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Research on Signal Frequency Estimation Based on Segmented Maximum Likelihood Estimation and NLMS

Summary

Due to accuracy issues in airspeed measurement, there is an urgent need for high-quality models to solve the frequency estimation problem of time series signals. This article combines **segmented maximum likelihood estimation** and **NLMS** algorithm to design and propose an efficient frequency estimation model.

For problem one: This article proposes four main steps: Extracting noise signals; Time domain analysis; **Fast Fourier Transform (FFT)**; Frequency domain analysis. In addition, a notch filter is used for noise filtering, and the FFT algorithm is used for time-frequency domain conversion to calculate the power spectral density. The signal is segmented and windowed using the Welch method to achieve smoothing of spectral estimation. The results indicate that the mean of the noise is **0.0058** and the standard deviation is **1.995**. Verified the non periodicity and Gaussian white noise characteristics of the noise signal z (t).

For problem two: Under unknown non noise signal frequencies, this article mainly uses the main frequency localization method and **Maximum Likelihood Estimation (MLE)** method. In addition, Hanning windowing and FFT transformation are also used. Through comparative analysis and frequency pre estimation, the final estimated main frequency value is **41 MHz**. The results indicate that the two methods complement each other and jointly verify the effectiveness and reliability of the frequency estimation method.

For problem three: We use a strategy that combines **bandpass filtering** and **moving average filtering** to preprocess the data. Then, the **NLMS** algorithm was used for frequency estimation, resulting in an estimated frequency of **34.98 MHz**.

For problem four: This question involves modeling intermittent reception mode. We use segmented maximum likelihood estimation method combined with frequency estimation method to estimate the frequency as **56.12 MHz**, amplitude as 0.8727, and phase as 2.5362.

Key word: Frequency Estimation, NLMS, FFT, Segmented Maximum Likelihood Estimation

Content

1. Introduction

1.1 Background

With the advancement of aviation technology, airplanes have become an essential mode of transportation in modern society. To ensure flight safety, real-time airspeed measurement is crucial. Airspeed refers to the velocity of an aircraft relative to the surrounding air, and it is a key parameter influencing flight performance. Among the various types of airspeed, calibrated airspeed (CAS) is particularly significant, as it is closely tied to flight conditions such as angle of attack and sideslip angle. CAS plays a vital role in determining the aircraft's maneuverability and overall flight safety. An inaccurate airspeed reading could lead pilots to operate the aircraft based on faulty data, which can result in severe accidents [1]. Consequently, enhancing the accuracy of airspeed measurement is of paramount importance for aviation safety. In 1974, Munioz et al. first proposed the laser Doppler velocimeter to address the limitations of traditional airspeed measurement methods. This technology is a direct precursor to modern laser velocimetry techniques [2]. Laser velocimetry is an advanced method for precisely measuring airspeed. It works by emitting a laser signal at a fixed frequency, which then interacts with aerosol particles in the air, causing a Doppler frequency shift due to the Mie scattering effect. By exploiting the principle of coherent interference, the system retrieves a signal containing the Doppler frequency shift information, which can be used to estimate the frequency. Using the estimated Doppler shift, the airspeed can be calculated. The schematic diagram of this process is shown in Figure.

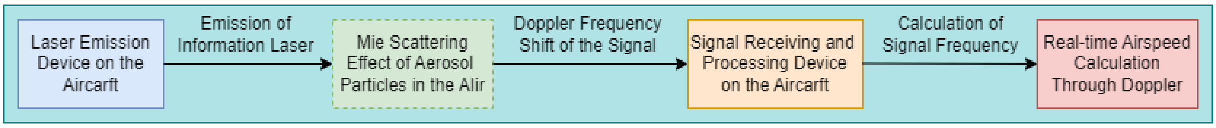

Figure 1 The Schematic Diagram of This Process

Assume the received signal by the aircraft is denoted as x(t), which contains a significant amount of Gaussian white noise z(t), making the frequency estimation of x(t) challenging. The signal can be expressed as follows:

$$x\,(t) = Asin(2\pi f_0 t + \varphi) + z(t) \tag{1}$$

Where A is the amplitude of the transmitted signal, f_0 is the frequency of the transmitted signal, φ is the phase of the transmitted signal, and z(t) represents the Gaussian white noise. This paper uses an aircraft flying in space as a case study. The sampling interval is known to be 2×10^{-9} seconds, and multiple Doppler frequency shift signals are received. The problem of estimating the frequency of the aircraft's laser speed measurement is explored under different flight cycles and environmental noise conditions.

1.2 Our work

The frequency estimation problem in aircraft laser velocity measurement primarily involves four key aspects: 1) analysis of noise characteristics; 2) signal frequency estimation under unknown frequency conditions; 3) frequency estimation under unknown amplitude and phase; and 4) the impact of intermittent reception mode on frequency estimation. This paper focuses on these four core issues and investigates the following four specific problems:

Question 1: To design an effective signal frequency estimation algorithm, it is essential to first analyze the noise characteristics of the received signal x(t). It is given that the amplitude A of the non-noise component of the received signal is 4, the frequency f_0 is 30×10^6 Hz, and the phase φ is 45°. Based on the data provided in Appendix 1, analyze the characteristics of the noise signal z(t) during Flight Cycle 1.

Question 2: In practical applications, the frequency of the non-noise component of the received signal is typically unknown. It is given that the amplitude A of the received signal x(t) is 2 and the phase φ is 0°. Based on the data provided in Appendix 1, analyze and estimate the frequency f_0 of the non-noise component in Flight Cycle 2. (Note: The noise characteristics in Flight Cycle 2 may differ from those in Flight Cycle 1.)

Question 3: In practical applications, the amplitude A and phase φ of the non-noise component of the received signal are unknown, but its frequency still needs to be estimated. Based on the data in Appendix 1, the frequency f of the received signal x(t) in Flight Cycle 3 is analyzed and estimated. (Note: The characteristics of the noise signal in Flight Cycle 3 may differ from those in Flight Cycles 1 and 2.)

Question 4: To avoid signal interference, intermittent reception is commonly employed in practical applications, which reduces the amount of available data. Based on the data in Annex 1, analyze and estimate the pattern of intermittent reception for the received signal x(t) during flight cycle 4, and determine its frequency f.

1.3 Problem Assumption

1. Hypothesis on Noise Characteristics: The noise z(t)exhibits independent statistical characteristics in different flight cycles, and the noise may follow a Gaussian distribution or possess specific frequency-domain features.

2. Hypothesis on Signal Frequency Estimation: When the signal amplitude and phase are known, frequency-domain analysis (such as Fast Fourier Transform, FFT) can effectively extract the primary frequency of the signal.

