Trigonometry

$(\cos\theta, \sin\theta)$ is the coordinate on the unit circle that makes angle θ with the positive x-axis.

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

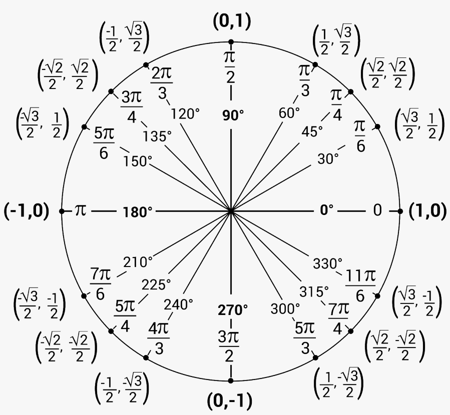
$$\text{Pythagorean identities} \begin{cases} \sin^2\theta + \cos^2\theta = 1 \\ \tan^2\theta + 1 = \sec^2\theta \\ 1 + \cot^2\theta = \csc^2\theta \end{cases}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$



Limits

Law	Let $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$.
Sum	$\lim_{x\to a}(f(x)+g(x)) = L + M$
Scalar	$\lim_{x\to a} cf(x) = cL$
Product	$\lim_{x\to a}(f(x)\cdot g(x)) = L \cdot M$
Quotient	$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{L}{M}$ for $M \neq 0$
Power	$\lim_{x\to a}(f(x))^n = L^n$
Root	$\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{L}$ for all L if n is odd, and for $L \geq 0$ if n is even and $f(x) \geq 0$.

Squeeze Theorem:
Let f, g, and h be functions with $g(x) \leq f(x) \leq h(x)$ for all x and $\lim_{x\to a} g(x) = L = \lim_{x\to a} h(x)$, then $\lim_{x\to a} f(x) = L$.

Indeterminate Forms:
$\frac{0}{0}$, $\frac{\infty}{\infty}$, 0^0, $\infty - \infty$, 1^∞, $0\cdot\infty$, ∞^0

$\varepsilon - \delta$ definition:
L is the limit of f as x approaches a if for all $\varepsilon > 0$, there is some $\delta > 0$, such that $|x - a| < \delta \implies |f(x) - L| < \varepsilon$.

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1 \qquad \lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x = e \qquad \lim_{x\to\infty} \frac{ax^n + \dots}{bx^m + \dots} = \begin{cases} 0 & m > n \\ \infty & n > m \\ a/b & n = m \end{cases}$$

Continuity

Definition: f is *continuous* at $x = a$ if $\lim_{x\to a} f(x) = f(a)$.

- The following functions are **continuous on their domains**: polynomials, rational functions, trig and inverse trig functions, exponential functions, logarithms.
- The sum, product, and composition of continuous functions is continuous.

Composite Function Theorem: If $f(x)$ is continuous at L and $\lim_{x\to a} g(x) = L$, then $\lim_{x\to a} f(g(x)) = f(L)$.

Intermediate Value Theorem: Let f be continuous over a closed, bounded interval $[a, b]$. If z is any real number between $f(a)$ and $f(b)$, then there is a number c in $[a, b]$ satisfying $f(c) = z$.

Finding Derivatives

Limit definition of the derivative:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

Tangent line to $f(x)$ at $x = a$:

$$L(x) = f(a) + f'(a)(x - a)$$

L'Hôpital's Rule:
If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ or ∞, then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

Scalar Rule	$[af]' = af'$
Sum Rule	$[f+g]' = f' + g'$
Product Rule	$[fg]' = f'g + fg'$
Quotient Rule	$\left[\frac{f}{g}\right]' = \frac{f'g - fg'}{g^2}$
Chain Rule	$[f(g(x))]' = f'(g(x))g'(x)$
Inverse Rule	$[f^{-1}(x)]' = \frac{1}{f'(f^{-1}(x))}$

Logarithmic Differentiation:
To find the derivative of $y = f(x)^{g(x)}$, take ln() of both sides, bring $g(x)$ down using the log rule ($\ln(a^b) = b\ln(a)$):

$$\ln(y) = \ln(f(x)^{g(x)}) = g(x)\ln(f(x))$$

Then implicitly differentiate and solve for y':

$$y' = f(x)^{g(x)}\left(g'(x)\ln(f(x)) + g(x)\frac{f'(x)}{f(x)}\right).$$

Power Rule	$[x^a]' = ax^{a-1}$	
Trig Rules	$[\sin(x)]' = \cos(x)$	$[\cos(x)]' = -\sin(x)$
(PSST!)	$[\tan(x)]' = \sec^2(x)$	$[\cot(x)]' = -\csc^2(x)$
	$[\sec(x)]' = \sec(x)\tan(x)$	$[\csc(x)]' = -\csc(x)\cot(x)$
Inverse Trig Rules	$[\arcsin(x)]' = \frac{1}{\sqrt{1-x^2}}$	$[\arccos(x)]' = \frac{-1}{\sqrt{1-x^2}}$
	$[\arctan(x)]' = \frac{1}{1+x^2}$	$[\text{arccot}(x)]' = \frac{-1}{1+x^2}$
	$[\text{arcsec}(x)]' = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$[\text{arccsc}(x)]' = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$
Exponent Rule	$[a^x]' = \ln(a)a^x$	
Logarithm Rule	$[\log_a(x)]' = \frac{1}{x\ln(a)}$	