

Rajeev Atla

~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

A hexagonal grid of 19 cells, numbered 0 to 18, arranged in a larger hexagonal shape. Each cell contains a red dot and a number. The cells are surrounded by 21 green dots, numbered 19 to 39, arranged in a larger hexagonal shape around the grid.

We wish to find the expected value of the number of turns in the game, which we denote  $N$ .

The dice is truly random, so there is no upper bound on  $N$ . We note that this game is really akin to a Markov chain, in that it doesn't matter what the past states are.



	P <sub>10</sub> = .45	P <sub>11</sub> = .16	P <sub>12</sub> = .16	P <sub>13</sub> = .16	P <sub>14</sub> = .16	P <sub>15</sub> = .16	P <sub>16</sub> = .16	P <sub>17</sub> = .16	P <sub>18</sub> = .16	P <sub>19</sub> = .16	P <sub>20</sub> = .16	P <sub>21</sub> = .16	P <sub>22</sub> = .16	P <sub>23</sub> = .16	P <sub>24</sub> = .16	P <sub>25</sub> = .16	P <sub>26</sub> = .16	P <sub>27</sub> = .16	P <sub>28</sub> = .16	P <sub>29</sub> = .16	P <sub>30</sub> = .16	P <sub>31</sub> = .16	P <sub>32</sub> = .16	P <sub>33</sub> = .16	P <sub>34</sub> = .16	P <sub>35</sub> = .16	P <sub>36</sub> = .16	P <sub>37</sub> = .16	P <sub>38</sub> = .16	P <sub>39</sub> = .16	P <sub>40</sub> = .16	P <sub>41</sub> = .16	P <sub>42</sub> = .16	P <sub>43</sub> = .16	P <sub>44</sub> = .16	P <sub>45</sub> = .16	P <sub>46</sub> = .16	P <sub>47</sub> = .16	P <sub>48</sub> = .16	P <sub>49</sub> = .16	P <sub>50</sub> = .16	P <sub>51</sub> = .16	P <sub>52</sub> = .16	P <sub>53</sub> = .16	P <sub>54</sub> = .16	P <sub>55</sub> = .16	P <sub>56</sub> = .16	P <sub>57</sub> = .16	P <sub>58</sub> = .16	P <sub>59</sub> = .16	P <sub>60</sub> = .16	P <sub>61</sub> = .16	P <sub>62</sub> = .16	P <sub>63</sub> = .16	P <sub>64</sub> = .16	P <sub>65</sub> = .16	P <sub>66</sub> = .16	P <sub>67</sub> = .16	P <sub>68</sub> = .16	P <sub>69</sub> = .16	P <sub>70</sub> = .16	P <sub>71</sub> = .16	P <sub>72</sub> = .16	P <sub>73</sub> = .16	P <sub>74</sub> = .16	P <sub>75</sub> = .16	P <sub>76</sub> = .16	P <sub>77</sub> = .16	P <sub>78</sub> = .16	P <sub>79</sub> = .16	P <sub>80</sub> = .16	P <sub>81</sub> = .16	P <sub>82</sub> = .16	P <sub>83</sub> = .16	P <sub>84</sub> = .16	P <sub>85</sub> = .16	P <sub>86</sub> = .16	P <sub>87</sub> = .16	P <sub>88</sub> = .16	P <sub>89</sub> = .16	P <sub>90</sub> = .16	P <sub>91</sub> = .16	P <sub>92</sub> = .16	P <sub>93</sub> = .16	P <sub>94</sub> = .16	P <sub>95</sub> = .16	P <sub>96</sub> = .16	P <sub>97</sub> = .16	P <sub>98</sub> = .16	P <sub>99</sub> = .16	P <sub>100</sub> = .16	P <sub>101</sub> = .16	P <sub>102</sub> = .16	P <sub>103</sub> = .16	P <sub>104</sub> = .16	P <sub>105</sub> = .16	P <sub>106</sub> = .16	P <sub>107</sub> = .16	P <sub>108</sub> = .16	P <sub>109</sub> = .16	P <sub>110</sub> = .16	P <sub>111</sub> = .16	P <sub>112</sub> = .16	P <sub>113</sub> = .16	P <sub>114</sub> = .16	P <sub>115</sub> = .16	P <sub>116</sub> = .16	P <sub>117</sub> = .16	P <sub>118</sub> = .16	