Subsumption, Recursive Split and Greedy Pebbling

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Third Workshop of the Amadeus Project on Proof Compression
Monday 16th September, 2013

Overview

Propopositional Resolution Calculus

Literal

- Variable v or negated variable $\bar{v}$

Clause

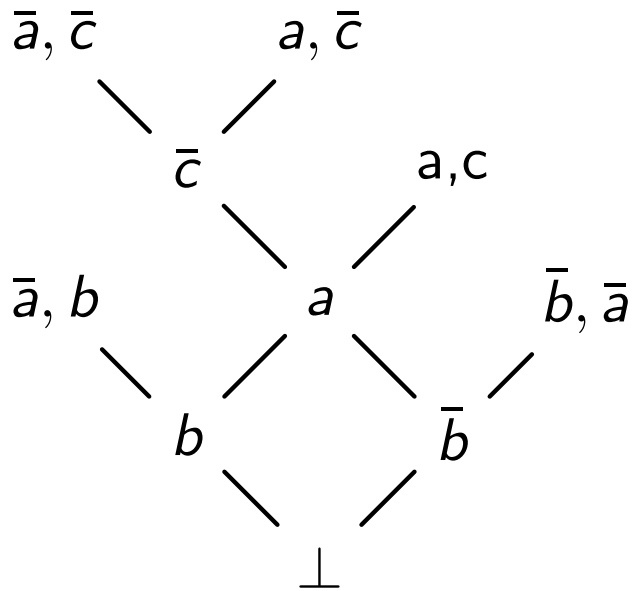
- Disjunction of literals
- Represented as a set

Resolution rule

$$\frac{\Gamma \vee v \qquad \bar{v} \vee \Delta}{\Gamma \vee \Delta}\, v$$

Proof as a directed acyclic graph (DAG)

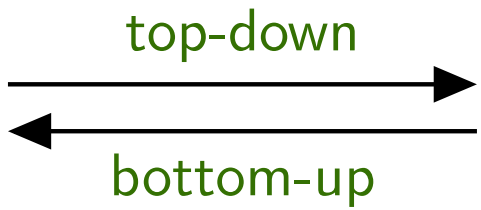
- node, conclusion, pivot, premise, child



Proof as sequence

- Use topological order

$\bar{a}, b$ $\bar{a}, \bar{c}$ $a, \bar{c}$ $\bar{c}$ a, c a b $\bar{b}, \bar{a}$ $\bar{b}$ $\bot$



Proof as a directed acyclic graph (DAG)

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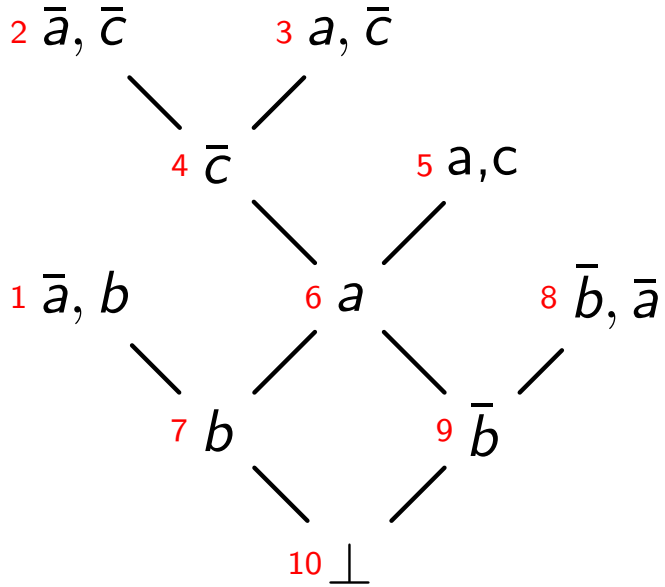


Proof as sequence

- Use topological order

$\bar{a}, b$ [1] $\bar{a}, \bar{c}$ [2] $a, \bar{c}$ [3] $\bar{c}$ [4] a, c [5] a [6] b [7] $\bar{b}, \bar{a}$ [8] $\bar{b}$ [9] $\bot$ [10]

top-down →

← bottom-up

Subsumption for Proof Compression

Idea

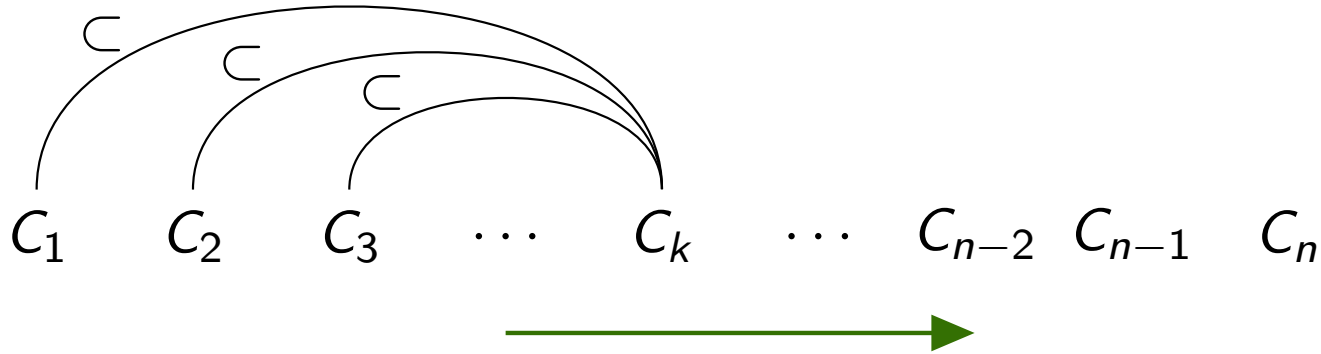
- Subsumption
  - C_1 subsumes C_2 iff $C_1 \subset C_2$
- Replace subsumed clauses by their subsumers
- Fix nodes with changed premises
  - Pivot in both premises $\rightarrow$ resolve premises
  - Pivot missing in a premise $\rightarrow$ use this premise

Performance

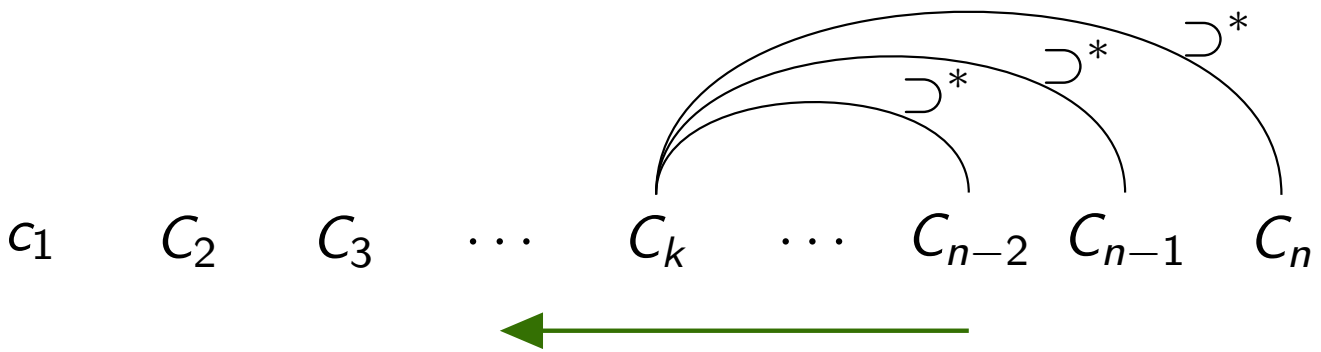
- Worst case quadratic runtime in the proof size
- Use literal-tree structure to store visited nodes and check subsumption

Top-down vs Bottom-up

Top-down

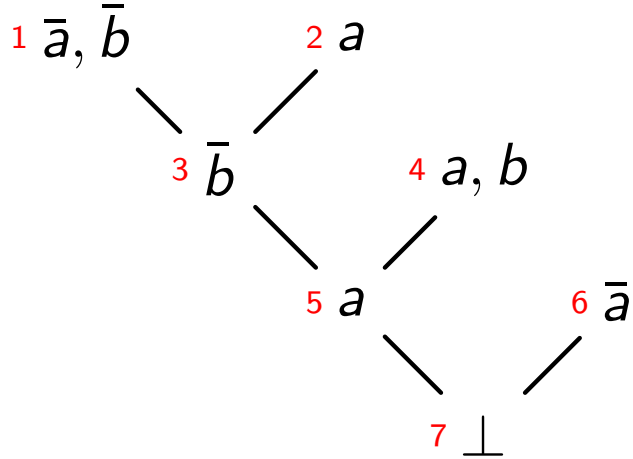


Bottom-up



$C \subset^* D$ iff $C \subset D$ and C is not an ancestor of D

Top-down Subsumption Example



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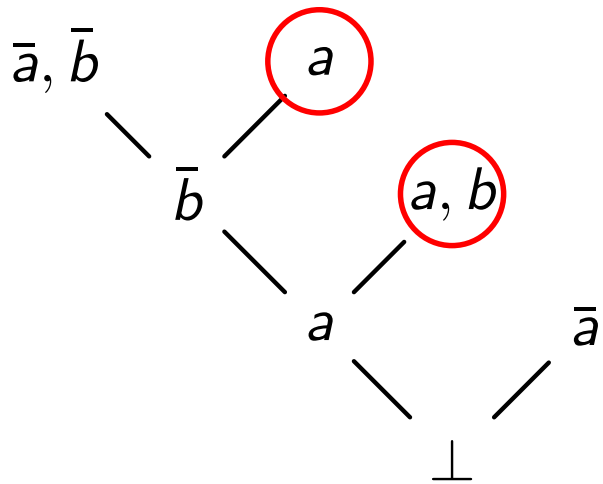

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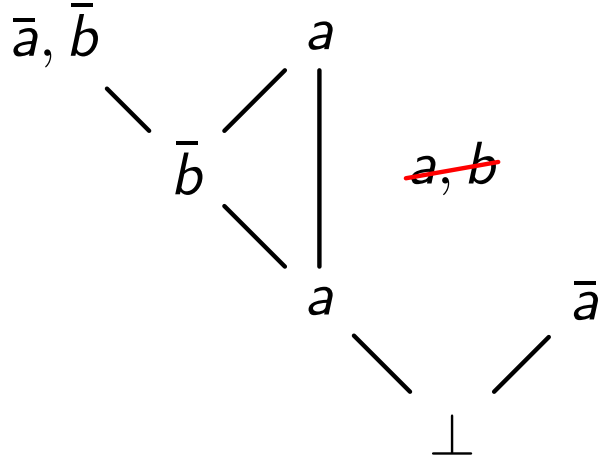

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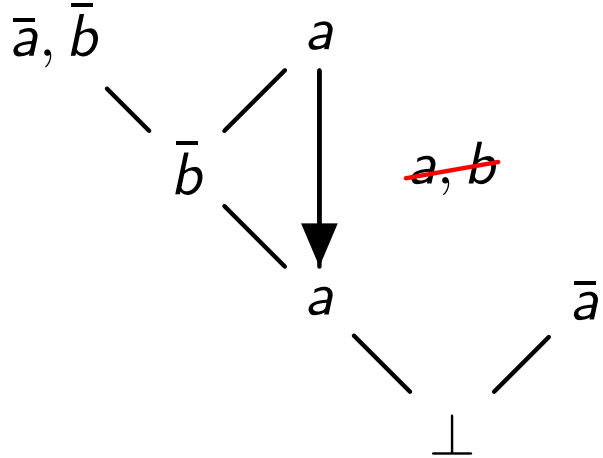

