

# Hybrid Intelligent Algorithm for Indoor Path Planning and Trajectory-Tracking Control of Wheeled Mobile Robot

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**Abstract** This paper presents a hybrid intelligent algorithm for a wheeled mobile robot (WMR) to implement both trajectory-tracking and path-following navigation missions. The novel control scheme combining the kinematic with TSK fuzzy control is developed to track the desired position, linear, and angular velocities, even though the WMR suffers from system uncertainties and disturbances. The proposed TSK fuzzy controller deals with a general dynamic model and has a good ability of disturbance rejection. For the path-following issue, the improved D\* lite algorithm determines an appropriate path between an initial position and a destination. The derived path is transformed into a tracking trajectory by a function of time. The asymptotic stability of the overall system is proven by Lyapunov theory. Finally, real-time experiments with the use of the proposed hybrid intelligent algorithm on an eight-shaped reference trajectory and long-distance movement demonstrate the feasibility of practical WMR maneuvers.

**Keywords** Wheeled mobile robot (WMR) · D\* lite algorithm · Trajectory-tracking control

## 1 Introduction

Recently, wheeled mobile robots (WMRs) have been widely developed to execute tasks in unstructured environments, where a high degree of autonomy is required, to make them suitable for applications of floor cleaning [1], patrolling [2, 3], transferring goods [4], autonomous wheelchairs [5, 6]. All of the applications are required for a stable navigation, and the navigation problems could be categorized into three basic problems [7], which are the trajectory-tracking, the path-following, and the point stabilization. The idea behind these problems is designing a velocity controller to stabilize the closed-loop system, in which a reference model generates desired motions that the WMRs are supposed to follow. The motions could be a desired time-parameterized trajectory for the trajectory-tracking, a desired xy-geometric path for the path-following, and a desired posture for the point stabilization.

The motion control can be simply divided into two parts: one utilizes a kinematic tracking controller to procure the tracking issue, while the other integrates a kinematic tracking controller with a dynamic tracking controller and expects to deal with a system with modeling uncertainties in the dynamic model. Carellia and Freire [8] proposed a kinematic-based tracking controller for both wall-following and hallway navigation issues, in which the sensing information is from an odometric sensor and sonar sensors. Kühn et al. [9] proposed nonlinear/linear model-based predictive controls in the sense of the kinematic model for the trajectory-tracking problem and the control effectiveness was shown in simulation results. Otherwise, most controller designs [10, 11] are based on the

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kinematics of the mobile robot. However, strategy-only-based kinematic models are difficult in controlling WMRs because, in the real world, there are uncertainties in their modeling and the “perfect velocity tracking” is impossible to be achieved. When high-speed movements and/or existences of external disturbances are required, it becomes essential to consider the robot dynamics.

Fierro and Lewis [12] proposed a dynamical extension that integrates a kinematic controller with a torque controller for nonholonomic WMRs and is applied it to the basic navigation problems, in which a backstepping control approach was considered. Fukao et al. [13] proposed a torque adaptive controller under the consideration of system uncertainties for the dynamical extension. However, only the simulation results were presented to verify the effectiveness. De La Cruz and Carelli [14] developed a complete dynamic model of WMRs for a multi-robot formation, in which the usual inputs in commercial robots,  $v$  and  $\omega$ , are considered as the input signals, and the inverse dynamics technique is then applied to design a centralized formation control. However, when the parameters of the dynamic model are unknown and/or change in time, the desired level of control system performance is difficult to be maintained. Therefore, providing a systematic approach for automatic adjustment of controllers in real-time is an important task. Chen et al. [15] proposed, for WMRs, a two-stage tracking controller, in which an adaptive sliding mode control was designed in order to eliminate external disturbances and uncertainty. However, the chattering phenomenon resulting in constant switch of the control law happens. Antonini et al. [16] designed an adaptive dynamic controller according to the linear and angular velocities for commercial robots, and the stability of the whole system was analyzed using Lyapunov theory. There were much effort in designing a dynamic controller [12–16] for trajectory-tracking in past, but design and implementation of a control system which provides both the trajectory-tracking and the path-following are few.

Considering the situations in which there are parametric uncertainties in the model of WMRs, having the design of a dynamic controller is an important issue. Over the past few decades, Tagaki–Sugeno–Kang (TSK) fuzzy control has attracted a great deal of attention due to its ability of coping with uncertainties. TSK fuzzy systems are nonlinear systems which are described by a set of IF–THEN rules. Because the technique of TSK fuzzy control is a very effective tool, some adaptive control schemes [17–19] via TSK fuzzy logic system have been proposed in the area of system dynamics modeling and stability analysis. In [17], a control scheme of online modeling and adaptive control via TSK fuzzy model was proposed for uncertain nonlinear systems, for which the parameters of the consequent part are updated online. Tanaka and Sugeno [18] proposed a useful theory using fuzzy block diagrams for analyzing the stability of a TSK fuzzy model.

Feng et al.’s [19] interpretation of the TSK fuzzy system can approximate any continuous function at any precision.

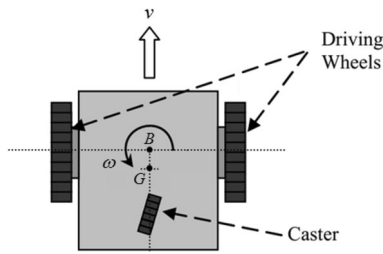
This paper considers not only the trajectory-tracking problem but also the path-following problem for the motion control. The path planning is also one of the major issues for autonomous robots. It is used to determine a moving path from the robot’s position to a destined position according to specific requirements, such as the shortest path. A\* [20], D\* [21], and D\* Lite [22] are commonly utilized in the path planning. A\* algorithm actively eliminates the paths with obvious errors so that the calculating speed increases. D\* algorithm takes most of the paths into consideration and it can recover the path immediately when the environment changes. However, it results in huge calculating time. On the one hand, D\* lite algorithm is a simplified version of D\*, and it executes with better efficiency by reducing the redundant parameters of D\*. On the other hand, some literatures [8, 23, 24] about wall-following control were proposed for WMR in indoor navigation problems. However, the wall-following path is impossible to be the shortest one and has serious limitations in an unstructured environment, e.g., stairwells.

On the whole, this paper develops a combined kinematic/TSK fuzzy control for both trajectory-tracking and path-following navigation missions in order to achieve a stable long-distance travel, in which the backstepping control technology and TSK fuzzy control system are utilized to design the kinematic controller and the dynamic controller, respectively. For commercial robots, the kinematic controller first generates desired values of the linear and angular velocities. Next, the proposed TSK fuzzy control makes the real velocities of WMRs reach the desired values, even though the WMR suffers from the system uncertainties and disturbances. Although studies about two-stage trajectory-tracking controllers [25, 26] have been made on WMR systems, little is known about both trajectory-tracking and path-following problems. Compared with the previous approaches, the proposed control scheme is suitable in real-world applications. Moreover, the improved D\* Lite algorithm executes with better efficiency and avoids the collision with the wall.

The rest of this paper is organized as follows. Section 2 describes the kinematic controller design of WMR. Section 3 discusses the dynamic controller design of WMR. The improved D\* Lite algorithm is described in Sect. 4. Some experimental results are shown in Sect. 5. Finally, some conclusions are presented in Sect. 6.

