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~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

A diagram showing a hexagonal grid of 19 cells, numbered 0 to 18, arranged in a roughly circular pattern. Each cell contains a red dot and a number. The cells are surrounded by 21 green dots, numbered 19 to 39, arranged in a larger hexagonal pattern around the central cluster.

The dice is truly random, so there is no upper bound on  $N$ . We note that this game is really akin to a Markov chain, in that it doesn't matter what the past states are.



|       |                           |  |                           |  |                           |  |                           |  |                           |  |                           |  |                           |  |                           |  |                           |  |                           |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |                            |  |            |
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| $N =$ | $P_{0,0} = \frac{45}{16}$ |  | $P_{0,1} = \frac{16}{16}$ |  | $P_{0,2} = \frac{16}{16}$ |  | $P_{0,3} = \frac{16}{16}$ |  | $P_{0,4} = \frac{16}{16}$ |  | $P_{0,5} = \frac{16}{16}$ |  | $P_{0,6} = \frac{16}{16}$ |  | $P_{0,7} = \frac{16}{16}$ |  | $P_{0,8} = \frac{16}{16}$ |  | $P_{0,9} = \frac{16}{16}$ |  | $P_{0,10} = \frac{16}{16}$ |  | $P_{0,11} = \frac{16}{16}$ |  | $P_{0,12} = \frac{16}{16}$ |  | $P_{0,13} = \frac{16}{16}$ |  | $P_{0,14} = \frac{16}{16}$ |  | $P_{0,15} = \frac{16}{16}$ |  | $P_{0,16} = \frac{16}{16}$ |  | $P_{0,17} = \frac{16}{16}$ |  | $P_{0,18} = \frac{16}{16}$ |  | $P_{0,19} = \frac{16}{16}$ |  | $P_{0,20} = \frac{16}{16}$ |  | $P_{0,21} = \frac{16}{16}$ |  | $P_{0,22} = \frac{16}{16}$ |  | $P_{0,23} = \frac{16}{16}$ |  | $P_{0,24} = \frac{16}{16}$ |  | $P_{0,25} = \frac{16}{16}$ |  | $P_{0,26} = \frac{16}{16}$ |  | $P_{0,27} = \frac{16}{16}$ |  | $P_{0,28} = \frac{16}{16}$ |  | $P_{0,29} = \frac{16}{16}$ |  | $P_{0,30} = \frac{16}{16}$ |  | $P_{0,31} = \frac{16}{16}$ |  | $P_{0,32} = \frac{16}{16}$ |  | $P_{0,33} = \frac{16}{16}$ |  | $P_{0,34} = \frac{16}{16}$ |  | $P_{0,35} = \frac{16}{16}$ |  | $P_{0,36} = \frac{16}{16}$ |  | $P_{0,37} = \frac{16}{16}$ |  | $P_{0,38} = \frac{16}{16}$ |  | $P_{0,39} = \frac{16}{16}$ |  | $P_{0,40} = \frac{16}{16}$ |  | $P_{0,41} = \frac{16}{16}$ |  | $P_{0,42} = \frac{16}{16}$ |  | $P_{0,43} = \frac{16}{16}$ |  | $P_{0,44} = \frac{16}{16}$ |  | $P_{0,45} = \frac{16}{16}$ |  | $P_{0,46} = \frac{16}{16}$ |  | $P_{0,47} = \frac{16}{16}$ |  | $P_{0,48} = \frac{16}{16}$ |  | $P_{0,49} = \frac{16}{16}$ |  | $P_{0,50} = \frac{16}{16}$ |  | $P_{0,51} = \frac{16}{16}$ |  | $P_{0,52} = \frac{16}{16}$ |  | $P_{0,53} = \frac{16}{16}$ |  | $P_{0,54} = \frac{16}{16}$ |  | $P_{0,55} = \frac{16}{16}$ |  | $P_{0,56} = \frac{16}{16}$ |  | $P_{0,57} = \frac{16}{16}$ |  | $P_{0,58} = \frac{16}{16}$ |  | $P_{0,59} = \frac{16}{16}$ |  | $P_{0,60} = \frac{16}{16}$ |  | $P_{0,61} = \frac{16}{16}$ |  | $P_{0,62} = \frac{16}{16}$ |  | $P_{0,63} = \frac{16}{16}$ |  | $P_{0,64} = \frac{16}{16}$ |  | $P_{0,65} = \frac{16}{16}$ |  | $P_{0,66} = \frac{16}{16}$ |  | $P_{0,67} = \frac{16}{16}$ |  | $P_{0,68} = \frac{16}{16}$ |  | $P_{0,69} = \frac{16}{16}$ |  | $P_{0,70} = \frac{16}{16}$ |  | $P_{0,71} = \frac{16}{16}$ |  | $P_{0,72} = \frac{16}{16}$ |  | $P_{0,73} = \frac{16}{16}$ |  | $P_{0,74} = \frac{16}{16}$ |  | $P_{0,75} = \frac{16}{16}$ |  | $P_{0,76} = \frac{16}{16}$ |  | $P_{0,77} = \frac{16}{16}$ |  | $P_{0,78} = \frac{16}{16}$ |  | $P_{0,79} = \frac{16}{16}$ |  | $P_{0,80} = \frac{16}{16}$ |  | $P_{0,81} = \frac{16}{16}$ |  | $P_{0,82} = \frac{16}{16}$ |  | $P_{0,83} = \frac{16}{16}$ |  | $P_{0,84} = \frac{16}{16}$ |  | $P_{0,85} = \frac{16}{16}$ |  | $P_{0,86} = \frac{16}{16}$ |  | $P_{0,87} = \frac{16}{16}$ |  | $P_{0,88} = \frac{16}{16}$ |  | $P_{0,89} = \frac{16}{16}$ |  | $P_{0,90} = \frac{16}{16}$ |  | $P_{0,91} = \frac{16}{16}$ |  | $P_{0,92} = \frac{16}{16}$ |  | $P_{0,93} = \frac{16}{16}$ |  | $P_{0,94} = \frac{16}{16}$ |  | $P_{0,95} = \frac{16}{16}$ |  | $P_{0,96}$ |
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$$t = N\mathbf{1}$$
[illegible]

Finally, we see that  $t_0 = \boxed{\frac{213}{29} \approx 7.345}$