Available online at www.sciencedirect.com

ScienceDirect

Transportation Research Procedia 25 (2017) 1416–1429

Transportation Research Procedia

www.elsevier.com/locate/procedia

World Conference on Transport Research - WCTR 2016 Shanghai. 10-15 July 2016

Methodology for the numerical calculation of racing lines and the virtual assessment of driving behavior for training circuits for the automobile industry

Wolfgang Kuhn, P. E., Ph.D first and second degree; Full Professor*

Zwickau University of Applied Sciences, Department of Transportation and Energy Engineering
Dr.-Friedrichs-Ring 2A, 08056 Zwickau, Germany

Abstract

The planning and design process for training circuits for the automobile industry has not used any standard methodology anywhere in the past. The new 3D alignment for a training course consists of measured individual elements from existing tracks combined with newly marked sequences of elements (generalized spline function).
To drive the training circuit at maximum speed along sections, the driver uses the course's full width and seeks a racing line that most closely matches the ideal theoretical .
Zwickau University has developed a new interactive design methodology by using driving simulation to prevent accidents on training circuits as far as possible.

Peer-review under responsibility of WORLD CONFERENCE ON TRANSPORT RESEARCH SOCIETY.

Keywords: Design Methodology for Training Circuits;

Calculating ideal Racing Line;

Checking Human Factors with Driving Simulation;

Safety Standard of Training Circuits;

* Corresponding author. Tel.: 0049-375-5363379; fax: 0049-375-53633288.
E-mail address: wolfgang.kuehn@fh-zwickau.de

2352-1465
Peer-review under responsibility of WORLD CONFERENCE ON TRANSPORT RESEARCH SOCIETY.
10.1016/j.trpro.2017.05.167

1. Introduction

There are no standard guidelines for planning and designing training circuits for automobile corporations – the route selection process for the road axis on the horizontal projection uses straights and different bend designs and combinations for each training circuit based on specific user requirements for the circuit in question. In addition to simple bends (circular arcs), combined curves (circular arcs with double-sided clothoids), the geometrical design also includes multiple three-center curves as the geometrical bend designs. In some cases, curves on well-known racing circuits are measured in detail and integrated in the horizontal projection as special designs alongside the bend designs already mentioned. The axis computed for the training circuit therefore consists of a combination of straights and bends with different geometrically defined design elements and bends at various points, as already described. The associated gradient course is computed using the classic design elements of longitudinal inclines (ascents, descents) and vertical curves (crests, sags, humps, basins) using quadratic parabolas. The paved roadway with its associated camber is determined horizontally and vertically based on the required width of the road.

Unlike the classic layout of a single lane road, traffic on training circuits only travels in one direction. The driver uses the width of the track and selects a racing line that is characterized by the transition from a straight to a curve and back to a straight again by the breaking point (P_{AB}), the turn-in point (P_{EL}), the apex (P_{KS}), the turn-out point (P_{AL}) and the acceleration point (P_{BS}). This real racing line differs slightly from the so-called ideal line, which, in the end, allows the driver to complete the course at the maximum possible speed. The calculation of the ideal line is therefore an important task during the planning and design process using a suitable mathematical model and an iteration process; this is designed to enable drivers to proceed along the training circuit at high speed and with great safety at the same time.

2. The aim

Based on the existing experience with routing real roads, a new kind of all-round methodology will be drawn up for the planning and design process for training circuits. In addition to developing a mathematical approach for calculating the racing line, virtual journeys on the driving simulator also offer the opportunity to assess the expected driving behavior. Comparative studies between virtual and real journeys will need to be performed to determine the validity. Finally, multi-stage iteration processes will be used to generate the ideal line, taking into account the major influencing factors. By introducing the new kind of methodology, safety levels at training circuits can be assessed during the design process and will increase during real operations at a later stage. This means that it should be possible to largely prevent subsequent, expensive corrections to the route or inserting additional chicanes as passive safety measures.

3. The current state of scientific knowledge and technology

A study of the relevant literature has clearly indicated that there are no special routing methods for training, test and racing circuits.

The routing process for circuits takes place in a similar way to normal roads according to the horizontal projection, the vertical projection and the cross section – i.e. the three-dimensional alignment is the result of the superimposition process of the three design levels. As each circuit is unique, there are no special rules for processing the design. Because the circuit also only has traffic moving in one direction and therefore the whole paved area can be used in contrast to normal roads, the ideal racing line needs to be calculated alongside tracing the paved roadway.

The simplest method for calculating the racing line consists of maximizing the radius when driving through a curve with the aim of being able to accelerate at an early stage at the end of the bend. The calculation of the relevant radius for the ideal line takes place in relation to the width of the driving area and the angle of change of direction in the curve. When driving along the ideal line, the driver brakes on the outer edge of the curve and drives through it using as even a movement of the steering wheel as possible so that the apex of the racing line coincides with the geometric apex of the curve. In contrast to this, the turn-in point with modern racing lines is selected much later, which leads to a shift in the apex and acceleration at a much earlier point (1, 2, 3).

Braghin (4) has developed an approach to calculating the racing line, taking into account the minimal curvature related to the driving path (4). In (5) Cardamone suggests an approach to calculating the racing line by determining the shortest route based on a number of consecutive points. The right-angle distances from any lane restriction line are optimized using a target function. Xiong finally calculates the ideal line with the help of the Euler spiral function (6).

