Space and Congruence Compression of Proofs

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European Master in Computational Logic

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Space and Congruence Compression of Proofs

Space and Congruence Compression of **Proofs**

Resolution

Resolution Rule

$$\frac{C \vee x \quad D \vee \neg x}{C \vee D}$$

Resolution

Resolution Rule

$$\begin{tikzcd}
C, x & & D, \neg x \\
& C, D \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

Example

$$(x_1 \lor x_2 \lor \neg x_3) \land (x_1 \lor x_2) \land (x_1 \lor x_3) \land (\neg x_1)$$

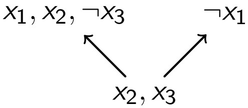
Example

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee x_2) \wedge (x_1 \vee x_3) \wedge (\neg x_1)$$

$\neg x_1$

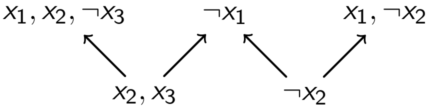
Example

$$(x_1 \lor x_2 \lor \neg x_3) \land (x_1 \lor x_2) \land (x_1 \lor x_3) \land (\neg x_1)$$



Example

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee x_2) \wedge (x_1 \vee x_3) \wedge (\neg x_1)$$



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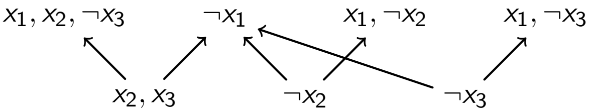

$x_1, x_2, \neg x_3$

$\neg x_1$

$x_1, \neg x_2$

$x_1, \neg x_3$

x_2, x_3

$\neg x_2$

$\neg x_3$

Example

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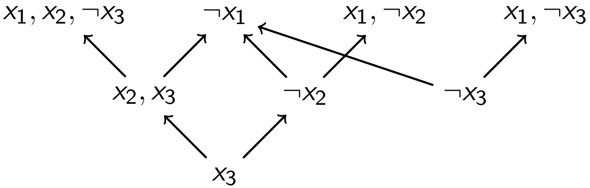


$x_1, x_2, \neg x_3$

$\neg x_1$

$x_1, \neg x_2$

$x_1, \neg x_3$

x_2, x_3

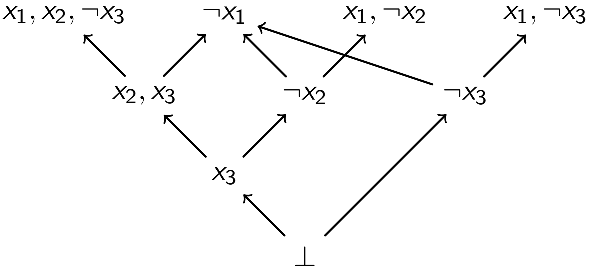
$\neg x_2$

$\neg x_3$

x_3

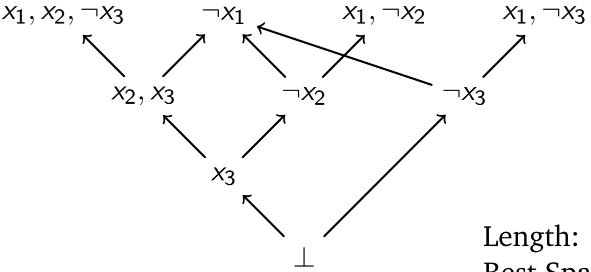
Example

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee x_2) \wedge (x_1 \vee x_3) \wedge (\neg x_1)$$



Example

$$(x_1 \lor x_2 \lor \neg x_3) \land (x_1 \lor x_2) \land (x_1 \lor x_3) \land (\neg x_1)$$

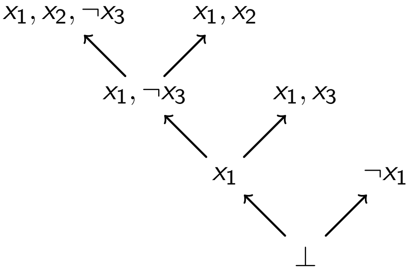


Length: 9

Best Space: 4

Example

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee x_2) \wedge (x_1 \vee x_3) \wedge (\neg x_1)$$



$x_1, x_2, \neg x_3$

x_1, x_2

$x_1, \neg x_3$

x_1, x_3

x_1

$\neg x_1$

$\perp$

Length: 7

Best Space: 3

Space and Congruence **Compression** of Proofs

Proof Compression

- Smaller proofs
- Smaller unsat cores, interpolants
- Easier proof processing
- Easier trusted interaction of deductive systems
- Proof generalization

Synthesizing Multiple Boolean Functions using Interpolation on a Single Proof

- Georg Hofferek, et al, 2013, TU Graz

Method

1. Obtain proof of unsatisfiability of a formula in the SMT theory of uninterpreted functions from SMT solver
2. Modify proof
3. Extract one interpolant from the proof
4. Obtain multiple interpolants from the one that represent the desired boolean functions

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- Worst case exponential runtime in proof size

Proof Compression using Skeptik

- Input proof: 1,870,407 nodes
- Output proof: 868,760 nodes (53,6% compression)

Space and **Congruence** Compression of Proofs

Example

Knowledge

1. $f(a) = a$
2. $a = b$
3. $b = f(b)$
4. $f(a) \neq f(b)$

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Proof

Equality is transitive, therefore from $f(a) = a$, $a = b$ and $b = f(b)$ follows $f(a) = f(b)$, which contradicts $f(a) \neq f(b)$

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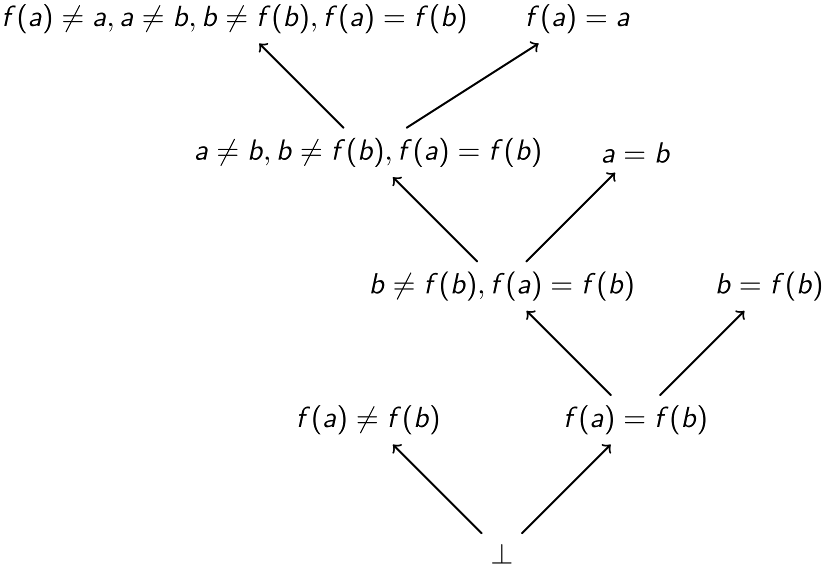
Proof

Equality is transitive, therefore from $f(a) = a$, $a = b$ and $b = f(b)$ follows $f(a) = f(b)$, which contradicts $f(a) \neq f(b)$

A different Proof

$f(.)$ is a function, therefore from $a = b$ follows $f(a) = f(b)$, which contradicts $f(a) \neq f(b)$

A proof



$f(a) \neq a, a \neq b, b \neq f(b), f(a) = f(b)$

$f(a) = a$

$a \neq b, b \neq f(b), f(a) = f(b)$

$a = b$

$b \neq f(b), f(a) = f(b)$

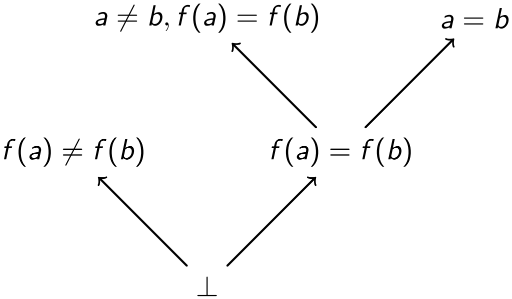
$b = f(b)$

$f(a) \neq f(b)$

$f(a) = f(b)$

$\perp$

A different proof



$a \neq b, f(a) = f(b)$

$a = b$

$f(a) \neq f(b)$

$f(a) = f(b)$

$\perp$

Definitions

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Ground Terms

- Constants $a, b, c, \dots$
- Compound Terms $f(t_1, \dots, t_n)$

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- Constants $a, b, c, \ldots$
- Compound Terms $f(t_1, \ldots, t_n)$

Congruence Relation R

- Reflexive: $\forall t (t, t) \in R$
- Symmetric: $(s, t) \in R \Rightarrow (t, s) \in R$
- Transitive: $(t_1, t_2) \in R \ldots (t_{m-1}, t_m) \in R \Rightarrow (t_1, t_m) \in R$
- Compatible: $\forall i (t_i, s_i) \in R \Rightarrow (f(t_1, \ldots, t_n), f(s_1, \ldots, s_n)) \in R$

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Congruence Closure E^* of set of equations E

- Smallest Congruence Relation containing E
- Computable in $O(n \log(n))$
- E is explanation for $(s, t) \in E^*$

Example revisited

Knowledge

1. $f(a) = a$
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1. $f(a) = a$
2. $a = b$
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Explanation for $f(a) = f(b)$

$\{\ f(a) = a,\ a = b\ ,\ b = f(b)\ \}$

Example revisited

Knowledge

1. $f(a) = a$
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Explanation for $f(a) = f(b)$

$\{ \qquad\qquad a = b\ , b = f(b)\ \}$

Example revisited

Knowledge

1. $f(a) = a$
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Explanation for $f(a) = f(b)$

$\{ \quad a = b \quad \}$

Example revisited

Knowledge

1. $f(a) = a$
2. $a = b$
3. $b = f(b)$
4. $f(a) \neq f(b)$

Explanation for $f(a) = f(b)$

$\{ \quad a = b \quad \}$

Short explanation ⇝ short (sub)proof

Short Explanation Decision Problem

Given a set of input equations E, a target equation $s = t$ and $k \in \mathbb{N}$, does there exist an explanation $E' \subseteq E$ of $s = t$ with $|E'| \leq k$?