## 2 Kinematic Controller Design for WMRs

A two-wheel driven mobile robot (unicycle-type) [27–29] is shown in Fig. 1  $v$  and  $\omega$  are the linear and angular velocities developed by the WMR, respectively.  $G$  is the center of mass of the WMR.  $B$  is the central point of the



**Fig. 1** Simple model of a WMR

virtual axis linking the two driving wheels. The motion and orientation are achieved by independent actuators providing necessary torques to the driving wheels. Under assumptions of in which the coordinate of mass of the WMR is located in the middle of the rear driving wheels, the kinematic model under the nonholonomic constraint of pure rolling and nonslipping is given as

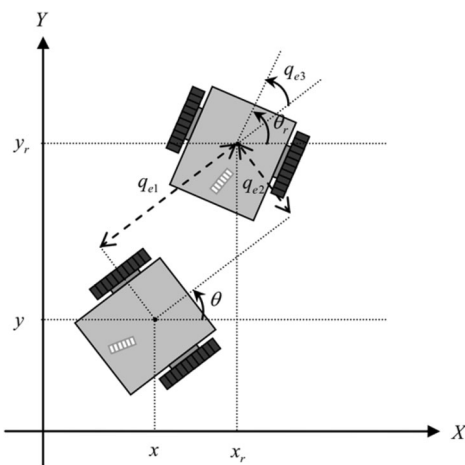
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \mathbf{V} \quad (1)$$

where  $\mathbf{V} = [v, \omega]^T$ .  $x(t)$ ,  $y(t)$ , and  $\theta(t)$ , respectively, represent as the ground truth position and orientation of the WMR at current time  $t$ .  $x_r(t)$ ,  $y_r(t)$ , and  $\theta_r(t)$ , respectively, represent the desired position and orientation of the WMR at current time  $t$ . The vectors of the ground truth posture  $\mathbf{q}$  and the reference posture  $\mathbf{q}_r$  are given as follows:

$$\mathbf{q}(t) = [x(t), y(t), \theta(t)]^T, \quad \mathbf{q}_r(t) = [x_r(t), y_r(t), \theta_r(t)]^T \quad (2)$$

Moreover, the posture error vector  $\mathbf{q}_e$  is defined as (3), as shown in Fig. 2.

$$\mathbf{q}_e = \begin{bmatrix} q_{e1} \\ q_{e2} \\ q_{e3} \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \quad (3)$$



**Fig. 2** Posture error coordinate

Differentiating (3) with respect to time, we can have the dynamic error equation as follows:

$$\begin{bmatrix} \dot{q}_{e1} \\ \dot{q}_{e2} \\ \dot{q}_{e3} \end{bmatrix} = v \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + \omega \begin{bmatrix} q_{e2} \\ -q_{e1} \\ -1 \end{bmatrix} + \begin{bmatrix} v_r \cos q_{e3} \\ v_r \sin q_{e3} \\ \omega_r \end{bmatrix} \quad (4)$$

where  $v_r$  and  $\omega_r$  denote the reference linear and angular velocities, respectively.  $\mathbf{V}_r = [v_r, \omega_r]^T$  is a vector of reference signals. A common velocities command using the well-known backstepping method is designed as (5). The kinematic control law (5) is recommended by many studies [15, 30] for the tracking problem of the WMR.

$$\begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} v_r \cos q_{e3} + \alpha_1 q_{e1} \\ \omega_r + \alpha_2 v_r q_{e2} + \alpha_3 v_r \sin q_{e3} \end{bmatrix} \quad (5)$$

where  $\mathbf{V}_c = [v_c, \omega_c]^T$  is the appropriate velocity control law.  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$  are chosen positive constants and  $v_r > 0$ .

### 3 Dynamic Adaptive TSK Fuzzy Controller Design for Wheeled Mobile Robot

By using the dynamic controller, the actual velocities of the WMR can track the predicted velocities. The well-known dynamic model [25] of WMR is described as follows:

$$\begin{cases} \dot{v} = f_1(v, \omega, v_d, \omega_d) = \frac{k_3}{k_1} \omega^2 - \frac{k_4}{k_1} v + \frac{1}{k_1} v_d \\ \dot{\omega} = f_2(v, \omega, v_d, \omega_d) = -\frac{k_5}{k_2} v \omega - \frac{k_6}{k_2} \omega + \frac{1}{k_2} \omega_d \end{cases} \quad (6)$$

where a vector  $\mathbf{K} = [k_1, k_2, \dots, k_6]$  was firstly presented in [31] and  $k_i$  are reproduced in (7)–(12). Table 1 shows the symbols and quantities of some physical parameters. It should be stressed that  $k_i > 0$ ,  $i = 1, 2, 4$ , and 6. The values of  $k_3$  and  $k_5$  can be influenced by the distance  $b$  between the point  $G$  and the point  $B$ . In this paper, because we consider a full equation of the dynamic model, the center G of WMR's mass is not exactly in the point  $B$  of the virtual axis linking the two driving wheels, i.e.,  $b \neq 0$ .  $\mathbf{V}_d = [v_d, \omega_d]^T$  is a vector of the dynamic control law. It is assumed that the WMR servos have PD controllers to control the velocities of each motor with  $k_{PT} > 0$ ,  $k_{PR} > 0$ ,  $k_{DT} \geq 0$ , and  $k_{DR} \geq 0$ . It is also assumed that the motors associated with the driven wheels are DC motors with identical characteristics and with negligible inductance.

$$k_1 = \frac{R_a}{k_a} (mR_t r + 2I_e) + 2rk_{DT} \quad (7)$$

$$k_2 = \frac{R_a}{k_a} (I_e d^2 + 2R_t r(I_z + mb^2)) + 2rdk_{DR} \quad (8)$$

**Table 1** List of notations

Symbol	Quantity
$R_a$	Electrical resistance of motors
$R_t$	Nominal radius of tires
$k_a$	Torque of motors
$k_b$	Electromotive constant of motors
$m$	Mass of wheeled mobile robot
$r$	Radius of the wheels
$I_e$	Moment of inertia of each group rotor-reduction gear-wheel
$k_{DT}$	Derivative gain
$k_{PT}$	Proportional gain
$k_{DR}$	Derivative gain
$k_{PR}$	Proportional gain
$b$	Distance between point $G$ and point $B$
$d$	Distance between two wheels
$I_z$	Moment of inertia at point $G$
$B_e$	Coefficient of friction

$$k_3 = \frac{R_a}{k_a} \left( \frac{mbR_t}{2k_{PT}} \right) \quad (9)$$

$$k_4 = \frac{\frac{R_a}{k_a} \left( \frac{k_a k_b}{R_a} + B_e \right)}{rk_{PT}} + 1 \quad (10)$$

$$k_5 = \frac{mbR_a R_t}{dk_a k_{PR}} \quad (11)$$

$$k_6 = \frac{R_a}{k_a} \left( \frac{k_a k_b}{R_a} + B_e \right) \frac{d}{2rk_{PR}} + 1 \quad (12)$$

Then we use Taylor series expansion of (6) around operation states  $V_o = [v_o, \omega_o]^T$  and  $V_{do} = [v_{do}, \omega_{do}]^T$ . Therefore, a virtual linearized system (VLS) [32] is written as

$$\dot{\mathbf{V}}_\delta = \mathbf{A}\mathbf{V}_\delta + \mathbf{B}\mathbf{V}_{d\delta} + \mathbf{d} \quad (13)$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} d_{h1} + f_1(\mathbf{V}_o, \mathbf{V}_{do}) \\ d_{h2} + f_2(\mathbf{V}_o, \mathbf{V}_{do}) \end{bmatrix}$$

where  $d_{hj}$  ( $j = 1, 2$ ) stands for high-order terms,

$$\mathbf{V}_\delta = [v_{\delta 1}, v_{\delta 2}]^T = \mathbf{V} - \mathbf{V}_o,$$

$$\mathbf{V}_{d\delta} = [v_{d\delta 1}, v_{d\delta 2}]^T = \mathbf{V}_d - \mathbf{V}_{do},$$

$$\mathbf{A} = \begin{bmatrix} a_{11}(\mathbf{V}_o, \mathbf{V}_{do}) & a_{12}(\mathbf{V}_o, \mathbf{V}_{do}) \\ a_{21}(\mathbf{V}_o, \mathbf{V}_{do}) & a_{22}(\mathbf{V}_o, \mathbf{V}_{do}) \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} b_{11}(\mathbf{V}_o, \mathbf{V}_{do}) & b_{12}(\mathbf{V}_o, \mathbf{V}_{do}) \\ b_{21}(\mathbf{V}_o, \mathbf{V}_{do}) & b_{22}(\mathbf{V}_o, \mathbf{V}_{do}) \end{bmatrix}$$

