

## Trigonometry

$(\cos \theta, \sin \theta)$  is the coordinate on the unit circle that makes angle  $\theta$  with the positive  $x$ -axis.

$$\sec \theta = \frac{1}{\cos \theta} \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

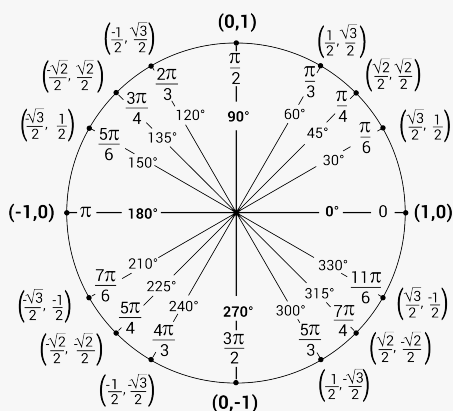
Pythagorean identities  $\begin{cases} \sin^2 \theta + \cos^2 \theta = 1 \\ \tan^2 \theta + 1 = \sec^2 \theta \\ 1 + \cot^2 \theta = \csc^2 \theta \end{cases}$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$



## Limits

**Law** Let  $\lim_{x \rightarrow a} f(x) = L$  and  $\lim_{x \rightarrow a} g(x) = M$ .

**Sum**  $\lim_{x \rightarrow a} (f(x) + g(x)) = L + M$

**Scalar**  $\lim_{x \rightarrow a} cf(x) = cL$

**Product**  $\lim_{x \rightarrow a} (f(x) \cdot g(x)) = L \cdot M$

**Quotient**  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{L}{M}$  for  $M \neq 0$

**Power**  $\lim_{x \rightarrow a} (f(x))^n = L^n$

**Root**  $\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{L}$  for all  $L$  if  $n$  is odd, and for  $L \geq 0$  if  $n$  is even and  $f(x) \geq 0$ .

### Squeeze Theorem:

Let  $f, g$ , and  $h$  be functions with  $g(x) \leq f(x) \leq h(x)$  for all  $x$  and  $\lim_{x \rightarrow a} g(x) = L = \lim_{x \rightarrow a} h(x)$ , then  $\lim_{x \rightarrow a} f(x) = L$ .

### Indeterminate Forms:

$$\frac{0}{0}, \frac{\infty}{\infty}, 0^0, \infty - \infty, 1^\infty, 0 \cdot \infty, \infty^0$$

### $\epsilon - \delta$ definition:

$L$  is the limit of  $f$  as  $x$  approaches  $a$  if for all  $\epsilon > 0$ , there is some  $\delta > 0$ , such that  $|x - a| < \delta \implies |f(x) - L| < \epsilon$ .

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1 \quad \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e \quad \lim_{x \rightarrow \infty} \frac{ax^n + \dots}{bx^m + \dots} = \begin{cases} 0 & m > n \\ \infty & n > m \\ a/b & n = m \end{cases}$$

## Continuity

**Definition:**  $f$  is continuous at  $x = a$  if  $\lim_{x \rightarrow a} f(x) = f(a)$ .

- The following functions are **continuous on their domains**: polynomials, rational functions, trig and inverse trig functions, exponential functions, logarithms.
- The sum, product, and composition of continuous functions is continuous.

### Composite Function Theorem:

If  $f(x)$  is continuous at  $L$  and  $\lim_{x \rightarrow a} g(x) = L$ , then  $\lim_{x \rightarrow a} f(g(x)) = f(L)$ .

### Intermediate Value Theorem:

Let  $f$  be continuous over a closed, bounded interval  $[a, b]$ . If  $z$  is any real number between  $f(a)$  and  $f(b)$ , then there is a number  $c$  in  $[a, b]$  satisfying  $f(c) = z$ .

## Finding Derivatives

**Limit definition** of the derivative:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

**Tangent line** to  $f(x)$  at  $x = a$ :

$$L(x) = f(a) + f'(a)(x - a)$$

### L'Hôpital's Rule:

If  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$  or  $\infty$ , then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}.$$

### Logarithmic Differentiation:

To find the derivative of  $y = f(x)^{g(x)}$ , take  $\ln()$  of both sides, bring  $g(x)$  down using the log rule ( $\ln(a^b) = b \ln(a)$ ):

$$\ln(y) = \ln(f(x)^{g(x)}) = g(x) \ln(f(x))$$

Then implicitly differentiate and solve for  $y'$ :

$$y' = f(x)^{g(x)} \left( g'(x) \ln(f(x)) + g(x) \frac{f'(x)}{f(x)} \right).$$

### Power Rule

$$[x^a]' = ax^{a-1}$$

### Trig Rules

$$[\sin(x)]' = \cos(x)$$

$$[\cos(x)]' = -\sin(x)$$

(PSST!)

$$[\tan(x)]' = \sec^2(x)$$

$$[\cot(x)]' = -\csc^2(x)$$

$$[\sec(x)]' = \sec(x) \tan(x)$$

$$[\csc(x)]' = -\csc(x) \cot(x)$$

### Inverse Trig Rules

$$[\arcsin(x)]' = \frac{1}{\sqrt{1-x^2}}$$

$$[\arccos(x)]' = \frac{-1}{\sqrt{1-x^2}}$$

$$[\arctan(x)]' = \frac{1}{1+x^2}$$

$$[\text{arccot}(x)]' = \frac{-1}{1+x^2}$$

$$[\text{arcsec}(x)]' = \frac{1}{|x|\sqrt{x^2-1}}$$

$$[\text{arccsc}(x)]' = \frac{-1}{|x|\sqrt{x^2-1}}$$

### Exponent Rule

$$[a^x]' = \ln(a)a^x$$

### Logarithm Rule

$$[\log_a(x)]' = \frac{1}{x \ln(a)}$$

### Scalar Rule

$$[af]' = af'$$

### Sum Rule

$$[f+g]' = f' + g'$$

### Product Rule

$$[fg]' = f'g + fg'$$

### Quotient Rule

$$\left[\frac{f}{g}\right]' = \frac{f'g - fg'}{g^2}$$

### The Chain Rule

$$[f(g(x))]' = f'(g(x))g'(x)$$

### Inverse Rule

$$[f^{-1}(x)]' = \frac{1}{f'(f^{-1}(x))}$$