By way of summary, it is possible to estimate that the modern calculation process for the racing line increasingly takes place on the basis of defined mathematical functions with constant coupling conditions for certain sections. In addition to the clearly defined straight function, different mathematical approaches are used for curves, taking into account the manipulation of fixed points and the modification of so-called free parameters. Mixed splines satisfy this requirement best of all from a mathematical point of view.

In addition to selecting a suitable mathematical model, the calculation of the ideal line should take place as an iteration process by virtually checking driving behavior on a driving simulator.

4. The design methodology

4.1. Procedure

The new kind of methodology that has been developed (7) for the design and planning process of training circuits is shown in Figure 1. It is basically subdivided into three stages.

During stage 1, the engineer sets the design elements related to the function of the training circuit and the route characteristics resulting from this. By taking into consideration the speed requirements for the individual sections of the route, the associated design elements and combinations of elements are basically set. On top of this, a route layout is worked out for the main route. Secondary routes are linked to the main route. The detailed work then takes place on the design elements and their sequence – i.e. the straight lengths and the curve designs are precisely set and the associated axis is calculated. The route model finally emerges as a result of superimposing the axis, the gradient and the roadway cross section. After superimposing the terrain and adding the necessary road area elements, stage 1 is complete once the 3D model of the complete route has been finished.

Stage 2 is used to virtually pre-check the developed route model with regard to whether it can be recognized and understood. A 3D projection using a power wall or a head-mounted display is used for this purpose. If any problem areas are discovered, the necessary corrections are made on the 3D route model.

Stage 3 now involves calculating the actual racing line using the new kind of mathematical approach by means of an iteration process. The safety checks on the calculated racing line are performed with the help of virtual journeys on a driving simulator. If the benchmark parameters are met, the racing line and the associated 3D route model are confirmed and the planning process for the main route has been completed.

4.2. Calculating the racing line

4.2.1 Principles

The basic principle for calculating the racing line is the paved road area that has been calculated. The following points are fixed in terms of their location for the calculation process (Fig. 2):
- The breaking point on the straights before the start of the curve (P_{AB})
- The turn-in point at the start of the curve (P_{EL})
- The apex at the middle of the curve (P_{KS})
- The turn-out point at the end of the curve (P_{AL})

The acceleration point on the straight after the end of the curve (P_{BS})

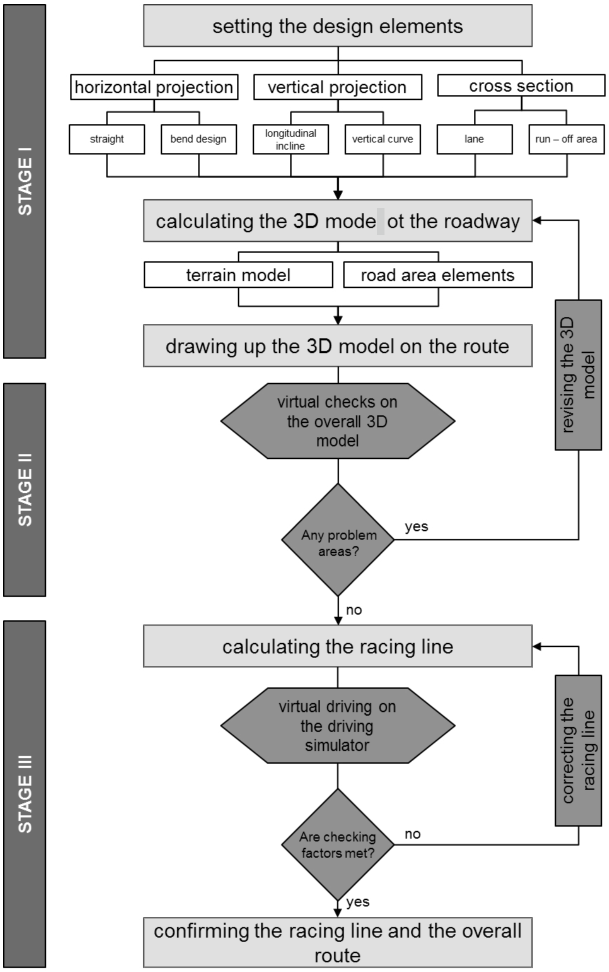


Fig. 1. New kind of multi-stage design methodology for training circuits

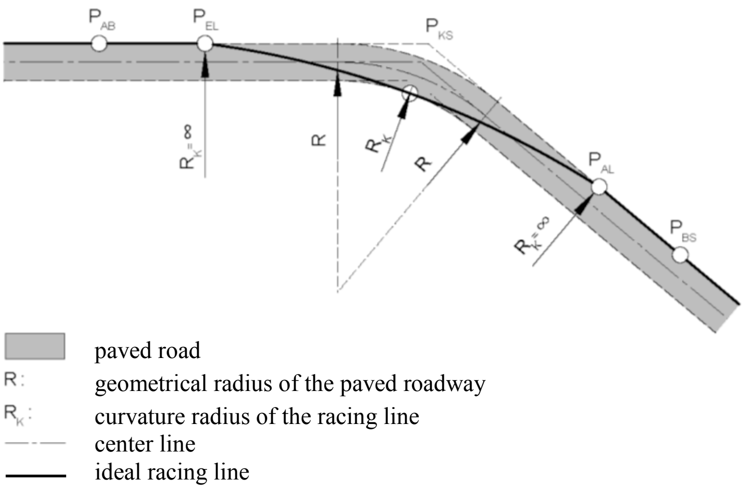


Fig. 2. Model approach to calculating the racing line

Generalized spline functions of a particular kind were selected as the mathematical approach for calculating the racing line because of their numerical and geometrical properties. By using this piecewise function a high grade of approximation between the theoretical and practical racing line in possible.

