Problem Chosen	2024 ShuWei Cup Summary Sheet	Team Control Number
A		2024100229348

Frequency estimation of aircraft airspeed measurement signals

Summary

The accurate measurement of airspeed during flight is a very important thing. When the aircraft receives the signal from the aerosols in the air, how to accurately estimate the signal frequency and extract the effective signal is a problem that needs to be discussed.

For problem 1, according to the known information of the subject, it can be concluded that the signal received by the aircraft is a sinusoidal signal with noise. When the amplitude, phase value and frequency are known, the noise signal is separated by introducing parameters, and the Gaussian white noise is obtained by observing its distribution, autocorrelation analysis and noise power spectrum density. Its distribution belongs to normal distribution, and then variance and mean analysis are carried out to test its statistical characteristics

For question 2: At this time, when the frequency is unknown, FFT is introduced, the signal main frequency is quickly found as the initial estimated frequency, and a bandpass filter is designed with this frequency to denoise the signal. Then, the signal frequency of the MLE model is further estimated by using the frequency initially estimated by FFT as the initial value. The model can almost perfectly estimate the parameters in problem 1, so the model has high accuracy and the frequency of flight period 2 is 41MHZ.

For question 3: When there were no signal parameters, MLE was considered to be used to estimate more parameters based on the model in question 2, and the autocorrelation method and Welch method were added to simultaneously estimate the frequency and cross-verify the results to improve the stability of the model. Finally, the data in question 1 was used to verify the accuracy of the model, and it was found that the model had high accuracy and the frequency of flight cycle 3 was 31MHZ.

For problem 4, under the premise of receiving signals in intermittent mode, the gap between signals is obtained by detecting the time interval and processed in segments using FFT and improved robust MLE model. Finally, the estimated frequency of all segmented signals is summarized, and the final estimated frequency is 55MHZ.

Key words: Frequency estimation FFT robust MLE

Catalogue

Ⅰ Problem restatement

Airspeed is an important parameter that needs accurate monitoring in aircraft flight, which is directly related to flight status and safety. Laser velocity measurement is an effective method to calculate airspeed by measuring the Doppler shift of laser signal.

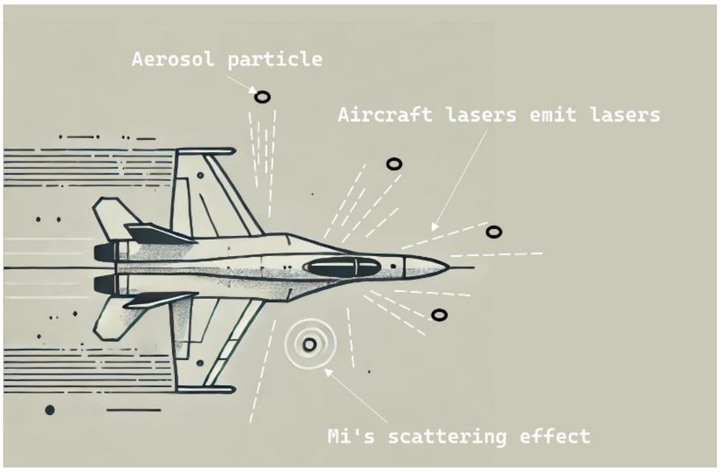


Fig. 1Overview of laser emission and signal reception in flight

The following issues need to be addressed:

- In flight time period 1, the amplitude, phase and frequency of the received signal are known, and the noise characteristics are solved and analyzed.
- In flight period 2, the amplitude and phase of the received signal are known, and the model is designed to estimate the signal frequency.
- During flight period 3, the received signal amplitude, phase and frequency are unknown, so please design a model to estimate the signal frequency.
- In flight time period 4, the signal is received intermittently, so it is necessary to analyze the intermittent receiving mode and design a method to estimate the frequency of the signal during this time period.

Ⅱ Problem analysis

For problem 1: Separate the noise from the known signal model and parameters, analyze its statistical characteristics, frequency characteristics, etc.

For problem 2: First, preprocess the data using known signal models and parameters, then find a model that can accurately estimate the signal frequency, such as

FFT, MLE, etc. to estimate the signal frequency, and verify it.

For problem 3: Since all parameters are unknown, further consider models that can accurately estimate parameters, such as FFT, MLE, autocorrelation method, Welch method, etc. Use multiple methods to verify each other and improve the estimation accuracy.

For problem 4: Since the signal is received intermittently and noisy, consider improving the robustness of the accurate MLE model to estimate it.

Ⅲ Model assumption

1. Within each flight time period, the power of the noise remains stable over time.
2. Within the same flight cycle, the frequency, amplitude and phase values of the signal remain unchanged.
3. The data quality within each received signal segment remains invariant, and the intermittent mode does not introduce additional nonlinear distortion.
4. It is hypothesized that the hardware errors of the signal receiving equipment can be neglected (such as frequency drift and phase jitter which do not significantly affect the signal quality).

Ⅳ Symbol specification

Table 1 Symbol specification table

symbol	explain
$x(t)$	Time domain signal
A	The amplitude of the signal
f	Frequency of signal
ϕ	Phase of signal
$z(t)$	Noise signal
$X(t)$	Frequency domain representation of the signal
T	Sampling interval
N	The total number of signal sampling points

f_z	Fourier transform maximum main frequency
n	Signal point ordinal number
τ	Time delay
K	Signal segment number
$w(t)$	Windowing function
r	Residual error
δ	Threshold parameter of the loss function

Ⅴ Model building and solving

5.1 Signal noise characteristics with known parameters

According to the known information of the topic, the signal satisfies the following expression:

$$x(t) = A\sin(2\pi ft + \phi) + z(t)$$

In problem 1, given the amplitude $A = 4$, phase $\phi = 45^\circ = \frac{\pi}{4}$, and frequency $f = 30 \times 10^6 HZ$, the expression of the noise sequence is as follows:

$$z(t) = x(t) - 4\sin(2\pi t \cdot 30 \times 10^6 + \frac{\pi}{4})$$

After removing the noise, the signal is compared with the original signal as follows:

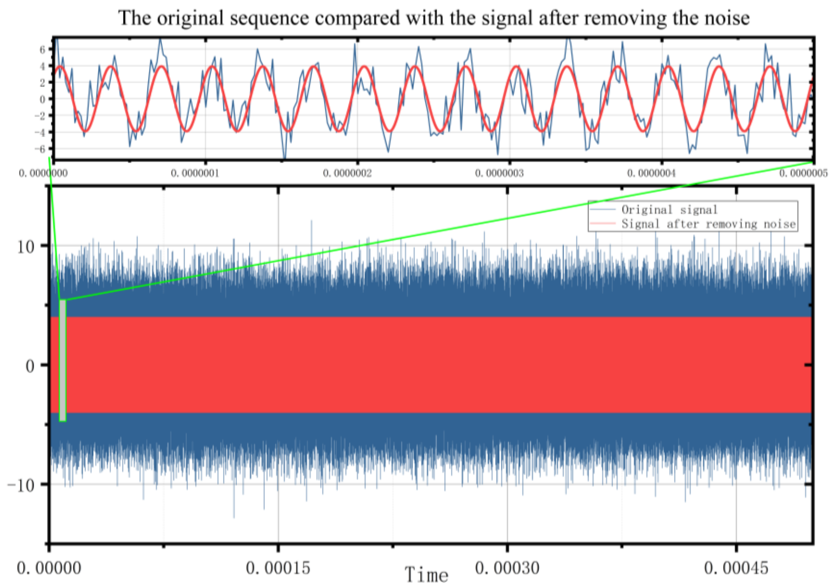


Fig. 2 The original sequence compared with the signal after removing the noise

As you can see, after removing the noise, the real signal appears as a clean sinusoidal fluctuation. The isolated noise sequence is shown in the figure below:

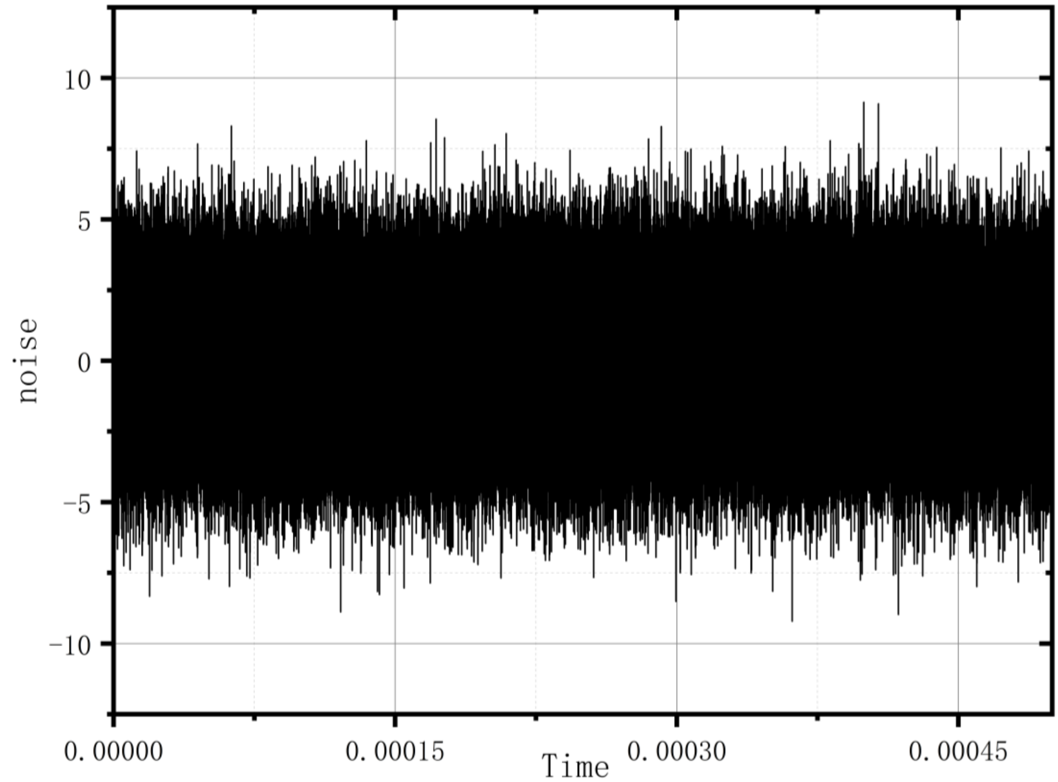


Fig. 3 Noise sequence

It can be roughly seen from the figure above that noise is basically random distribution. By calculating the probability density of noise, the histogram of noise is drawn, and the Gaussian distribution is tried to be fitted to check whether it belongs to Gaussian distribution:

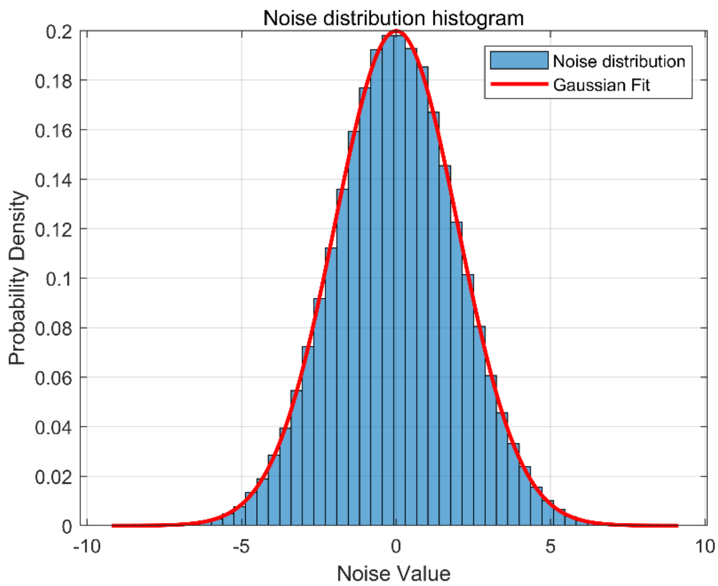


Fig. 4 Noise distribution

It can be seen from the above figure that the noise belongs to Gaussian distribution. In order to further know the noise characteristics, Matlab is used to draw the noise autocorrelation analysis diagram and noise power spectral density analysis diagram:

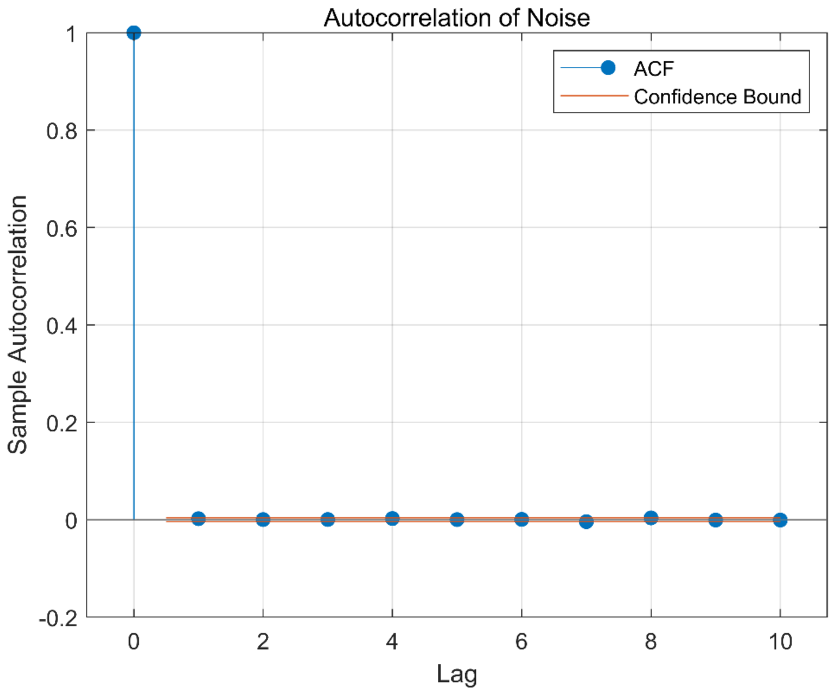


Fig. 5 Noise autocorrelation images

It can be seen that ACF rapidly decays in the direction of 0 in the autocorrelation analysis diagram of noise, and the subsequent ACF value is stable, which indicates that the noise may belong to white noise.