3. Hypothesis on Signals with Unknown Amplitude and Phase: For signals with unknown amplitude and phase, it can be assumed that methods based on autocorrelation functions or spectral estimation can determine the frequency.

4. Hypothesis on Intermittent Signal Reception: Under intermittent reception modes, the frequency of the signal can be estimated by combining data reconstruction with time-frequency analysis methods, such as Short-Time Fourier Transform (STFT).

5. Hypothesis on Noise Differences Across Multiple Flight Cycles: The noise characteristics in different flight cycles can be assumed to result from external environmental factors (such as pressure and temperature variations) and can be modeled and distinguished through statistical analysis.

2. Model for Problem One

2.1 Analysis of Problem One

The main task of problem one is to analyze and extract the noise signal z(t) from the signal x(t) received by the aircraft. This process is crucial for accurately estimating the signal frequency. By identifying and quantifying the characteristics of noise, not only can the accuracy of signal processing be improved, but the robustness and anti-interference ability of the laser speed measurement system can also be enhanced, thereby optimizing the stability and reliability of the flight system. According to the conditions of question one, it can be inferred that the received signal consists of two parts: a non noise signal and a noise signal

$$x\ (t) = Asin(2\pi f_0 t + \varphi) + z(t) \tag{2}$$

Among them, the amplitude A of the non noise signal is 4, the frequency $f_0 = 30 \times 10^6$ Hz, and the phase $\varphi = 45^{\circ}$, while the noise signal z (t) may include Gaussian white noise signals and other forms of interference signals. To accurately analyze the characteristics of the noise signal z (t) in flight cycle 1, the noise signal should be extracted first. Non noise signals should be removed from the total received signal x (t) in equation (1) through signal separation techniques, and an approximate noise signal z (t) can be obtained using effective methods; Secondly, perform time-domain analysis on the extracted noise signal z (t), determine the statistical characteristics and randomness test of the noise signal, and test the distribution characteristics of the noise signal; Finally, by analyzing the spectral characteristics of the noise signal, interference bands in the spectrum are identified, providing a basis for the design of subsequent frequency estimation algorithms. The flowchart is shown in Figure 2:

2.2 Resolution of Problem Two

2.2.1 Extract Noise

It is known that during high-speed flight, aircraft receive signals that are affected by Doppler frequency shift and various complex interference signals. The received laser velocimetry signal x (t) is the superposition of the emission signal s (t) and the noise signal z (t), and its mathematical expression is:

$$x(t) = A sin(2\pi f_0 t + \varphi) + z(t) \tag{3}$$

According to the known formula and non noise signal parameters, data preprocessing is first performed in flight cycle 1. Subtracting the transmitted signal s (t) from the received signal x (t) yields an approximate noise signal z (t), which is expressed as:

$$z(t) = x(t) - A sin(2\pi f_0 t + \varphi) \tag{4}$$

Based on the frequency domain characteristics, the approximate noise signal z (t) is filtered using a notch filter to further extract the pure noise signal. A notch filter is a type of band stop filter composed of a high pass filter and a low-pass filter, also known as a ”band suppression filter”. It can effectively suppress signals of specific frequencies and their vicinity, while retaining the energy of other frequency signals, making it very suitable for eliminating interference signals. In this article, a band stop filter was implemented using Python custom FIR filter to achieve notch effect.

2.2.2 Time Domain Analysis

To analyze the time-domain characteristics of the noise signal z (t), the signal is first visualized, as shown in Figure 3, which shows the variation of the received noise signal in the time domain. The following characteristics can be observed from Figure 3:

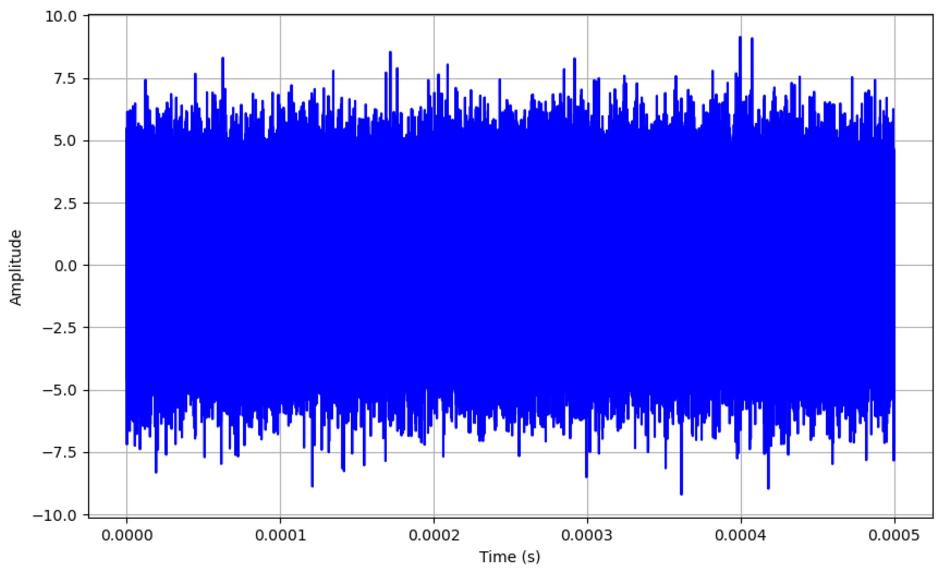

Figure 2 Noise Signal z(t) in Time Domain

(1) Random volatility

The noise signal z (t) exhibits significant random fluctuations in the time domain, which is consistent with the typical characteristics of random noise signals. By carefully observing the distribution changes of amplitude, it was found that the amplitude varied between about -10 and+10, indicating that the noise signal z (t) has high randomness and dynamic volatility. This phenomenon may be related to high-frequency interference in the system or complex noise factors in the external environment.

(2) Stationarity

From the time distribution characteristics, it can be seen that the noise signal z (t) does not show a significant trend change throughout the entire observation period, nor does it show a significant upward or downward trend, as well as a long-term average deviation. It belongs to the category of stationary noise and can be considered as stationary noise.

(3) Non periodic

Through further observation, it can be seen that the fluctuation characteristics of the noise signal z (t) do not exhibit periodicity or repeatability. This characteristic is consistent with the definition of random noise, indicating that the noise signal z (t) belongs to the non periodic random noise category, further confirming that the signal has non periodic random characteristics and belongs to the non periodic random noise signal.

(4) The statistical mean is close to zero

By calculating the mean of the z (t) signal, it can be found that the average value of the signal is close to zero, which conforms to the typical characteristics of Gaussian white noise. The mean of Gaussian white noise signals is usually zero and the amplitude fluctuates randomly between positive and negative values. This discovery provides important basis for noise modeling and classification.