P <sub>119</sub> = .16	P <sub>120</sub> = .16	P <sub>121</sub> = .16	P <sub>122</sub> = .16	P <sub>123</sub> = .16	P <sub>124</sub> = .16	P <sub>125</sub> = .16	P <sub>126</sub> = .16	P <sub>127</sub> = .16	P <sub>128</sub> = .16	P <sub>129</sub> = .16	P <sub>130</sub> = .16	P <sub>131</sub> = .16	P <sub>132</sub> = .16	P <sub>133</sub> = .16	P <sub>134</sub> = .16	P <sub>135</sub> = .16	P <sub>136</sub> = .16	P <sub>137</sub> = .16	P <sub>138</sub> = .16	P <sub>139</sub> = .16	P <sub>140</sub> = .16	P <sub>141</sub> = .16	P <sub>142</sub> = .16	P <sub>143</sub> = .16	P <sub>144</sub> = .16	P <sub>145</sub> = .16	P <sub>146</sub> = .16	P <sub>147</sub> = .16	P <sub>148</sub> = .16	P <sub>149</sub> = .16	P <sub>150</sub> = .16	P <sub>151</sub> = .16	P <sub>152</sub> = .16	P <sub>153</sub> = .16	P <sub>154</sub> = .16	P <sub>155</sub> = .16	P <sub>156</sub> = .16	P <sub>157</sub> = .16	P <sub>158</sub> = .16	P <sub>159</sub> = .16	P <sub>160</sub> = .16	P <sub>161</sub> = .16	P <sub>162</sub> = .16	P <sub>163</sub> = .16	P <sub>164</sub> = .16	P <sub>165</sub> = .16	P <sub>166</sub> = .16	P <sub>167</sub> = .16	P <sub>168</sub> = .16	P <sub>169</sub> = .16	P <sub>170</sub> = .16	P <sub>171</sub> = .16	P <sub>172</sub> = .16	P <sub>173</sub> = .16	P <sub>174</sub> = .16	P <sub>175</sub> = .16	P <sub>176</sub> = .16	P <sub>177</sub> = .16	P <sub>178</sub> = .16	P <sub>179</sub> = .16	P <sub>180</sub> = .16	P <sub>181</sub> = .16	P <sub>182</sub> = .16	P <sub>183</sub> = .16	P <sub>184</sub> = .16	P <sub>185</sub> = .16	P <sub>186</sub> = .16	P <sub>187</sub> = .16	P <sub>188</sub> = .16	P <sub>189</sub> = .16	P <sub>190</sub> = .16	P <sub>191</sub> = .16	P <sub>192</sub> = .16	P <sub>193</sub> = .16	P <sub>194</sub> = .16	P <sub>195</sub> = .16	P <sub>196</sub> = .16	P <sub>197</sub> = .16	P <sub>198</sub> = .16	P <sub>199</sub> = .16	P <sub>200</sub> = .16	P <sub>201</sub> = .16	P <sub>202</sub> = .16	P <sub>203</sub> = .16	P <sub>204</sub> = .16	P <sub>205</sub> = .16	P <sub>206</sub> = .16	P <sub>207</sub> = .16	P <sub>208</sub> = .16	P <sub>209</sub> = .16	P <sub>210</sub> = .16	P <sub>211</sub> = .16	P <sub>212</sub> = .16	P <sub>213</sub> = .16	P <sub>214</sub> = .16	P <sub>215</sub> = .16	P <sub>216</sub> = .16	P <sub>217</sub> = .16	P <sub>218</sub> = .16	P <sub>219</sub> = .16	P <sub>220</sub> = .16	P <sub>221</sub> = .16	P <sub>222</sub> = .16	P <sub>223</sub> = .16	P <sub>224</sub> = .16	P <sub>225</sub> = .16	P <sub>226</sub> = .16	P <sub>227</sub> = .16	P <sub>228</sub> = .16	P <sub>229</sub> = .16	P <sub>230</sub> = .16	P <sub>231</sub> = .16	P <sub>232</sub> = .16	P <sub>233</sub> = .16	P <sub>234</sub> = .16	P <sub>235</sub> = .16	P <sub>236</sub> = .16	P <sub>237</sub> = .16	P <sub>238</sub> = .16	P <sub>239</sub> = .16	P <sub>240</sub>
--	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------------	------------------

$$t = N\mathbf{1}$$
[illegible]

Finally, we see that  $t_0 = \boxed{\frac{213}{29} \approx 7.345}$