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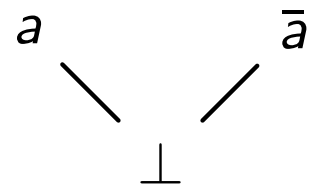
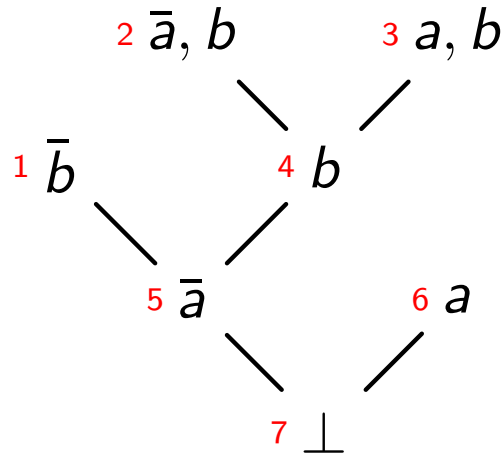
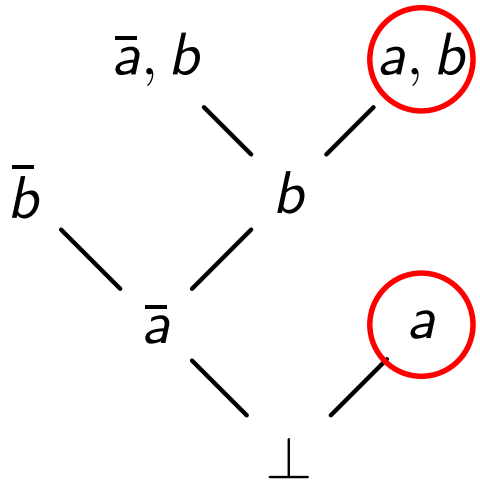


Bottom-up Subsumption Example



Bottom-up Subsumption Example



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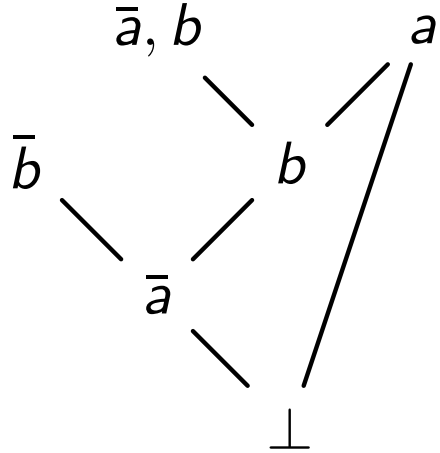

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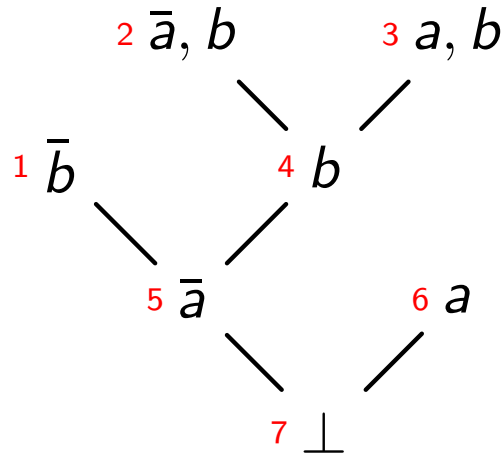

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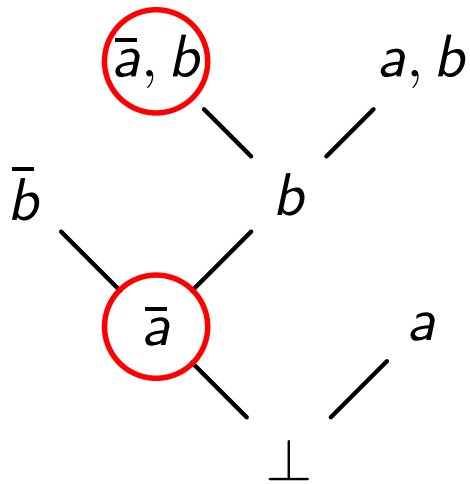

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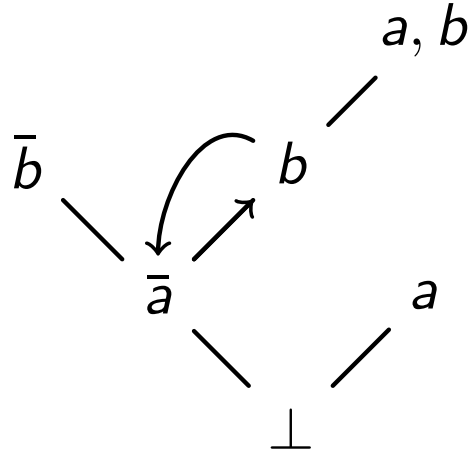


Problems with Bottom-up Subsumption

Performance

- Ancestor check is costly

Copy node issue

- Fixing has to be done top-down
- Nodes are replaced bottom-up

RecycleUnits

Omer Bar-Ilan et al., 2009

- IBM Haifa Research Laboratory

Special case of bottom-up subsumption

- Check only for subsuming unit clauses
- Replace subsumption check by comparing units to pivots
- Worst case quadratic runtime in the amount of unit clauses

Experiments

Setting

- 500 proofs, provided by VeriT SMT Solver
- 659,584 nodes in total
- Average 1320 nodes per proof

Results

Algorithm	Length Compression	Speed
Top-down Subsumption	3.7 %	2.3 nodes/ms
Bottom-up Subsumption	1%	0.4 nodes/ms
RecycleUnits	2%	1.0 nodes/ms
DAGify	0.6 %	7.3 nodes/ms

Split Algorithm

Author

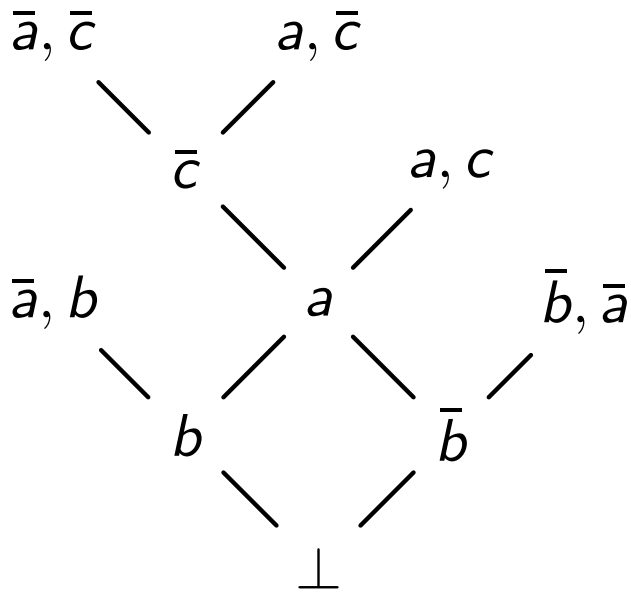
- Scott Cotton, 2010

Idea

- Let P be a proof with root clause C
- Choose a variable v, occuring in P, heuristically
- Extract proofs for $C \vee v$ and $C \vee \bar{v}$
  - By deleting positive/negative branches of nodes with pivot v
  - And fixing nodes like at subsumption algorithms
- Combine proofs by resolving roots