4.2.2 Rational spline approach

Mathematical model:
The mathematical model for a generalized spline function *f* consists of n-1 parts of f_k of the following form (8):

$$f_k(x) = a_k g_1(x) + b_k g_2(x) + c_k g_3(x) + d_k g_4(x) \tag{1}$$

The g_i may, however, be at least functions that are constantly differentiable twice. Their structure may be very different and may even depend on the interval $[x_k, x_{k+1}]$. In order to obtain smooth interpolation curves and restrict the calculation work, certain requirements must be made on the g_i. The most important feature of the generalized spline functions consists of the fact that the g_i functions depend on selectable parameters. Normally, one or two parameters are used. The course of the interpolation curve can be steered in each interval by varying the parameters until the desired geometrical course of the curve is found.

Rational spline interpolation is one form of generalized spline interpolation with selectable parameters, which can be used for the interactive dialogue to interpolate between set interpolation points and generate smooth curves.

The rational spline function f consists of n-1 parts f_k in the following form:

$$f_k(x) = a_k + b_k(x - x_k) + c_k(x - x_k)^2 + \frac{d_k}{x - x_k + p_k} \tag{2}$$

$$p_k \in [-\infty, -\Delta x_k] \ oder \ p_k \in [0, \infty], \quad (k = 1, \dots, n-1) \tag{3}$$

As a result, each interval $[x_k, x_{k+1}]$ is a parabola, which is superimposed by a hyperbola, the pole of which lies outside the interval $[x_k, x_{k+1}]$. The parameters p_k determine the location of the poles. Approach (2) is, however, numerically unstable, as differences of almost equally large numbers occur. Using the equivalent approach

$$\begin{aligned} f_k(x) &= a_k(x - x_k) + b_k(x_{k+1} - x_k) \\ &+ \frac{p_k(x - x_k)(x_{k+1} - x)}{x - x_k + p_k}[c_k(x - x_k) + d_k(x_{k+1} - x)] \end{aligned} \tag{4}$$

with the same requirements for the p_k, this negative phenomenon does not occur.

Using the familiar conditions,

$$\begin{aligned} &f(x_k) = y_K, \ f(x_{K+1}) = y_{K+1} \\ &f'(x_k) = y'_K, \ f'(x_{K+1}) = y'_{K+1} \\ &\Delta x_K = x_{K+1} - x_k, \ \Delta y_K = y_{K+1} - y_K \end{aligned} \tag{5}$$

the equations for the coefficients a_k, b_k, c_k, d_k depending on the values of x_k, y_k and y'_k can be determined using formula (4).

It is possible to summarize the results of the experiments with geometrical test examples as follows:

- In the case of rational spline interpolation, it is possible to vary the parameter p_k in each interval $[x_k, x_{k+1}]$ in line with formula (4). A cubic spline occurs where $p_k \rightarrow \pm\infty$. But if we aim for $p_k \rightarrow 0$, different curve courses occur than with cubic spline interpolation. The curves often have less strong curvature at the interpolation points.

- The rational spline function reacts most sensitively if we select negative figures for p_k close to $-\Delta x_k$. The reaction is, however, weak in the intervals where the cubic spline function has already produced an elongated curve line.

Parameter description:
A suitable description of the parameters is essential for calculating the racing line in order to be able to maintain the strict monotony of the abscissa regardless of the coordinate system. For this purpose, a parameter t is introduced and it grows monotonously with the course of the curve:

$$t_1 < t_2 < \ldots < t_n. \tag{6}$$

The rational spline function s is represented by the two functions $x = x(t)$ und $y = y(t)$ in terms of parameters. The ideal values for the parameters t_k, (k=1,…,n) would be those of the accumulated arc length of the curve, which, however, first has to be calculated. Theoretical and practical experiments have demonstrated that the most appropriate representation for t is the straight line connection between the interpolation points (cord length) (Fig. 3).

