

Rajeev Atla

~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

A hexagonal grid of 19 cells, numbered 0 to 18. Cell 0 is the central cell. Cells 1 through 6 form a ring around cell 0. Cells 7 through 12 form a second ring. Cells 13 through 18 form a third ring. The grid is surrounded by 19 green dots, numbered 20 through 38, which are arranged in a larger hexagonal pattern around the central grid.

$$\mathbb{E}(N) = \sum N \mathbb{P}(N)$$

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|       |                                       |                                       |                                       |                                       |                                       |                                       |                                       |                                       |                                    |                                     |                                     |                                     |                                     |                                     |                                     |                                     |                                     |                                     |                           |
|-------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|---------------------------|
| $N =$ | $P_{0,0} = \frac{45}{16}$             | $P_{1,0} = \frac{15}{16}$             | $P_{2,0} = \frac{15}{16}$             | $P_{3,0} = \frac{15}{16}$             | $P_{4,0} = \frac{15}{16}$             | $P_{5,0} = \frac{15}{16}$             | $P_{6,0} = \frac{15}{16}$             | $P_{7,0} = \frac{7}{2}$               | $P_{8,0} = \frac{5}{16}$           | $P_{9,0} = \frac{7}{2}$             | $P_{10,0} = \frac{5}{16}$           | $P_{11,0} = \frac{7}{2}$            | $P_{12,0} = \frac{5}{16}$           | $P_{13,0} = \frac{7}{2}$            | $P_{14,0} = \frac{5}{16}$           | $P_{15,0} = \frac{7}{2}$            | $P_{16,0} = \frac{5}{16}$           | $P_{17,0} = \frac{7}{2}$            | $P_{18,0} = \frac{5}{16}$ |
|       | $P_{1,1} = \frac{10771}{3136}$        | $P_{2,1} = \frac{10771}{3136}$        | $P_{3,1} = \frac{10771}{3136}$        | $P_{4,1} = \frac{10771}{3136}$        | $P_{5,1} = \frac{10771}{3136}$        | $P_{6,1} = \frac{10771}{3136}$        | $P_{7,1} = \frac{10771}{3136}$        | $P_{8,1} = \frac{10771}{3136}$        | $P_{9,1} = \frac{10771}{3136}$     | $P_{10,1} = \frac{10771}{3136}$     | $P_{11,1} = \frac{10771}{3136}$     | $P_{12,1} = \frac{10771}{3136}$     | $P_{13,1} = \frac{10771}{3136}$     | $P_{14,1} = \frac{10771}{3136}$     | $P_{15,1} = \frac{10771}{3136}$     | $P_{16,1} = \frac{10771}{3136}$     | $P_{17,1} = \frac{10771}{3136}$     | $P_{18,1} = \frac{10771}{3136}$     |                           |
|       | $P_{1,2} = \frac{3135}{1024}$         | $P_{2,2} = \frac{3135}{1024}$         | $P_{3,2} = \frac{3135}{1024}$         | $P_{4,2} = \frac{3135}{1024}$         | $P_{5,2} = \frac{3135}{1024}$         | $P_{6,2} = \frac{3135}{1024}$         | $P_{7,2} = \frac{3135}{1024}$         | $P_{8,2} = \frac{3135}{1024}$         | $P_{9,2} = \frac{3135}{1024}$      | $P_{10,2} = \frac{3135}{1024}$      | $P_{11,2} = \frac{3135}{1024}$      | $P_{12,2} = \frac{3135}{1024}$      | $P_{13,2} = \frac{3135}{1024}$      | $P_{14,2} = \frac{3135}{1024}$      | $P_{15,2} = \frac{3135}{1024}$      | $P_{16,2} = \frac{3135}{1024}$      | $P_{17,2} = \frac{3135}{1024}$      | $P_{18,2} = \frac{3135}{1024}$      |                           |
