradoxika/Skeptik

Bibliography

- Fontaine, P., Merz, S., Woltzenlogel Paleo, B.: Compression of propositional resolution proofs via partial regularization. In: CADE. Lecture Notes in Computer Science, vol. 6803, pp. 237–251. Springer (2011)