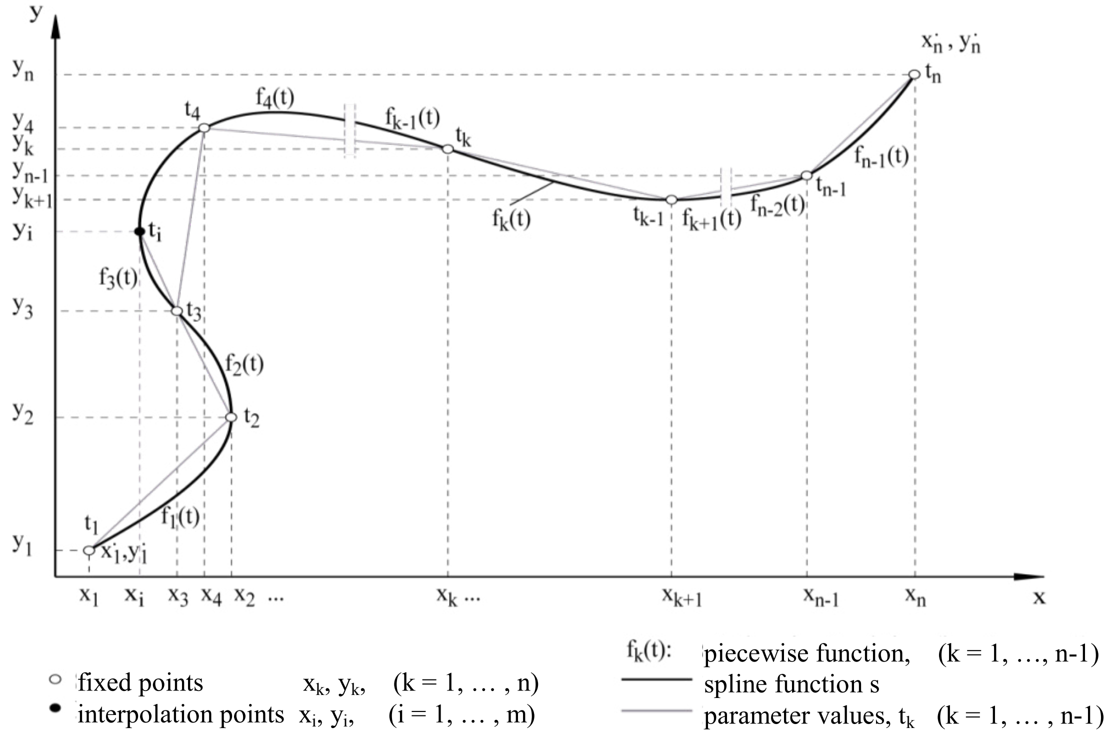


Fig. 3. Parameterization of a rational spline function s

The parameters t_k can be determined recursively as follows:

$$\begin{aligned} t_1 &= t_a \\ t_k &= t_{k-1} + \sqrt{\Delta x_{k-1}^2 + \Delta y_{k-1}^2}, \qquad (k = 2, \ldots, n) \end{aligned} \tag{7}$$

In the end, there are two separate equations for the spline approach in parameter form:

$$x_i = ax_k(t_i - t_k) + bx_k(t_{i+1} - t_k) \tag{8}$$

$$+\frac{p_k\,(t-t_k)(t_{k+1}-t)}{t-\,t_k+p_k}\qquad [cx_k(t-t_k)+dx_k(t_{k+1}-t)]$$

$$y_i = ay_k(t_i-t_k)+by_k(t_{i+1}-t_k) \tag{9}$$

$$+\frac{p_k\,(t-t_k)(t_{k+1}-t)}{t-\,t_k+p_k}\qquad [cy_k(t-t_k)+dy_k(t_{k+1}-t)]$$

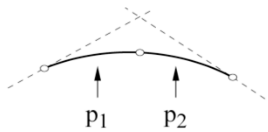
In contrast to the rational spline interpolation in explicit form, a separate calculation of the features of the two rational splines takes place with the points (tk, xk) or (tk, yk). The constellation of the fixed points is depended of the kind of the design elements and the number and the position of the interpolation points from the curvature graph of the several elements.

Design elements:
Based on the theoretical experiments performed (10) with the rational spline approach, so-called mathematical design elements have been defined, which can be combined to form racing lines.

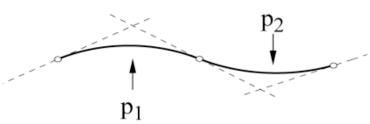
Fixed elements are all the straight sections within the racing line, the location of which is set by the beginning and end points. In order to be able to control the course of the racing line in the curve areas too regardless of the fixed points, dialogue elements based on the rational spline approach have been defined and introduced. As a result of the mathematical approach, the dialogue elements have free parameters. By varying the parameters, the course of the racing line can be changed for certain sections in dialogue with the computer.

Generally, the following dialogue elements have been defined, depending on the constellation of fixed points (Fig. 4):

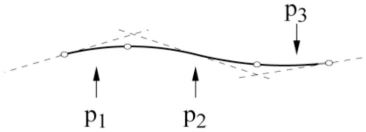
- Dialogue element for a simple curve: (DVS 2),
- Free dialogue element for a double curve: (DVS 3),
- Bound dialogue element for a double curve: (DVS 4),



a) Dialog element for a simple curve (DVS 2)



b) free dialog element for a double curve (DVS 3)



c) bound dialog element for a double curve (DVS 4)

Fig. 4. Defined dialogue elements

The number of dialogue sections and scope for introducing variations are set by the constellation of fixed points. As the number of fixed points increases, the number of variation areas increases, but in the end the scope for making corrections to the racing line decreases.

Test example:

The developed model for calculating the racing line was checked to see whether it can be used by means of test examples (Fig. 5).

Racing line 1 (continuous line) was calculated on the basis of the pre-set fixed points. There is a straight section at the beginning and the end. There are three curves arranged between two straight sections. By manipulating the free parameter p_k along individual sections, racing lines 2 and 3 are created. The manipulation of the racing line can take place with the help of the parameters p_k and also with the constellation of fixed points.