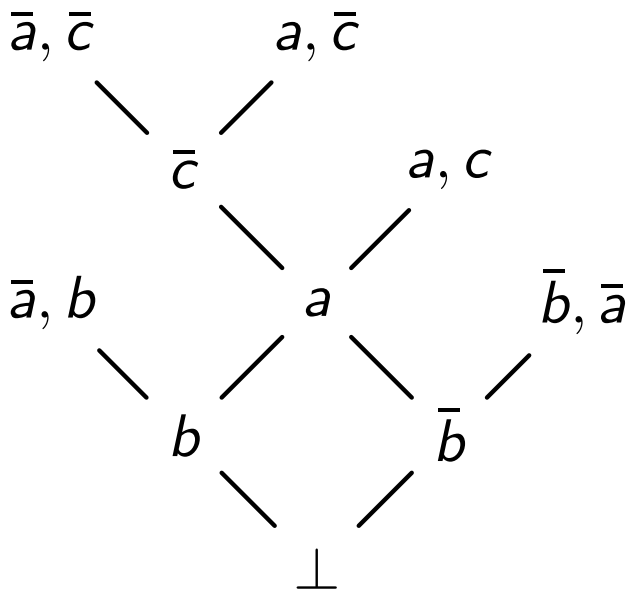
Split Algorithm Example

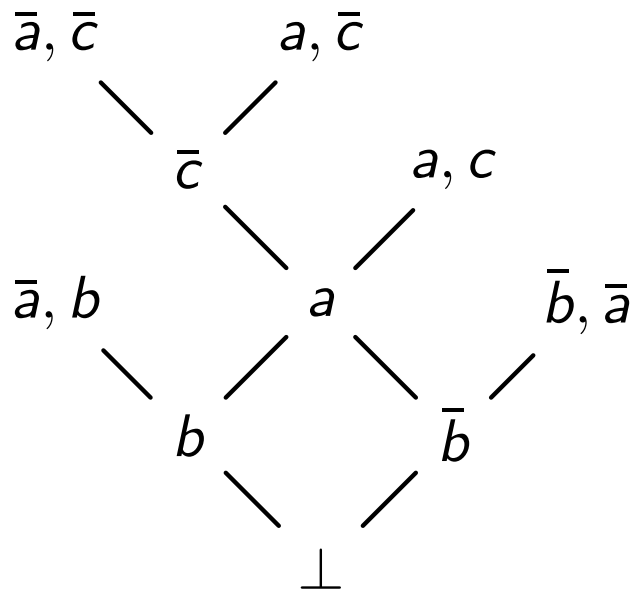
Split variable a



Split Algorithm Example

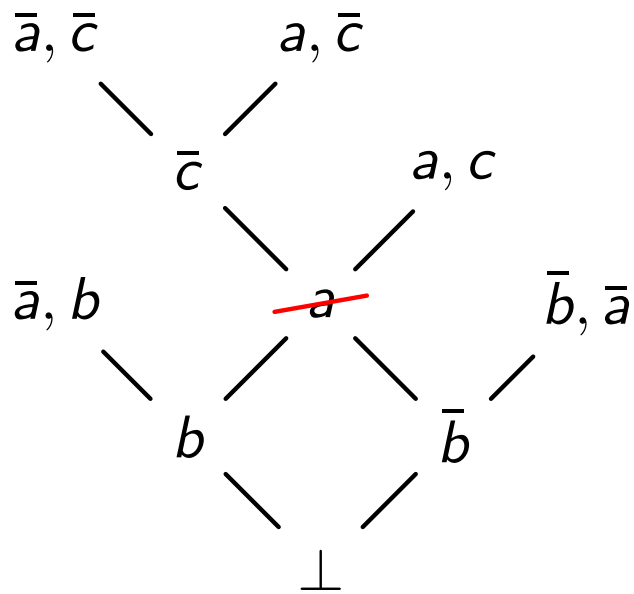
Duplicate the proof



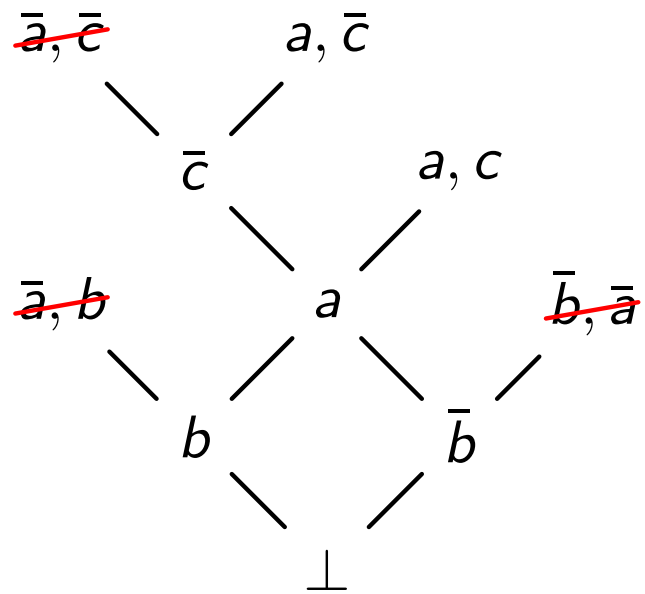


Split Algorithm Example

Delete positive branch

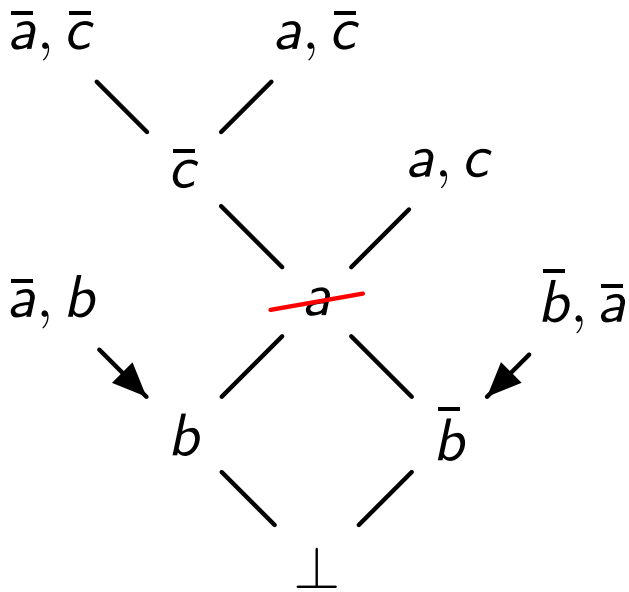


Delete negative branch



Split Algorithm Example

Fix positive branch



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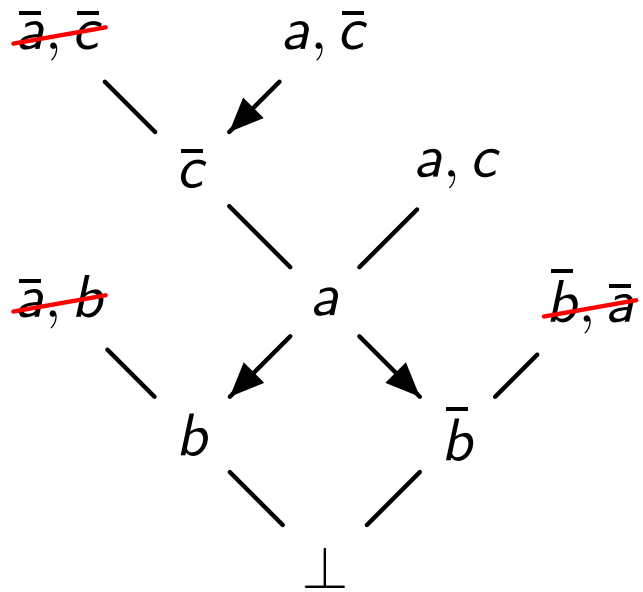

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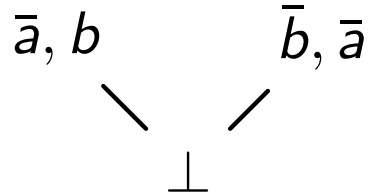
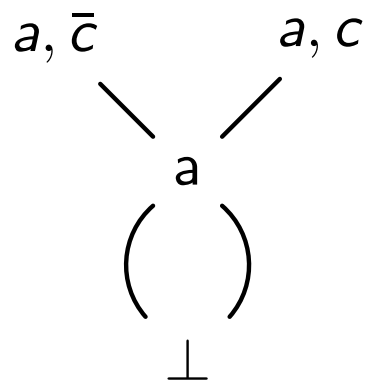


Fix negative branch



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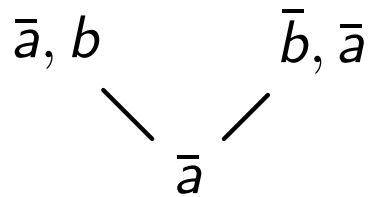

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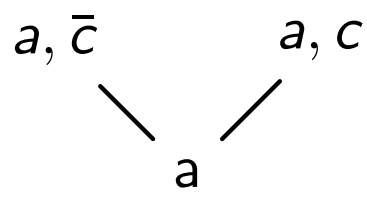
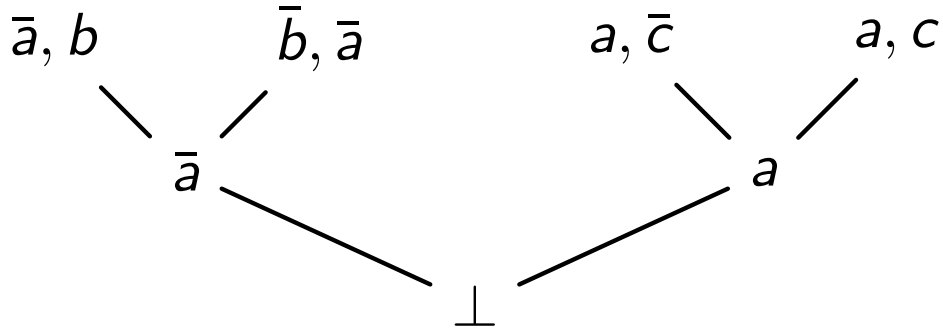


Split Algorithm Example

Combine branches by resolving



Iterative Split

Idea

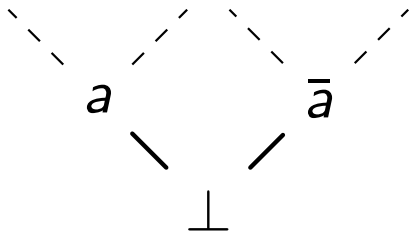
- Apply split to result of split

Issues

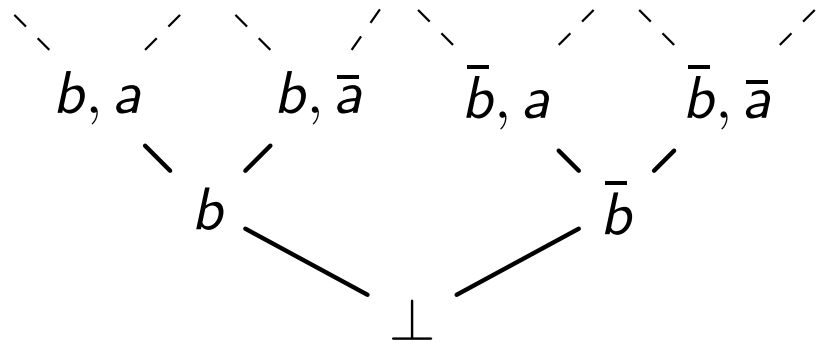
- Best variables don't end up lowest
- Same variables for positive and negative branches

Example

Split once, best variable: a



Split again, best variable: b



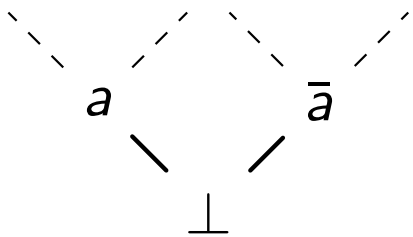
Recursive Split

Idea

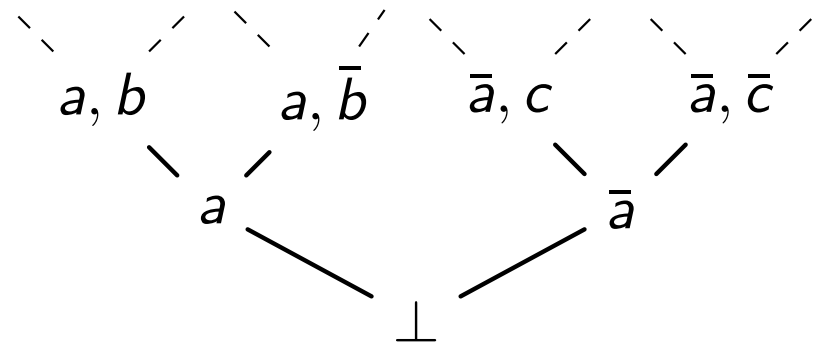
- Apply split to positive and negative branches before combining
- Use stopping criteria (depth or time)

Example

Split once, best variable: a



Split positive/negative branches, best variables: b/c;



Issue

- Equal nodes may be computed in both branches

Experiments

Results

Algorithm	Length Compression	Speed
Recursive Split (depth 3)	2.3%	3.0 nodes/ms
Recursive Split (depth 5)	2.0%	2.0 nodes/ms
Iterative Split (depth 3)	2.9%	3.2 nodes/ms
Iterative Split (depth 5)	3.4%	2.3 nodes/ms

Space Compression

Space measure

- Maximal amount of nodes that have to be kept in memory at once

Deletion information

- Extra lines in proof output
- Example: y is the last child of x
  - Read and check node x

    ...
  - Read and check node y
  - Delete node x

    ...

Interesting scenario

- Proof checker has much less memory than proof producer

Black Pebbling Game

A pebble is a small stone

Rules

- If all premises of a node p are pebbled, p may be pebbled
  - In this case, a pebble may be moved from a premise to p
- Nodes can be unpebbled at any time

Goal

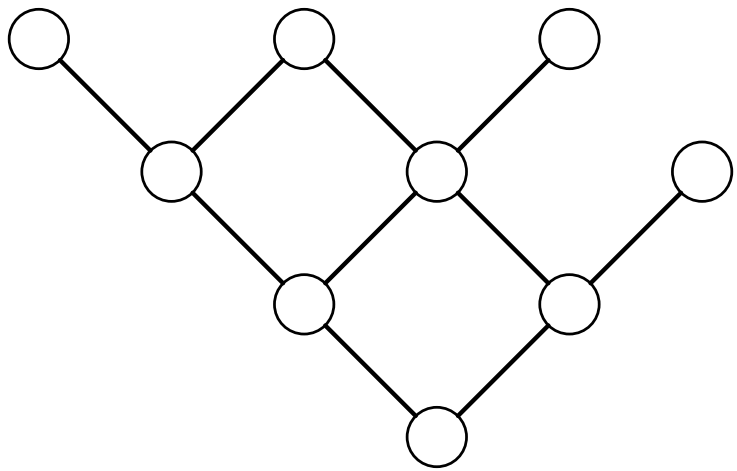
- Pebble some node v

Pebbling problem

- For a given DAG and a node v, can v be pebbled using no more than n pebbles in total?
- PSPACE-complete (John R. Gilbert et al., 1980)

