

CENTER FOR MACHINE PERCEPTION



## Minimal Problem Solver Generator

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Acknowledge grants here. Use centering if the text is too short.

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Text of acknowledgements. . .

### **Abstract**

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### Resumé

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### **Abbreviations**

AHA! Some optional explanation before the list. Indentation can be set by the command \setlength{\AbbrvIndent}{5em}.

1D one dimension(al)

2D, 3D, ... two dimension(al), three dimension(al), two dimension(al), three di-

mension(al), two dimension(al), three dimension(al), two dimension(al),

three dimension(al),  $\dots$ 

AAM active appearance model
AI artificial intelligence
ASM active shape model
B-rep boundary representation
BBN Bayesian belief networks

### 1 Introduction

Here comes introduction.

### 2 Polynomial system solving

Firstly we review the state of the art algorithms for computing Gröbner basis. Better understanding of these algorithms helps us to more efficiently integrate them into polynomial solving algorithms based on Gröbner basis computation.

### 2.1 Buchberger's Algorithm

Buchberger's Algorithm was the first algorithm for computing Gröbener basis and it was invented by Bruno Buchberger.

#### 2.1.1 First implementation

The first and easy, but very inefficient, implementation of this algorithm says that we can extend a set F of polynomials to a Gröbner basis only by adding all nonzero remainders  $\overline{S(f_i, f_j)}^F$  of all pairs from F into F. The pseudocode of this algorithm can be found as Theorem 2 in Section 2, §7 in [1]. Gröbner basis computed by this algorithm are often bigger than necessary.

### 2.2 $F_4$ Algorithm

The  $F_4$  Algorithm [2] by Jean-Charles Faugère is an improved version of the Buchberger's Algorithm. The  $F_4$  replaces the classical polynomial reduction found in the Buchberger's Algorithm by the simultaneous reduction of several polynomials. This reduction mechanism is achieved by a symbolic precomputation and by use of sparse linear algebra methods.  $F_4$  speeds up the reduction step by exchanging multiple polynomial divisions for row-reduction of single matrix.

MAIN UPDATE

#### 2.2.1 Function Symbolic Preprocessing

In the first part of the function Symbolic Preprocessing we get set L of tuples containing monomial and polynomial. These tuples were made from selected pairs. Then are these tuples simplified by function Simplify and after multiplying polynomials with corresponding monomials are results put into the set F.

After that the function goes through all monomials in the set F and for each monomial m looks for some polynomial f from G (future Gröbner basis) such  $m = m' \times \mathrm{LM}(f)$  where m' is a some monomial. Found polynomial f and monomial m' are after simplification multiplied and put into set F. The goal of this search is to have for each monomial in F some polynomial in F with the same leading monomial. This will ensure that after polynomial division (using linear algebra) all added polynomials will be reduced for G.

## **2.3** $F_5$ Algorithm

## 3 Automatic generator

- 3.1 Reimplementation
- 3.2 Multiple eliminations solver
- 3.3 Removing unnecessary polynomials
- 3.4 Matrix partitioning
- 3.5 F4 strategy

# 4 Experiments

## 5 Conclusion

### **Bibliography**

- [1] David Cox, John Little, and Donald O'Shea. *Ideals, Varieties, and Algorithms:* An Introduction to Computational Algebraic Geometry and Commutative Algebra. Undergraduate Texts in Mathematics. Springer, New York, USA, 2nd edition, 1997.
- [2] Jean-Charles Faugère. A new efficient algorithm for computing gröbner bases  $(f_4)$ . Journal of pure and applied algebra, 139(1-3):61-88, 7 1999. 4