

Rajeev Atla

~~Settlers of Catan~~ A board game is played on a hexagonal grid of 19 tiles. A 'traveler' token starts on the center tile. Each turn a die is rolled to determine what neighboring tile the traveler moves to (all six directions equally likely). The turn that the traveler leaves the board, the game ends. What is the expected number of turns of the game?

A diagram showing a hexagonal grid of 19 cells, numbered 0 to 18, arranged in a honeycomb pattern. Each cell contains a red dot and a number. The cells are surrounded by 21 green dots, numbered 19 to 39, arranged in a larger hexagonal pattern around the central cluster.

$$\mathbb{E}(N) = \sum N \mathbb{P}(N)$$

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$$N = \begin{array}{c} \begin{array}{c} P_{0,0} = \frac{45}{16} \\ P_{0,1} = \frac{34506}{107144} \\ P_{0,2} = \frac{116568}{345066} \\ P_{0,3} = \frac{249996}{1071440} \\ P_{0,4} = \frac{479996}{1607160} \\ P_{0,5} = \frac{699996}{2142880} \\ P_{0,6} = \frac{919996}{2678600} \\ P_{0,7} = \frac{1139996}{3214320} \\ P_{0,8} = \frac{1359996}{3750040} \\ P_{0,9} = \frac{1579996}{4285760} \\ P_{0,10} = \frac{1799996}{4821480} \\ P_{0,11} = \frac{2019996}{5357200} \\ P_{0,12} = \frac{2239996}{5892920} \\ P_{0,13} = \frac{2459996}{6428640} \\ P_{0,14} = \frac{2679996}{6964360} \\ P_{0,15} = \frac{2899996}{7500080} \\ P_{0,16} = \frac{3119996}{8035800} \\ P_{0,17} = \frac{3339996}{8571520} \\ P_{0,18} = \frac{3559996}{9107240} \\ P_{0,19} = \frac{3779996}{9642960} \\ P_{0,20} = \frac{3999996}{10178680} \\ P_{0,21} = \frac{4219996}{10714400} \\ P_{0,22} = \frac{4439996}{11250120} \\ P_{0,23} = \frac{4659996}{11785840} \\ P_{0,24} = \frac{4879996}{12321560} \\ P_{0,25} = \frac{5099996}{12857280} \\ P_{0,26} = \frac{5319996}{13393000} \\ P_{0,27} = \frac{5539996}{13928720} \\ P_{0,28} = \frac{5759996}{14464440} \\ P_{0,29} = \frac{5979996}{15000160} \\ P_{0,30} = \frac{6199996}{15535880} \\ P_{0,31} = \frac{6419996}{16071600} \\ P_{0,32} = \frac{6639996}{16607320} \\ P_{0,33} = \frac{6859996}{17143040} \\ P_{0,34} = \frac{7079996}{17678760} \\ P_{0,35} = \frac{7299996}{18214480} \\ P_{0,36} = \frac{7519996}{18750200} \\ P_{0,37} = \frac{7739996}{19285920} \\ P_{0,38} = \frac{7959996}{19821640} \\ P_{0,39} = \frac{8179996}{20357360} \\ P_{0,40} = \frac{8399996}{20893080} \\ P_{0,41} = \frac{8619996}{21428800} \\ P_{0,42} = \frac{8839996}{21964520} \\ P_{0,43} = \frac{9059996}{22500240} \\ P_{0,44} = \frac{9279996}{23035960} \\ P_{0,45} = \frac{9499996}{23571680} \\ P_{0,46} = \frac{9719996}{24107400} \\ P_{0,47} = \frac{9939996}{24643120} \\ P_{0,48} = \frac{10159996}{25178840} \\ P_{0,49} = \frac{10379996}{25714560} \\ P_{0,50} = \frac{10599996}{26250280} \\ P_{0,51} = \frac{10819996}{26786000} \\ P_{0,52} = \frac{11039996}{27321720} \\ P_{0,53} = \frac{11259996}{27857440} \\ P_{0,54} = \frac{11479996}{28393160} \\ P_{0,55} = \frac{11699996}{28928880} \\ P_{0,56} = \frac{11919996}{29464600} \\ P_{0,57} = \frac{12139996}{29999320} \\ P_{0,58} = \frac{12359996}{30535040} \\ P_{0,59} = \frac{12579996}{31070760} \\ P_{0,60} = \frac{12799996}{31606480} \\ P_{0,61} = \frac{13019996}{32142200} \\ P_{0,62} = \frac{13239996}{32677920} \\ P_{0,63} = \frac{13459996}{33213640} \\ P_{0,64} = \frac{13679996}{33749360} \\ P_{0,65} = \frac{13899996}{34285080} \\ P_{0,66} = \frac{14119996}{34820800} \\ P_{0,67} = \frac{14339996}{35356520} \\ P_{0,68} = \frac{14559996}{35892240} \\ P_{0,69} = \frac{14779996}{36427960} \\ P_{0,70} = \frac{14999996}{36963680} \\ P_{0,71} = \frac{15219996}{37499400} \\ P_{0,72} = \frac{15439996}{38035120} \\ P_{0,73} = \frac{15659996}{38570840} \\ P_{0,74} = \frac{15879996}{39106560} \\ P_{0,75} = \frac{16099996}{39642280} \\ P_{0,76} = \frac{16319996}{40178000} \\ P_{0,77} = \frac{16539996}{40713720} \\ P_{0,78} = \frac{16759996}{41249440} \\ P_{0,79} = \frac{16979996}{41785160} \\ P_{0,80} = \frac{17199996}{42320880} \\ P_{0,81} = \frac{17419996}{42856600} \\ P_{0,82} = \frac{17639996}{43392320} \\ P_{0,83} = \frac{17859996}{43928040} \\ P_{0,84} = \frac{18079996}{44463760} \\ P_{0,85} = \frac{18299996}{45000000} \\ P_{0,86} = \frac{18519996}{45535720} \\ P_{0,87} = \frac{18739996}{46071440} \\ P_{0,88} = \frac{18959996}{46607160} \\ P_{0,89} = \frac{19179996}{47142880} \\ P_{0,90} = \frac{19399996}{47678600} \\ P_{0,91} = \frac{19619996}{48214320} \\ P_{0,92} = \frac{19839996}{48750040} \\ P_{0,93} = \frac{20059996}{49285760} \\ P_{0,94} = \frac{20279996}{49821480} \\ P_{0,95} = \frac{20499996}{50357200} \\ P_{0,96} = \frac{20719996}{50892920} \\ P_{0,97} = \frac{20939996}{51428640} \\ P_{0,98} = \frac{21159996}{51964360} \\ P_{0,99} = \frac{21379996}{52500080} \\ P_{0,100} = \frac{21599996}{53035800} \\ P_{0,101} = \frac{21819996}{53571520} \\ P_{0,102} = \frac{22039996}{54107240} \\ P_{0,103} = \frac{22259996}{54642960} \\ P_{0,104} = \frac{22479996}{55178680} \\ P_{0,105} = \frac{22699996}{55714400} \\ P_{0,106} = \frac{22919996}{56250120} \\ P_{0,107} = \frac{23139996}{56785840} \\ P_{0,108} = \frac{23359996}{57321560} \\ P_{0,109} = \frac{23579996}{57857280} \\ P_{0,110} = \frac{23799996}{58393000} \\ P_{0,111} = \frac{24019996}{58928720} \\ P_{0,112} = \frac{24239996}{59464440} \\ P_{0,113} = \frac{24459996}{59999320} \\ P_{0,114} = \frac{24679996}{60535040} \\ P_{0,115} = \frac{24899996}{61070760} \\ P_{0,116} = \frac{25119996}{61606480} \\ P_{0,117} = \frac{25339996}{62142200} \\ P_{0,118} = \frac{25559996}{62677920} \\ P_{0,119} = \frac{25779996}{63213640} \\ P_{0,120} = \frac{25999996}{63749360} \\ P_{0,121} = \frac{26219996}{64285080} \\ P_{0,122} = \frac{26439996}{64820800} \\ P_{0,123} = \frac{26659996}{65356520} \\ P_{0,124} = \frac{26879996}{65892240} \\ P_{0,125} = \frac{27099996}{66427960} \\ P_{0,126} = \frac{27319996}{66963680} \\ P_{0,127} = \frac{27539996}{67499400} \\ P_{0,128} = \frac{27759996}{68035120} \\ P_{0,129} = \frac{27979996}{68570840} \\ P_{0,130} = \frac{28199996}{69106560} \\ P_{0,131} = \frac{28419996}{69642280} \\ P_{0,132} = \frac{28639996}{70178000} \\ P_{0,133} = \frac{28859996}{70713720} \\ P_{0,134} = \frac{29079996}{71249440} \\ P_{0,135} = \frac{29299996}{71785160} \\ P_{0,136} = \frac{29519996}{7232$$

$$t \equiv N\mathbf{1}$$
[illegible]

Finally, we see that $t_0 = \boxed{\frac{213}{29} \approx 7.345}$