Derivatives Exercises

A. Are the following true or false? If true, explain why. If false, give a counter-example.

1. If a function is continuous at a, then $f'(a)$ exists.
2. If a function is differentiable, then its derivative is differentiable.

B. For each f, find f' using the limit definition of the derivative.

1. $f(x) = 3$
2. $f(x) = \sqrt{x+4}$
3. $f(x) = 2x^2 + 3x$
4. $f(x) = \sin(x)$

C. Which functions' derivative is given by the following limits?

1. $\lim\limits_{h \to 0} \dfrac{\tan(x+h) - \tan(x)}{h}$
2. $\lim\limits_{h \to 0} \dfrac{\sqrt{2x+2h-3} - \sqrt{2x-3}}{h}$
3. $\lim\limits_{h \to 0} \dfrac{3x^2 + 6xh + 3h^2 - 1 - 3x^2 - 1}{h}$

D. For each f, find f' and f'' using any method you want.

1. $f(x) = 3x^3 - \frac{4}{x^2}$
2. $f(x) = x\sin(x)$
3. $f(x) = \frac{x^2+3}{x-4}$
4. $f(x) = x^2\cos(x) + x\tan(x)$
5. $f(x) = \sin(x)\cos(x)e^x$

E. When are the following functions increasing/decreasing? When are they concave up/down?

1. $f(x) = 2x^2 + 3x$
2. $f(x) = x^3 + 17x^2 + 2$

F. Find the 100th derivative of each function.

1. $f(x) = x^{70}$
2. $f(x) = xe^x$

Answers (in no particular order)

- $f'(x) = \frac{1}{2}(x+4)^{-1/2}$
- increasing: $x > \frac{-3}{4}$, decreasing: $x < \frac{-3}{4}$, concave up: all of $\mathbb{R}$, concave down: nowhere
- $f(x) = 3x^2 - 1$
- $f'(x) = 4x + 3$
- $f'(x) = 0$
- False ($f(x) = \frac{|x|^3}{2x}$ is differentiable and has derivative $f'(x) = |x|$, which is not differentiable)
- $f'(x) = \cos(x)$
- $f(x) = \tan(x)$
- $f'(x) = 2x\cos(x) - x^2\sin(x) + \tan(x) + x\sec^2(x)$, $f''(x) = \left(2 - x^2\right)\cos(x) - 4x\sin(x) + \sec^2(x)\left(2 + 2x\tan(x)\right)$
- $f'(x) = \sin(x) + x\cos(x)$, $f''(x) = \cos(x) + \cos(x) - x\sin(x)$
- $f^{(100)}(x) = 0$
- $f(x) = \sqrt{2x-3}$
- False ($f(x) = |x|$ is continuous at 0, but $f'(0)$ DNE)
- $f'(x) = e^x(\cos^2(x) + \sin(x)\cos(x) - \sin^2(x))$, $f''(x) = f'(x) + e^x\left(-4\cos(x)\sin(x) + \cos^2(x) - \sin^2(x)\right)$
- increasing: $x > 0$ and $x < \frac{-3}{4}$, decreasing: $\frac{-3}{4} < x < 0$, concave up: $x > \frac{-17}{3}$, concave down: $x < \frac{-17}{3}$
- $f'(x) = 9x^2 + 8x^{-3}$, $f''(x) = 18x - 24x^{-4}$
- $f^{(100)}(x) = 100e^x + xe^x$
- $f'(x) = \frac{(2x)(x-4)-(x^2+3)}{(x-4)^2}$, $f''(x) = \frac{(2x-8)(x^2-8x+16)-(x^2-8x-3)(2x-8)}{(x^2-8x+16)^2}$