# Derivatives Exercises

**A. Are the following true or false? If true, explain why. If false, give a counter-example.**

1. If a function is continuous at $a$, then $f'(a)$ exists.
2. If a function is differentiable, then its derivative is differentiable.

**B. For each $f$, find $f'$ using the limit definition of the derivative.**

1. $f(x) = 3$
2. $f(x) = \sqrt{x+4}$
3. $f(x) = 2x^2 + 3x$
4. $f(x) = \sin(x)$

**C. Which functions' derivative is given by the following limits?**

1. $\lim_{h\to 0} \dfrac{\tan(x+h) - \tan(x)}{h}$
2. $\lim_{h\to 0} \dfrac{\sqrt{2x+2h-3} - \sqrt{2x-3}}{h}$
3. $\lim_{h\to 0} \dfrac{3x^2+6xh+3h^2-1-3x^2+1}{h}$

**D. Differentiate each of the following.**

1. $4x^{3/4}$
2. $\cot(\sin(x))$
3. $x^{\ln(2x)}$
4. $\ln \frac{(x^2-4x+1)^3}{(3x+5x^2)^8}$
5. $\sin^{-1}(\pi x)$
6. $e^{\frac{1}{\cos(\sqrt{x})}}$
7. $\csc(4x^2)$
8. $\sqrt{1+x^2}$
9. $\cos(\ln(\sin(x)))$
10. $\arctan(\pi + \ln(x))$
11. $\left(\frac{1}{x}\right)^x + \left(\frac{1}{x}\right)^2 + \left(\frac{1}{2}\right)^x$
12. $(x^3+2x^5)(x^9-5x^7+3)$
13. $\frac{\tan(3x)}{\sin(3x)}$
14. $\arccos(\sqrt[3]{x} + 2^x)$
15. $\sin(x^x)$

**E. For each $f$, find $f'$ and $f''$ using any method you want.**

1. $f(x) = 3x^3 - \frac{4}{x^2}$
2. $f(x) = x\sin(x)$
3. $f(x) = \frac{x^2+3}{x-4}$
4. $f(x) = x^2\cos(x) + x\tan(x)$
5. $f(x) = \sin(x)\cos(x)e^x$
6. $f(x) = x^5+x^4+\pi^3+x^2+x+1$

**F. Suppose you know the following information: $f(2) = -3$, $f'(2) = 4$, $g(2) = 2$, $g'(2) = 3$, $h(2) = -2$, and $h'(2) = -4$. Evaluate the <u>derivative</u> of each of the following at $x = 2$.**

1. $3f(x) - 6h(x)$
2. $(f(x)+g(x))^4$
3. $f(g(x))$
4. $(g(x))^{h(x)}$
5. $e^{h(x)}$

**G. Find the 100th derivative of each function.**

1. $f(x) = x^{70}$
2. $f(x) = xe^x$
3. $f(x) = -\cos(x)$
4. $f(x) = 2^{-x}$

**H. Find the tangent line to the following curves at the given point.**

1. $y^3x = 3x + 4y$, $(0,0)$
2. $\tan(x+y) = \tan(xy)$, $(0,0)$
3. $\frac{(x-2)^2}{9} + \frac{(y-5)^2}{4} = 1$, $(2,7)$
4. $y^x = 4x$, $(4,2)$
5. $y^4 - 4y^2 = x^4 - 9x^2$, $(3,-2)$
6. $\sin\left(\frac{\cos(y)}{\pi}\right) = 3xy$, $(0,\pi/2)$

**I. Where is the tangent line to the curve defined by $xy^2 = x^2 + 2y$...**

1. parallel to the $x$-axis?
2. vertical?
3. perpendicular to $3y = 5x + 1$ and $x$ and $y$ are both integers less than 3 in absolute value?