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Given a set of input equations E, a target equation $s = t$ and $k \in \mathbb{N}$, does there exist an explanation $E' \subseteq E$ of $s = t$ with $|E'| \leq k$?

NP-complete

NP-completeness proof sketch

From a propositional logic formula Φ obtain ...

- a set of equations E_Φ
- a target equation $s_\Phi = t_\Phi$
- $k_\Phi \in \mathbb{N}$

such that ...

Φ is satisfiable if and only if there is an explanation $E' \subseteq E_\Phi$ of $s_\Phi = t_\Phi$ with $|E'| \leq k_\Phi$

NP-completeness proof sketch example

Formula

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2)$$

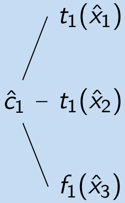
Translation to equations

NP-completeness proof sketch example

Formula

$$(x_1 \lor x_2 \lor \neg x_3) \land (\neg x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2)$$

Translation to equations



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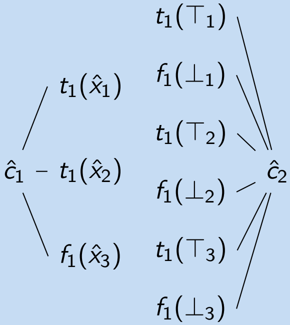

$\hat{c}_1$ – $t_1(\hat{x}_1)$, $t_1(\hat{x}_2)$, $f_1(\hat{x}_3)$

$t_1(\top_1)$, $f_1(\bot_1)$, $t_1(\top_2)$, $f_1(\bot_2)$, $t_1(\top_3)$, $f_1(\bot_3)$ – $\hat{c}_2$

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Formula

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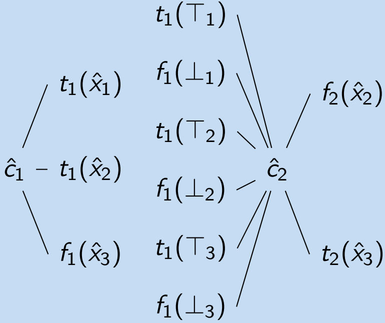
Translation to equations



$\hat{c}_1$

$t_1(\hat{x}_1)$

$t_1(\hat{x}_2)$

$f_1(\hat{x}_3)$

$t_1(\top_1)$

$f_1(\bot_1)$

$t_1(\top_2)$

$f_1(\bot_2)$

$t_1(\top_3)$

$f_1(\bot_3)$

$\hat{c}_2$

$f_2(\hat{x}_2)$

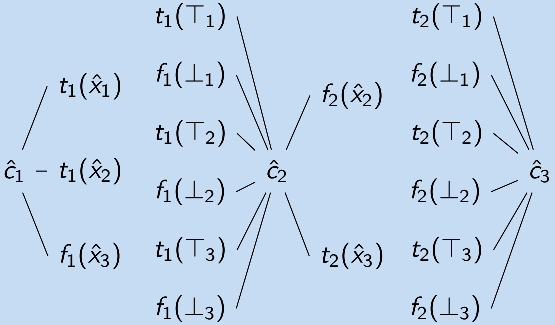
$t_2(\hat{x}_3)$

NP-completeness proof sketch example

Formula

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2)$$

Translation to equations

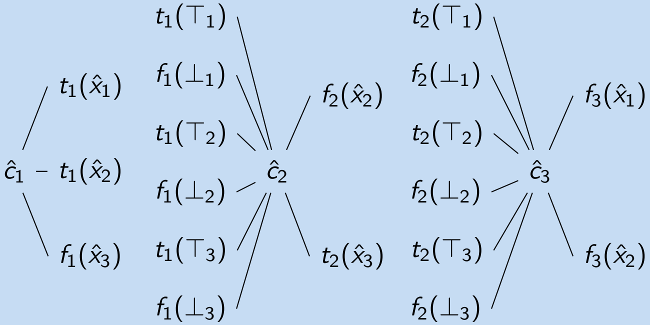


NP-completeness proof sketch example

Formula

$$(x_1 \lor x_2 \lor \neg x_3) \land (\neg x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2)$$

Translation to equations



NP-completeness proof sketch example

Formula

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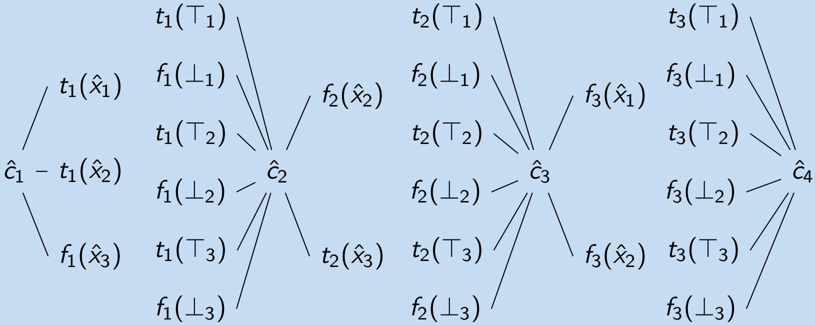
Translation to equations



$\hat{c}_1$

$t_1(\hat{x}_1)$ $t_1(\hat{x}_2)$ $f_1(\hat{x}_3)$

$t_1(\top_1)$ $f_1(\bot_1)$ $t_1(\top_2)$ $f_1(\bot_2)$ $t_1(\top_3)$ $f_1(\bot_3)$

$\hat{c}_2$

$f_2(\hat{x}_2)$ $t_2(\hat{x}_3)$

$t_2(\top_1)$ $f_2(\bot_1)$ $t_2(\top_2)$ $f_2(\bot_2)$ $t_2(\top_3)$ $f_2(\bot_3)$

$\hat{c}_3$

$f_3(\hat{x}_1)$ $f_3(\hat{x}_2)$

$t_3(\top_1)$ $f_3(\bot_1)$ $t_3(\top_2)$ $f_3(\bot_2)$ $t_3(\top_3)$ $f_3(\bot_3)$

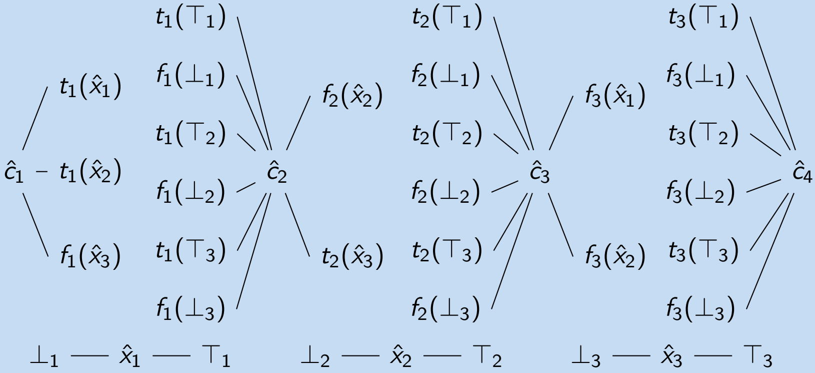
$\hat{c}_4$

NP-completeness proof sketch example

Formula

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2)$$

Translation to equations



$\hat{c}_1$ — $t_1(\hat{x}_1)$, $t_1(\hat{x}_2)$, $f_1(\hat{x}_3)$

$t_1(\top_1)$, $f_1(\bot_1)$, $t_1(\top_2)$, $f_1(\bot_2)$, $t_1(\top_3)$, $f_1(\bot_3)$ — $\hat{c}_2$ — $f_2(\hat{x}_2)$, $t_2(\hat{x}_3)$

$t_2(\top_1)$, $f_2(\bot_1)$, $t_2(\top_2)$, $f_2(\bot_2)$, $t_2(\top_3)$, $f_2(\bot_3)$ — $\hat{c}_3$ — $f_3(\hat{x}_1)$, $f_3(\hat{x}_2)$

$t_3(\top_1)$, $f_3(\bot_1)$, $t_3(\top_2)$, $f_3(\bot_2)$, $t_3(\top_3)$, $f_3(\bot_3)$ — $\hat{c}_4$

$\bot_1$ — $\hat{x}_1$ — $\top_1$ $\quad$ $\bot_2$ — $\hat{x}_2$ — $\top_2$ $\quad$ $\bot_3$ — $\hat{x}_3$ — $\top_3$

NP-completeness proof sketch example

Formula

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2)$$

Small subset corresponding to satisfying assignment

$$\hat{c}_1 - t_1(\hat{x}_1) \quad t_1(\top_1) - \hat{c}_2 - f_2(\hat{x}_2) \quad f_2(\bot_2) - \hat{c}_3 - f_3(\hat{x}_2) \quad f_3(\bot_2) - \hat{c}_4$$

$$\hat{x}_1 \text{ —— } \top_1 \qquad \bot_2 \text{ —— } \hat{x}_2 \qquad \hat{x}_3 \text{ ———— } \top_3$$

Space and Congruence Compression of Proofs

Is it possible to remove parts of the proof from memory during proof processing?

