

Rajeev Atla

~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

The diagram shows a hexagonal lattice with 36 numbered cells (0-35) and 36 numbered vertices (1-36). The cells are arranged in a 6x6 grid. The vertices are marked with green dots and numbered 1-36. The cells are numbered 0-35, with 0 in the center. The numbering of cells follows a specific pattern, and the numbering of vertices follows a different pattern.

The dice is truly random, so there is no upper bound on  $N$ . We note that this game is really akin to a Markov chain, in that it doesn't matter what the past states are.

Page 2 of 4

$P_{0,0} = 45$	$P_{0,1} = 16$	$P_{0,2} = 3456$	$P_{0,3} = 16$	$P_{0,4} = 10774$	$P_{0,5} = 16$	$P_{0,6} = 17222$	$P_{0,7} = 16$	$P_{0,8} = 7$	$P_{0,9} = 5$	$P_{0,10} = 5$	$P_{0,11} = 7$	$P_{0,12} = 5$	$P_{0,13} = 7$	$P_{0,14} = 5$	$P_{0,15} = 7$	$P_{0,16} = 5$	$P_{0,17} = 7$	$P_{0,18} = 5$
$P_{1,0} = 16$	$P_{1,1} = 16$	$P_{1,2} = 3456$	$P_{1,3} = 16$	$P_{1,4} = 10774$	$P_{1,5} = 16$	$P_{1,6} = 17222$	$P_{1,7} = 16$	$P_{1,8} = 7$	$P_{1,9} = 5$	$P_{1,10} = 5$	$P_{1,11} = 7$	$P_{1,12} = 5$	$P_{1,13} = 7$	$P_{1,14} = 5$	$P_{1,15} = 7$	$P_{1,16} = 5$	$P_{1,17} = 7$	$P_{1,18} = 5$
$P_{2,0} = 16$	$P_{2,1} = 16$	$P_{2,2} = 3456$	$P_{2,3} = 16$	$P_{2,4} = 10774$	$P_{2,5} = 16$	$P_{2,6} = 17222$	$P_{2,7} = 16$	$P_{2,8} = 7$	$P_{2,9} = 5$	$P_{2,10} = 5$	$P_{2,11} = 7$	$P_{2,12} = 5$	$P_{2,13} = 7$	$P_{2,14} = 5$	$P_{2,15} = 7$	$P_{2,16} = 5$	$P_{2,17} = 7$	$P_{2,18} = 5$
$P_{3,0} = 16$	$P_{3,1} = 16$	$P_{3,2} = 3456$	$P_{3,3} = 16$	$P_{3,4} = 10774$	$P_{3,5} = 16$	$P_{3,6} = 17222$	$P_{3,7} = 16$	$P_{3,8} = 7$	$P_{3,9} = 5$	$P_{3,10} = 5$	$P_{3,11} = 7$	$P_{3,12} = 5$	$P_{3,13} = 7$	$P_{3,14} = 5$	$P_{3,15} = 7$	$P_{3,16} = 5$	$P_{3,17} = 7$	$P_{3,18} = 5$
$P_{4,0} = 16$	$P_{4,1} = 16$	$P_{4,2} = 3456$	$P_{4,3} = 16$	$P_{4,4} = 10774$	$P_{4,5} = 16$	$P_{4,6} = 17222$	$P_{4,7} = 16$	$P_{4,8} = 7$	$P_{4,9} = 5$	$P_{4,10} = 5$	$P_{4,11} = 7$	$P_{4,12} = 5$	$P_{4,13} = 7$	$P_{4,14} = 5$	$P_{4,15} = 7$	$P_{4,16} = 5$	$P_{4,17} = 7$	$P_{4,18} = 5$