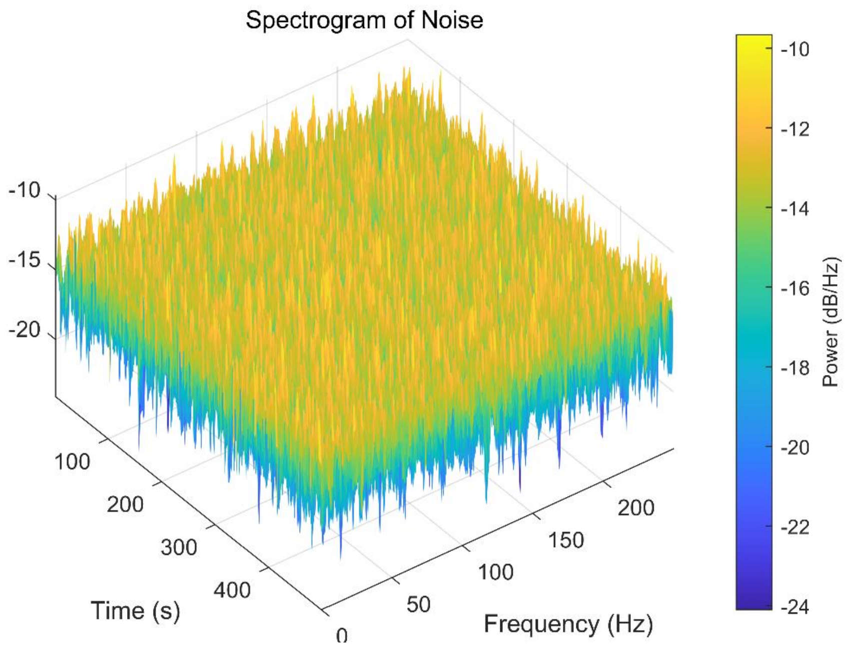


Fig. 6 Noise power spectrum density analysis diagram

As can be seen from the noise power spectrum density analysis diagram, the energy distribution of noise is uniform, which further proves that noise is white noise. Therefore, the noise under this signal data is Gaussian white noise, and its statistical characteristics can be calculated and analyzed:

Table 2 Statistical characteristic

Statistical characteristic	Characteristic value	Nature
Noise Mean	0.0058	It shows that the noise signal has no significant deviation on the whole.
Noise Variance	3.9819	It indicates that the energy intensity of noise is relatively moderate and there is no large anomaly.

5.2 Frequency estimation method with known parameters

5.2.1 Fast Fourier transform (FFT) model is introduced

FFT is a fast method for calculating discrete Fourier transform (DFT). By performing FFT on the signal, the spectrum can be obtained and the main frequency of the signal can be estimated initially.

First, the data is converted from the time domain to the frequency domain, and the conversion formula is as follows:

$$X(f)=\sum_{n=0}^{N-1} x(nT)\cdot e^{-j2\pi fnT}$$

Find the largest peak in the spectrum in the frequency domain to determine the main frequency of the signal, expressed as:

$$f_z=\arg\max\left|X(f)\right|$$

On this basis, the resolution can also be improved by interpolation method to obtain a more accurate estimated frequency. The interpolation operation is as follows:

$$\hat{f}_z=f_z+\frac{\alpha-\gamma}{2(\alpha-2\beta+\gamma)}\cdot\frac{1}{TN}$$

Where $\alpha = |X(f_z - 1)|$、$\beta = |X(f_z)|$、$\gamma = |X(f_z + 1)|$.

5.2.2 Maximum likelihood estimation (MLE) model is introduced

Maximum likelihood estimation is a parameter estimation method. On the premise that the known signal model is a sinusoidal signal model, the likelihood function is constructed:

$$S(f) = \prod_{t=0}^{N-1} \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^N e^{-\frac{1}{2\sigma^2}\sum_{t=0}^{N-1}[x(t) - A\sin(2\pi ft + \phi)]^2}$$

Take the logarithmic likelihood function:

$$\ln S(f) = -\frac{N}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{n=0}^{N-1}[x(t_n) - A\sin(2\pi ft_n + \phi)]^2$$

It can be obtained from the above formula：

$$\ln S(f) \propto -\frac{1}{2\sigma^2}\sum_{n=0}^{N-1}[x(t_n) - A\sin(2\pi ft_n + \phi)]^2$$

Then maximizing the log-likelihood function is equivalent to minimizing the sum of squares of residuals, constructing the objective function:

$$O(f) = \sum_{n=1}^{N}[x(t_n) - A\sin(2\pi ft_n + \phi)]^2$$

The final estimated frequency can be obtained by using the nonlinear optimizer fminsearch in Matlab. MLE can estimate not only the frequency but also other parameters, such as amplitude and phase. Since the distribution of noise will affect the estimation accuracy of MLE, it is necessary to preprocess the data to remove some noise to increase the estimation accuracy.

5.2.3 Data preprocessing

In order to ensure that the dynamic part of the signal will not be covered up during spectrum analysis, the data is first processed to remove the DC component:

$$x_m(t) = x(t) - \overline{x}$$

Next, the band-pass filter is designed to retain the main frequency component of the signal and suppress the influence of noise. In order to ensure that the cut-off frequency of the band-pass filter is effective, the fast Fourier transform is performed on the data to obtain the preliminary estimate of the frequency. 20% of the initial estimated frequency is set as the cut-off frequency:

The lower cut-off frequency is: $f_{low} = \max(0.01, \frac{T \cdot f_z \cdot 0.8}{2})$

The upper cut-off frequency is: $f_{high} = \min(0.5, \frac{T \cdot f_z \cdot 1.2}{2})$

Call the Butterworth filter in Matlab, the order is set to 4, and the pre-processed signal is shown below (in order to show clearly only the signal with the time of $0-5\times10^{-7}$) :

