Fast Polynomial Reduction

Nikolay Vassiliev
St.Petersburg department of Steklov Institute of Mathewmatics
St.Petersburg, Russia

The algorithm of polynomial reduction is one of the key points of any algorithm for finding Gröbner bases from Buchberger's one up to Gröbner walk [1] or Faugerre's algorithms [2]. Standard algorithm for polynomial reduction eliminates leading terms of polynomials step by step using leading terms of polynomials from a Gröbner basis while it is possible. During this process the number of terms in the intermediate polynomials can be very huge even in the case if the result is zero.

We suggest an algorithm for polynomial reduction which uses fast evaluation of the images of polynomials in the quotient algebra of an ideal. The algorithm is based on some generalization of Horner's method for the case of sparse multivariate polynomials [4, 3]. It uses a special representation of polynomials by marked trees. The vertices of the trees are marked by coefficients of polynomials and the edges by some monomials, which are defined recursively when the tree constructs. The number of different such monomials is much less than the number of terms of polynomial corresponded the tree. When the tree is constructed we associate multiplication operators in the quotient algebra to the monomials used as marks for edges. These operators evaluates recursively also using monomial basis of the quotient algebra. After all we evaluate the result using algorithm with closed to minimal value of number of multiplications in the quotient algebra. All the intermediate results are presented by their normal forms therefore the complexity of this algorithm is much more less than of standard one. We give also upper bound of the complexity in the case of zero dimensional ideals. This algorithm was imlemented in C package by A.Kliskunov. In this implementation it works from 100 up to 10000 times faster than normalization procedure implemented in FGb package in Maple.

References

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