



Bayesian LDA for categorical mixture model clustering.

Pedro Albuquerque
University of Brasília

Denis Ribeiro do Valle
University of Florida

Daijiang Li
University of Florida

Abstract

The goal of this paper is to describe the Bayesian LDA model for fuzzy clustering based on different types of data (i.e., Multinomial, Bernoulli, and Binomial entries) and provide some examples of the use of this model in R. These types of data frequently emerge in fields like as disparate as ecology, remote sensing, marketing, and finance and others fields and, as a result, we believe this package will be of broad interest for pattern recognition, especially fuzzy clustering for categorical data.

Keywords: LDA, fuzzy clustering.

1. Introduction.

The Latent Dirichlet Allocation model (LDA), first proposed by Blei, Ng, and Jordan (2003), has been extensively used for text-mining in multiple fields. Tsai (2011) used LDA to study the tag pattern representation in blogs to construct clusters of tags that represents the most common topics. Lee et al. (2010) compared LDA against three other text mining methods that are frequently used: latent semantic analysis, probabilistic latent semantic analysis and the correlated topic model. The authors described the limitations of each method, asserting that the LDA limitations were essentially two: the method does not consider relationships between topics and cannot allocate a word to multiple topics. However, beside these limitations the LDA continues to be used in multiple disciplines. For instance, Griffiths and Steyvers (2004) used LDA to identify the main scientific topics in a large corpus of Proceedings of the National Academy of Science (PNAS) articles. In conservation biology, LDA has been used to identify research gaps in the conservation literature (Westgate et al. 2015). LDA has also been proposed as a promising method for the automatic annotation of remote sensing imagery (Lienou, Maître, and Datcu 2010). In marketing, LDA has been used to extract information from product reviews across 15 firms in five markets over four years, enabling the identification

of the most important latent dimensions of consumer decision making in each studied market (Tirunillai and Tellis 2014). In finance, a stock market analysis system based on LDA is used to analyze financial news items together with market data to identify and characterize major events that impact the market. This system is then used to predict whether the stock market will fall or rise, based on news items identified by LDA (Mahajan, Dey, and Haque 2008).

Despite its success in text mining across multiple fields, LDA is a model that need not be restricted to text-mining. More specifically, LDA can be viewed as a mixture model since each element in the sample can belong to more than one cluster (or state) simultaneously. There are a few examples of LDA being used for other purposes than text-mining. For instance, a modified version of LDA has been extensively used on genetic data to identify populations and admixture probabilities of individuals (Pritchard, Stephens, and Donnelly 2000). Similarly, LDA has been used in Ecology to identify plant communities from tree data for the eastern United States and from a tropical forest chronosequence (Valle et al. 2014).

The aim of this paper is to describe a Bayesian LDA model for mixture model clustering based on different types of data (i.e., multinomial, Bernoulli and Binomial), illustrating its use in a diverse set of examples. The innovative features of this model are that it generalizes LDA for other types of categorical data that commonly appear in Life and Social Science and it enables the selection of the optimal number of clusters based on a truncated stick-breaking prior approach which regularizes model results. Finally, we provide a package to estimate the quantities of interest.

This paper is organized as follows. Section 2 describes the mathematical formulation for the Bayesian LDA model for fuzzy clustering analysis and section 3 shows how the model was implemented in R. Sections 4 and 5 present examples of the use of the package and the conclusions, respectively.

2. Methods.

In the Bayesian LDA model for fuzzy clustering we postulate that exists a latent state for which each element can be allocated (not exclusively) in one cluster. Specifically, consider a latent matrix \mathbf{Z} with dimension equals to $L \times C$ where each row represent a sampling unit ($l = 1, \dots, L$) and each column a possible state or cluster ($c = 1, \dots, C$). The Data Generating Process postulated for this latent matrix is given by:

$$Z_{lc} \sim \text{Multinomial}(n_l, \boldsymbol{\theta}_l) \quad (1)$$

where n_l is total number of elements drawn for location l and $\boldsymbol{\theta}_l = (\theta_{l1}, \dots, \theta_{lC})$ is a vector of parameters representing the probability of allocation in each cluster. Following Occam's razor, we intend to create the least number of clusters as possible, which is achieved by assuming a truncated stick-breaking prior:

$$\theta_{lc^*} = V_{lc^*} \prod_{c < c^*}^C (1 - V_c) \quad (2)$$

where $V_{lc} \sim \text{Beta}(1, \alpha)$ for $c = 1, \dots, C - 1$ and $V_{lC} = 1$ by definition. This truncated stick-breaking prior will force the elements to be aggregated in the minimum number of clusters, given that θ_{lc^*} is stochastically exponentially decreasing.