The TSK fuzzy model [33–35] is designed as

$R^{(i)}$ : If  $z_1$  is  $F_1^i$  and  $z_2$  is  $F_2^i$  and  $z_3$  is  $F_3^i$  and  $z_4$  is  $F_4^i$

$$\text{Then } y_l = p_{l1}^i z_1 + p_{l2}^i z_2 + p_{l3}^i z_3 + p_{l4}^i z_4$$

where  $\mathbf{z} = [z_1, z_2, z_3, z_4]^T \in \mathbb{R}^4$  is a vector of states,  $y_l$  is the system output,  $F_j^i$  are fuzzy sets, and  $p_{lk}^i$  are adjustable parameters. The output,  $p_{lk}$ , of the TSK fuzzy is

$$p_{lk} = \frac{\sum_{i=1}^h p_{lk}^i \left( \prod_{j=1}^4 \mu_{F_j^i}(z_j) \right)}{\sum_{i=1}^h \left( \prod_{j=1}^4 \mu_{F_j^i}(z_j) \right)} \quad (14)$$

where  $\mu_{F_j^i}(z_j)$  is the value of the membership function. For the tuning of the weighting factors  $p_{lk}^i$ ,  $w^i$  ( $i = 1, 2, \dots, h$ ) are defined as

$$w^i \equiv \frac{\prod_{j=1}^4 \mu_{F_j^i}(z_j)}{\sum_{i=1}^h \left( \prod_{j=1}^4 \mu_{F_j^i}(z_j) \right)} \quad (15)$$

Based on the above assumptions, for the purpose of approximating the VLS in (13), the  $i$ th fuzzy implication can be described as

$R^i$ : If  $v_{\delta 1}$  is  $F_1^i$  and  $v_{\delta 2}$  is  $F_2^i$  and  $v_{d\delta 1}$  is  $F_3^i$  and  $v_{d\delta 2}$  is  $F_4^i$

$$\text{Then } \dot{\mathbf{V}}_\delta = \hat{\mathbf{A}}^i \mathbf{V}_\delta + \hat{\mathbf{B}}^i \mathbf{V}_{d\delta}$$

(16)

where

$$\hat{\mathbf{A}}^i = \begin{bmatrix} p_{11}^i & p_{12}^i \\ p_{21}^i & p_{22}^i \end{bmatrix} \quad \text{and} \quad \hat{\mathbf{B}}^i = \begin{bmatrix} p_{13}^i & p_{14}^i \\ p_{23}^i & p_{24}^i \end{bmatrix}$$

After applying some commonly used defuzzification strategies, we can obtain

$$\dot{\mathbf{V}}_\delta = \sum_{i=1}^h w^i \{ \hat{\mathbf{A}}^i \mathbf{V}_\delta + \hat{\mathbf{B}}^i \mathbf{V}_{d\delta} \}$$

$$= \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \mathbf{V}_\delta + \begin{bmatrix} p_{13} & p_{14} \\ p_{23} & p_{24} \end{bmatrix} \mathbf{V}_{d\delta} \quad (17)$$

where  $p_{ij}$  is used to approximate  $a_{ij}$  and  $b_{ij}$  of (13).

In order to compensate for the total effect of high-order terms, unmodeled dynamics, and modeling errors  $\mathbf{d}_f = [d_{f1}, d_{f2}]^T$ , we redefine the fuzzy approximator including total error

$$\tilde{\mathbf{d}} = \begin{bmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{bmatrix} = \begin{bmatrix} d_{h1} + f_1(\mathbf{V}_o, \mathbf{V}_{do}) + d_{f1} \\ d_{h2} + f_2(\mathbf{V}_o, \mathbf{V}_{do}) + d_{f2} \end{bmatrix}$$

as follows:

$$\dot{\mathbf{V}}_\delta = \sum_{i=1}^h w^i \{ \hat{\mathbf{A}}^i \mathbf{V}_\delta + \hat{\mathbf{B}}^i \mathbf{V}_{d\delta} \} + \tilde{\mathbf{d}} \quad (18)$$

We define the velocity tracking error  $\mathbf{e}_v = \mathbf{V} - \mathbf{V}_c$  and a coefficient matrix [36–39]  $\mathbf{\Lambda} = [-\lambda_1, 0; 0, -\lambda_2]$ , where the coefficients,  $\lambda_1$  and  $\lambda_2$ , are selected such that  $\mathbf{\Lambda}$  is a Hurwitz matrix. Then the dynamic control law is designed as

$$\mathbf{V}_d = \left( \sum_{i=1}^h w^i \hat{\mathbf{B}}^i \right)^{-1} \left( - \sum_{i=1}^h w^i \hat{\mathbf{A}}^i \mathbf{V}_\delta + \dot{\mathbf{V}}_c + \mathbf{\Lambda} \mathbf{e}_v - \mathbf{V}_{ds} \right) + \mathbf{V}_{do} \quad (19)$$

where  $\mathbf{V}_{ds} = \text{Diag}[\text{sign}(\mathbf{e}_v^T \mathbf{\Gamma})] \mathbf{K}_v$ ,  $\mathbf{\Gamma} > 0$  is a Lyapunov matrix and  $\mathbf{K}_v = [k_{v1}, k_{v2}]^T$  is a control gain vector. Using  $\dot{\mathbf{e}}_v = \dot{\mathbf{V}} - \dot{\mathbf{V}}_c$  and the virtual linearized system (VLS) (13), the error dynamic equation of the VLS is

$$\dot{\mathbf{e}}_v = \mathbf{\Lambda} \mathbf{e}_v + \sum_{i=1}^h w^i \tilde{\mathbf{A}}^i \mathbf{V}_\delta + \sum_{i=1}^h w^i \tilde{\mathbf{B}}^i \mathbf{V}_{d\delta} - \mathbf{V}_{ds} + \tilde{\mathbf{d}} \quad (20)$$

where  $\tilde{\mathbf{A}}^i = \hat{\mathbf{A}}^{i*} - \hat{\mathbf{A}}^i$  and  $\tilde{\mathbf{B}}^i = \hat{\mathbf{B}}^{i*} - \hat{\mathbf{B}}^i$ .

**Lemma 1** [40] Suppose that a matrix  $\mathbf{\Lambda} \in \mathbb{R}^{2 \times 2}$  is given. For every symmetric positive definite matrix  $\mathbf{Q} \in \mathbb{R}^{2 \times 2}$ , the Lyapunov matrix equation  $\mathbf{\Lambda}^T \mathbf{\Gamma} + \mathbf{\Gamma} \mathbf{\Lambda} = -\mathbf{Q}$  has a unique solution for  $\mathbf{\Gamma} = \mathbf{\Gamma}^T > 0$  if and only if  $\mathbf{\Lambda} \in \mathbb{R}^{2 \times 2}$  is a Hurwitz matrix. ♦

**Theorem 1** Consider a kinematic and dynamic models for a WMR under the nonholonomic constraint of pure rolling and nonslipping as (1) and (6), respectively. The kinematic controller is designed as (5). The dynamic controller is designed as (19) with update laws (21) and (22).

$$\dot{\hat{\mathbf{A}}}^i = \eta_1 w^i \mathbf{e}_v \mathbf{V}_\delta^T \quad (21)$$

$$\dot{\hat{\mathbf{B}}}^i = \eta_2 w^i \mathbf{e}_v \mathbf{V}_{d\delta}^T \quad (22)$$

where the control gain  $\mathbf{K}_v$  is selected such  $\sum_{j=1}^2 k_{vj} |\mathbf{e}_v^T \mathbf{\Gamma}| > \bar{k}_v$ ,  $\eta_i$  and  $\bar{k}_v$  are positive constants. Then, the actual position, linear velocity, and angular velocity of the WMR can reach the predicted ones.

