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Report 1: Benchmarks in Numerical Methods
for single particle trajectories

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In this text the most important results of my work at GoLP to this date are presented. This includes *benchmarking* well-known results from classical radiation theory through simple codes written in *Octave*. The scenarios considered for the study are synchrotron motion of a charged particle (part 1) and interaction with a laser plane wave (part 2).

Contents

1 Introduction

The internship's initial stages were the following:

1. Individual particle and counter propagating laser: first harmonics as functions of a_0 and w_0, with $a_0 w_0 = c^{te}$.

2. Can individual particles and plasmas in magnetic bottles be used as radiation sources?

3. What are the radiation spectra of individual particles and plasma in magnetic dipoles and how can they be deduced by the plasma properties.

Not all parts were complete at the time of the writing of these notes.
In simulation, the following normalization is made

$$\begin{array}{ll} \omega' = \omega/\omega_p & \mathbf{p}' = p/(m_e c) \\ t' = \omega_p t & \mathbf{A}' = e\mathbf{A}/(m_e c^2) \\ \mathbf{x}' = k_p \mathbf{x} & \mathbf{E}' = e\mathbf{E}/(m_e \omega_p c) \\ \mathbf{v}' = \mathbf{v}/c & \mathbf{B}' = e\mathbf{B}/(m_e \omega_p c) \end{array}$$

where e is the absolute electron charge, m_e is the electron mass, ω_p is a reference frequency and k_p is the associated wave vector. Once this ω_p is chosen, all mentioned physical variables can be computed through these relations. In plots, the variables should appear with labels $x[c/\omega_p]$, $t[1/\omega_p]$ and $u[c]$, although many times the frequency will appear as ω_0, which should not be confused with the laser frequency. Hopefully context will help to solve this ambiguity in notation. A very important variable in the second part of this text is the normalized laser intensity a_0, which also appears under the notation a_L. In general, analytical/theoretical results will appear in black continuous line, while simulation results will appear in color.

2 Synchrotron

The studied configuration is that of a particle's motion in a constant magnetic field. Here, the frequency of motion is $\omega_{synch} = qB/m\gamma$, which means that in general the fundamental frequency is not the reference frequency. For comparison with other configurations, it is more practical if the reference frequency is held constant. Through some approximations we can express the intensity distribution with the help of modified Bessel functions of the second kind. In the case of pure circular motion, we have $\rho = c/\omega_1$ (cf. (1)).
The intensity distribution with which simulation will be compared to is

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c}\left(\frac{\omega}{\omega_1 \gamma}\right)^2 (1+(\gamma\theta)^2)\left[K^2_{2/3}(\xi) + \frac{(\gamma\theta)^2}{1+(\gamma\theta)^2}K^2_{1/3}(\xi)\right]$$

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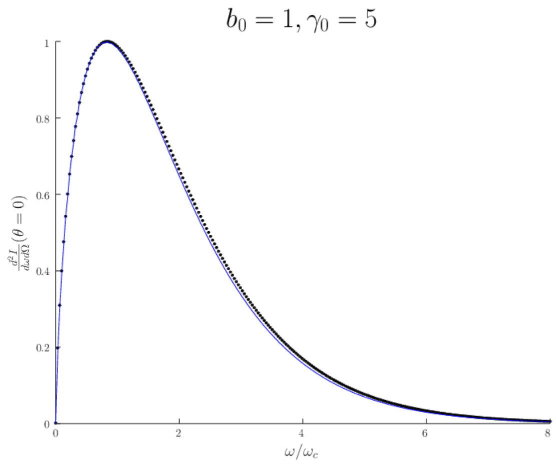
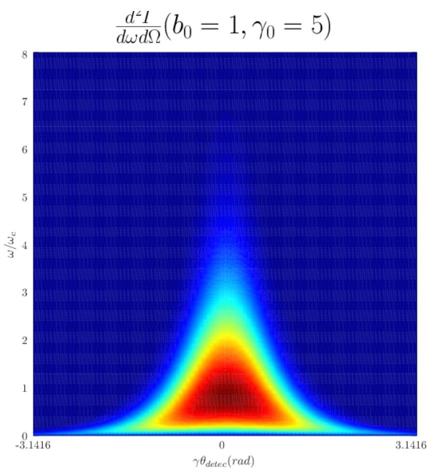
where $\xi = ((\omega/\omega_p)/3\gamma^3)(1+(\gamma\theta)^2)^{3/2}$. This means that the critical frequency is $\omega_c = 1.5\gamma^3\omega_0$. Thus, the horizontal axis will be plotted in units of $\omega/\omega_c = (1/1.5\gamma^3)(\omega/\omega_p)$.



