Towards the Compression of First-Order Resolution Proofs by Lowering Unit Clauses

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Proof Compression Motivation

an accessible, good motivational example for proof compression

(Propositional) Proofs

Definition (Proof)

A directed acyclic graph $\langle V, E, \Gamma \rangle$, where

- V is a set of nodes
- E is a set of edges labeled by literals
- Γ (the proof clause) is inductively constructible using *axiom* and *resolution* nodes

Definition (Axiom)

A proof with a single node (so $E = \emptyset$)

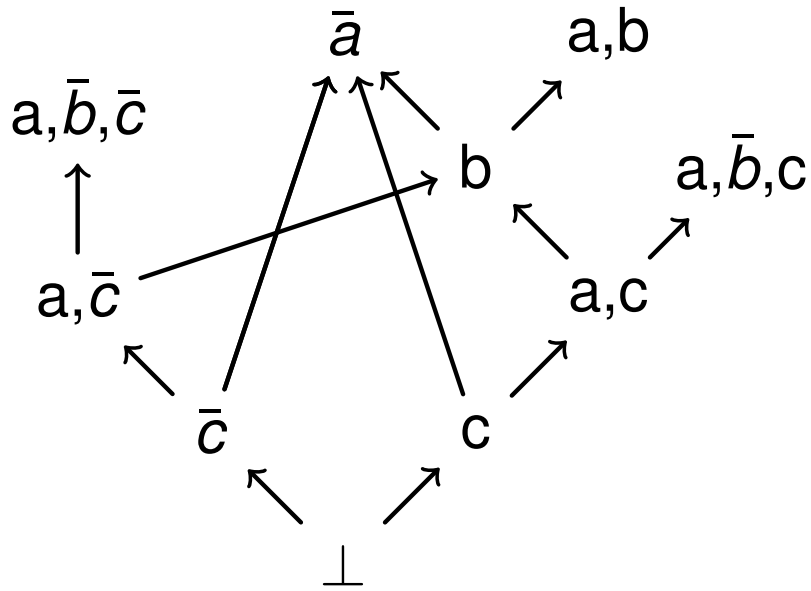
(Propositional) Resolution

Definition (Resolution)

Given two proofs ψ_L and ψ_R with conclusions Γ_L and Γ_R with some literal l such that $\bar{l} \in \Gamma_L$ and $l \in \Gamma_R$, the resolution proof ψ of ψ_L and ψ_R on l, denoted $\psi = \psi_L \psi_R$ is such that:

- ψ's nodes are the union of the nodes of ψ_L and ψ_R, and a new root node
- there is an edge from $\rho(\psi)$ to $\rho(\psi_L)$ labeled with $\bar{l}$
- there is an edge from $\rho(\psi)$ to $\rho(\psi_R)$ labeled with l
- ψ's conclusion is $(\Gamma_L \setminus \{\bar{l}\}) \cup (\Gamma_R \setminus \{l\})$

A Propositional Proof



Deletion

how deleting subproofs or edges in proofs affect them

Redundancy

types of redundancy we hope to remove, small examples (before/after proofs; not animated)

First-Order Proofs

Definition (First-Order Proof)

A directed acyclic graph $\langle V, E, \Gamma \rangle$, where

- V is a set of nodes
- E is a set of edges labeled by literals **and substitutions**
- Γ (the proof clause) is inductively constructible using *axiom*, ***(first order) resolution***, **and** ***contraction*** nodes

Axioms are unchanged

Substitutions and Unifiers

Definition (Substitution)

A mapping $\{X_1 \setminus t_1, X_2 \setminus t_2, \ldots\}$ from variables $X_1, X_2, \ldots$ to terms $t_1, t_2, \ldots$

Definition (Unifier)

First Order (Unifying) Resolution

Definition (First Order Resolution)

Given two proofs ψ_L and ψ_R with conclusions Γ_L and Γ_R with some literal l such that $l_L \in \Gamma_L$ and $l_R \in \Gamma_R$, and σ_L and σ_R are substitutions usch that $l_L\sigma_L = \overline{l_R}\sigma_R$, and the variables in $(\Gamma_L \setminus l_L)\sigma_L$ and $(\Gamma_R \setminus l_R)\sigma_R$ are disjoint, then the resolution proof ψ of ψ_L and ψ_R on l, denoted $\psi = \psi_L\psi_R$ is such that:

- ψ's nodes are the union of the nodes of ψ_L and ψ_R, and a new root node
- there is an edge from $\rho(\psi)$ to $\rho(\psi_L)$ labeled with l_L and σ_L
- there is an edge from $\rho(\psi)$ to $\rho(\psi_R)$ labeled with l_R and σ_R
- ψ's conclusion is $(\Gamma_L \setminus l_L)\sigma_L \cup (\Gamma_R \setminus l_R)\sigma_R$

Contraction

Definition (Contraction)

If ψ' is a proof and σ is a unifier of $\{l_1, \ldots, l_n\} \subset \Gamma'$, then a contraction ψ is a proof where

- ψ's nodes are the union of the nodes of ψ' and a new node v
- There is an edge from $\rho(\psi')$ to v labeled with $\{l_1, \ldots, l_n\}$ and σ
- The conclusion is $(\Gamma' \setminus \{l_1, \ldots, l_n\})\sigma \cup \{l\}$, where $l = l_k\sigma$ for $k \in \{1, \ldots, n\}$

LowerUnits

Definition (Unit)

A unit is a subproof with a conclusion having exactly 1 literal

Theorem

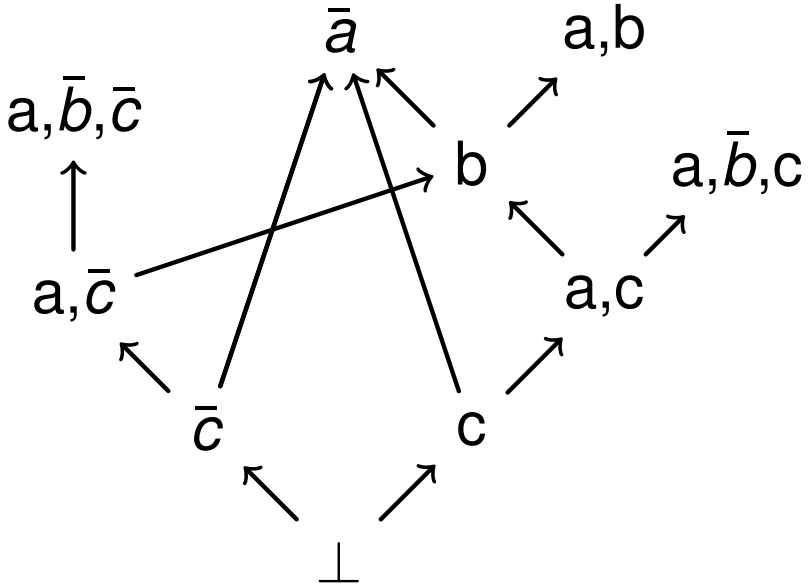
A unit can always be lowered

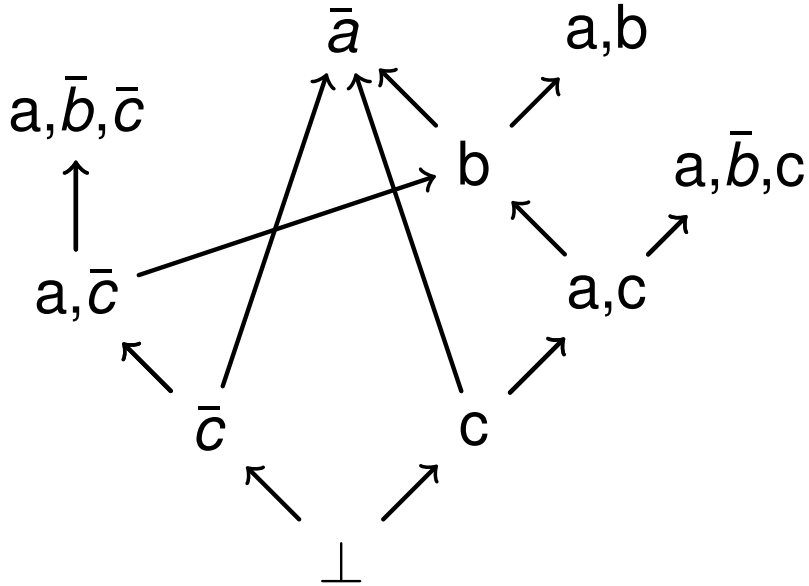
Compression is achieved by delaying resolution with unit subproofs.