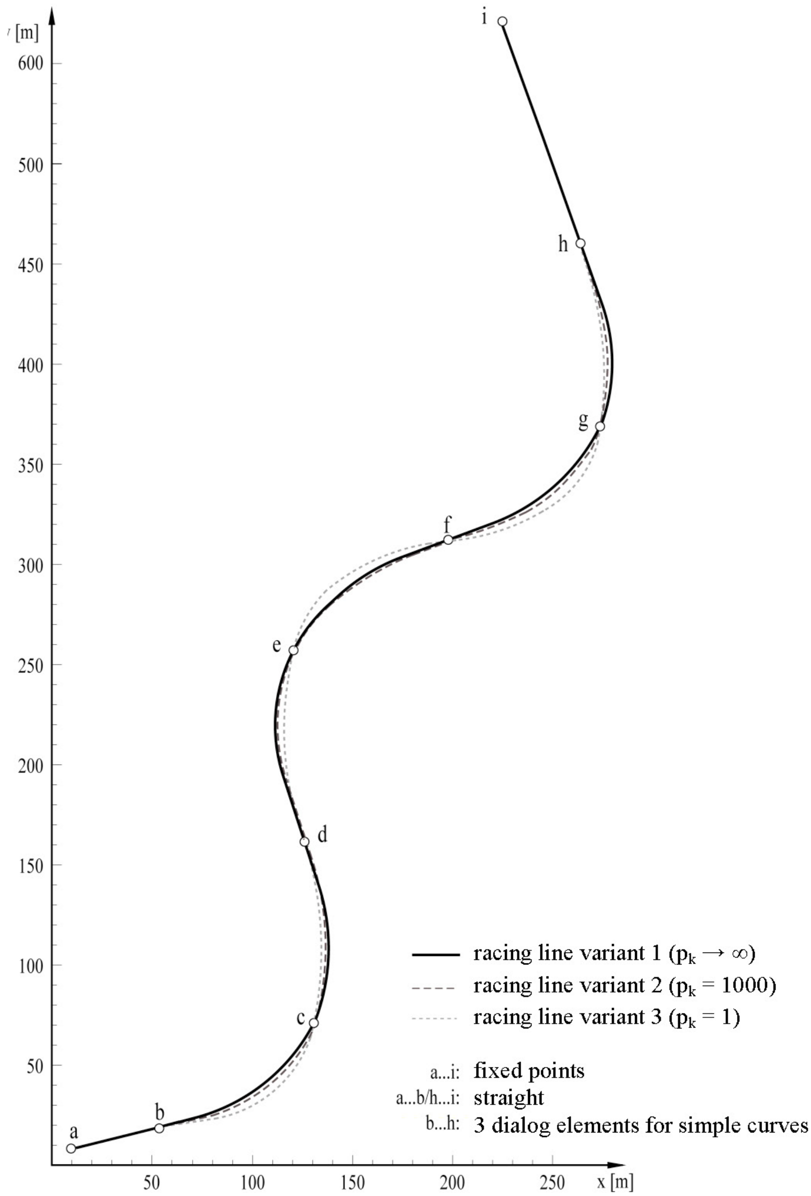
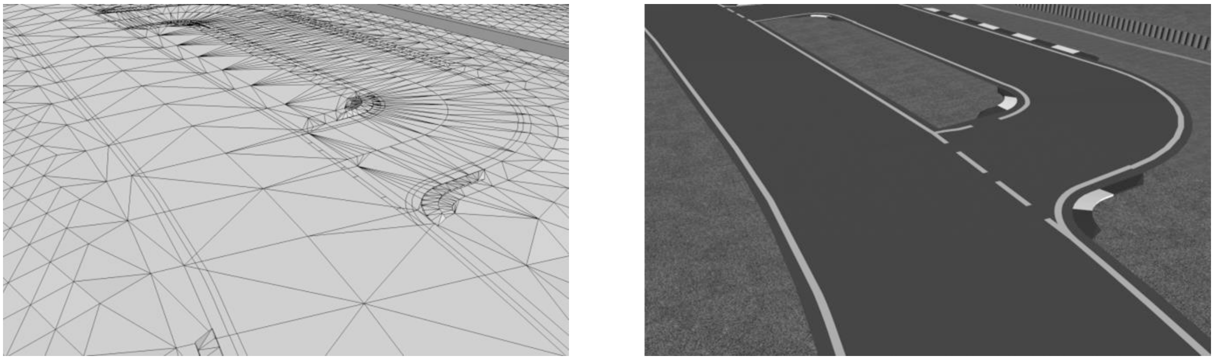


Fig. 5. Selected test example for calculating the racing line in the iteration process

4.3 Virtual preliminary check

Experiments performed have clearly demonstrated that a virtual assessment of the developed 3D route model with all the necessary road area elements for a training circuit definitely makes sense (Fig. 6). Test drivers can derive any necessary changes to the course of the route and the necessary markings and signs etc. just from visual observation. For this reason, the route model is subjected to a preliminary visual check using a power wall or a head-mounted display. The test driver moves dynamically within the 3D route model and observes the route from different perspectives. As a result, it is then possible to make changes to the route layout or the road area elements.



a) 3D grid model b) Textured 3D model

Fig. 6. Virtual preliminary check of the 3D route model

4.4 Comparison between virtual and real journeys

Driving simulators with different configurations have not been used in the past to support the planning and design process for training circuits. Experiments carried out (11), however, have demonstrated that test drivers with static driving simulators can handle things well, because they quickly learn the special handling that is required and can therefore focus completely on the driving course during the journeys. In order to be able to prove the fundamental suitability of the driving simulator used to make a comparison between virtual and real journeys, various test and training circuits were travelled along virtually and in reality using a team of guinea pigs consisting of test drivers. The same kind of vehicle (a Porsche Panamera) was used for the simulated and real journeys.

The differences in the maximum speed v_{max} were approx. 4% despite the high speed of travel; the figure was about 6% at moderate speeds $\bar{v}$ and the standard deviation σ_v was about 5% (in the literature the differences are specified between 5 to 10%).