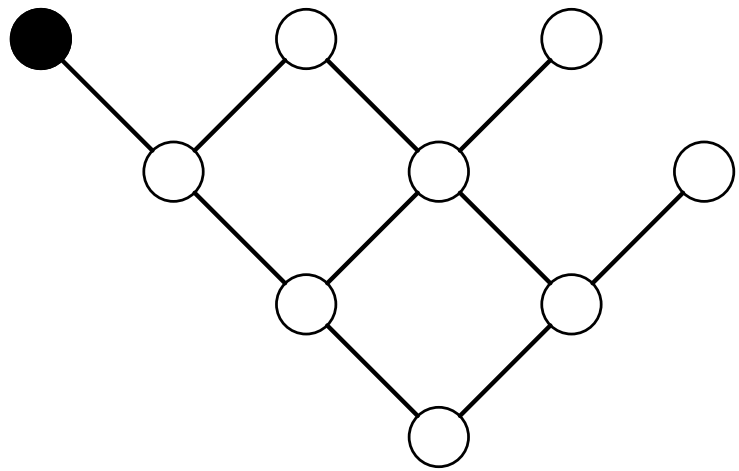
Pebbling Game Example

Maximum pebbles used: 0



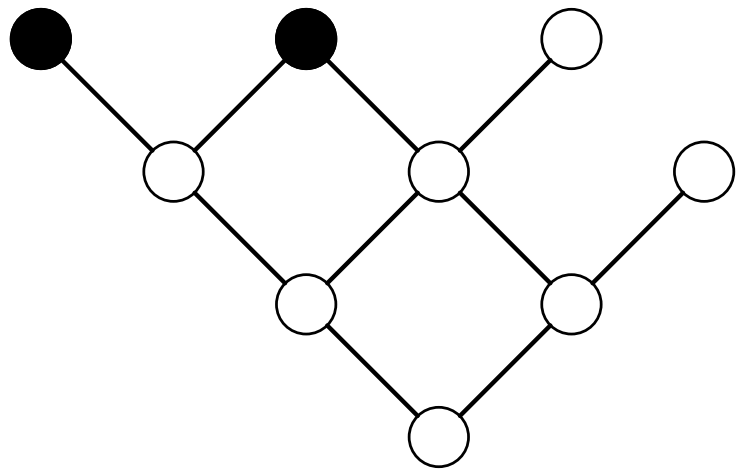
Pebbling Game Example

Maximum pebbles used: 1



Pebbling Game Example

Maximum pebbles used: 2



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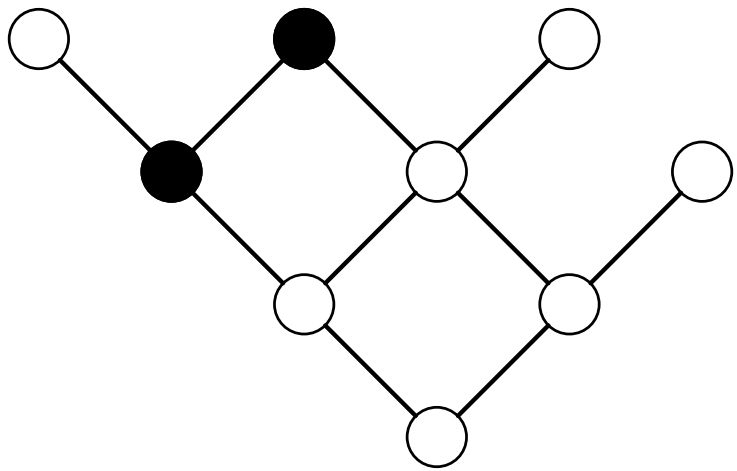
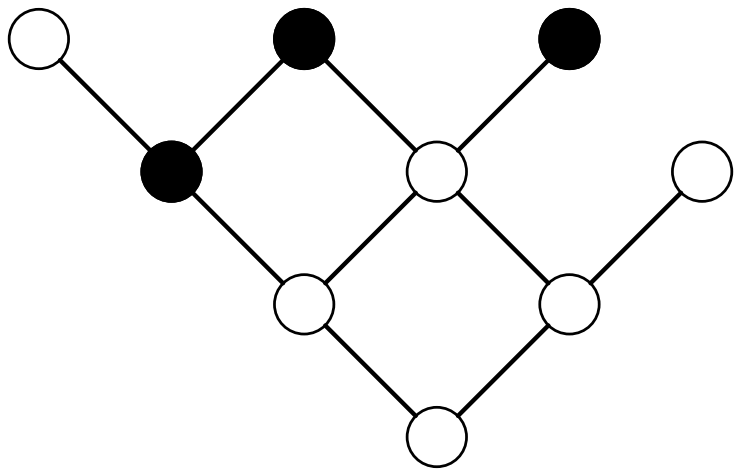


Pebbling Game Example

Maximum pebbles used: 3



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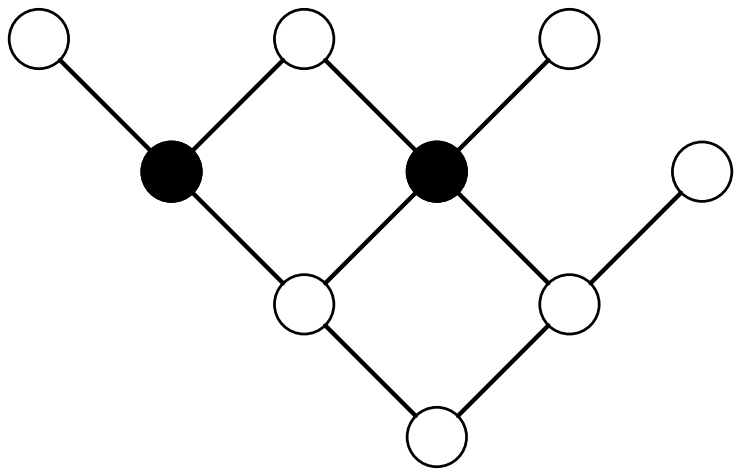

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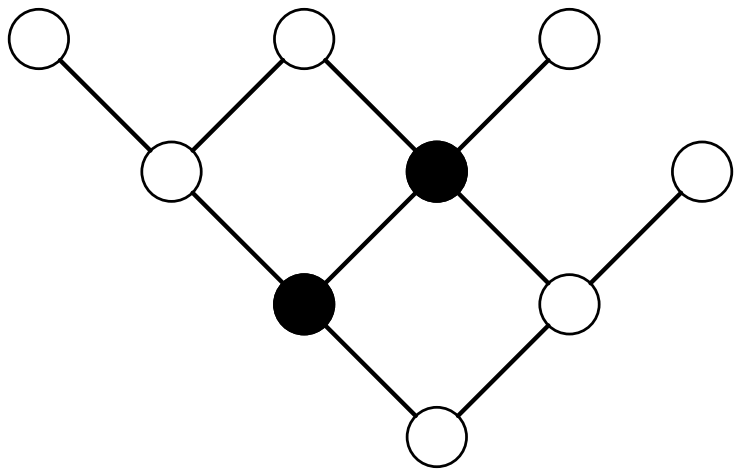

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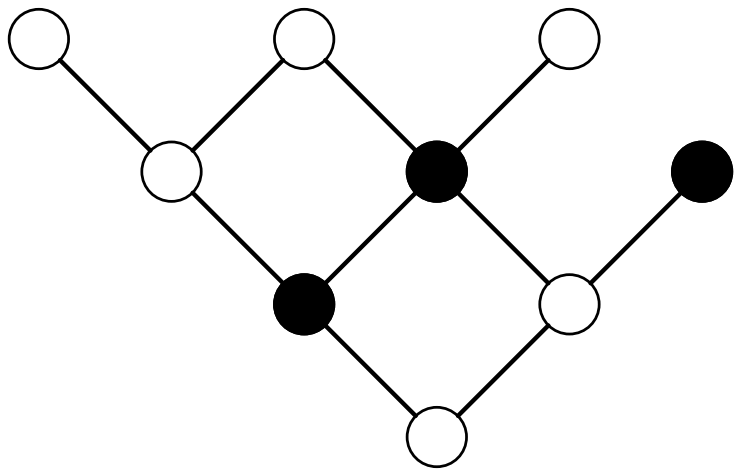
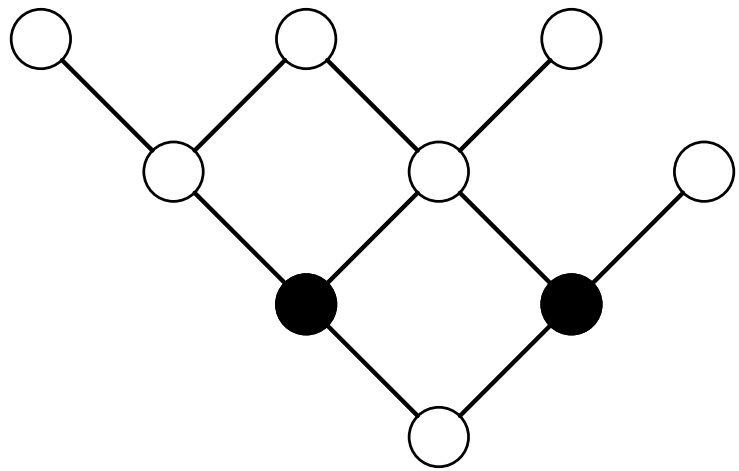


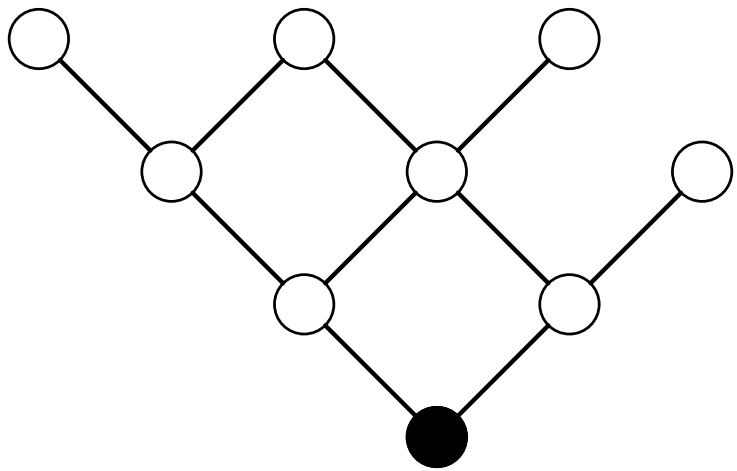
Pebbling Game Example

Maximum pebbles used: 3



Pebbling Game Example

Maximum pebbles used: 3



Greedy Pebbling

Topological Order + Deletion Information

- Correspond to a strategy for the pebbling game

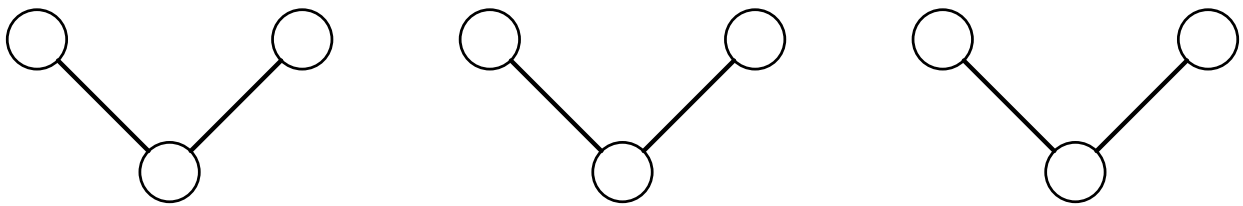
Greedy heuristics

- Find better order w.r.t. space measure
- Choose next node w.r.t.:
  - Number of pebbles that can be removed
  - Pebbled premises
  - Children with pebbled premises
  - Number of children
  - Number of premises, which the node is the last children of
  - ...

