

Series and Sequences Exercises

A. Determine whether each geometric series converges or diverges. If it converges, find its sum; If not, say why.

1. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

4. $\sum_{n=0}^{\infty} (-1)^n \frac{5}{4^n}$

7. $\sum_{n=1}^{\infty} \frac{1}{n^2 + 5n + 6}$

2. $\sum_{k=0}^{\infty} \frac{2^{k+1}}{3^k}$

5. $\sum_{n=3}^{\infty} (-1)^n \frac{3}{2^n}$

3. $\sum_{k=10}^{\infty} \frac{3^{k-1}}{2^k}$

6. $\sum_{n=1}^{\infty} \left(\frac{1}{n^2 + n} + \frac{3^{2n}}{14^n} \right)$

B. True or False? Explain.

1. There exists an infinite geometric series with sum -4 and first term -1.

C. Determine whether each series converges or diverges.

1. $\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k}}$

4. $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$

8. $\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$

12. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

2. $\sum_{n=0}^{\infty} \frac{\sin^2(n)}{1 + n^2}$

5. $\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$

9. $\sum_{n=1}^{\infty} (\arctan(n) + n)$

13. $\sum_{n=2}^{\infty} \left(\frac{\sin(100)}{\pi} \right)^n$

3. $\sum_{k=0}^{\infty} \frac{k}{e^k}$

6. $\sum_{n=1}^{\infty} \frac{n}{n^3 + 4n - 3}$

10. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$

7. $\sum_{n=1}^{\infty} \frac{2^{2n} + (-1)^n}{5^n}$

11. $\sum_{n=2}^{\infty} \frac{(2n)!}{2^n n^2}$

14. $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^3}$

D. Find the interval of convergence for the following power series.

1. $\sum_{n=0}^{\infty} \left(\frac{x}{4e} \right)^n$

2. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{3^n (n+1)}$

E. Taylor and Maclaurin Series.

1. Use Taylor series to evaluate $\int_0^1 x \cos(x)^3 dx$.

2. Find all antiderivatives of $\cos(x^2)$ using Maclaurin series.

3. Write a third degree Taylor Polynomial for \sqrt{x} centered at $x = 4$.