As a result, the differences are very low, so there is a high degree of validity. This fundamental statement on the suitability of driving simulators for comparative observations with real journeys was not expected for training circuits and is therefore an important basis for using simulator trips to assess driving behavior and the safety levels even during the planning and design process of training circuits in general.

5. Practical example

5.1 Marginal conditions

The methodology described has been used to design the alignment and calculate the ideal line for a training circuit in a practical situation (12). The following design parameters were used to establish the layout of the route model for the main route:

- Route length: 2,020 m
- Road width: 2 m
- Individual elements on the horizontal projection: 4 straight sections measuring between 50 m and 360 m
  17 curve sections with radii between 5 m and 477 m
- Individual elements on the vertical projection: Longitudinal incline: $s = 0\,\% - 21\,\%$
  Vertical curve diameter: $H_K = 260\text{ m} - 33000\text{ m}$
  $H_W = 150\text{ m} - 39600\text{ m}$
- Camber: $q = 2.5\% - 5\%$

Figure 7 illustrates the marked main route on the training circuit on the horizontal projection with the associated design elements for the road axis and the calculated ideal line based on the model that has been developed. The associated cross section with the 3D route model is also shown.

The team of guinea pigs consisted of 10 test drivers with roughly the same practical experience. The selection of the test driver took place us the result of a complex proband analysis. Each of them drove an introductory round on the driving simulator and on the real route. The associated average figures were formed from three subsequent test drives. There was a time lag of 12 months (construction period for the circuit) between the virtual journeys on the driving simulator and the real trips. Figure 7 illustrates a cockpit photo of the simulation journey and the real trip.

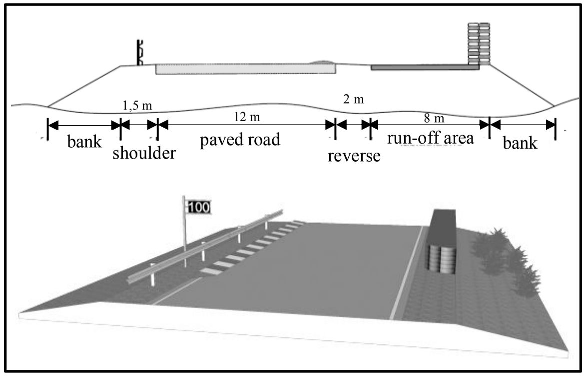
5.2 Results

The ideal line was generated in an iteration process using 6 stages by changing the interpolation points and the associated free parameter p_K. With each iteration stage, checks on the driving behavior were made using virtual journeys on the driving simulator. After the sixth iteration stage, the ideal line and the geometrical design parameters for the main route were finally set (Fig. 7). One stop criterion is the associated variation $\sigma_{\bar{v}}$ between the given speed graph and the achieved driving speed inside of each racing line element. This completed the planning and design process and the necessary marking documents (axis, gradient, cross section profiles etc.) could be calculated for the building work.

The comparative experiments between the virtual and real journeys were made on the basis of the following parameters: speed, transverse acceleration and the racing line. The maximum figure (max. v, max. a_q, max. Δs), the average figure ($\bar{v}$, $\bar{a}_q, \overline{\Delta s}$) and the associated variations $\sigma_v, \sigma_{aq}, \sigma_{\Delta s}$ were determined for all three parameters as assessment parameters. Table 1 summarizes the results.

Tab. 1. Summary of the assessment parameters

driving scenarios	assessment parameter								
	speed [km/h]			lateral acceleration [m/s²]			deviation from the racing line [m]		
	v_{max}	$\bar{v}$	$\sigma_{\bar{v}}$	$\alpha_{q_{max}}$	$\bar{\alpha}_q$	$\bar{\alpha}_q$	s_{max}	$\overline{\Delta s}$	$\sigma_{\overline{\Delta s}}$
virtually on the driving simulator	166,0	108,0	30,4	9,92	3,43	2,56	1,70	0,33	0,40
real trip on the training circuit	170,0	104,0	32,0	10,15	3,96	3,42	1,58	0,15	0,26