For a more complete view of the intensity distribution we extend the range of observation angles. This is shown in the following plot.



Angular distribution of radiation emission

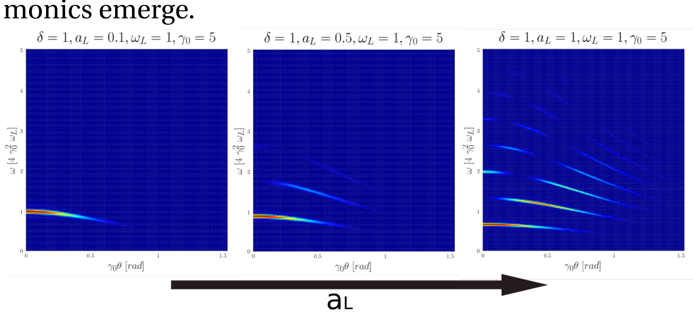
As can be seen from the plot, most power is emitted in an angular range of $[-1/\gamma, 1/\gamma]$.

3 Laser Plane Wave

In this section we study numerically the radiation emitted by an electron interacting with a counter-propagating laser plane wave. We essentially follow the paper (3), but also the book (2). First we describe the matching between harmonic distribution and later we test the hypothesis from classical theory that power should only depend on $\chi \equiv a_0\omega_0$.

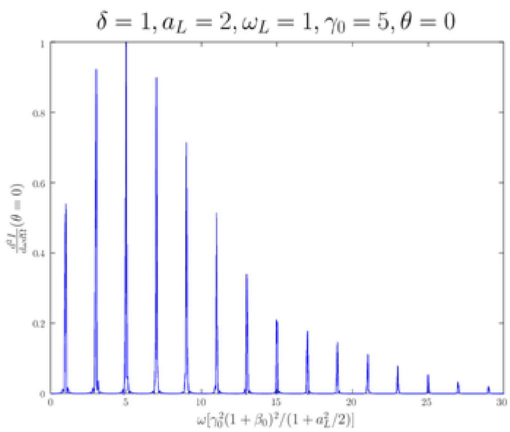
A. Linear polarization

Number of harmonics. The number of harmonics is a monotonic increasing function of a_0. In the first case, only one harmonic is visible. As a_0 increases more harmonics emerge.

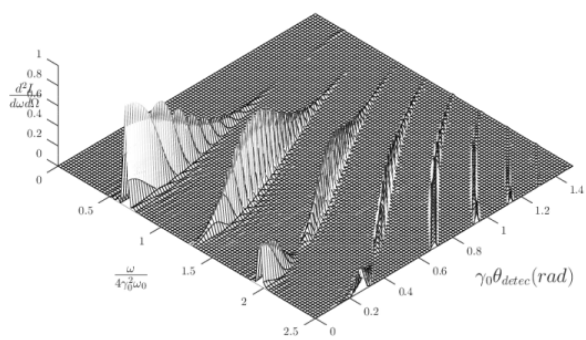


A1. Distribution of harmonics

In linearly polarized only odd harmonics are visible on axis. Plotting the spectrum in units of $\gamma_0^2(1+\beta_0)^2/(1+a_0^2/2)$, the harmonics are located exactly on odd numbers.

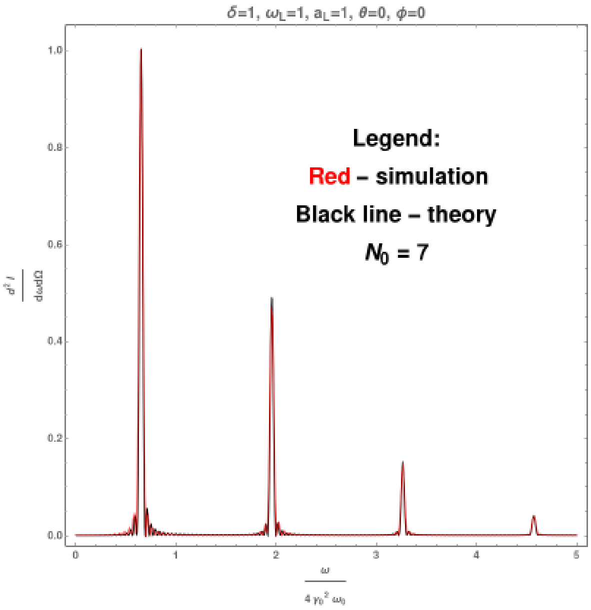


3d View. Parameters $a_0 = 1, \omega_0 = 1$. At some angles some harmonics might not be visible (take for example $\theta \sim 0.3$ for the third harmonic).

