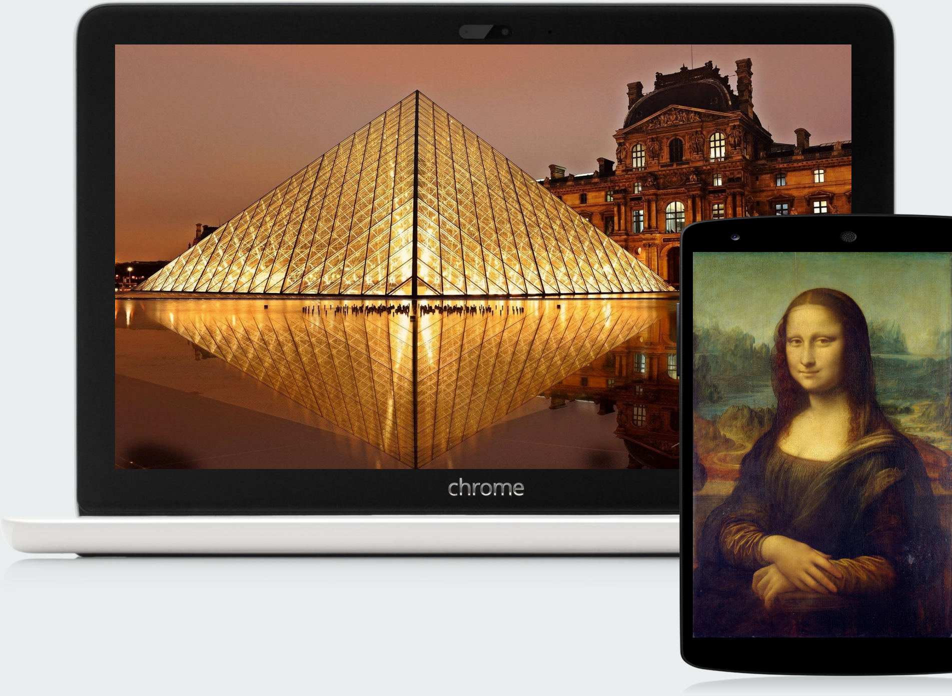
The Museum Problem

Édition du musée du Louvre

Team - Sic Mundus



Outline

The Problem we face - a statement

Design Variables we understand

The simplest we begin

The complexities we include

The algorithms we employ

Taking the next step

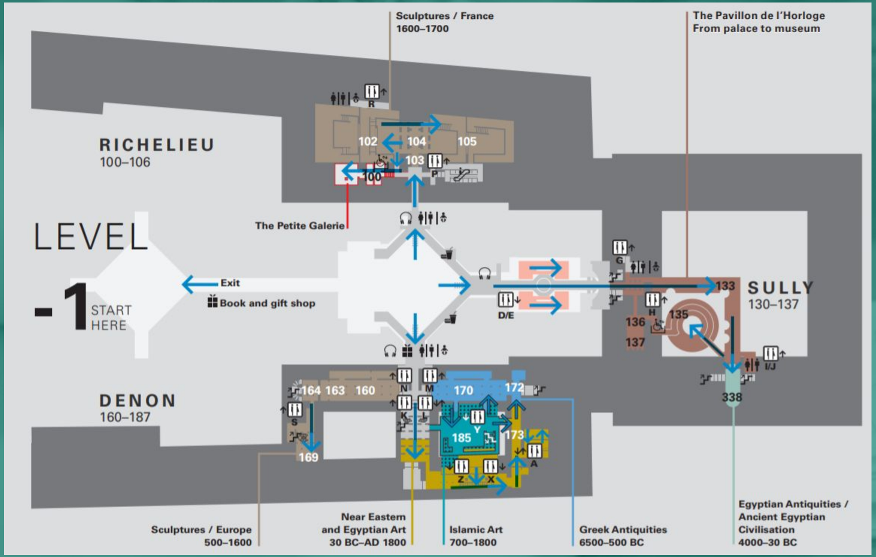
The Problem

The problem started with someone at the gate wondering:

“What if I could avoid crowds, see lots of exhibits and still get back on time?”

Bringing us to their rescue

Problem Statement



Optimize the route for a tourist visiting The Louvre Museum, such that the satisfaction level is maximised by visiting all/select exhibits in a single working day. With coinciding exit and entry points.