*Proof* Consider the Lyapunov-like function candidate as  $M = M_1 + M_2$

where

$$M_1 = \frac{1}{2} (q_{e1}^2 + q_{e2}^2) + \frac{1 - \cos q_{e3}}{\alpha_2} \quad (23)$$

$$M_2 = \frac{1}{2} \mathbf{e}_v^T \mathbf{\Gamma} \mathbf{e}_v + \frac{1}{2\eta_1} \sum_{i=1}^h \text{tr}(\tilde{\mathbf{A}}^{iT} \mathbf{\Gamma} \tilde{\mathbf{A}}^i) + \frac{1}{2\eta_2} \sum_{i=1}^h \text{tr}(\tilde{\mathbf{B}}^{iT} \mathbf{\Gamma} \tilde{\mathbf{B}}^i). \quad (24)$$

Differentiating (23) with respect to time, we get

$$\dot{M}_1 = q_{e1} \dot{q}_{e1} + q_{e2} \dot{q}_{e2} + \frac{\sin q_{e3}}{\alpha_2} \dot{q}_{e3}. \quad (25)$$

Inserting (4) in the above equation yields

$$\begin{aligned} \dot{M}_1 &= q_{e1} (-v_c + \omega_c q_{e2} + v_r \cos q_{e3}) \\ &\quad + q_{e2} (-\omega_c q_{e1} + v_r \sin q_{e3}) + \frac{\sin q_{e3}}{\alpha_2} (\omega_r - \omega_c) \\ &= -v_c q_{e1} + v_r q_{e1} \cos q_{e3} + v_r q_{e2} \sin q_{e3} \\ &\quad + \frac{\sin q_{e3}}{\alpha_2} (\omega_r - \omega_c) \end{aligned} \quad (26)$$

Inserting (5) in the above equation yields

$$\begin{aligned} \dot{M}_1 &= -\alpha_1 q_{e1}^2 + v_r q_{e2} \sin q_{e3} \\ &\quad + \frac{\sin q_{e3}}{\alpha_2} (-\alpha_2 v_r q_{e2} - \alpha_3 v_r \sin q_{e3}) \\ &= -\alpha_1 q_{e1}^2 - \frac{\alpha_3 v_r}{\alpha_2} \sin^2 q_{e3} \end{aligned} \quad (27)$$

Therefore, if the reference velocity  $v_r \geq 0$  then  $\dot{M}_1 \leq 0$ . Differentiating (24) with respect to time, we get

$$\begin{aligned} \dot{M}_2 &= \frac{1}{2} \dot{\mathbf{e}}_v^T \mathbf{\Gamma} \mathbf{e}_v + \frac{1}{2} \mathbf{e}_v^T \mathbf{\Gamma} \dot{\mathbf{e}}_v + \frac{1}{2\eta_1} \sum_{i=1}^h \text{tr}(\dot{\tilde{\mathbf{A}}}^{iT} \mathbf{\Gamma} \tilde{\mathbf{A}}^i) \\ &\quad + \frac{1}{2\eta_1} \sum_{i=1}^h \text{tr}(\tilde{\mathbf{A}}^{iT} \mathbf{\Gamma} \dot{\tilde{\mathbf{A}}}^i) + \frac{1}{2\eta_2} \sum_{i=1}^h \text{tr}(\dot{\tilde{\mathbf{B}}}^{iT} \mathbf{\Gamma} \tilde{\mathbf{B}}^i) \\ &\quad + \frac{1}{2\eta_2} \sum_{i=1}^h \text{tr}(\tilde{\mathbf{B}}^{iT} \mathbf{\Gamma} \dot{\tilde{\mathbf{B}}}^i) \end{aligned} \quad (28)$$

Inserting (20) in the above equation yields

$$\begin{aligned} \dot{M}_2 &= \frac{1}{2} \mathbf{e}_v^T (\mathbf{\Lambda}^T \mathbf{\Gamma} + \mathbf{\Gamma} \mathbf{\Lambda}) \mathbf{e}_v + \mathbf{e}_v^T \mathbf{\Gamma} \sum_{i=1}^h w^i \dot{\tilde{\mathbf{A}}}^i \mathbf{V}_\delta \\ &\quad + \mathbf{e}_v^T \mathbf{\Gamma} \sum_{i=1}^h w^i \dot{\tilde{\mathbf{B}}}^i \mathbf{V}_{d\delta} + \frac{1}{\eta_1} \sum_{i=1}^h \text{tr}(\tilde{\mathbf{A}}^{iT} \mathbf{\Gamma} \dot{\tilde{\mathbf{A}}}^i) \\ &\quad + \frac{1}{\eta_2} \sum_{i=1}^h \text{tr}(\tilde{\mathbf{B}}^{iT} \mathbf{\Gamma} \dot{\tilde{\mathbf{B}}}^i) + \mathbf{e}_v^T \mathbf{\Gamma} \tilde{\mathbf{d}} - \mathbf{e}_v^T \mathbf{\Gamma} \mathbf{V}_{ds}. \end{aligned} \quad (29)$$

From Lemma 1, (29) can be rewritten as

$$\begin{aligned} \dot{M}_2 &= -\frac{1}{2}\mathbf{e}_v^T\mathbf{Q}\mathbf{e}_v + \mathbf{e}_v^T\Gamma\sum_{i=1}^h w^i\dot{\hat{\mathbf{A}}}^i\mathbf{V}_\delta + \mathbf{e}_v^T\Gamma\sum_{i=1}^h w^i\dot{\hat{\mathbf{B}}}^i\mathbf{V}_{d\delta} \\ &\quad + \frac{1}{\eta_1}\sum_{i=1}^h \text{tr}\left(\tilde{\mathbf{A}}^T\Gamma\dot{\hat{\mathbf{A}}}^i\right) + \frac{1}{\eta_2}\sum_{i=1}^h \text{tr}\left(\tilde{\mathbf{B}}^T\Gamma\dot{\hat{\mathbf{B}}}^i\right) \\ &\quad + \mathbf{e}_v^T\Gamma\tilde{\mathbf{d}} - \mathbf{e}_v^T\Gamma\mathbf{V}_{ds} \\ &= \Delta + \mathbf{e}_v^T\Gamma\tilde{\mathbf{d}} - \mathbf{e}_v^T\Gamma\text{Diag}[\text{sign}(\mathbf{e}_\Delta)]\mathbf{K}_v \\ &\leq \Delta + \bar{k} - \sum_{j=1}^n k_j|e_{\Delta j}| \end{aligned} \quad (30)$$

where

$$\begin{aligned} \Delta &= -\frac{1}{2}\mathbf{e}_v^T\mathbf{Q}\mathbf{e}_v + \mathbf{e}_v^T\Gamma\sum_{i=1}^h w^i\dot{\hat{\mathbf{A}}}^i\mathbf{V}_\delta + \mathbf{e}_v^T\Gamma\sum_{i=1}^h w^i\dot{\hat{\mathbf{B}}}^i\mathbf{V}_{d\delta} \\ &\quad + \frac{1}{\eta_1}\sum_{i=1}^h \text{tr}\left(\tilde{\mathbf{A}}^T\Gamma\dot{\hat{\mathbf{A}}}^i\right) + \frac{1}{\eta_2}\sum_{i=1}^h \text{tr}\left(\tilde{\mathbf{B}}^T\Gamma\dot{\hat{\mathbf{B}}}^i\right) \\ &= -\frac{1}{2}\mathbf{e}_v^T\mathbf{Q}\mathbf{e}_v + \text{tr}\left(\sum_{i=1}^h w^i\tilde{\mathbf{A}}^T\Gamma\mathbf{e}_v\mathbf{V}_\delta^T - \sum_{i=1}^h \frac{\tilde{\mathbf{A}}^T\Gamma\dot{\hat{\mathbf{A}}}^i}{\eta_1}\right. \\ &\quad \left.+ \sum_{i=1}^h w^i\tilde{\mathbf{B}}^T\Gamma\mathbf{e}_v\mathbf{V}_{d\delta}^T - \sum_{i=1}^h \frac{\tilde{\mathbf{B}}^T\Gamma\dot{\hat{\mathbf{B}}}^i}{\eta_2}\right) \end{aligned} \quad (31)$$

If we select  $\hat{\mathbf{A}}^i$  and  $\hat{\mathbf{B}}^i$  as (21) and (22), (30) becomes

$$\dot{M}_2 \leq -\frac{1}{2}\mathbf{e}_v^T\mathbf{Q}\mathbf{e}_v + \bar{k} - \sum_{j=1}^2 k_j|e_{\Delta j}|. \quad (32)$$

Choose values of  $k_j$ ,  $j = 1, 2$ , such that  $\sum_{j=1}^2 k_j|e_{\Delta j}| > \bar{k}$ , then

$$\dot{M}_2 = -\frac{1}{2}\mathbf{e}_v^T\mathbf{Q}\mathbf{e}_v \leq 0. \quad (33)$$

From (27) and (33), we can conclude that  $\dot{M} = \dot{M}_1 + \dot{M}_2$  is negative semi-definite. Then by using Barbalat lemma [41], it is obvious that the posture tracking error  $\mathbf{q}_e$  and the velocity tracking error  $\mathbf{e}_v$  approach zero vectors. This completes the proof of the theorem. ♦

