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~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

A hexagonal grid of 19 cells, numbered 0 to 18. Cell 0 is the central cell. Cells 1 through 6 form a ring around cell 0. Cells 7 through 12 form another ring. Cells 13 through 18 form the outermost ring. Each cell has a red dot in the center. The grid is surrounded by 21 green dots, numbered 19 through 39, which are arranged in a larger hexagonal pattern around the central grid.

The dice is truly random, so there is no upper bound on  $N$ . We note that this game is really akin to a Markov chain, in that it doesn't matter what the past states are.



$N =$	$P_{0,0} = \frac{45}{16}$	$P_{1,0} = \frac{15}{16}$	$P_{2,0} = \frac{15}{16}$	$P_{3,0} = \frac{15}{16}$	$P_{4,0} = \frac{15}{16}$	$P_{5,0} = \frac{15}{16}$	$P_{6,0} = \frac{15}{16}$	$P_{7,0} = \frac{7}{2}$	$P_{8,0} = \frac{5}{16}$	$P_{9,0} = \frac{7}{2}$	$P_{10,0} = \frac{5}{16}$	$P_{11,0} = \frac{7}{2}$	$P_{12,0} = \frac{5}{16}$	$P_{13,0} = \frac{7}{2}$	$P_{14,0} = \frac{5}{16}$	$P_{15,0} = \frac{7}{2}$	$P_{16,0} = \frac{5}{16}$	$P_{17,0} = \frac{7}{2}$	$P_{18,0} = \frac{5}{16}$
	$P_{1,1} = \frac{10771}{3136}$	$P_{2,1} = \frac{10771}{3136}$	$P_{3,1} = \frac{10771}{3136}$	$P_{4,1} = \frac{10771}{3136}$	$P_{5,1} = \frac{10771}{3136}$	$P_{6,1} = \frac{10771}{3136}$	$P_{7,1} = \frac{10771}{3136}$	$P_{8,1} = \frac{10771}{3136}$	$P_{9,1} = \frac{10771}{3136}$	$P_{10,1} = \frac{10771}{3136}$	$P_{11,1} = \frac{10771}{3136}$	$P_{12,1} = \frac{10771}{3136}$	$P_{13,1} = \frac{10771}{3136}$	$P_{14,1} = \frac{10771}{3136}$	$P_{15,1} = \frac{10771}{3136}$	$P_{16,1} = \frac{10771}{3136}$	$P_{17,1} = \frac{10771}{3136}$	$P_{18,1} = \frac{10771}{3136}$	
	$P_{1,2} = \frac{3135}{1024}$	$P_{2,2} = \frac{3135}{1024}$	$P_{3,2} = \frac{3135}{1024}$	$P_{4,2} = \frac{3135}{1024}$	$P_{5,2} = \frac{3135}{1024}$	$P_{6,2} = \frac{3135}{1024}$	$P_{7,2} = \frac{3135}{1024}$	$P_{8,2} = \frac{3135}{1024}$	$P_{9,2} = \frac{3135}{1024}$	$P_{10,2} = \frac{3135}{1024}$	$P_{11,2} = \frac{3135}{1024}$	$P_{12,2} = \frac{3135}{1024}$	$P_{13,2} = \frac{3135}{1024}$	$P_{14,2} = \frac{3135}{1024}$	$P_{15,2} = \frac{3135}{1024}$	$P_{16,2} = \frac{3135}{1024}$	$P_{17,2} = \frac{3135}{1024}$	$P_{18,2} = \frac{3135}{1024}$	
	$P_{1,3} = \frac{6999}{16384}$	$P_{2,3} = \frac{6999}{16384}$	$P_{3,3} = \frac{6999}{16384}$	$P_{4,3} = \frac{6999}{16384}$	$P_{5,3} = \frac{6999}{16384}$	$P_{6,3} = \frac{6999}{16384}$	$P_{7,3} = \frac{6999}{16384}$	$P_{8,3} = \frac{6999}{16384}$	$P_{9,3} = \frac{6999}{16384}$	$P_{10,3} = \frac{6999}{16384}$	$P_{11,3} = \frac{6999}{16384}$	$P_{12,3} = \frac{6999}{16384}$	$P_{13,3} = \frac{6999}{16384}$	$P_{14,3} = \frac{6999}{16384}$	$P_{15,3} = \frac{6999}{16384}$	$P_{16,3} = \frac{6999}{16384}$	$P_{17,3} = \frac{6999}{16384}$	$P_{18,3} = \frac{6999}{16384}$	
