Robust control design by scenario optimization

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DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

STOCHASTIC OPERATIONS RESEARCH

Acknowledge Dr Luis G. Crespo, Dr Sean P. Kenny for the kind support

B. Wie and D. S. Bernstein, "*Benchmark Problems for Robust Control Design*," in 1991 American Control Conference, 1991

Problem statement

Given a numerical model of a dynamic system $\mathrm{y} = M_y(d, \delta)$, a reliability model $\mathrm{g} = M_g(d, \delta)$ and a set of N of experimental observations

$$\{\delta_i\}_{i=1,..,N}$$

We want to find a reliable design $d^\star$ that maximizes the probability of satisfactory performance (reliability) while avoids catastrophic failures (robustness against worst-case scenarios)

In contrast with d, the random factors δ are inherently variable and non-tunable.

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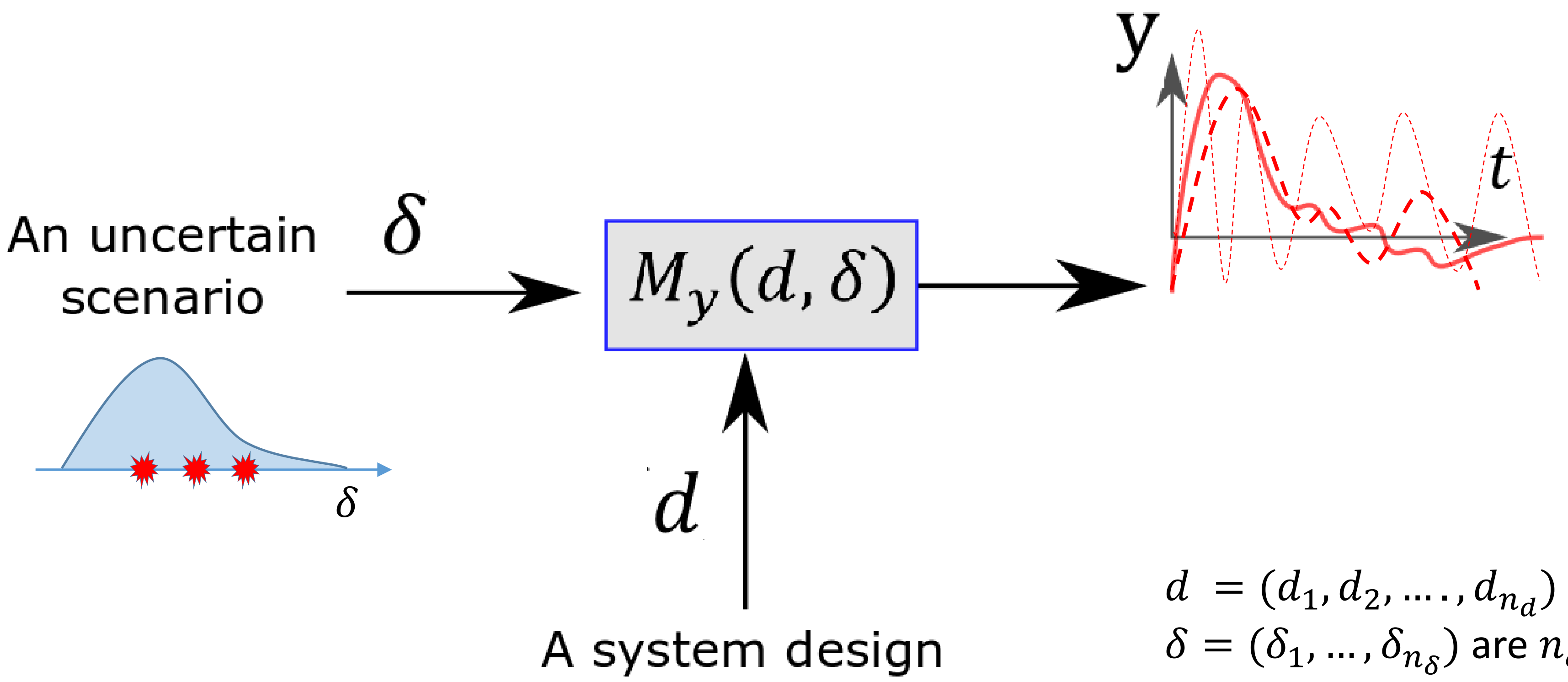

Different uncertain scenarios affect the system's response

$d = (d_1, d_2, \dots, d_{n_d})$ are n_d design parameters,
$\delta = (\delta_1, \dots, \delta_{n_\delta})$ are n_δ uncertain factors
$y = (y_1, \dots, y_T)$ response at time $t = 1, 2, \dots, T$.