|       | $P_{1,3} = \frac{6999}{16384}$        | $P_{2,3} = \frac{6999}{16384}$        | $P_{3,3} = \frac{6999}{16384}$        | $P_{4,3} = \frac{6999}{16384}$        | $P_{5,3} = \frac{6999}{16384}$        | $P_{6,3} = \frac{6999}{16384}$        | $P_{7,3} = \frac{6999}{16384}$        | $P_{8,3} = \frac{6999}{16384}$        | $P_{9,3} = \frac{6999}{16384}$     | $P_{10,3} = \frac{6999}{16384}$     | $P_{11,3} = \frac{6999}{16384}$     | $P_{12,3} = \frac{6999}{16384}$     | $P_{13,3} = \frac{6999}{16384}$     | $P_{14,3} = \frac{6999}{16384}$     | $P_{15,3} = \frac{6999}{16384}$     | $P_{16,3} = \frac{6999}{16384}$     | $P_{17,3} = \frac{6999}{16384}$     | $P_{18,3} = \frac{6999}{16384}$     |                           |
|       | $P_{1,4} = \frac{16399}{262144}$      | $P_{2,4} = \frac{16399}{262144}$      | $P_{3,4} = \frac{16399}{262144}$      | $P_{4,4} = \frac{16399}{262144}$      | $P_{5,4} = \frac{16399}{262144}$      | $P_{6,4} = \frac{16399}{262144}$      | $P_{7,4} = \frac{16399}{262144}$      | $P_{8,4} = \frac{16399}{262144}$      | $P_{9,4} = \frac{16399}{262144}$   | $P_{10,4} = \frac{16399}{262144}$   | $P_{11,4} = \frac{16399}{262144}$   | $P_{12,4} = \frac{16399}{262144}$   | $P_{13,4} = \frac{16399}{262144}$   | $P_{14,4} = \frac{16399}{262144}$   | $P_{15,4} = \frac{16399}{262144}$   | $P_{16,4} = \frac{16399}{262144}$   | $P_{17,4} = \frac{16399}{262144}$   | $P_{18,4} = \frac{16399}{262144}$   |                           |
|       | $P_{1,5} = \frac{36879}{4194304}$     | $P_{2,5} = \frac{36879}{4194304}$     | $P_{3,5} = \frac{36879}{4194304}$     | $P_{4,5} = \frac{36879}{4194304}$     | $P_{5,5} = \frac{36879}{4194304}$     | $P_{6,5} = \frac{36879}{4194304}$     | $P_{7,5} = \frac{36879}{4194304}$     | $P_{8,5} = \frac{36879}{4194304}$     | $P_{9,5} = \frac{36879}{4194304}$  | $P_{10,5} = \frac{36879}{4194304}$  | $P_{11,5} = \frac{36879}{4194304}$  | $P_{12,5} = \frac{36879}{4194304}$  | $P_{13,5} = \frac{36879}{4194304}$  | $P_{14,5} = \frac{36879}{4194304}$  | $P_{15,5} = \frac{36879}{4194304}$  | $P_{16,5} = \frac{36879}{4194304}$  | $P_{17,5} = \frac{36879}{4194304}$  | $P_{18,5} = \frac{36879}{4194304}$  |                           |
|       | $P_{1,6} = \frac{81839}{68437760}$    | $P_{2,6} = \frac{81839}{68437760}$    | $P_{3,6} = \frac{81839}{68437760}$    | $P_{4,6} = \frac{81839}{68437760}$    | $P_{5,6} = \frac{81839}{68437760}$    | $P_{6,6} = \frac{81839}{68437760}$    | $P_{7,6} = \frac{81839}{68437760}$    | $P_{8,6} = \frac{81839}{68437760}$    | $P_{9,6} = \frac{81839}{68437760}$ | $P_{10,6} = \frac{81839}{68437760}$ | $P_{11,6} = \frac{81839}{68437760}$ | $P_{12,6} = \frac{81839}{68437760}$ | $P_{13,6} = \frac{81839}{68437760}$ | $P_{14,6} = \frac{81839}{68437760}$ | $P_{15,6} = \frac{81839}{68437760}$ | $P_{16,6} = \frac{81839}{68437760}$ | $P_{17,6} = \frac{81839}{68437760}$ | $P_{18,6} = \frac{81839}{68437760}$ |                           |
|       | $P_{1,7} = \frac{184199}{1048576000}$ | $P_{2,7} = \frac{184199}{1048576000}$ | $P_{3,7} = \frac{184199}{1048576000}$ | $P_{4,7} = \frac{184199}{1048576000}$ | $P_{5,7} = \frac{184199}{1048576000}$ | $P_{6,7} = \frac{184199}{1048576000}$ | $P_{7,7} = \frac{184199}{1048576000}$ | $P_{8,7} = \frac{184199}{1048576000}$ | $P_{9,7} = \frac{1$                |                                     |                                     |                                     |                                     |                                     |                                     |                                     |                                     |                                     |                           |

$$t = N\mathbf{1}$$
[illegible]

Finally, we see that  $t_0 = \boxed{\frac{213}{29} \approx 7.345}$