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~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

A diagram showing a hexagonal grid graph with 37 nodes. The nodes are arranged in a honeycomb lattice. Nodes 0 through 18 are black with a red dot in the center. Nodes 19 through 36 are green with a green dot in the center. The nodes are numbered as follows:

- Row 1 (top): 21
- Row 2: 22, 20
- Row 3: 23, 8, 19
- Row 4: 24, 10, 9, 7, 36
- Row 5: 25, 11, 2, 6, 18, 35
- Row 6: 26, 12, 3, 5, 17, 34
- Row 7: 27, 13, 4, 15, 16, 33
- Row 8: 28, 14, 32
- Row 9: 29, 31
- Row 10 (bottom): 30

The dice is truly random, so there is no upper bound on N . We note that this game is really akin to a Markov chain, in that it doesn't matter what the past states are.

$P_{0,0} = 45$	$P_{0,1} = 16$	$P_{0,2} = 3456$	$P_{0,3} = 16$	$P_{0,4} = 106714$	$P_{0,5} = 16$	$P_{0,6} = 10722$	$P_{0,7} = 16$	$P_{0,8} = 7$	$P_{0,9} = 5$	$P_{0,10} = 7$	$P_{0,11} = 5$	$P_{0,12} = 7$	$P_{0,13} = 7$	$P_{0,14} = 5$	$P_{0,15} = 7$	$P_{0,16} = 5$	$P_{0,17} = 7$	$P_{0,18} = 5$
$P_{1,0} = 16$	$P_{1,1} = 16$	$P_{1,2} = 3456$	$P_{1,3} = 16$	$P_{1,4} = 106714$	$P_{1,5} = 16$	$P_{1,6} = 10722$	$P_{1,7} = 16$	$P_{1,8} = 7$	$P_{1,9} = 5$	$P_{1,10} = 7$	$P_{1,11} = 5$	$P_{1,12} = 7$	$P_{1,13} = 7$	$P_{1,14} = 5$	$P_{1,15} = 7$	$P_{1,16} = 5$	$P_{1,17} = 7$	$P_{1,18} = 5$
$P_{2,0} = 16$	$P_{2,1} = 16$	$P_{2,2} = 3456$	$P_{2,3} = 16$	$P_{2,4} = 106714$	$P_{2,5} = 16$	$P_{2,6} = 10722$	$P_{2,7} = 16$	$P_{2,8} = 7$	$P_{2,9} = 5$	$P_{2,10} = 7$	$P_{2,11} = 5$	$P_{2,12} = 7$	$P_{2,13} = 7$	$P_{2,14} = 5$	$P_{2,15} = 7$	$P_{2,16} = 5$	$P_{2,17} = 7$	$P_{2,18} = 5$
$P_{3,0} = 16$	$P_{3,1} = 16$	$P_{3,2} = 3456$	$P_{3,3} = 16$	$P_{3,4} = 106714$	$P_{3,5} = 16$	$P_{3,6} = 10722$	$P_{3,7} = 16$	$P_{3,8} = 7$	$P_{3,9} = 5$	$P_{3,10} = 7$	$P_{3,11} = 5$	$P_{3,12} = 7$	$P_{3,13} = 7$	$P_{3,14} = 5$	$P_{3,15} = 7$	$P_{3,16} = 5$	$P_{3,17} = 7$	$P_{3,18} = 5$
$P_{4,0} = 16$	$P_{4,1} = 16$	$P_{4,2} = 3456$	$P_{4,3} = 16$	$P_{4,4} = 106714$	$P_{4,5} = 16$	$P_{4,6} = 10722$	$P_{4,7} = 16$	$P_{4,8} = 7$	$P_{4,9} = 5$	$P_{4,10} = 7$	$P_{4,11} = 5$	$P_{4,12} = 7$	$P_{4,13} = 7$	$P_{4,14} = 5$	$P_{4,15} = 7$	$P_{4,16} = 5$	$P_{4,17} = 7$	$P_{4,18} = 5$