Design Variables

1. $(2\ X\ {}^{N}C_{2})$ - Path indicator variables
2. (N) - Time to be spent at each exhibit

List of Symbols

$$y_{ij} = \begin{cases} 1, \text{If path goes from exhibit } i \text{ to exhibit } j \\ 0, \text{otherwise} \end{cases}$$

$$c_{ij} := \text{Penalty incurred for going from } i \text{ to } j$$

$$c_{ij} > 0$$

$$G := (V, \mathbb{E})$$

$$V := \text{Vertices}$$

$$\mathbb{E} := \{(x, y) | x, y \in V\}$$

$$S := \text{Set of all tours of } G$$

$$E := \text{Subset of permitted paths in } \mathbb{E}$$

$$E' := \text{Complement of E}$$

$$v = \text{Walking speed}$$

$$\tau = \text{Time spent at each exhibit}$$

$$T_o = \text{Total available time}$$

Problem 1

- This is the simplest version of the museum path optimization problem, which requires the tourist to visit all the exhibits located at lattice points separated by known distances.
- The objective is to find an optimized sequence of visiting the exhibits such that the total path length is minimised.

$$min \sum_{i} \sum_{j=1} c_{ij} y_{ij}$$

$$s.t \sum_{i<k} y_{ik} + \sum_{j>k} y_{kj} = 2, k \in V$$

$$\sum_{i} \sum_{j} y_{ij} \leq |S| - 1, S \subset V, 3 \leq |S| \leq n - 3$$

$$y_{ij} \in 0, 1, \forall [i, j] \in E$$

Problem 2

- Consider a model, where certain points cannot be reached by all of the points in the space of lattice points
  - Models exhibits in museum situated at different floors; can be accessed only from certain entry/exit points.
- Problem is asymmetric, represents paths which don't exist in both directions.
  - Models one-way routes, and/or routes with different departure and arrival rates.

$$min \sum_{i} \sum_{j=1} c_{ij} y_{ij}$$

$$s.t \sum_{j} y_{ij} = 1, i = 0, 1, \ldots, n-1$$

$$\sum_{i} y_{ij} = 1, j = 0, 1, \ldots, n-1$$

$$\sum_{i} \sum_{j} y_{ij} \leq |S| - 1, S \subset V, 2 \leq |S| \leq n-2$$

$$y_{ij} \in 0, 1, \forall [i, j] \in E$$

$$y_{ij} \in 0, \forall [i, j] \in E'$$

Problem 3

- Consider a model, where the satisfaction level of the tourist needs to maximised over a fixed interval of time
    - Models a tourist who prioritises visiting exhibits with higher popularity index in order to leave the museum at the end of the day with maximum satisfaction

Note: Satisfaction level based on popularity index of an exhibit is approximated to be a deterministic variable in this case, complexities can be added in the future by considering it to be a probabilistic variable.

$$max \sum_{i=1}^{n} s_i \sum_{j} y_{ij}$$

$$\sum_{i=1}^{n-1} y_{ij} = \sum_{k=2}^{n} y_{jk} \leq 1, s.t. j = 2, \ldots, n-1$$

