

0

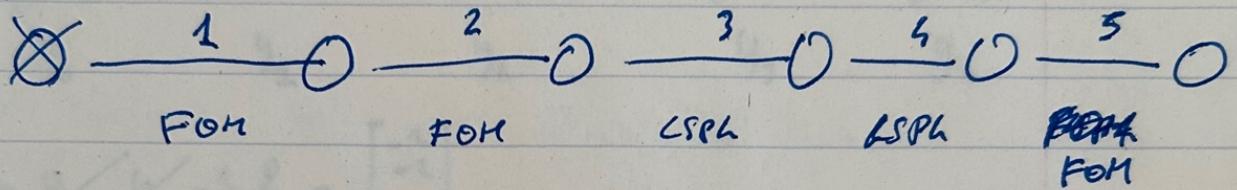
FIRING SEQUENCE

MAIN - BDF2.cc

# MIXED FOM - ROM (LBPL)

BDF2

Start



$$\phi = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \\ \vdots & \vdots & \vdots \\ 1 & 2 & 3 \end{bmatrix}$$

$dt = 2$ . (called h hence fourth)

$$y_0 = \begin{bmatrix} 10 \\ \vdots \\ 10 \end{bmatrix}$$

initial condition

FOM size = 8

$f(\cdot)$  always yields  $\begin{bmatrix} 1 \\ \vdots \\ i \end{bmatrix}$

(1)

BDF1  
Step 1  
Form

BDF2  
Step 2  
Form

$$\begin{array}{c}
 \text{BDF1} \\
 \text{Step 1} \\
 \text{Form} \rightarrow
 \end{array}
 \quad
 \begin{array}{c}
 \text{BDF2} \\
 \text{Step 2} \\
 \text{Form} \rightarrow
 \end{array}$$

$$\begin{matrix}
 \cancel{0} & \xrightarrow{\hspace{1cm}} & 1 \\
 \left[ \begin{array}{c} 10 \\ \vdots \\ 10 \end{array} \right] & \xrightarrow{\text{solve 1}} & \left[ \begin{array}{c} 11 \\ \vdots \\ 11 \end{array} \right] & \xrightarrow{\text{solve 2}} & \left[ \begin{array}{c} 12 \\ 12 \\ \vdots \\ 12 \end{array} \right] & \xrightarrow{\text{solve}} & \left[ \begin{array}{c} 13 \\ 13 \\ \vdots \\ 13 \end{array} \right] & \xrightarrow{\text{solve}} & \left[ \begin{array}{c} 14 \\ 14 \\ \vdots \\ 14 \end{array} \right] \\
 y_0 & & y_1 & & y_2 & & y_3 & & y_4
 \end{matrix}$$

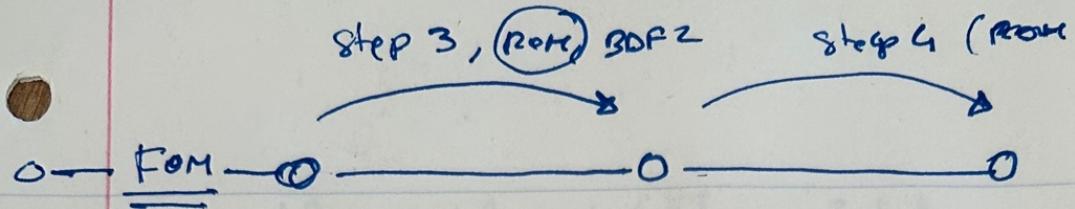
$$R_1 = y_1 - y_0 - hf = \begin{bmatrix} -2 \\ -2 \\ \vdots \\ -2 \end{bmatrix}$$

$$R_2 = y_2 - y_1 - hf = \begin{bmatrix} -1 \\ -1 \\ \vdots \\ -1 \end{bmatrix}$$

$$R_3 = y_3 - \frac{1}{3}y_2 + \frac{1}{3}y_0 - \frac{2}{3}hf = \begin{bmatrix} -2 \\ -2 \\ \vdots \\ -2 \end{bmatrix}$$

$$R_4 = y_4 - \frac{4}{3}y_3 + \frac{1}{3}y_0 - \frac{2}{3}hf = \begin{bmatrix} -1 \\ -1 \\ \vdots \\ -1 \end{bmatrix}$$