$P_{5,0} = 16$	$P_{5,1} = 16$	$P_{5,2} = 3456$	$P_{5,3} = 16$	$P_{5,4} = 106714$	$P_{5,5} = 16$	$P_{5,6} = 10722$	$P_{5,7} = 16$	$P_{5,8} = 7$	$P_{5,9} = 5$	$P_{5,10} = 7$	$P_{5,11} = 5$	$P_{5,12} = 7$	$P_{5,13} = 7$	$P_{5,14} = 5$	$P_{5,15} = 7$	$P_{5,16} = 5$	$P_{5,17} = 7$	$P_{5,18} = 5$
$P_{6,0} = 16$	$P_{6,1} = 16$	$P_{6,2} = 3456$	$P_{6,3} = 16$	$P_{6,4} = 106714$	$P_{6,5} = 16$	$P_{6,6} = 10722$	$P_{6,7} = 16$	$P_{6,8} = 7$	$P_{6,9} = 5$	$P_{6,10} = 7$	$P_{6,11} = 5$	$P_{6,12} = 7$	$P_{6,13} = 7$	$P_{6,14} = 5$	$P_{6,15} = 7$	$P_{6,16} = 5$	$P_{6,17} = 7$	$P_{6,18} = 5$
$P_{7,0} = 16$	$P_{7,1} = 16$	$P_{7,2} = 3456$	$P_{7,3} = 16$	$P_{7,4} = 106714$	$P_{7,5} = 16$	$P_{7,6} = 10722$	$P_{7,7} = 16$	$P_{7,8} = 7$	$P_{7,9} = 5$	$P_{7,10} = 7$	$P_{7,11} = 5$	$P_{7,12} = 7$	$P_{7,13} = 7$	$P_{7,14} = 5$	$P_{7,15} = 7$	$P_{7,16} = 5$	$P_{7,17} = 7$	$P_{7,18} = 5$
$P_{8,0} = 16$	$P_{8,1} = 16$	$P_{8,2} = 3456$	$P_{8,3} = 16$	$P_{8,4} = 106714$	$P_{8,5} = 16$	$P_{8,6} = 10722$	$P_{8,7} = 16$	$P_{8,8} = 7$	$P_{8,9} = 5$	$P_{8,10} = 7$	$P_{8,11} = 5$	$P_{8,12} = 7$	$P_{8,13} = 7$	$P_{8,14} = 5$	$P_{8,15} = 7$	$P_{8,16} = 5$	$P_{8,17} = 7$	$P_{8,18} = 5$
$P_{9,0} = 16$	$P_{9,1} = 16$	$P_{9,2} = 3456$	$P_{9,3} = 16$	$P_{9,4} = 106714$	$P_{9,5} = 16$	$P_{9,6} = 10722$	$P_{9,7} = 16$	$P_{9,8} = 7$	$P_{9,9} = 5$	$P_{9,10} = 7$	$P_{9,11} = 5$	$P_{9,12} = 7$	$P_{9,13} = 7$	$P_{9,14} = 5$	$P_{9,15} = 7$	$P_{9,16} = 5$	$P_{9,17} = 7$	$P_{9,18} = 5$
$P_{10,0} = 16$	$P_{10,1} = 16$	$P_{10,2} = 3456$	$P_{10,3} = 16$	$P_{10,4} = 106714$	$P_{10,5} = 16$	$P_{10,6} = 10722$	$P_{10,7} = 16$	$P_{10,8} = 7$	$P_{10,9} = 5$	$P_{10,10} = 7$	$P_{10,11} = 5$	$P_{10,12} = 7$	$P_{10,13} = 7$	$P_{10,14} = 5$	$P_{10,15} = 7$	$P_{10,16} = 5$	$P_{10,17} = 7$	$P_{10,18} = 5$
$P_{11,0} = 16$	$P_{11,1} = 16$	$P_{11,2} = 3456$	$P_{11,3} = 16$	$P_{11,4} = 106714$	$P_{11,5} = 16$	$P_{11,6} = 10722$												

$$t \equiv N\mathbf{1}$$
[illegible]

Finally, we see that $t_0 = \boxed{\frac{213}{29} \approx 7.345}$