Two Traversals

- $\uparrow$ Collect units with more than one resolvent
- $\downarrow$ Delete units and reintroduce them at the bottom of the proof

Propositional Example





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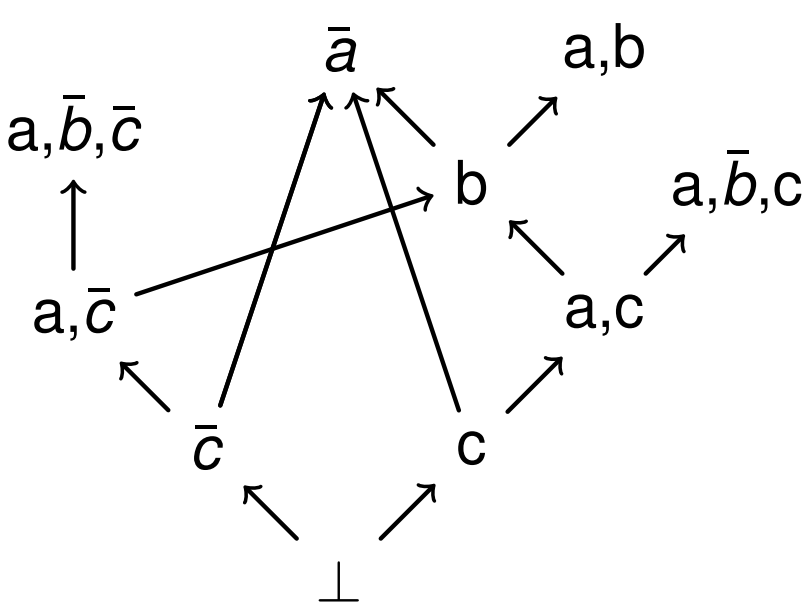

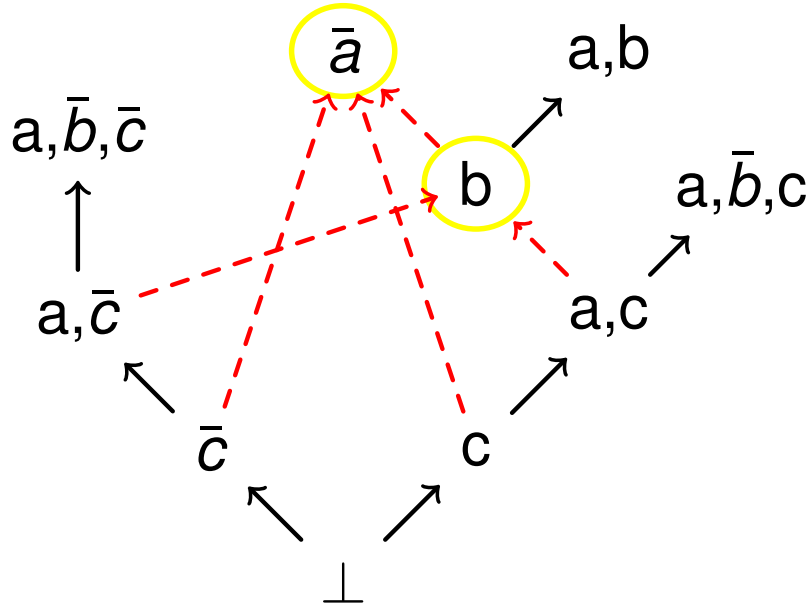

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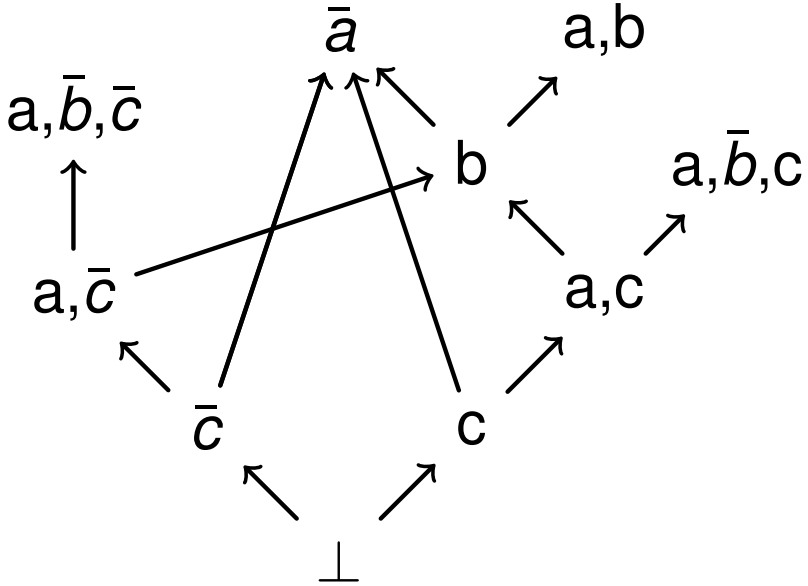

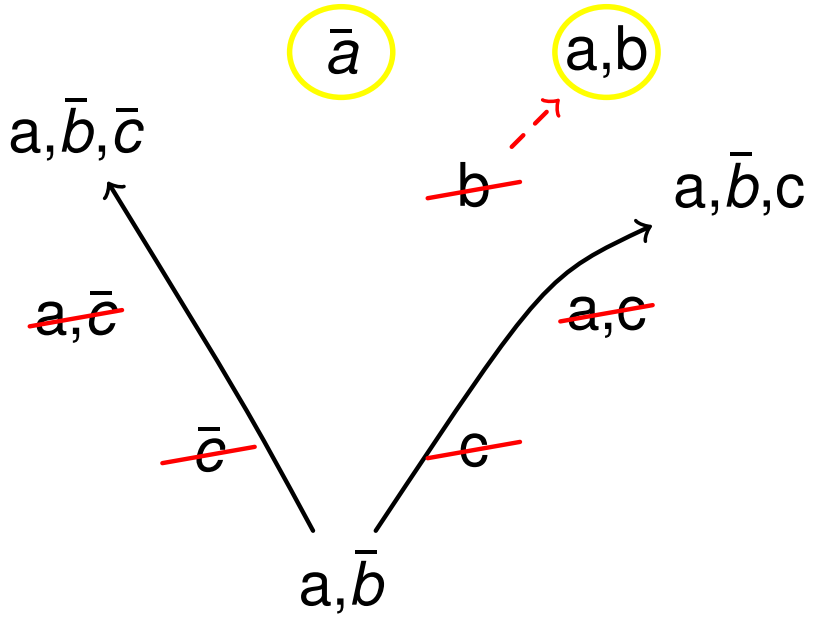

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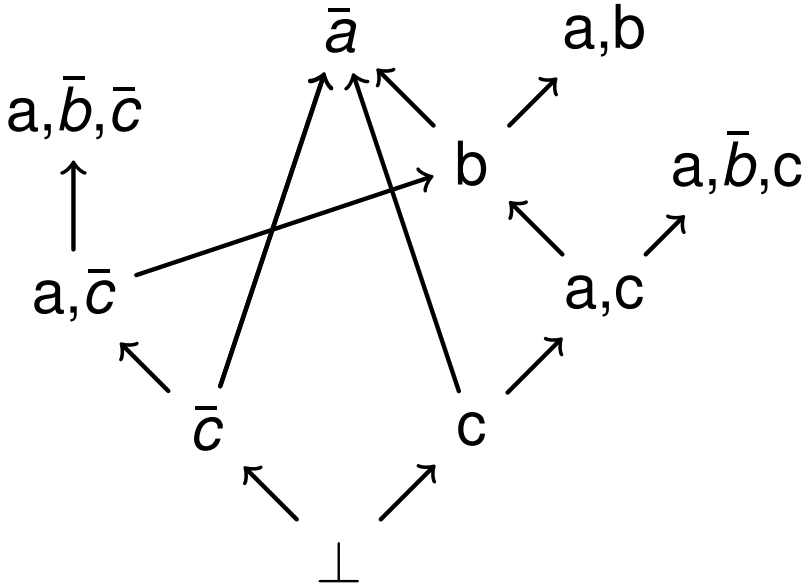

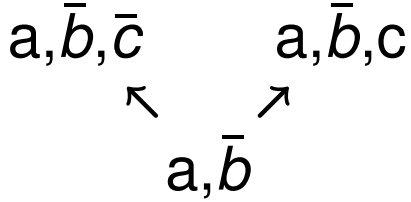

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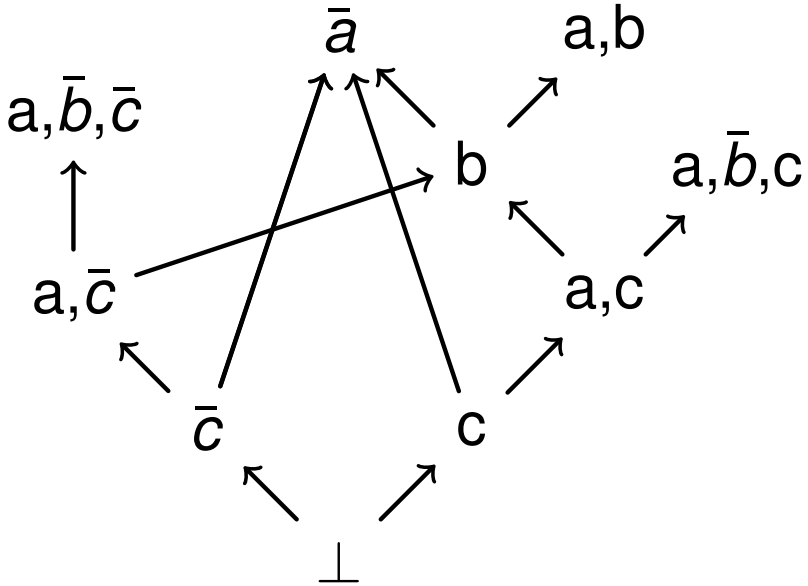