$$y_{ij} \in 0, 1 \forall i, j \in V$$

$$\sum_{i} \sum_{j} y_{ij} \leq |S| - 1, S \subset V, 2 \leq |S| \leq n-2$$

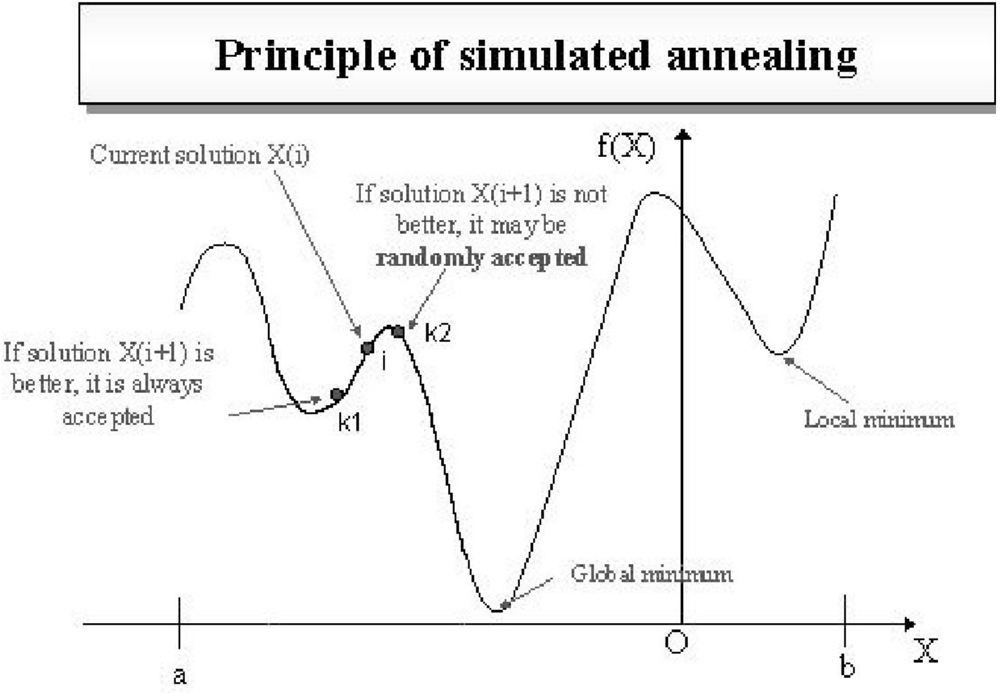
$$\sum_{i} \sum_{j} (\frac{c_{ij}}{v} + \tau) y_{ij} \leq T_0$$

Pseudo Codes

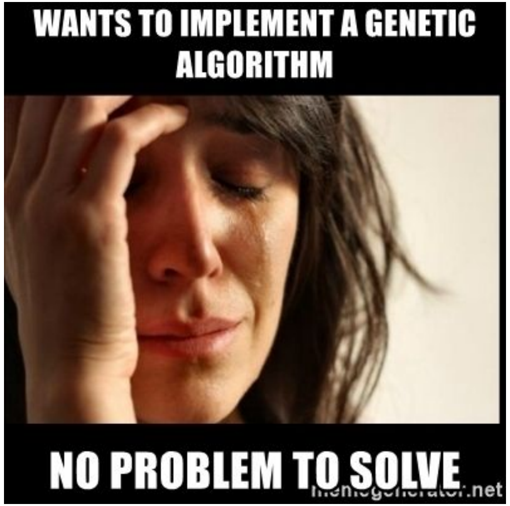
Simulated Annealing

[Link](#) to Pseudo Code.

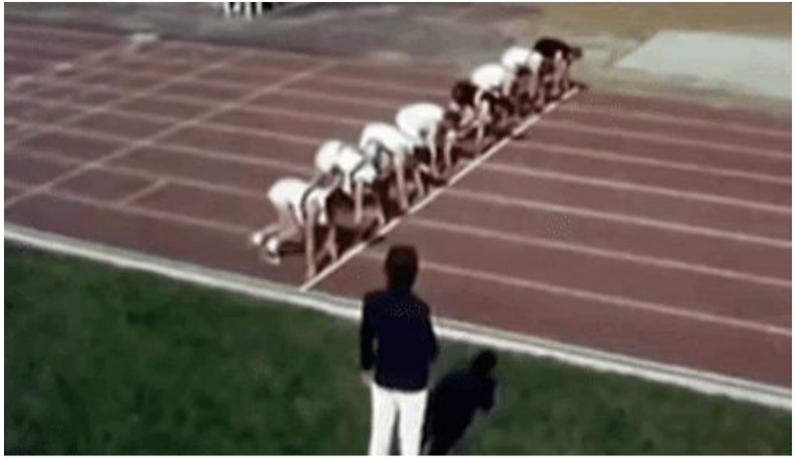
$$S_c = \{[1, 8, 2, 5], [9, 7, 10]\}$$
$$QueueList = [3, 4, 6]$$