In the second hierarchical level, we consider a matrix \mathbf{Y} with dimension equals to $L \times S$ where each row represents a sampling unit (e.g., locations, firms, individuals, plots) and each column a variable that describes these elements. In the Bayesian LDA model for fuzzy clustering, after integrating over the latent vector \mathbf{Z}_l , Y_{ls} can follow one of these distributions:

$$\begin{cases} \mathbf{Y}_l \sim \text{Multinomial}(n_l, \boldsymbol{\theta}_l^t \Phi) \\ Y_{ls} \sim \text{Bernoulli}(\boldsymbol{\theta}_l^t \boldsymbol{\phi}_s) \\ Y_{ls} \sim \text{Binomial}(n_{ls}, \boldsymbol{\theta}_l^t \boldsymbol{\phi}_s) \end{cases} \quad (3)$$

for $l = 1, \dots, L$ and $s = 1, \dots, S$ possible states. Y_{ls} represents a random variable, n_l is the total number of elements in sampling unit l and variable s . In these models, $\boldsymbol{\phi}_s = (\phi_{1s}, \dots, \phi_{Cs})$ is a vector of parameters, while:

$$\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \dots & \phi_{1S} \\ \phi_{21} & \phi_{22} & \dots & \phi_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{C1} & \phi_{C2} & \dots & \phi_{CS} \end{bmatrix}$$

is a $C \times S$ matrix of parameters.

In the last step, we specify the priors for ϕ_{cs} . For the multinomial model, we adopt a Dirichlet prior (i.e. $\boldsymbol{\phi}_c \sim \text{Dirichlet}(\boldsymbol{\beta})$ where $\boldsymbol{\beta} = (\beta_1, \dots, \beta_S)$ is the hyperparameter vector). For the Bernoulli and Binomial representations, we assume that ϕ_{cs} comes from a Beta distribution, (i.e., $\phi_{cs} \sim \text{Beta}(\alpha_0, \alpha_1)$).

These models are fit using Gibbs Sampling where parameters draws are iteratively made from each full conditional distribution. From a conceptual perspective, all of these models assume the following matrix decomposition:

$$\mathbb{E}[\mathbf{Y}_{L \times S}] = \mathbf{K} \circ [\boldsymbol{\Theta}_{L \times C} \Phi_{C \times S}] \quad (4)$$

where \mathbf{K} is a matrix of constants and \circ is the Hadamard product. Sparseness is ensured by forcing large c in the $\boldsymbol{\Theta}_{L \times C}$ matrix to be close to zero. For the multinomial model, the \mathbf{K} matrix contains the total number of elements in each row whereas for the Bernoulli model, this matrix is equal to the identity matrix. Finally, for the Binomial model, the \mathbf{K} matrix has the total number of trials of each binomial distribution (i.e., n_{ls}). Although there are many ways matrices can be decomposed, the key characteristic of this particular form of matrix decomposition is that each row of \mathbf{K} is comprised of probabilities that sum to one. As a result, one can interpret $\Phi_{C \times S}$ as the matrix that contain the “pure” features of the data, which are then mixed by the matrix $\boldsymbol{\Theta}_{L \times C}$ and multiplied by \mathbf{K} to generate the expected data.

2.1. Full Conditional Distributions - FCD.

Bernoulli model.

The probability of community membership status Z_{ls} is given by:

$$p(Z_{lc} = c^* | Y_{ls} = y_{ls}, \phi_{c^*s}, \theta_l) = \frac{\theta_{lc^*} \phi_{c^*s}^{y_{ls}} (1 - \phi_{c^*s})^{1-y_{ls}}}{\sum_{c=1}^C \theta_{lc} \phi_{cs}^{y_{ls}} (1 - \phi_{cs})^{1-y_{ls}}} \quad (5)$$

Therefore, Z_{ls} can be drawn from a categorical distribution. The FCD for ϕ_{cs} is given by:

$$p(V_{lc^*} | \mathbf{Y}) = \text{Beta}(N_{lc^*} + 1, N_{l(c>c^*)} + \gamma)$$

where N_{lc^*} is the total number of elements in location l classified into cluster c^* , and $N_{l(c>c^*)}$ is the total number of elements in location l classified in clusters larger than c^* . These quantities are given by $N_{lc^*} = \sum_{s=1}^S \mathbb{1}(z_{ls} = c^*)$ and $N_{l(c>c^*)} = \sum_{s=1}^S \sum_{c=c^*+1}^C \mathbb{1}(z_{ls} = c)$, respectively.

Binomial model.

