

\mathbb{R}^n Bonus Problem #3

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§1 Problem

~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

§2 Diagram



§3 Solution

We wish to find the expected value of the number of turns in the game, which we denote N .

$$\mathbb{E}(N) = \sum N \mathbb{P}(N)$$

The dice is truly random, so there is no upper bound on N . We note that this game is really akin to a Markov chain, in that it doesn't matter what the past states are.

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$$N = \begin{array}{c} \begin{array}{c} P_{0,0} = 45 \\ P_{0,1} = 34506 \\ P_{0,2} = 10714 \\ P_{0,3} = 249999 \\ P_{0,4} = 57876 \\ P_{0,5} = 11792 \\ P_{0,6} = 18354 \\ P_{0,7} = 24792 \\ P_{0,8} = 30744 \\ P_{0,9} = 36216 \\ P_{0,10} = 41688 \\ P_{0,11} = 47160 \\ P_{0,12} = 52632 \\ P_{0,13} = 58104 \\ P_{0,14} = 63576 \\ P_{0,15} = 69048 \\ P_{0,16} = 74520 \\ P_{0,17} = 79992 \\ P_{0,18} = 85464 \\ P_{0,19} = 90936 \\ P_{0,20} = 96408 \\ P_{0,21} = 101880 \\ P_{0,22} = 107352 \\ P_{0,23} = 112824 \\ P_{0,24} = 118296 \\ P_{0,25} = 123768 \\ P_{0,26} = 129240 \\ P_{0,27} = 134712 \\ P_{0,28} = 140184 \\ P_{0,29} = 145656 \\ P_{0,30} = 151128 \\ P_{0,31} = 156600 \\ P_{0,32} = 162072 \\ P_{0,33} = 167544 \\ P_{0,34} = 173016 \\ P_{0,35} = 178488 \\ P_{0,36} = 183960 \\ P_{0,37} = 189432 \\ P_{0,38} = 194904 \\ P_{0,39} = 200376 \\ P_{0,40} = 205848 \\ P_{0,41} = 211320 \\ P_{0,42} = 216792 \\ P_{0,43} = 222264 \\ P_{0,44} = 227736 \\ P_{0,45} = 233208 \\ P_{0,46} = 238680 \\ P_{0,47} = 244152 \\ P_{0,48} = 249624 \\ P_{0,49} = 255096 \\ P_{0,50} = 260568 \\ P_{0,51} = 266040 \\ P_{0,52} = 271512 \\ P_{0,53} = 276984 \\ P_{0,54} = 282456 \\ P_{0,55} = 287928 \\ P_{0,56} = 293400 \\ P_{0,57} = 298872 \\ P_{0,58} = 304344 \\ P_{0,59} = 309816 \\ P_{0,60} = 315288 \\ P_{0,61} = 320760 \\ P_{0,62} = 326232 \\ P_{0,63} = 331704 \\ P_{0,64} = 337176 \\ P_{0,65} = 342648 \\ P_{0,66} = 348120 \\ P_{0,67} = 353592 \\ P_{0,68} = 359064 \\ P_{0,69} = 364536 \\ P_{0,70} = 370008 \\ P_{0,71} = 375480 \\ P_{0,72} = 380952 \\ P_{0,73} = 386424 \\ P_{0,74} = 391896 \\ P_{0,75} = 397368 \\ P_{0,76} = 402840 \\ P_{0,77} = 408312 \\ P_{0,78} = 413784 \\ P_{0,79} = 419256 \\ P_{0,80} = 424728 \\ P_{0,81} = 430200 \\ P_{0,82} = 435672 \\ P_{0,83} = 441144 \\ P_{0,84} = 446616 \\ P_{0,85} = 452088 \\ P_{0,86} = 457560 \\ P_{0,87} = 463032 \\ P_{0,88} = 468504 \\ P_{0,89} = 473976 \\ P_{0,90} = 479448 \\ P_{0,91} = 484920 \\ P_{0,92} = 490392 \\ P_{0,93} = 495864 \\ P_{0,94} = 501336 \\ P_{0,95} = 506808 \\ P_{0,96} = 512280 \\ P_{0,97} = 517752 \\ P_{0,98} = 523224 \\ P_{0,99} = 528696 \\ P_{0,100} = 534168 \\ P_{0,101} = 539640 \\ P_{0,102} = 545112 \\ P_{0,103} = 550584 \\ P_{0,104} = 556056 \\ P_{0,105} = 561528 \\ P_{0,106} = 566992 \\ P_{0,107} = 572464 \\ P_{0,108} = 577936 \\ P_{0,109} = 583408 \\ P_{0,110} = 588880 \\ P_{0,111} = 594352 \\ P_{0,112} = 599824 \\ P_{0,113} = 605296 \\ P_{0,114} = 