(2)



$$\begin{array}{c}
 \left[ \begin{array}{c} 12 \\ 12 \\ \vdots \\ 12 \\ y_2 \end{array} \right] \xrightarrow{\text{Solve } R_S} \left[ \begin{array}{c} 14 \\ 14 \\ \vdots \\ 14 \\ y_4 \end{array} \right] \xrightarrow{\text{value } R_G} \left[ \begin{array}{c} 14 \\ 14 \\ \vdots \\ 14 \\ y_6 \end{array} \right] \xrightarrow{\text{value } R_B} \left[ \begin{array}{c} \hat{y}_3 \\ \hat{y}_4 \end{array} \right] \xrightarrow{\text{value } R_B} \left[ \begin{array}{c} \hat{y}_8 \end{array} \right] \\
 \hat{y}_7 = \left[ \begin{array}{c} 115 \\ 222 \\ 339 \end{array} \right] \quad \hat{y}_7 = \left[ \begin{array}{c} 113 \\ 225 \\ 337 \end{array} \right] \quad \hat{y}_6 = \left[ \begin{array}{c} 114 \\ 226 \\ 333 \end{array} \right] \quad \hat{y}_8 = \left[ \begin{array}{c} 116 \\ 228 \\ 340 \end{array} \right]
 \end{array}$$

$y_4, \hat{y}_4 = \phi^T y_4$

At Step 3, we switch from FOM to ROM.

So we have :

$$\begin{aligned}
 R_S &= \phi \phi^T y_4 - \frac{1}{3} \phi \phi^T y_4 + \frac{1}{3} \phi \hat{y}_3 \phi^T y_2 - \frac{2}{3} h f \\
 &= -\frac{1}{3} \begin{bmatrix} 1568 \\ \vdots \\ 1568 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1344 \\ \vdots \\ 1344 \end{bmatrix} - \frac{2}{3} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} -76 \\ \vdots \\ -76 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 R_G &= \phi \hat{y}_5 - \frac{1}{3} \phi \phi^T y_4 + \frac{1}{3} \phi \phi^T y_2 - \frac{2}{3} h f \\
 &= \begin{bmatrix} 1574 \\ \vdots \\ 1574 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} 1568 \\ \vdots \\ 1568 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1344 \\ \vdots \\ 1344 \end{bmatrix} - \frac{2}{3} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} -70 \\ \vdots \\ -70 \end{bmatrix}
 \end{aligned}$$

(3)

$$R_7 = \phi \hat{g}_5 - \frac{4}{3} \phi \hat{g}_6 + \frac{1}{3} \phi \phi^T g_4 - \frac{2}{3} h \neq$$

$$= \frac{1}{3} \begin{bmatrix} 1580 \\ \vdots \\ 1580 \end{bmatrix} + \frac{4}{3} \begin{bmatrix} 1568 \\ \vdots \\ 1568 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} 16/3 \\ \vdots \\ 16/3 \end{bmatrix}$$

$$R_8 = \phi \hat{g}_7 - \frac{4}{3} \phi \hat{g}_6 + \frac{1}{3} \phi \phi^T g_4 - \frac{2}{3} h \neq$$

$$= \begin{bmatrix} 1586 \\ \vdots \\ 1586 \end{bmatrix} - \frac{4}{3} \begin{bmatrix} 1580 \\ \vdots \\ 1580 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1568 \\ \vdots \\ 1568 \end{bmatrix} - \frac{4}{3} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} 2/3 \\ \vdots \\ 2/3 \end{bmatrix}$$

(power page)

step 4

step 5 (FORC)

④



$$\begin{bmatrix} 116 \\ 228 \\ 340 \end{bmatrix} \xrightarrow{\text{step } R_9} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \xrightarrow{\text{step } R_{10}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \xrightarrow{\text{step } R_{10}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\hat{y}_6 = \begin{bmatrix} 116 \\ 226 \\ 338 \end{bmatrix}$$

$$y_6 = \phi \hat{y}_6$$

$$R_9 = y_8 - \frac{1}{3}y_8 + \frac{1}{3}y_6 - \frac{2}{3}h^f$$

$$= -\frac{1}{3} \begin{bmatrix} 1592 \\ \vdots \\ 1592 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1580 \\ \vdots \\ 1580 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} -16/3 \\ \vdots \\ -16/3 \end{bmatrix}$$

$$R_{10} = y_9 - \frac{1}{3}y_8 + \frac{1}{3}y_6 - \frac{2}{3}h^f$$

$$= \frac{1}{3} \begin{bmatrix} 1593 \\ \vdots \\ 1593 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} 1592 \\ \vdots \\ 1592 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1580 \\ \vdots \\ 1580 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} -13/3 \\ \vdots \\ -13/3 \end{bmatrix}$$