(5) Autocorrelation analysis

As shown in Figure 3, the autocorrelation of the z (t) signal is low, and it does not exhibit significant repeatability or similarity over time, further verifying the random and non periodic nature of the noise signal z (t). In summary, the time-domain analysis in Figure 3 indicates that the noise signal z (t) is a stationary and highly random noise signal with zero mean, exhibiting distinct Gaussian white noise characteristics.

Generate the statistical probability density distribution map of the noise signal using the hist function in Python software, as shown in the green part of Figure 4. Observing its distribution pattern, it can be seen that it is highly consistent with the normal distribution function. Meanwhile, the standard normal distribution function is plotted in the figure, as shown by the red curve in Figure 4. Comparing the probability density function (PDF) plotted by the hist function, it was found that the two are highly consistent, which verifies that the noise mean in the data is 0 and has typical Gaussian white noise characteristics.

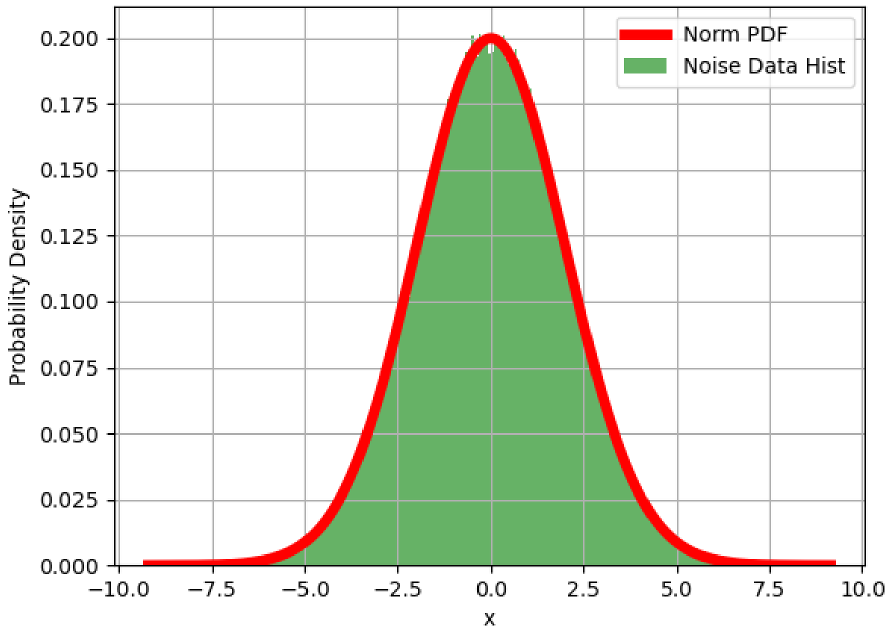

Figure 3 Normal Distribution PDF

2.2.3 Fast Fourier Transform

To further study the spectral characteristics of signals, this paper adopts the method of Fast Fourier Transform (FFT). FFT is an efficient algorithm that optimizes the calculation process of Discrete Fourier Transform (DFT) and is widely used for fast conversion of signals from time domain to frequency domain.Compared to the $o(N^2)$ computational complexity of DFT, FFT reduces the complexity to O (NlogN) through recursive methods, which not only significantly improves computational efficiency, but also effectively identifies and analyzes signal spectral features including noise signals, thereby supporting real-time data processing and signal characteristic analysis[3].

The FFT method can directly process discrete signals and also approximate continuous time signals. Based on the above advantages, this article uses FFT to analyze the frequency domain of z (t) signal. Firstly, the time-domain signal z (t) is sampled to obtain a discrete signal sequence z [n] of length N. Calculate its frequency domain representation Z (k) through FFT transformation:

$$Z(k) = \sum_{n=0}^{N-1} z[n]e^{-j\frac{2\pi}{N}kn}, k = 0, 1, ..., N - 1 \tag{5}$$

Among them, Z (k) represents the complex amplitude of the kth frequency component, including amplitude and phase information. By using the FFT algorithm, the time-domain signal z (t) is converted to the frequency domain to generate a spectrogram, as shown in Figure 4. The following characteristics can be observed from Figure 4:

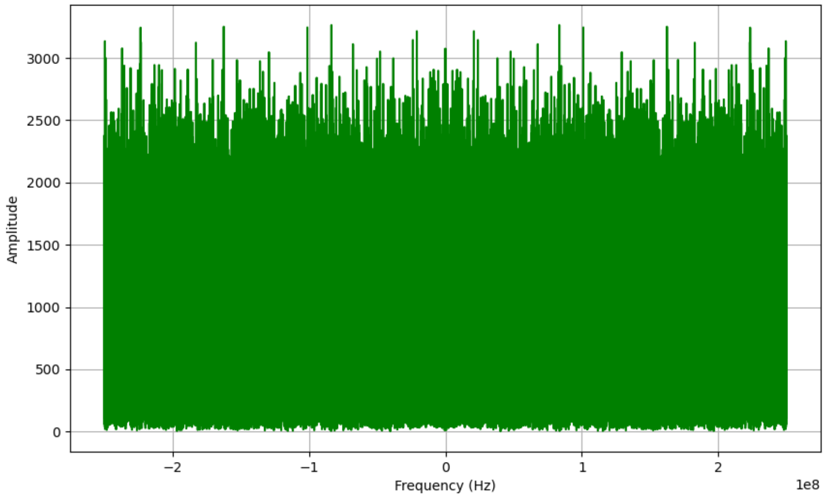

Figure 4 Frequency Spectrum of Noise Z(f)(FFT)

(1) The spectrum diagram displays the spectral distribution characteristics of the z(t) signal, with the horizontal axis representing frequency in units of 10^8 Hz and the vertical axis representing amplitude.

(2) The frequency spectrum of the noise signal is evenly distributed throughout the entire frequency range, mainly concentrated in the range of 2500 to 3000, exhibiting broadband white noise characteristics.

(3) No significant main frequency component or energy peak was observed, indicating that the signal has randomness and non periodicity.

(4) The symmetrical distribution of the spectrum conforms to the theoretical results of DFT, indicating that the signal contains both positive and negative frequency components.From the perspective of signal processing, noise signals are widely distributed in the frequency domain, indicating that their interference to the system involves multiple frequency bands rather than being concentrated in a certain frequency range. This also indicates that it is difficult to effectively suppress noise through single band filtering alone, and multi band filtering or adaptive noise suppression techniques are needed. In addition, the randomness and broadband characteristics of the spectrum may originate from external complex interference or internal random noise effects.