$P_{5,0} = 16$	$P_{5,1} = 16$	$P_{5,2} = 3456$	$P_{5,3} = 16$	$P_{5,4} = 10774$	$P_{5,5} = 16$	$P_{5,6} = 17222$	$P_{5,7} = 16$	$P_{5,8} = 7$	$P_{5,9} = 5$	$P_{5,10} = 5$	$P_{5,11} = 7$	$P_{5,12} = 5$	$P_{5,13} = 7$	$P_{5,14} = 5$	$P_{5,15} = 7$	$P_{5,16} = 5$	$P_{5,17} = 7$	$P_{5,18} = 5$
$P_{6,0} = 16$	$P_{6,1} = 16$	$P_{6,2} = 3456$	$P_{6,3} = 16$	$P_{6,4} = 10774$	$P_{6,5} = 16$	$P_{6,6} = 17222$	$P_{6,7} = 16$	$P_{6,8} = 7$	$P_{6,9} = 5$	$P_{6,10} = 5$	$P_{6,11} = 7$	$P_{6,12} = 5$	$P_{6,13} = 7$	$P_{6,14} = 5$	$P_{6,15} = 7$	$P_{6,16} = 5$	$P_{6,17} = 7$	$P_{6,18} = 5$
$P_{7,0} = 16$	$P_{7,1} = 16$	$P_{7,2} = 3456$	$P_{7,3} = 16$	$P_{7,4} = 10774$	$P_{7,5} = 16$	$P_{7,6} = 17222$	$P_{7,7} = 16$	$P_{7,8} = 7$	$P_{7,9} = 5$	$P_{7,10} = 5$	$P_{7,11} = 7$	$P_{7,12} = 5$	$P_{7,13} = 7$	$P_{7,14} = 5$	$P_{7,15} = 7$	$P_{7,16} = 5$	$P_{7,17} = 7$	$P_{7,18} = 5$
$P_{8,0} = 16$	$P_{8,1} = 16$	$P_{8,2} = 3456$	$P_{8,3} = 16$	$P_{8,4} = 10774$	$P_{8,5} = 16$	$P_{8,6} = 17222$	$P_{8,7} = 16$	$P_{8,8} = 7$	$P_{8,9} = 5$	$P_{8,10} = 5$	$P_{8,11} = 7$	$P_{8,12} = 5$	$P_{8,13} = 7$	$P_{8,14} = 5$	$P_{8,15} = 7$	$P_{8,16} = 5$	$P_{8,17} = 7$	$P_{8,18} = 5$
$P_{9,0} = 16$	$P_{9,1} = 16$	$P_{9,2} = 3456$	$P_{9,3} = 16$	$P_{9,4} = 10774$	$P_{9,5} = 16$	$P_{9,6} = 17222$	$P_{9,7} = 16$	$P_{9,8} = 7$	$P_{9,9} = 5$	$P_{9,10} = 5$	$P_{9,11} = 7$	$P_{9,12} = 5$	$P_{9,13} = 7$	$P_{9,14} = 5$	$P_{9,15} = 7$	$P_{9,16} = 5$	$P_{9,17} = 7$	$P_{9,18} = 5$
$P_{10,0} = 16$	$P_{10,1} = 16$	$P_{10,2} = 3456$	$P_{10,3} = 16$	$P_{10,4} = 10774$	$P_{10,5} = 16$	$P_{10,6} = 17222$	$P_{10,7} = 16$	$P_{10,8} = 7$	$P_{10,9} = 5$	$P_{10,10} = 5$	$P_{10,11} = 7$	$P_{10,12} = 5$	$P_{10,13} = 7$	$P_{10,14} = 5$	$P_{10,15} = 7$	$P_{10,16} = 5$	$P_{10,17} = 7$	$P_{10,18} = 5$
$P_{11,0} = 16$	$P_{11,1} = 16$	$P_{11,2} = 3456$	$P_{11,3} = 16$	$P_{11,4} = 10774$	$P_{11,5} = 16$	$P_{11,6} = 17222$	$P_{11,7$											

$$t \equiv N\mathbf{1}$$
[illegible]

Finally, we see that  $t_0 = \boxed{\frac{213}{29} \approx 7.345}$