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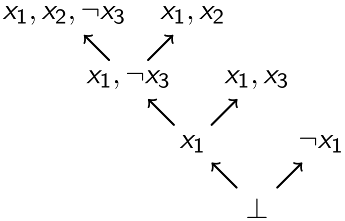
Which parts?

Space Measure Example



$x_1, x_2, \neg x_3$

x_1, x_2

$x_1, \neg x_3$

x_1, x_3

x_1

$\neg x_1$

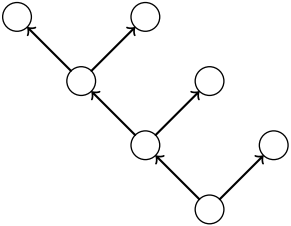
$\bot$

Space Measure Example

Maximum number of nodes in memory: 0

○ Not in memory

● In memory

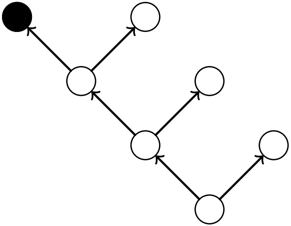


Space Measure Example

Maximum number of nodes in memory: 1

○ Not in memory

● In memory

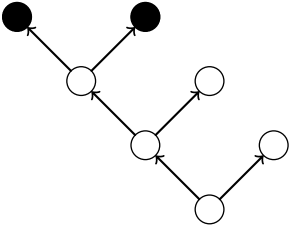


Space Measure Example

Maximum number of nodes in memory: 2

○ Not in memory

● In memory

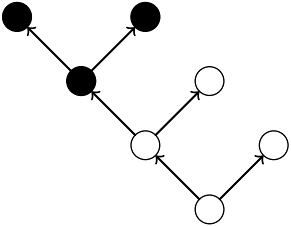


Space Measure Example

Maximum number of nodes in memory: 3

○ Not in memory

● In memory

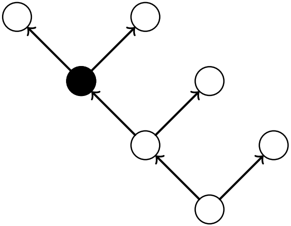


Space Measure Example

Maximum number of nodes in memory: 3

○ Not in memory

● In memory

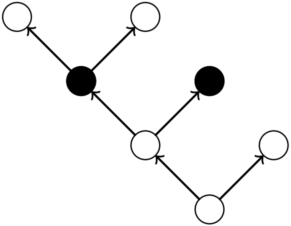


Space Measure Example

Maximum number of nodes in memory: 3

○ Not in memory

● In memory

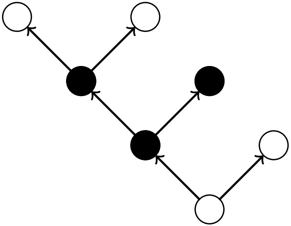


Space Measure Example

Maximum number of nodes in memory: 3

○ Not in memory

● In memory



Space Measure Example

Maximum number of nodes in memory: 3

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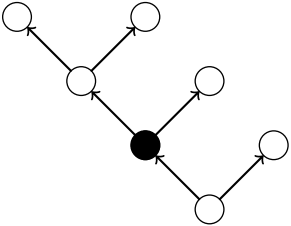
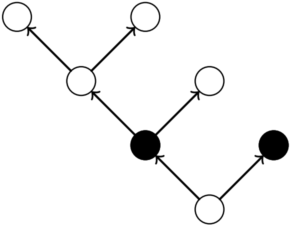


Space Measure Example

Maximum number of nodes in memory: 3

○ Not in memory

● In memory

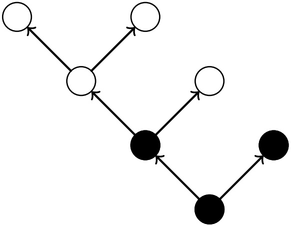


Space Measure Example

Maximum number of nodes in memory: 3

○ Not in memory

● In memory

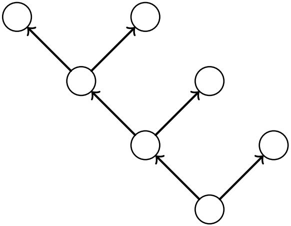


Space Measure Example

Maximum number of nodes in memory: 0

○ Not in memory

● In memory

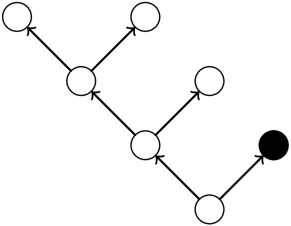


Space Measure Example

Maximum number of nodes in memory: 1

○ Not in memory

● In memory

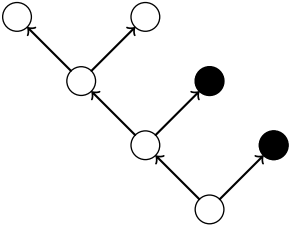


Space Measure Example

Maximum number of nodes in memory: 2

○ Not in memory

● In memory

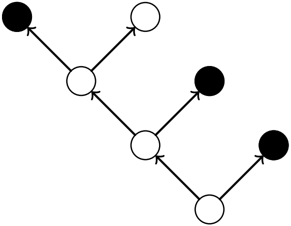


Space Measure Example

Maximum number of nodes in memory: 3

○ Not in memory

● In memory

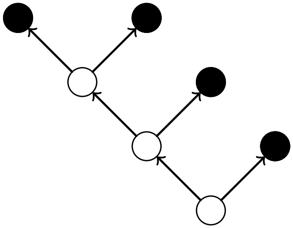


Space Measure Example

Maximum number of nodes in memory: 4

○ Not in memory

● In memory

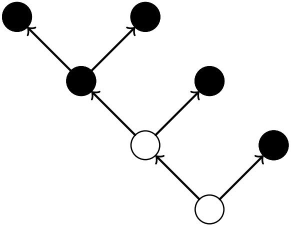


Space Measure Example

Maximum number of nodes in memory: 5

○ Not in memory

● In memory

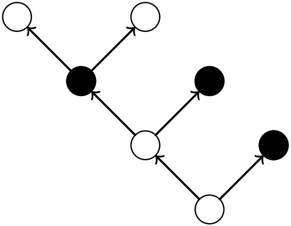


Space Measure Example

Maximum number of nodes in memory: 5

○ Not in memory

● In memory

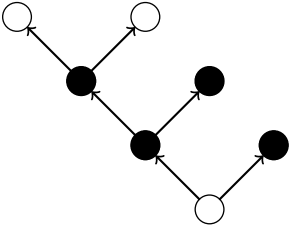


Space Measure Example

Maximum number of nodes in memory: 5

○ Not in memory

● In memory

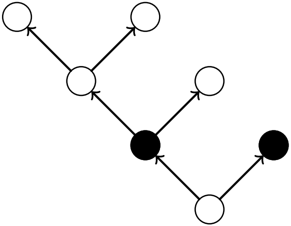


Space Measure Example

Maximum number of nodes in memory: 5

○ Not in memory

● In memory

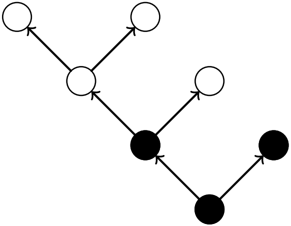


Space Measure Example

Maximum number of nodes in memory: 5

○ Not in memory

● In memory

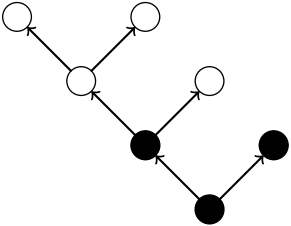


Space Measure Example

Maximum number of nodes in memory: 5

○ Not in memory

● In memory



Good traversal orders are essential!

