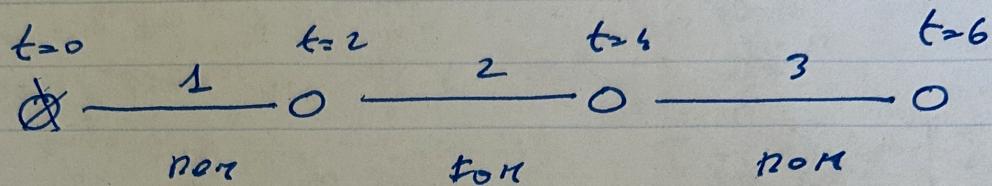


MIXED FEM-NOM (<sup>Default</sup>  
GALERKIN) with BDF1

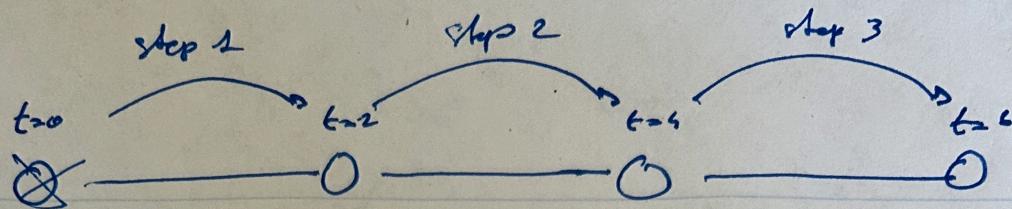


$$\phi = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}, \quad h=2 \quad (dt), \quad \dot{\phi}_0 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

- FEM size = 8

$$- f(y, t) = y + t$$

MAIN1-BDF1.cc



$$\hat{y}_0 \xrightarrow{R_1} \hat{y}_1 \xrightarrow{R_2} \hat{y}_2$$

$$y_2 \xrightarrow{R_3} y_3 \xrightarrow{R_4} y_4$$

$$\hat{y}_4 \xrightarrow{R_5} \hat{y}_5 \xrightarrow{R_6} \hat{y}_6$$

$$\hat{y}_0 = [222]^T$$

$$\hat{y}_1 = [333]^T$$

$$\hat{y}_2 = [444]^T$$

$$y_2 = \phi \hat{y}_2 = [26 \dots 26]^T$$

$$y_3 = [25 \dots 25]^T$$

$$y_4 = [26 \dots 26]^T$$

$$\hat{y}_4 = \phi^T y_4 = \begin{bmatrix} 208 \\ 516 \\ -624 \end{bmatrix}$$

$$\hat{y}_5 = \hat{y}_4 + 1$$

$$\hat{y}_6 = \hat{y}_5 + 1$$

$$R_1 = \hat{y}_0 - \hat{y}_0 - h \phi^T f = -h \phi^T f(y_0, t=2) = \begin{bmatrix} -226 & -448 & -672 \end{bmatrix}$$

$$R_2 = \hat{y}_1 - \hat{y}_0 - h \phi^T f = [111]^T - h \phi^T f(y_1, t=2) = \begin{bmatrix} -319 & -638 & -958 \\ -319 & -639 & -959 \end{bmatrix}$$

$$R_3 = y_2 - y_1 - h f = -h f(y_2, t=4) = [-56 \dots -56]^T$$

$$R_4 = y_3 - y_2 - h f = [1 \dots 1]^T - h f(y_3, t=4) = [-57 \dots -57]^T$$

$$R_5 = \hat{y}_4 - \hat{y}_3 - h \phi^T f = -h \phi^T f(\phi \hat{y}_4, t=6) = \begin{bmatrix} -46688 \\ -93376 \\ -140064 \end{bmatrix}$$

$$R_6 = \hat{y}_5 - \hat{y}_4 - h \phi^T f(\phi \hat{y}_5, t=6) = [111]^T - h \phi^T f = \begin{bmatrix} -46789 \\ -93567 \\ -140351 \end{bmatrix}$$