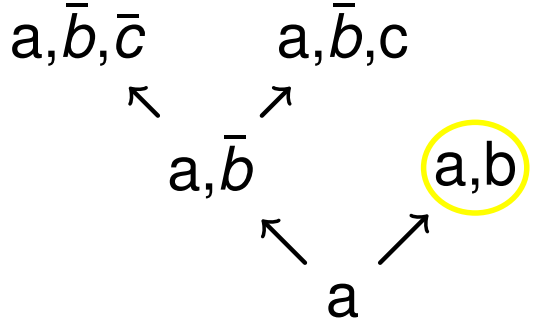

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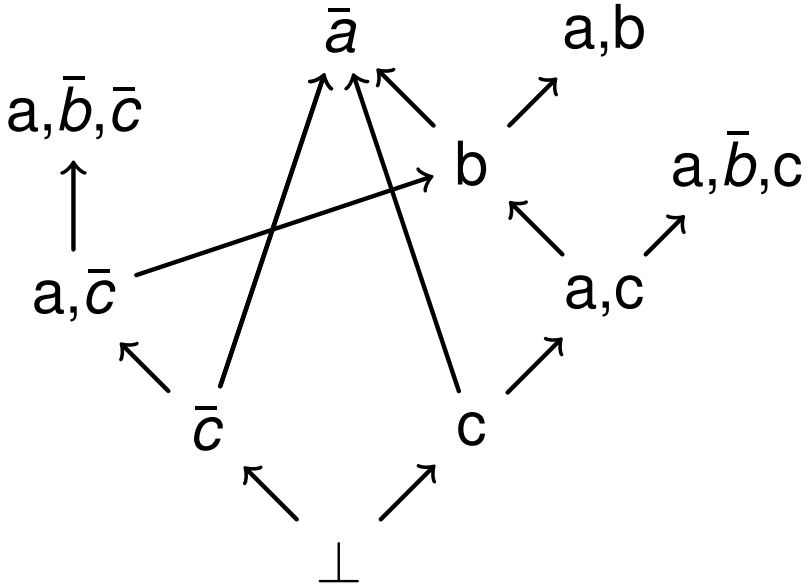
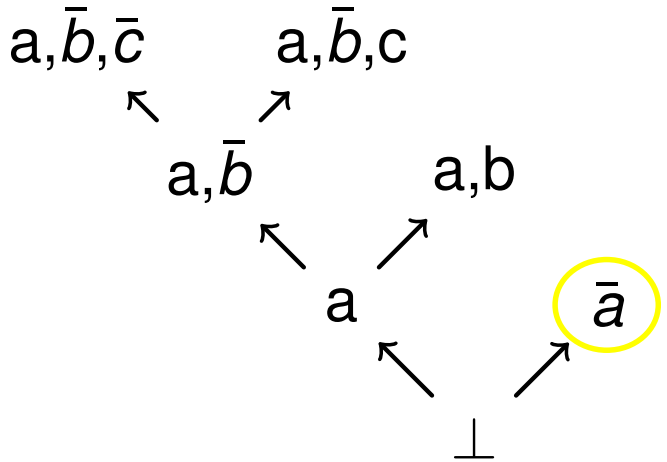
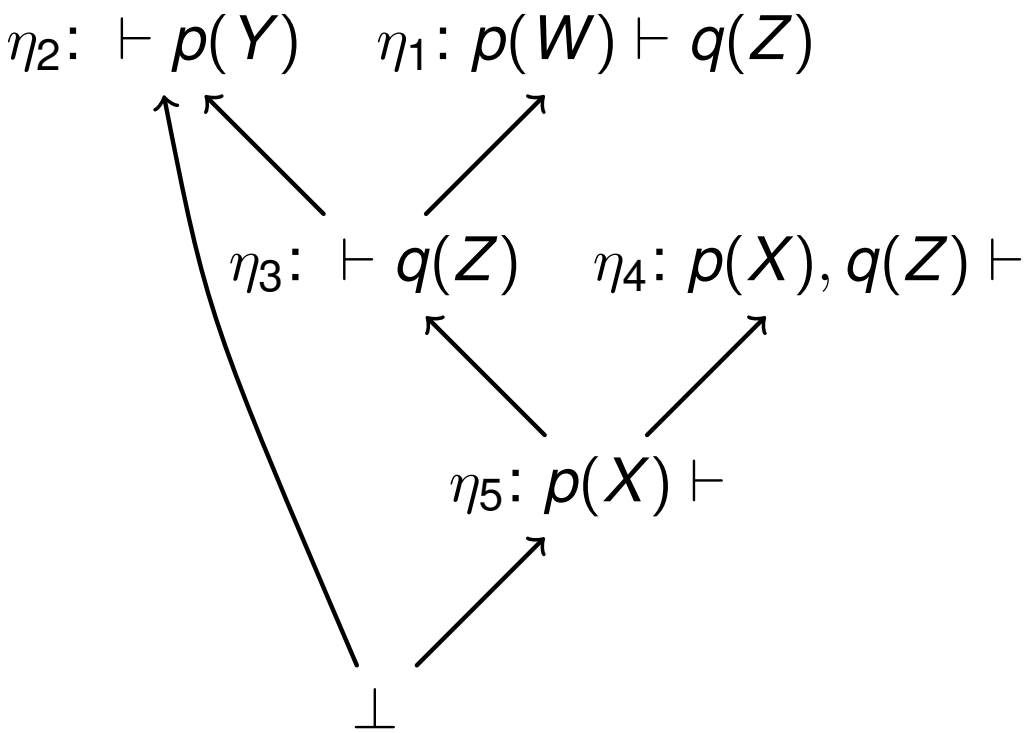




First Order Change: Helpful Contractions



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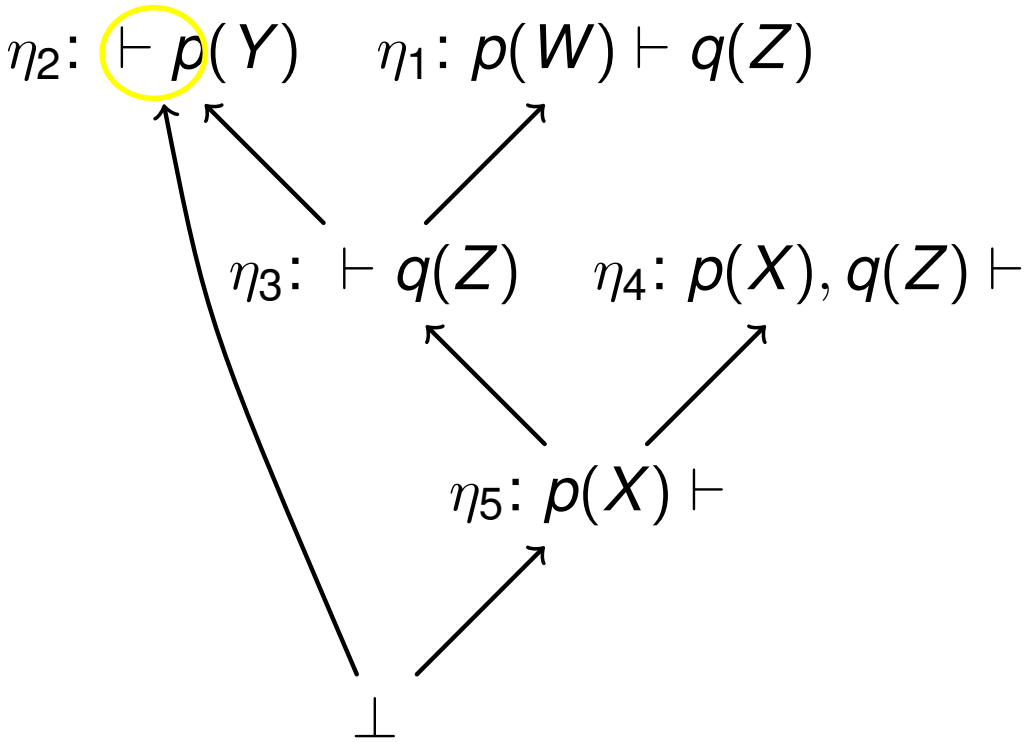

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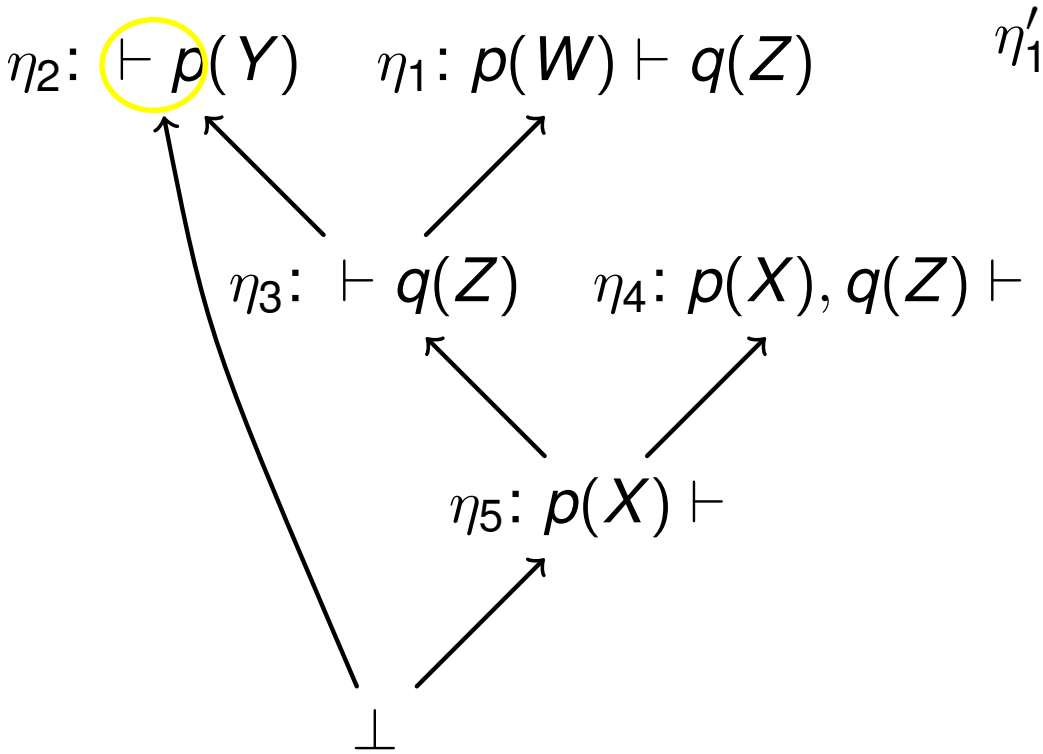