Top-down vs Bottom-up

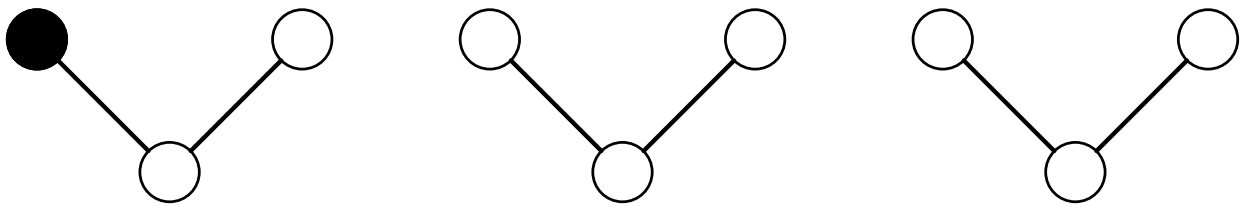
Greedy Top-down Pebbling

Maximum pebbles used: 0



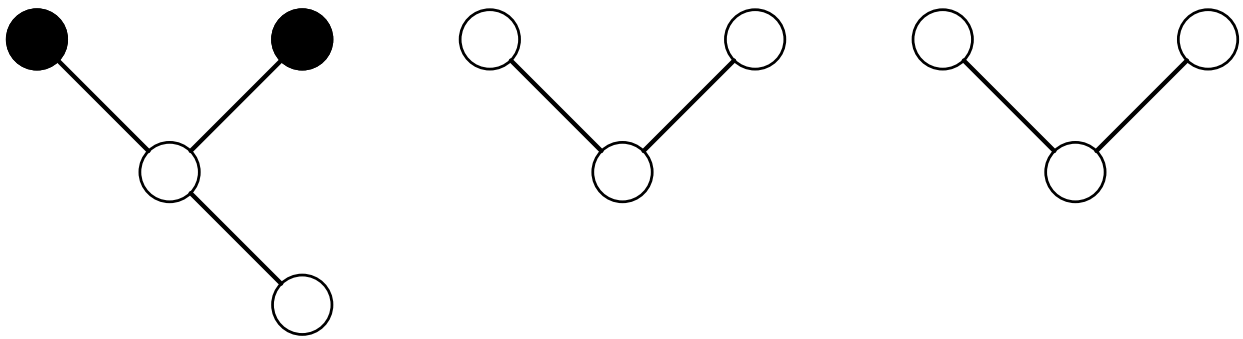
Greedy Top-down Pebbling

Maximum pebbles used: 1



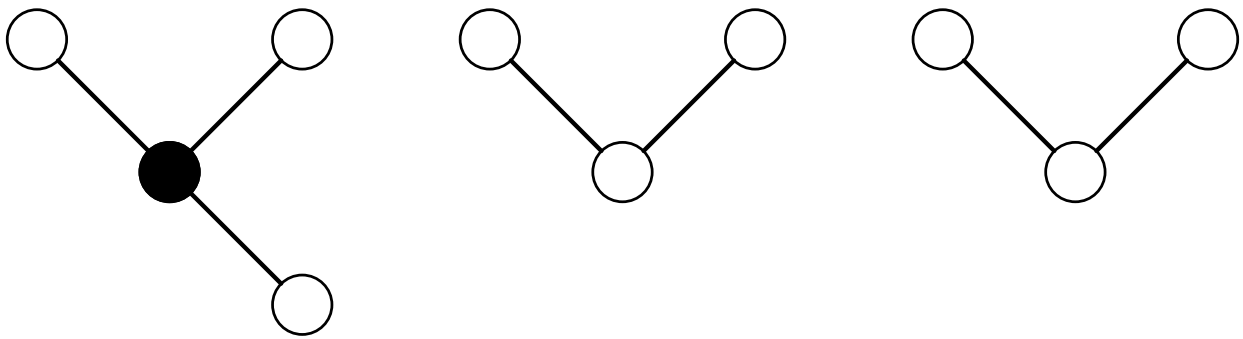
Greedy Top-down Pebbling

Maximum pebbles used: 2



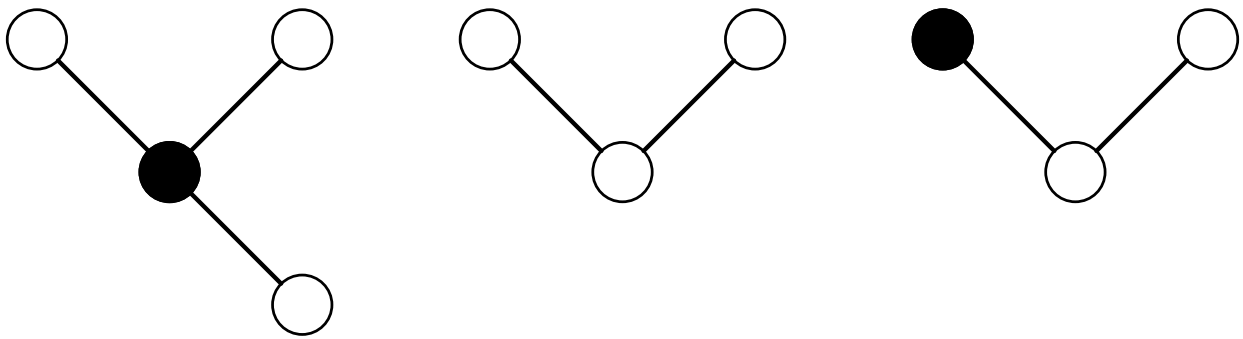
Greedy Top-down Pebbling

Maximum pebbles used: 2



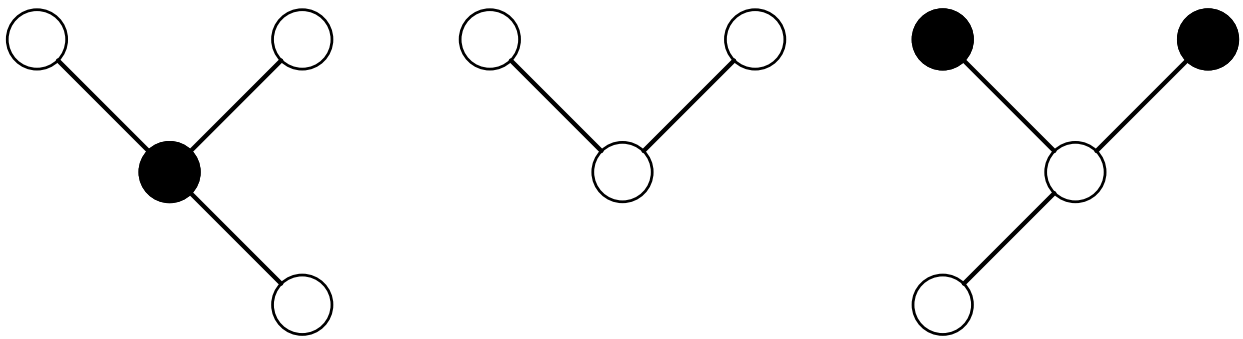
Greedy Top-down Pebbling

Maximum pebbles used: 2



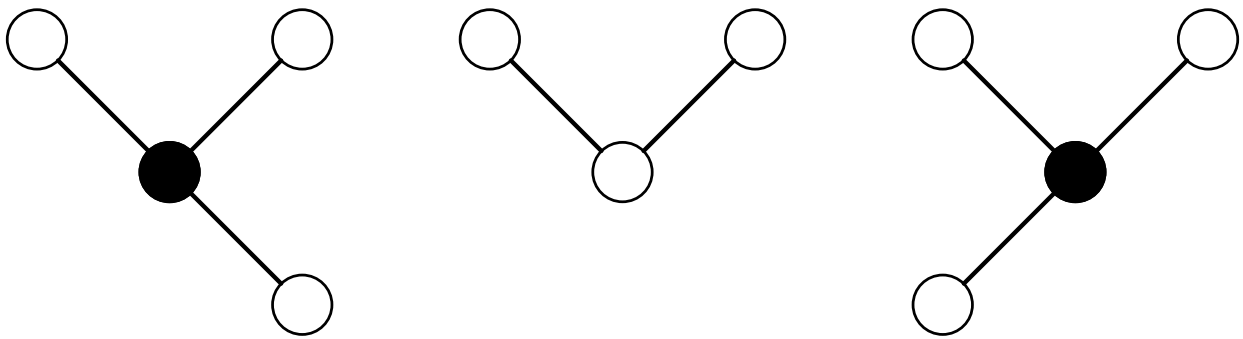
Greedy Top-down Pebbling

Maximum pebbles used: 3



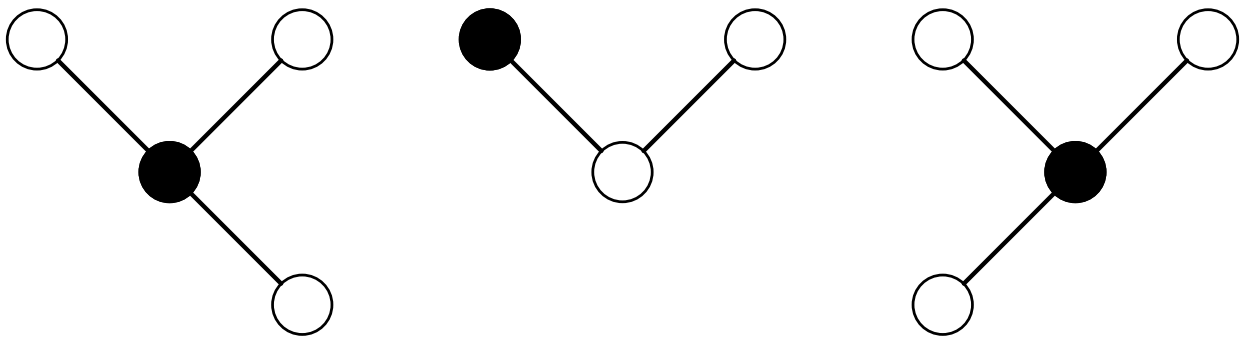
Greedy Top-down Pebbling

Maximum pebbles used: 3



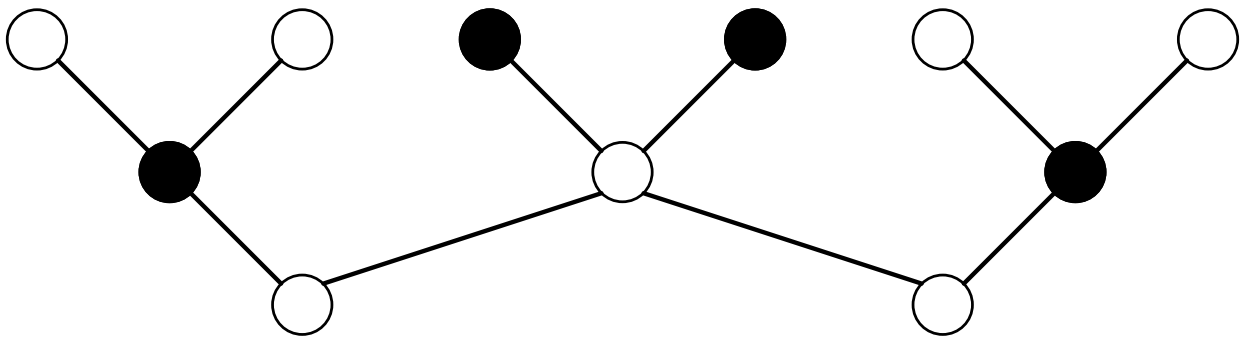
Greedy Top-down Pebbling

Maximum pebbles used: 3