For this model, we have n_{ls} elements for each sampling unit l and community z . The i -th element is denoted as Z_{ils} and its probability is similar to the one for the Bernoulli model:

$$p(Z_{lc} = c^* | Y_{ls} = y_{ls}, \phi_s, \theta_l) = \frac{\theta_{lc^*} \phi_{c^*s}^{x_{ils}} (1 - \phi_{c^*s})^{1-x_{ils}}}{\sum_{c=1}^C \theta_{lc} \phi_{cs}^{x_{ils}} (1 - \phi_{cs})^{1-x_{ils}}} \quad (6)$$

where x_{ils} are binary random variables such that $\sum_{i=1}^{n_{ls}} x_{ils} = y_{ls}$. Therefore, Z_{ils} can be drawn from a multinomial distribution. The FCD is given by:

$$p(\phi_{cs} | \mathbf{Z}, \mathbf{Y}) = \text{Beta} \left(\sum_{l=1}^L \sum_{i=1}^{n_{ls}} \mathbb{1}(x_{ils} = 1, z_{ils} = c) + \alpha_0, \sum_{l=1}^L \sum_{i=1}^{n_{ls}} \mathbb{1}(x_{ils} = 0, z_{ils} = c) + \alpha_1 \right) \quad (7)$$

Finally, the final FCD for V_{lc^*} is given by:

$$p(V_{lc^*} | Z_{lc^*}) = \text{Beta}(N_{lc^*} + 1, N_{l(c>c^*)} + \gamma) \quad (8)$$

where N_{lc^*} is the total number of elements in location l classified into cluster c^* and $N_{l(c>c^*)}$ is the total number of elements in location l classified in clusters larger than c^* . These quantities are given by $N_{lc^*} = \sum_{s=1}^S \sum_{i=1}^{n_{ls}} \mathbb{1}(z_{ils} = c^*)$ and $N_{l(c>c^*)} = \sum_{s=1}^S \sum_{i=1}^{n_{ls}} \sum_{c=c^*+1}^C \mathbb{1}(z_{ils} = c)$.

Multinomial model.

For the Multinomial case, if unit i in location l is associated with variable s (i.e., $x_{il} = s$ such that $y_{ls} = \sum_{i=1}^{n_l} \mathbb{1}(x_{il} = s)$), we have that:

$$p(Z_{lc} = c^* | \mathbf{Y}_{ls} = s, \phi_s, \theta_l) = \frac{\theta_{lc^*} \phi_{sc^*}}{(\theta_{1l} \phi_{s1} + \dots + \theta_{Cl} \phi_{sC})} \quad (9)$$

Therefore, Z_{il} can be sampled from a categorical distribution. Since we assumed a conjugate prior for ϕ_{c^*} with $c^* \in \{1, \dots, C\}$ the Full Conditional Distribution for this vector of parameters is a straight-forward Dirichlet distribution:

$$p(\phi_{c^*} | \mathbf{Y}) = \text{Dirichlet}([n_{c^*1} + \beta_1, \dots, n_{c^*S} + \beta_S]) \quad (10)$$

where n_{c^*s} is the total number of observations classified in cluster c^* in all locations for the s -th variable (i.e., $n_{c^*s} = \sum_{l=1}^L \sum_{i=1}^{n_l} \mathbb{1}(z_{il} = c^*, x_{il} = s)$).

Finally, the FCD for V_{lc^*} is given by:

$$p(V_{lc^*} | Z_{lc^*}) = \text{Beta}(N_{lc^*} + 1, N_{l(c>c^*)} + \gamma) \quad (11)$$

where N_{lc^*} is the total number of elements in observation l classified into cluster c^* and $N_{l(c>c^*)}$ is the total number of elements in observation l classified in clusters larger than c^* . These quantities are given by $N_{lc^*} = \sum_{i=1}^{n_l} \mathbb{1}(z_{il} = c^*)$ and $N_{l(c>c^*)} = \sum_{i=1}^{n_l} \sum_{c=c^*+1}^C \mathbb{1}(z_{il} = c)$.

3. The package.

The package is not the only one package that can be able to perform the Latent Dirichlet Allocation model. We found four packages that propose model a Latent Dirichlet Allocation structure in R. Hornik and Grün (2011) developed the **topicmodels** package for which there are two LDA implementations, one using the variational inference (as described in Blei, Ng, and Jordan (2003)) and the other implementation using Gibbs Sampling based on Phan and Nguyen (2013). Similarly, Jones (2016) proposed the **textmineR** which has a Gibbs Sampling method to estimate the topics in a corpus structure.

