

# Limits and Continuity Exercises

**A. Are the following true or false? If true, explain why. If false, give a counter-example.**

1. If  $\lim_{x \rightarrow a} f(x)$  does not exist, then  $f$  is undefined at the point  $x = a$ .
2. If a function is not defined at  $x = a$ , then  $\lim_{x \rightarrow a} f(x)$  does not exist.
3. If  $f$  and  $g$  are continuous on their domains which contain  $a$ , then  $\lim_{x \rightarrow a} f(x) + g(x) = f(a) + g(a)$ .
4. If  $\lim_{x \rightarrow a} f(x)$  exists, then  $f$  is continuous at  $a$ .

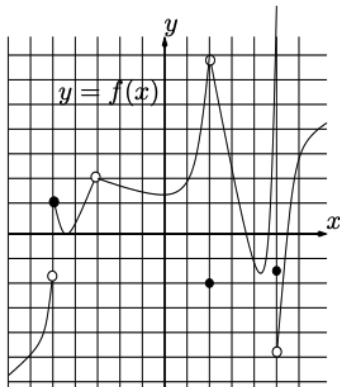
**B. Evaluate the following limits (or say that the limit DNE):**

1.  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x + 3}$
2.  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$
3.  $\lim_{x \rightarrow \pi/2} \frac{\cot(x)}{\cos(x)}$
4.  $\lim_{x \rightarrow 6} \frac{10}{x^2 - 36}$
5.  $\lim_{x \rightarrow \infty} \tan(x)$
6.  $\lim_{x \rightarrow \pi/2^+} \tan(x)$
7.  $\lim_{x \rightarrow \infty} \frac{x^3 + 3x^2 + 4}{1 - x^2}$
8.  $\lim_{x \rightarrow \infty} \frac{\cos(x)}{x^2}$
9.  $\lim_{x \rightarrow \infty} \frac{4x^4 + 3x^3}{7x^4 + x}$
10.  $\lim_{x \rightarrow \infty} \frac{10000x^3 - x^2}{8x^4 + 2x + 1}$
11.  $\lim_{x \rightarrow 1^+} \frac{x^2 + x + 1}{x^2 - 1}$
12.  $\lim_{x \rightarrow 0} \frac{(\cos^2(x) - 1)(x + 3)}{x}$
13.  $\lim_{x \rightarrow 5} x^3 + e^x \sin(x)$
14.  $\lim_{x \rightarrow 5} \frac{6 \sin(x - 5)}{x - 5}$

**C. For each function  $f$ , find a value of  $c$  so that  $f$  is continuous on  $\mathbb{R}$ :**

1.  $f(x) = \begin{cases} 2x & x \leq c \\ x^2 + 1 & x > c \end{cases}$
2.  $f(x) = \begin{cases} 2x + c & x < 2 \\ x^2 + cx + 1 & x \geq 2 \end{cases}$

**D. Answer the following questions based on the graph (each box has width 1).**



1. At what points  $a$  does  $\lim_{x \rightarrow a} f(x) = L$  but  $L \neq f(a)$ ?
2. At which points is  $f$  not continuous?
3. Does  $\lim_{x \rightarrow 2^-} f(x)$  exist? If it does, what is its value?
4. Does  $\lim_{x \rightarrow 2^+} f(x)$  exist? If it does, what is its value?
5. Does  $\lim_{x \rightarrow 2} f(x)$  exist? If it does, what is its value?
6. What is  $f(2)$ ?

**E. Answer the following questions based on the function  $f$  defined below.**

$$f(t) = \begin{cases} 1 + t & t < 0 \\ t^2 + 1 & 0 \leq t < 1 \\ 3 & t = 1 \\ t + 4 & t > 1 \end{cases}$$

1. What is  $\lim_{t \rightarrow 0} f(t)$ ?
2. What is  $\lim_{t \rightarrow 0^+} f(t)$ ?
3. What is  $\lim_{t \rightarrow 0^-} f(t)$ ?
4. Where is  $f$  continuous?

## Answers (in no particular order)

- $-5, -3, 2, 5$
- $0$
- $6$
- $-1$
- False ( $f(x) = \frac{x^2}{x}$  is not defined at  $0$ , but  $\lim_{x \rightarrow 0} f(x) = 0$ )
- $1$
- $6$
- $1$
- $-3, 2$
- DNE
- False ( $f(x) = \frac{|x|}{x}$  is not continuous at  $0$ , but  $\lim_{x \rightarrow 0} f(x) = 0$ )
- $-\infty$
- $1$
- $125 + e^5 \sin(5)$
- DNE
- $\infty$
- $-2$
- True (Since  $f$  and  $g$  are continuous, so is  $f + g$ . Then by the def. of continuity,  $\lim_{x \rightarrow a} f(x) + g(x) = f(a) + g(a)$ )
- $0$
- $(\infty, 1) \cup (1, \infty)$
- False (If  $f(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$  then  $\lim_{x \rightarrow a} f(x)$  but  $f(0) = 1$ )
- $1$
- yes,  $6.8$
- $0$
- yes,  $6.8$
- $1$
- yes,  $6.8$
- $\infty$
- $\frac{4}{7}$