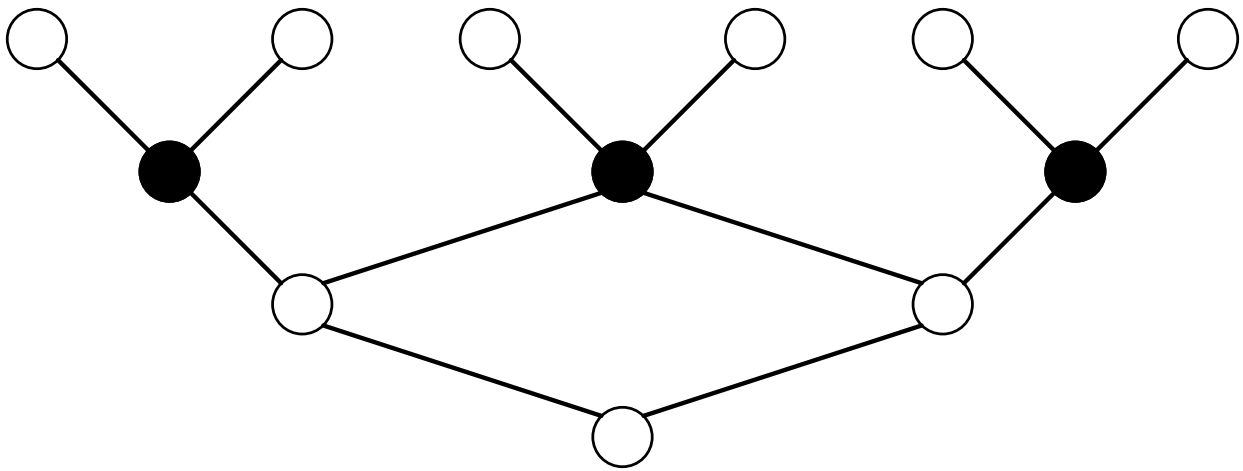
Greedy Top-down Pebbling

Maximum pebbles used: 4



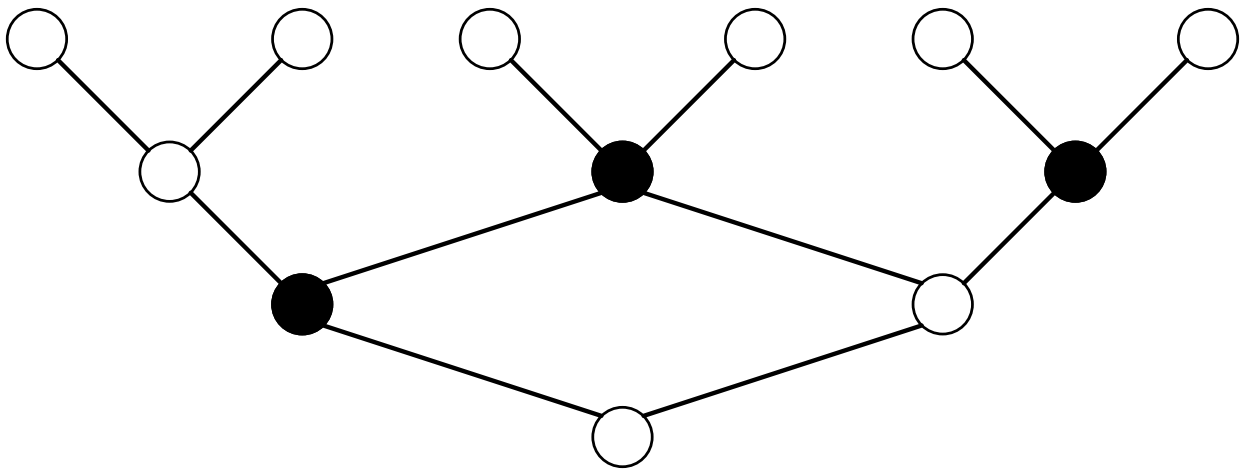
Greedy Top-down Pebbling

Maximum pebbles used: 4



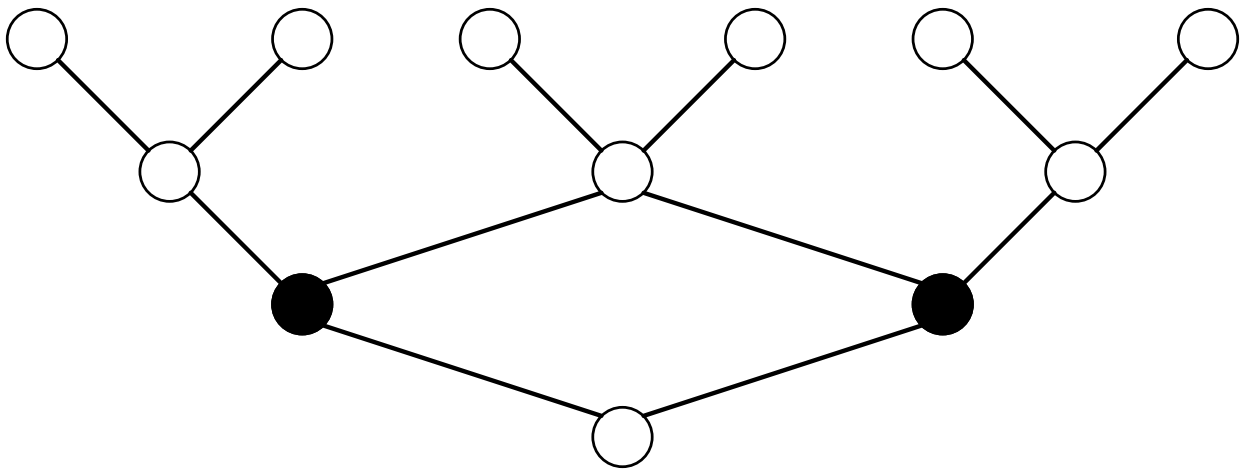
Greedy Top-down Pebbling

Maximum pebbles used: 4



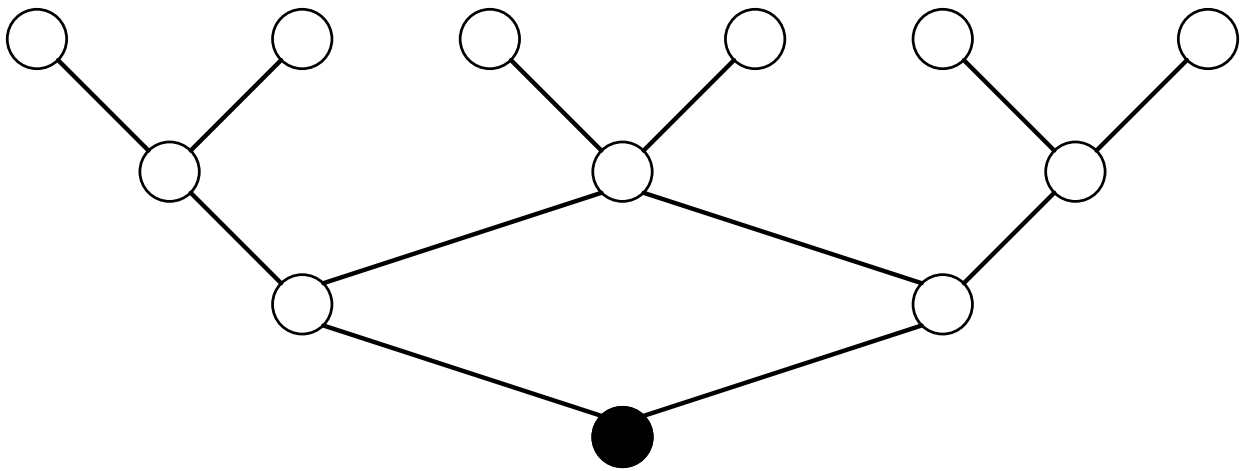
Greedy Top-down Pebbling

Maximum pebbles used: 4



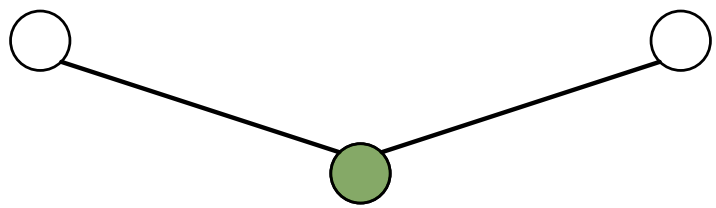
Greedy Top-down Pebbling

Maximum pebbles used: 4



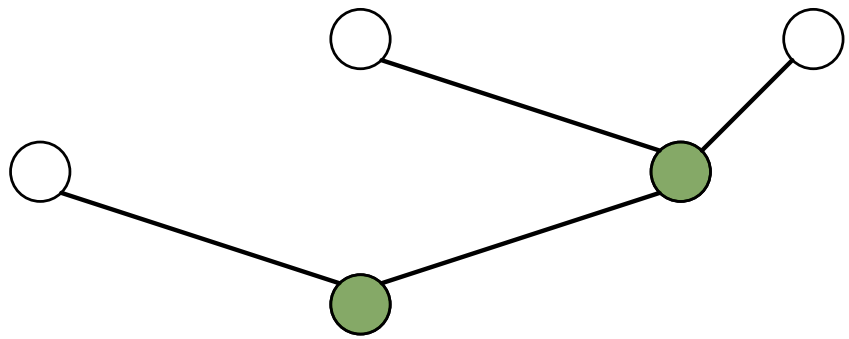
Greedy Bottom-up Pebbling

Maximum pebbles used: 0



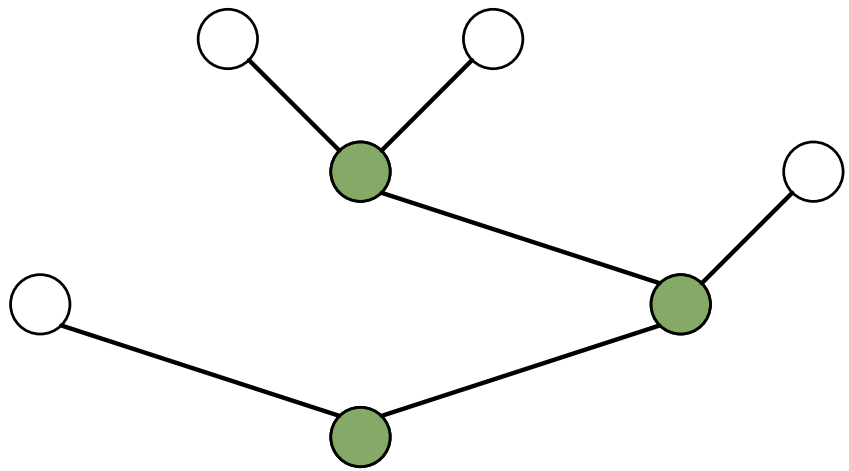
Greedy Bottom-up Pebbling

Maximum pebbles used: 0



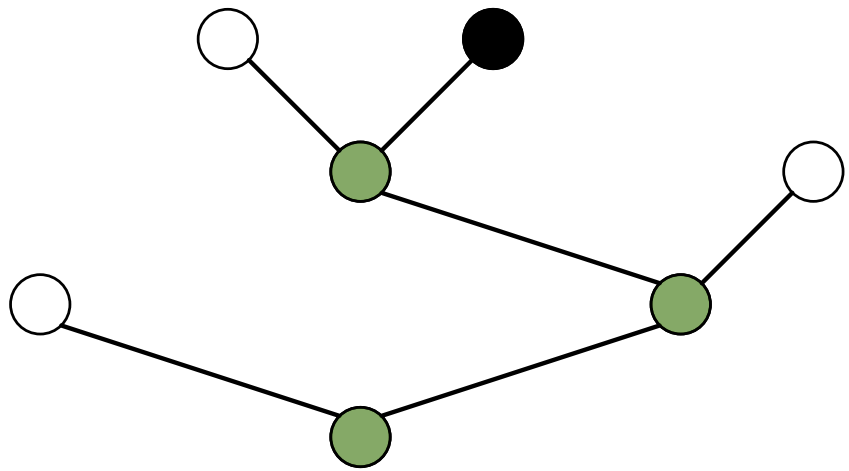
Greedy Bottom-up Pebbling

Maximum pebbles used: 0



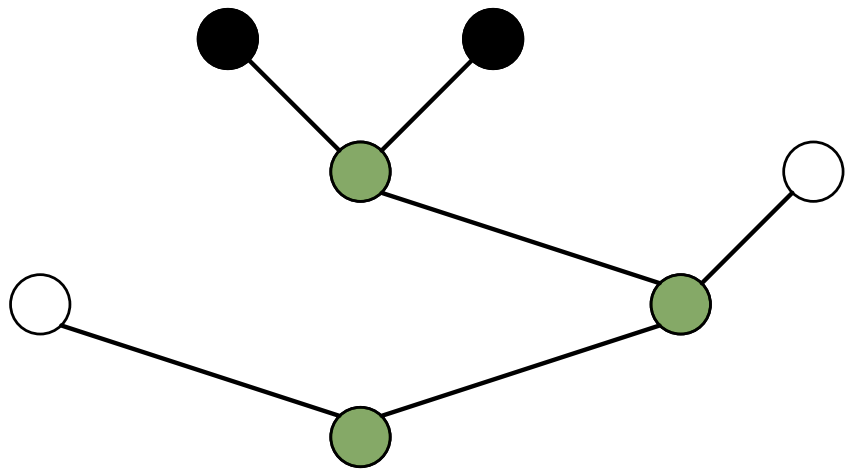