Chang (2012) developed the **lda** package which includes the mixed-membership stochastic blockmodel (Airoldi et al. 2008), supervised Latent Dirichlet Allocation - sLDA (McAuliffe and Blei 2008) and Correspondence-Latent Dirichlet Allocation - corrLDA (Blei and Jordan 2003).

More recently, Roberts, Stewart, and Tingley (2014) created the **stm** which has some unsupervised functions to determine the optimal number of clusters. This unsupervised method is based on likelihood metrics obtained by the EM Algorithm, which are then used for a backward model selection approach.

None of these packages adopt the truncated stick-breaking prior and none of them use others distributions besides the Multinomial outcome for the dependent variable. Thus, our package complements current LDA approaches already available in R.

4. Examples.

In this section we present some applications for Latent Dirichlet Allocation in marketing, finance and ecology.

4.1. Marketing.

The first application considers the classical LDA for a Multinomial entry in the field of Marketing. More specifically, we are interested in characterizing firms based on their consumers' complaints.

It is well known that attracting a new customer is often considerably more costly than keeping current customer (Kotler and Armstrong 2006). For this reason, firms can better retain their customers if they pay careful attention to their consumers' complaints and work to solve them in a satisfactory way.

The data used is from the 2015 **Consumer Complaint Database** which are complaints received for the **Bureau of Consumer Financial Protection** in US about financial products and services. Specifically in this example we work with only credit card complaint's and it contains information on the number of complaints for each firm ($L = 226$), categorized according to the specific type of issue ($S = 30$). Examples of issues include: Billing disputes, Identity theft / Fraud, Unsolicited issuance of credit card and so on. Each sampling unit in this case represents a firm and each variable the total number of complaints'complaints associated with each issue.

The characterization of firms provided by our analysis can be useful to reveal communalities and differences across different firms. This can then be used by managers to identify and potentially adopt the solutions that are employed by other firms to deal with these issues.

To use the **Rlda** package for the Multinomial entry is necessary to create a matrix where each cell represents the total number of cases observed for each variable and sampling unit.

```
library(Rlda)
#Read the Complaints data
data(complaints)
#Create the abundance matrix
library(reshape2)
mat1<-dcast(complaints[,c("Company","Issue")],
            Company~Issue, length,
            value.var="Issue")
#Create the rowname
rownames(mat1)<-mat1[,1]
#Remove the ID variable
mat1<-mat1[,-1]
```

To use the `lda_multinomial` method we need to specify some arguments:

```
#Set seed
set.seed(9292)
#Hyperparameters for each prior distribution
beta<-rep(1,ncol(mat1))
gamma<-0.01
#Execute the LDA for the Multinomial entry
res<-lda_multinomial(mat1,30,beta,gamma,1000,TRUE,FALSE)
```

In the above code we defined the maximum number of clusters as 30 and 1000 Gibbs Sampling iterations with the likelihood computed with priors and not showing the progress bar.

We can visually evaluate the convergence analysing the Figure 1:

```
#Get the logLikelihood
ll<-res[["logLikelihood"]]
#Plot the log-likelihood
plot(ll,type="l",xlab="Iterations",
      ylab="Log-likelihood")
```

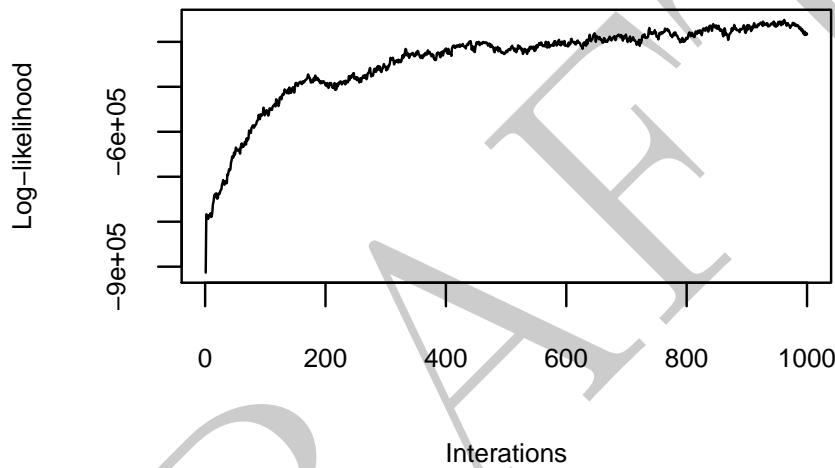


Figure 1: Log-likelihood iterations.

