

\mathbb{R}^n Bonus Problem #3

Rajeev Atla

§1 Problem

~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

§2 Diagram



§3 Solution

We wish to find the expected value of the number of turns in the game, which we denote N .

$$\mathbb{E}(N) = \sum N \mathbb{P}(N)$$

The dice is truly random, so there is no upper bound on N . We note that this game is really akin to a Markov chain, in that it doesn't matter what the past states are.

Let $X_i \in [0, 36]$ be the current state, or position of the traveler. The traveler always starts at position $X_0 = 0$. The final state must be $X_N \in [19, 36]$.

Now that we've defined some notation, we can write the transition matrix P . Because a 37×37 matrix is cumbersome, we combine the states $[19, 36]$ into a

$$P = \begin{pmatrix} p_{0,0} = 0 & p_{0,1} = \frac{1}{6} & p_{0,2} = \frac{1}{6} & p_{0,3} = \frac{1}{6} & p_{0,4} = \frac{1}{6} & p_{0,5} = \frac{1}{6} & p_{0,6} = \frac{1}{6} & p_{0,7} = 0 & p_{0,8} = 0 & p_{0,9} = 0 & p_{0,10} = 0 & p_{0,11} = 0 & p_{0,12} = 0 & p_{0,13} = 0 & p_{0,14} = 0 & p_{0,15} = 0 & p_{0,16} = 0 & p_{0,17} = 0 & p_{0,18} = 0 & p_{0,19} = 0 \\ p_{1,0} = \frac{1}{6} & p_{1,1} = 0 & p_{1,2} = \frac{1}{6} & p_{1,3} = 0 & p_{1,4} = 0 & p_{1,5} = 0 & p_{1,6} = \frac{1}{6} & p_{1,7} = \frac{1}{6} & p_{1,8} = \frac{1}{6} & p_{1,9} = \frac{1}{6} & p_{1,10} = 0 & p_{1,11} = 0 & p_{1,12} = 0 & p_{1,13} = 0 & p_{1,14} = 0 & p_{1,15} = 0 & p_{1,16} = 0 & p_{1,17} = 0 & p_{1,18} = 0 & p_{1,19} = 0 \\ p_{2,0} = 0 & p_{2,1} = \frac{1}{6} & p_{2,2} = 0 & p_{2,3} = \frac{1}{6} & p_{2,4} = 0 & p_{2,5} = 0 & p_{2,6} = 0 & p_{2,7} = 0 & p_{2,8} = 0 & p_{2,9} = \frac{1}{6} & p_{2,10} = \frac{1}{6} & p_{2,11} = \frac{1}{6} & p_{2,12} = 0 & p_{2,13} = 0 & p_{2,14} = 0 & p_{2,15} = 0 & p_{2,16} = 0 & p_{2,17} = 0 & p_{2,18} = 0 & p_{2,19} = 0 \\ p_{3,0} = \frac{1}{6} & p_{3,1} = 0 & p_{3,2} = \frac{1}{6} & p_{3,3} = 0 & p_{3,4} = \frac{1}{6} & p_{3,5} = 0 & p_{3,6} = 0 & p_{3,7} = 0 & p_{3,8} = 0 & p_{3,9} = 0 & p_{3,10} = 0 & p_{3,11} = \frac{1}{6} & p_{3,12} = \frac{1}{6} & p_{3,13} = \frac{1}{6} & p_{3,14} = 0 & p_{3,15} = 0 & p_{3,16} = 0 & p_{3,17} = 0 & p_{3,18} = 0 & p_{3,19} = 0 \\ p_{4,0} = \frac{1}{6} & p_{4,1} = 0 & p_{4,2} = 0 & p_{4,3} = \frac{1}{6} & p_{4,4} = 0 & p_{4,5} = \frac{1}{6} & p_{4,6} = 0 & p_{4,7} = 0 & p_{4,8} = 0 & p_{4,9} = 0 & p_{4,10} = 0 & p_{4,11} = 0 & p_{4,12} = 0 & p_{4,13} = \frac{1}{6} & p_{4,14} = \frac{1}{6} & p_{4,15} = \frac{1}{6} & p_{4,16} = 0 & p_{4,17} = 0 & p_{4,18} = 0 & p_{4,19} = 0 \\ p_{5,0} = 0 & p_{5,1} = 0 & p_{5,2} = 0 & p_{5,3} = 0 & p_{5,4} = \frac{1}{6} & p_{5,5} = 0 & p_{5,6} = \frac{1}{6} & p_{5,7} = 0 & p_{5,8} = 0 & p_{5,9} = 0 & p_{5,10} = 0 & p_{5,11} = 0 & p_{5,12} = 0 & p_{5,13} = 0 & p_{5,14} = 0 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P_{12,4} = 0 & P_{12,5} = 0 & P_{12,6} = 0 & P_{12,7} = 0 & P_{12,8} = 0 & P_{12,9} = 0 & P_{12,10} = 0 & P_{12,11} = \frac{1}{6} & P_{12,12} = 0 & P_{12,13} = \frac{1}{6} & P_{12,14} = 0 & P_{12,15} = 0 & P_{12,16} = 0 & P_{12,17} = 0 & P_{12,18} = 0 \\ P_{13,0} = 0 & P_{13,1} = 0 & P_{13,2} = 0 & P_{13,3} = \frac{1}{6} & P_{13,4} = \frac{1}{6} & P_{13,5} = 0 & P_{13,6} = 0 & P_{13,7} = 0 & P_{13,8} = 0 & P_{13,9} = 0 & P_{13,10} = 0 & P_{13,11} = 0 & P_{13,12} = \frac{1}{6} & P_{13,13} = 0 & P_{13,14} = \frac{1}{6} & P_{13,15} = 0 & P_{13,16} = 0 & P_{13,17} = 0 & P_{13,18} = 0 \\ P_{14,0} = 0 & P_{14,1} = 0 & P_{14,2} = 0 & P_{14,3} = 0 & P_{14,4} = \frac{1}{6} & P_{14,5} = 0 & P_{14,6} = 0 & P_{14,7} = 0 & P_{14,8} = 0 & P_{14,9} = 0 & P_{14,10} = 0 & P_{14,11} = 0 & P_{14,12} = 0 & P_{14,13} = \frac{1}{6} & P_{14,14} = \frac{1}{6} & P_{14,15} = \frac{1}{6} & P_{14,16} = 0 & P_{14,17} = 0 & P_{14,18} = 0 \\ P_{15,0} = 0 & P_{15,1} = 0 & P_{15,2} = 0 & P_{15,3} = 0 & P_{15,4} = \frac{1}{6} & P_{15,5} = \frac{1}{6} & P_{15,6} = 0 & P_{15,7} = 0 & P_{15,8} = 0 & P_{15,$$

