# Integration Application Exercises

## A. Kinematics

1. If a particle's position at time $t$ is $x(t) = \cos(t)\sin(t)$, what is the total distance it travels from $t = 0$ to $t = 2\pi$?

## B. Average Values

1. Find the average value of the function $f(x) = \cos^3(x)$ on the interval $[0, \pi]$.

## C. Probability

1. Prove that the Cauchy distribution

$$c(x) = \frac{1}{\pi\gamma\left(1 + \left(\frac{x-x_0}{\gamma}\right)^2\right)}$$

is a PDF for any $x_0$ and any $\gamma > 0$. Find its CDF. Then prove that its expected value and variance do not exist.

## D. Volumes and Surface Area

1. Let $R$ be the region above the graph of $x^2 + 2x - 3$ and below the $x$-axis. Find the volume of the solid obtained by rotating the region $R$ around the $x$-axis.
2. A cylindrical hole of radius 3 cm is bored through the center of a wooden sphere of radius 5 cm. Find the volume of the resulting wooden object.
3. Find the volume of the finite region between $x^2 - 2$ and $2x - 2$ revolved around the $x$-axis. Then compute the same for the $y$-axis.
4. Find the surface area and volume of the solid obtained by rotating the graph of $f(x) = \frac{1}{x}$ around the $x$-axis for $x \geq 1$.
5. The finite region bounded by the curves $x = 0$, $y = x^2$, and $y = 1$ is rotated about the the $y$-axis. Find the volume of the resulting solid by using the cylindrical shells method.
6. What is the volume of the solid obtained by rotating the region enclosed by $x^2 - x$ and $2x^2$ around the line $y = 2$.
7. Derive the volume of a hemisphere.
8. What is the largest volume of solid with a (2,1)-elliptical base and whose cross sections are squares?

## E. Arc Length

1. Find the length of the parameterized curve in the plane given by $x = t^2$ and $y = \frac{2}{3}t^3$ with $t \in [0, 1]$.
2. Find the length of the curve given by the equation $9y^2 = 4(x + 1)^3$ and the constraints $0 \leq x \leq 1$, $y \geq 0$.

## F. Moments and Centroids

1. Find the centroid of the finite region bounded by the functions $f(x) = x^2$ and $g(x) = x$.

## G. Work

1. A 10m-long metal chain of total mass 20 kilograms is lying flat on the floor. How much work would be done if someone grabbed one end of the chain and lifted it 5 meters above the floor? Assume that the acceleration due to gravity is $9.8\text{m/sec}^2$, and that the lifted portion of the chain always hangs vertically.