#### 4 Improved D\* Lite Algorithm for Path Planning

In robot path planning, many algorithms, for example A\*, D\*, and D\* Lite, need to use grid graphs to represent the map. D\* Lite is a simplified version of the D\*. First, we set the robot's position as the first node. Choose the node that has the smallest cost to the destination as the robot's next position. Repeat the above actions until the robot reaches

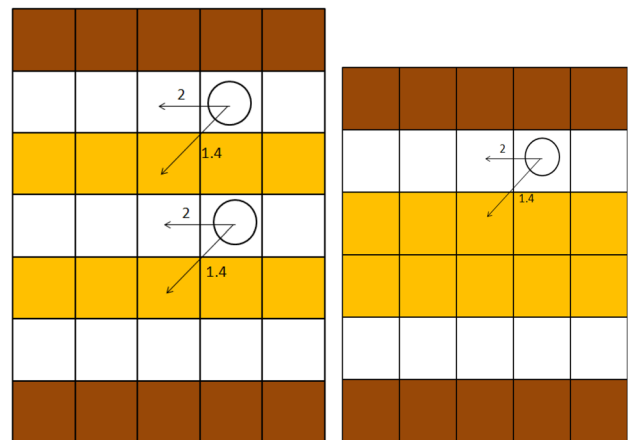


Fig. 4 The diagram for cost of grids

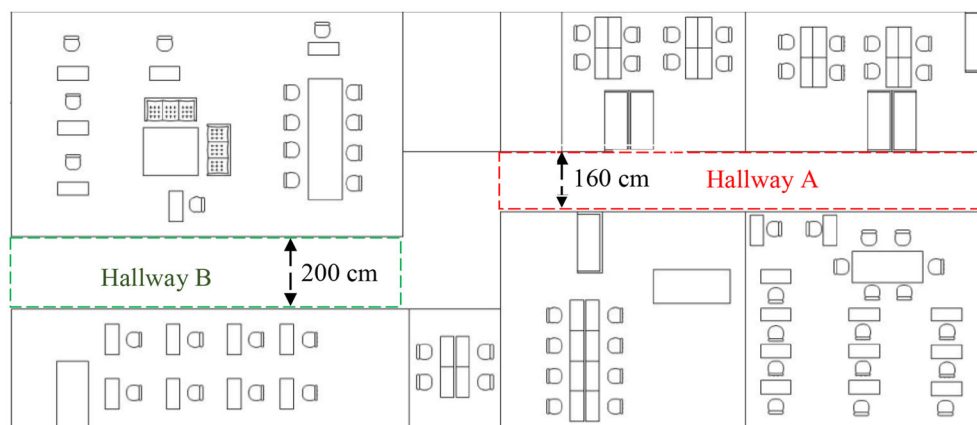
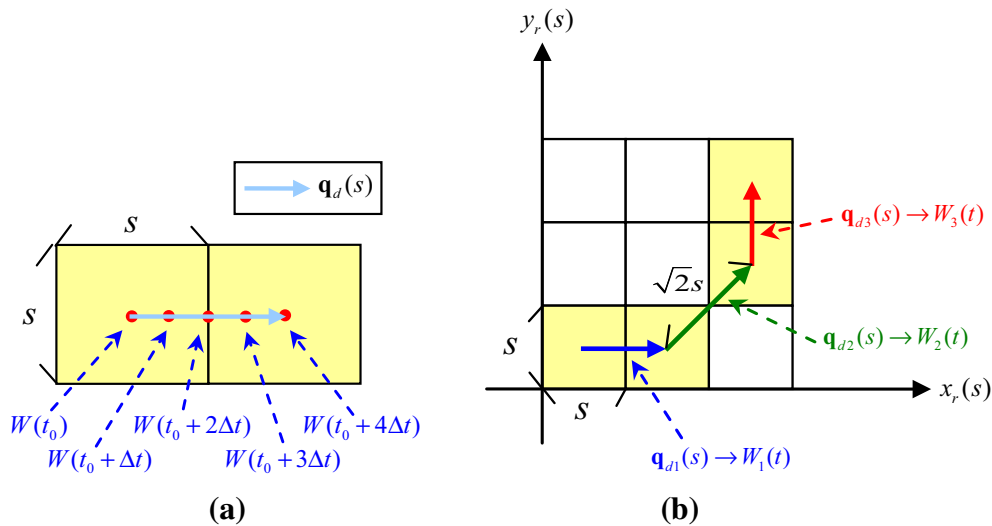


Fig. 3 A site map for the experiment





**Fig. 5** Diagram of path transformed into a tracking trajectory. **a** The neighbor grid points. **b** The horizontal, vertical and diagonal paths

the destination. The path found is the shortest one. We define  $g$  as a type of cost-to-goal function and function  $rhs$  [21] as

$$rhs(u) = \min_{s' \in Succ(u)} (c(u, s') + g(s')) \quad (34)$$

where  $u$  is the source node and  $s' \in Succ(u)$ .  $Succ(u)$  is the successor.  $c(u, s')$  is the cost from  $u$  to  $s'$ . There is an open list sometimes called a priority queue as

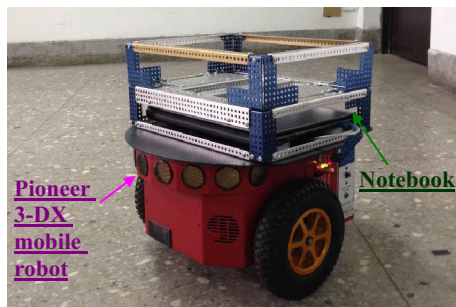
1. When  $g = rhs$ , a node is consistent.
2. When  $g > rhs$ , a node is over-consistent.
3. When  $g < rhs$ , a node is under-consistent.

We hope that the path planned by D\* Lite can be adjusted within 60 cm from the wall. Figure 3 is a site map for the experiment. There are two diagrams of the hallway, which are 200 and 160 cm in width. Because each grid is 40 cm in width, the hallway A has four grids and the hallway B has five. The brown grids are obstacles (the walls) and we would like to have the path planned by D\* Lite fall on the yellow grids. In D\* Lite algorithm, 1 is added to the cost of grids in white, as shown in Fig. 4. If the robot is on the white grids, the cost of yellow grids is smaller than that of the white grids. Therefore, the planned path will fall on the yellow grids, which is the one we are hoping for.

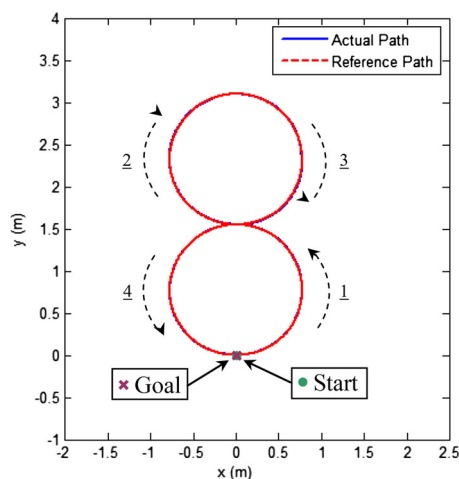
There are four steps in the D\* Lite algorithm to plan path.

**Step 1:** Initially,  $rhs$  and  $g$  of all the node's are set to infinity. The open list is set to empty. Then, the goal's  $g$  value is set to infinity and its  $rhs$  value is set to zero. The goal is put on the open list and the goal is the current node.

