Series and Sequences Exercises

A. Determine whether each geometric series converges or diverges. If it converges, find its sum; If not, say why.

1. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$
2. $\sum_{k=0}^{\infty} \frac{2^{k+1}}{3^k}$
3. $\sum_{k=10}^{\infty} \frac{3^{k-1}}{2^k}$
4. $\sum_{n=0}^{\infty} (-1)^n \frac{5}{4^n}$
5. $\sum_{n=3}^{\infty} (-1)^n \frac{3}{2^n}$
6. $\sum_{n=1}^{\infty} \left(\frac{1}{n^2+n} + \frac{3^{2n}}{14^n} \right)$
7. $\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6}$

B. True or False? Explain.

1. There exists an infinite geometric series with sum -4 and first term -1.
2. There exists a real function f for which $\int_{-\infty}^{\infty} f(x)\, dx$ is finite but $\lim_{x\to\infty} f(x) \neq 0$.
3. If $f(x) = \sum_{n=0}^{\infty} a_n x^n$ and $b > 0$ is in the domain of f, then $-b$ is also in the domain of f.

C. Determine whether each series converges or diverges.

1. $\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k}}$
2. $\sum_{n=0}^{\infty} \frac{\sin^2(n)}{1+n^2}$
3. $\sum_{k=0}^{\infty} \frac{k}{e^k}$
4. $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$
5. $\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$
6. $\sum_{n=1}^{\infty} \frac{n}{n^3+4n-3}$
7. $\sum_{n=1}^{\infty} \frac{2^{2n}+(-1)^n}{5^n}$
8. $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$
9. $\sum_{n=1}^{\infty} (\arctan(n)+n)$
10. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$
11. $\sum_{n=2}^{\infty} \frac{(2n)!}{2^n n^2}$
12. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$
13. $\sum_{n=2}^{\infty} \left(\frac{\sin(100)}{\pi} \right)^n$
14. $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^3}$

D. Find the interval of convergence for the following power series.

1. $\sum_{n=0}^{\infty} \left(\frac{x}{4e} \right)^n$
2. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{3^n(n+1)}$
3. $\sum_{n=0}^{\infty} \frac{n^{3n+1}}{4^{3n}} (2x-7)^n$
4. $\sum_{n=1}^{\infty} \frac{n+1}{(2n+1)!} x^n$

E. Taylor and Maclaurin Series.

1. Use Taylor series to evaluate $\int_0^1 x\cos(x)^3\, dx$.
2. Find all antiderivatives of $\cos(x^2)$ using Maclaurin series.
3. Write a third degree Taylor Polynomial for $\sqrt{x}$ centered at $x = 4$.

F. Fractals

You may have to look up definitions of the fractals in this section.

1. Find the area and perimeter of the Koch Snowflake.
2. Find the area of the Sierpinski Triangle.
3. Estimate the area under the Weierstrass function $w(x) = \displaystyle\sum_{k=1}^{\infty} \frac{\sin\left(2^k x\right)}{\sqrt{2}^k}$ on the interval $[0, 1]$.
4. Find the area of the T-square fractal.