

Limits and Continuity Exercises

A. Are the following true or false? If true, explain why. If false, give a counter-example.

1. If $\lim_{x \rightarrow a} f(x)$ does not exist, then f is undefined at the point $x = a$.
2. If a function is not defined at $x = a$, then $\lim_{x \rightarrow a} f(x)$ does not exist.
3. If f and g are continuous on their domains which contain a , then $\lim_{x \rightarrow a} f(x) + g(x) = f(a) + g(a)$.

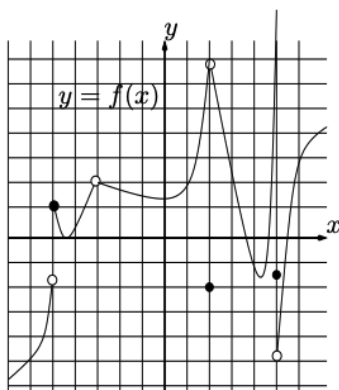
B. Evaluate the following limits (or say that the limit DNE):

1. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x + 3}$
2. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$
3. $\lim_{x \rightarrow \pi/2} \frac{\cot(x)}{\cos(x)}$
4. $\lim_{x \rightarrow 6} \frac{10}{x^2 - 36}$
5. $\lim_{x \rightarrow \infty} \tan(x)$
6. $\lim_{x \rightarrow \pi/2^+} \tan(x)$
7. $\lim_{x \rightarrow \infty} \frac{x^3 + 3x^2 + 4}{1 - x^2}$
8. $\lim_{x \rightarrow \infty} \frac{\cos(x)}{x^2}$
9. $\lim_{x \rightarrow \infty} \frac{4x^4 + 3x^3}{7x^4 + x}$
10. $\lim_{x \rightarrow \infty} \frac{10000x^3 - x^2}{8x^4 + 2x + 1}$
11. $\lim_{x \rightarrow 1^+} \frac{x^2 + x + 1}{x^2 - 1}$
12. $\lim_{x \rightarrow 0} \frac{(\cos^2(x) - 1)(x + 3)}{x}$
13. $\lim_{x \rightarrow 5} x^3 + e^x \sin(x)$
14. $\lim_{x \rightarrow 5} \frac{6 \sin(x - 5)}{x - 5}$

C. For each function f , find a value of c so that f is continuous on \mathbb{R} :

1. $f(x) = \begin{cases} 2x & x \leq c \\ x^2 + 1 & x > c. \end{cases}$
2. $f(x) = \begin{cases} 2x + c & x < 2 \\ x^2 + cx + 1 & x \geq 2. \end{cases}$

D. Answer the following questions based on the graph (each box has width 1).



1. At what points a does $\lim_{x \rightarrow a} f(x) = L$ but $L \neq f(a)$?
2. At which points is f not continuous?
3. Does $\lim_{x \rightarrow 2^-} f(x)$ exist? If it does, what is its value?
4. Does $\lim_{x \rightarrow 2^+} f(x)$ exist? If it does, what is its value?
5. Does $\lim_{x \rightarrow 2} f(x)$ exist? If it does, what is its value?
6. What is $f(2)$?

E. Answer the following questions based on the function f defined below.

$$f(t) = \begin{cases} 1 + t & t < 0 \\ t^2 + 1 & 0 \leq t < 1 \\ 3 & t = 1 \\ t + 4 & t > 1 \end{cases}$$

1. What is $\lim_{t \rightarrow 0} f(t)$?
2. What is $\lim_{t \rightarrow 0^+} f(t)$?
3. What is $\lim_{t \rightarrow 0^-} f(t)$?
4. Where is f continuous?

Answers (in no particular order)

- $-5, -3, 2, 5$
- 0
- 6
- -1
- False ($f(x) = \frac{x^2}{x}$ is not defined at 0 , but $\lim_{x \rightarrow 0} f(x) = 0$)
- 1
- 6
- 1
- $-3, 2$
- DNE
- $-\infty$
- 1
- $125 + e^5 \sin(5)$
- DNE
- ∞
- -2
- True (Since f and g are continuous, so is $f + g$. Then by the def. of continuity, $\lim_{x \rightarrow a} f(x) + g(x) = f(a) + g(a)$)
- 0
- $(\infty, 1) \cup (1, \infty)$
- False (If $f(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$ then $\lim_{x \rightarrow a} f(x)$ but $f(0) = 1$)
- 1
- yes, 6.8
- 0
- yes, 6.8
- 1
- yes, 6.8
- ∞
- $\frac{4}{7}$