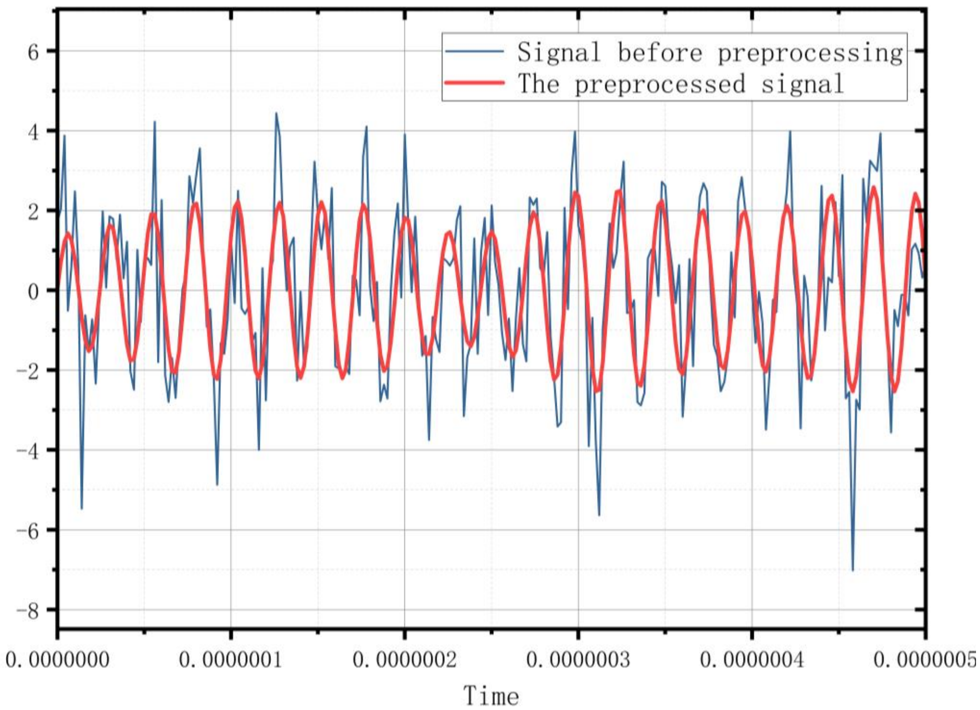


Fig. 7 Signal comparison before and after filtering

It can be seen that the pre-processed data is smoother and the influence of noise is reduced.

5.2.4 Model solving

The pre-processed data is used to solve the model. The operation process is as follows:

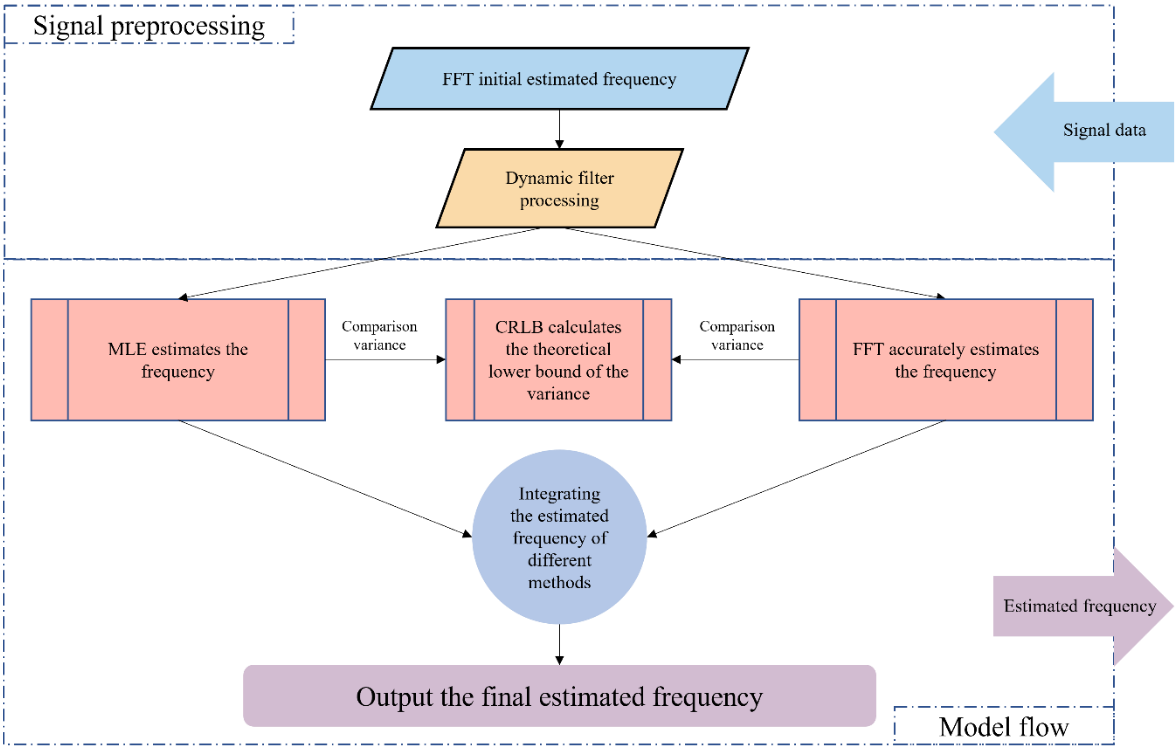


Fig. 8 Model flow chart

FFT estimated frequency results are as follows:

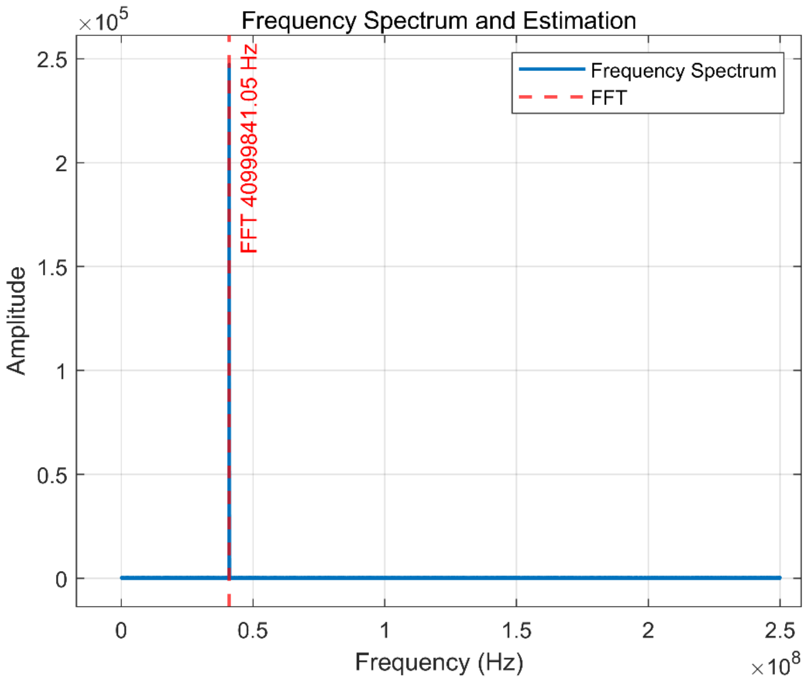


Fig. 9 FFT spectrum diagram

The estimated frequency of the FFT is 40999841.054336 Hz, and the variance is $0HZ^2$, This may be due to a very high signal-to-noise ratio or a very ideal signal, resulting in unobservable errors. MLE estimated frequency results are as follows:

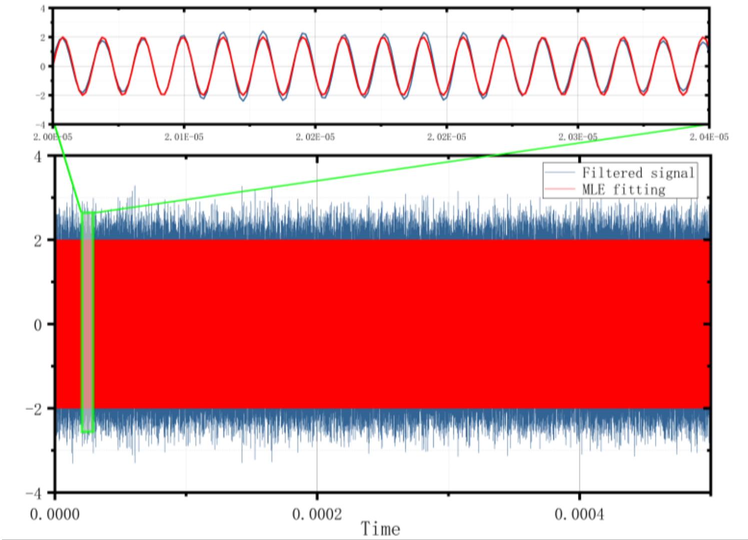


Fig. 10 MLE fitting effect

As can be seen from the figure above, MLE fitting effect is very good, MLE estimated frequency is 41000000.179917Hz, variance is $0HZ^2$。

CRLB provides the lower bound of theoretical variance for parameter estimation, which is an important indicator for evaluating algorithm performance. For sinusoidal signals with known sampling interval noise, CRLB for frequency estimation is as follows:

$$Var(\hat{f}) \geq \frac{12\sigma^2}{2\pi^2 A^2 N(N^2-1)T^2}$$

After calculation, CRLB=$0.00140282HZ^2$， It can be concluded that the model has a good effect, and the residual noise distribution is shown in the figure below:

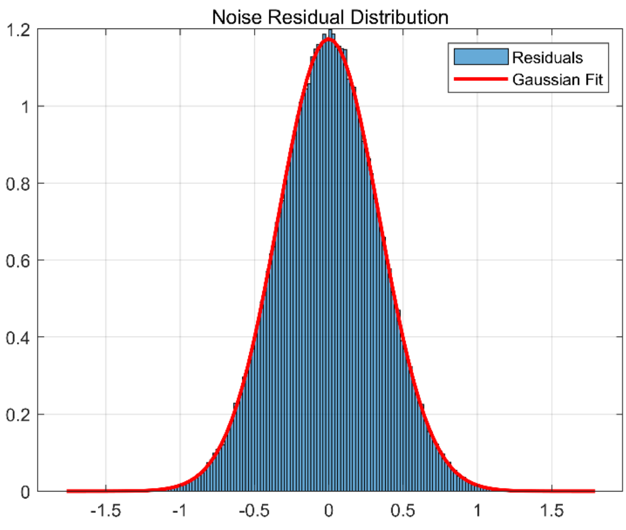


Fig. 11 Noise residual distribution

It can be seen from the above figure that the noise is Gaussian distribution.

5.2.5 Model verification

Bring the data of flight period 1 into the model and only give the amplitude and phase to see if the model can accurately estimate the frequency of flight period 1. The results are as follows:

The FFT result is estimated frequency: 29999884.591124Hz, the effect diagram is as follows.

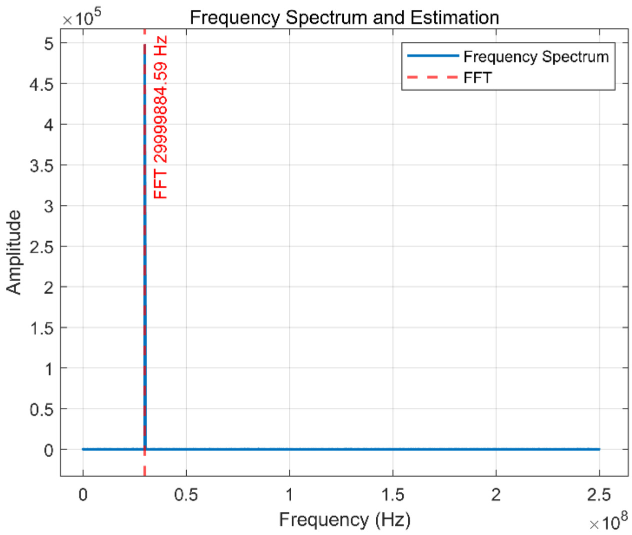


Fig. 12 FFT spectrum

The MLE result is estimated frequency: 29999999.080615 Hz, and the fitting image is as follows:

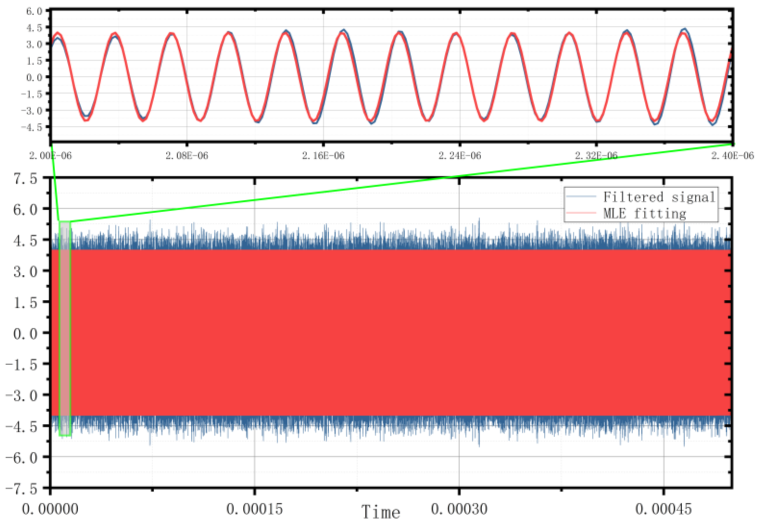


Fig. 13 MLE fitting

It can be seen that the model can achieve a good estimation effect for the data of flight cycle 1, and the estimation accuracy is 99%, almost exactly correct, indicating that the model has a good effect, and the frequency estimation accuracy of flight cycle 2 is high. Therefore, the final estimated frequency of flight cycle 2 is 41MHZ by integrating the results of FFT and MLE.

5.3 Frequency estimation method for unknown parameters

In the case of unknown amplitude and phase, due to greater uncertainty, in order to increase the robustness of the model, other algorithms are considered to be added to the model in question 2 to estimate the frequency at the same time, and finally, a joint evaluation is carried out.

5.3.1 Welch method

Welch is an improved method of period-graph estimation. First, for data segmentation:

$$x_k(t_n) = x[t_n + (k-1)(L-D)] \quad k = 1, 2, ..., K$$

Windowing function:

$$x_k^c(t_n) = x_k(t_n) \cdot w(t_n) \qquad n = 0, 1, ..., L-1$$

For each windowed subsegment, calculate its power spectral density estimation:

$$P_k(f) = \frac{1}{C}\left|\sum_{n=1}^{L-1} x_k^c(t_n) e^{-j2\pi fnT}\right|^2$$

The period plots of all subsegments are averaged to obtain power spectral density estimates:

$$P_{xx}^c(f) = \frac{1}{K}\sum_{k=1}^{K} P_k(f)$$

By finding the maximum value in the PSD vector and its corresponding frequency vector as the frequency estimate.

