

Rajeev Atla

~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

The diagram shows a hexagonal lattice with 19 cells labeled 0 to 18. Cell 0 is the central cell. Cells 1 through 18 are arranged in concentric rings around cell 0. Each cell contains a red dot and a black number. The cells are surrounded by 19 green dots, each labeled with a number from 20 to 38. The green dots are positioned at the midpoints of the edges of the hexagonal lattice.

We wish to find the expected value of the number of turns in the game, which we denote N .

The dice is truly random, so there is no upper bound on N . We note that this game is really akin to a Markov chain, in that it doesn't matter what the past states are.

Let $X_i \in [0, 36]$ be the current state, or position of the traveler. The traveler always starts at position $X_0 = 0$. The final state must be $X_N \in [19, 36]$.

Now that we've defined some notation, we can write the transition matrix P . Because a 37×37 matrix is cumbersome, we combine the states $[19, 36]$ into a

$$P = \begin{pmatrix} p_{0,0}=0 & p_{0,1}=\frac{1}{6} & p_{0,2}=\frac{1}{6} & p_{0,3}=\frac{1}{6} & p_{0,4}=\frac{1}{6} & p_{0,5}=\frac{1}{6} & p_{0,6}=\frac{1}{6} & p_{0,7}=0 & p_{0,8}=0 & p_{0,9}=0 & p_{0,10}=0 & p_{0,11}=0 & p_{0,12}=0 & p_{0,13}=0 & p_{0,14}=0 & p_{0,15}=0 & p_{0,16}=0 & p_{0,17}=0 & p_{0,18}=0 & p_{0,19}=0 \\ p_{1,0}=\frac{1}{6} & p_{1,1}=0 & p_{1,2}=\frac{1}{6} & p_{1,3}=0 & p_{1,4}=0 & p_{1,5}=0 & p_{1,6}=\frac{1}{6} & p_{1,7}=\frac{1}{6} & p_{1,8}=\frac{1}{6} & p_{1,9}=\frac{1}{6} & p_{1,10}=0 & p_{1,11}=0 & p_{1,12}=0 & p_{1,13}=0 & p_{1,14}=0 & p_{1,15}=0 & p_{1,16}=0 & p_{1,17}=0 & p_{1,18}=0 & p_{1,19}=0 \\ p_{2,0}=0 & p_{2,1}=\frac{1}{6} & p_{2,2}=0 & p_{2,3}=\frac{1}{6} & p_{2,4}=0 & p_{2,5}=0 & p_{2,6}=0 & p_{2,7}=0 & p_{2,8}=0 & p_{2,9}=\frac{1}{6} & p_{2,10}=\frac{1}{6} & p_{2,11}=\frac{1}{6} & p_{2,12}=0 & p_{2,13}=0 & p_{2,14}=0 & p_{2,15}=0 & p_{2,16}=0 & p_{2,17}=0 & p_{2,18}=0 & p_{2,19}=0 \\ p_{3,0}=\frac{1}{6} & p_{3,1}=0 & p_{3,2}=\frac{1}{6} & 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$$N^{-1} = \begin{pmatrix} p_{0,0} = 1 & p_{0,1} = \frac{1}{6} & p_{0,2} = \frac{1}{6} & p_{0,3} = \frac{1}{6} & p_{0,4} = \frac{1}{6} & p_{0,5} = \frac{1}{6} & p_{0,6} = \frac{1}{6} & p_{0,7} = 0 & p_{0,8} = 0 & p_{0,9} = 0 & p_{0,10} = 0 & p_{0,11} = 0 & p_{0,12} = 0 & p_{0,13} = 0 & p_{0,14} = 