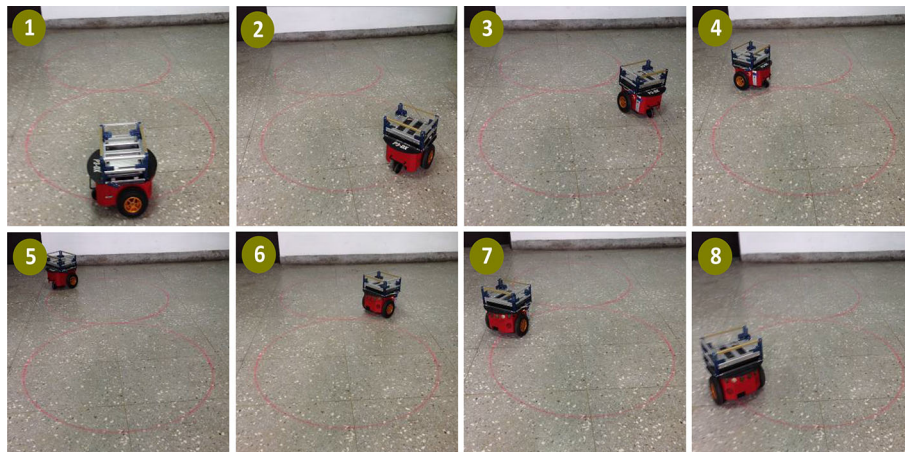
**Step 2:** If current node is over-consistent, its  $g$  value is equal to its  $rhs$  value. Then all of the neighbors are put on the open list. Their  $rhs$  value is updated. The node is removed from the open list. If current node is under-consistent, its  $g$  value is set to infinity. Then all of the



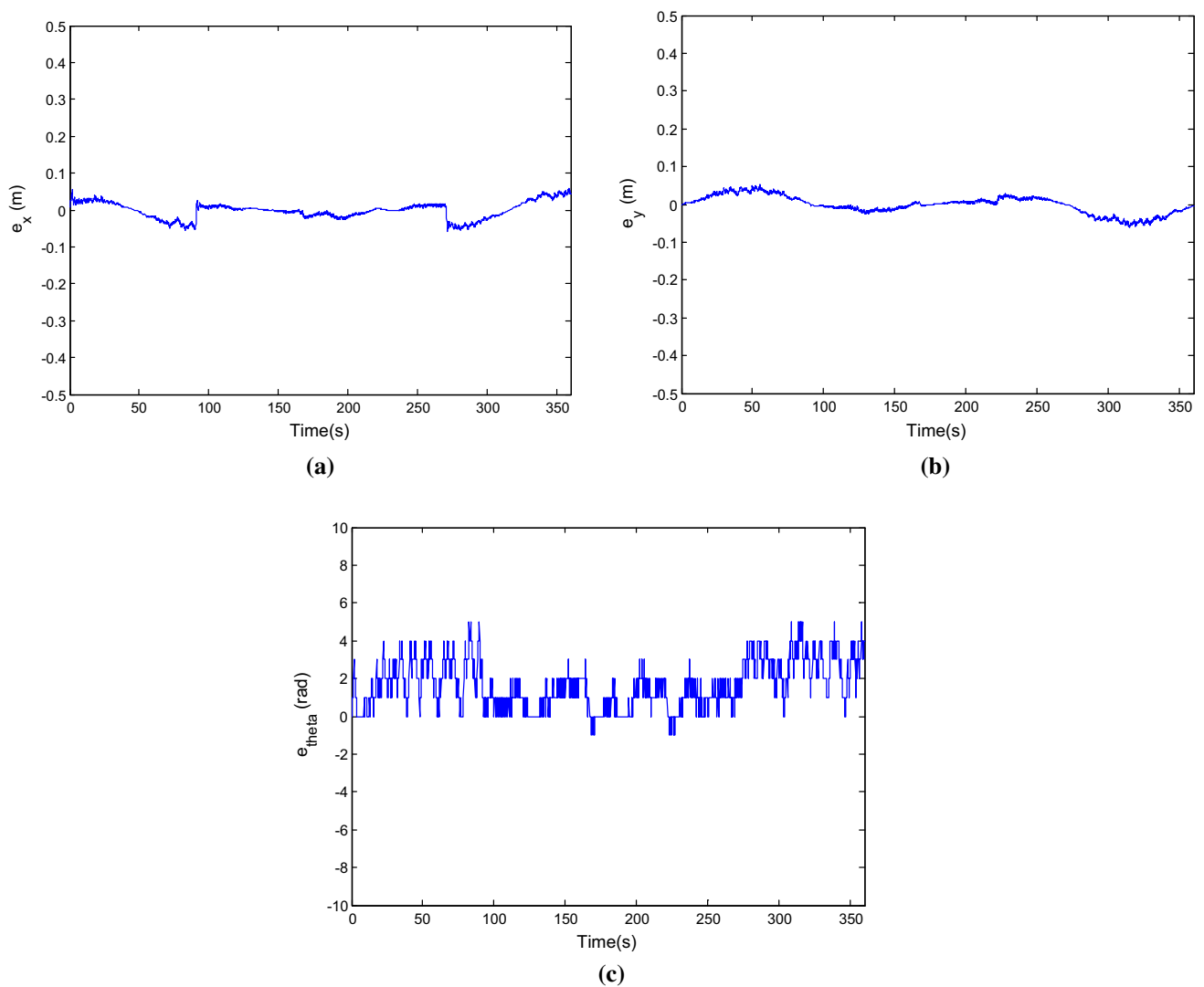
**Fig. 6** The WMR used in the experiment



**Fig. 7** The reference and the actual trajectories in Example 1



**Fig. 8** Experimental results of tracking an eight-shaped reference trajectory in Example 1



**Fig. 9** Trajectory-tracking errors in Example 1. **a** The position error along the x-axis. **b** The position error along the y-axis. **c** The angular error



neighbors are put on the open list. And their  $rhs$  value is updated. The node is removed from the open list. Choose the next node to determine the priority of the nodes in the open list.

**Step 3:** If the robot detects an obstacle, its  $rhs$  value is set to infinity. The costs of neighbors of the obstacles are updated to infinity. This node cannot be passed through, and hence the current path is not feasible. Moreover, we re-plan an optimal path from current node to goal node.

**Step 4:** Repeat the step 2 and 3 until the current states are equal to the start states or no path exists. The path is the shortest path.

According to the improved D\* lite algorithm, a problem of forcing the WMR to track a desired path  $\mathbf{q}_d(s)$  is considered. The given path  $\mathbf{q}_d(s)$  is in a two-dimension plane parameterized by  $(x_r(s), y_r(s))$  with  $s$  being the path parameter. It is noted that the path parameter,  $s$ , is not a time variable. Because we want to change the path-following issue to the trajectory-tracking issue, the derived path is represented as a function  $W$  of time. The function  $W(t)$  depends on the desired speed that the WMR moves on the desired path. Figure 5 shows the diagram of path transformed into a tracking trajectory. In Fig. 5a,  $t_0$  and  $\Delta t$  mean the initial value and the sampling time, respectively. The expected route of the WMR changes from the center of the left side grid to that of the right side grid and from the red dot on the left side to that on the right side subsequently. Figure 5b shows a simple path including horizontal, vertical, and diagonal paths. The derived path  $\mathbf{q}_d(s)$  is composed of three paths  $\mathbf{q}_{d1}(s)$ ,  $\mathbf{q}_{d2}(s)$ , and  $\mathbf{q}_{d3}(s)$ . Then we transform these three paths to three functions (i.e.,  $W_1(t)$ ,  $W_2(t)$ , and  $W_3(t)$ ) of time. The distance of the green path equals to

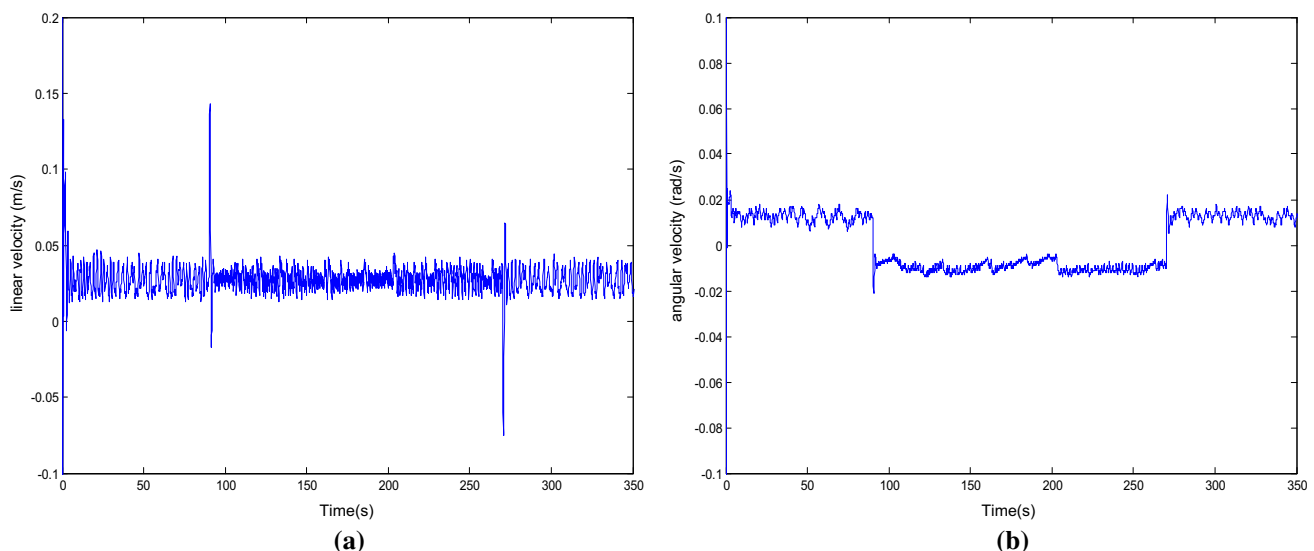
$\sqrt{2}s$  and is longer than that of the other two paths. When transforming the green path into the function  $W_2(t)$  of time, we can choose a little sampling time for the function  $W_2(t)$  to make the WMR track the tracking trajectory more effectively.