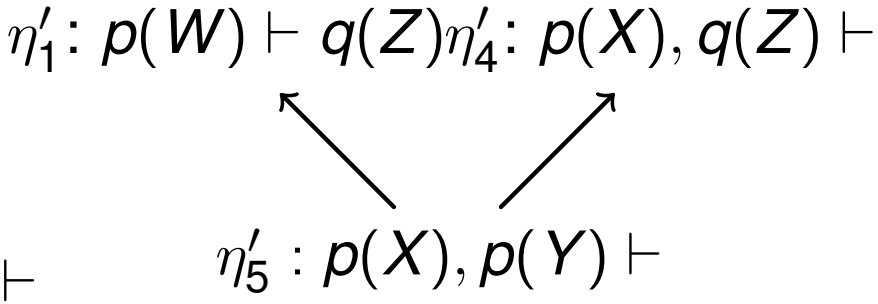

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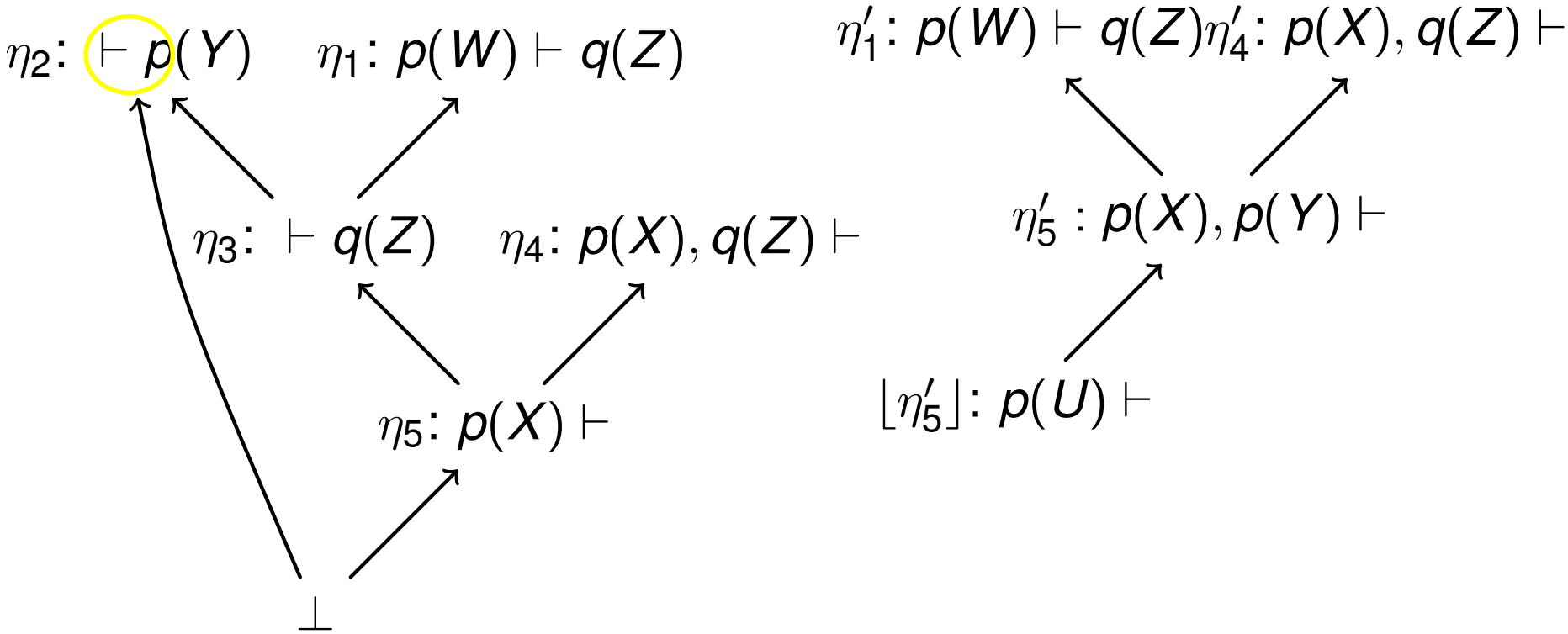

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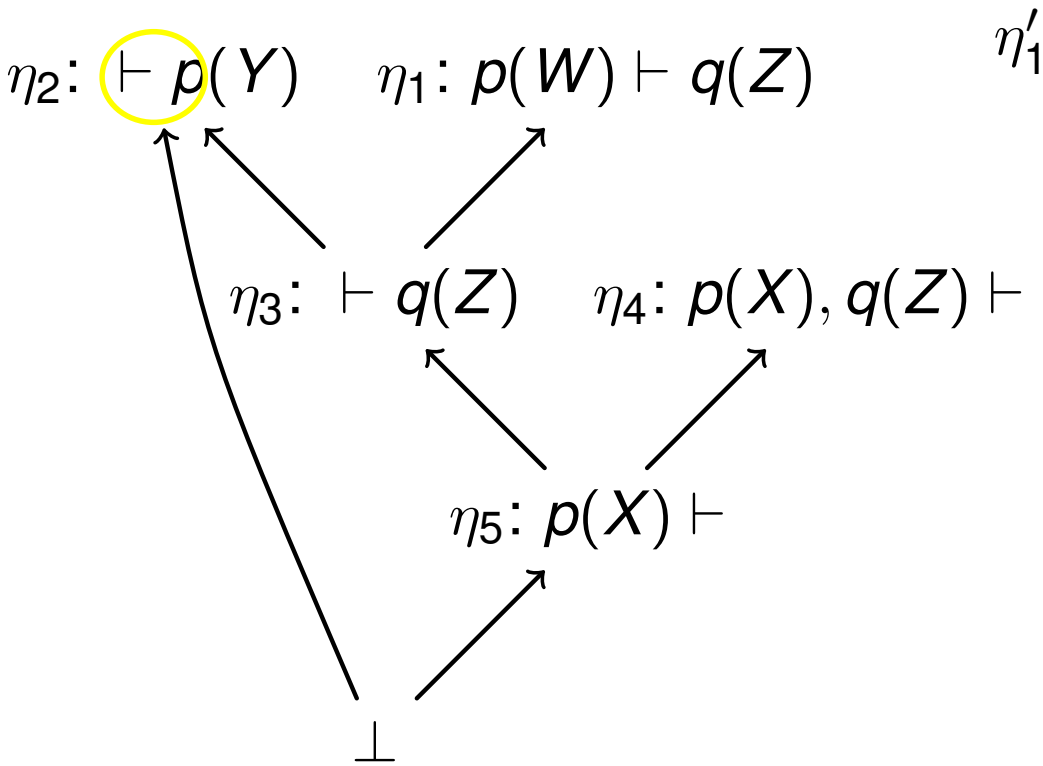

