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~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

The diagram shows a hexagonal lattice structure. The internal nodes are numbered 0 to 18, and the external nodes are numbered 19 to 30. The nodes are arranged in a hexagonal pattern, with each node connected to its six neighbors. The nodes are represented by red dots with corresponding numbers. The lattice is composed of blue hexagons.

$$\mathbb{E}(N) = \sum N \mathbb{P}(N)$$

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$N =$	$P_{0,0} = \frac{45}{16}$	$P_{0,1} = \frac{15}{16}$	$P_{0,2} = \frac{15}{16}$	$P_{0,3} = \frac{15}{16}$	$P_{0,4} = \frac{15}{16}$	$P_{0,5} = \frac{15}{16}$	$P_{0,6} = \frac{15}{16}$	$P_{0,7} = \frac{7}{2}$	$P_{0,8} = \frac{5}{4}$	$P_{0,9} = \frac{7}{2}$	$P_{0,10} = \frac{15}{16}$	$P_{0,11} = \frac{7}{2}$	$P_{0,12} = \frac{5}{4}$	$P_{0,13} = \frac{7}{2}$	$P_{0,14} = \frac{5}{4}$	$P_{0,15} = \frac{7}{2}$	$P_{0,16} = \frac{5}{4}$	$P_{0,17} = \frac{7}{2}$	$P_{0,18} = \frac{5}{4}$
	$P_{1,0} = \frac{15}{16}$	$P_{1,1} = \frac{10771}{16384}$	$P_{1,2} = \frac{62509}{16384}$	$P_{1,3} = \frac{62509}{16384}$	$P_{1,4} = \frac{10771}{16384}$	$P_{1,5} = \frac{10771}{16384}$	$P_{1,6} = \frac{10771}{16384}$	$P_{1,7} = \frac{13133}{16384}$	$P_{1,8} = \frac{22965}{16384}$	$P_{1,9} = \frac{13133}{16384}$	$P_{1,10} = \frac{10771}{16384}$	$P_{1,11} = \frac{13133}{16384}$	$P_{1,12} = \frac{22965}{16384}$	$P_{1,13} = \frac{13133}{16384}$	$P_{1,14} = \frac{22965}{16384}$	$P_{1,15} = \frac{13133}{16384}$	$P_{1,16} = \frac{10771}{16384}$	$P_{1,17} = \frac{13133}{16384}$	$P_{1,18} = \frac{10771}{16384}$
	$P_{2,0} = \frac{15}{16}$	$P_{2,1} = \frac{15}{16}$	$P_{2,2} = \frac{15}{16}$	$P_{2,3} = \frac{15}{16}$	$P_{2,4} = \frac{15}{16}$	$P_{2,5} = \frac{15}{16}$	$P_{2,6} = \frac{15}{16}$	$P_{2,7} = \frac{15}{16}$	$P_{2,8} = \frac{15}{16}$	$P_{2,9} = \frac{15}{16}$	$P_{2,10} = \frac{15}{16}$	$P_{2,11} = \frac{15}{16}$	$P_{2,12} = \frac{15}{16}$	$P_{2,13} = \frac{15}{16}$	$P_{2,14} = \frac{15}{16}$	$P_{2,15} = \frac{15}{16}$	$P_{2,16} = \frac{15}{16}$	$P_{2,17} = \frac{15}{16}$	$P_{2,18} = \frac{15}{16}$
	$P_{3,0} = \frac{15}{16}$	$P_{3,1} = \frac{15}{16}$	$P_{3,2} = \frac{15}{16}$	$P_{3,3} = \frac{15}{16}$	$P_{3,4} = \frac{15}{16}$	$P_{3,5} = \frac{15}{16}$	$P_{3,6} = \frac{15}{16}$	$P_{3,7} = \frac{15}{16}$	$P_{3,8} = \frac{15}{16}$	$P_{3,9} = \frac{15}{16}$	$P_{3,10} = \frac{15}{16}$	$P_{3,11} = \frac{15}{16}$	$P_{3,12} = \frac{15}{16}$	$P_{3,13} = \frac{15}{16}$	$P_{3,14} = \frac{15}{16}$	$P_{3,15} = \frac{15}{16}$	$P_{3,16} = \frac{15}{16}$	$P_{3,17} = \frac{15}{16}$	$P_{3,18} = \frac{15}{16}$