0 & p_{0,15} = 0 & p_{0,16} = 0 & p_{0,17} = 0 & p_{0,18} = 0 & p_{0,19} = 0 \\ p_{1,0} = \frac{1}{6} & p_{1,1} = 1 & p_{1,2} = \frac{1}{6} & p_{1,3} = 0 & p_{1,4} = 0 & p_{1,5} = 0 & p_{1,6} = \frac{1}{6} & p_{1,7} = \frac{1}{6} & p_{1,8} = \frac{1}{6} & p_{1,9} = \frac{1}{6} & p_{1,10} = 0 & p_{1,11} = \frac{1}{6} & p_{1,12} = 0 & p_{1,13} = 0 & p_{1,14} = 0 & p_{1,15} = 0 & p_{1,16} = 0 & p_{1,17} = 0 & p_{1,18} = 0 & p_{1,19} = 0 \\ p_{2,0} = \frac{1}{6} & p_{2,1} = \frac{1}{6} & p_{2,2} = 1 & p_{2,3} = \frac{1}{6} & p_{2,4} = 0 & p_{2,5} = 0 & p_{2,6} = 0 & p_{2,7} = 0 & p_{2,8} = 0 & p_{2,9} = 0 & p_{2,10} = \frac{1}{6} & p_{2,11} = \frac{1}{6} & p_{2,12} = 0 & p_{2,13} = 0 & p_{2,14} = 0 & p_{2,15} = 0 & p_{2,16} = 0 & p_{2,17} = 0 & p_{2,18} = 0 & p_{2,19} = 0 \\ p_{3,0} = \frac{1}{6} & p_{3,1} = 0 & p_{3,2} = \frac{1}{6} & p_{3,3} = 1 & p_{3,4} = \frac{1}{6} & p_{3,5} = 0 & p_{3,6} = 0 & p_{3,7} = 0 & p_{3,8} = 0 & p_{3,9} = 0 & p_{3,10} = 1 & p_{3,11} = \frac{1}{6} & p_{3,12} = \frac{1}{6} & p_{3,13} = 0 & p_{3,14} = 0 & p_{3,15} = 0 & p_{3,16} = 0 & p_{3,17} = 0 & p_{3,18} = 0 & p_{3,19} = 0 \\ p_{4,0} = \frac{1}{6} & p_{4,1} = 0 & p_{4,2} = 0 & p_{4,3} = \frac{1}{6} & p_{4,4} = 1 & p_{4,5} = \frac{1}{6} & p_{4,6} = 0 & p_{4,7} = 0 & p_{4,8} = 0 & p_{4,9} = 0 & p_{4,10} = 0 & p_{4,11} = 0 & p_{4,12} = 0 & p_{4,13} = \frac{1}{6} & p_{4,14} = \frac{1}{6} & p_{4,15} = \frac{1}{6} & p_{4,16} = 0 & p_{4,17} = 0 & p_{4,18} = 0 & p_{4,19} = 0 \\ p_{5,0} = 0 & p_{5,1} = 0 & p_{5,2} = 0 & p_{5,3} = 0 & p_{5,4} = \frac{1}{6} & p_{5,5} = \frac{1}{6} & p_{5,6} = 0 & p_{5,7} = 0 & p_{5,8} = 0 & p_{5,9} = 0 & p_{5,10} = 0 & p_{5,11} = 0 & p_{5,12} = 0 & p_{5,13} = 0 & p_{5,14} = 0 & p_{5,15} = \frac{1}{6} & p_{5,16} = \frac{1}{6} & p_{5,17} = \frac{1}{6} & p_{5,18} = 0 & p_{5,19} = 0 \\ p_{6,0} = \frac{1}{6} & p_{6,1} = \frac{1}{6} & p_{6,2} = 0 & p_{6,3} = 0 & p_{6,4} = 0 & p_{6,5} = \frac{1}{6} & p_{6,6} = 1 & p_{6,7} = \frac{1}{6} & p_{6,8} = 0 & p_{6,9} = 0 & p_{6,10} = 0 & p_{6,11} = 0 & p_{6,12} = 0 & p_{6,13} = 0 & p_{6,14} = 0 & p_{6,15} = \frac{1}{6} & p_{6,16} = 0 & p_{6,17} = 0 & p_{6,18} = \frac{1}{6} & p_{6,19} = 0 \\ p_{7,0} = 0 & p_{7,1} = \frac{1}{6} & p_{7,2} = 0 & p_{7,3} = 0 & p_{7,4} = 0 & p_{7,5} = 0 & p_{7,6} = \frac{1}{6} & p_{7,7} = 1 & p_{7,8} = \frac{1}{6} & p_{7,9} = 0 & p_{7,10} = 0 & p_{7,11} = 0 & p_{7,12} = 0 & p_{7,13} = 0 & p_{7,14} = 0 & p_{7,15} = 0 & p_{7,16} = 0 & p_{7,17} = 0 & p_{7,18} = \frac{1}{6} & p_{7,19} = \frac{1}{6} \\ p_{8,0} = 0 & p_{8,1} = \frac{1}{6} & p_{8,2} = 0 & p_{8,3} = 0 & p_{8,4} = 0 & p_{8,5} = 0 & p_{8,6} = 0 & p_{8,7} = \frac{1}{6} & p_{8,8} = 1 & p_{8,9} = \frac{1}{6} & p_{8,10} = 0 & p_{8,11} = 0 & p_{8,12} = 0 & p_{8,13} = 0 & p_{8,14} = 0 & p_{8,15} = 0 & p_{8,16} = 0 & p_{8,17} = 0 & p_{8,18} = 0 & p_{8,19} = 0 \\ p_{9,0} = 0 & p_{9,1} = \frac{1}{6} & p_{9,2} = \frac{1}{6} & p_{9,3} = 0 & p_{9,4} = 0 & p_{9,5} = 0 & p_{9,6} = 0 & p_{9,7} = 0 & p_{9,8} = \frac{1}{6} & p_{9,9} = 1 & p_{9,10} = \frac{1}{6} & p_{9,11} = 0 & p_{9,12} = 0 & p_{9,13} = 0 & p_{9,14} = 0 & p_{9,15} = 0 & p_{9,16} = 0 & p_{9,17} = 0 & p_{9,18} = 0 & p_{9,19} = 0 \\ p_{10,0} = 0 & p_{10,1} = 0 & p_{10,2} = \frac{1}{6} & p_{10,3} = 0 & p_{10,4} = 0 & p_{10,5} = 0 & p_{10,6} = 0 & p_{10,7} = 0 & p_{10,8} = 0 & p_{10,9} = \frac{1}{6} & p_{10,10} = 1 & p_{10,11} = 0 & p_{10,12} = 0 & p_{10,13} = 0 & p_{10,14} = 0 & p_{10,15} = 0 & p_{10,16} = 0 & p_{10,17} = 0 & p_{10,18} = 0 & p_{10,19} = 0 \\ p_{11,0} = 0 & p_{11,1} = 0 & p_{11,2} = \frac{1}{6} & p_{11,3} = \frac{1}{6} & p_{11,4} = 0 & p_{11,5} = 0 & p_{11,6} = 0 & p_{11,7} = 0 & p_{11,8} = 0 & p_{11,9} = 0 & p_{11,10} = 0 & p_{11,11} = 1 & p_{11,12} = 0 & p_{11,13} = 0 & p_{11,14} = 0 & p_{11,15} = 0 & p_{11,16} = 0 & p_{11,17} = 0 & p_{11,18} = 0 & p_{11,19} = 0 \\ p_{12,0} = 0 & p_{12,1} = 0 & p_{12,2} = 0 & p_{12,3} = \frac{1}{6} & p_{12,4} = 0 & p_{12,5} = 0 & p_{12,6} = 0 & p_{12,7} = 0 & p_{12,8} = 0 & p_{12,9} = 0 & p_{12,10} = 0 & p_{12,11} = \frac{1}{6} & p_{12,12} = \frac{1}{6} & p_{12,13} = \frac{1}{6} & p_{12,14} = 0 & p_{12,15} = 0 & p_{12,16} = 0 & p_{12,17} = 0 & p_{12,18} = 0 & p_{12,19} = 0 \\ p_{13,0} = 0 & p_{13,1} = 0 & p_{13,2} = 0 & p_{13,3} = \frac{1}{6} & p_{13,4} = \frac{1}{6} & p_{13,5} = 0 & p_{13,6} = 0 & p_{13,7} = 0 & p_{13,8} = 0 & p_{13,9} = 0 & p_{13,10} = 0 & p_{13,11} = 0 & p_{13,12} = 0 & p_{13,13} = 1 & p_{13,14} = \frac{1}{6} & p_{13,15} = 0 & p_{13,16} = 0 & p_{13,17} = 0 & p_{13,18} = 0 & p_{13,19} = 0 \\ p_{14,0} = 0 & p_{14,1} = 0 & p_{14,2} = 0 & p_{14,3} = 0 & p_{14,4} = \frac{1}{6} & p_{14,5} = \frac{1}{6} & p_{14,6} = 0 & p_{14,7} = 0 & p_{14,8} = 0 & p_{14,9} = 0 & p_{14,10} = 0 & p_{14,11} = 0 & p_{14,12} = 0 & p_{14,13} = \frac{1}{6} & p_{14,14} = 1$$