Genetic

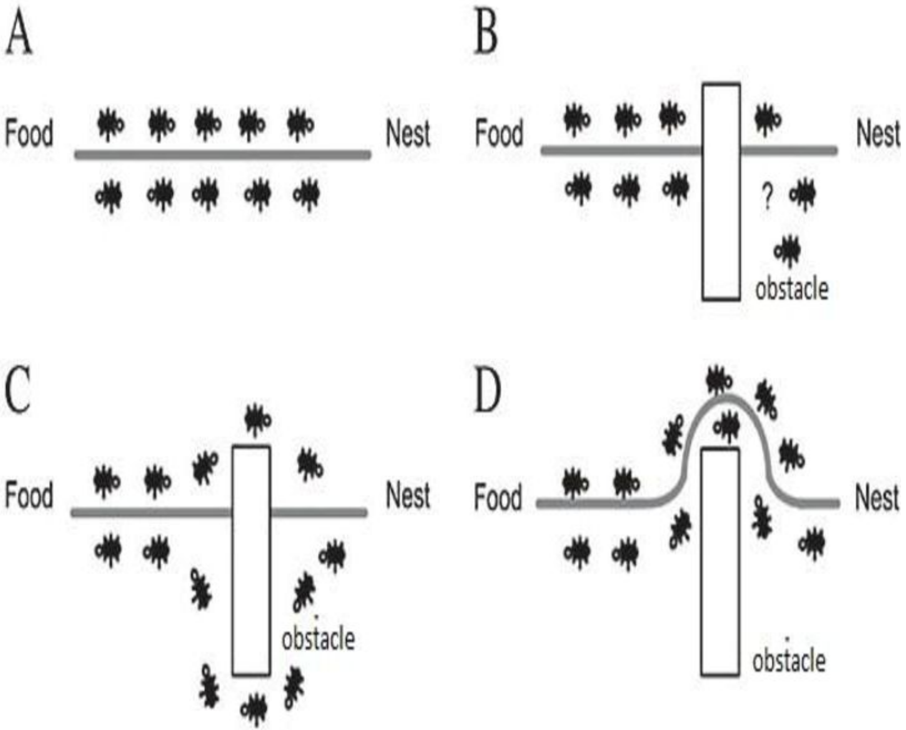


[Link](#) to Pseudo Code.



When you watch the first generation of your genetic algorithm

Ant Colony



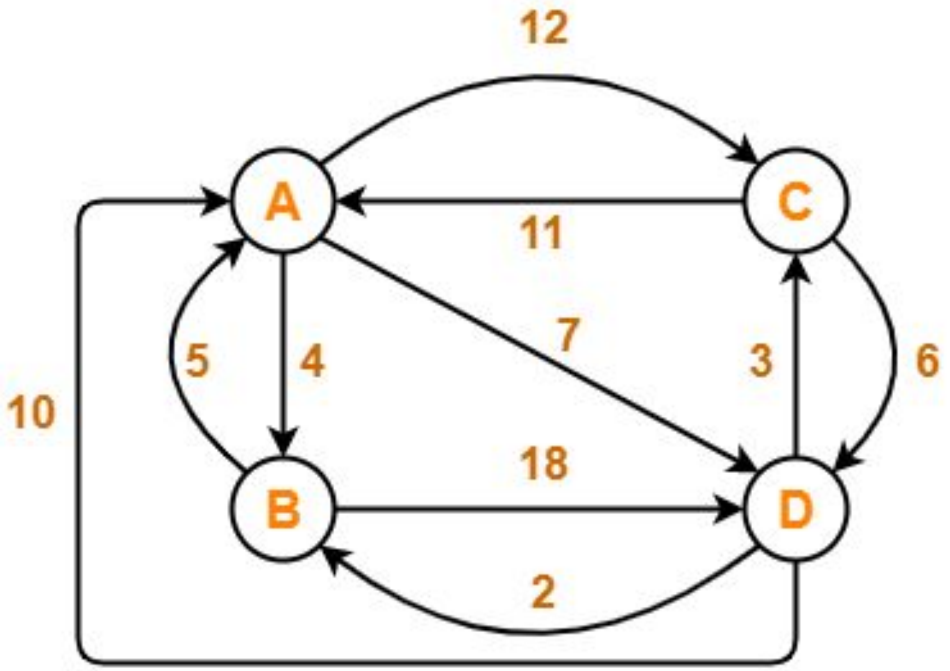
Link to Pseudo Code.

Plan of Action - Analysis

- Solve each problem separately using all the selected algorithms to understand the nuances of the problem and search algorithms
- Use Branch-and-Bound algorithms to obtain the global optimal solution.
- Compare the results with available literature and TSP libraries
- Solve the complete problem statement with all of the complexities

Branch-and-Bound

Link to Pseudo Code.