Space Measure

Space measure of a proof and a traversal order

Maximal amount of nodes that have to be kept in memory at once while processing the proof following the traversal order

Construct Traversal Orders

Construct Optimal Order

- NP-complete
- Optimal strategy in some pebbling game

Construct Good Order

- Top-Down
- Bottom-Up
- Heuristic choices

Construct Order Top-Down

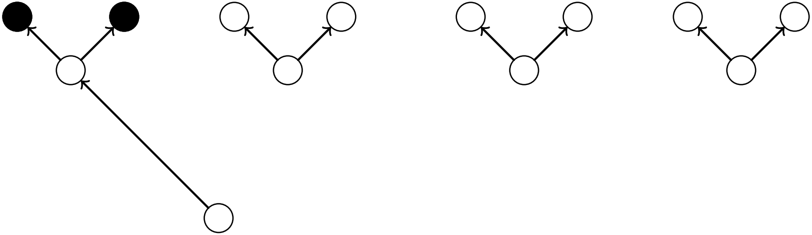
Maximum number of nodes in memory: 0

Construct Order Top-Down

Maximum number of nodes in memory: 1

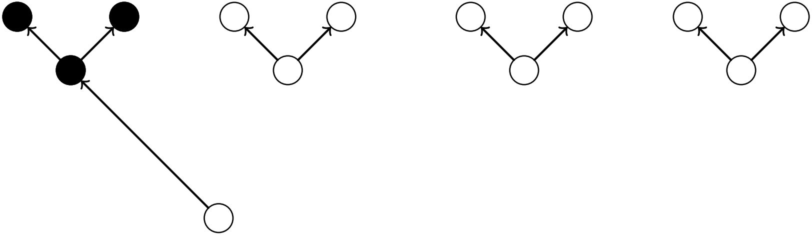
Construct Order Top-Down

Maximum number of nodes in memory: 2



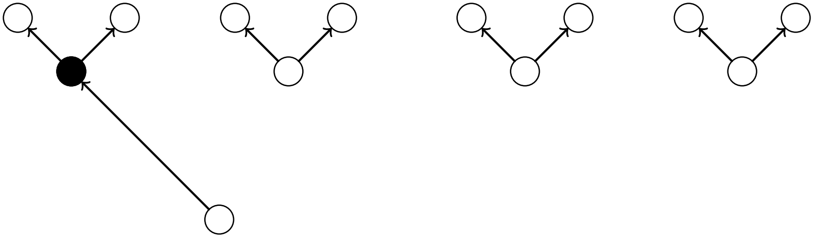
Construct Order Top-Down

Maximum number of nodes in memory: 3



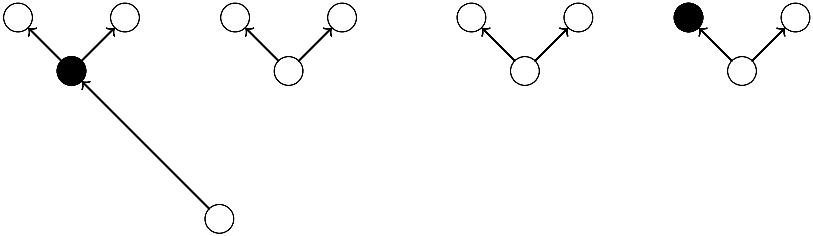
Construct Order Top-Down

Maximum number of nodes in memory: 3



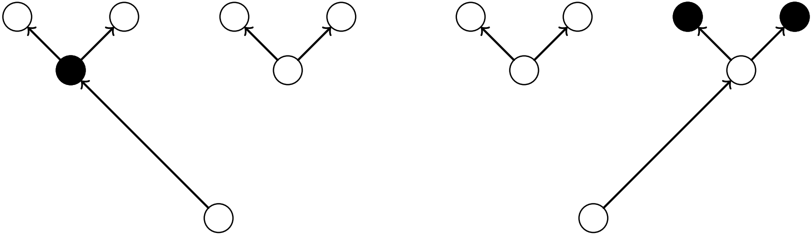
Construct Order Top-Down

Maximum number of nodes in memory: 3



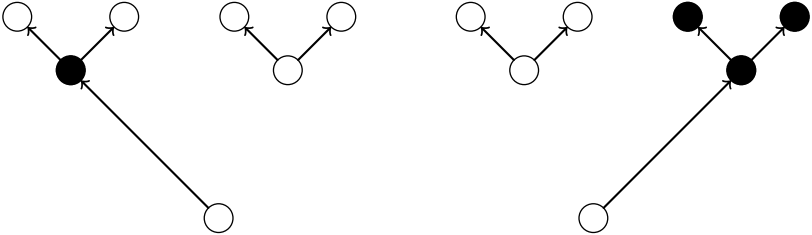
Construct Order Top-Down

Maximum number of nodes in memory: 3



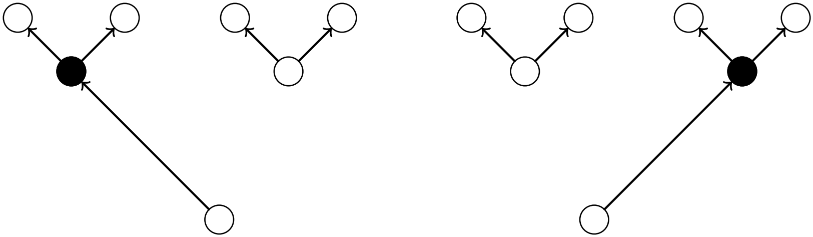
Construct Order Top-Down

Maximum number of nodes in memory: 4



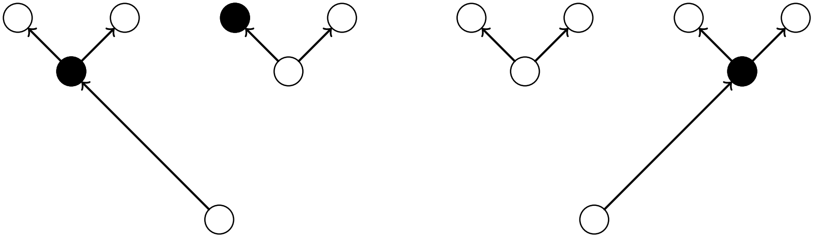
Construct Order Top-Down

Maximum number of nodes in memory: 4



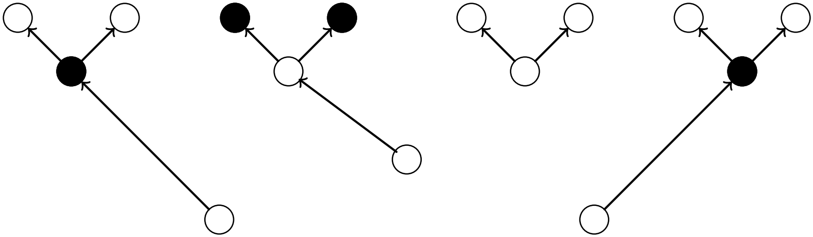
