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~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

The diagram shows a hexagonal lattice structure. The central cell is labeled 0. Surrounding it are cells 1 through 18. The vertices of the lattice are marked with green dots and numbered 19 through 37. The numbering of the vertices follows a specific pattern, likely representing a coordinate system or a specific labeling scheme for the lattice.

$$\mathbb{E}(N) = \sum N \mathbb{P}(N)$$

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Let $X_i \in [0, 36]$ be the current state, or position of the traveler. The traveler always starts at position $X_0 = 0$. The final state must be $X_N \in [19, 36]$.

Now that we've defined some notation, we can write the transition matrix P . Because a 37×37 matrix is cumbersome, we combine the states $[19, 36]$ into a

$$P = \begin{pmatrix} p_{0,0}=0 & p_{0,1}=\frac{1}{6} & p_{0,2}=\frac{1}{6} & p_{0,3}=\frac{1}{6} & p_{0,4}=\frac{1}{6} & p_{0,5}=\frac{1}{6} & p_{0,6}=\frac{1}{6} & p_{0,7}=0 & p_{0,8}=0 & p_{0,9}=0 & p_{0,10}=0 & p_{0,11}=0 & p_{0,12}=0 & p_{0,13}=0 & p_{0,14}=0 & p_{0,15}=0 & p_{0,16}=0 & p_{0,17}=0 & p_{0,18}=0 & p_{0,19}=0 \\ p_{1,0}=\frac{1}{6} & p_{1,1}=0 & p_{1,2}=\frac{1}{6} & p_{1,3}=0 & p_{1,4}=0 & p_{1,5}=0 & p_{1,6}=\frac{1}{6} & p_{1,7}=\frac{1}{6} & p_{1,8}=\frac{1}{6} & p_{1,9}=\frac{1}{6} & p_{1,10}=0 & p_{1,11}=0 & p_{1,12}=0 & p_{1,13}=0 & p_{1,14}=0 & p_{1,15}=0 & p_{1,16}=0 & p_{1,17}=0 & p_{1,18}=0 & p_{1,19}=0 \\ p_{2,0}=0 & p_{2,1}=\frac{1}{6} & p_{2,2}=0 & p_{2,3}=\frac{1}{6} & p_{2,4}=0 & p_{2,5}=0 & p_{2,6}=0 & p_{2,7}=0 & p_{2,8}=0 & p_{2,9}=\frac{1}{6} & p_{2,10}=\frac{1}{6} & p_{2,11}=\frac{1}{6} & p_{2,12}=0 & p_{2,13}=0 & p_{2,14}=0 & p_{2,15}=0 & p_{2,16}=0 & p_{2,17}=0 & p_{2,18}=0 & p_{2,19}=0 \\ p_{3,0}=\frac{1}{6} & p_{3,1}=0 & p_{3,2}=\frac{1}{6} & 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$$Q = \begin{pmatrix} P_{0,0} = 0 & P_{0,1} = \frac{1}{6} & P_{0,2} = \frac{1}{6} & P_{0,3} = \frac{1}{6} & P_{0,4} = \frac{1}{6} & P_{0,5} = \frac{1}{6} & P_{0,6} = \frac{1}{6} & P_{0,7} = 0 & P_{0,8} = 0 & P_{0,9} = 0 & P_{0,10} = 0 & P_{0,11} = 0 & P_{0,12} = 0 & P_{0,13} = 0 & P_{0,14} = 0 & P_{0,15} = 0 & P_{0,16} = 0 & P_{0,17} = 0 & P_{0,18} = 0 \\ P_{1,0} = \frac{1}{6} & P_{1,1} = 0 & P_{1,2} = 0 & P_{1,3} = 0 & P_{1,4} = 0 & P_{1,5} = 0 & P_{1,6} = \frac{1}{6} & P_{1,7} = \frac{1}{6} & P_{1,8} = \frac{1}{6} & P_{1,9} = \frac{1}{6} & P_{1,10} = 0 & P_{1,11} = 0 & P_{1,12} = 0 & P_{1,13} = 0 & P_{1,14} = 0 & P_{1,15} = 0 & P_{1,16} = 0 & P_{1,17} = 0 & P_{1,18} = 0 \\ P_{2,0} = \frac{1}{6} & P_{2,1} = \frac{1}{6} & P_{2,2} = 0 & P_{2,3} = \frac{1}{6} & P_{2,4} = 0 & P_{2,5} = 0 & P_{2,6} = 0 & P_{2,7} = 0 & P_{2,8} = 0 & P_{2,9} = \frac{1}{6} & P_{2,10} = \frac{1}{6} & P_{2,11} = \frac{1}{6} & P_{2,12} = 0 & P_{2,13} = 0 & P_{2,14} = 0 & P_{2,15} = 0 & P_{2,16} = 0 & P_{2,17} = 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\frac{1}{6} & P_{15,6} = 0 & P_{15,7} = 0 & P_{15,8} = 0 & P_{15,9} = 0 & P_{15,10$$