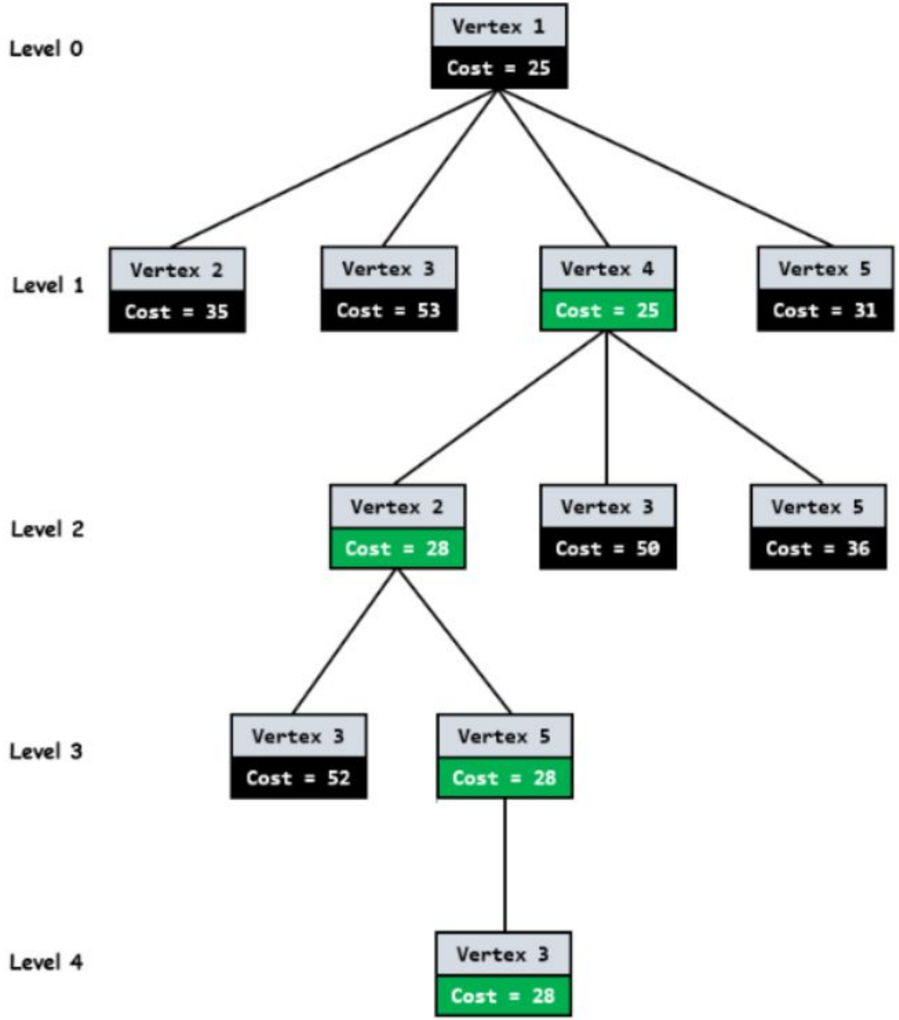


Matrix Reduction

$$\begin{matrix} INF & 20 & 30 & 10 & 11 \\ 15 & INF & 16 & 4 & 2 \\ 3 & 5 & INF & 2 & 4 \\ 19 & 6 & 18 & INF & 3 \\ 16 & 4 & 7 & 16 & INF \end{matrix}$$

$$\begin{matrix} INF & 10 & 20 & 0 & 1 \\ 13 & INF & 14 & 2 & 0 \\ 1 & 3 & INF & 0 & 2 \\ 16 & 3 & 15 & INF & 0 \\ 12 & 0 & 3 & 12 & INF \end{matrix}$$

$$\begin{matrix} INF & 10 & 17 & 0 & 1 \\ 12 & INF & 11 & 2 & 0 \\ 0 & 3 & INF & 0 & 2 \\ 15 & 3 & 12 & INF & 0 \\ 11 & 0 & 0 & 12 & INF \end{matrix}$$



Performance Metrics

- Once the algorithms are implemented they can be compared using performance metrics
- The performance metrics to be used are Efficiency, Reliability and Quality of Solution
- Efficiency involves measuring number of fundamental evaluations, running time and memory consumption of the algorithms
- Reliability can be checked by considering aspects like success rate, number of constraint violations, if any, and effect of starting point choice.
- Quality of Solution is determined by measuring computational accuracy, time taken to find optimal value within a fixed range of the solution or accuracy of the solution after running for a certain number of iterations or a certain time.