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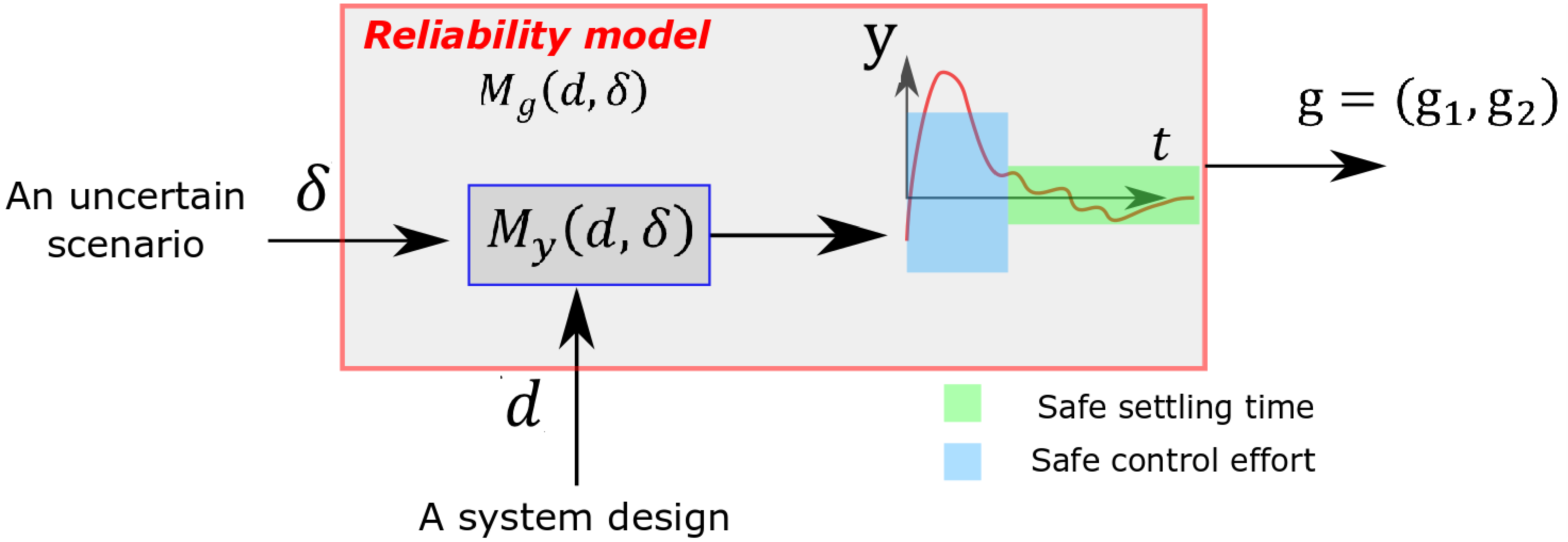

$g = (g_1, \ldots, g_{n_g})$ is a vector of performance scores for n_g reliability requirements.

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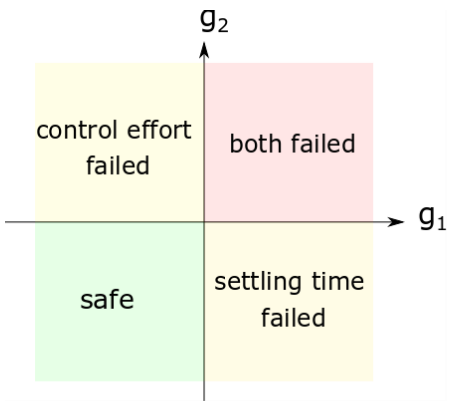

