Towards the Compression of First-Order Resolution Proofs by Lowering Unit Clauses

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Proof Compression Motivation

an accessible, good motivational example for proof compression

(Propositional) Proofs

Definition (Proof)

A directed acyclic graph $\langle V, E, \Gamma \rangle$, where

- V is a set of nodes
- E is a set of edges labeled by literals
- Γ (the proof clause) is inductively constructible using *axiom* and *resolution* nodes

Definition (Axiom)

A proof with a single node (so $E = \emptyset$)

(Propositional) Resolution

Definition (Resolution)

Given two proofs ψ_L and ψ_R with conclusions Γ_L and Γ_R with some literal l such that $\bar{l} \in \Gamma_L$ and $l \in \Gamma_R$, the resolution proof ψ of ψ_L and ψ_R on l, denoted $\psi = \psi_L \psi_R$ is such that:

- ψ's nodes are the union of the nodes of ψ_L and ψ_R, and a new root node
- there is an edge from $\rho(\psi)$ to $\rho(\psi_L)$ labeled with $\bar{l}$
- there is an edge from $\rho(\psi)$ to $\rho(\psi_R)$ labeled with l
- ψ's conclusion is $(\Gamma_L \setminus \{\bar{l}\}) \cup (\Gamma_R \setminus \{l\})$

A Propositional Proof

a small example to illustrate the definitions from the last two slides
the example should be redundant, so that we can show it again after the next slide in it's more minimal state
ideally minimized via LU, so that we can show the transformation later

Deletion

how deleting subproofs or edges in proofs affect them

Redundancy

types of redundancy we hope to remove, small examples (before/after proofs; not animated)

First-Order Proofs

Definition (First-Order Proof)

A directed acyclic graph $\langle V, E, \Gamma \rangle$, where

- V is a set of nodes
- E is a set of edges labeled by literals **and substitutions**
- Γ (the proof clause) is inductively constructible using *axiom*, ***(first order) resolution***, **and *contraction*** nodes

Axioms are unchanged

Substitutions and Unifiers

Definition (Substitution)

A mapping $\{X_1 \setminus t_1, X_2 \setminus t_2, \ldots\}$ from variables $X_1, X_2, \ldots$ to terms $t_1, t_2, \ldots$

Definition (Unifier)

A set of literals in a substitution that makes all literals in the set equal

First Order (Unifying) Resolution

Definition (First Order Resolution)

Given two proofs ψ_L and ψ_R with conclusions Γ_L and Γ_R with some literal l such that $l_L \in \Gamma_L$ and $l_R \in \Gamma_R$, and σ_L and σ_R are substitutions usch that $l_L\sigma_L = \overline{l_R}\sigma_R$, and the variables in $(\Gamma_L \setminus l_L)\sigma_L$ and $(\Gamma_R \setminus l_R)\sigma_R$ are disjoint, then the resolution proof ψ of ψ_L and ψ_R on l, denoted $\psi = \psi_L\psi_R$ is such that:

- ψ's nodes are the union of the nodes of ψ_L and ψ_R, and a new root node
- there is an edge from $\rho(\psi)$ to $\rho(\psi_L)$ labeled with l_L and σ_L
- there is an edge from $\rho(\psi)$ to $\rho(\psi_R)$ labeled with l_R and σ_R
- ψ's conclusion is $(\Gamma_L \setminus l_L)\sigma_L \cup (\Gamma_R \setminus l_R)\sigma_R$

Contraction

Definition (Contraction)

If ψ' is a proof and σ is a unifier of $\{l_1, \ldots, l_n\} \subset \Gamma'$, then a contraction ψ is a proof where

- ψ's nodes are the union of the nodes of ψ' and a new node v
- There is an edge from $\rho(\psi')$ to v labeled with $\{l_1, \ldots, l_n\}$ and σ
- The conclusion is $(\Gamma' \setminus \{l_1, \ldots, l_n\})\sigma \cup \{l\}$, where $l = l_k\sigma$ for $k \in \{1, \ldots, n\}$

LowerUnits

brief high level description; complexity
probably not pseudo-code

Propositional Example

quick, clear example of LU (animated), perhaps showing how one of the redundancies described before is fixed

First Order Challenges I

example 1 demonstrated

First Order Challenges II

example 2 demonstrated; definition of pre-deletion unification property

First Order Challenges III

example 2 demonstrated; definition of post-deletion unification property

First Order Lower Units Ideas/Principles

briefly mention all ideas, e.g. quadratic time naive approach to deal with both properties

Simple/Greedy First Order Lower Units

introduce simpler idea, make compromises explicit and list benefits
high level description
(probably not pseudo-code, but list # of traversals, complexity, etc)

First Order Example

small, animated example

Experiment Setup

proof sources, systems used, etc.

Results I

at least one or two of the more informative graphs

Results II

text summary of results (numbers, percentages, times, etc)

Conclusion

summary
future work (FORPI)
source link