Construct Order Top-Down

Maximum number of nodes in memory: 4



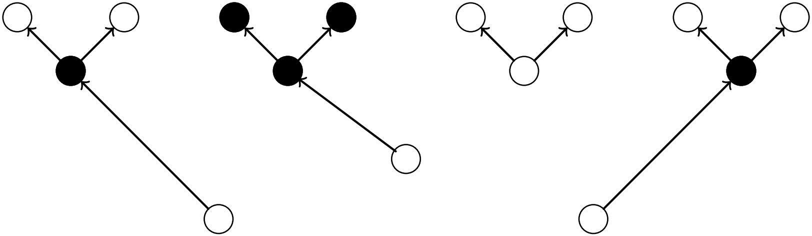
Construct Order Top-Down

Maximum number of nodes in memory: 4



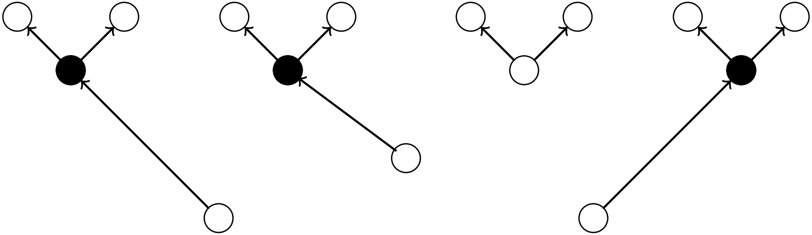
Construct Order Top-Down

Maximum number of nodes in memory: 5



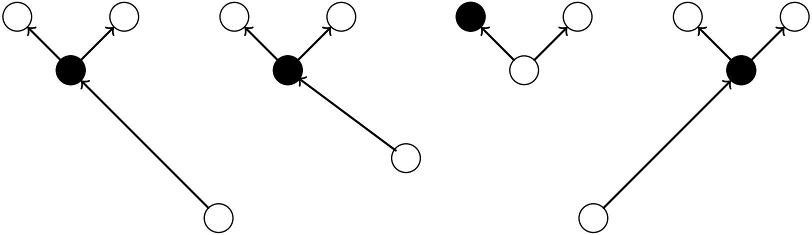
Construct Order Top-Down

Maximum number of nodes in memory: 5



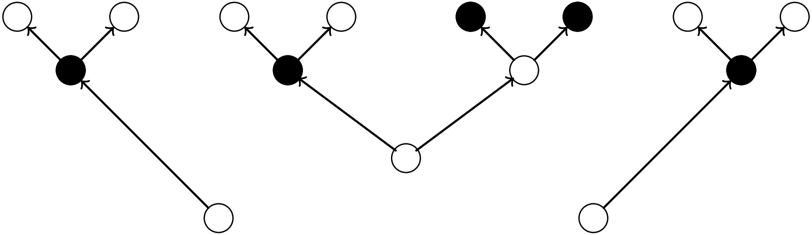
Construct Order Top-Down

Maximum number of nodes in memory: 5



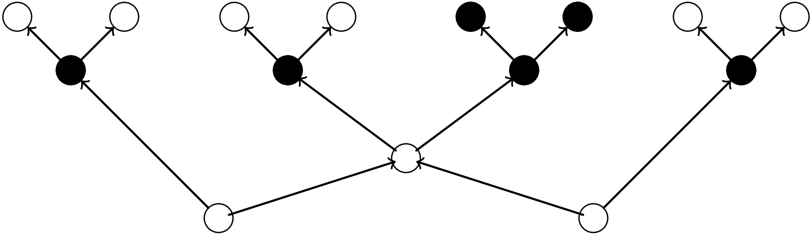
Construct Order Top-Down

Maximum number of nodes in memory: 5



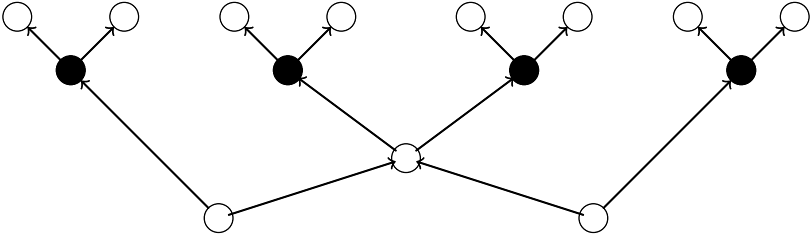
Construct Order Top-Down

Maximum number of nodes in memory: 6



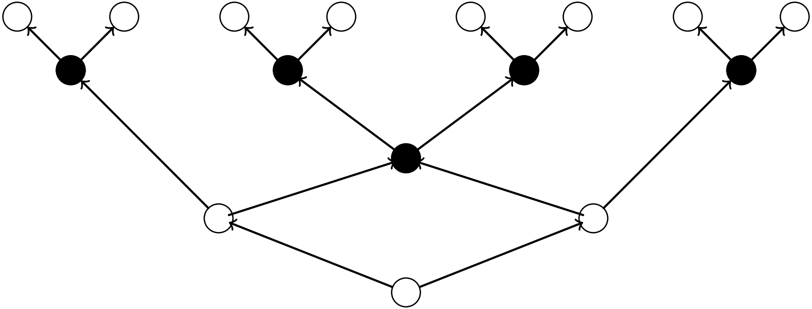
Construct Order Top-Down

Maximum number of nodes in memory: 6



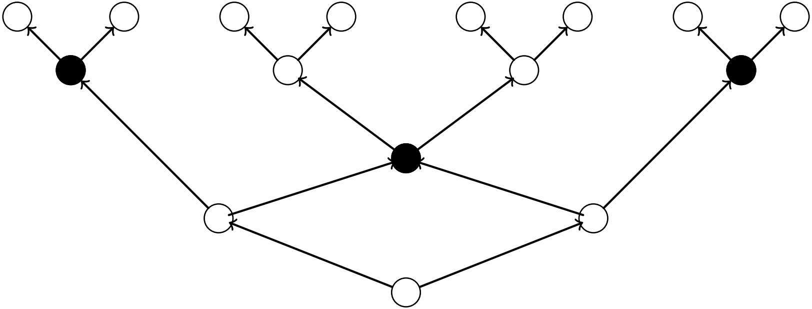
Construct Order Top-Down

Maximum number of nodes in memory: 6



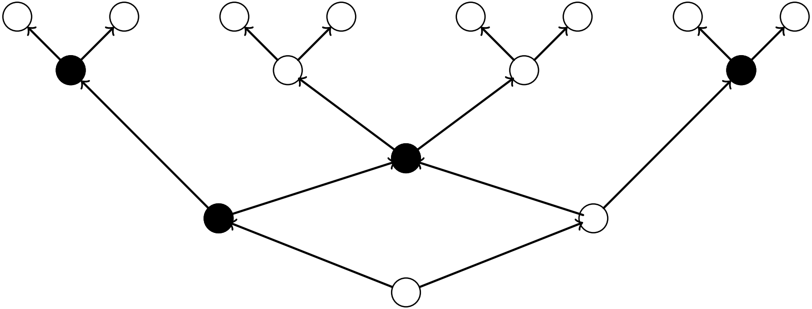
Construct Order Top-Down

Maximum number of nodes in memory: 6



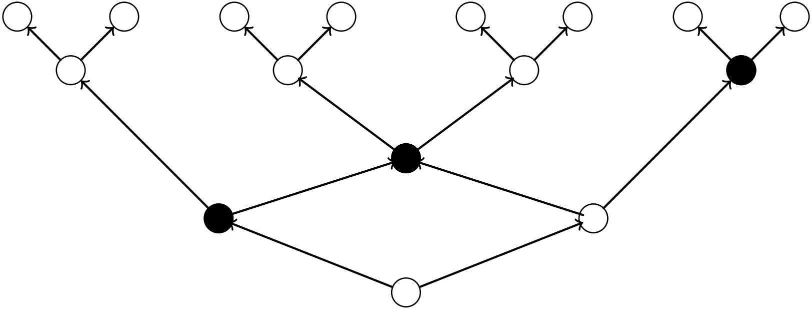
Construct Order Top-Down

Maximum number of nodes in memory: 6



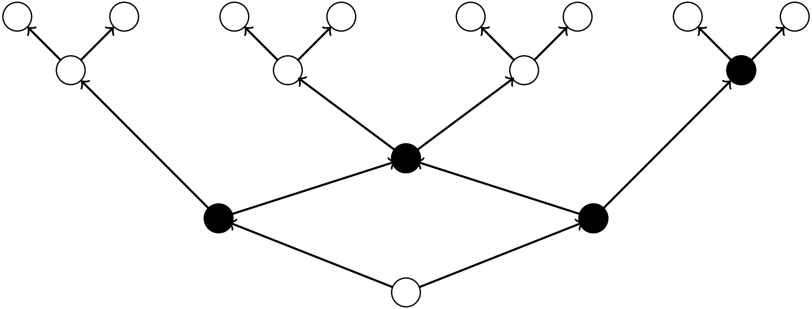
Construct Order Top-Down

Maximum number of nodes in memory: 6



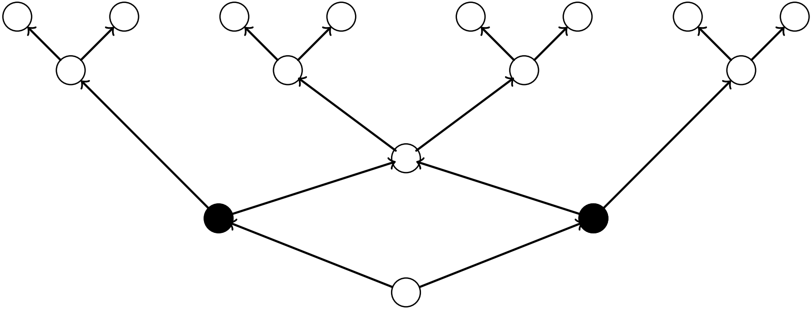