2.2.4 Frequency Domain

In order to further explore the spectral characteristics of signals, this paper adopts the calculation and analysis method of Power Spectral Density (PSD). PSD is a key indicator for describing the energy distribution of noise signals in the frequency domain. By using the Welch method to estimate PSD, we obtained the spectrum estimation results shown in Figure 5. The

Welch method divides the signal into multiple segments, applies a window function to each segment of data and performs Fourier transform, and then calculates the square average of these transform results, effectively reducing the influence of random noise in a single Fourier transform and providing smoother and more stable power spectrum estimation. This method is particularly suitable for spectral analysis of random signals, achieving a good balance between frequency resolution and estimation stability [4]. The following key characteristics can be observed from

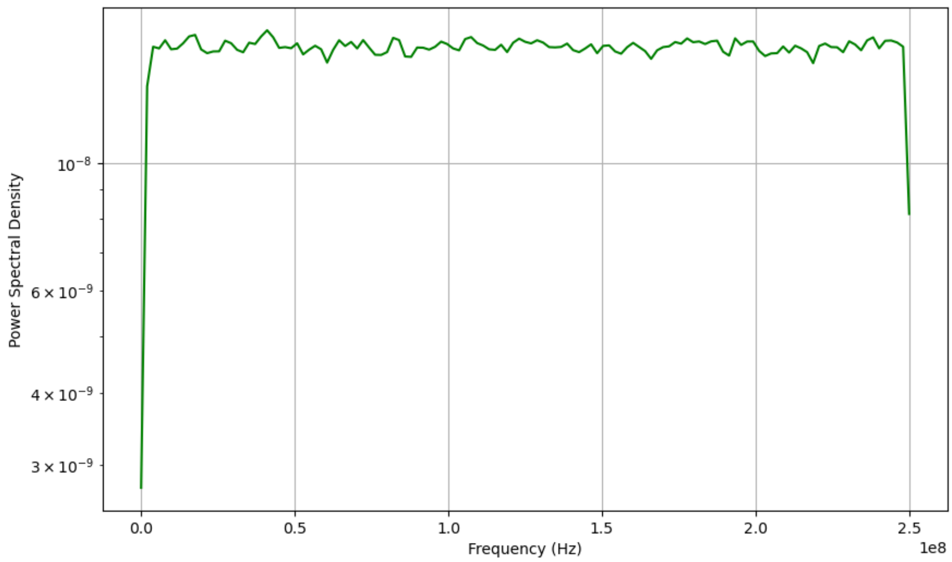

Figure 5 Power Spectral Density of Noise(Welch Method)

Figure 5:

(1) The vertical axis of Figure 5 represents power spectral density in W/Hz, and the horizontal axis represents frequency in 10^8 Hz[5].

(2) The PSD curve is relatively flat throughout the entire frequency range, with amplitudes mainly maintained at around 6×10^{-9}. This indicates that the noise signal exhibits the characteristics of broadband white noise, that is, it has a uniform power distribution at all frequencies.

(3) At both ends of the frequency (close to 0 Hz and 2.5×10^8 Hz), there is a sharp decrease in PSD values, which may be due to sampling boundary effects or signal bandwidth limitations[6].

(4) Through the Welch method, smoothing of spectrum estimation has been achieved, significantly reducing the spectral fluctuations caused by randomness in traditional spectrum estimation. The smoothing characteristics of the spectrum in Figure 5 further confirm this point. In addition, no obvious main frequency components or sharp energy peaks were found in the noise spectrum, indicating that the interference characteristics of the noise are random and widespread[7][8].

Based on data calculations, we have come to the following conclusion:

(1) The mean of the noise is about 0.0058, close to zero, indicating that the positive and negative parts of the noise are roughly symmetrical. This characteristic is consistent with typical zero mean noise.

(2) The noise mean is not completely zero, and there is a certain deviation, which may be caused by systematic errors in the data acquisition process or bias of the noise signal. The standard deviation of the noise is 1.995, indicating that the fluctuation amplitude of the noise signal is between approximately plus or minus 2. This fluctuation range is consistent with the observed positive and negative fluctuation range of about 7.5 in the time domain graph. A larger standard deviation means that the noise signal has significant volatility, and the larger the standard deviation, the stronger the energy of the noise and the more significant its impact[9].

3. Model for Problem Two

3.1 Analysis of Problem Two

The core difficulty of this section lies in how to analyze the received signal x(t) when the frequency of the non noise signal is unknown. The goal of problem two is to extract the main frequency component of the non noise signal. To accurately estimate the frequency of the non noise part in flight cycle 2, the main frequency localization method and maximum likelihood estimation method are used to solve the problem. The main solution idea is as follows[10]:

Firstly, preprocess the received signal data in flight cycle 2. In order to accurately estimate frequency, it is necessary to sample the data to obtain a discrete time series x[n], and remove the DC component from it, which can effectively eliminate the mean shift of the signal.

Secondly, smoothing processing is adopted to weaken the interference of high-frequency noise using a window function, while preserving the main frequency characteristics of the signal, with the aim of reducing the impact of noise on subsequent frequency estimation.

Thirdly, FFT transform is used to convert the preprocessed signal from the time domain to the frequency domain, obtaining a spectrogram that can visually display the frequency distribution of the signal. Fourthly, by analyzing the spectrogram and finding the point of maximum spectral amplitude, the main frequency component of the signal can be quickly extracted. Finally, the estimated signal is compared and analyzed with the smoothed signal, and the results of the pre-estimation are compared using the main frequency localization method and maximum likelihood estimation method, respectively. The flowchart is shown in Figure[11]:

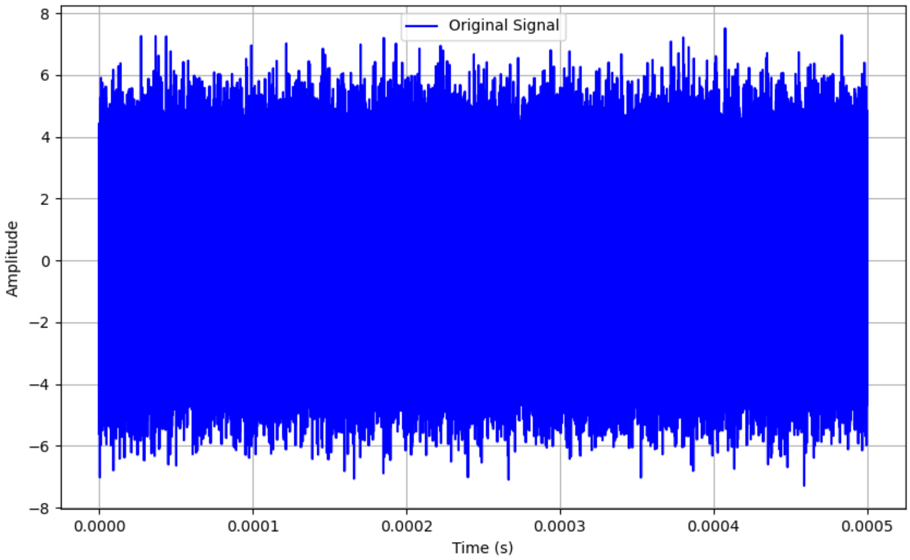

Figure 6 Time Domain of the Original Signal

3.2 Resolution of Problem Two

3.2.1 DC Component Removal

In order to solve the frequency estimation problem of the non noise part of the received signal in problem two, it is necessary to preprocess and analyze the received signal in flight cycle two. Firstly, conduct a preliminary time-domain analysis of the received signals during flight cycle 2. The initial time domain diagram of the received signal is shown in Figure[12]. Secondly, remove the DC component from the signal to eliminate the mean shift of the signal.The DC component is the average value of the signal, and its presence may interfere with spectral analysis.The process of removing the DC component can be achieved through the following formula:

$$x_{new}[n] = x[n] - \frac{1}{N}\sum_{n=0}^{N-1} x[n] \tag{6}$$

Where x[n] is the original signal, $\frac{1}{N}\sum_{n=0}^{N-1} x[n]$ is the mean of the signal, which is the DC component, and $x_{new}[n]$ is the signal after removing the DC component[13]. The time-domain diagram of the signal after removing the DC component is shown in the figure:

3.2.2 Smoothing

Smooth the signal after removing the DC component and introduce a Hanning window as the window function to weaken the influence of high-frequency noise interference while preserving

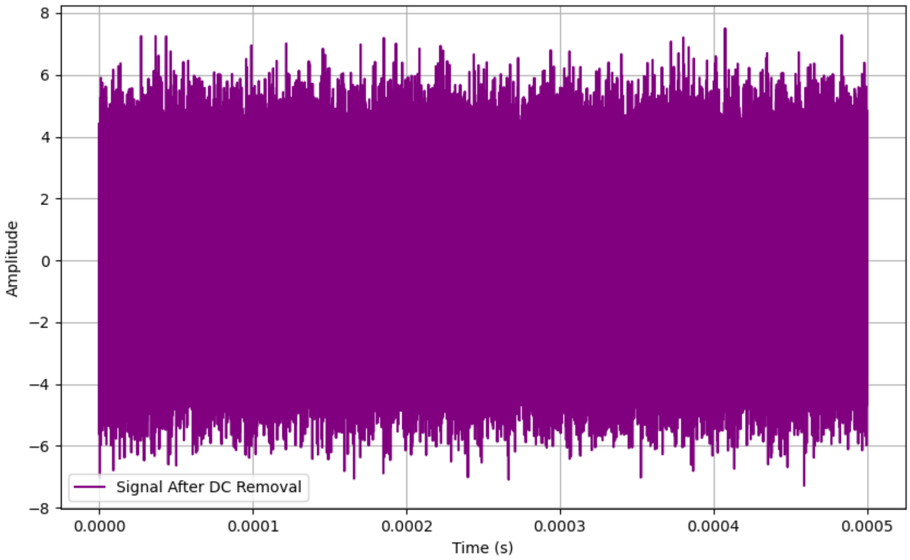

Figure 7 Time Domain of the Original Signal

the main characteristics of the signal. The mathematical expression for Hanning window is:

$$\omega(n) = 0.5 - 0.5cos(\frac{2\pi n}{N-1}) \tag{7}$$

W (n) is the value of the window function at position n, where n is the index of the sample points in the window, starting from 0. N is the length of the window, which is the number of sample points. The time-domain diagram after windowing the signal is shown in the figure:

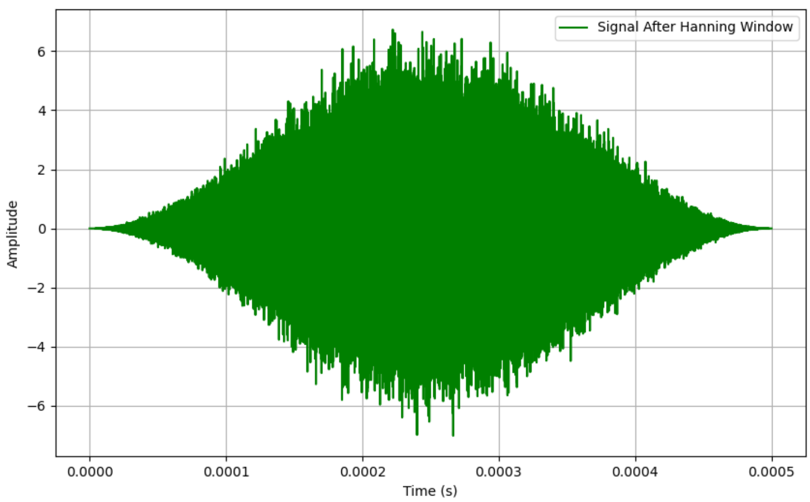

Figure 8 Signal after Applying Window Function

3.2.3 Maximum Likelihood Estimation

In problem two, in order to estimate the frequency of the non noise part in the received signal, we introduced the method of maximum likelihood estimation. The advantage of maximum likelihood estimation is that it can utilize the statistical properties of noise to accurately estimate the frequency components in signals, especially in situations where noise interference is strong and signal amplitude changes are complex. This method not only improves the robustness of frequency estimation, but also provides a reliable theoretical basis for subsequent modeling of multi flight period noise characteristics and frequency estimation[14]. In signal models, the likelihood function can be expressed as

$$L(f_0, A, \phi \mid x(t)) = \prod_{t=1}^{T} p(x(t) \mid f_0, A, \phi) \tag{8}$$

To simplify the calculation, the logarithmic likelihood function is usually taken:

$$lnL(f_0, A, \phi \mid x(t)) = \sum_{t=1}^{T} lnp(x(t) \mid f_0, A, \phi) \tag{9}$$

By maximizing the logarithmic likelihood function, we can obtain the optimal estimate of the frequency f_0 for the non noise portion. Finally, we obtained the estimated frequency result as 41MHz[15].

