

Condensed Calculus

Trigonometry

$(\cos \theta, \sin \theta)$ is the coordinate on the unit circle that makes angle θ with the positive x -axis.

$$\sec \theta = \frac{1}{\cos \theta} \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

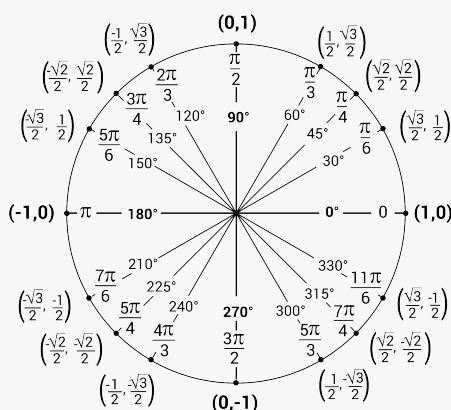
Pythagorean identities $\begin{cases} \sin^2 \theta + \cos^2 \theta = 1 \\ \tan^2 \theta + 1 = \sec^2 \theta \\ 1 + \cot^2 \theta = \csc^2 \theta \end{cases}$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$



Limits

Law Let $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = M$.

Sum $\lim_{x \rightarrow a} (f(x) + g(x)) = L + M$

Scalar $\lim_{x \rightarrow a} cf(x) = cL$

Product $\lim_{x \rightarrow a} (f(x) \cdot g(x)) = L \cdot M$

Quotient $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{L}{M}$ for $M \neq 0$

Power $\lim_{x \rightarrow a} (f(x))^n = L^n$

Root $\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{L}$ for all L if n is odd, and for $L \geq 0$ if n is even and $f(x) \geq 0$.

Squeeze Theorem:

Let f , g , and h be functions with $g(x) \leq f(x) \leq h(x)$ for all x and $\lim_{x \rightarrow a} g(x) = L = \lim_{x \rightarrow a} h(x)$, then $\lim_{x \rightarrow a} f(x) = L$.

Indeterminate Forms:

$\frac{0}{0}, \frac{\infty}{\infty}, 0^0, \infty - \infty, 1^\infty, 0 \cdot \infty, \infty^0$

$\epsilon - \delta$ definition:

L is the limit of f as x approaches a if for all $\epsilon > 0$, there is some $\delta > 0$, such that

$$|x - a| < \delta \implies |f(x) - L| < \epsilon.$$

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1 \quad \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e \quad \lim_{x \rightarrow \infty} \frac{ax^n + \dots}{bx^m + \dots} = \begin{cases} 0 & m > n \\ \infty & n > m \\ a/b & n = m \end{cases}$$

Continuity

Definition: f is continuous at $x = a$ if $\lim_{x \rightarrow a} f(x) = f(a)$.

- The following functions are **continuous on their domains**: polynomials, rational functions, trig and inverse trig functions, exponential functions, logarithms.
- The sum, product and composition of continuous functions is continuous.

Composite Function Theorem:

If $f(x)$ is continuous at L and $\lim_{x \rightarrow a} g(x) = L$, then $\lim_{x \rightarrow a} f(g(x)) = f(L)$.

The Intermediate Value Theorem:

Let f be continuous over a closed, bounded interval $[a, b]$. If z is any real number between $f(a)$ and $f(b)$, then there is a number c in $[a, b]$ satisfying $f(c) = z$.

Derivatives

Limit definition of the derivative:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

Tangent line to $f(x)$ at $x = a$:

$$L(x) = f(a) + f'(a)(x - a)$$

L'Hôpital's Rule:

If $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$ or ∞ , then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}.$$

Logarithmic Differentiation:

To take the derivative of $y = f(x)^{g(x)}$, take $\ln()$ of both sides, bring $g(x)$ down using the log rule ($\ln(a^b) = b \ln(a)$), implicitly differentiate, and then solve for y' :

$$y' = f(x)^{g(x)} \left(g'(x) \ln(f(x)) + g(x) \frac{f'(x)}{f(x)} \right).$$

The Scalar Multiple Rule $[af]' = af'$

The Sum Rule $[f + g]' = f' + g'$

The Product Rule $[fg]' = f'g + fg'$

The Quotient Rule $\left[\frac{f}{g}\right]' = \frac{f'g - fg'}{g^2}$

The Chain Rule $[f(g(x))]' = f'(g(x))g'(x)$

The Inverse Rule $[f^{-1}(x)]' = \frac{1}{f'(f^{-1}(x))}$

The Constant Rule $[a]' = 0$

The Power Rule $[x^a]' = ax^{a-1}$

Trig Rules $[\sin(x)]' = \cos(x)$ $[\cos(x)]' = -\sin(x)$

(PSST!) $[\tan(x)]' = \sec^2(x)$ $[\cot(x)]' = -\csc^2(x)$

$[\sec(x)]' = \sec(x) \tan(x)$ $[\csc(x)]' = -\csc(x) \cot(x)$

Exponent Rule $[a^x]' = \ln(a)a^x$

Logarithm Rule $[\log_a(x)]' = \frac{1}{x \ln(a)}$