610768 \\ P_{0,115} = 616240 \\ P_{0,116} = 621712 \\ P_{0,117} = 627184 \\ P_{0,118} = 632656 \\ P_{0,119} = 638128 \\ P_{0,120} = 643600 \\ P_{0,121} = 649072 \\ P_{0,122} = 654544 \\ P_{0,123} = 660016 \\ P_{0,124} = 665488 \\ P_{0,125} = 670960 \\ P_{0,126} = 676432 \\ P_{0,127} = 681904 \\ P_{0,128} = 687376 \\ P_{0,129} = 692848 \\ P_{0,130} = 698320 \\ P_{0,131} = 703792 \\ P_{0,132} = 709264 \\ P_{0,133} = 714736 \\ P_{0,134} = 720208 \\ P_{0,135} = 725680 \\ P_{0,136} = 731152 \\ P_{0,137} = 736624 \\ P_{0,138} = 742096 \\ P_{0,139} = 747568 \\ P_{0,140} = 753040 \\ P_{0,141} = 758512 \\ P_{0,142} = 763984 \\ P_{0,143} = 769456 \\ P_{0,144} = 774928 \\ P_{0,145} = 780400 \\ P_{0,146} = 785872 \\ P_{0,147} = 791344 \\ P_{0,148} = 796816 \\ P_{0,149} = 802288 \\ P_{0,150} = 807760 \\ P_{0,151} = 813232 \\ P_{0,152} = 818704 \\ P_{0,153} = 824176 \\ P_{0,154} = 829648 \\ P_{0,155} = 835120 \\ P_{0,156} = 840592 \\ P_{0,157} = 846064 \\ P_{0,158} = 851536 \\ P_{0,159} = 857008 \\ P_{0,160} = 862480 \\ P_{0,161} = 867952 \\ P_{0,162} = 873424 \\ P_{0,163} = 878896 \\ P_{0,164} = 884368 \\ P_{0,165} = 889840 \\ P_{0,166} = 895312 \\ P_{0,167} = 900784 \\ P_{0,168} = 906256 \\ P_{0,169} = 911728 \\ P_{0,170} = 917200 \\ P_{0,171} = 922672 \\ P_{0,172} = 928144 \\ P_{0,173} = 933616 \\ P_{0,174} = 939088 \\ P_{0,175} = 944560 \\ P_{0,176} = 950032 \\ P_{0,177} = 955504 \\ P_{0,178} = 960976 \\ P_{0,179} = 966448 \\ P_{0,180} = 971920 \\ P_{0,181} = 977392 \\ P_{0,182} = 982864 \\ P_{0,183} = 988336 \\ P_{0,184} = 993808 \\ P_{0,185} = 999280 \\ P_{0,186} = 1004752 \\ P_{0,187} = 1010224 \\ P_{0,188} = 1015696 \\ P_{0,189} = 1021168 \\ P_{0,190} = 1026640 \\ P_{0,191} = 1032112 \\ P_{0,192} = 1037584 \\ P_{0,193} = 1043056 \\ P_{0,194} = 1048528 \\ P_{0,195} = 1053992 \\ P_{0,196} = 1059456 \\ P_{0,197} = 1064920 \\ P_{0,198} = 1070384 \\ P_{0,199} = 1075848 \\ P_{0,200} = 1081312 \\ P_{0,201} = 1086776 \\ P_{0,202} = 1092240 \\ P_{0,203} = 1097704 \\ P_{0,204} = 1103168 \\ P_{0,205} = 1108632 \\ P_{0,206} = 1114096 \\ P_{0,207} = 1119560 \\ P_{0,208} = 1125024 \\ P_{0,209} = 1130488 \\ P_{0,210} = 1135952 \\ P_{0,211} = 1141416 \\ P_{0,212} = 1146880 \\ P_{0,213} = 1152344 \\ P_{0,214} = 1157808 \\ P_{0,215} = 1163272 \\ P_{0,216} = 1168736 \\ P_{0,217} = 1174200 \\ P_{0,218} = 1179664 \\ P_{0,219} = 1185128 \\ P_{0,220} = 1190592 \\ P_{0,221} = 1196056 \\ P_{0,222} = 1201520 \\ P_{0,223} = 1206984 \\ P_{0,224} = 1212448 \\ P_{0,225} = 1217912 \\ P_{0,226} = 1223376 \\ P_{0,227} = 1228840 \\ P_{0,228} = 1234304 \\ P_{0,229} = 1239768 \\ P_{0,230} = 1245232 \\ P_{0,231} = 1250696 \\ P_{0,232} = 1256160 \\ P_{0,233} = 1261624 \\ P_{0,234} = 1267088 \\ P_{0,235} = 1272552 \\ P_{0,236} = 1278016 \\ P_{0,237} = 1283480 \\ P_{0,238} = 1288944 \\ P_{0,239} = 1294408 \\ P_{0,240} = 1299872 \\ P_{0,241} = 1305336 \\ P_{0,242$$

$$t = N\mathbf{1}$$
[illegible]

Finally, we see that $t_0 = \boxed{\frac{213}{29} \approx 7.345}$