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$P_{0,0} = 45$	$P_{0,1} = 16$	$P_{0,2} = 3456$	$P_{0,3} = 16$	$P_{0,4} = 106714$	$P_{0,5} = 16$	$P_{0,6} = 10722$	$P_{0,7} = 16$	$P_{0,8} = 7$	$P_{0,9} = 5$	$P_{0,10} = 7$	$P_{0,11} = 5$	$P_{0,12} = 7$	$P_{0,13} = 5$	$P_{0,14} = 7$	$P_{0,15} = 5$	$P_{0,16} = 7$	$P_{0,17} = 5$	$P_{0,18} = 7$
$P_{1,0} = 16$	$P_{1,1} = 16$	$P_{1,2} = 3456$	$P_{1,3} = 16$	$P_{1,4} = 106714$	$P_{1,5} = 16$	$P_{1,6} = 10722$	$P_{1,7} = 16$	$P_{1,8} = 7$	$P_{1,9} = 5$	$P_{1,10} = 7$	$P_{1,11} = 5$	$P_{1,12} = 7$	$P_{1,13} = 5$	$P_{1,14} = 7$	$P_{1,15} = 5$	$P_{1,16} = 7$	$P_{1,17} = 5$	$P_{1,18} = 7$
$P_{2,0} = 16$	$P_{2,1} = 16$	$P_{2,2} = 3456$	$P_{2,3} = 16$	$P_{2,4} = 106714$	$P_{2,5} = 16$	$P_{2,6} = 10722$	$P_{2,7} = 16$	$P_{2,8} = 7$	$P_{2,9} = 5$	$P_{2,10} = 7$	$P_{2,11} = 5$	$P_{2,12} = 7$	$P_{2,13} = 5$	$P_{2,14} = 7$	$P_{2,15} = 5$	$P_{2,16} = 7$	$P_{2,17} = 5$	$P_{2,18} = 7$
$P_{3,0} = 16$	$P_{3,1} = 16$	$P_{3,2} = 3456$	$P_{3,3} = 16$	$P_{3,4} = 106714$	$P_{3,5} = 16$	$P_{3,6} = 10722$	$P_{3,7} = 16$	$P_{3,8} = 7$	$P_{3,9} = 5$	$P_{3,10} = 7$	$P_{3,11} = 5$	$P_{3,12} = 7$	$P_{3,13} = 5$	$P_{3,14} = 7$	$P_{3,15} = 5$	$P_{3,16} = 7$	$P_{3,17} = 5$	$P_{3,18} = 7$
$P_{4,0} = 16$	$P_{4,1} = 16$	$P_{4,2} = 3456$	$P_{4,3} = 16$	$P_{4,4} = 106714$	$P_{4,5} = 16$	$P_{4,6} = 10722$	$P_{4,7} = 16$	$P_{4,8} = 7$	$P_{4,9} = 5$	$P_{4,10} = 7$	$P_{4,11} = 5$	$P_{4,12} = 7$	$P_{4,13} = 5$	$P_{4,14} = 7$	$P_{4,15} = 5$	$P_{4,16} = 7$	$P_{4,17} = 5$	$P_{4,18} = 7$
$P_{5,0} = 16$	$P_{5,1} = 16$	$P_{5,2} = 3456$	$P_{5,3} = 16$	$P_{5,4} = 106714$	$P_{5,5} = 16$	$P_{5,6} = 10722$	$P_{5,7} = 16$	$P_{5,8} = 7$	$P_{5,9} = 5$	$P_{5,10} = 7$	$P_{5,11} = 5$	$P_{5,12} = 7$	$P_{5,13} = 5$	$P_{5,14} = 7$	$P_{5,15} = 5$	$P_{5,16} = 7$	$P_{5,17} = 5$	$P_{5,18} = 7$