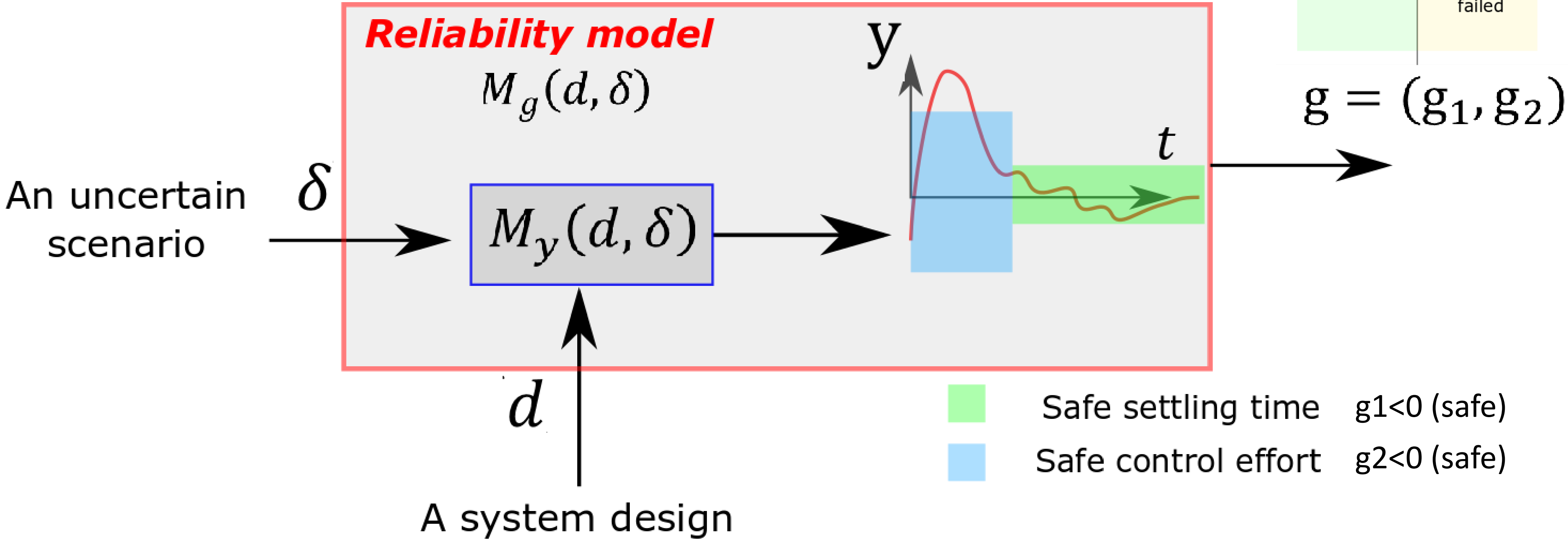

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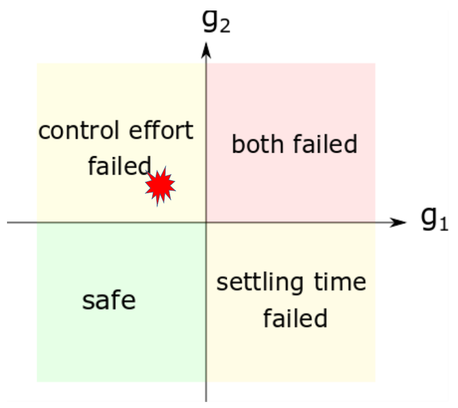

