Series and Sequences Exercises

A. Determine whether each geometric series converges or diverges. If it converges, find its sum; If not, say why.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$
2. $\displaystyle\sum_{k=0}^{\infty} \frac{2^{k+1}}{3^k}$
3. $\displaystyle\sum_{k=10}^{\infty} \frac{3^{k-1}}{2^k}$
4. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{5}{4^n}$
5. $\displaystyle\sum_{n=3}^{\infty} (-1)^n \frac{3}{2^n}$
6. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^2+n} + \frac{3^{2n}}{14^n}\right)$
7. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6}$

B. True or False? Explain.

1. There exists an infinite geometric series with sum -4 and first term -1.
2. There esists a function whose integral is finite but does not limit to 0 at ∞.

C. Determine whether each series converges or diverges.

1. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k}}$
2. $\displaystyle\sum_{n=0}^{\infty} \frac{\sin^2(n)}{1+n^2}$
3. $\displaystyle\sum_{k=0}^{\infty} \frac{k}{e^k}$
4. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$
5. $\displaystyle\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3+4n-3}$
7. $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}+(-1)^n}{5^n}$
8. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2-1}$
9. $\displaystyle\sum_{n=1}^{\infty} (\arctan(n)+n)$
10. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$
11. $\displaystyle\sum_{n=2}^{\infty} \frac{(2n)!}{2^n n^2}$
12. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$
13. $\displaystyle\sum_{n=2}^{\infty} \left(\frac{\sin(100)}{\pi}\right)^n$
14. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^3}$

D. Find the interval of convergence for the following power series.

1. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{x}{4e}\right)^n$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{3^n(n+1)}$
3. $\displaystyle\sum_{n=0}^{\infty} \frac{n^{3n+1}}{4^{3n}}(2x-7)^n$
4. $\displaystyle\sum_{n=0}^{\infty} \frac{n+1}{(2n+1)!}x^n$

E. Taylor and Maclaurin Series.

1. Use Taylor series to evaluate $\displaystyle\int_0^1 x\cos(x)^3\, dx$.
2. Find all antiderivatives of $\cos(x^2)$ using Maclaurin series.
3. Write a third degree Taylor Polynomial for $\sqrt{x}$ centered at $x=4$.