The Bayesian estimate is given by the **Theta** and **Phi** matrix. Each line for those objects represent a Gibbs iteration, in this way we can find estimates based on the mean of the matrices only for those iterations after the *burn-in phase*.

```
#Get the Theta Estimate
Theta<-res[["Theta"]]
#Burnout
Theta<-colMeans(Theta[300:1000,])
#Create the matrix
Theta<-matrix(Theta,nrow = nrow(mat1),ncol=30)
#Rownames
rownames(Theta)<-rownames(mat1)
```

The **Theta** has a sparse structure since our truncated stick-breaking prior tends reduce the total number dominant clusters. We can think that each probability estimated in each cell represent the probability of the l -th firm be allocated in cluster s , in this case $s = 1, \dots, S$.

To describe each cluster, we can look the **Phi** matrix:

```

#Get the Phi Estimate
Phi<-res[["Phi"]]
#Burnout
Phi<-colMeans(Phi[300:1000,])
#Create the matrix
Phi<-matrix(Phi,nrow =30,ncol=ncol(mat1))
#Colnames
colnames(Phi)<-colnames(mat1)
#Rownames
rownames(Phi)<-paste0("Cluster ",seq(1,30))
#Get the most likely issues
ids<-which(Phi > 0.2, arr.ind = TRUE)

```

The most likely issues and its clusters are summarized as follow:

Cluster	Issue
7	Advertising and marketing
7	Rewards
17	Closing/Cancelling account
26	Unsolicited issuance of credit card

Table 1: Clusters description.

Based on that information we can now analyze the firms with highest probability to belong these clusters:

Firm	Cluster	Probability
U.S. Bancorp	Cluster 7	0.5239
Amex	Cluster 7	0.1878
Barclays PLC	Cluster 7	0.0974
Discover	Cluster 7	0.0511
PayPal Holdings, Inc.	Cluster 7	0.0153
U.S. Bancorp	Cluster 17	0.0000
Amex	Cluster 17	0.0000
Barclays PLC	Cluster 17	0.0000
Discover	Cluster 17	0.2957
PayPal Holdings, Inc.	Cluster 17	0.0088
U.S. Bancorp	Cluster 26	0.0000
Amex	Cluster 26	0.0000
Barclays PLC	Cluster 26	0.0000
Discover	Cluster 26	0.0000
PayPal Holdings, Inc.	Cluster 26	0.3958

Table 2: Probability of belonging for each company.

The results are consistent with the type of product studied, in this case *Credit card*. It is interesting to note that the Cluster 7 is mostly represented by financial services corporation such as U.S. Bancorp and Amex. This cluster is mostly represented by *Advertising and marketing* and *Rewards* issues which may represent a larger cluster of complaints, for instance *Deceptive advertising complaints*.

Managers of those companies can use this information to create, for example, a specific department to solve this disputes and/or clarify their consumers about the *Advertising* and *Rewards* rules and conditions, aiming to avoid misunderstood about the programs.

Cluster 17 on the other hand is mostly represented by firms with *Closing/Cancelling account* complaints and its most representative firm is Discover which is also a financial services corporation but, differently of the firms with large participation in Cluster 7, this firm has complaints about *Closing/Cancelling account*. For these companies that belong to Cluster 17 they could focus their attention in provide easy steps to *Closing/Cancelling account* in a way to avoid more complaints and improve their Customer Relationship Management (CRM).

The final cluster is represented by financial services corporation that act in most part on web, and its representative firm is PayPal Holdings, Inc. The most likely issue associated with these companies that belongs to Cluster 26 is *Unsolicited issuance of credit card*.

These results can be explained by a change in the PayPal operations. The complaints are related with the type of account provided by PayPal which were converted from standard PayPal account to a revolving credit account and some consumers claims to be unaware of this change, and again, a good communication system between the company and its consumers could avoid and easily solve complaints about those issues.

4.2. Finance.

In Financial Market is usually common analyze firms based on their liquidity, i.e., liquidity of a firm refers to its ability to meet short-term obligations using firm's assets can be quickly converted to cash (Audretsch and Elston 2002).

In this example we studied how firms can be clustered based on two main factor: The firms' free float¹ and the negotiated volume².

We can approximate the volume random variable for each firm in each day as Binomial trial, since the volume should be less than free float³ for the same firm and day.

In this example we worked with all daily transactions for 46 firms of the S&P500 Index in 2015 obtained by the Reuters Terminal, the data was treated to represent the expected value of daily transactions for a free float equal 100:

```
library(Rlda)
#Read the SP500 data
data(sp500)
#Create size
spSize<-as.data.frame(matrix(100,
                             ncol=ncol(sp500),
                             nrow=nrow(sp500)))
```

In this cases since we are postulated a Binomial trial we should use the `lda_binomial`, for which we need to specify the follow arguments:

```
#Set seed
set.seed(5874)
#Hyperparameters for each prior distribution
gamma <-0.01
```

¹Free float is the number of outstanding shares that are available for trading by the public.