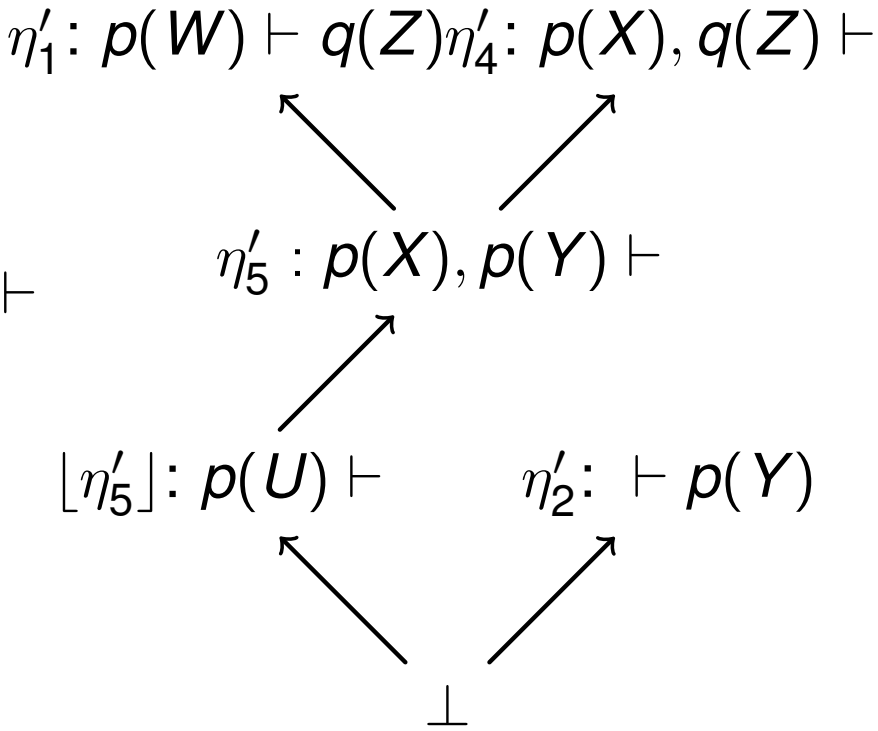

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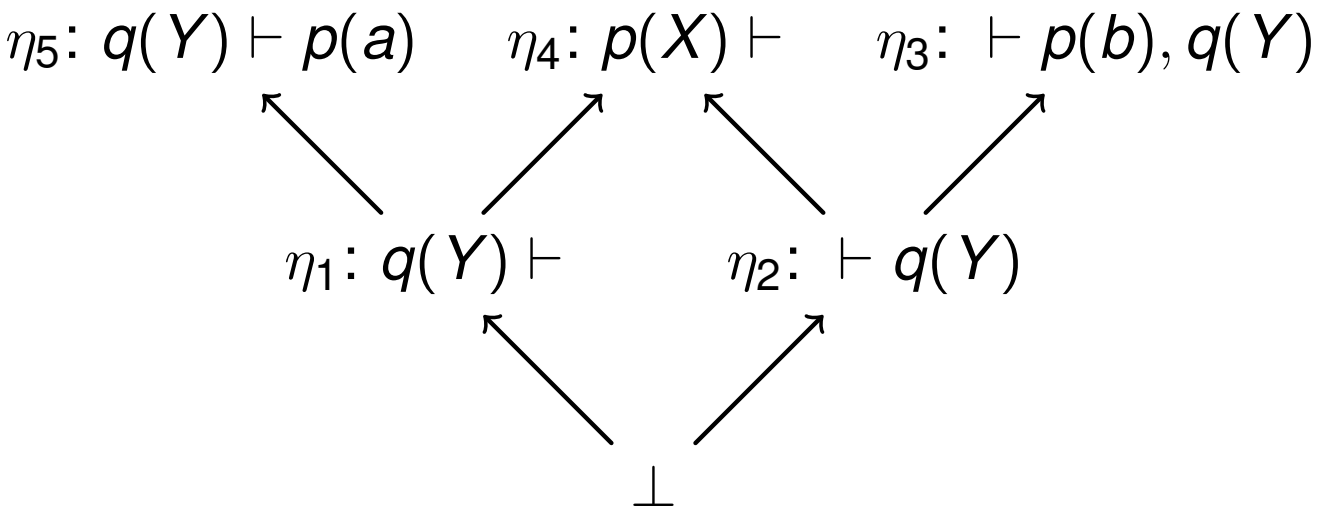

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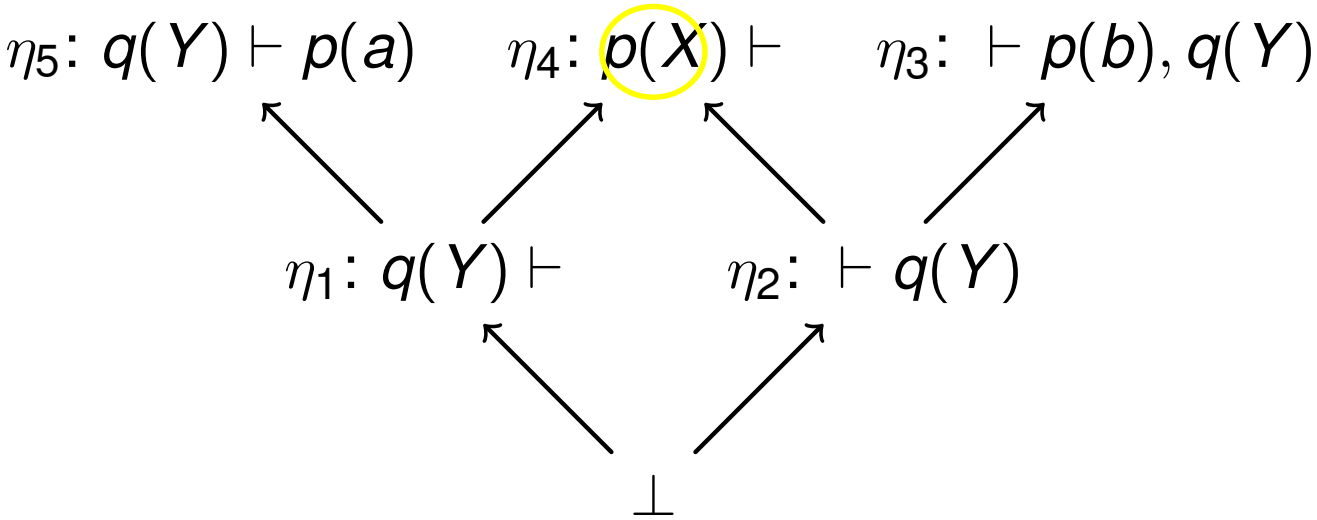

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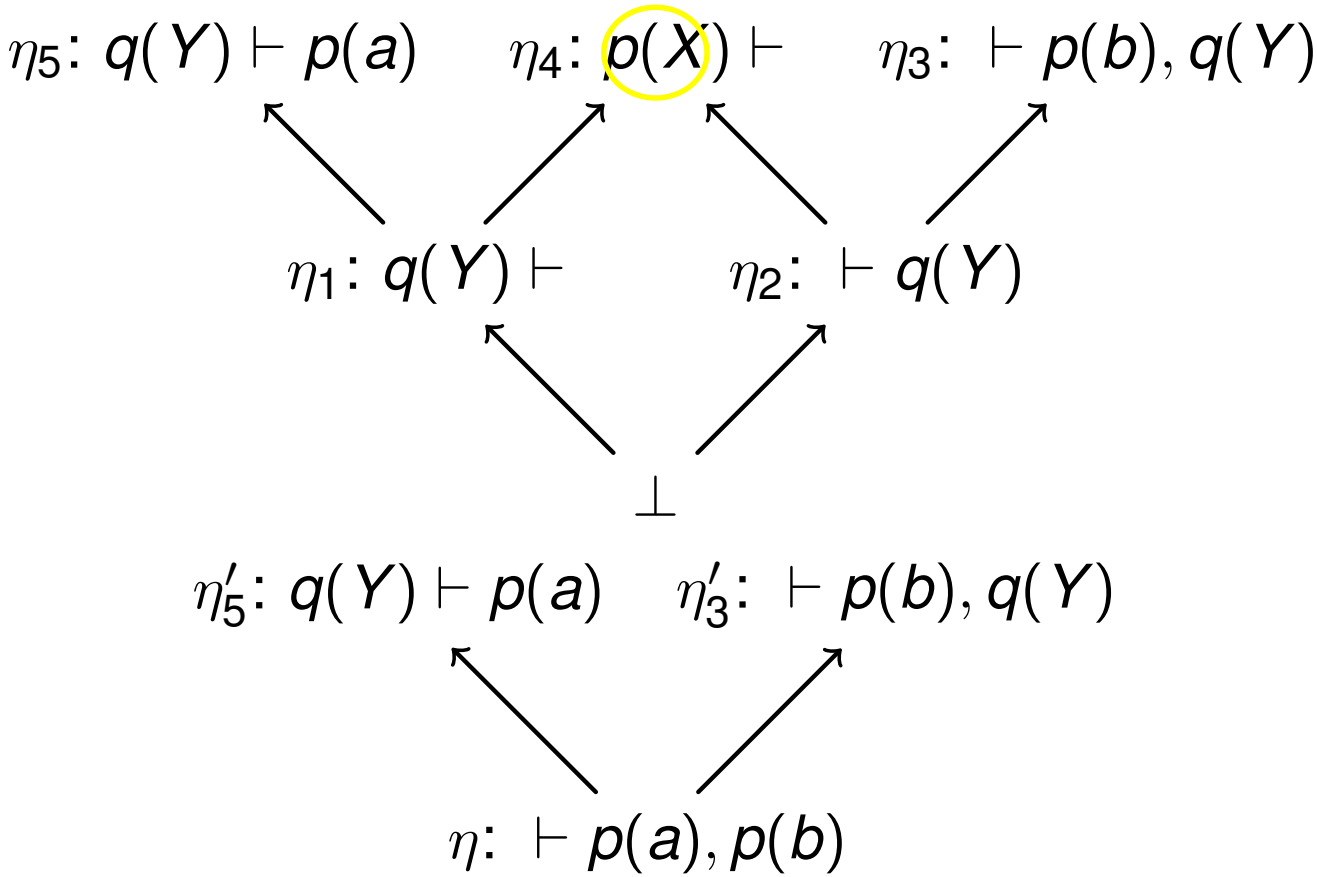

