

Rajeev Atla

~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

A hexagonal lattice graph with 19 internal nodes and 28 boundary nodes. The internal nodes are numbered 0 through 18, and the boundary nodes are numbered 19 through 46. The nodes are colored red (internal) or green (boundary). The graph is a portion of a larger hexagonal lattice.

The dice is truly random, so there is no upper bound on N . We note that this game is really akin to a Markov chain, in that it doesn't matter what the past states are.

Let $X_i \in [0, 36]$ be the current state, or position of the traveler. The traveler always starts at position $X_0 = 0$. The final state must be $X_N \in [19, 36]$.

Now that we've defined some notation, we can write the transition matrix P . Because a 37×37 matrix is cumbersome, we combine the states [19, 36] into a

$$P = \begin{pmatrix} p_{0,0} = 0 & p_{0,1} = \frac{1}{6} & p_{0,2} = \frac{1}{6} & p_{0,3} = \frac{1}{6} & p_{0,4} = \frac{1}{6} & p_{0,5} = \frac{1}{6} & p_{0,6} = \frac{1}{6} & p_{0,7} = 0 & p_{0,8} = 0 & p_{0,9} = 0 & p_{0,10} = 0 & p_{0,11} = 0 & p_{0,12} = 0 & p_{0,13} = 0 & p_{0,14} = 0 & p_{0,15} = 0 & p_{0,16} = 0 & p_{0,17} = 0 & p_{0,18} = 0 & p_{0,19} = 0 \\ p_{1,0} = \frac{1}{6} & p_{1,1} = 0 & p_{1,2} = 0 & p_{1,3} = 0 & p_{1,4} = 0 & p_{1,5} = 0 & p_{1,6} = \frac{1}{6} & p_{1,7} = \frac{1}{6} & p_{1,8} = \frac{1}{6} & p_{1,9} = \frac{1}{6} & p_{1,10} = 0 & p_{1,11} = 0 & p_{1,12} = 0 & p_{1,13} = 0 & p_{1,14} = 0 & p_{1,15} = 0 & p_{1,16} = 0 & p_{1,17} = 0 & p_{1,18} = 0 & p_{1,19} = 0 \\ p_{2,0} = \frac{1}{6} & p_{2,1} = \frac{1}{6} & p_{2,2} = 0 & p_{2,3} = \frac{1}{6} & p_{2,4} = 0 & p_{2,5} = 0 & p_{2,6} = 0 & p_{2,7} = 0 & p_{2,8} = 0 & p_{2,9} = \frac{1}{6} & p_{2,10} = \frac{1}{6} & p_{2,11} = \frac{1}{6} & p_{2,12} = 0 & p_{2,13} = 0 & p_{2,14} = 0 & p_{2,15} = 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P_{15,2} = 0 & P_{15,3} = 0 & P_{15,4} = \frac{1}{6} & P_{15,5} = \frac{1}{6} & P_{15,6} = 0 & P_{15,7$$

