

General parameter-shift rules for quantum gradients

Original authors of paper: David Wierichs, Josh Izaac, Cody Wang, and Cedric Yen-Yu Lin

Paper Quantum-journal (2022): <https://quantum-journal.org/papers/q-2022-03-30-677/#>

e-print: arXiv:2107.12390v3

Notebook: Óscar Amaro, September 2023 @ GoLP-EPP

Introduction

In this notebook we reproduce some results from the paper.

Official repo from paper from the original authors: <https://github.com/dwierichs/General-Parameter-Shift-Rules>

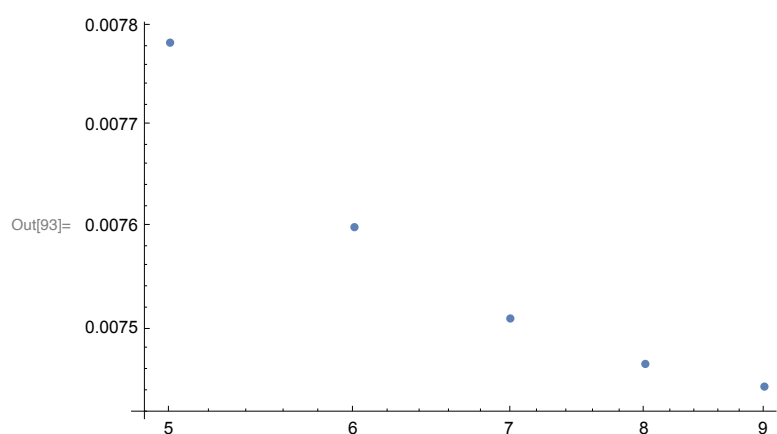
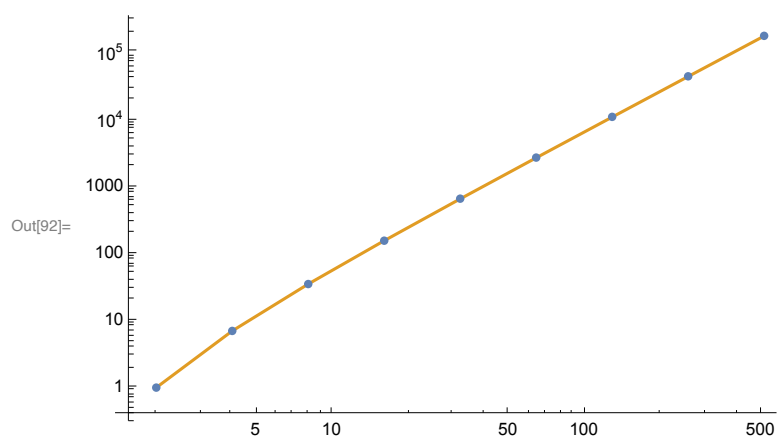
$$\text{Proof } \sum_{k=1}^{n-1} \tan^2 \left(\frac{k\pi}{2n} \right) = \frac{(n-1)(2n-1)}{3}$$

```
In[87]:= Clear[k, n, getSum, tab, tab2, nmax]

getSum[n_] := NSum[Tan[ $\frac{k\pi}{2n}$ ]^2, {k, 1, n-1}] // Quiet

nmax = 9;
tab = ParallelTable[{2^n, getSum[2^n]}, {n, 1, nmax, 1}];
tab2 = ParallelTable[{2^n,  $\frac{(2^n - 1)(2 \times 2^n - 1)}{3}$ }, {n, 1, nmax, 1}];

ListLogLogPlot[{tab, tab2}, Joined -> {False, True}]
(* there is some numerical error for larger n, but it is negligible *)
ListLogLogPlot[ $\left( \frac{\text{tab2} - \text{tab}}{\text{tab2}} \right)$ [[All, 2]] // Chop // Abs]
```



Appendix A

```
In[*]:= Clear[R, x, l]
```

```
(* R needs to be integer > 1*)
```

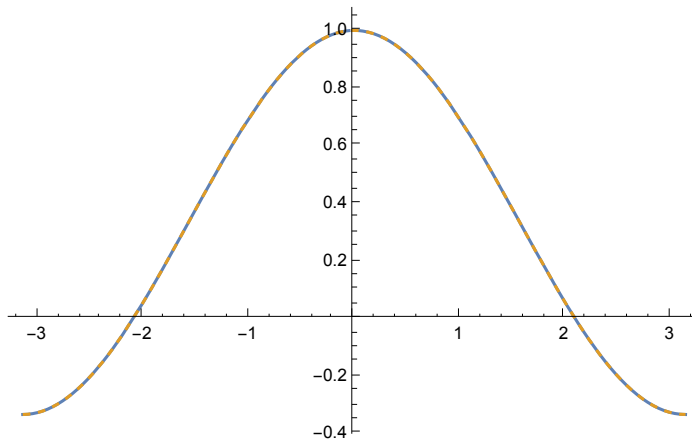
```
R = 1;
```

```
Plot[ {  $\frac{\sin\left[\frac{2R+1}{2}x\right]}{(2R+1)\sin\left[\frac{1}{2}x\right]}$ ,  $\frac{1}{2R+1} + \frac{2}{2R+1} \text{NSum}[\text{Cos}[l x], \{l, 1, R\}]$  },  
 {x, - $\pi$ ,  $\pi$ }, PlotPoints  $\rightarrow$  3, PlotStyle  $\rightarrow$  {Default, Dashed}, PlotRange  $\rightarrow$  All]
```

```
R = 10;
```

```
Plot[ {  $\frac{\sin\left[\frac{2R+1}{2}x\right]}{(2R+1)\sin\left[\frac{1}{2}x\right]}$ ,  $\frac{1}{2R+1} + \frac{2}{2R+1} \text{NSum}[\text{Cos}[l x], \{l, 1, R\}]$  },  
 {x, - $\pi$ ,  $\pi$ }, PlotPoints  $\rightarrow$  3, PlotStyle  $\rightarrow$  {Default, Dashed}, PlotRange  $\rightarrow$  All]
```

```
Out[*]:=
```



```
Out[*]:=
```