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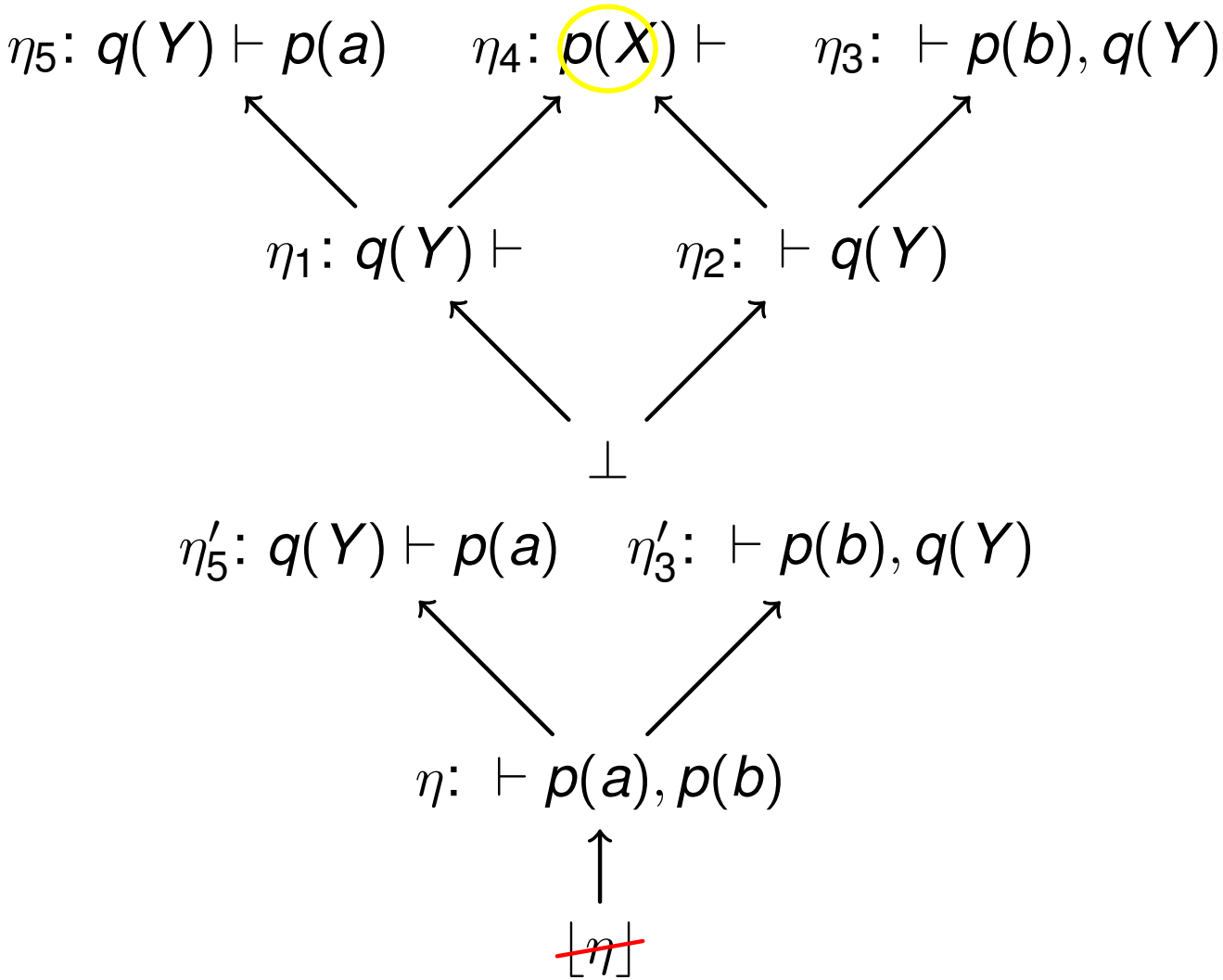

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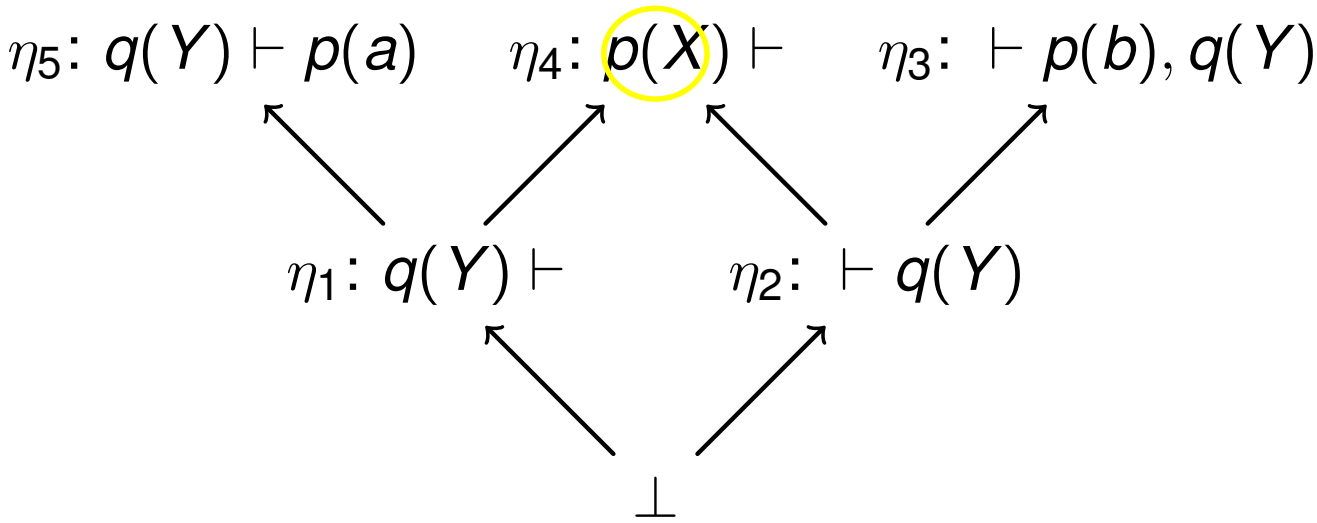
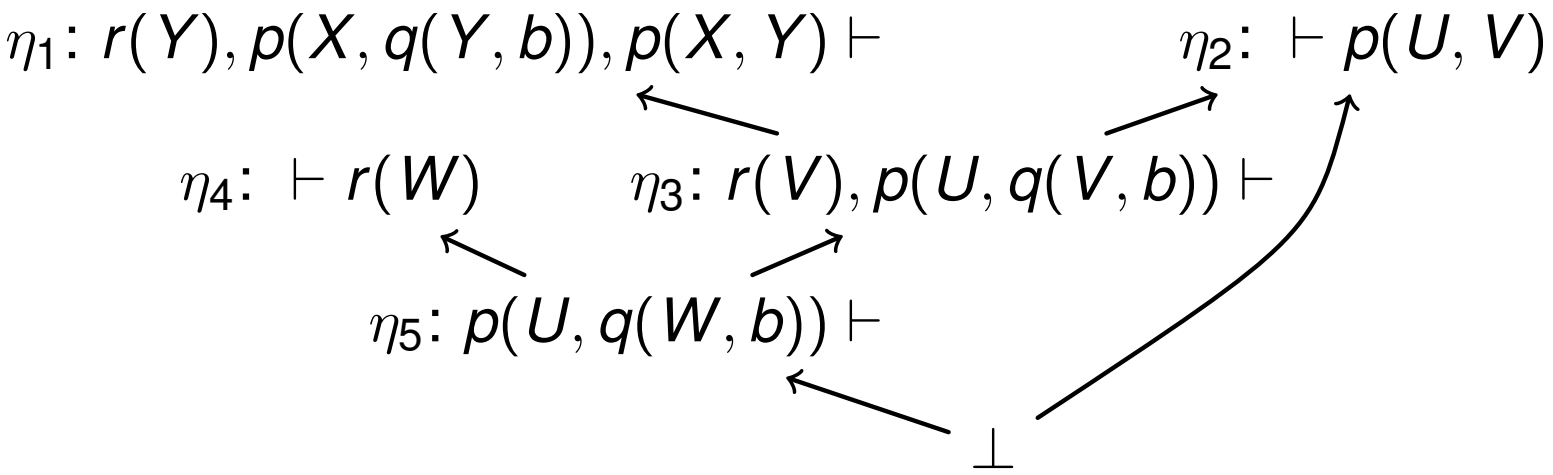


Definition (Pre-Deletion Property)

η unit, $l \in \eta$, such that l is resolved with literals $l_1, \ldots, l_n$ in a proof ψ. η satisfies the *pre-deletion unifiability* property in ψ if $l_1, \ldots, l_n$ and $\bar{l}$ are unifiable.

First Order Challenge: Post-Deletion Check



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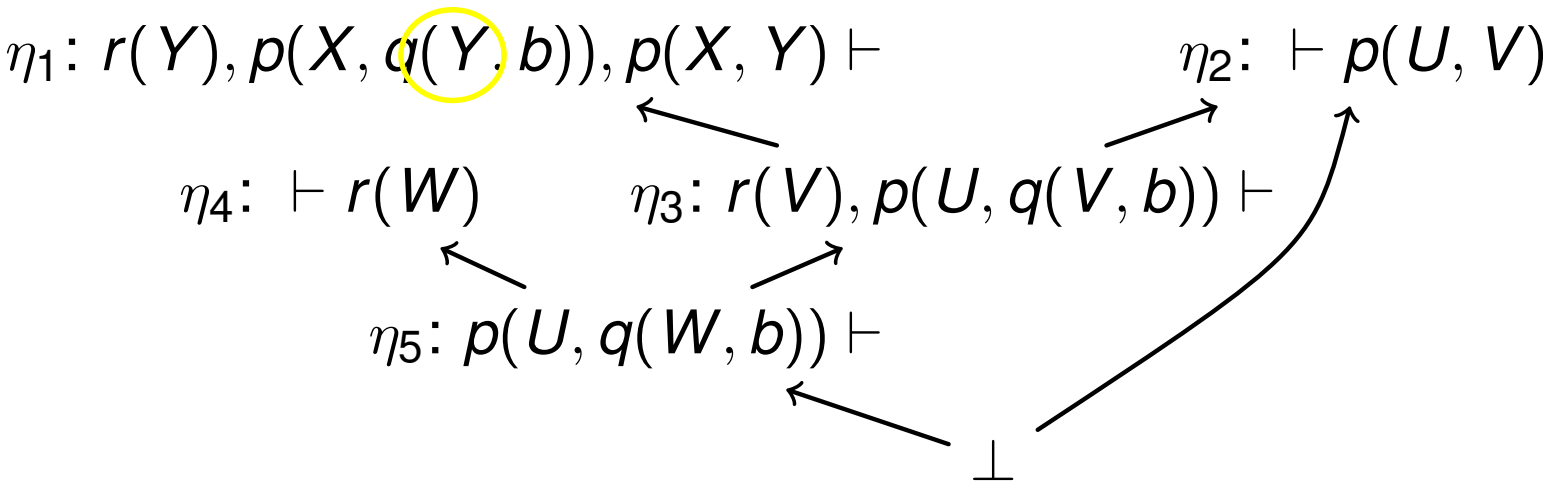