Construct Order Top-Down

Maximum number of nodes in memory: 6



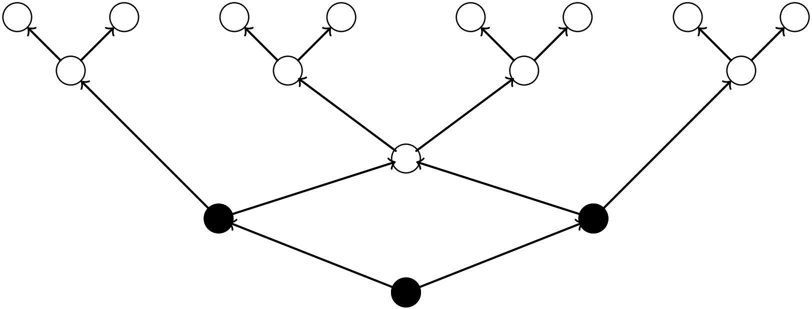
Construct Order Top-Down

Maximum number of nodes in memory: 6



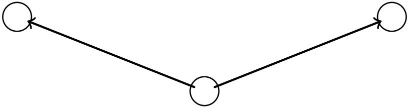
Construct Order Top-Down

Maximum number of nodes in memory: 6



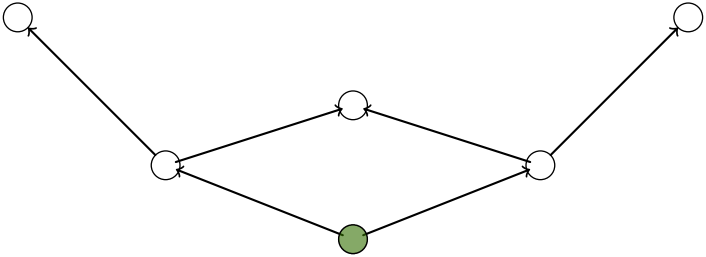
Construct Order Bottom-Up

Maximum number of nodes in memory: 0



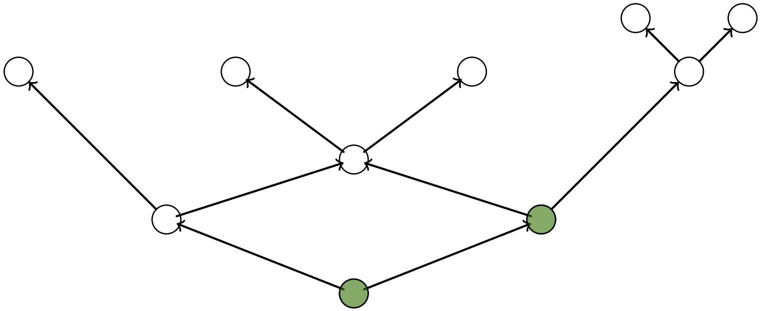
Construct Order Bottom-Up

Maximum number of nodes in memory: 0



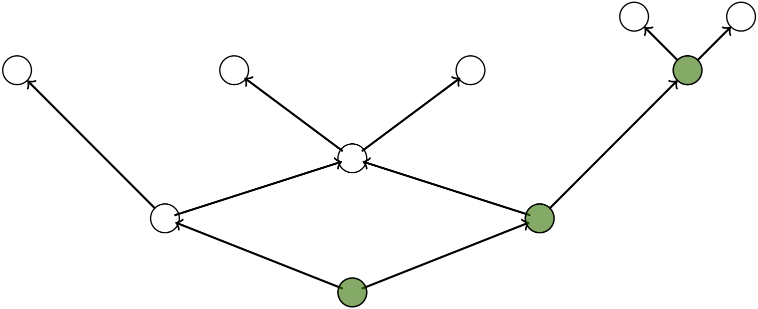
Construct Order Bottom-Up

Maximum number of nodes in memory: 0



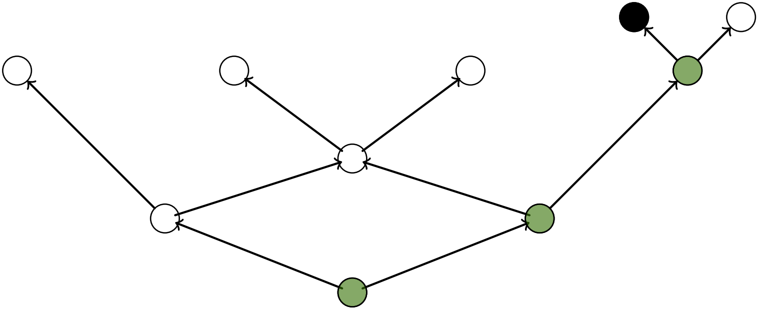
Construct Order Bottom-Up

Maximum number of nodes in memory: 0



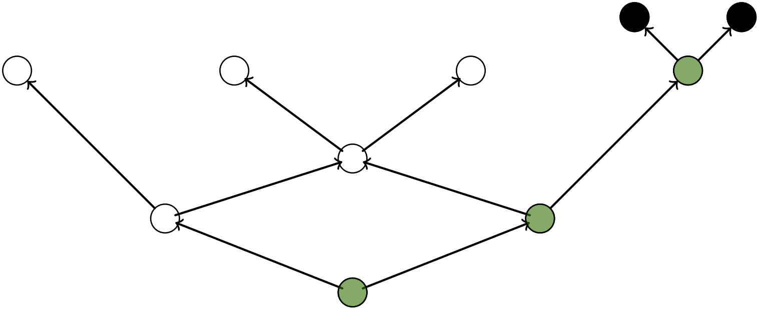
Construct Order Bottom-Up

Maximum number of nodes in memory: 1



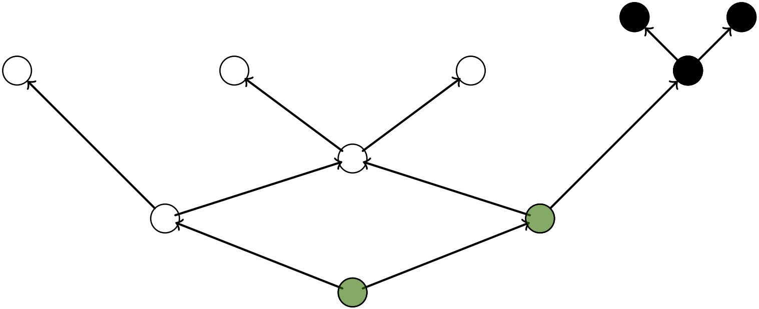
Construct Order Bottom-Up

Maximum number of nodes in memory: 2



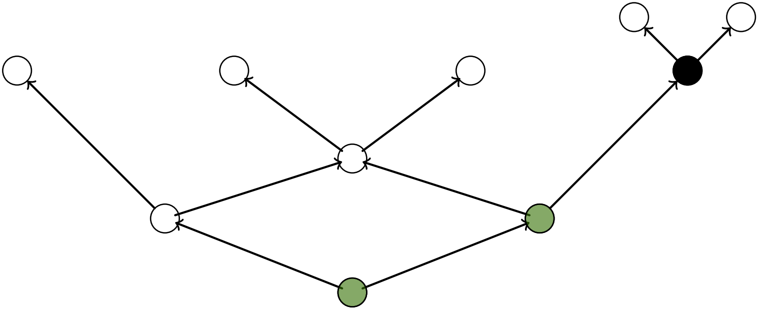
Construct Order Bottom-Up

Maximum number of nodes in memory: 3



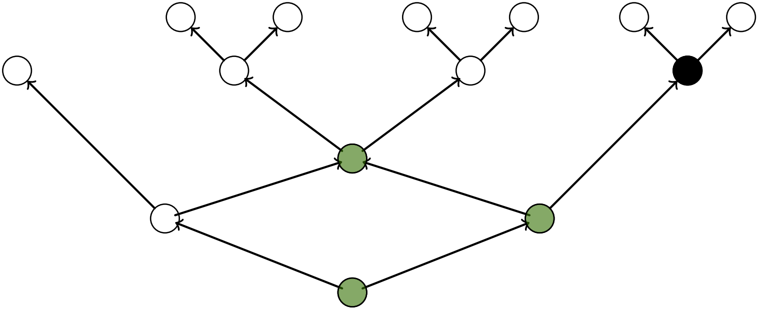
Construct Order Bottom-Up

Maximum number of nodes in memory: 3



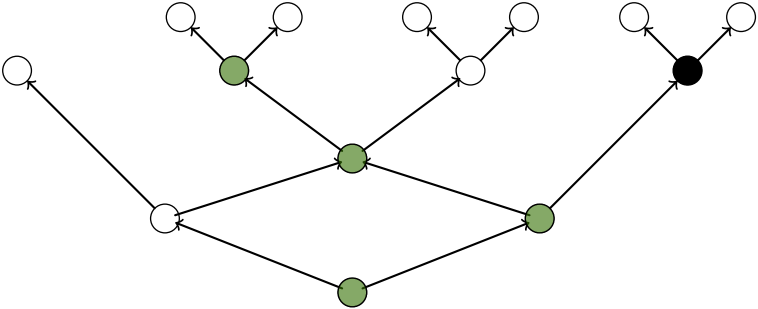
Construct Order Bottom-Up

Maximum number of nodes in memory: 3



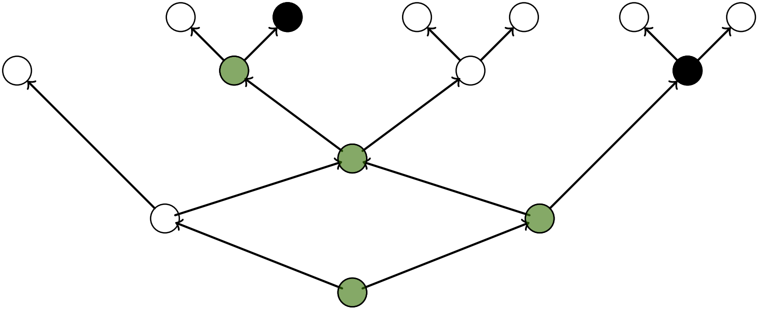