$P_{0,0} = 45$	$P_{0,1} = 16$	$P_{0,2} = 16$	$P_{0,3} = 16$	$P_{0,4} = 16$	$P_{0,5} = 16$	$P_{0,6} = 16$	$P_{0,7} = 7$	$P_{0,8} = \frac{5}{2}$	$P_{0,9} = 7$	$P_{0,10} = 16$	$P_{0,11} = 7$	$P_{0,12} = 5$	$P_{0,13} = 7$	$P_{0,14} = 5$	$P_{0,15} = 7$	$P_{0,16} = 5$	$P_{0,17} = 7$	$P_{0,18} = 5$
$P_{1,0} = 16$	$P_{1,1} = 16$	$P_{1,2} = 16$	$P_{1,3} = 16$	$P_{1,4} = 16$	$P_{1,5} = 16$	$P_{1,6} = 16$	$P_{1,7} = 16$	$P_{1,8} = 16$	$P_{1,9} = 16$	$P_{1,10} = 16$	$P_{1,11} = 16$	$P_{1,12} = 16$	$P_{1,13} = 16$	$P_{1,14} = 16$	$P_{1,15} = 16$	$P_{1,16} = 16$	$P_{1,17} = 16$	$P_{1,18} = 16$
$P_{2,0} = 16$	$P_{2,1} = 16$	$P_{2,2} = 16$	$P_{2,3} = 16$	$P_{2,4} = 16$	$P_{2,5} = 16$	$P_{2,6} = 16$	$P_{2,7} = 16$	$P_{2,8} = 16$	$P_{2,9} = 16$	$P_{2,10} = 16$	$P_{2,11} = 16$	$P_{2,12} = 16$	$P_{2,13} = 16$	$P_{2,14} = 16$	$P_{2,15} = 16$	$P_{2,16} = 16$	$P_{2,17} = 16$	$P_{2,18} = 16$
$P_{3,0} = 16$	$P_{3,1} = 16$	$P_{3,2} = 16$	$P_{3,3} = 16$	$P_{3,4} = 16$	$P_{3,5} = 16$	$P_{3,6} = 16$	$P_{3,7} = 16$	$P_{3,8} = 16$	$P_{3,9} = 16$	$P_{3,10} = 16$	$P_{3,11} = 16$	$P_{3,12} = 16$	$P_{3,13} = 16$	$P_{3,14} = 16$	$P_{3,15} = 16$	$P_{3,16} = 16$	$P_{3,17} = 16$	$P_{3,18} = 16$
$P_{4,0} = 16$	$P_{4,1} = 16$	$P_{4,2} = 16$	$P_{4,3} = 16$	$P_{4,4} = 16$	$P_{4,5} = 16$	$P_{4,6} = 16$	$P_{4,7} = 16$	$P_{4,8} = 16$	$P_{4,9} = 16$	$P_{4,10} = 16$	$P_{4,11} = 16$	$P_{4,12} = 16$	$P_{4,13} = 16$	$P_{4,14} = 16$	$P_{4,15} = 16$	$P_{4,16} = 16$	$P_{4,17} = 16$	$P_{4,18} = 16$
$P_{5,0} = 16$	$P_{5,1} = 16$	$P_{5,2} = 16$	$P_{5,3} = 16$	$P_{5,4} = 16$	$P_{5,5} = 16$	$P_{5,6} = 16$	$P_{5,7} = 16$	$P_{5,8} = 16$	$P_{5,9} = 16$	$P_{5,10} = 16$	$P_{5,11} = 16$	$P_{5,12} = 16$	$P_{5,13} = 16$	$P_{5,14} = 16$	$P_{5,15} = 16$	$P_{5,16} = 16$	$P_{5,17} = 16$	$P_{5,18} = 16$
$P_{6,0} = 16$	$P_{6,1} = 16$	$P_{6,2} = 16$	$P_{6,3} = 16$	$P_{6,4} = 16$	$P_{6,5} = 16$	$P_{6,6} = 16$	$P_{6,7} = 16$	$P_{6,8} = 16$	$P_{6,9} = 16$	$P_{6,10} = 16$	$P_{6,11} = 16$	$P_{6,12} = 16$	$P_{6,13} = 16$	$P_{6,14} = 16$	$P_{6,15} = 16$	$P_{6,16} = 16$	$P_{6,17} = 16$	$P_{6,18} = 16$