Cross section and 3D model

Photo of a simulator journey

Photo of a real trip

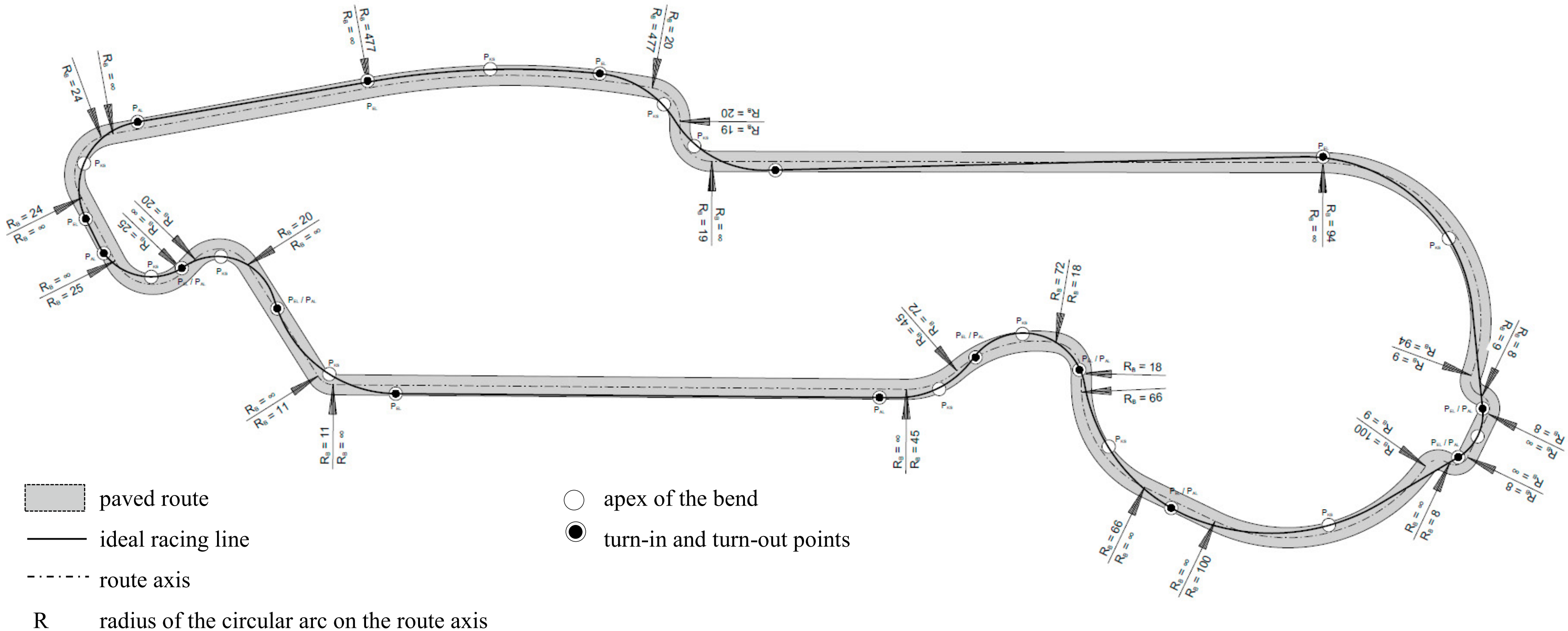


Fig. 7. Paved roadway of the main route on the training circuit with a calculated ideal line

In terms of the speed parameter, the curve courses were very similar and the assessment parameters only varied slightly from each other. The speed differences between the subsequent individual elements were considerable at v = 90 km/h. As expected, the problems in the speed courses in the curves occurred in the bends with small radii (Fig. 8).

As far as the transverse acceleration parameter was concerned, the curve courses were also very similar and therefore the assessment parameters only varied slightly from each other. With maximum figures for a_q of approx. 10 m/s^2, the figures were definitely above the feel-good values for drivers on normal roads, but were within the permissible stress range for test drivers. The highest transverse acceleration figures naturally occurred in the curves with small radii (Fig. 8).

In terms of the lane deviation parameter, there were no comparable curve courses and even the individual figures for the assessment parameters were barely comparable (Fig. 8). The instances of lane deviation in the curves were much greater with the virtual simulator journeys than with the real trips, i.e. the handling, particularly with the steering wheel, tends to be difficult with the speed differences. This result contrasts starkly with the previous simulator journeys on classic rural roads, where the maximum speed is 100 km/h. The given tolerance of $\pm$ 0.5 m from the ideal line was largely complied with on the real trips, while there were fairly large deviations on the simulator journeys.

In conclusion, it is possible to state that the simulator journeys and real trips reflect roughly similar driving behavior. But there are significant differences in the lane deviation and they are clearly due to the very high speeds and the significant differences in speed when driving along a training circuit virtually and the associated problems with handling the steering.

6. Summary

A new kind of multi-stage methodology has been developed for planning and designing training circuits and it will allow a standard approach and a preliminary assessment of driving behavior and safety levels in the future. By using the special design elements a harmoniously racing line will be calculated. During the training process the driver get in real time all necessary data of the permissible speed and the difference from the ideal line.

Based on a classic calculation of the alignment of the training circuit, a mixed spline approach with fixed elements (straight sections) and dialogue elements (rational splines with variable parameters p_k) are used to generate the associated ideal line as part of an iteration process. After each iteration stage, the assessment of the driving behavior takes place using special assessment factors, which are derived from the speed, transverse acceleration and lane position parameters.

Comparative studies between virtual journeys on the driving simulator and real trips on the completed circuit have demonstrated that driving simulators are suitable for assessing the driving behavior on training circuits where high speeds and huge differences in speed occur between the consecutive elements.

Problems, however, did emerge with handling the steering wheel – i.e. it was only possible to handle the necessary extreme steering wheel movements on the driving simulator in a way that bore little relation to normal driving. The results of the comparison of the racing line illustrate this. The deviations between the virtual racing line and the ideal line were significant, particularly in the bends.

Initial results for planning and designing the training circuit have, however, demonstrated that an assessment of driving behavior and safety is possible and this should be integrated in the overall process in the future.