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$N =$	$P_{0,0} = \frac{45}{16}$	$P_{1,0} = \frac{15}{16}$	$P_{2,0} = \frac{15}{16}$	$P_{3,0} = \frac{15}{16}$	$P_{4,0} = \frac{15}{16}$	$P_{5,0} = \frac{15}{16}$	$P_{6,0} = \frac{15}{16}$	$P_{7,0} = \frac{7}{2}$	$P_{8,0} = \frac{5}{4}$	$P_{9,0} = \frac{7}{2}$	$P_{10,0} = \frac{15}{16}$	$P_{11,0} = \frac{7}{8}$	$P_{12,0} = \frac{5}{4}$	$P_{13,0} = \frac{7}{8}$	$P_{14,0} = \frac{5}{4}$	$P_{15,0} = \frac{7}{8}$	$P_{16,0} = \frac{7}{8}$	$P_{17,0} = \frac{7}{8}$	$P_{18,0} = \frac{5}{4}$
	$P_{1,1} = \frac{15}{16}$	$P_{2,1} = \frac{15}{16}$	$P_{3,1} = \frac{15}{16}$	$P_{4,1} = \frac{15}{16}$	$P_{5,1} = \frac{15}{16}$	$P_{6,1} = \frac{15}{16}$	$P_{7,1} = \frac{7}{2}$	$P_{8,1} = \frac{5}{4}$	$P_{9,1} = \frac{7}{2}$	$P_{10,1} = \frac{15}{16}$	$P_{11,1} = \frac{7}{8}$	$P_{12,1} = \frac{5}{4}$	$P_{13,1} = \frac{7}{8}$	$P_{14,1} = \frac{5}{4}$	$P_{15,1} = \frac{7}{8}$	$P_{16,1} = \frac{7}{8}$	$P_{17,1} = \frac{7}{8}$	$P_{18,1} = \frac{5}{4}$	
	$P_{2,2} = \frac{15}{16}$	$P_{3,2} = \frac{15}{16}$	$P_{4,2} = \frac{15}{16}$	$P_{5,2} = \frac{15}{16}$	$P_{6,2} = \frac{15}{16}$	$P_{7,2} = \frac{7}{2}$	$P_{8,2} = \frac{5}{4}$	$P_{9,2} = \frac{7}{2}$	$P_{10,2} = \frac{15}{16}$	$P_{11,2} = \frac{7}{8}$	$P_{12,2} = \frac{5}{4}$	$P_{13,2} = \frac{7}{8}$	$P_{14,2} = \frac{5}{4}$	$P_{15,2} = \frac{7}{8}$	$P_{16,2} = \frac{7}{8}$	$P_{17,2} = \frac{7}{8}$	$P_{18,2} = \frac{5}{4}$		
	$P_{3,3} = \frac{15}{16}$	$P_{4,3} = \frac{15}{16}$	$P_{5,3} = \frac{15}{16}$	$P_{6,3} = \frac{15}{16}$	$P_{7,3} = \frac{7}{2}$	$P_{8,3} = \frac{5}{4}$	$P_{9,3} = \frac{7}{2}$	$P_{10,3} = \frac{15}{16}$	$P_{11,3} = \frac{7}{8}$	$P_{12,3} = \frac{5}{4}$	$P_{13,3} = \frac{7}{8}$	$P_{14,3} = \frac{5}{4}$	$P_{15,3} = \frac{7}{8}$	$P_{16,3} = \frac{7}{8}$	$P_{17,3} = \frac{7}{8}$	$P_{18,3} = \frac{5}{4}$			
	$P_{4,4} = \frac{15}{16}$	$P_{5,4} = \frac{15}{16}$	$P_{6,4} = \frac{15}{16}$	$P_{7,4} = \frac{7}{2}$	$P_{8,4} = \frac{5}{4}$	$P_{9,4} = \frac{7}{2}$	$P_{10,4} = \frac{15}{16}$	$P_{11,4} = \frac{7}{8}$	$P_{12,4} = \frac{5}{4}$	$P_{13,4} = \frac{7}{8}$	$P_{14,4} = \frac{5}{4}$	$P_{15,4} = \frac{7}{8}$	$P_{16,4} = \frac{7}{8}$	$P_{17,4} = \frac{7}{8}$	$P_{18,4} = \frac{5}{4}$				
	$P_{5,5} = \frac{15}{16}$	$P_{6,5} = \frac{15}{16}$	$P_{7,5} = \frac{7}{2}$	$P_{8,5} = \frac{5}{4}$	$P_{9,5} = \frac{7}{2}$	$P_{10,5} = \frac{15}{16}$	$P_{11,5} = \frac{7}{8}$	$P_{12,5} = \frac{5}{4}$	$P_{13,5} = \frac{7}{8}$	$P_{14,5} = \frac{5}{4}$	$P_{15,5} = \frac{7}{8}$	$P_{16,5} = \frac{7}{8}$	$P_{17,5} = \frac{7}{8}$	$P_{18,5} = \frac{5}{4}$					
	$P_{6,6} = \frac{15}{16}$	$P_{7,6} = \frac{7}{2}$	$P_{8,6} = \frac{5}{4}$	$P_{9,6} = \frac{7}{2}$	$P_{10,6} = \frac{15}{16}$	$P_{11,6} = \frac{7}{8}$	$P_{12,6} = \frac{5}{4}$	$P_{13,6} = \frac{7}{8}$	$P_{14,6} = \frac{5}{4}$	$P_{15,6} = \frac{7}{8}$	$P_{16,6} = \frac{7}{8}$	$P_{17,6} = \frac{7}{8}$	$P_{18,6} = \frac{5}{4}$						
	$P_{7,7} = \frac{7}{2}$	$P_{8,7} = \frac{5}{4}$	$P_{9,7} = \frac{7}{2}$	$P_{10,7} = \frac{15}{16}$	$P_{11,7} = \frac{7}{8}$	$P_{12,7} = \frac{5}{4}$	$P_{13,7} = \frac{7}{8}$	$P_{14,7} = \frac{5}{4}$	$P_{15,7} = \frac{7}{8}$	$P_{16,7} = \frac{7}{8}$	$P_{17,7} = \frac{7}{8}$	$P_{18,7} = \frac{5}{4}$							
	$P_{8,8} = \frac{5}{4}$	$P_{9,8} = \frac{7}{2}$	$P_{10,8} = \frac{15}{16}$	$P_{11,8} = \frac{7}{8}$	$P_{12,8} = \frac{5}{4}$	$P_{13,8} = \frac{7}{8}$	$P_{14,8} = \frac{5}{4}$	$P_{15,8} = \frac{7}{8}$	$P_{16,8} = \frac{7}{8}$	$P_{17,8} = \frac{7}{8}$	$P_{18,8} = \frac{5}{4}$								
	$P_{9,9} = \frac{7}{2}$	$P_{10,9} = \frac{15}{16}$	$P_{11,9} = \frac{7}{8}$	$P_{12,9} = \frac{5}{4}$	$P_{13,9} = \frac{7}{8}$	$P_{14,9} = \frac{5}{4}$	$P_{15,9} = \frac{7}{8}$	$P_{16,9} = \frac{7}{8}$	$P_{17,9} = \frac{7}{8}$	$P_{18,9} = \frac{5}{4}$									
$N =$	$P_{1,1} = \frac{15}{16}$	$P_{2,1} = \frac{15}{16}$	$P_{3,1} = \frac{15}{16}$	$P_{4,1} = \frac{15}{16}$	$P_{5,1} = \frac{15}{16}$	$P_{6,1} = \frac{15}{16}$	$P_{7,1} = \frac{7}{2}$	$P_{8,1} = \frac{5}{4}$	$P_{9,1} = \frac{7}{2}$	$P_{10,1} = \frac{15}{16}$	$P_{11,1} = \frac{7}{8}$	$P_{12,1} = \frac{5}{4}$	$P_{13,1} = \frac{7}{8}$	$P_{14,1} = \frac{5}{4}$	$P_{15,1} = \frac{7}{8}$	$P_{16,1} = \frac{7}{8}$	$P_{17,1} = \frac{7}{8}$	$P_{18,1} = \frac{5}{4}$	
	$P_{2,2} = \frac{15}{16}$	$P_{3,2} = \frac{15}{16}$	$P_{4,2} = \frac{15}{16}$	$P_{5,2} = \frac{15}{16}$ </															

$$t = N\mathbf{1}$$
[illegible]

Finally, we see that $t_0 = \boxed{\frac{213}{29} \approx 7.345}$