3.2.4 Spectrum Estimation

Perform FFT transformation on the preprocessed signal, converting it from the time domain to the frequency domain to obtain its spectral distribution. In the spectrogram, by analyzing the peak position of the amplitude, the main frequency components of the non noise part are extracted using the main frequency positioning method. Specifically, by locating the maximum point of spectral amplitude, the main frequency of the signal can be quickly determined. In order to verify the accuracy and robustness of frequency estimation, the main frequency localization method and maximum likelihood estimation method were used to analyze the signal and draw a comparison chart between the estimated signal and the smoothed signal, as shown in Figure.

Based on comparative analysis, the following conclusion is drawn: the estimation results of these two methods are highly consistent, indicating that under different algorithm applications, the frequency estimation results can effectively reflect the main frequency characteristics of the non noise part. The main frequency positioning method performs well in extracting signal main frequencies due to its intuitiveness and computational efficiency. Meanwhile, the maximum likelihood estimation method further validated the accuracy and robustness of the results through statistical optimization. Overall, these two methods complement each other and jointly demonstrate the effectiveness and reliability of frequency estimation methods[16].

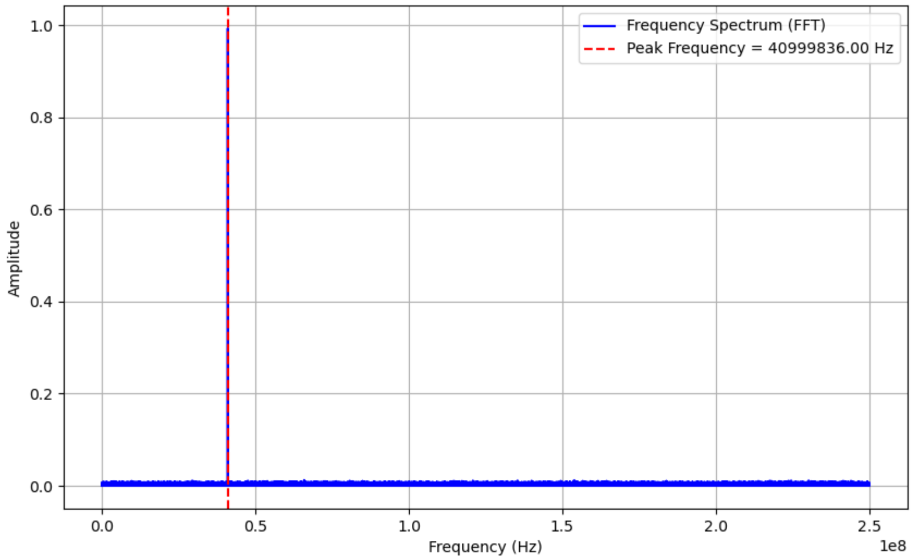

Figure 9 Frequency Estimation for Flight Period 2

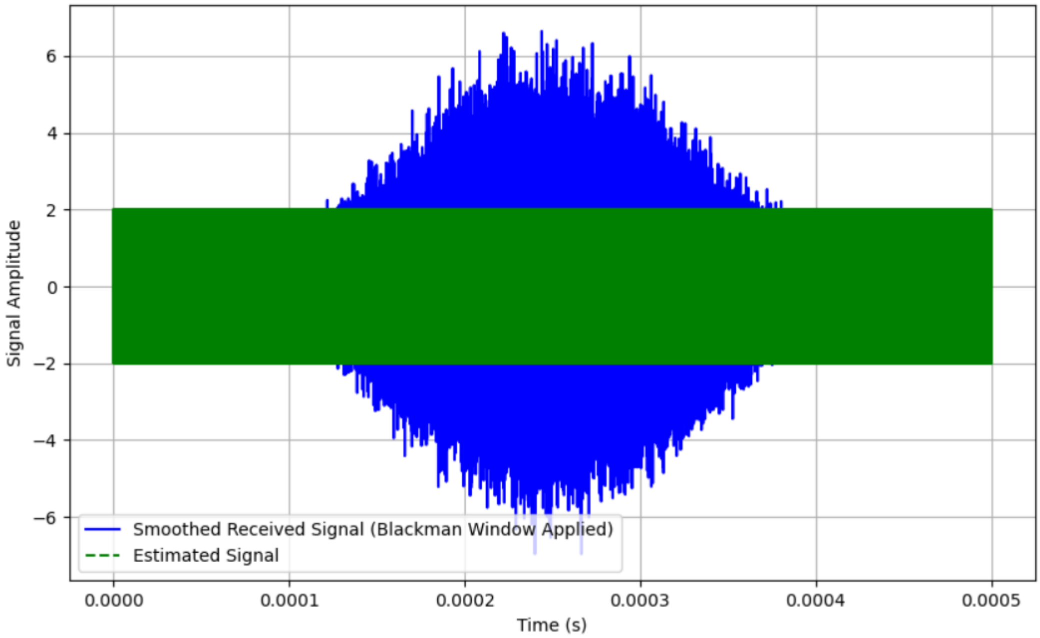

Figure 10 Comparison between Smoothed Signal and Estimated Signal

4. Model for Problem Three

4.1 Analysis of Problem Three

The core challenge of this problem lies in the statistical properties of the noise signal z(t), which may differ from the first two questions, coupled with the unknown fundamental parameters of the signal. These variations not only increase the complexity of frequency estimation but also impose higher demands on the robustness of the algorithm. The frequency estimation algorithm must extract the dominant frequency information under a strong noise background while avoiding interference from spurious peaks caused by noise, despite unknown amplitude and phase[17].

To address this issue, we employ a combination of bandpass filtering and the normalized least mean square (NLMS) algorithm. First, bandpass and moving average filters are applied for preprocessing to extract the effective frequency components of the signal. Subsequently, the NLMS algorithm dynamically adjusts to minimize the prediction error and iteratively optimizes the model coefficients. Finally, the roots of the linear predictive polynomial are solved to identify the characteristic root closest to the unit circle, and the corresponding frequency is calculated[18].

4.2 Resolution of Problem Three

4.2.1 Data Preprocessing

In the preprocessing phase of this problem, we employ a combined strategy of bandpass filtering and moving average filtering.

Bandpass Filter A bandpass filter can clean signals in the frequency domain by removing irrelevant frequency noise. It allows signals within a specific frequency range to pass through while suppressing those outside the range. Its design is based on the theoretical foundation of frequency domain analysis, using a frequency response function H(f) to selectively amplify or attenuate the frequency components of the input signal x(t) . The frequency response of a bandpass filter is defined as:

$$H(f) = \begin{cases} 1, & f_L \leq f \leq f_H \\ 0, & otherwise \end{cases} \tag{10}$$

f_L represents the lower cutoff frequency, which filters out signal components below f_L (such as drift). f_H represents the higher cutoff frequency, which filters out signal components above f_H (such as high-frequency noise). The actual frequency response of a filter is not an ideal ”rectangular” shape but is instead approximated using functions such as Butterworth, Chebyshev, or Bessel. In this study, we adopt the Butterworth function.