Construct Order Bottom-Up

Maximum number of nodes in memory: 3



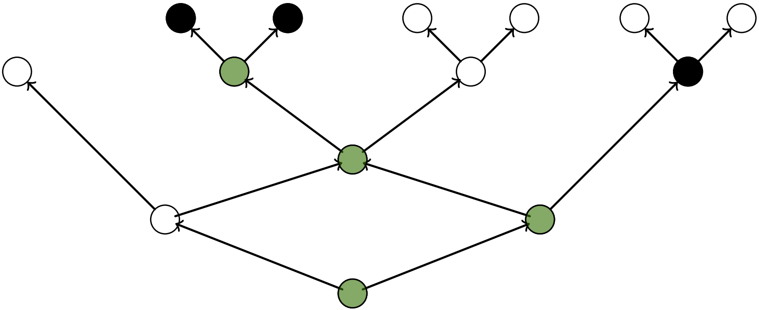
Construct Order Bottom-Up

Maximum number of nodes in memory: 3



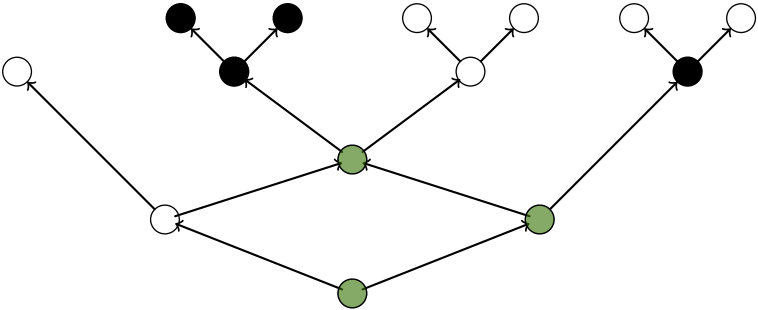
Construct Order Bottom-Up

Maximum number of nodes in memory: 3



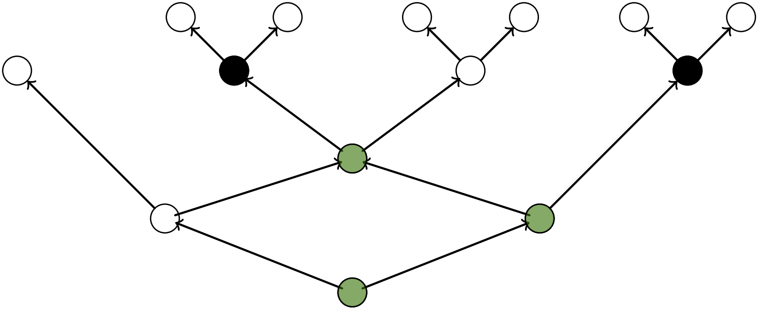
Construct Order Bottom-Up

Maximum number of nodes in memory: 4



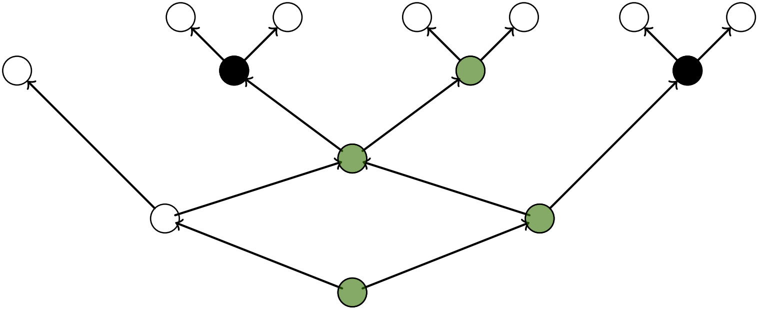
Construct Order Bottom-Up

Maximum number of nodes in memory: 4



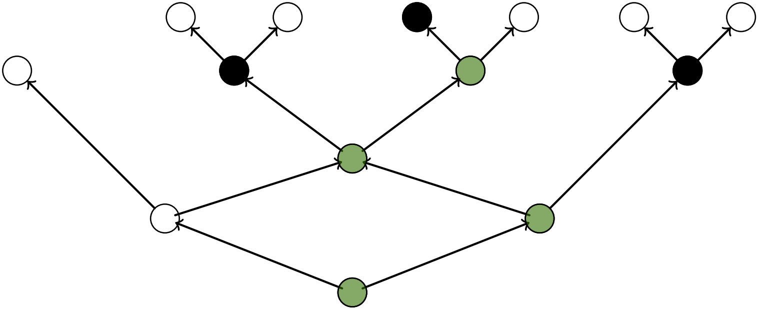
Construct Order Bottom-Up

Maximum number of nodes in memory: 4



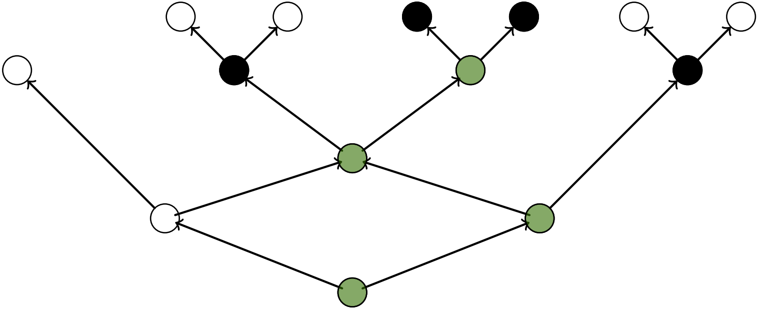
Construct Order Bottom-Up

Maximum number of nodes in memory: 4



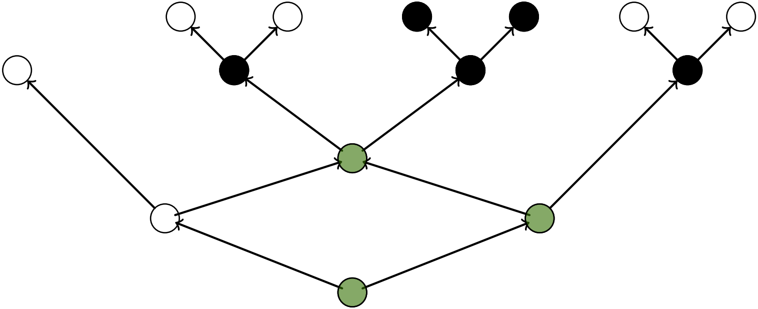
Construct Order Bottom-Up

Maximum number of nodes in memory: 4



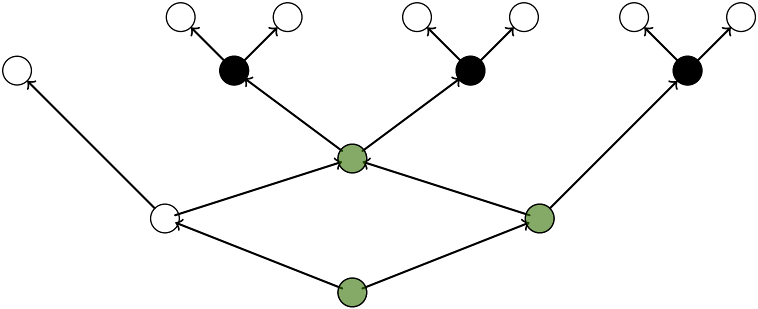
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



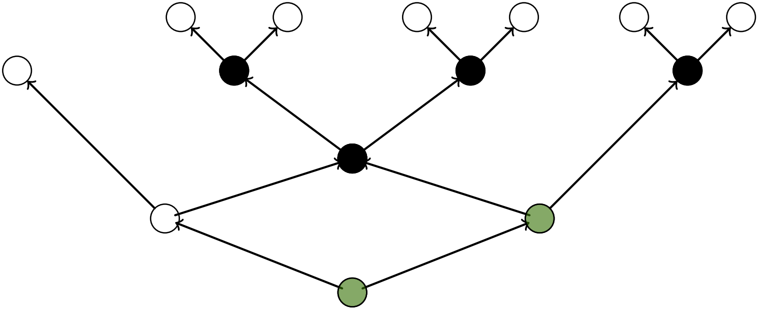
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



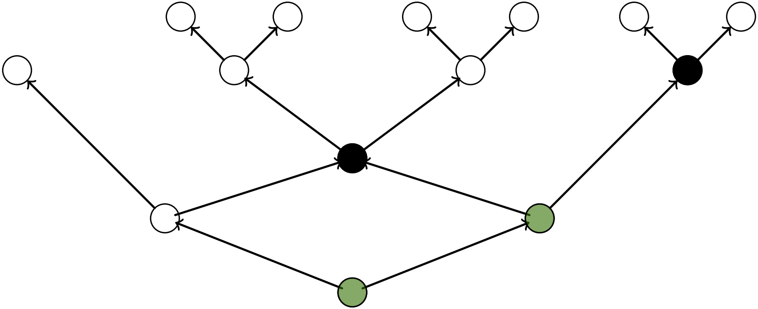
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



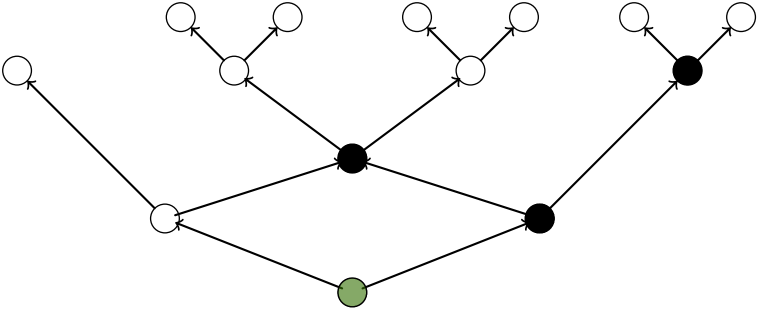
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



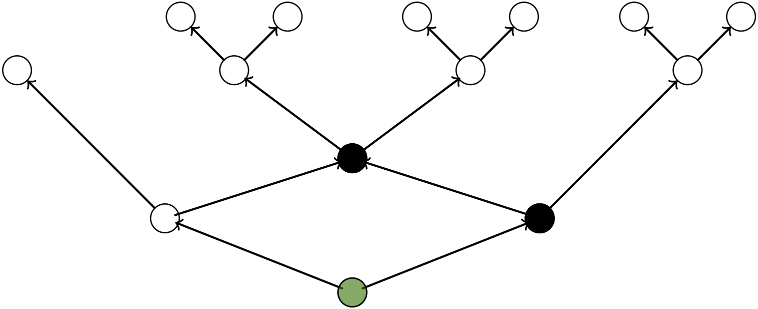
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