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$N =$	$P_{0,0} = \frac{45}{16}$	$P_{1,0} = \frac{15}{16}$	$P_{2,0} = \frac{15}{16}$	$P_{3,0} = \frac{15}{16}$	$P_{4,0} = \frac{15}{16}$	$P_{5,0} = \frac{15}{16}$	$P_{6,0} = \frac{15}{16}$	$P_{7,0} = \frac{7}{2}$	$P_{8,0} = \frac{5}{4}$	$P_{9,0} = \frac{7}{2}$	$P_{10,0} = \frac{15}{16}$	$P_{11,0} = \frac{7}{2}$	$P_{12,0} = \frac{5}{4}$	$P_{13,0} = \frac{7}{2}$	$P_{14,0} = \frac{5}{4}$	$P_{15,0} = \frac{7}{2}$	$P_{16,0} = \frac{5}{4}$	$P_{17,0} = \frac{7}{2}$	$P_{18,0} = \frac{5}{4}$
	$P_{1,1} = \frac{10571}{16384}$	$P_{2,1} = \frac{6595}{16384}$	$P_{3,1} = \frac{6595}{16384}$	$P_{4,1} = \frac{10571}{16384}$	$P_{5,1} = \frac{10571}{16384}$	$P_{6,1} = \frac{10571}{16384}$	$P_{7,1} = \frac{10571}{16384}$	$P_{8,1} = \frac{21905}{262144}$	$P_{9,1} = \frac{21905}{262144}$	$P_{10,1} = \frac{10571}{16384}$	$P_{11,1} = \frac{10571}{16384}$	$P_{12,1} = \frac{6595}{16384}$	$P_{13,1} = \frac{10571}{16384}$	$P_{14,1} = \frac{21905}{262144}$	$P_{15,1} = \frac{6595}{16384}$	$P_{16,1} = \frac{21905}{262144}$	$P_{17,1} = \frac{10571}{16384}$	$P_{18,1} = \frac{10571}{16384}$	$P_{19,1} = \frac{10571}{16384}$
	$P_{2,2} = \frac{10571}{16384}$	$P_{3,2} = \frac{6595}{16384}$	$P_{4,2} = \frac{6595}{16384}$	$P_{5,2} = \frac{10571}{16384}$	$P_{6,2} = \frac{10571}{16384}$	$P_{7,2} = \frac{10571}{16384}$	$P_{8,2} = \frac{21905}{262144}$	$P_{9,2} = \frac{21905}{262144}$	$P_{10,2} = \frac{10571}{16384}$	$P_{11,2} = \frac{10571}{16384}$	$P_{12,2} = \frac{6595}{16384}$	$P_{13,2} = \frac{10571}{16384}$	$P_{14,2} = \frac{21905}{262144}$	$P_{15,2} = \frac{6595}{16384}$	$P_{16,2} = \frac{21905}{262144}$	$P_{17,2} = \frac{10571}{16384}$	$P_{18,2} = \frac{10571}{16384}$	$P_{19,2} = \frac{10571}{16384}$	$P_{20,2} = \frac{10571}{16384}$
	$P_{3,3} = \frac{10571}{16384}$	$P_{4,3} = \frac{6595}{16384}$	$P_{5,3} = \frac{6595}{16384}$	$P_{6,3} = \frac{10571}{16384}$	$P_{7,3} = \frac{10571}{16384}$	$P_{8,3} = \frac{21905}{262144}$	$P_{9,3} = \frac{21905}{262144}$	$P_{10,3} = \frac{10571}{16384}$	$P_{11,3} = \frac{10571}{16384}$	$P_{12,3} = \frac{6595}{16384}$	$P_{13,3} = \frac{10571}{16384}$	$P_{14,3} = \frac{21905}{262144}$	$P_{15,3} = \frac{6595}{16384}$	$P_{16,3} = \frac{21905}{262144}$	$P_{17,3} = \frac{10571}{16384}$	$P_{18,3} = \frac{10571}{16384}$	$P_{19,3} = \frac{10571}{16384}$	$P_{20,3} = \frac{10571}{16384}$	$P_{21,3} = \frac{10571}{16384}$