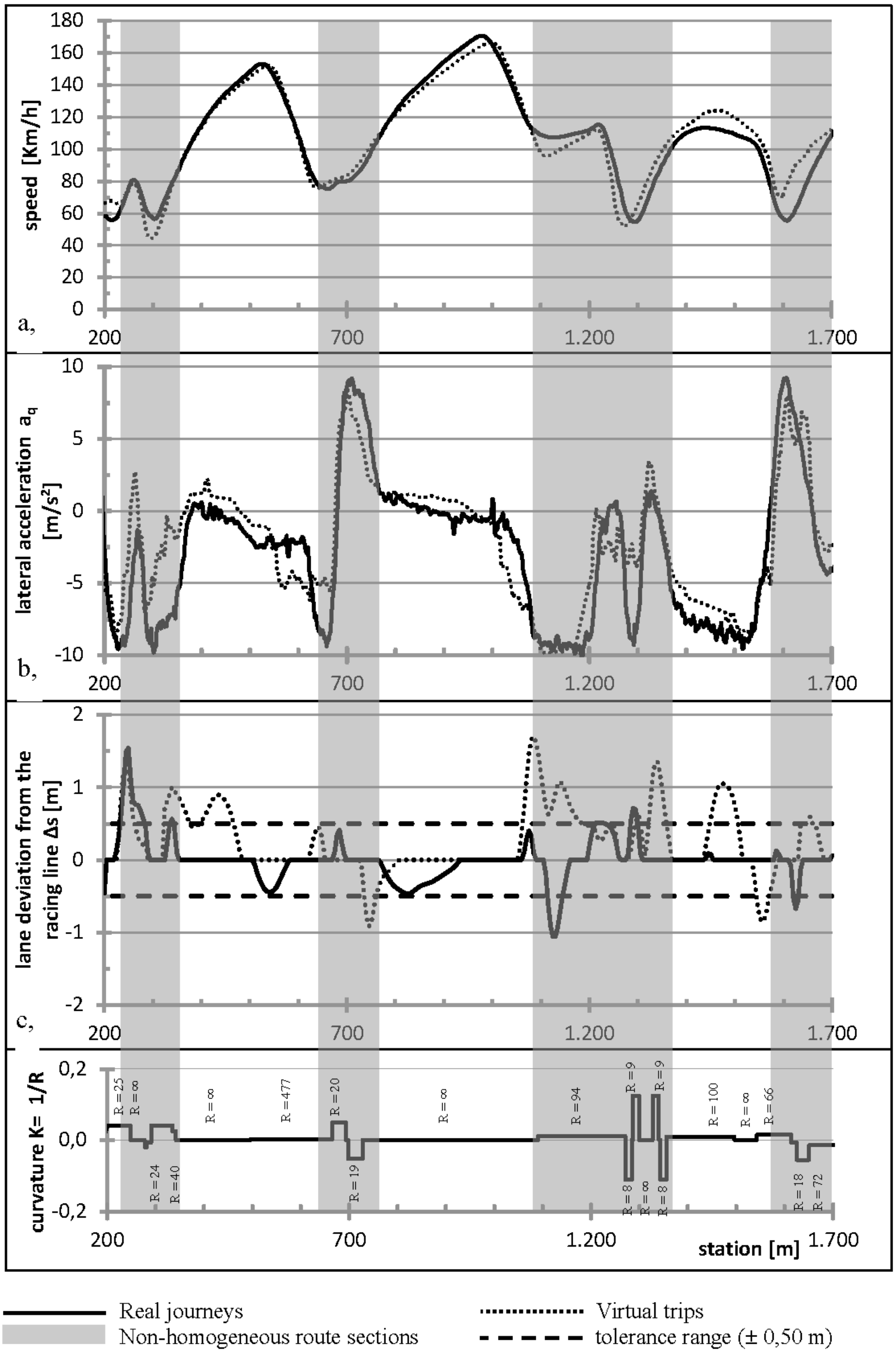


Fig. 8. Comparison of the individual assessment parameters between simulator journeys and real trips (average figures for the test persons)

References

(1) Online-Referenz: http://www.drivingfast.net/techniques/racing-line.htm, accessed on 9 Oct. 2014

(2) Online-Referenz: http://www.zweiradsicherheit.de/Sonderheft.pdf, accessed on 9 Oct. 2014

(3) Online-Referenz: http://www.driversdomainuk.com/motorsport-2/racing.line-2/, accessed on 9 Oct. 2014

(4) Braghin, F., Cheli, F., Melzi, S., and Sabbioni, E.: Race driver model., Comput. Struct., Heft 86, 2008

(5) Cardamone, L., Loiacono, D., Lanzi, P.L., Bardelli, A.P.: Searching for the Optimal Racing Line Using Genetic Algorithms, IEEE Conference on Computational Intelligence and Games, 2010

(6) XIONG, Y.: Racing Line Optimization, Master's thesis, Jiao Tong University, Shanghai, 2009

(7) Kuhn, W.; Muller, M.; Anforderungen an den Entwurfs- und Planungsprozess von Trainingsstrecken, Forschungsbericht (unveröffentlicht), Universität Zwickau, 2014

(8) Spath, H.: Rationale Spline-Interpolation, Angewandte Informatik Heft 8, Oldenburg 1971

(9) Kuhn, W.; Untersuchungen zu neuartigen Modellvorstellungen und Verfahren – Ein Beitrag zur Weiterentwicklung der Entwurfsmethodik für Straßen, Habilitationsschrift, TU Dresden, 2002

(10) Kuhn, W.; GRUTRA – ein neues Verfahren für den rechnergestützten Straßenentwurf, Die Straße, Heft 5, 1987

(11) Kuhn, W.; Müller, M.; Nutzung der Fahrsimulation für den Entwurfs- und Planungsprozess von Trainingsstrecken, Forschungsbericht (unveröffentlicht), Universität Zwickau, 2014

(12) Muller, M.; Untersuchungen zur Visualisierung und Fahrverhaltensbewertung von Teststrecken, Diplomarbeit, FH Zwickau, 2014