	$P_{1,4} = \frac{16399}{262144}$	$P_{2,4} = \frac{16399}{262144}$	$P_{3,4} = \frac{16399}{262144}$	$P_{4,4} = \frac{16399}{262144}$	$P_{5,4} = \frac{16399}{262144}$	$P_{6,4} = \frac{16399}{262144}$	$P_{7,4} = \frac{16399}{262144}$	$P_{8,4} = \frac{16399}{262144}$	$P_{9,4} = \frac{16399}{262144}$	$P_{10,4} = \frac{16399}{262144}$	$P_{11,4} = \frac{16399}{262144}$	$P_{12,4} = \frac{16399}{262144}$	$P_{13,4} = \frac{16399}{262144}$	$P_{14,4} = \frac{16399}{262144}$	$P_{15,4} = \frac{16399}{262144}$	$P_{16,4} = \frac{16399}{262144}$	$P_{17,4} = \frac{16399}{262144}$	$P_{18,4} = \frac{16399}{262144}$	
	$P_{1,5} = \frac{36879}{4194304}$	$P_{2,5} = \frac{36879}{4194304}$	$P_{3,5} = \frac{36879}{4194304}$	$P_{4,5} = \frac{36879}{4194304}$	$P_{5,5} = \frac{36879}{4194304}$	$P_{6,5} = \frac{36879}{4194304}$	$P_{7,5} = \frac{36879}{4194304}$	$P_{8,5} = \frac{36879}{4194304}$	$P_{9,5} = \frac{36879}{4194304}$	$P_{10,5} = \frac{36879}{4194304}$	$P_{11,5} = \frac{36879}{4194304}$	$P_{12,5} = \frac{36879}{4194304}$	$P_{13,5} = \frac{36879}{4194304}$	$P_{14,5} = \frac{36879}{4194304}$	$P_{15,5} = \frac{36879}{4194304}$	$P_{16,5} = \frac{36879}{4194304}$	$P_{17,5} = \frac{36879}{4194304}$	$P_{18,5} = \frac{36879}{4194304}$	
	$P_{1,6} = \frac{8199}{65536}$	$P_{2,6} = \frac{8199}{65536}$	$P_{3,6} = \frac{8199}{65536}$	$P_{4,6} = \frac{8199}{65536}$	$P_{5,6} = \frac{8199}{65536}$	$P_{6,6} = \frac{8199}{65536}$	$P_{7,6} = \frac{8199}{65536}$	$P_{8,6} = \frac{8199}{65536}$	$P_{9,6} = \frac{8199}{65536}$	$P_{10,6} = \frac{8199}{65536}$	$P_{11,6} = \frac{8199}{65536}$	$P_{12,6} = \frac{8199}{65536}$	$P_{13,6} = \frac{8199}{65536}$	$P_{14,6} = \frac{8199}{65536}$	$P_{15,6} = \frac{8199}{65536}$	$P_{16,6} = \frac{8199}{65536}$	$P_{17,6} = \frac{8199}{65536}$	$P_{18,6} = \frac{8199}{65536}$	
	$P_{1,7} = \frac{18479}{1048576}$	$P_{2,7} = \frac{18479}{1048576}$	$P_{3,7} = \frac{18479}{1048576}$	$P_{4,7} = \frac{18479}{1048576}$	$P_{5,7} = \frac{18479}{1048576}$	$P_{6,7} = \frac{18479}{1048576}$	$P_{7,7} = \frac{18479}{1048576}$	$P_{8,7} = \frac{18479}{1048576}$	$P_{9,7} = \frac{18479}{1048576}$	$P_{10,7} = \frac{18479}{1048576}$	$P_{11,7} = \frac{18479}{1048576}$	$P_{12,7} = \frac{18479}{104$							

$$t = N\mathbf{1}$$
[illegible]

Finally, we see that  $t_0 = \boxed{\frac{213}{29} \approx 7.345}$