	$P_{4,0} = \frac{15}{16}$	$P_{4,1} = \frac{15}{16}$	$P_{4,2} = \frac{15}{16}$	$P_{4,3} = \frac{15}{16}$	$P_{4,4} = \frac{15}{16}$	$P_{4,5} = \frac{15}{16}$	$P_{4,6} = \frac{15}{16}$	$P_{4,7} = \frac{15}{16}$	$P_{4,8} = \frac{15}{16}$	$P_{4,9} = \frac{15}{16}$	$P_{4,10} = \frac{15}{16}$	$P_{4,11} = \frac{15}{16}$	$P_{4,12} = \frac{15}{16}$	$P_{4,13} = \frac{15}{16}$	$P_{4,14} = \frac{15}{16}$	$P_{4,15} = \frac{15}{16}$	$P_{4,16} = \frac{15}{16}$	$P_{4,17} = \frac{15}{16}$	$P_{4,18} = \frac{15}{16}$
	$P_{5,0} = \frac{15}{16}$	$P_{5,1} = \frac{15}{16}$	$P_{5,2} = \frac{15}{16}$	$P_{5,3} = \frac{15}{16}$	$P_{5,4} = \frac{15}{16}$	$P_{5,5} = \frac{15}{16}$	$P_{5,6} = \frac{15}{16}$	$P_{5,7} = \frac{15}{16}$	$P_{5,8} = \frac{15}{16}$	$P_{5,9} = \frac{15}{16}$	$P_{5,10} = \frac{15}{16}$	$P_{5,11} = \frac{15}{16}$	$P_{5,12} = \frac{15}{16}$	$P_{5,13} = \frac{15}{16}$	$P_{5,14} = \frac{15}{16}$	$P_{5,15} = \frac{15}{16}$	$P_{5,16} = \frac{15}{16}$	$P_{5,17} = \frac{15}{16}$	$P_{5,18} = \frac{15}{16}$
	$P_{6,0} = \frac{15}{16}$	$P_{6,1} = \frac{15}{16}$	$P_{6,2} = \frac{15}{16}$	$P_{6,3} = \frac{15}{16}$	$P_{6,4} = \frac{15}{16}$	$P_{6,5} = \frac{15}{16}$	$P_{6,6} = \frac{15}{16}$	$P_{6,7} = \frac{15}{16}$	$P_{6,8} = \frac{15}{16}$	$P_{6,9} = \frac{15}{16}$	$P_{6,10} = \frac{15}{16}$	$P_{6,11} = \frac{15}{16}$	$P_{6,12} = \frac{15}{16}$	$P_{6,13} = \frac{15}{16}$	$P_{6,14} = \frac{15}{16}$	$P_{6,15} = \frac{15}{16}$	$P_{6,16} = \frac{15}{16}$	$P_{6,17} = \frac{15}{16}$	$P_{6,18} = \frac{15}{16}$
	$P_{7,0} = \frac{7}{2}$	$P_{7,1} = \frac{14355}{16384}$	$P_{7,2} = \frac{14355}{16384}$	$P_{7,3} = \frac{14355}{16384}$	$P_{7,4} = \frac{14355}{16384}$	$P_{7,5} = \frac{14355}{16384}$	$P_{7,6} = \frac{14355}{16384}$	$P_{7,7} = \frac{14355}{16384}$	$P_{7,8} = \frac{14355}{16384}$	$P_{7,9} = \frac{14355}{16384}$	$P_{7,10} = \frac{14355}{16384}$	$P_{7,11} = \frac{14355}{16384}$	$P_{7,12} = \frac{14355}{16384}$	$P_{7,13} = \frac{14355}{16384}$	$P_{7,14} = \frac{14355}{16384}$	$P_{7,15} = \frac{14355}{16384}$	$P_{7,16} = \frac{14355}{16384}$	$P_{7,17} = \frac{14355}{16384}$	$P_{7,18} = \frac{14355}{16384}$
	$P_{8,0} = \frac{5}{4}$	$P_{8,1} = \frac{15}{16}$	$P_{8,2} = \frac{15}{16}$	$P_{8,3} = \frac{15}{16}$	$P_{8,4} = \frac{15}{16}$	$P_{8,5} = \frac{15}{16}$	$P_{8,6} = \frac{15}{16}$	$P_{8,7} = \$											

$$t = N\mathbf{1}$$
[illegible]

Finally, we see that  $t_0 = \boxed{\frac{213}{29} \approx 7.345}$