5.3.2 Autocorrelation method

According to the information given by the title, the real signal is a sinusoidal signal with noise, so it has a certain periodicity, while the autocorrelation method is based on the periodic characteristics of the signal, and estimates the frequency by calculating the similarity between the signal and its own delayed version.

For a signal $x(t_n)$, the autocorrelation function can be expressed as:

$$R_{xx}(\tau) = \frac{1}{N}\sum_{n=0}^{N-1} x(t_n)x(t_n+\tau)$$

The autocorrelation function will show a periodic peak value, and the period presented by the autocorrelation function is T_z corresponding to the signal period, so the frequency is estimated as:

$$f = \frac{1}{T_z}$$

5.3.3 Model solving

The solution steps of the model are similar to problem 2. Welch and autocorrelation method are only added for comparison verification, and the functions of MLE estimation amplitude and phase are added. Then the variance of MLE estimation amplitude and phase is used to calculate MLE estimated frequency.

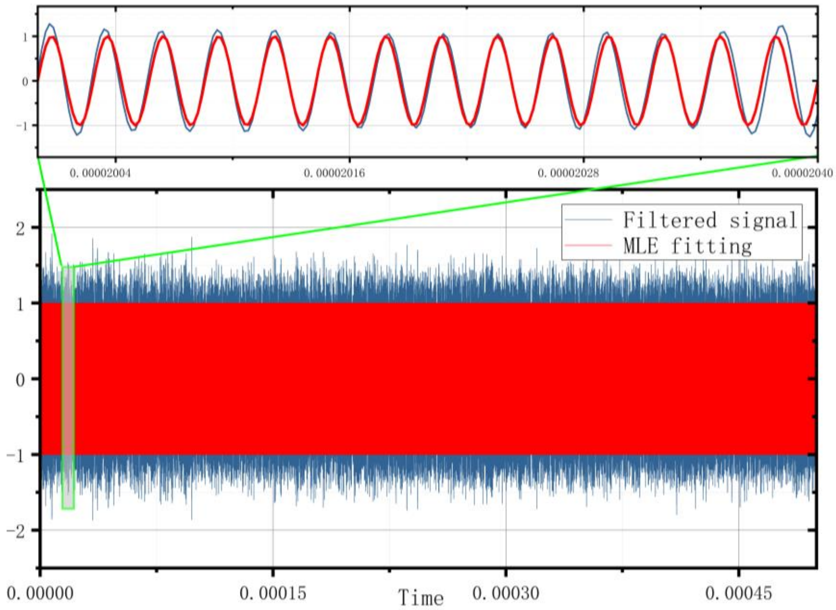


Fig. 14 MLE fitting

MLE's estimated amplitude is 1.0015, phase is 0°, frequency is 34999998.68 Hz, variance is $4.94\times10^{-2}\,HZ^2$, FFT estimated frequency results are as follows:

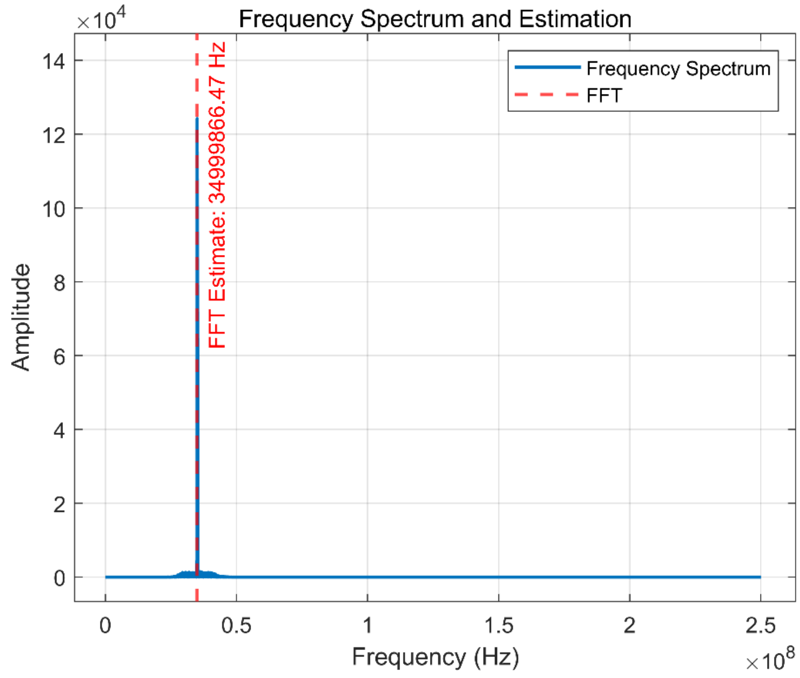


Fig. 15 FFT spectrum

The estimated frequency of FFT is 34999866.47336 Hz, variance is $7.80\times10^{-2}\,HZ^2$, The estimated effect of Welch method is as follows:

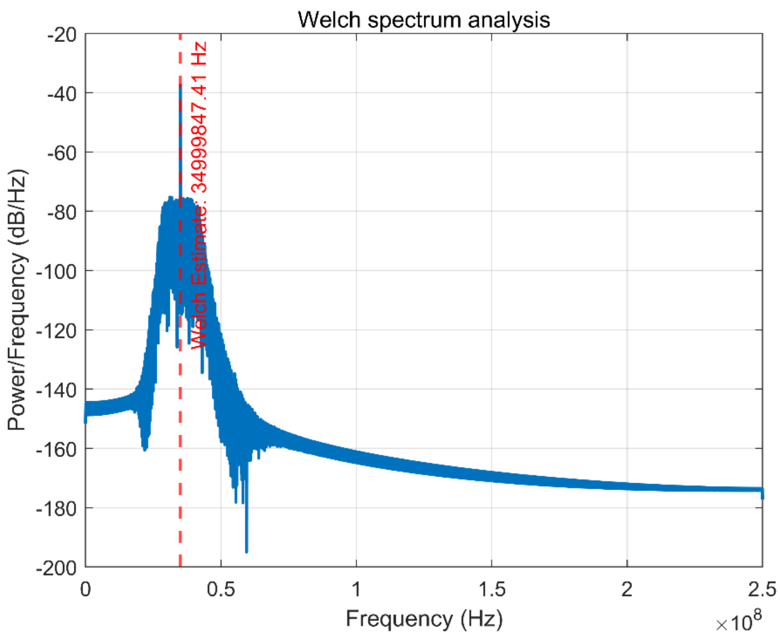


Fig. 16 Welch method estimated the effect

Welch method estimated the frequency as 34999847.41211 Hz and the variance as $8.68\times10^{-2}\,HZ^2$, The estimated effect of autocorrelation method is as follows:

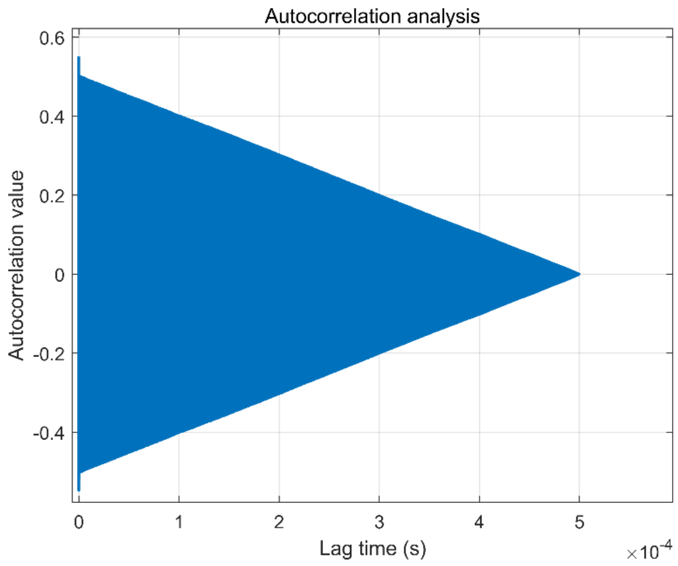
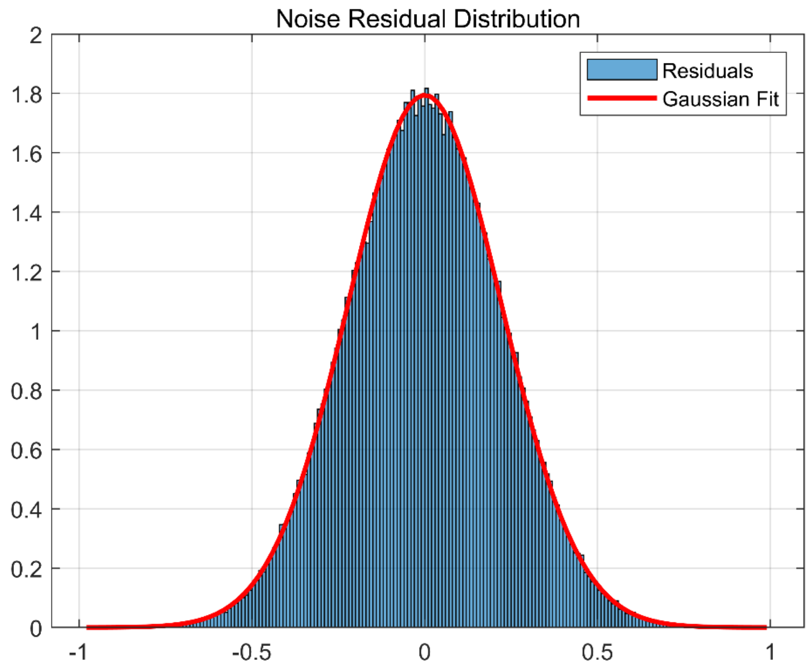


Fig. 17 The autocorrelation method estimates the effect

The frequency estimated by the autocorrelation method is 34999919.99104 Hz, and the variance is $5.96\times10^{-2}\,HZ^2$, After calculation, CRLB= $0.2396094HZ^2$, As a result, the model has a good effect. The estimated frequencies of all methods are around 35MHZ and the deviation is very small. The residual noise distribution is shown in the figure below:



5.3.4 Model verification

Bring the data of flight period 1 into the model, without giving the amplitude and phase, to see whether the model can accurately estimate the frequency of flight period

1 and other parameters. The image here only shows the MLE fitting image:

The MLE fitting diagram is as follows:

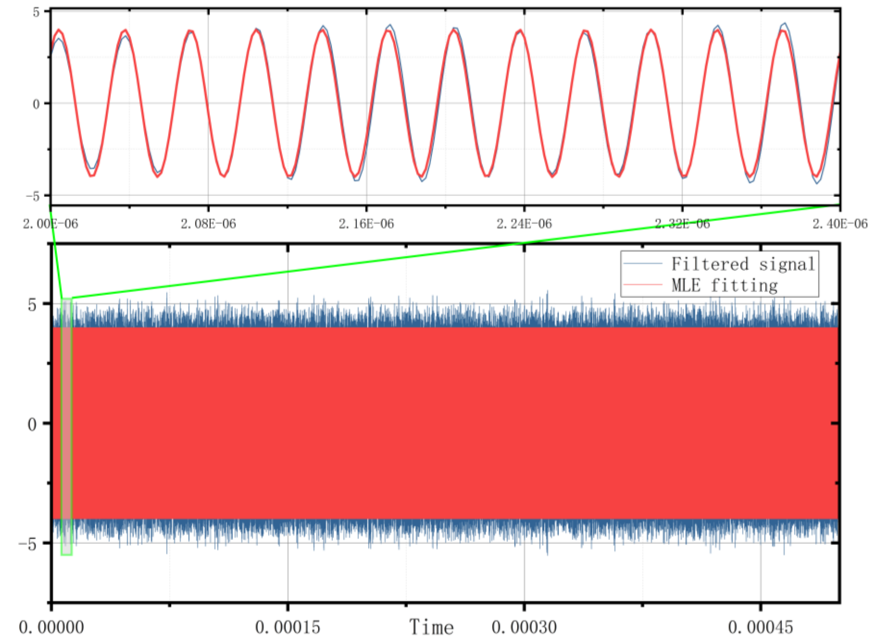


Fig. 18 MLE fitting

Table 3 Model validation data estimate parameters

method	Estimated parameter	variance
MLE	frequency：30000001.02444 Hz、 amplitude：3.99739、 phase：44.77°	$1.71\times10^{-1}HZ^2$
FFT	frequency：29999884.59112 Hz	$5.25\times10^{-1}HZ^2$
Welch method	frequency：30000686.64551 Hz	$9.96HZ^2$
Autocorrelation method	frequency：30000400.03200 Hz	$4.03HZ^2$

It can be seen from the figure above that MLE has a good effect in fitting the signal, and its prediction of frequency, amplitude and phase is almost the same as that of the known conditions in question 1, with high accuracy. Therefore, after rounding the parameters, the parameters of flight period 3 can be directly written as follows: frequency: 35MHZ, phase: 0°, and amplitude: 1.

5.4 Frequency estimation method for intermittent receiving signal

5.4.1 Robust maximum likelihood estimation

In the first two problems, MLE has achieved relatively high precision estimation of frequency estimation, phase and amplitude parameters. However, when the signal is received intermittently, due to the loss of data in the middle, the signal dispersion degree is larger, and traditional MLE is very sensitive to outliers. To improve the robustness of the estimate, consider introducing a robust loss function, here Huber loss function is introduced:

$$\rho_\delta(r)=\begin{cases}\frac{1}{2}r^2 & |r|\le\delta \\ \delta|r|-\frac{1}{2}\delta^2 & |r|>\delta\end{cases}$$

Where the residual:

$$r=x(t_n)-A\sin(2\pi ft_n+\phi)$$

The Huber loss function is a secondary loss when the residuals are small, and has an efficient gradient descent property, which can reduce the sensitivity to outliers.