Greedy Bottom-up Pebbling

Maximum pebbles used: 1



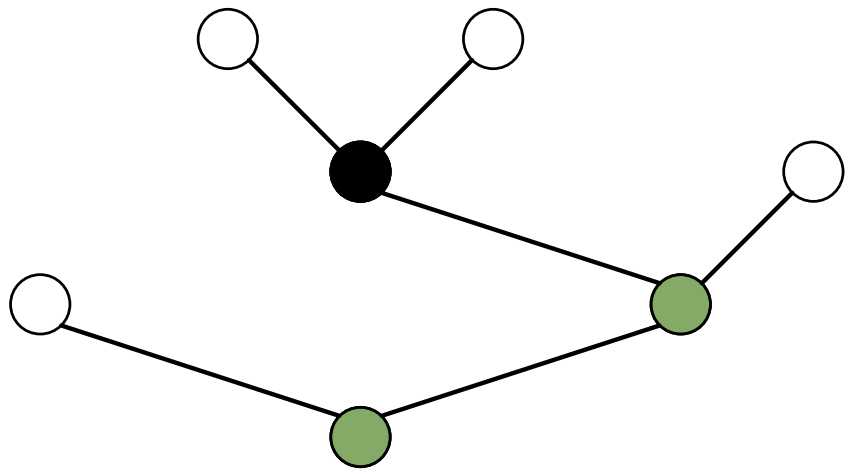
Greedy Bottom-up Pebbling

Maximum pebbles used: 2



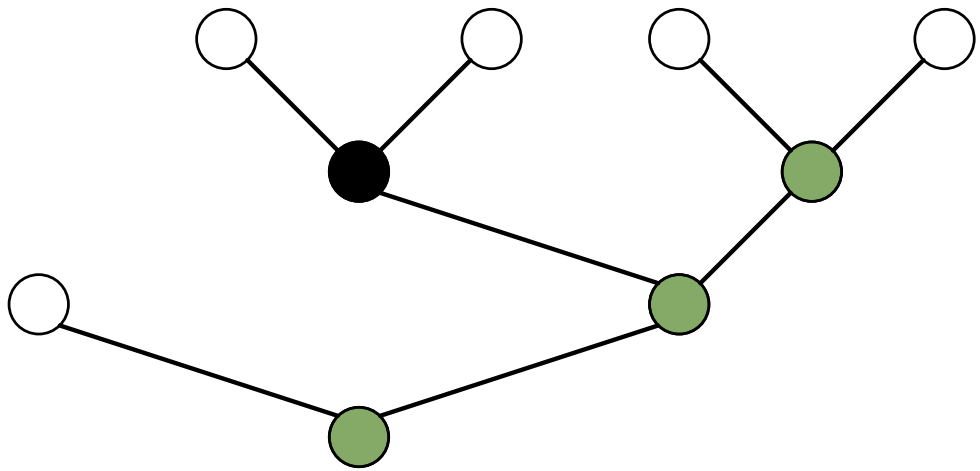
Greedy Bottom-up Pebbling

Maximum pebbles used: 2



Greedy Bottom-up Pebbling

Maximum pebbles used: 2



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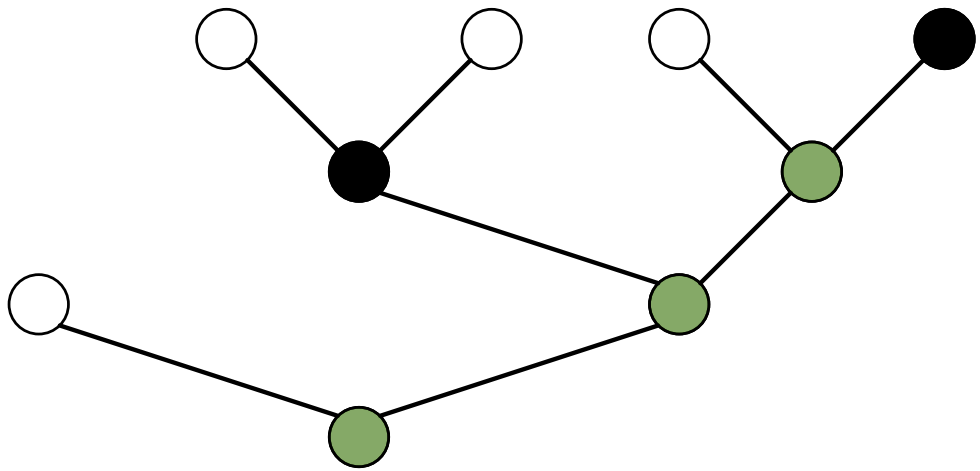
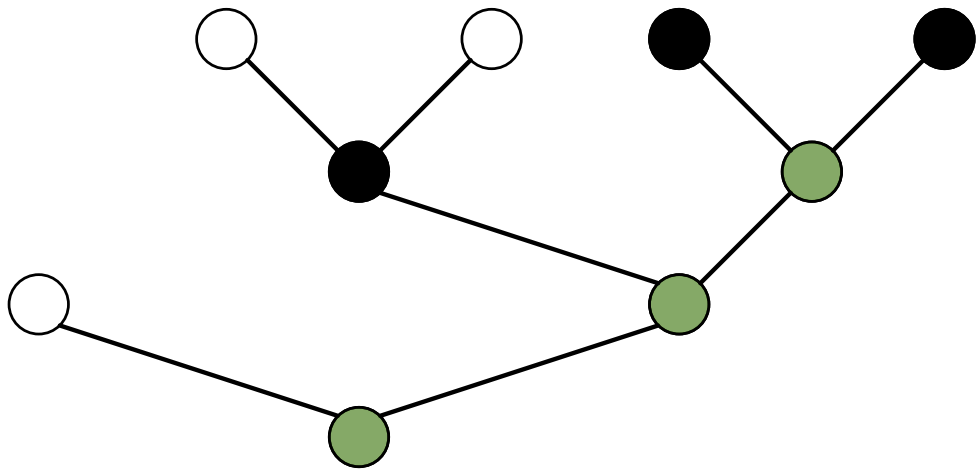
Maximum pebbles used: 2



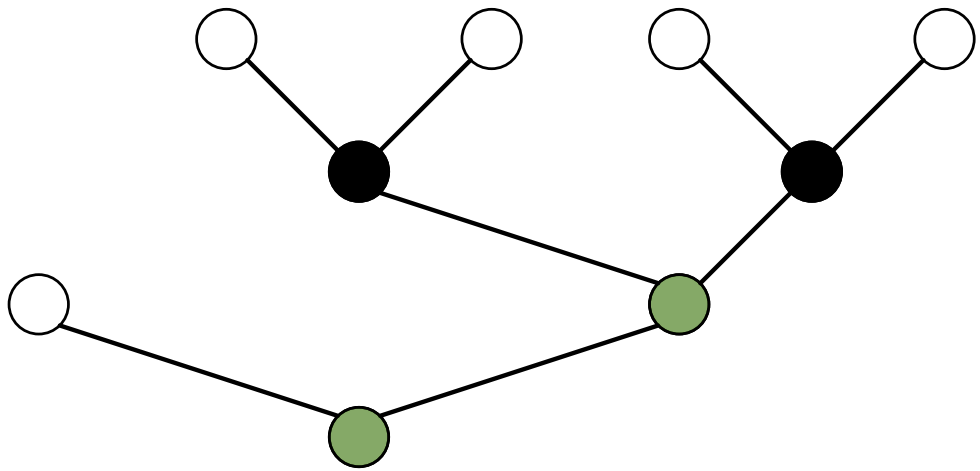
Greedy Bottom-up Pebbling

Maximum pebbles used: 3



Greedy Bottom-up Pebbling

Maximum pebbles used: 3



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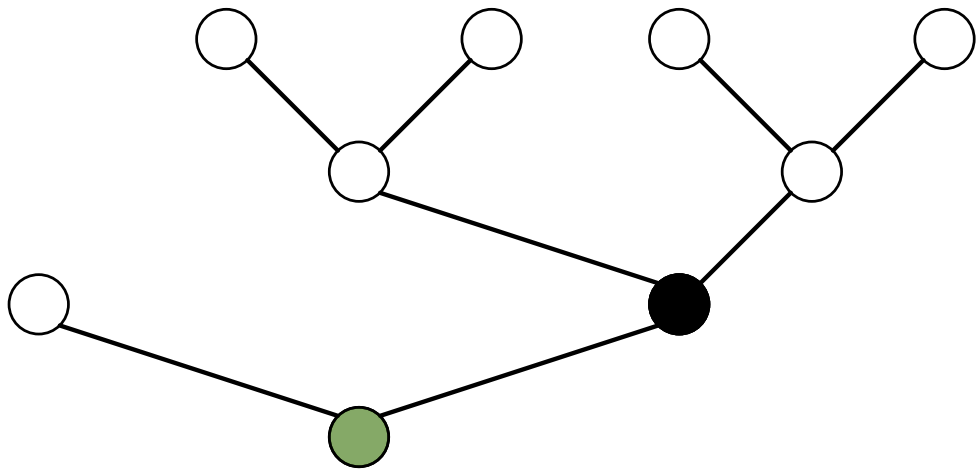

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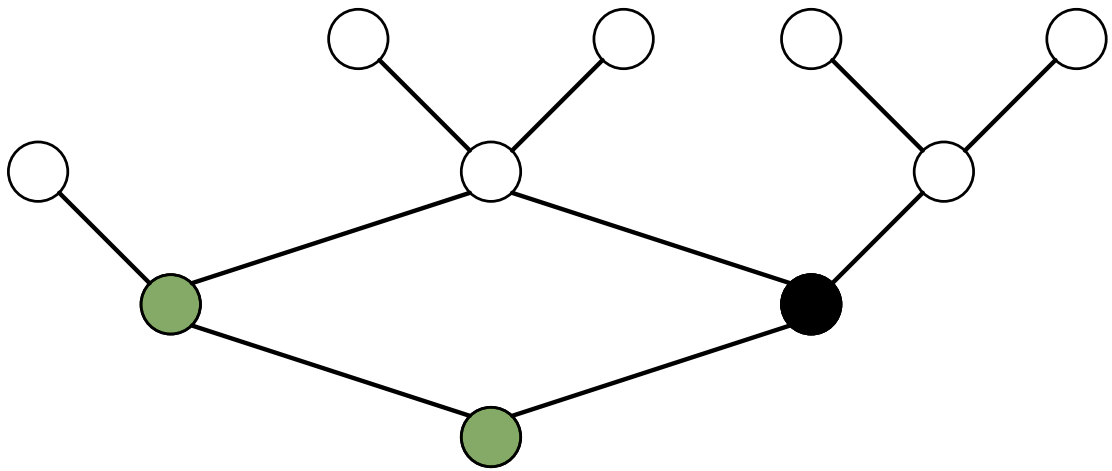

