The Museum Problem

Édition du musée du Louvre

Team - Sic Mundus

Outline

The Problem we face - a statement

Design Variables we understand

The simplest we begin

The complexities we include

The algorithms we employ

Taking the next step

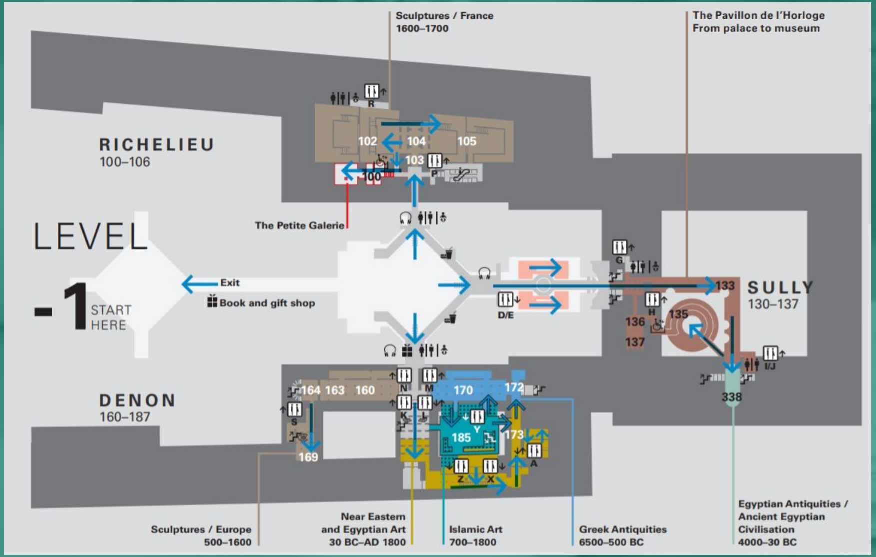
The Problem

The problem started with someone at the gate wondering:

“What if I could avoid crowds, see lots of exhibits and still get back on time?”

Bringing us to their rescue

Problem Statement



Optimize the route for a tourist visiting The Louvre Museum, such that the satisfaction level is maximised by visiting all/select exhibits in a single working day. With coinciding exit and entry points.

Design Variables

1. $(2 \text{ X } {}^{N}C_2)$ - Path indicator variables
2. (N) - Time to be spent at each exhibit

The Team with a Plan

- Break down the problem into its simplest objectives:
    - Minimise path length
    - Minimise travel time
    - Introduce obstacles/one-way routes in the path
    - Maximise satisfaction level
- Solve each objective separately in the beginning to understand the nuances of the problem and search algorithms
- Research mixed integer programming algorithms
- Solve the complete problem statement with all of the complexities

List of Symbols

$$y_{ij} = \left\{ \begin{array}{l} 1, \text{If path goes from exhibit } i \text{ to exhibit } j \\ 0, \text{otherwise} \end{array} \right\}$$

$$c_{ij} := \text{Penalty incurred for going from } i \text{ to } j$$

$$c_{ij} > 0$$

$$G := (V, \mathbb{E})$$

$$V := \text{Vertices}$$

$$\mathbb{E} := \{(x, y) | x, y \in V\}$$

$$S := \text{Set of all tours of } G$$

$$E := \text{Subset of permitted paths in } \mathbb{E}$$

$$E' := \text{Complement of E}$$

$$v = \text{Walking speed}$$

$$\tau = \text{Time spent at each exhibit}$$

$$T_o = \text{Total available time}$$

Problem 1

- The simplest model of the museum path optimization problem, which requires the tourist to visit all the exhibits located at lattice points separated by known distances.
- The total path is minimised in this problem

$$\min \sum_i \sum_j c_{ij} y_{ij}$$

$$\text{s.t} \sum_{i<k} y_{ik} + \sum_{j>k} y_{kj} = 2, \quad k \in V$$

$$\sum_i \sum_j y_{ij} \leq |S| - 1 \quad S \subset V, 3 \leq |S| \leq n-3$$

$$y_{ij} \in \{0,1\} \; \forall i,j \in E$$

Problem 2

- Consider a model, where certain points cannot be reached by all of the points in the space of lattice points
  - Models exhibits in museum situated at different floors; can be accessed only from certain entry/exit points.
- Problem is asymmetric, represents paths which don't exist in both directions.
  - Models one-way routes, and/or routes with different departure and arrival rates.