²Volume is number of shares that traded on the trade date.

³In fact this is just an approximation, since one share can be negotiated multiple times in the same day.

```

alpha0<-0.01
alpha1<-0.01
#Execute the LDA for the Binomial entry
res<-lda_binomial(sp500,spSize,10,alpha0,alpha1,gamma,5000,TRUE,FALSE)

```

The firms can be represented by the Phi matrix which presents the probability of belonging (not exclusively) by cluster.

```

#Get the Phi Estimate
Phi<-res[["Phi"]]
#Bayes Estimation
Phi<-colMeans(Phi)
#Create the matrix
Phi<-matrix(Phi,nrow =10,ncol=ncol(sp500))
#Colnames
colnames(Phi)<-colnames(sp500)
#Rownames
rownames(Phi)<-paste0("C",seq(1,10))

```

Hence, one way to summarize the results is showing which cluster has the highest probability to appear for each possible firm, in this case only for clusters with probability greater than 50%:

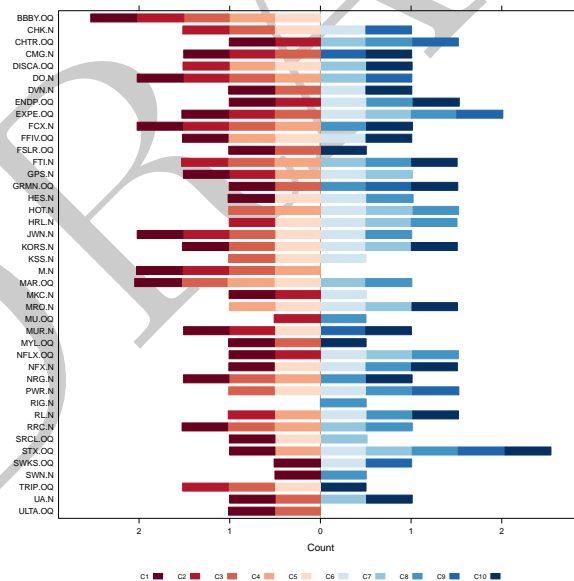


Figure 2: Accumulated probability of belonging.

Financial investors can, for example, using the information presented in Figure 2 to construct portfolios based on these clusters, aiming to diversify their assets or studied the similar behavior associated with the liquidity clusters.

This is another way to evaluate the results of the package's functions. In the first example we presented the results using only tables of frequency, now in this example we present the

results in a more visually way, where each cluster is represented by a color and the size of the bar is associated with the probability of belonging.

We can also note that for most of the firms, the probability of belonging is very similar and distributed across the clusters, but this not the general case. Usually, when data can be partitioned in a hard way, some clusters will have more probability than others.

4.3. Ecology.

One of the most challenge situation in study ecological data is the fact the in some applications, it is not possible to observe the total number of species in each sampled plot. However, it is possible to know if some is species is presented (i.e. overserved) or absente (i.e. not observed) (Pearce and Boyce 2006).

For this kind of data, we can use the the `lda_bernoulli` function. Specifically, in this example we used `data("SPDATA")` from **PresenceAbsence** package which is referred to 13 species that could be observed (value equal 1) and not observed or absent (value equal 0) at 386 forested locations (Moisen et al. 2006).

```
library(PresenceAbsence)
#Read SPDATA data
data(SPDATA)
PresAbs<-SPDATA[,1:2]
#Location
library(data.table)
PresAbs <- data.table(PresAbs)
PresAbs[, Location := sequence(.N), by = c("SPECIES")]
#Create the binary matrix
library(reshape2)
mat1<-dcast(PresAbs,
             Location~SPECIES,
             value.var="OBSERVED")
#Remove the Location variable
matPres<-as.data.frame(mat1[,-1])
```

In this example we can postulated a Bernouli trial for each Specie and Location, and we are able to use the `lda_bernoulli` function, for which we need to specify the follow arguments:

```
#Set seed
set.seed(9842)
#Hyperparameters for each prior distribution
gamma <-0.01
alpha0<-0.01
alpha1<-0.01
#Execute the LDA for the Binomial entry
res<-lda_bernoulli(matPres,10,
                   alpha0,alpha1,gamma,
                   5000,TRUE,FALSE)
```

We can visually evaluate the cluster distribution across species, after the *burn-in phase* in Figure 3:

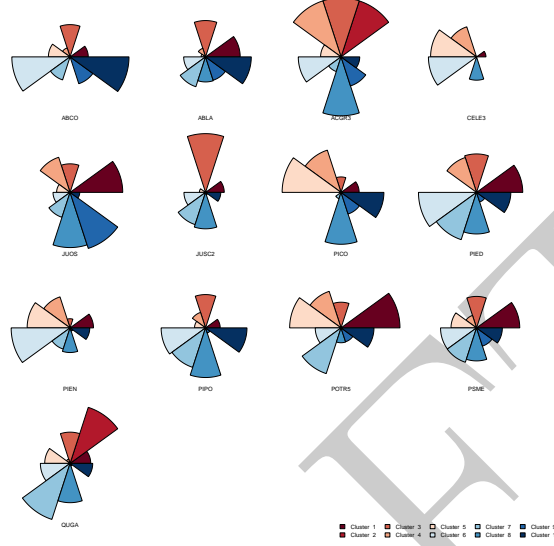


Figure 3: Cluster distribution across species.

In this type of graph each slice size is proportional to the probability of belonging, and it is possible to note that some species of trees are more or less associated with some clusters. For example, QUGA specie is more associated with Clusters 7 and 2 as well ACGR3 specie.

5. Conclusion.

The goal of this paper was describe the Bayesian LDA model for fuzzy clustering based on different types of data, specifically we demonstrated how to use the model for Multinomial entry in the Marketing example, Binomial trial using the financial dataset and Bernoulli trial using ecological data.

Also, we present the results in three different ways showing the flexibility to use the results and represent the information numerically and/or visually which is important when we faced large datasets and a great number of possible clusters.

One of the main properties of the model presented here is the fact that this model adopts the truncated stick-breaking prior which enables the selection of the optimal number of clusters which regularizes model results. The next step in the development of the model presented here is the possibility to work with explanatory variables in the same structured described in this article, which can be useful not to model the cluster but also explain how the clusters are created.

Appendix.

Here in the Appendix we will show how the Full Conditional Distributions were constructed.

Consider initially the Full Conditional Distributions for $Z_{lc} = c^*$ in all three cases the kernel of the distribution derived is the same:

$$\begin{aligned} p(Z_{lc} = c^* | Y_{ls} = s, \zeta_{c^*}, \theta_l) &= \kappa \left[\phi_{c^*1}^{\mathbb{1}(Y_{l1}=s, Z_{lc}=c^*)} \times \dots \times \phi_{c^*S}^{\mathbb{1}(Y_{lS}=s, Z_{lc}=c^*)} \right] \\ &\quad \times \left[\theta_{l1}^{\mathbb{1}(Z_{l1}=c^*)} \times \dots \times \theta_{lC}^{\mathbb{1}(Z_{lC}=c^*)} \right] \\ &= \kappa \zeta_{c^*s} \theta_{lc^*} \end{aligned}$$

where $\mathbb{1}(Y_{l1} = s, Z_{lc} = c^*)$ is the indicator function which assumes one only for the l -th observation, s -th variable and has been identified as belong to cluster c^* . In a similar way, $\mathbb{1}(Z_{lC} = c^*)$ assumes one only for observations classified as belong to cluster c^* .

Since Z_{lc} is a categorical random variable with support in $\mathcal{Z} = (1, 2, \dots, C)$ the sum of all probabilities for all elements must one, then the constant κ is given by:

$$\kappa = (\theta_{l1}\zeta_{1s} + \dots + \theta_{lC}\zeta_{Cs})^{-1}$$

Then, each category for Z_{lc} can be draw from a categorical distribution with probabilities equal to $\kappa \zeta_{c^*s} \theta_{lc^*}$ with $c^* \in \mathcal{Z}$, where $\zeta_{c^*s} = \phi_{sc^*}$ in the multinomial case and $\zeta_{c^*s} = \phi_{c^*s}^{y_{ls}}(1 - \phi_{c^*s})^{1-y_{ls}}$ and $\zeta_{c^*s} = \phi_{c^*s}^{x_{ils}}(1 - \phi_{c^*s})^{1-x_{ils}}$ for Bernoulli and Binomial cases respectively.

