Solving nonlinear differential equations with differentiable quantum circuits

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Phys. Rev. A **103**, 052416 (2021)

Notebook: Óscar Amaro, September 2023

Figure 1

```
(* since the target function is even,
the Chebyshev approximation should also have this symmetry*)
ChebyshevApprox[n_Integer?Positive, f_Function, x_] :=
  Module[{c, xk}, xk = Pi (Range[n] - 1 / 2) / n;
   c[j_] = 2 * Total[Cos[j * xk] * (f /@ Cos[xk])] / n;
   Total[Table[c[k] * ChebyshevT[k, x], {k, 0, n - 1}]] - c[0] / 2];
f = Exp[-1/(25 #^2)] Cos[20 #] &
```

Out[6]= $\mathrm{Exp}\left[-\frac{1}{25\,\#1^2}\right] \mathrm{Cos}[20\,\#1]\ \&$

```
Clear[x]
f2 = ChebyshevApprox[2, f, x] // N;
f16 = ChebyshevApprox[16, f, x] // N;
f64 = ChebyshevApprox[64, f, x] // N // Simplify;
(* some numerical instability,
works better but slower if Simplify is applied before N *)
```

In[50]:=
```
Plot[{f[x], f2, f16, f64}, {x, -0.8, 0.8}, Frame → True, Axes → False,
 PlotRange → {-2, 2}, PlotLegends → {"y(x)", "n=2", "n=16", "n=64"}, PlotStyle →
  {{LightBlue, Thickness[0.03]}, {Dashed, Red}, {Dashed, Pink}, {Dashed, Blue}}]
```

Out[50]=