$P_{7,0} = 7$	$P_{7,1} = 16$	$P_{7,2} = 7$	$P_{7,3} = 16$	$P_{7,4} = 16$	$P_{7,5} = 7$	$P_{7,6} = 16$	$P_{7,7} = 7$	$P_{7,8} = 16$	$P_{7,9} = 7$	$P_{7,10} = 16$	$P_{7,11} = 7$	$P_{7,12} = 16$	$P_{7,13} = 16$	$P_{7,14} = 16$	$P_{7,15} = 16$	$P_{7,16} = 16$	$P_{7,17} = 7$	$P_{7,18} = 16$
$P_{8,0} = \frac{5}{2}$	$P_{8,1} = 16$	$P_{8,2} = \frac{5}{2}$	$P_{8,3} = 16$	$P_{8,4} = 16$	$P_{8,5} = \frac{5}{2}$	$P_{8,6} = 16$	$P_{8,7} = 16$	$P_{8,8} = 16$	$P_{8,9} = 16$	$P_{8,10} = 16$	$P_{8,11} = 16$	$P_{8,12} = 16$	$P_{8,13} = 16$	$P_{8,14} = 16$	$P_{8,15} = 16$	$P_{8,16} = 16$	$P_{8,17} = 16$	$P_{8,18} = 16$
$P_{9,0} = 7$	$P_{9,1} = 16$	$P_{9,2} = 7$	$P_{9,3} = 16$	$P_{9,4} = 16$	$P_{9,5} = 7$	$P_{9,6} = 16$	$P_{9,7} = 7$	$P_{9,8} = 16$	$P_{9,9} = 7$	$P_{9,10} = 16$	$P_{9,11} = 7$	$P_{9,12} = 16$	$P_{9,13} = 16$	$P_{9,14} = 16$	$P_{9,15} = 16$	$P_{9,16} = 16$	$P_{9,17} = 7$	$P_{9,18} = 16$
$P_{10,0} = 16$	$P_{10,1} = 16$	$P_{10,2} = 16$	$P_{10,3} = 16$	$P_{10,4} = 16$	$P_{10,5} = 16$	$P_{10,6} = 16$	$P_{10,7} = 16$	$P_{10,8} = 16$	$P_{10,9} = 16$	$P_{10,10} = 16$	$P_{10,11} = 16$	$P_{10,12} = 16$	$P_{10,13} = 16$	$P_{10,14} = 16$	$P_{10,15} = 16$	$P_{10,16} = 16$	$P_{10,17} = 16$	$P_{10,18} = 16$
$P_{11,0} = 7$	$P_{11,1} = 16$	$P_{11,2} = 7$	$P_{11,3} = 16$	$P_{11,4} = 16$	$P_{11,5} = 7$	$P_{11,6} = 16$												

In order to get the expected number of steps, we find t_0 , where

$$t \equiv N\mathbf{1}$$

Here, $\mathbf{1}$ is a vector whose entries are all 1.

$$t = \left(\begin{array}{c} 213 \\ \frac{29}{184} \\ \frac{29}{184} \\ \frac{29}{184} \\ \frac{29}{184} \\ \frac{29}{184} \\ \frac{29}{124} \\ \frac{29}{101} \\ \frac{29}{124} \\ \frac{29}{101} \\ \frac{29}{124} \\ \frac{29}{101} \\ \frac{29}{124} \\ \frac{29}{101} \\ \frac{29}{124} \\ \frac{29}{101} \end{array} \right)$$

Finally, we see that $t_0 = \boxed{\frac{213}{29} \approx 7.345}$