$P_{6,0} = 16$	$P_{6,1} = 16$	$P_{6,2} = 3456$	$P_{6,3} = 16$	$P_{6,4} = 106714$	$P_{6,5} = 16$	$P_{6,6} = 10722$	$P_{6,7} = 16$	$P_{6,8} = 7$	$P_{6,9} = 5$	$P_{6,10} = 7$	$P_{6,11} = 5$	$P_{6,12} = 7$	$P_{6,13} = 5$	$P_{6,14} = 7$	$P_{6,15} = 5$	$P_{6,16} = 7$	$P_{6,17} = 5$	$P_{6,18} = 7$
$P_{7,0} = 16$	$P_{7,1} = 16$	$P_{7,2} = 3456$	$P_{7,3} = 16$	$P_{7,4} = 106714$	$P_{7,5} = 16$	$P_{7,6} = 10722$	$P_{7,7} = 16$	$P_{7,8} = 7$	$P_{7,9} = 5$	$P_{7,10} = 7$	$P_{7,11} = 5$	$P_{7,12} = 7$	$P_{7,13} = 5$	$P_{7,14} = 7$	$P_{7,15} = 5$	$P_{7,16} = 7$	$P_{7,17} = 5$	$P_{7,18} = 7$
$P_{8,0} = 16$	$P_{8,1} = 16$	$P_{8,2} = 3456$	$P_{8,3} = 16$	$P_{8,4} = 106714$	$P_{8,5} = 16$	$P_{8,6} = 10722$	$P_{8,7} = 16$	$P_{8,8} = 7$	$P_{8,9} = 5$	$P_{8,10} = 7$	$P_{8,11} = 5$	$P_{8,12} = 7$	$P_{8,13} = 5$	$P_{8,14} = 7$	$P_{8,15} = 5$	$P_{8,16} = 7$	$P_{8,17} = 5$	$P_{8,18} = 7$
$P_{9,0} = 16$	$P_{9,1} = 16$	$P_{9,2} = 3456$	$P_{9,3} = 16$	$P_{9,4} = 106714$	$P_{9,5} = 16$	$P_{9,6} = 10722$	$P_{9,7} = 16$	$P_{9,8} = 7$	$P_{9,9} = 5$	$P_{9,10} = 7$	$P_{9,11} = 5$	$P_{9,12} = 7$	$P_{9,13} = 5$	$P_{9,14} = 7$	$P_{9,15} = 5$	$P_{9,16} = 7$	$P_{9,17} = 5$	$P_{9,18} = 7$
$P_{10,0} = 16$	$P_{10,1} = 16$	$P_{10,2} = 3456$	$P_{10,3} = 16$	$P_{10,4} = 106714$	$P_{10,5} = 16$	$P_{10,6} = 10722$	$P_{10,7} = 16$	$P_{10,8} = 7$	$P_{10,9} = 5$	$P_{10,10} = 7$	$P_{10,11} = 5$	$P_{10,12} = 7$	$P_{10,13} = 5$	$P_{10,14} = 7$	$P_{10,15} = 5$	$P_{10,16} = 7$	$P_{10,17} = 5$	$P_{10,18} = 7$
$P_{11,0} = 16$	$P_{11,1} = 16$	$P_{11,2} = 3456$	$P_{11,3} = 16$	$P_{11,4} = 106714$	$P_{11,5} = 16$	$P_{11,6} = 10722$												

$$t \equiv N\mathbf{1}$$
[illegible]

Finally, we see that $t_0 = \boxed{\frac{213}{29} \approx 7.345}$