Greedy Bottom-up Pebbling

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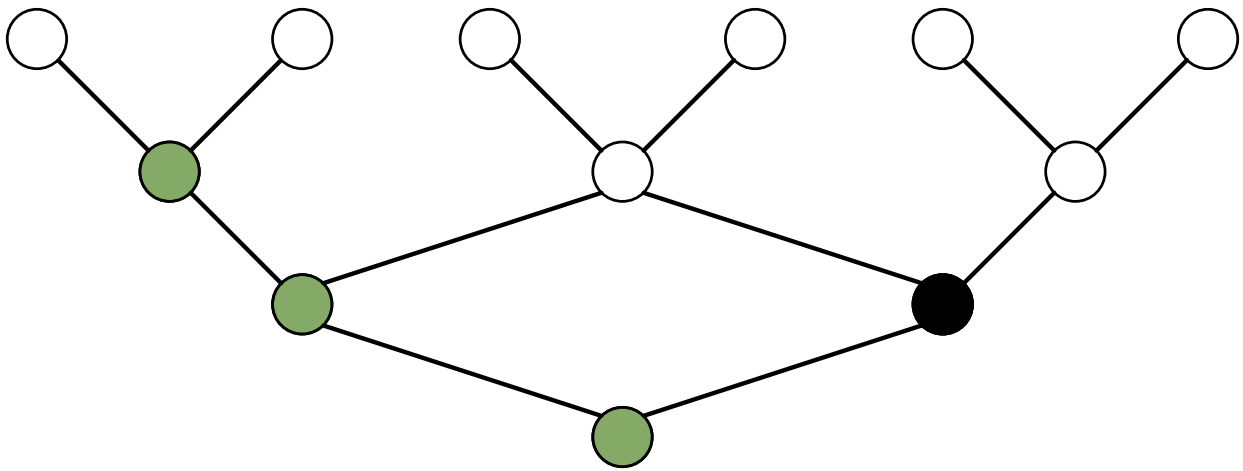
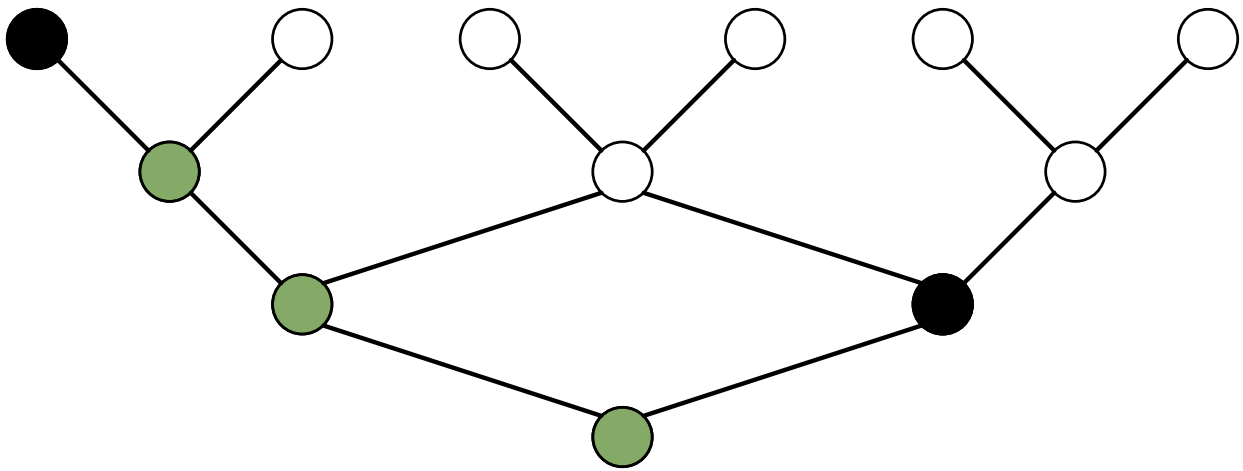


Greedy Bottom-up Pebbling

Maximum pebbles used: 3



Greedy Bottom-up Pebbling

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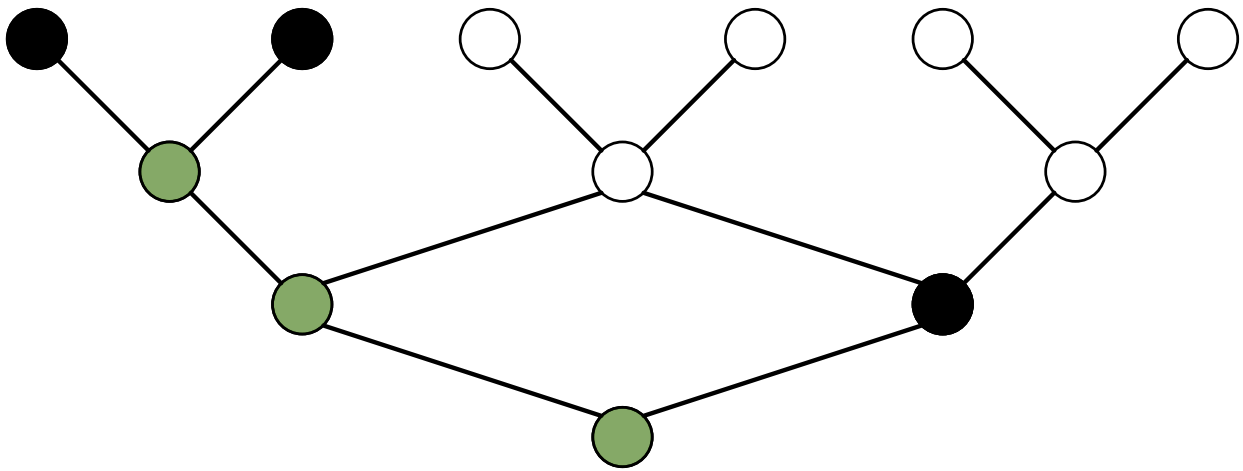

Greedy Bottom-up Pebbling

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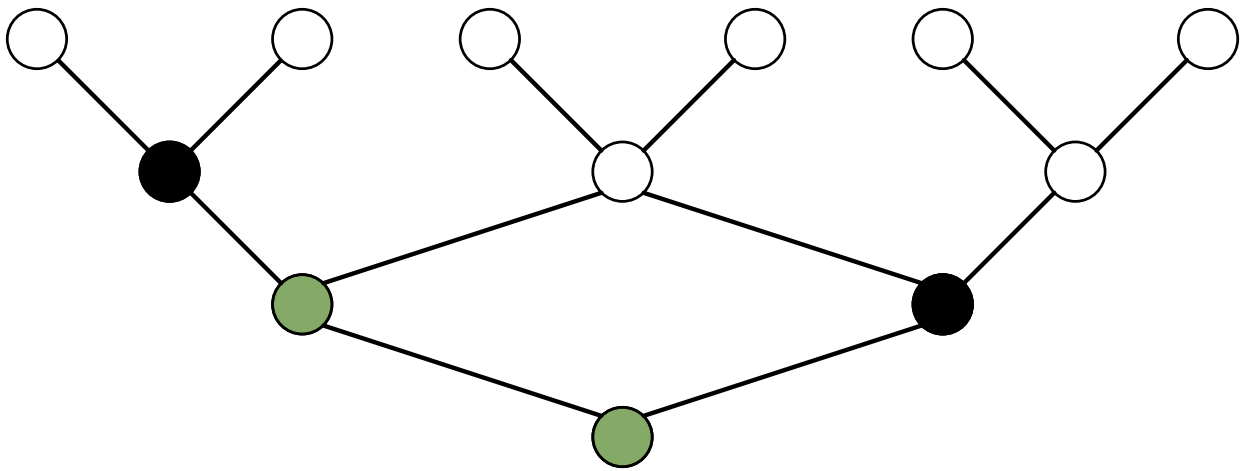
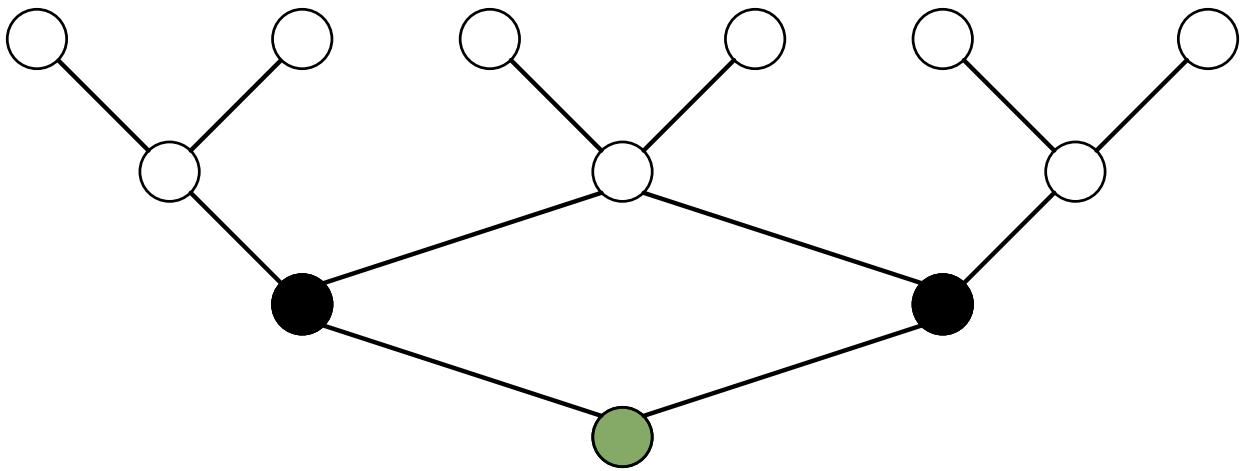


Greedy Bottom-up Pebbling

Maximum pebbles used: 3



Greedy Bottom-up Pebbling

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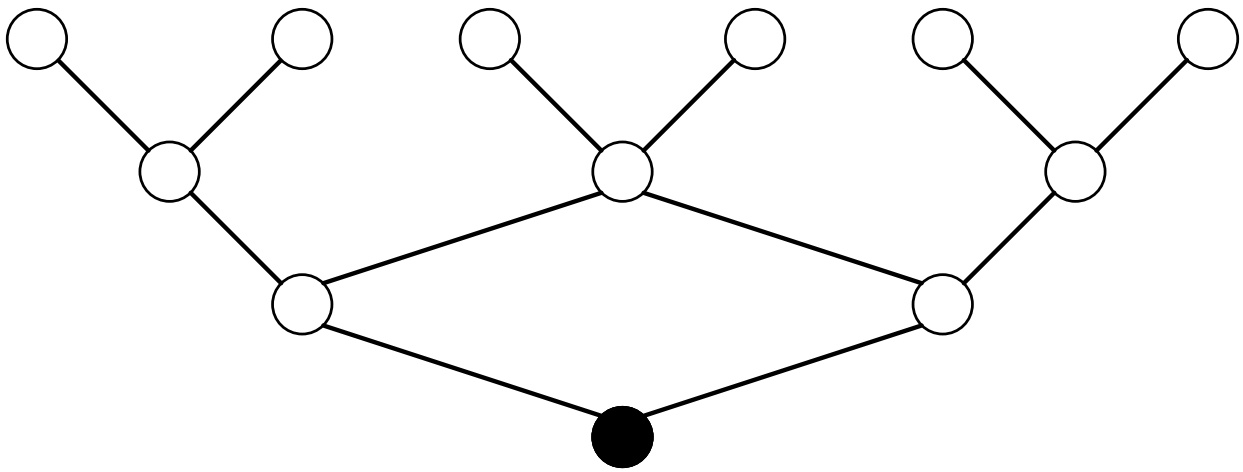

Experiments

Results

Algorithm	Space Compression	Speed
Top-down Pebbling*	-40.4776%	1.4 nodes/ms
Bottom-up Pebbling**	6.8%	12.2 nodes/ms

Used heuristics

- *: removes pebbles $>$ pebbled premises $>$ children with pebbled premises $>$ index in the original proof
- **: last child of a node $>$ number of children $>$ index in the original proof

Conclusion & Future Work

Subsumption

- Top-down-S interesting replacement for DAGification
- Bottom-up-S could possibly do much
- Boost performance
- Fix problems with Bottom-up-S

Recursive Split

- Promising Idea
- Still needs some fine tuning

Greedy Pebbling

- Top-down version is not clever enough
- Bottom-up version shows nice results
- Find better heuristics for Bottom-up

Special Thanks to:

Google &

European Master's Program in Computational Logic (EMCL)

for financial support

Thank you for your attention

Feel free to ask questions

http://github.com/Paradoxika/Skeptik