MIXED   FOM- ROM (LSPG) with <u>BDF2</u>
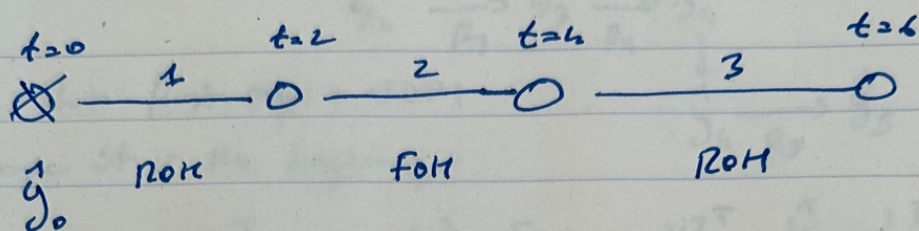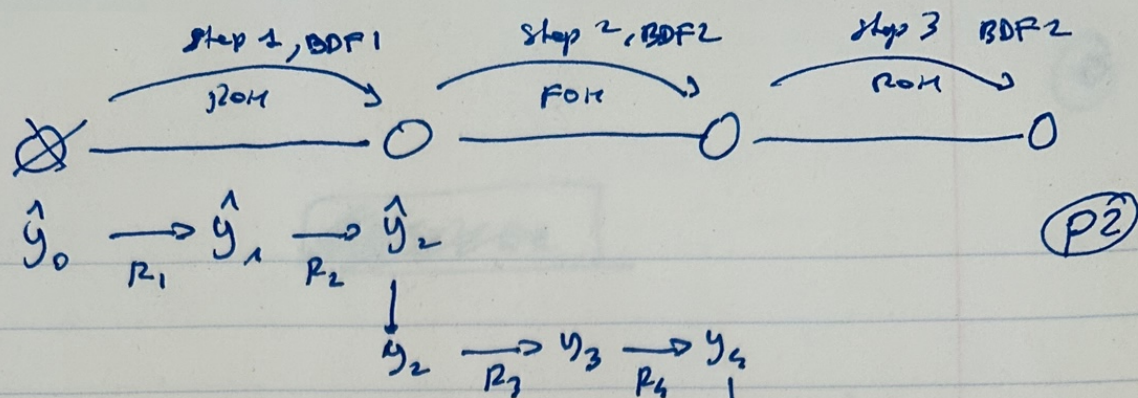

$$- \phi = \begin{bmatrix} 1 & 23 \end{bmatrix} \quad , \quad h=2 \quad , \quad \hat{y}_0 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

- FOM   size = 8

- f() always returns $\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$

- force solve by adding 1 to the given state

$$\boxed{\text{MAIN1\_BDF2.cc}}$$

$\hat{y}_0 \xrightarrow{R_1} \hat{y}_1 \xrightarrow{R_2} \hat{y}_2$

$y_2 \xrightarrow{R_3} y_3 \xrightarrow{R_4} y_4$

Note that first step is BDF1 because it is the beginning

$y_4 \xrightarrow{R_5} \hat{y}_5 \xrightarrow{R_6} \hat{y}_6$

$\hat{y}_0 = [2\ 2\ 2]^T$

$\hat{y}_1 = [3\ 3\ 3]^T$

$\hat{y}_2 = [5\ 4\ 4]^T$

$y_2 = \phi \hat{y}_2 = [24 \ \cdots \ 24]^T$

$y_3 = [25 \cdots 25]^T$

$y_4 = [26 \cdots 26]^T$

$\hat{y}_4 = \phi^T y_4 = \begin{bmatrix} 208 \\ 416 \\ 624 \end{bmatrix}$

$\hat{y}_5 = \frac{5}{3}\hat{y}_4 + 1$

$\hat{y}_6 = \hat{y}_5 + 1$

$R_1 = \phi \hat{y}_0 - \phi \hat{y}_0 - hf = [-2 \cdots -2]^T$

$R_2 = \phi \hat{y}_1 - \phi \hat{y}_0 - hf = [64 \cdots 4]^T$

$R_3 = \phi \hat{y}_2 - \frac{4}{3}\phi \hat{y}_2 + \frac{1}{3}\phi \hat{y}_0 - \frac{2}{3}hf \ast$

$= -\frac{1}{3}\phi \hat{y}_2 + \frac{1}{3}\phi \hat{y}_0 - \frac{2}{3}hf = [-16/3 \cdots -16/3]^T$

$R_4 = y_3 - \frac{4}{3}\phi \hat{y}_2 + \frac{1}{3}\phi \hat{y}_0 - \frac{2}{3}hf = [-13/3 \cdots -13/3]^T$

$\frac{\downarrow}{=}$

$R_5 = \phi \phi^T y_4 - \frac{4}{3}\phi \phi^T y_4 + \frac{1}{3}\phi \phi^T y_2 - \frac{2}{3}hf$

$\frac{\downarrow}{=} [-76 \cdots -76]^T$

$R_6 = \phi \hat{y}_5 - \frac{4}{3}\phi \phi^T y_4 + \frac{1}{3}\phi \phi^T y_2 - \frac{2}{3}hf$

$\frac{\downarrow}{=} [-70 \cdots -70]^T$