Minimize the sum of Huber losses to construct the objective function:

$$\min_{A,f,\phi}\sum_{n=1}^{N}\rho_\delta[x(t_n)-A\sin(2\pi ft_n+\phi)]$$

Then fminsearch function in Matlab is used to optimize the solution.

5.4.2 Model solving

For the case of intermittent receiving signals, it is considered to perform segmentation and simultaneous estimation, and then extract the estimated frequencies of all segments to check the variation degree of estimated frequencies among data of different segments. Since the data is short, the accuracy of autocorrelation method and Welch may be affected, so only FFT and robust MLE methods are used for estimation in this problem. The results of robust MLE estimated frequency are as follows. The figure here only shows the fitting effect of the last signal segment:

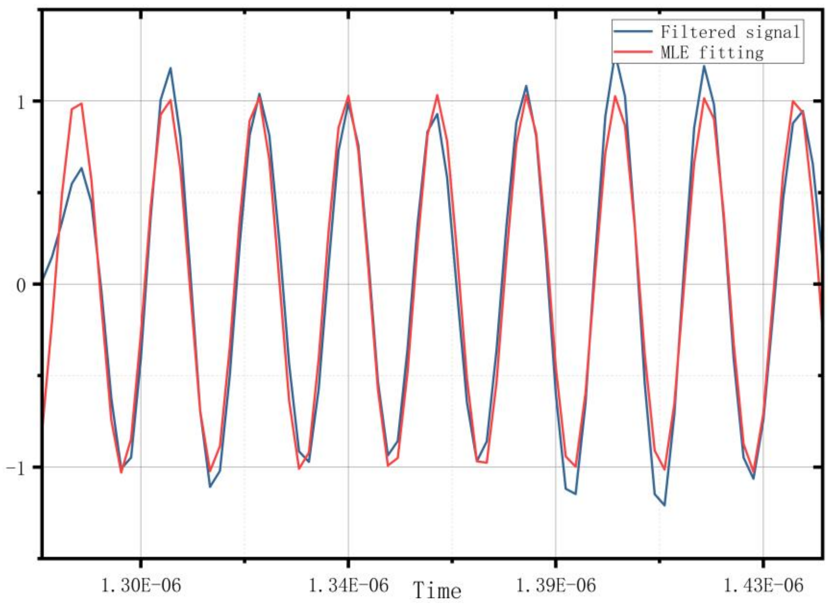
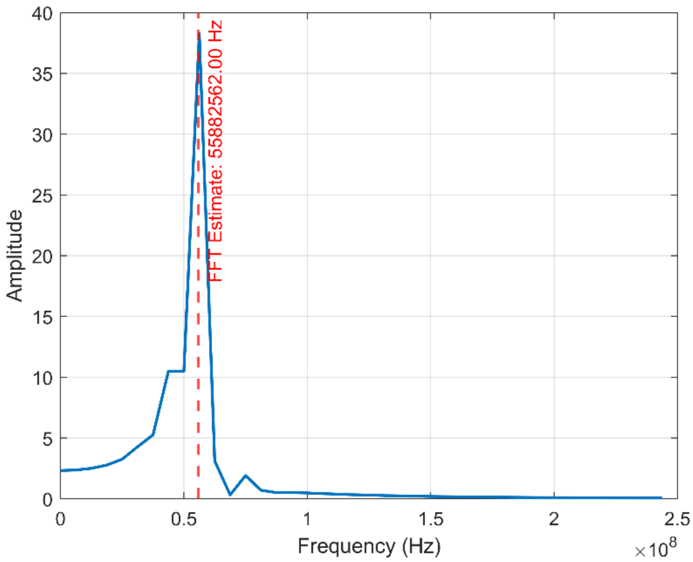


Fig. 19 MLE fitting

It can be seen from the figure that robust MLE has a better signal fitting effect. The estimated frequency of FFT is as follows, and the figure here only shows the estimated effect of the last signal segment:



The results of each section are summarized:

Table 4 Summary of solving parameters for each paragraph

Estimation method	FFT estimated frequency	Robust MLE estimated frequency
	55180398.22	54815499.24
	55198155.03	54138614.88

	55498184.73	54682778.31
	55949703.9	54786398.57
	56371437.4	55087831.09
	55680516.3	55869627.59
	55147084.37	53870843.12
	55932900.09	55044330.16
	55882562	54944492.57
Mean value	55648993.56	54804490.62

It can be seen that FFT is close to the estimated frequency of robust MLE, and the graph drawn is as follows:

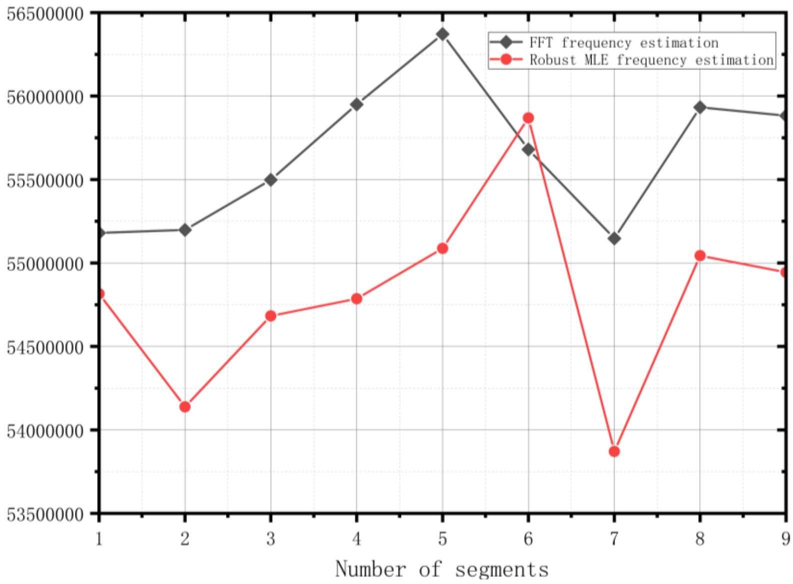


Fig. 20 FFT and MLE solution parameters comparison diagram

The mean value of the two methods is taken as the final result of estimation, that is, the estimated frequency of flight cycle 4 is 55226742.09HZ, and the rounding is 55MHZ.

Ⅵ Model advantages and disadvantages

6.1 Model advantage

1, Through the fast Fourier transform (FFT) analysis of the signal spectrum, can quickly locate the dominant frequency.

2. Based on the main frequency obtained by FFT, the band-pass filter is designed

to effectively reduce the influence of non-target frequency noise.

3. Huber loss function is used to improve the robustness of the model and the fitting accuracy of the model.

6.2 Model defect

1. The filter design is completely dependent on the main frequency estimated by the FFT.

2. The MLE fitting process is highly dependent on the initial value. If the initial value is not accurate, it may fall into the local optimal solution or lead to fitting failure.

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