Moving Average Filtering We will use moving average filtering for preprocessing. Thus smoothing the time series data and reducing the impact of noise. It generates a smooth output by calculating the average value of the signal within a certain window, thereby eliminating unnecessary high-frequency components.

We use weighted moving average here, which is superior to simple moving average in that each value in the window is assigned a different weight[19]. Recent data points are usually given higher weights. The formula is as follows:

$$y_t = \frac{\sum_{i=0}^{N-1} w_i \cdot x_{t-i}}{\sum_{i=0}^{N-1} w_i} \tag{11}$$

Among them, w_i is the weights of each data point in the window.After that, we successfully reduced the noise or high-frequency components in the measurement data.

4.2.2 NLMS Algorithm

The initial step involves constructing an estimated value for the current sample using linear prediction, whose core idea lies in utilizing a linear combination approach to optimize model parameters by minimizing prediction errors. The mathematical expression for the linear prediction model is:

$$\hat{X}[n] = \sum_{k=1}^{M} C_k X[n-k] \tag{12}$$

$\hat{X}[n]$ is the predicted value of the current sample, X[n-k] is the k-th sample from the past, C_k is a predictive coefficient that needs to be optimized to obtain, M is the order of the model that determines the number of historical samples used in the model. This formula represents that the current sample value is predicted by the weighted sum of M past samples. We use prediction error to measure the estimated value. Prediction error is an important indicator for optimizing prediction models, and its size directly reflects the accuracy of the model. It is defined as the difference between the true value of the current sample and the predicted value:

$$e[n] = x[n] - \hat{x}[n] \tag{13}$$

Among them, e[n] is the prediction error in this problem, $x[n]$ is the true value of the current sample, and $\hat{x}[n]$ is the predicted value corresponding to the current sample. Our coefficient update for this question uses the NLMS algorithm. The NLMS algorithm is an online optimization algorithm based on gradient descent, used for iteratively updating prediction coefficients. Compared with the standard LMS algorithm, NLMS avoids the problem of unstable learning step size caused by excessive signal amplitude by normalizing the input signal.

$$c_k^{new} = c_k^{old} + \mu \frac{e[n]x[n-k]}{\|x[n-k]\|^2 + \epsilon} \tag{14}$$

Among them, μ is a step size factor that can be used to control the learning speed, $\|x[n-k]\|^2$ is the normalization factor of the input signal to prevent divergence caused by a large step size.Iteration is the parameter used to control the change of step size factor based on the current iteration number scalefactor, which gradually decreases the step size factor according to the iteration number[20]. This problem also involves a dynamic step size factor, which gradually decreases according to the number of iterations:

$$\mu_{dynamic} = \mu_{base} \cdot scalefactor^{[iteration/500]} \tag{15}$$

4.2.3 Frequency Estimation

Next, we perform frequency estimation with the aim of extracting the primary frequency information of the signal from the linear prediction model. The core method is to determine the dominant frequency of the signal by solving the roots of the prediction polynomial and finding the roots closest to the unit circle. Based on the final prediction coefficient c_k obtained from the linear prediction model, a prediction polynomial can be constructed:

$$P(z) = 1 - \sum_{k=1}^{M} c_k z^{-k} \tag{16}$$

Among them, z is a complex variable representing a point on the complex plane, c_k is the predictive coefficient of this problem, which can be trained by the model, and M is the order of the model, which can determine the order of the polynomial. Next, we search for characteristic roots and select the root r that is closest to the unit circle, whose position determines the main frequency of the signal. The degree to which the roots of a polynomial are close to the unit circle reflects the stability of the signal, while the angle of the roots corresponds to the frequency components of the signal. Assuming the root found is $r = |r|\, e^{j\theta_0}$, $|r| \approx 1$, indicates that the root is close to the unit circle, θ_0 is the angle of the root can represent the main frequency characteristics of the signal. According to the angle θ_0 of the eigenvalues, the frequency of the signal can be considered as:

$$f_{estimated} = \frac{\theta_0}{2\pi \cdot T} \tag{17}$$

Among them, $f_{estimated}$ is the estimated frequency of this problem, θ_0 is the angle of the eigenvalues, represented in radians here, and T is the sampling time interval in this problem, used to measure the sampling frequency.

4.2.4 Experimental Result

We set the moving average window size to 5, the length (order) of the linear prediction model to M=40, and dynamically adjust the step size factor u to 0.0005. Thus, we estimate the frequency to be 34.98 MHz.

Next, we will analyze the chart. The initial stage has a large error: At the beginning, the prediction error is large, which may be due to the random setting of the initial linear prediction

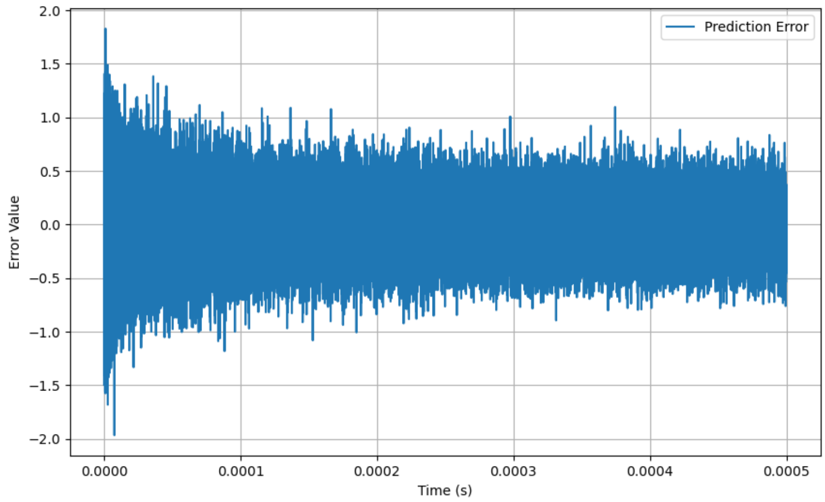

Figure 11 Prediction Error using NLMS Algorithm (After Further Improvements)

coefficient, which does not match the signal. The error gradually decreases: As the iteration progresses, the NLMS algorithm updates the prediction coefficients, and the error gradually decreases. In the figure, the error amplitude remains stable within a small range, indicating the convergence of the algorithm. Error range: The error range in the stable stage is about [-1.0, 1.0], indicating that the prediction model can fit the signal well, but there is residual noise influence. The estimated frequency value (34.98 MHz) is very close to the actual center frequency, indicating that the linear prediction model is effective on the filtered signal.