For the Bernoulli and Binomial cases we have, respectively.:

$$p(Z_{lc} = c^* | Y_{ls} = s, \phi_{c^*}, \theta_l) \propto \theta_{lc^*} \phi_{c^*s}^{y_{ls}} (1 - \phi_{c^*s})^{1-y_{ls}}$$

$$p(Z_{lc} = c^* | Y_{ls} = s, \phi_{c^*}, \theta_l) \propto \theta_{lc^*} \phi_{c^*s}^{x_{ils}} (1 - \phi_{c^*s})^{1-x_{ils}}$$

where each cluster label can be draw from a categorical distribution with probabilities equal to (Multinomial, Bernoulli and Binomial):

$$p(Z_{lc} = c^* | Y_{ls} = s, \phi_{c^*}, \theta_l) = \frac{\theta_{lc^*} \phi_{c^*s}}{\sum_{c=1}^C \theta_{lc} \phi_{cs}}$$

$$p(Z_{lc} = c^* | Y_{ls} = s, \phi_{c^*}, \theta_l) = \frac{\theta_{lc^*} \phi_{c^*s}^{y_{ls}} (1 - \phi_{c^*s})^{1-y_{ls}}}{\sum_{c=1}^C \theta_{lc} \phi_{cs}^{y_{ls}} (1 - \phi_{cs})^{1-y_{ls}}}$$

$$p(Z_{lc} = c^* | Y_{ls} = s, \phi_{c^*}, \theta_l) = \frac{\theta_{lc^*} \phi_{c^*s}^{x_{ils}} (1 - \phi_{c^*s})^{1-x_{ils}}}{\sum_{c=1}^C \theta_{lc} \phi_{cs}^{x_{ils}} (1 - \phi_{cs})^{1-x_{ils}}}$$

For the ϕ_{c^*} its Full Conditional is given by:

$$\begin{aligned}
p(\phi_{c^*} | \mathbf{Z}_{\cdot c^*}, \{\mathbf{Y}_l | Z_{lc} = c^*\}, \beta) &\propto \prod_{l=1}^L \phi_{c^*1}^{\mathbb{1}(Y_{l1}=1, Z_{lc}=c^*)} \times \dots \times \phi_{c^*S}^{\mathbb{1}(Y_{lS}=S, Z_{lc}=c^*)} \\
&\times \phi_{c^*1}^{\beta_1-1} \times \dots \times \phi_{c^*S}^{\beta_S-1} \\
&\propto \prod_{s=1}^S \phi_{c^*s}^{n_{c^*s} + \beta_s - 1}
\end{aligned}$$

where n_{c^*s} is the total number of elements classified in community c^* and independent variable s . In the Binomial or Bernoulli case the Full Conditional is given by:

$$\begin{aligned}
p(\phi_{c^*s} | \mathbf{Z}_{\cdot c^*}, \{\mathbf{Y}_l | Z_{lc} = c^*\}, \beta) &\propto \prod_{l=1}^L \phi_{c^*s}^{\mathbb{1}(Y_{ls}=s, Z_{lc}=c^*)} \phi_{c^*s}^{\alpha_0-1} (1 - \phi_{c^*s})^{\alpha_1-1} \\
&\propto \phi_{c^*s}^{n_{c^*s} + \alpha_0 - 1} (1 - \phi_{c^*s})^{n_{-c^*s} + \alpha_1 - 1}
\end{aligned}$$

where n_{-c^*s} is the total number of elements that **not** belong to either community c^* and independent variable s .

The last Full Conditional described here is the Full Conditional for $p(V_{lc} | Z_{lc})$, in this case we have:

$$\begin{aligned}
p(V_{lc} | Z_{lc}) &\propto \text{Binom}(n_{lc} | N = N_{lc} + N_{l(c>c^*)}, V_{lc}) \text{Beta}(V_{lc} | 1, \gamma) \\
&\propto \text{Beta}(1 + N_{lc}, \gamma + N_{l(c>c^*)}) \\
&= \frac{\Gamma(N_{lc}+1) \Gamma(N_{l(c>c^*)} + \gamma)}{\Gamma(N_{lc}+1) \Gamma(N_{l(c>c^*)} + \gamma)} V_{lc}^{N_{lc}} (1 - V_{lc})^{N_{l(c>c^*)} + \gamma}
\end{aligned}$$

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Affiliation:

Pedro Albuquerque

University of Brasília

Faculdade de Economia, Administração e Contabilidade, Building A-2 - Office A1-54/7,
Brasília, DF 70910-900.

E-mail: pedroa@unb.br

URL: <http://pedrounb.blogspot.com/>

Denis Ribeiro do Valle

University of Florida

408 McCarty Hall C, PO Box 110339, Gainesville, FL 32611-0410.

E-mail: drvalle@ufl.edu

URL: <http://denisvalle.weebly.com/>

Daijiang Li

University of Florida

408 McCarty Hall C, PO Box 110339, Gainesville, FL 32611-0410.

E-mail: daijianglee@gmail.com