

\mathbb{R}^n Bonus Problem #3

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§1 Problem

~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

§2 Diagram



§3 Solution

We wish to find the expected value of the number of turns in the game, which we denote N .

$$\mathbb{E}(N) = \sum N \mathbb{P}(N)$$

The dice is truly random, so there is no upper bound on N . We note that this game is really akin to a Markov chain, in that it doesn't matter what the past states are.

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$N =$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608	16777216	33554432	67108864	134217728	268435456	536870912	1073741824	2147483648	4294967296	8589934592	17179869184	34359738368	68719476736	137438953472	274877906944	549755813888	1099511627776	2199023255552	4398046511104	8796093022208	17592186044416	35184372088832	70368744177664	140737488355328	281474976710656	562949953421312	1125899906842624	2251799813685248	4503599627370496	9007199254740992	18014398509481984	36028797018963968	72057594037927936	144115188075855872	288230376151711744	576460752303423488	1152921504606846976	2305843009213693952	4611686018427387904	9223372036854775808	18446744073709551616	36893488147419103232	73786976294838206464	147573952589676412928	295147905179352825856	590295810358705651712	1180591620717411303424	2361183241434822606848	4722366482869645213696	9444732965739290427392	18889465931478580854784	37778931862957161709568	75557863725914323419136	151115727451828646838272	302231454903657293676544	604462909807314587353088	$1208925819614629174706176$	$2417851639229258349412352$	$4835703278458516698824704$	$9671406556917033397649408$	$19342813113834066795298816$	$38685626227668133590597632$	$77371252455336267181195264$	$154742504910672534362390528$	$309485009821345068724781056$	$618970019642690137449562112$	$1237940039285380274899124224$	$2475880078570760549798248448$	$4951760157141521099596496896$	$9903520314283042199192993792$	$19807040628566084398385987584$	$39614081257132168796771975168$	$79228162514264337593543950336$	$158456325028528675187087900672$	$316912650057057350374175801344$	$633825300114114700748351602688$	$1267650600228229401496703205376$	$2535301200456458802993406410752$	$5070602400912917605986812821504$	$10141204801825835211973625643008$	$20282409603651670423947251286016$	$40564819207303340847894502572032$	$81129638414606681695789005144064$	$162259276829213363391578010288128$	$324518553658426726783156020576256$	$649037107316853453566312041152512$	$1298074214633706907132624082305024$	$2596148429267413814265248164610048$	$5192296858534827628530496329220096$	$10384593717069655257060992658440192$	$20769187434139310514121985316880384$	$41538374868278621028243970633760768$	$83076749736557242056487941267521536$	$166153499473114484112975882535043072$	$332306998946228968225951765070086144$	$664613997892457936451903530140172288$	$1329227995784915872903807060280344576$	$2658455991569831745807614120560689152$	$5316911983139663491615228241121378304$	$10633823966279326983230456482242756608$	$21267647932558653966460912964485513216$	$42535295865117307932921825928971026432$	$85070591730234615865843651857942052864$	$170141183460469231731687303715884105728$	$340282366920938463463374607431768211456$	$680564733841876926926749214863536422912$	$1361129467683753853853498429727072845824$	$272225893536750770770699685945414569152$	$544451787073501541541399371890829138304$	$1088903574147003083082798743781658276608$	$217780714829400616616559748756331655321$
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$$t = N\mathbf{1}$$
[illegible]

Finally, we see that $t_0 = \boxed{\frac{213}{29} \approx 7.345}$