Derivatives Exercises

A. Are the following true or false? If true, explain why. If false, give a counter-example.

1. If a function is continuous at a, then $f'(a)$ exists.

2. If a function is differentiable, then its derivative is differentiable.

B. For each f, find f' using the limit definition of the derivative.

1. $f(x) = 3$
2. $f(x) = \sqrt{x+4}$
3. $f(x) = 2x^2 + 3x$
4. $f(x) = \sin(x)$

C. Which functions' derivative is given by the following limits?

1. $\lim\limits_{h \to 0} \dfrac{\tan(x+h) - \tan(x)}{h}$

2. $\lim\limits_{h \to 0} \dfrac{\sqrt{2x+2h-3} - \sqrt{2x-3}}{h}$

3. $\lim\limits_{h \to 0} \dfrac{3x^2 + 6xh + 3h^2 - 1 - 3x^2 + 1}{h}$

D. Differentiate each of the following.

1. $4x^{3/4}$
2. $\cot(\sin(x))$
3. $x^{\ln(2x)}$
4. $\ln \dfrac{(x^2 - 4x + 1)^3}{(3x + 5x^2)^8}$
5. $\sin^{-1}(\pi x)$
6. $e^{\frac{1}{\cos(\sqrt{x})}}$
7. $\csc(4x^2)$
8. $\sqrt{1 + x^2}$
9. $\cos(\ln(\sin(x)))$
10. $\arctan(\pi + \ln(x))$
11. $\left(\frac{1}{x}\right)^x + \left(\frac{1}{x}\right)^2 + \left(\frac{1}{2}\right)^x$
12. $(x^3 + 2x^5)(x^9 - 5x^7 + 3)$

E. For each f, find f' and f'' using any method you want.

1. $f(x) = 3x^3 - \frac{4}{x^2}$
2. $f(x) = x\sin(x)$
3. $f(x) = \frac{x^2 + 3}{x - 4}$
4. $f(x) = x^2 \cos(x) + x\tan(x)$
5. $f(x) = \sin(x)\cos(x)e^x$
6. $f(x) = x^5 + x^4 + \pi^3 + x^2 + x + 1$

F. Suppose you know the following information: $f(2) = -3$, $f'(2) = 4$, $g(2) = 2$, $g'(2) = 3$, $h(2) = -2$, and $h'(2) = -4$. Evaluate the _derivative_ of each of the following at $x = 2$.

1. $3f(x) - 6h(x)$
2. $(f(x) + g(x))^4$
3. $f(g(x))$
4. $(g(x))^{h(x)}$
5. $e^{h(x)}$

G. Find the 100th derivative of each function.

1. $f(x) = x^{70}$
2. $f(x) = xe^x$
3. $f(x) = -\cos(x)$
4. $f(x) = 2^{-x}$

H. Find the tangent line to the following curves at the given point.

1. $y^3 x = 3x + 4y$, $(0, 0)$
2. $\tan(x + y) = \tan(xy)$, $(0, 0)$
3. $\frac{(x-2)^2}{9} + \frac{(y-5)^2}{4} = 1$, $(2, 7)$
4. $y^x = 4x$, $(4, 2)$
5. $y^4 - 4y^2 = x^4 - 9x^2$, $(3, -2)$
6. $\sin\left(\frac{\cos(y)}{\pi}\right) = 3xy$, $(0, \pi/2)$

I. Where is the tangent line to the curve defined by $xy^2 = x^2 + 2y$...

1. parallel to the x-axis?

2. vertical?

3. perpendicular to $3y = 5x + 1$ and x and y are both integers less than 3 in absolute value?