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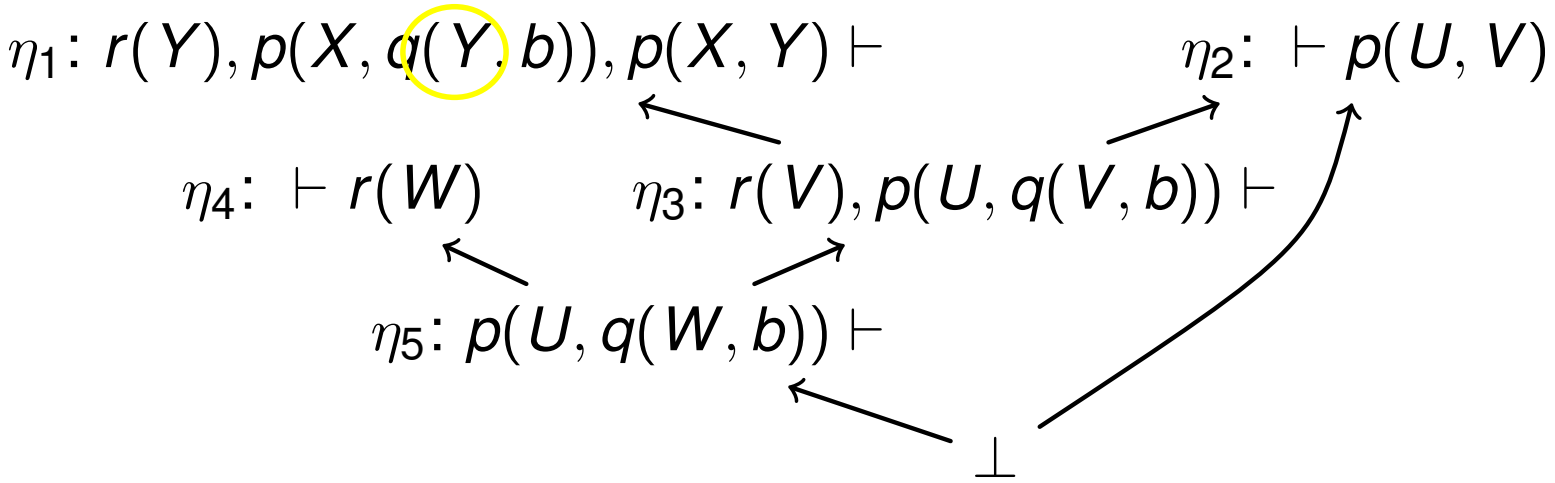

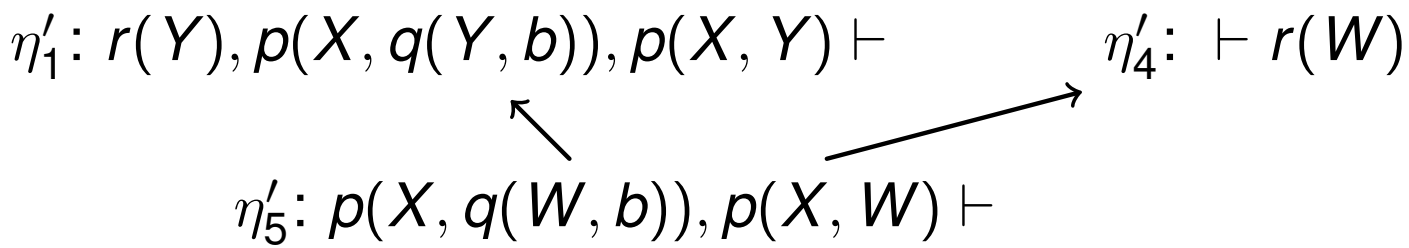

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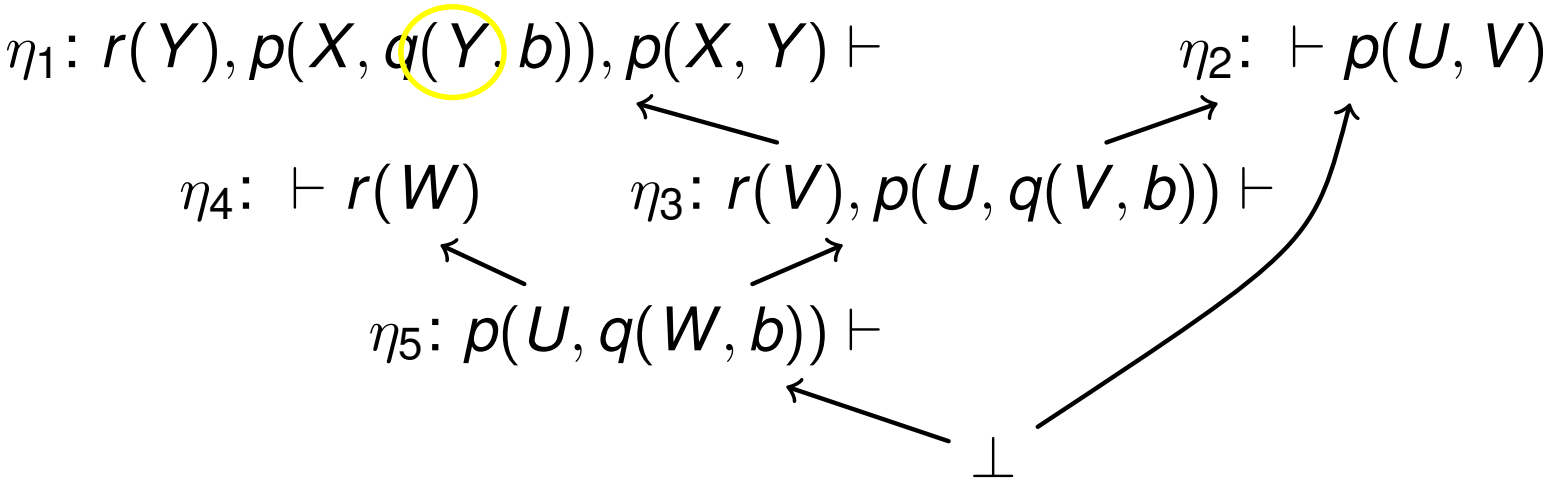