	$P_{4,4} = \frac{10571}{16384}$	$P_{5,4} = \frac{6595}{16384}$	$P_{6,4} = \frac{6595}{16384}$	$P_{7,4} = \frac{10571}{16384}$	$P_{8,4} = \frac{21905}{262144}$	$P_{9,4} = \frac{21905}{262144}$	$P_{10,4} = \frac{10571}{16384}$	$P_{11,4} = \frac{10571}{16384}$	$P_{12,4} = \frac{6595}{16384}$	$P_{13,4} = \frac{10571}{16384}$	$P_{14,4} = \frac{21905}{262144}$	$P_{15,4} = \frac{6595}{16384}$	$P_{16,4} = \frac{21905}{262144}$	$P_{17,4} = \frac{10571}{16384}$	$P_{18,4} = \frac{10571}{16384}$	$P_{19,4} = \frac{10571}{16384}$	$P_{20,4} = \frac{10571}{16384}$	$P_{21,4} = \frac{10571}{16384}$	$P_{22,4} = \frac{10571}{16384}$
	$P_{5,5} = \frac{10571}{16384}$	$P_{6,5} = \frac{6595}{16384}$	$P_{7,5} = \frac{6595}{16384}$	$P_{8,5} = \frac{10571}{16384}$	$P_{9,5} = \frac{21905}{262144}$	$P_{10,5} = \frac{21905}{262144}$	$P_{11,5} = \frac{10571}{16384}$	$P_{12,5} = \frac{10571}{16384}$	$P_{13,5} = \frac{6595}{16384}$	$P_{14,5} = \frac{10571}{16384}$	$P_{15,5} = \frac{21905}{262144}$	$P_{16,5} = \frac{6595}{16384}$	$P_{17,5} = \frac{10571}{16384}$	$P_{18,5} = \frac{10571}{16384}$	$P_{19,5} = \frac{10571}{16384}$	$P_{20,5} = \frac{10571}{16384}$	$P_{21,5} = \frac{10571}{16384}$	$P_{22,5} = \frac{10571}{16384}$	$P_{23,5} = \frac{10571}{16384}$
	$P_{6,6} = \frac{10571}{16384}$	$P_{7,6} = \frac{6595}{16384}$	$P_{8,6} = \frac{6595}{16384}$	$P_{9,6} = \frac{10571}{16384}$	$P_{10,6} = \frac{21905}{262144}$	$P_{11,6} = \frac{21905}{262144}$	$P_{12,6} = \frac{10571}{16384}$	$P_{13,6} = \frac{10571}{16384}$	$P_{14,6} = \frac{6595}{16384}$	$P_{15,6} = \frac{10571}{16384}$	$P_{16,6} = \frac{21905}{262144}$	$P_{17,6} = \frac{6595}{16384}$	$P_{18,6} = \frac{10571}{16384}$	$P_{19,6} = \frac{10571}{16384}$	$P_{20,6} = \frac{10571}{16384}$	$P_{21,6} = \frac{10571}{16384}$	$P_{22,6} = \frac{10571}{16384}$	$P_{23,6} = \frac{10571}{16384}$	$P_{24,6} = \frac{10571}{16384}$
	$P_{7,7} = \frac{10571}{16384}$	$P_{8,7} = \frac{6595}{16384}$	$P_{9,7} = \frac{6595}{16384}$	$P_{10,7} = \frac{10571}{16384}$	$P_{11,7} = \frac{21905}{262144$														

$$t = N\mathbf{1}$$
[illegible]

Finally, we see that $t_0 = \boxed{\frac{213}{29} \approx 7.345}$