TSPLIB Test Cases

- TSPLIB is a library of sample instances for the TSP (and related problems) from various sources and of various types.
- TSPLIB has test case problems of Symmetric and Asymmetric TSP Problems, with varying number of nodes, which will be used for performance evaluation of the implemented algorithms

References

Optimal walk around path in a museum to view all exhibits

- Optimal Museum Traversal Using Graph Theory
  - Explains basics of Hamiltonian path
- Warehouse Optimization - Algorithms For Picking Path Optimization
  - Gives a brief about all kinds of algorithms which can be employed for Path Optimization

Travelling Salesman Problem (TSP)

- Travelling salesman problem - Wikipedia
  - Explains the problem, formulations and constraints
  - Talks about the different algorithms as well
- Travelling Salesman Problem | Set 1 (Naive and Dynamic Programming)
  - There a lot of implementations of different algorithms for solving TSP in the Related Articles section
- Chapter 10 The Traveling Salesman Problem
  - Detailed paper talking about various solutions to TSP, and their analysis and their time complexity
- Particle swarm optimization for traveling salesman problem
- Traveling Salesman Problem: an Overview of Applications, Formulations, and Solution Approaches

References

Problem Formulation

- Travelling Salesman Formulation
  - Traveling salesman problems
- School Bus Problem - This paper has a lot of variants
  - https://dspace.mit.edu/bitstream/handle/1721.1/5363/OR-078-78.pdf?sequence=1&isAllowed=y
- An Exact Algorithm for the Time-Constrained Traveling Salesman Problem
  - https://scihub.wikicn.top/10.1287/opre.31.5.938

Algorithms

- Dynamic Programming, Simulated Annealing, and 2-opt:
  - How to Solve Traveling Salesman Problem — A Comparative Analysis
- Genetic Algorithm, Simulated Annealing, Particle Swarm Optimization, Ant Colony Optimization, Bacteria Foraging Optimization, and Bee Colony Optimization
  - (PDF) Optimization Techniques for Solving Travelling Salesman Problem

References

Genetic Algorithm

- A Genetic Algorithm for Solving Travelling Salesman Problem
- A genetic algorithm for the orienteering problem - IEEE Conference Publication
- A genetic algorithm to design touristic routes in a bike sharing system | Request PDF

Simulated Annealing

- A simulated annealing heuristic for the team orienteering problem with time windows
- A simulated annealing heuristic for the multiconstraint team orienteering problem with multiple time windows
- Solving tourist trip planning problem via a simulated annealing algorithm

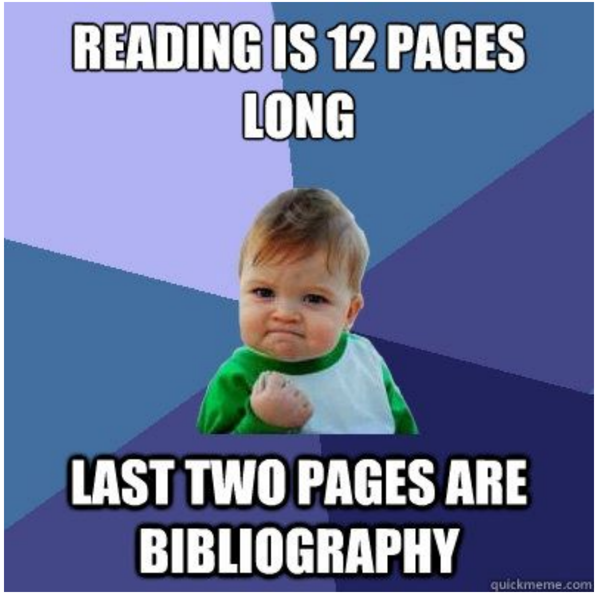
Ant Colony Algorithm

- (PDF) An ant colony approach to the orienteering problem
- Ant colony approach to the orienteering problem

Branch-and-Bound

- A Proposed Solution to Travelling Salesman Problem using Branch and Bound
- Traveling Salesman Problem: an Overview of Applications, Formulations, and Solution Approaches
- Branch and Bound for TSP

🙂


READING IS 12 PAGES LONG
LAST TWO PAGES ARE BIBLIOGRAPHY
quickmeme.com