## 5 Experimental Results

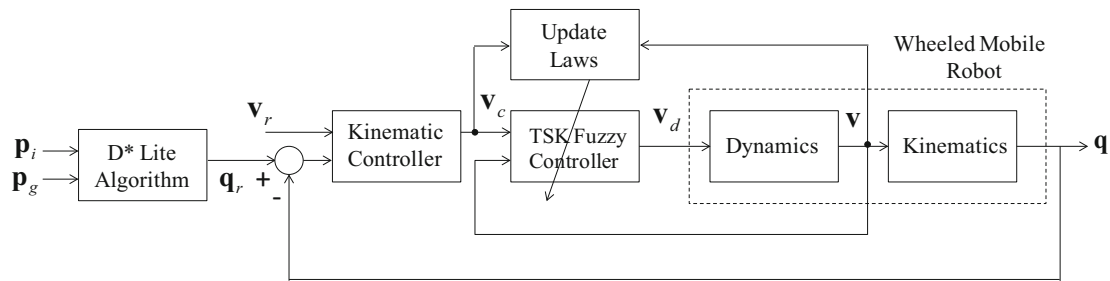
In this section, Pioneer 3-DX mobile robot shown in Fig. 6 is used to illustrate the effectiveness and the feasibility of the improved D\* Lite algorithm, the kinematic control, and



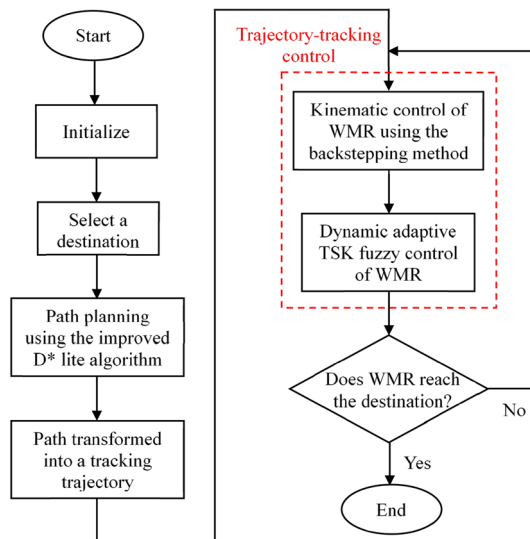
**Fig. 11** The simulated map and actual surroundings



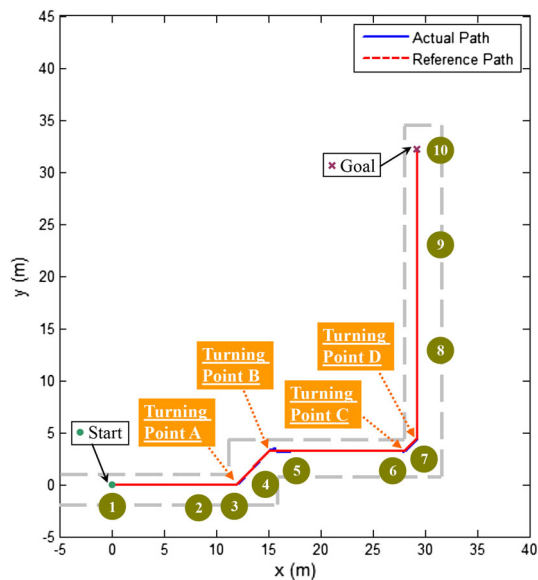
**Fig. 10** Linear velocity and angular velocity in Example 1. **a** The linear velocity. **b** The angular velocity



**Fig. 12** Overall scheme of the proposed controller for WMR system



**Fig. 13** The flowchart of the proposed hybrid intelligent algorithm for WMR

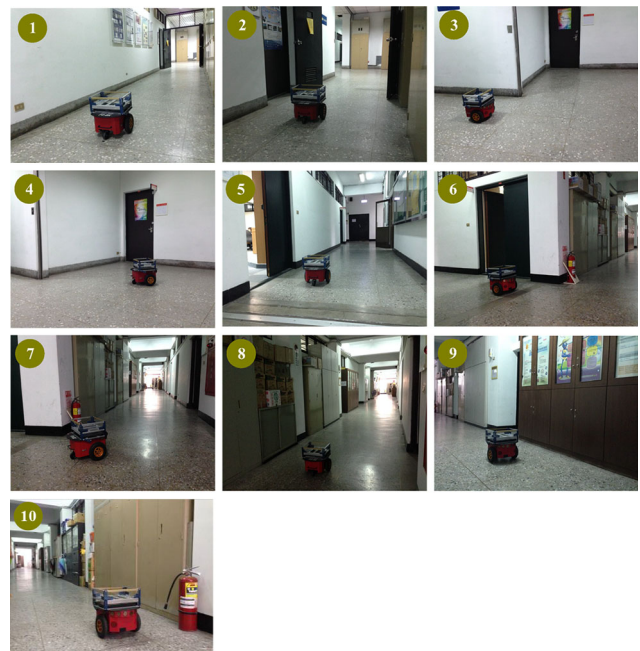


**Fig. 14** The reference and the actual trajectories in Example 2

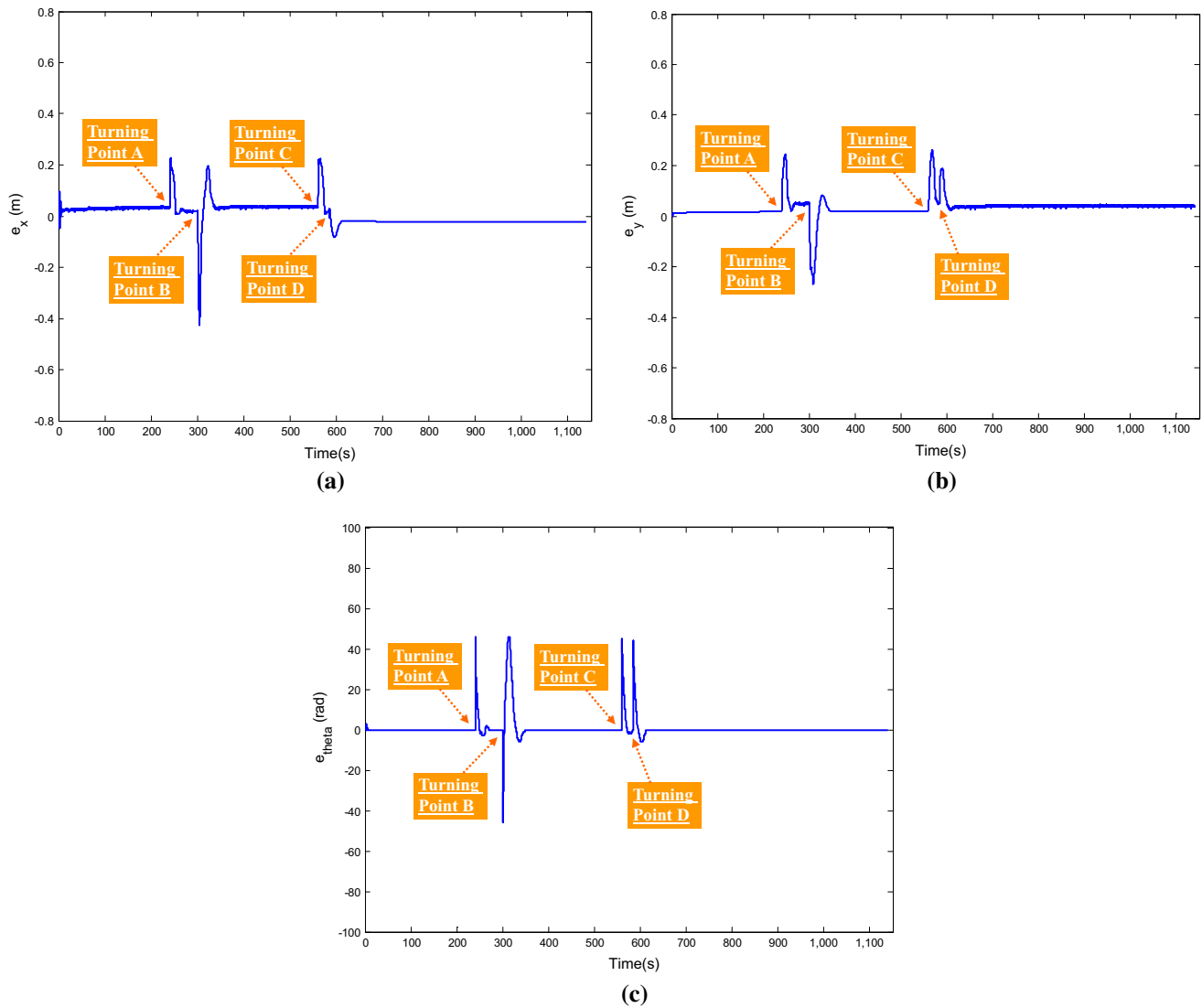
the dynamic control. There are two examples to confirm the proposed control scheme. In Example 1, experimental results are utilized to verify the ability of trajectory-tracking control. In Example 2, experimental results are utilized to verify the abilities of long-distance path planning and trajectory-tracking control.