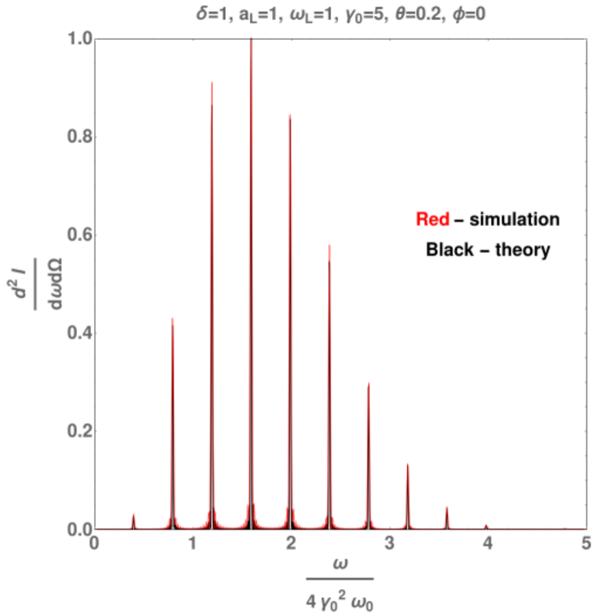


A3. Direct comparison

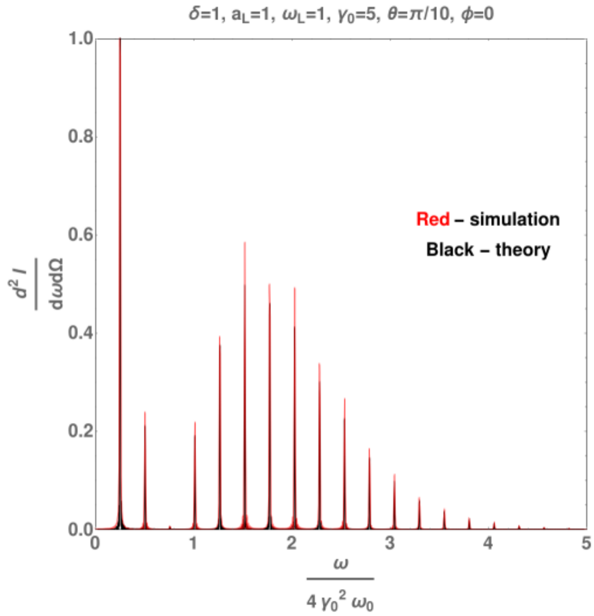
Spectra for different observation angles. In linear polarization, the odd harmonic are visible on axis (but not even). Here is a comparison between theory and simulation. In the following plot, both spectra were normalized by their respective maxima.



Off axis for an angle of $\theta = 0.2$, we have



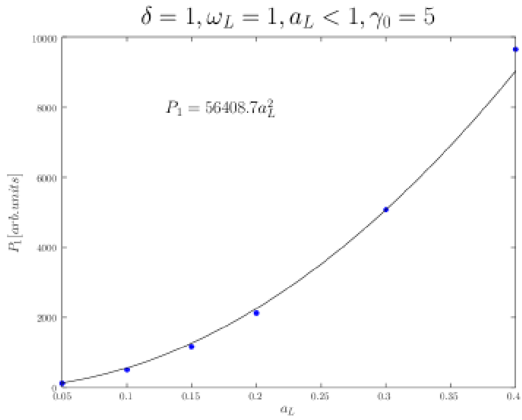
in good agreement with theory. However, some peaks show different amplitudes than expected. This should be due to precision in "theory" (cf. equations (36) to (38) of (3), in particular the parameters n and m in the summations) and in simulation (only $d\omega$, smaller dt didn't increase the resolution significantly). Similarly, for $\theta = \pi/10$, we have



with less agreement than in the previous plot, but still retains the overall relation between peak amplitude.