5. Model for Problem Four

5.1 Analysis of Problem Four

In intermittent reception mode, the signal is discontinuous due to intermittent reception, which limits the amount of available information. This poses a challenge for frequency estimation, requiring the design of suitable frequency estimation algorithms under limited data volume and intermittent noise characteristics. So in this article, we adopt the frequency domain analysis fast Fourier model. We will first conduct time-domain analysis to observe its overall fluctuations and cycles.

Then, the signal is transformed from the time domain to the frequency domain using FFT to obtain the spectral information of the signal. Then perform MLE to find the optimal parameter combination that makes the model closest to the actual signal segment. Calculate the mean of the parameters estimated for all segments to obtain the final frequency, amplitude, and phase estimates. This segmented method can reduce the impact of local noise and improve the estimation accuracy of global parameters.

5.2 Resolution of Problem Four

5.2.1 Segmented Maximum Likelihood Estimation

We first provide the problem model for this question:

$$Y(f) = \int_{-\infty}^{+\infty} x(t)e^{-2j\pi ft}dt \tag{18}$$

Then calculate the unilateral amplitude spectrum to find the maximum value and index of the amplitude spectrum. After performing spectral analysis, assuming that the signal is a sine wave with noise, we estimate the specific parameters using the maximum likelihood estimation method

Assuming that the signal can be modeled as a sine wave model, the goal is to find the optimal parameters A, f, to make the model closest to the observed data. The model is expressed as follows:

$$x(t) = Asin(2\pi ft + \theta) \tag{19}$$

To achieve maximum likelihood estimation by minimizing the squared error between the signal and the model, the goal is to minimize the following objective function

$$L(A, f, \theta) = -\sum_{i=0}^{N-1}(x_i - Asin(2\pi ft_i + \theta))^2 \tag{20}$$

The final parameter estimate is

$$\begin{cases} \hat{A} = \frac{1}{M}\sum_{k=1}^{M} A_k \\ \hat{f} = \frac{1}{M}\sum_{k=1}^{M} f_k \\ \hat{\theta} = \frac{1}{M}\sum_{k=1}^{M} \theta_k \end{cases} \tag{21}$$

5.2.2 Experimental Result

The received signal value and time are data in the dataset In the MLE algorithm, the data is segmented into 10 segments, each containing 80 sampling points. We have estimated that the frequency f is 56.12 MHz, the amplitude A is 0.87, and the phase is 2.54.

Next, we will draw a time-domain signal diagram.

From the graph, it can be seen that the signal exhibits a periodic fluctuation with significant amplitude changes. This indicates that the signal has a certain periodicity in the time domain, and there are also some high-frequency components or noise present The signal has some obvious periodic fluctuations, but there are subtle irregular oscillations within each cycle, which may be due to interference from noise or other high-frequency components

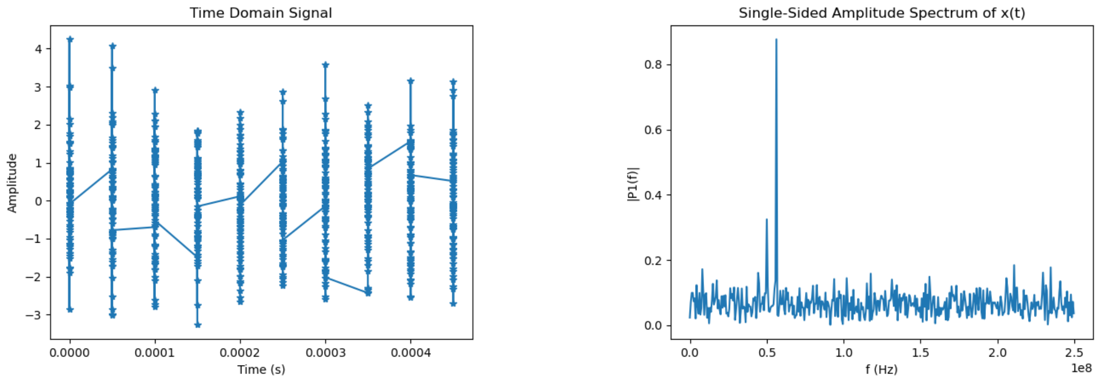

Figure 12

Then, we will draw a time-domain signal diagram:

In the frequency domain diagram, the frequency corresponding to the most obvious peak is the main frequency of the signal. From the single-sided amplitude spectrum, it can be seen that the frequency is about 50 MHz, indicating that the signal has a very high main frequency component. However, from the overall shape of the spectrum, in addition to the main frequency, there are also some lower amplitude frequency components, which may be caused by noise, harmonics, or other secondary frequency components.

In further analysis, the frequency obtained through maximum likelihood estimation method is approximately 56.12 MHz, which is consistent with the frequency near the maximum peak in the spectrum. On this basis, maximum likelihood estimation more accurately determines the frequency of the main sine components of the signal, while also describing the main oscillation characteristics of the signal, including frequency, amplitude, and phase.

Overall, both the spectrogram and maximum likelihood estimation results confirm the existence of the main frequency of the signal and provide more accurate frequency estimates, further supporting the results and reliability of signal analysis.

6. Model Evaluation

In this article, we comprehensively used segmented maximum likelihood estimation and NLMS algorithm to solve the problem of signal frequency estimation, and achieved good results. However, there are still directions that can be optimized:

1. The NLMS algorithm relies on the selection of initial parameters. If the initial value is chosen improperly, it may cause the algorithm to converge to a local optimal solution. You can try the following methods to improve: Multiple random initialization: Perform multiple random initialization on the initial parameters to select the optimal result. Heuristic method: Combining

prior knowledge or domain knowledge to select a more reasonable initial value.

2. The NLMS algorithm may converge slowly, especially when dealing with complex models. The following methods may help improve convergence efficiency: adaptive learning rate: dynamically adjust the learning rate to improve iteration efficiency. Preconditioned method: Use preconditioned techniques to improve the convergence of algorithms, especially in large datasets or high-dimensional problems.

3. In traditional segmented maximum likelihood estimation, segmentation is usually manually specified and may be influenced by subjective factors. The following improvements can be considered: data-driven segmentation method: using clustering algorithms (such as K-means or DBSCAN) or change point detection methods (such as CUSUM or Pelt algorithm) to automatically identify the change points of the data and segment them accordingly. Adaptive segmentation: dynamically adjust the segmentation method and quantity based on the statistical characteristics of the data or the fitting error of the model to avoid over simplification or over refinement.

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