

Rajeev Atla

~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

$$\mathbb{E}(N) = \sum N \mathbb{P}(N)$$

Page 1 of 4

$\rho_{0,0} = 45$	$\rho_{0,1} = 16$	$\rho_{0,2} = 16$	$\rho_{0,3} = 16$	$\rho_{0,4} = 16$	$\rho_{0,5} = 16$	$\rho_{0,6} = 16$	$\rho_{0,7} = 7$	$\rho_{0,8} = 5$	$\rho_{0,9} = 7$	$\rho_{0,10} = 5$	$\rho_{0,11} = 7$	$\rho_{0,12} = 5$	$\rho_{0,13} = 7$	$\rho_{0,14} = 5$	$\rho_{0,15} = 7$	$\rho_{0,16} = 5$	$\rho_{0,17} = 7$	$\rho_{0,18} = 5$
$\rho_{1,0} = 34566$	$\rho_{1,1} = 107714$	$\rho_{1,2} = 34566$	$\rho_{1,3} = 107714$	$\rho_{1,4} = 34566$	$\rho_{1,5} = 107714$	$\rho_{1,6} = 34566$	$\rho_{1,7} = 14425$	$\rho_{1,8} = 22485$	$\rho_{1,9} = 14425$	$\rho_{1,10} = 22485$	$\rho_{1,11} = 14425$	$\rho_{1,12} = 22485$	$\rho_{1,13} = 14425$	$\rho_{1,14} = 22485$	$\rho_{1,15} = 14425$	$\rho_{1,16} = 22485$	$\rho_{1,17} = 14425$	$\rho_{1,18} = 22485$
$\rho_{2,0} = 191995$	$\rho_{2,1} = 618085$	$\rho_{2,2} = 191995$	$\rho_{2,3} = 618085$	$\rho_{2,4} = 191995$	$\rho_{2,5} = 618085$	$\rho_{2,6} = 191995$	$\rho_{2,7} = 249991$	$\rho_{2,8} = 399991$	$\rho_{2,9} = 249991$	$\rho_{2,10} = 399991$	$\rho_{2,11} = 249991$	$\rho_{2,12} = 399991$	$\rho_{2,13} = 249991$	$\rho_{2,14} = 399991$	$\rho_{2,15} = 249991$	$\rho_{2,16} = 399991$	$\rho_{2,17} = 249991$	$\rho_{2,18} = 399991$
$\rho_{3,0} = 1099995$	$\rho_{3,1} = 3499995$	$\rho_{3,2} = 1099995$	$\rho_{3,3} = 3499995$	$\rho_{3,4} = 1099995$	$\rho_{3,5} = 3499995$	$\rho_{3,6} = 1099995$	$\rho_{3,7} = 4499995$	$\rho_{3,8} = 6999995$	$\rho_{3,9} = 4499995$	$\rho_{3,10} = 6999995$	$\rho_{3,11} = 4499995$	$\rho_{3,12} = 6999995$	$\rho_{3,13} = 4499995$	$\rho_{3,14} = 6999995$	$\rho_{3,15} = 4499995$	$\rho_{3,16} = 6999995$	$\rho_{3,17} = 4499995$	$\rho_{3,18} = 6999995$
$\rho_{4,0} = 6199995$	$\rho_{4,1} = 19199995$	$\rho_{4,2} = 6199995$	$\rho_{4,3} = 19199995$	$\rho_{4,4} = 6199995$	$\rho_{4,5} = 19199995$	$\rho_{4,6} = 6199995$	$\rho_{4,7} = 7799995$	$\rho_{4,8} = 11999995$	$\rho_{4,9} = 7799995$	$\rho_{4,10} = 11999995$	$\rho_{4,11} = 7799995$	$\rho_{4,12} = 11999995$	$\rho_{4,13} = 7799995$	$\rho_{4,14} = 11999995$	$\rho_{4,15} = 7799995$	$\rho_{4,16} = 11999995$	$\rho_{4,17} = 7799995$	$\rho_{4,18} = 11999995$
$\rho_{5,0} = 34999995$	$\rho_{5,1} = 107999995$	$\rho_{5,2} = 34999995$	$\rho_{5,3} = 107999995$	$\rho_{5,4} = 34999995$	$\rho_{5,5} = 107999995$	$\rho_{5,6} = 34999995$	$\rho_{5,7} = 139999995$	$\rho_{5,8} = 219999995$	$\rho_{5,9} = 139999995$	$\rho_{5,10} = 219999995$	$\rho_{5,11} = 139999995$	$\rho_{5,12} = 219999995$	$\rho_{5,13} = 139999995$	$\rho_{5,14} = 219999995$	$\rho_{5,15} = 139999995$	$\rho_{5,16} = 219999995$	$\rho_{5,17} = 139999995$	$\rho_{5,18} = 219999995$
$\rho_{6,0} = 199999995$	$\rho_{6,1} = 619999995$	$\rho_{6,2} = 199999995$	$\rho_{6,3} = 619999995$	$\rho_{6,4} = 199999995$	$\rho_{6,5} = 619999995$	$\rho_{6,6} = 199999995$	$\rho_{6,7} = 259999995$	$\rho_{6,8} = 399999995$	$\rho_{6,9} = 259999995$	$\rho_{6,10} = 399999995$	$\rho_{6,11} = 259999995$	$\rho_{6,12} = 399999995$	$\rho_{6,13} = 259999995$	$\rho_{6,14} = 399999995$	$\rho_{6,15} = 259999995$	$\rho_{6,16} = 399999995$	$\rho_{6,17} = 259999995$	$\rho_{6,18} = 399999995$
$\rho_{7,0} = 1199999995$	$\rho_{7,1} = 3499999995$	$\rho_{7,2} = 1199999995$	$\rho_{7,3} = 3499999995$	$\rho_{7,4} = 1199999995$	$\rho_{7,5} = 3499999995$	$\rho_{7,6} = 1199999995$	$\rho_{7,7} = 1499999995$	$\rho_{7,8} = 2299999995$	$\rho_{7,9} = 1499999995$	$\rho_{7,10} = 2299999995$	$\rho_{7,11} = 1499999995$	$\rho_{7,12} = 2299999995$	$\rho_{7,13} = 1499999995$	$\rho_{7,14} = 2299999995$	$\rho_{7,15} = 1499999995$	$\rho_{7,16} = 2299999995$	$\rho_{7,17} = 1499999995$	$\rho_{7,18} = 2299999995$
$\rho_{8,0} = 6799999995$	$\rho_{8,1} = 19999999995$	$\rho_{8,2} = 6799999995$	$\rho_{8,3} = 19999999995$	$\rho_{8,4} = 6799999995$	$\rho_{8,5} = 19999999995$	$\rho_{8,6} = 6799999995$	$\rho_{8,7} = 8799999995$	$\rho_{8,8} = 13199999995$	$\rho_{8,9} = 8799999995$	$\rho_{8,10} = 13199999995$	$\rho_{8,11} = 87$							

$$t \equiv N\mathbf{1}$$
[illegible]

Finally, we see that $t_0 = \boxed{\frac{213}{29} \approx 7.345}$