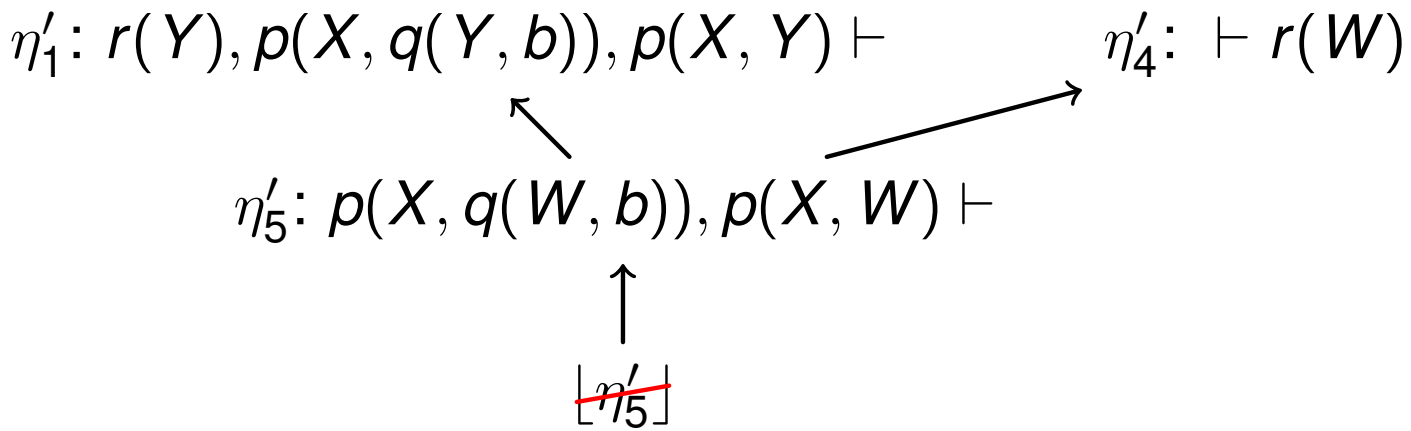

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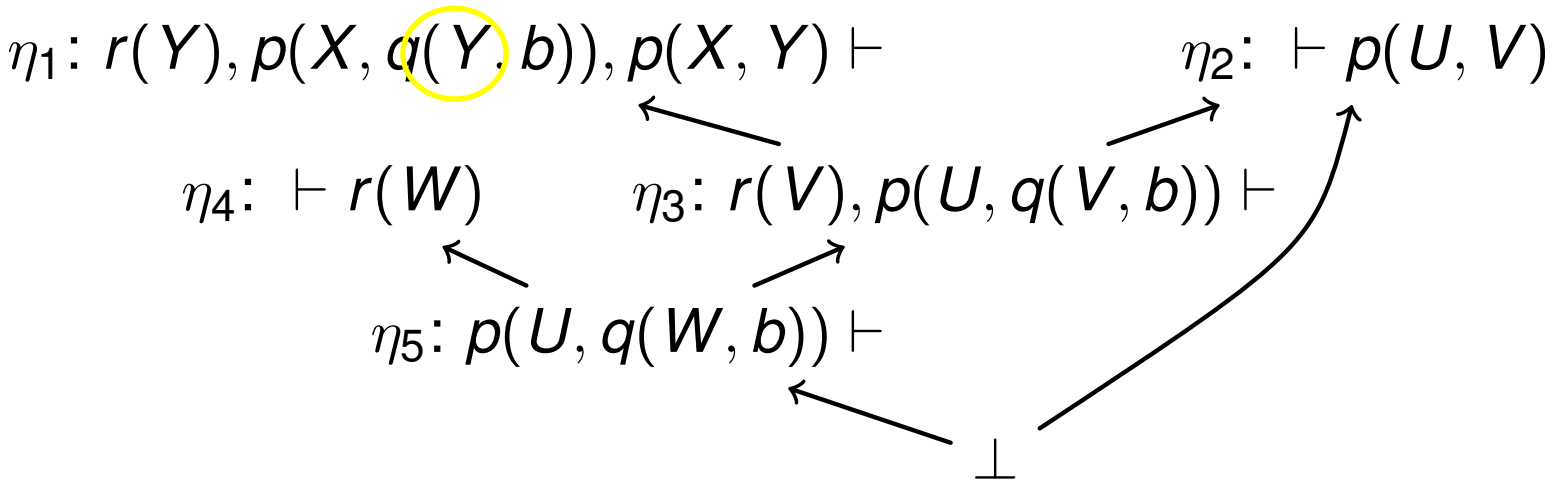

Definition (Post-Deletion Property)

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First Order Lower Units Challenges

- Deletion changes literals
- Unit collection depends on if contraction is possible after propagation down the proof

Deletion of units require knowledge of proof after deletion, and deletion depends on what will be lowered.

- $O(n^2)$ solution to have full knowledge
- Difficult bookkeeping required for implementation

Greedy First Order Lower Units - A Quicker Alternative

- Ignore post-deletion satisfaction
- Focus on pre-deletion satisfaction
- Greedy contraction

Faster run-time (linear; one traversal)
Easier to implement

- Doesn't always compress (returns original proof sometimes)

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First Order Example

small, animated example

Experiment Setup

- Simple First Order Lower units implemented as part of Skeptik (in Scala)
- 308 real first-order proofs generated by SPASS from problems from TPTP Problem Library
- proofs *generated* on cluster at the University of Victoria
- proofs *compressed* on *this laptop*

Time to generate proofs: $\approx$ 40 minutes
Time to compress proofs: $\approx$ 5 seconds

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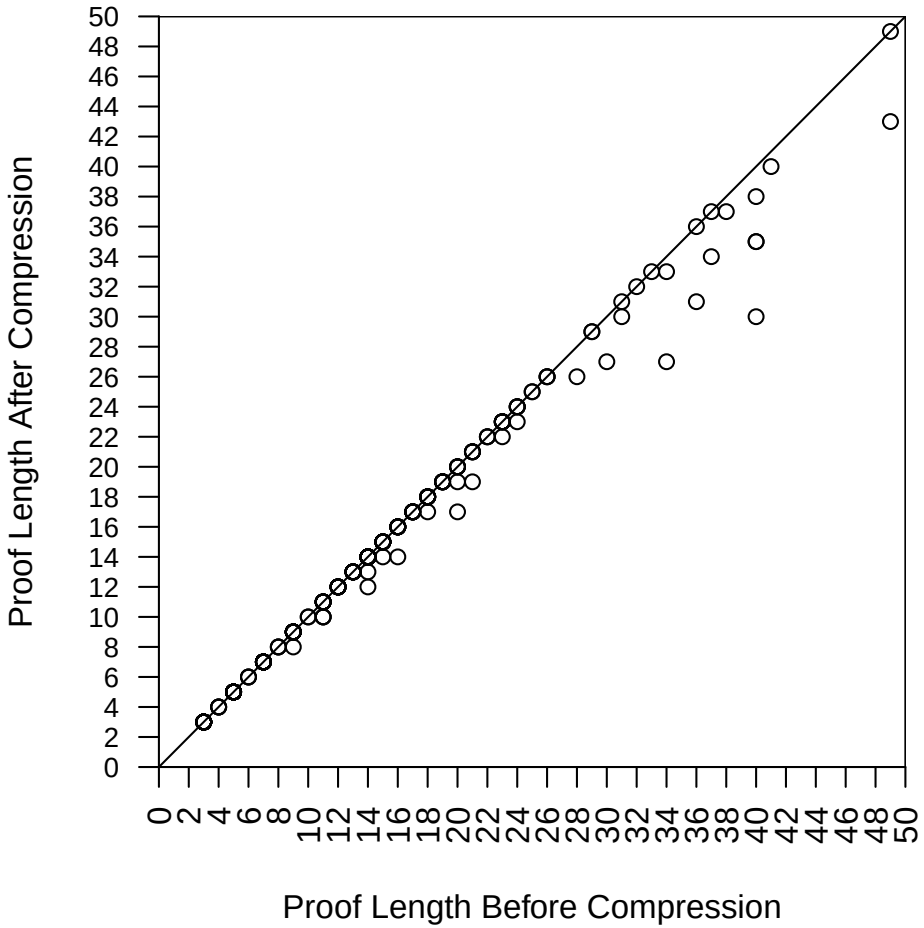
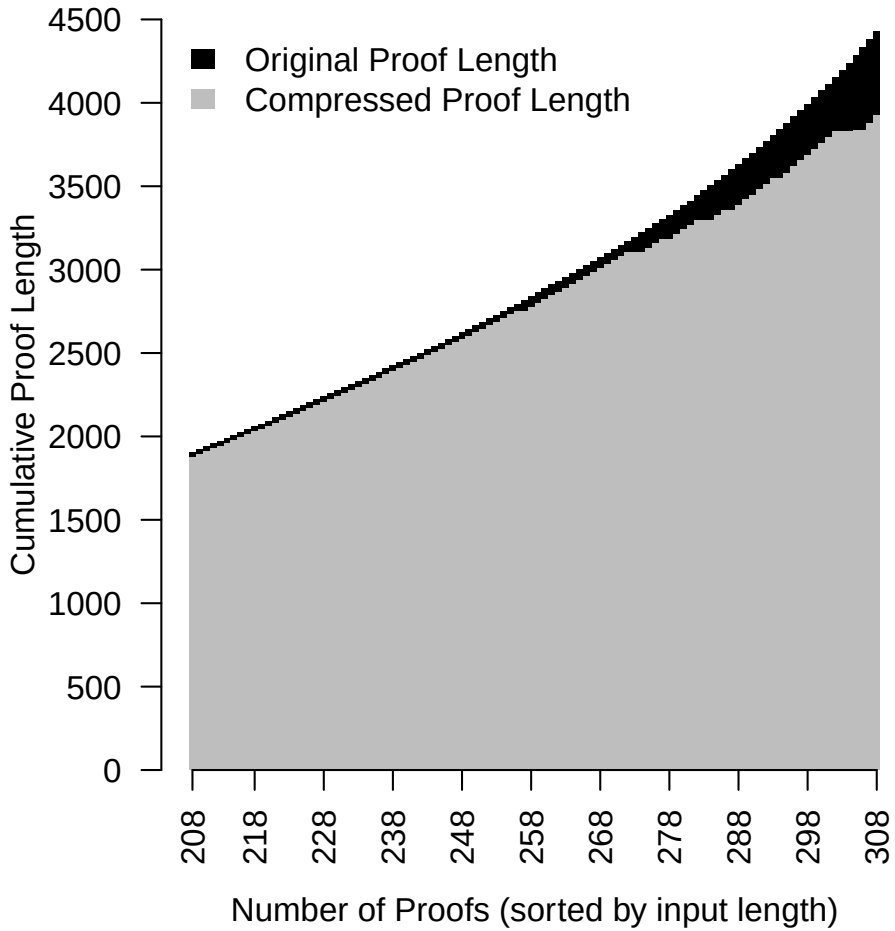
Results


Proof Length After Compression
Proof Length Before Compression
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
42
44
46
48
50


Original Proof Length
Compressed Proof Length
Cumulative Proof Length
Number of Proofs (sorted by input length)
0
500
1000
1500
2000
2500
3000
3500
4000
4500
208
218
228
238
248
258
268
278
288
298
308

Results

Higher compression in longer proofs: 13/18 proofs with length ≥ 30 nodes successfully compressed.

Total compression ratio 11.3%: 4429 vs. 3929 nodes.
18.4% for 100 longest proofs.

Only 14/308 proofs were returned untouched

Conclusion

- Simple First Order Lower Units is a quick algorithm for first order proof compression
- Future work:
  - Explore other proof compression algorithms, e.g. Recycle Pivots with Intersection
  - Explore ways of dealing with the post-deletion property quickly

Thank you for your attention.
Any questions?

- Source code: `https://github.com/jgorzny/Skeptik`
- Data: `http://www.math.uvic.ca/~jgorzny/data/`