A2. Power dependence on χ

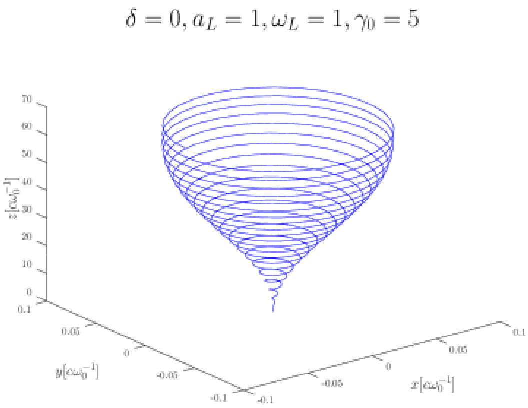
Finally, we test whether power follows a_0^2 for low values of laser intensity in linear polarization. This is achieved in simulation by approximating the integral $P = \int\int dP/d\omega d\Omega \sim \sum_\theta \sum_\omega d^2I/d\Omega d\theta d\omega$. Of course, as a_0 approaches 1, the higher harmonics also contribute, so we expect a deviation from the quadratic rule, as when $a_0 = 0.4$. The curve plotted below comes from manually fitting a quadratic curve.



B. Circular polarization

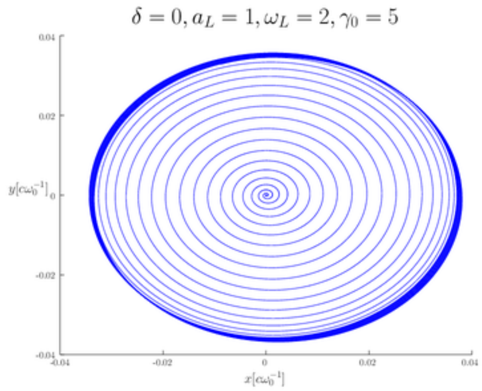
B1. Trajectory

A charged particle will describe a helical trajectory in a laser with circular polarization. In this example we show such a motion but with an envelope function modeling the laser pulse.

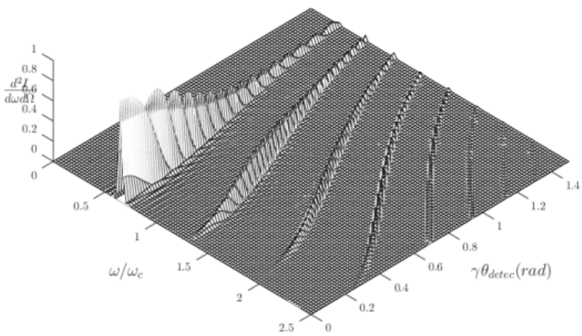


B1. Numerics

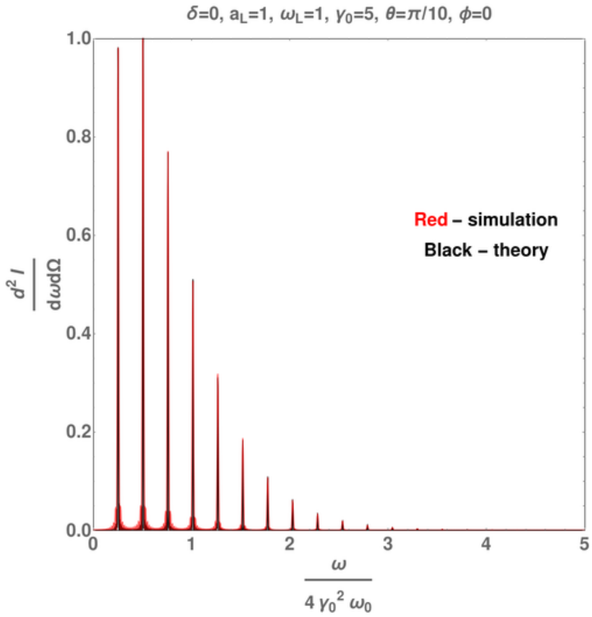
Attention, the number of periods for which the envelope function is on must be enough so that no drift motion occurs. For example, in the next plot one can see some darker regions than others, which indicate this kind of drift.



3d View. Parameters $a_0 = 1, \omega_0 = 1$. No harmonics are visible on axis besides the fundamental.



