

# Derivatives Exercises

**A. Are the following true or false? If true, explain why. If false, give a counter-example.**

1. If a function is continuous at  $a$ , then  $f'(a)$  exists.
2. If a function is differentiable, then its derivative is differentiable.

**B. For each  $f$ , find  $f'$  using the limit definition of the derivative.**

1.  $f(x) = 3$
2.  $f(x) = \sqrt{x+4}$
3.  $f(x) = 2x^2 + 3x$
4.  $f(x) = \sin(x)$

**C. Which functions' derivative is given by the following limits?**

1.  $\lim_{h \rightarrow 0} \frac{\tan(x+h) - \tan(x)}{h}$
2.  $\lim_{h \rightarrow 0} \frac{\sqrt{2x+2h-3} - \sqrt{2x-3}}{h}$
3.  $\lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 1 - 3x^2 - 1}{h}$

**D. Differentiate each of the following.**

1.  $4x^{3/4}$
2.  $\cot(\sin(x))$
3.  $x^{\ln(2x)}$
4.  $\ln \frac{(x^2-4x+1)^3}{(3x+5x^2)^8}$
5.  $\sin^{-1}(\pi x)$
6.  $e^{\frac{1}{\cos(\sqrt{x})}}$
7.  $\csc(4x^2)$
8.  $\sqrt{1+x^2}$
9.  $\cos(\ln(\sin(x)))$
10.  $\arctan(\pi + \ln(x))$
11.  $\left(\frac{1}{x}\right)^x + \left(\frac{1}{x}\right)^2 + \left(\frac{1}{2}\right)^x$
12.  $(x^3 + 2x^5)(x^9 - 5x^7 + 3)$

**E. For each  $f$ , find  $f'$  and  $f''$  using any method you want.**

1.  $f(x) = 3x^3 - \frac{4}{x^2}$
2.  $f(x) = x \sin(x)$
3.  $f(x) = \frac{x^2+3}{x-4}$
4.  $f(x) = x^2 \cos(x) + x \tan(x)$
5.  $f(x) = \sin(x) \cos(x) e^x$
6.  $f(x) = x^5 + x^4 + \pi^3 + x^2 + x + 1$

**F. Suppose you know the following information:  $f(2) = -3$ ,  $f'(2) = 4$ ,  $g(2) = 2$ ,  $g'(2) = 3$ ,  $h(2) = -2$ , and  $h'(2) = -4$ . Evaluate the derivative of each of the following at  $x = 2$ .**

1.  $3f(x) - 6h(x)$
2.  $(f(x) + g(x))^4$
3.  $f(g(x))$
4.  $(g(x))^{h(x)}$
5.  $e^{h(x)}$

**G. Find the 100th derivative of each function.**

1.  $f(x) = x^{70}$
2.  $f(x) = xe^x$
3.  $f(x) = -\cos(x)$
4.  $f(x) = 2^{-x}$

**H. Find the tangent line to the following curves at the given point.**

1.  $y^3x = 3x + 4y$ ,  $(0, 0)$
2.  $\tan(x+y) = \tan(xy)$ ,  $(0, 0)$
3.  $\frac{(x-2)^2}{9} + \frac{(y-5)^2}{4} = 1$ ,  $(2, 7)$
4.  $y^x = 4x$ ,  $(4, 2)$
5.  $y^4 - 4y^2 = x^4 - 9x^2$ ,  $(3, -2)$
6.  $\sin\left(\frac{\cos(y)}{\pi}\right) = 3xy$ ,  $(0, \pi/2)$

**I. Where is the tangent line to the curve defined by  $xy^2 = x^2 + 2y$ ...**

1. parallel to the  $x$ -axis?
2. vertical?
3. perpendicular to  $3y = 5x + 1$  and  $x$  and  $y$  are both integers less than 3 in absolute value?

## Answers (in no particular order)

- $f'(x) = \frac{1}{2}(x+4)^{-1/2}$
- $f(x) = 3x^2 - 1$
- $y = -\frac{3}{4}x$
- 36
- $x^{\ln(2x)-1}(\ln(x) + \ln(2x))$
- $f'(x) = 4x + 3$
- $\frac{1}{2}(1+x^2)^{-1/2}$
- $f'(x) = 0$
- False ( $f(x) = \frac{|x|^3}{2x}$  is differentiable and has derivative  $f'(x) = |x|$ , which is not differentiable)
- $f'(x) = \cos(x)$
- $f(x) = \tan(x)$
- $y = 7$
- $\frac{-4}{e^2}$
- $f^{(100)} = \ln(2)^{100}2^{-x}$
- $-\csc(4x^2)\cot(4x^2)8x$
- $-\csc^2(\sin x)\cos(x)$
- $f'(x) = 2x\cos(x) - x^2\sin(x) + \tan(x) + x\sec^2(x)$ ,  
 $f''(x) = (2 - x^2)\cos(x) - 4x\sin(x) + \sec^2(x)(2 + 2x\tan(x))$
- -28
- $y = \frac{\pi}{2} - \frac{3\pi^2}{2}x$
- $f^{(100)} = -\cos(x)$
- $y = -x$
- $3x^{-1/4}$
- $(3x^2 + 10x^9)(x^9 - 5x^7 + 3) + (x^3 + 2x^5)(9x^8 - 35x^6)$
- $f'(x) = \sin(x) + x\cos(x)$ ,  $f''(x) = \cos(x) + \cos(x) - x\sin(x)$
- $e^{\frac{1}{\cos(\sqrt{x})}}\cos(\sqrt{x})^{-2}\sin(\sqrt{x})\frac{1}{2}x^{-1/2}$
- $-\ln(3) - \frac{3}{4}$
- $-\sin(\ln(\sin(x)))\frac{1}{\sin(x)}\cos(x)$
- $f^{(100)}(x) = 0$
- $(2, -1)$
- $y = -2 - \frac{27}{8}(x-3)$
- $f(x) = \sqrt{2x-3}$
- False ( $f(x) = |x|$  is continuous at 0, but  $f'(0)$  DNE)

- $f'(x) = e^x(\cos^2(x) + \sin(x)\cos(x) - \sin^2(x))$ ,  
 $f''(x) = f'(x) + e^x(-4\cos(x)\sin(x) + \cos^2(x) - \sin^2(x))$
- 12
- $y = 2 + \left(\frac{1}{8} + \ln(1/\sqrt{x})\right)(x - 4)$
- $(0, 0), (2, 2)$
- $\frac{(3x+5x^2)^8}{(x^2-4x+1)^3} \cdot \frac{3(x^2-4x+1)^2(2x-4)(3x+5x^2)^8 - 8(x^2-4x+1)^3(3x+5x^2)(3+10x)}{(3x+5x^2)^{16}}$
- $f'(x) = 5x^4 + 4x^3 + 2x + 1, f''(x) = 20x^3 + 12x^2 + 2$
- $\left(\frac{1}{x}\right)^x \left(\ln\left(\frac{1}{x}\right)\right) - 2x^{-3} + \ln\left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^x$
- $f'(x) = 9x^2 + 8x^{-3}, f''(x) = 18x - 24x^{-4}$
- $f^{(100)}(x) = 100e^x + xe^x$
- $(-1, -1)$
- $\frac{\pi}{\sqrt{1-(\pi x)^2}}$
- $f'(x) = \frac{(2x)(x-4)-(x^2+3)}{(x-4)^2}, f''(x) = \frac{(2x-8)(x^2-8x+16)-(x^2-8x-3)(2x-8)}{(x^2-8x+16)^2}$