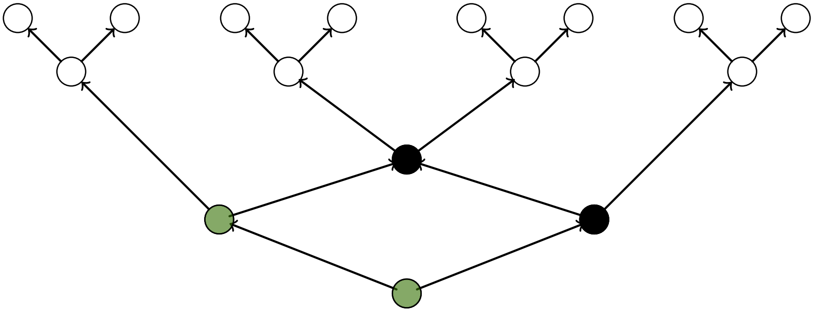
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



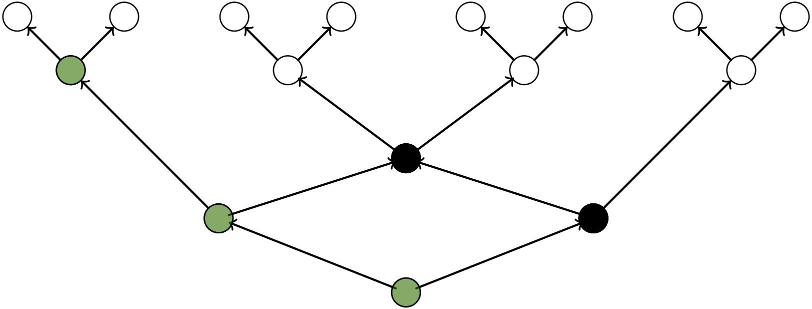
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



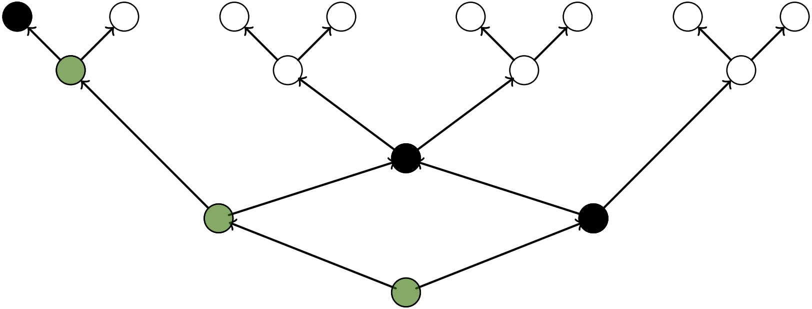
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



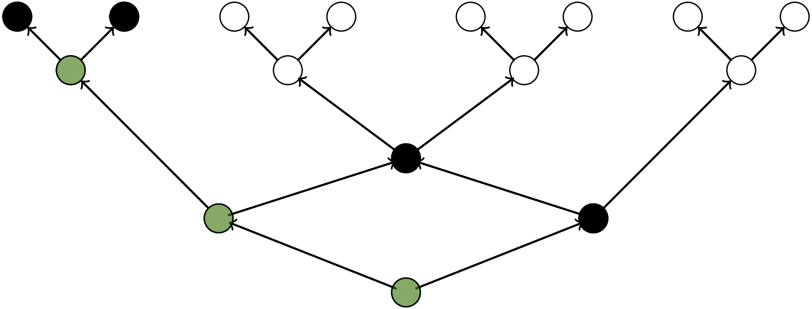
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



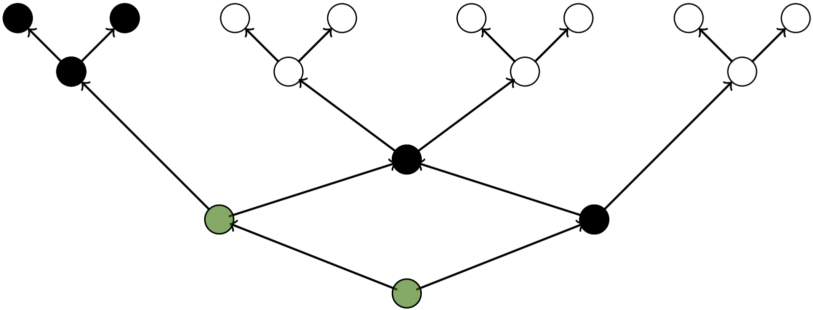
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



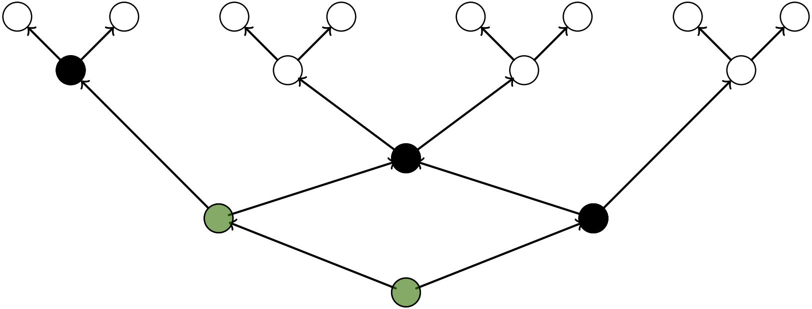
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



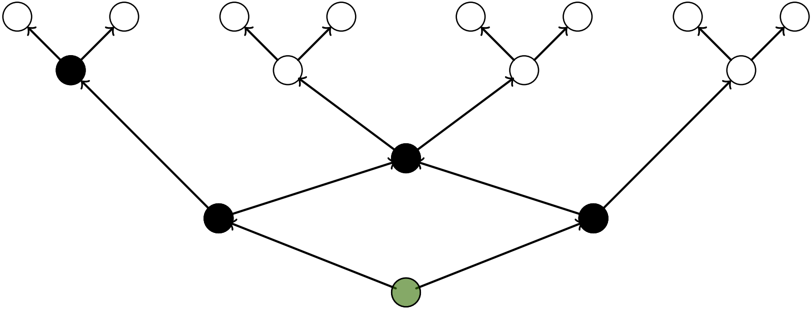
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



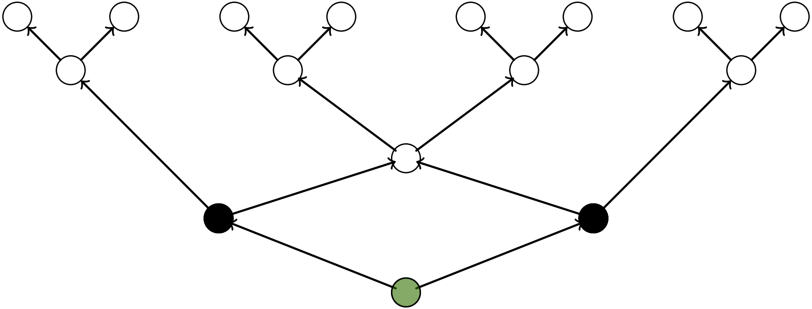
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



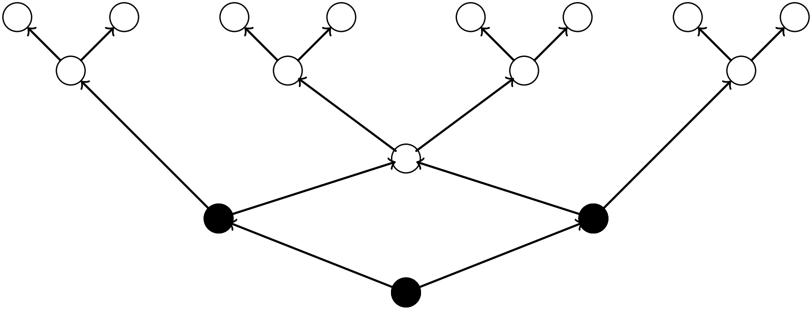
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



Construct Order Bottom-Up

Maximum number of nodes in memory: 5



Experiments, Unsung Heroes & Conclusion

Experimental Results

Congruence Compression

- 2% average effective compression in proof length
- 28% compression in explanation length

Space Compression

- Bottom-Up outperforms Top-Down
- Average space measure is 44.1 times smaller than proof length

Unsung Heroes

- Explanation producing congruence closure algorithm
  - Using immutable data structures
  - Modified version of Dijkstra's shortest path algorithm
- Proof producing algorithm
- Resolution calculus extended with equality
- SAT translation of optimal traversal order
- Correct- & soundness proofs
- Implementation of all presented methods

Conclusion

- Proofs can be compressed in length and space
- Finding the shortest explanation is NP-complete
- Proof production is tricky
- Construct traversal orders Bottom-Up

Thank you for your attention!