*Example 1* An eight-shaped reference trajectory is considered as follows to show the ability in tracking smooth path:

$$x_r = \begin{cases} 0.775 \cos\left(\frac{\pi t}{60} - \frac{\pi}{2}\right), & \text{if } (120\kappa \leq t < 60 + 120\kappa) \\ 0.775 \cos\left(\frac{\pi}{2} - \frac{\pi t}{60}\right), & \text{if } (60 + 120\kappa \leq t < 120 + 120\kappa) \end{cases} \quad (35)$$



**Fig. 15** Experimental results of tracking a long-distance trajectory in Example 2



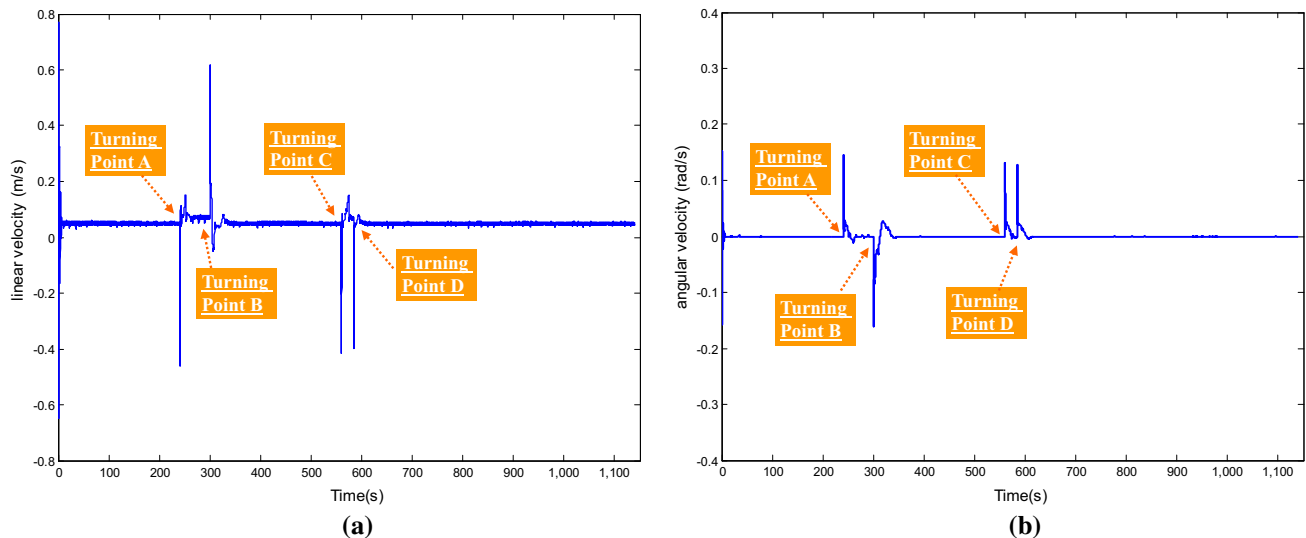
**Fig. 16** Trajectory-tracking errors in Example 2. **a** The position error along the x-axis. **b** The position error along the y-axis. **c** The angular error

$$y_r = \begin{cases} 0.775 \left( 1 + \sin \left( \frac{\pi t}{60} - \frac{\pi}{2} \right) \right), & \text{if } (240\kappa \leq t < 60 + 240\kappa) \\ \text{or } (180 + 240\kappa \leq t < 240 + 240\kappa) \\ 0.775 \left( 3 + \sin \left( \frac{\pi}{2} - \frac{\pi t}{60} \right) \right), & \text{if } (60 + 240\kappa \leq t < 180 + 240\kappa) \end{cases} \quad (36)$$

where  $\kappa = 0, 1$ . Fig. 7 shows the reference and the actual trajectories of the WMR. The red line is the planning path and the blue line is the value recorded from the code wheel. Figure 8 shows the movement when the WMR tracks an eight-shaped reference trajectory. We use the proposed kinematic and dynamic controllers in (5) and (19) to track the reference signals  $x_r$ ,  $y_r$ , and  $\theta_r$  of the WMR. Figure 9 shows the position errors  $e_x$ ,  $e_y$ , and angular error  $e_\theta$ . Figure 10 shows the linear velocity  $v$  and angular velocity  $\omega$ .

**Example 2** The working area for the wheeled robot is the entire fifth floor at National Taiwan Normal University, where only the hallways are designed to be passable and other areas are regarded as obstacles. Figure 11 shows the simulated map and actual surroundings of the floor for the experiment. Due to the tires' wear, the value from the encoder does not entirely correspond to the coordinates of the real map. Therefore, the real position of the robot needs to be adjusted. Figure 11 shows four points to correct the position of the robot.

Consider a kinematic model (1) and a dynamic model (6); the overall scheme of the proposed controller is shown in Fig. 12.  $\mathbf{p}_i$  and  $\mathbf{p}_g$  denote the initial position and the destination position of the WMR, respectively. The desired trajectory is planned by the improved D\* Lite algorithm.



**Fig. 17** Linear velocity and angular velocity in Example 2. **a** The linear velocity. **b** The angular velocity

Figure 13 shows the flowchart of the proposed hybrid intelligent algorithm for WMR. Figure 14 shows the reference and the actual trajectories of the WMR. The red line is the planning path and the blue line is the value recorded from the code wheel. We use the proposed kinematic controller in (5) to track the reference signals  $x_r$ ,  $y_r$ , and  $\theta_r$  of the WMR. Moreover, we use the proposed dynamic controller in (19) to track the reference velocity  $v_r$  and angular velocity  $\omega_r$  of the WMR. Figure 15 shows the movement when the WMR tracks a long-distance trajectory. Figure 16 shows the position errors  $e_x, e_y$ , and the angular error  $e_\theta$ . Figure 17 shows the linear velocity  $v$  and the angular velocity  $\omega$ .

## 6 Conclusion

In this paper, a novel path planning and an adaptive trajectory-tracking algorithm for WMR are developed. Firstly, the improved D\* lite algorithm is utilized to design a moving path for WMR. Then, we design a two-stage trajectory-tracking controller, which are the kinematic controller and dynamic controller. By using the kinematic controller, the position and angle errors of the WMR converge to zero efficiently. Moreover, the linear and angular velocity errors of the WMR converge to zero using the adaptive TSK fuzzy controller. Finally, the effectiveness and the feasibility of the proposed scheme are verified by experimental results.

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