

# $\mathbb{R}^n$ Bonus Problem #3

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## §1 Problem

~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

## §2 Diagram



## §3 Solution

We wish to find the expected value of the number of turns in the game, which we denote  $N$ .

$$\mathbb{E}(N) = \sum N \mathbb{P}(N)$$

The dice is truly random, so there is no upper bound on  $N$ . We note that this game is really akin to a Markov chain, in that it doesn't matter what the past states are.

Let  $X_i \in [0, 36]$  be the current state, or position of the traveler. The traveler always starts at position  $X_0 = 0$ . The final state must be  $X_N \in [19, 36]$ .

Now that we've defined some notation, we can write the transition matrix  $P$ . Because a  $37 \times 37$  matrix is cumbersome, we combine the states  $[19, 36]$  into a

$$P = \begin{pmatrix} p_{0,0}=0 & p_{0,1}=\frac{1}{6} & p_{0,2}=\frac{1}{6} & p_{0,3}=\frac{1}{6} & p_{0,4}=\frac{1}{6} & p_{0,5}=\frac{1}{6} & p_{0,6}=\frac{1}{6} & p_{0,7}=0 & p_{0,8}=0 & p_{0,9}=0 & p_{0,10}=0 & p_{0,11}=0 & p_{0,12}=0 & p_{0,13}=0 & p_{0,14}=0 & p_{0,15}=0 & p_{0,16}=0 & p_{0,17}=0 & p_{0,18}=0 & p_{0,19}=0 \\ p_{1,0}=\frac{1}{6} & p_{1,1}=0 & p_{1,2}=\frac{1}{6} & p_{1,3}=0 & p_{1,4}=0 & p_{1,5}=0 & p_{1,6}=\frac{1}{6} & p_{1,7}=\frac{1}{6} & p_{1,8}=\frac{1}{6} & p_{1,9}=\frac{1}{6} & p_{1,10}=0 & p_{1,11}=0 & p_{1,12}=0 & p_{1,13}=0 & p_{1,14}=0 & p_{1,15}=0 & p_{1,16}=0 & p_{1,17}=0 & p_{1,18}=0 & p_{1,19}=0 \\ p_{2,0}=0 & p_{2,1}=\frac{1}{6} & p_{2,2}=0 & p_{2,3}=\frac{1}{6} & p_{2,4}=0 & p_{2,5}=0 & p_{2,6}=0 & p_{2,7}=0 & p_{2,8}=0 & p_{2,9}=\frac{1}{6} & p_{2,10}=\frac{1}{6} & p_{2,11}=\frac{1}{6} & p_{2,12}=0 & p_{2,13}=0 & p_{2,14}=0 & p_{2,15}=0 & p_{2,16}=0 & p_{2,17}=0 & p_{2,18}=0 & p_{2,19}=0 \\ p_{3,0}=\frac{1}{6} & p_{3,1}=0 & p_{3,2}=\frac{1}{6} & 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| $N =$ | $P_{0,0} = \frac{45}{16}$ |  | $P_{0,1} = \frac{15}{16}$ |  | $P_{0,2} = \frac{15}{16}$ |  | $P_{0,3} = \frac{15}{16}$ |  | $P_{0,4} = \frac{15}{16}$ |  | $P_{0,5} = \frac{15}{16}$ |  | $P_{0,6} = \frac{15}{16}$ |  | $P_{0,7} = \frac{15}{16}$ |  | $P_{0,8} = \frac{15}{16}$ |  | $P_{0,9} = \frac{15}{16}$ |  | $P_{0,10} = \frac{15}{16}$ |  | $P_{0,11} = \frac{15}{16}$ |  | $P_{0,12} = \frac{15}{16}$ |  | $P_{0,13} = \frac{15}{16}$ |  | $P_{0,14} = \frac{15}{16}$ |  | $P_{0,15} = \frac{15}{16}$ |  | $P_{0,16} = \frac{15}{16}$ |  | $P_{0,17} = \frac{15}{16}$ |  | $P_{0,18} = \frac{15}{16}$ |  | $P_{0,19} = \frac{15}{16}$ |  | $P_{0,20} = \frac{15}{16}$ |  | $P_{0,21} = \frac{15}{16}$ |  | $P_{0,22} = \frac{15}{16}$ |  | $P_{0,23} = \frac{15}{16}$ |  | $P_{0,24} = \frac{15}{16}$ |  | $P_{0,25} = \frac{15}{16}$ |  | $P_{0,26} = \frac{15}{16}$ |  | $P_{0,27} = \frac{15}{16}$ |  | $P_{0,28} = \frac{15}{16}$ |  | $P_{0,29} = \frac{15}{16}$ |  | $P_{0,30} = \frac{15}{16}$ |  | $P_{0,31} = \frac{15}{16}$ |  | $P_{0,32} = \frac{15}{16}$ |  | $P_{0,33} = \frac{15}{16}$ |  | $P_{0,34} = \frac{15}{16}$ |  | $P_{0,35} = \frac{15}{16}$ |  | $P_{0,36} = \frac{15}{16}$ |  | $P_{0,37} = \frac{15}{16}$ |  | $P_{0,38} = \frac{15}{16}$ |  | $P_{0,39} = \frac{15}{16}$ |  | $P_{0,40} = \frac{15}{16}$ |  | $P_{0,41} = \frac{15}{16}$ |  | $P_{0,42} = \frac{15}{16}$ |  | $P_{0,43} = \frac{15}{16}$ |  | $P_{0,44} = \frac{15}{16}$ |  | $P_{0,45} = \frac{15}{16}$ |  | $P_{0,46} = \frac{15}{16}$ |  | $P_{0,47} = \frac{15}{16}$ |  | $P_{0,48} = \frac{15}{16}$ |  | $P_{0,49} = \frac{15}{16}$ |  | $P_{0,50} = \frac{15}{16}$ |  | $P_{0,51} = \frac{15}{16}$ |  | $P_{0,52} = \frac{15}{16}$ |  | $P_{0,53} = \frac{15}{16}$ |  | $P_{0,54} = \frac{15}{16}$ |  | $P_{0,55} = \frac{15}{16}$ |  | $P_{0,56} = \frac{15}{16}$ |  | $P_{0,57} = \frac{15}{16}$ |  | $P_{0,58} = \frac{15}{16}$ |  | $P_{0,59} = \frac{15}{16}$ |  | $P_{0,60} = \frac{15}{16}$ |  | $P_{0,61} = \frac{15}{16}$ |  | $P_{0,62} = \frac{15}{16}$ |  | $P_{0,63} = \frac{15}{16}$ |  | $P_{0,64} = \frac{15}{16}$ |  | $P_{0,65} = \frac{15}{16}$ |  | $P_{0,66} = \frac{15}{16}$ |  | $P_{0,67} = \frac{15}{16}$ |  | $P_{0,68} = \frac{15}{16}$ |  | $P_{0,69} = \frac{15}{16}$ |  | $P_{0,70} = \frac{15}{16}$ |  | $P_{0,71} = \frac{15}{16}$ |  | $P_{0,72} = \frac{15}{16}$ |  | $P_{0,73} = \frac{15}{16}$ |  | $P_{0,74} = \frac{15}{16}$ |  | $P_{0,75} = \frac{15}{16}$ |  | $P_{0,76} = \frac{15}{16}$ |  | $P_{0,77} = \frac{15}{16}$ |  | $P_{0,78} = \frac{15}{16}$ |  | $P_{0,79} = \frac{15}{16}$ |  | $P_{0,80} = \frac{15}{16}$ |  | $P_{0,81} = \frac{15}{16}$ |  | $P_{0,82} = \frac{15}{16}$ |  | $P_{0,83} = \frac{15}{16}$ |  | $P_{0,84} = \frac{15}{16}$ |  | $P_{0,85} = \frac{15}{16}$ |  | $P_{0,86} = \frac{15}{16}$ |  | $P_{0,87} = \frac{15}{16}$ |  | $P_{0,88} = \frac{15}{16}$ |  | $P_{0,89} = \frac{15}{16}$ |  | $P_{0,90} = \frac{15}{16}$ |  | $P_{0,91} = \frac{15}{16}$ |  | $P_{0,92} = \frac{15}{16}$ |  | $P_{0,93} = \frac{15}{16}$ |  | $P_{0,94} = \frac{15}{16}$ |  | $P_{0,95} = \frac{15}{16}$ |  | $P_{0,96}$ |
|-------|---------------------------|--|---------------------------|--|---------------------------|--|---------------------------|--|---------------------------|--|---------------------------|--|---------------------------|--|---------------------------|--|---------------------------|--|---------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|----------------------------|--|------------|

$$t = N\mathbf{1}$$
[illegible]

Finally, we see that  $t_0 = \boxed{\frac{213}{29} \approx 7.345}$