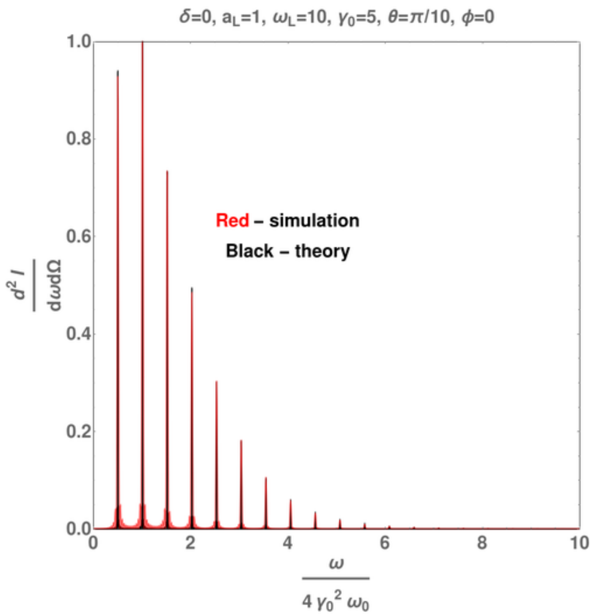
Spectra for different observation angles. The spectra (simulation and theoretical) normalized to the maximum peak are



which agree quite well.

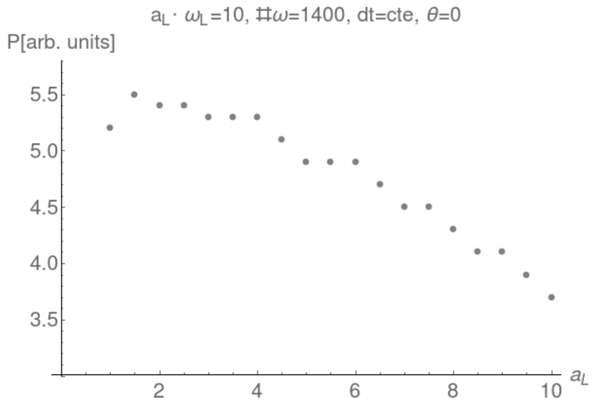
B3. Scaling with ω_0

Distributions off axis follow the synchrotron-type curve for maxima of peaks. An increase of C in laser frequency shifts all harmonics $\omega_n \to C\,\omega_n$.

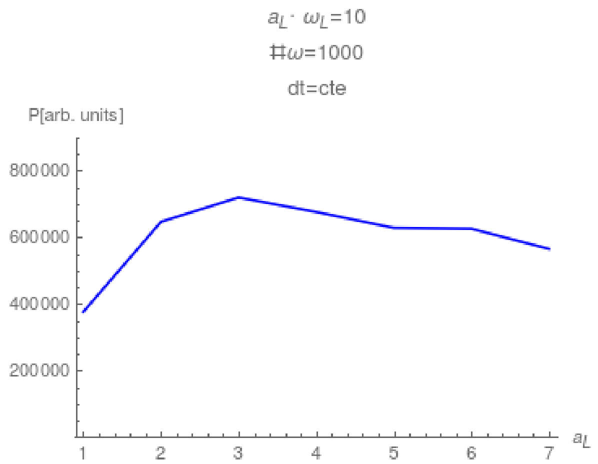


B2. Power dependence on χ

Total emitted power invariance on axis. We expect the total power to be invariant for $a_0\omega_0 = c^{te}$ in the classical regime. In this parameter scan, we chose $a_0\omega_0 = 10$. We fixed the dimension of the frequency array so as to have a reference. Because peaks get sharper as a_0 increases, one should use better resolved frequencies.



As one could expected, the emitted power on axis decreases as a_0 increases, which could already be seen in the first plot of this text. More power is off of axis, so in order to test our hypothesis we need to include several values of angle. Integrating in frequency and angle (not forgetting that the detector is spherical, so no factor of $sin\theta$ is needed for the calculation), we get.



Numerical precision might not have been the best suited, but one can see that the total power does not change significantly provided the constraint $\chi = c^{te}$ is followed.

4 Conclusions

The development of the code that supports these results let to the creation of some very simple yet important tools for trajectory and radiation analysis for single particle configurations.

References

[1] Jackson, J.D., *Classical Electrodynamics*, Wiley, New York, 3rd edition, 1999

[2] Gibbon, P., *Short pulse laser interactions with matter: An introduction.*, Imperial College Press, London, 2005

[3] Esarey, E., Sprangle, P. Ride, S.K., *Nonlinear Thomson Scattering of Intense Laser Pulses from Beams and Plasmas*, Naval Research Laboratory, 1993