$$\min \sum_i \sum_j c_{ij} y_{ij}$$

$$\text{s.t} \sum_j y_{ij} = 1, \quad i = 0, 1, ..., n-1$$

$$\sum_i y_{ij} = 1, \quad j = 0, 1, ..., n-1$$

$$\sum_i \sum_j y_{ij} \leq |S| - 1 \quad S \subset V, 2 \leq |S| \leq n-2$$

$$y_{ij} \in \{0, 1\} \; \forall i, j \in E$$

$$y_{ij} = 0, \forall i, j \in E'$$

Problem 3

- Consider a model, where the satisfaction level of the tourist needs to maximised over a fixed interval of time
  - Models a tourist who prioritises visiting exhibits with higher popularity index in order to leave the museum at the end of the day with maximum satisfaction

$$\max \sum_{i=1}^{n} s_i \sum_{j} y_{ij}$$

$$\sum_{j}^{n} y_{ij} \in \{0,1\}, i = \{0,1,\ldots n-1\}$$

$$\sum_{i}^{n} y_{ij} \in \{0,1\}, j = \{0,1,\ldots n-1\}$$

$$y_{ij} \in \{0,1\} \forall i,j \in V$$

$$\sum_{i}\sum_{j} y_{ij} \leq |S|-1, S \subset V, 2 \leq |S| \leq n-2$$

$$\sum_{i}\sum_{j} \left(\frac{c_{ij}}{v} + \tau\right) y_{ij} \leq T_o$$

Search Algorithms

- **Exact:**
  - Branch-and-bound Algorithm
  - Simplex Method (LP)
- **Heuristic algorithms**
  - The Held-Karp lower bound to be used to judge the performance of a heuristic algorithm
  - **Tour construction procedures:**
    - Greedy
  - **Tour improvement procedures:**
    - 2-opt and 3-opt
    - Simulated Annealing
    - Genetic
    - Ant Colony/ Particle Swarm

References

Optimal walk around path in a museum to view all exhibits

- Optimal Museum Traversal Using Graph Theory
    - Explains basics of Hamiltonian path
- Warehouse Optimization - Algorithms For Picking Path Optimization
    - Gives a brief about all kinds of algorithms which can be employed for Path Optimization

Travelling Salesman Problem (TSP)

- Travelling salesman problem - Wikipedia
    - Explains the problem, formulations and constraints
    - Talks about the different algorithms as well
- Travelling Salesman Problem | Set 1 (Naive and Dynamic Programming)
    - There a lot of implementations of different algorithms for solving TSP in the Related Articles section
- Chapter 10 The Traveling Salesman Problem
    - Detailed paper talking about various solutions to TSP, and their analysis and their time complexity
- Particle swarm optimization for traveling salesman problem
- Traveling Salesman Problem: an Overview of Applications, Formulations, and Solution Approaches

References

Problem Formulation

- Travelling Salesman Formulation
  - Traveling salesman problems
- School Bus Problem - This paper has a lot of variants
  - https://dspace.mit.edu/bitstream/handle/1721.1/5363/OR-078-78.pdf?sequence=1&isAllowed=y
- An Exact Algorithm for the Time-Constrained Traveling Salesman Problem
  - https://scihub.wikicn.top/10.1287/opre.31.5.938

Algorithms

- Dynamic Programming, Simulated Annealing, and 2-opt:
  - How to Solve Traveling Salesman Problem — A Comparative Analysis
- Genetic Algorithm, Simulated Annealing, Particle Swarm Optimization, Ant Colony Optimization, Bacteria Foraging Optimization, and Bee Colony Optimization
  - (PDF) Optimization Techniques for Solving Travelling Salesman Problem