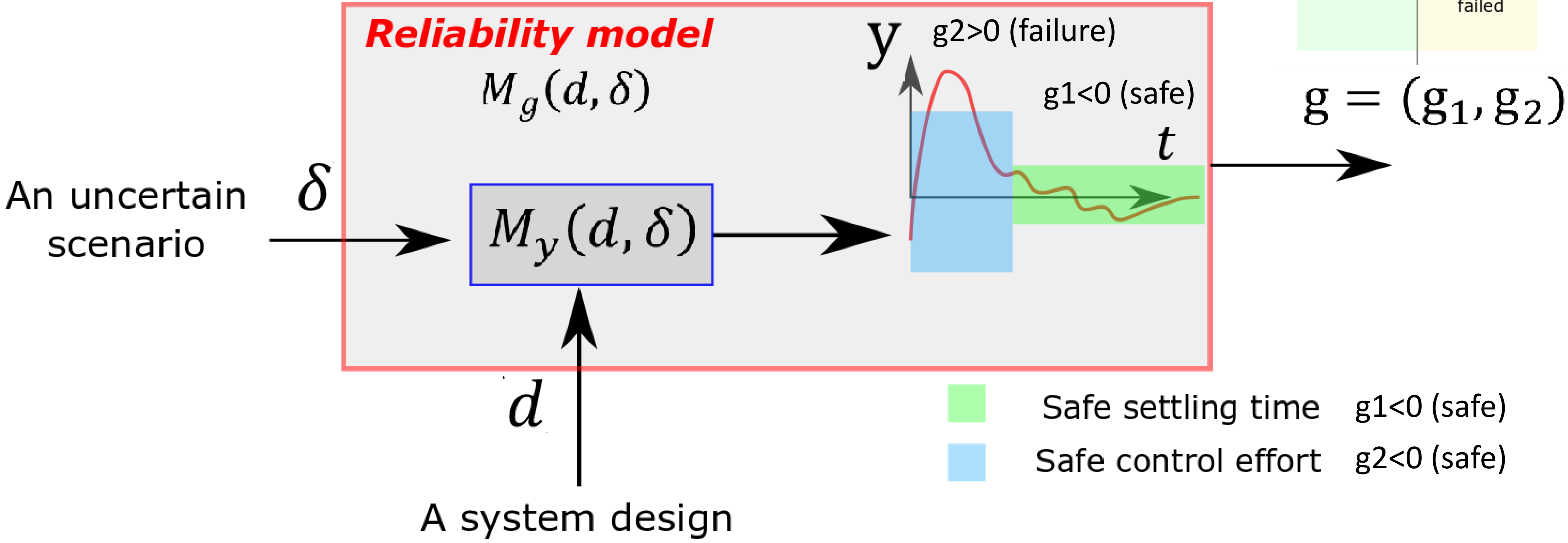

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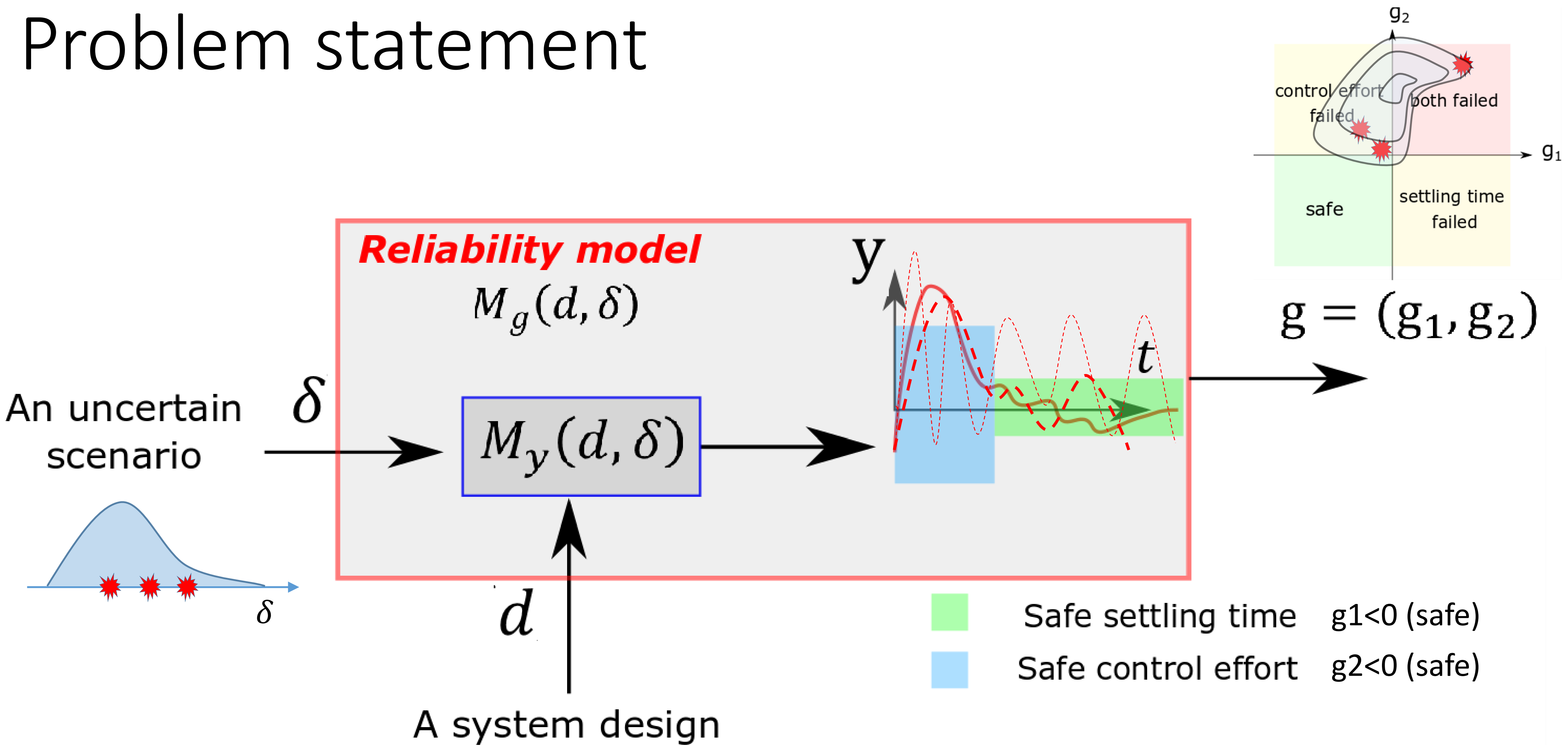

Ideally, an optimal design $d^{\star}$ pushes the realization of g in the safe region, (and far away from severe failure)

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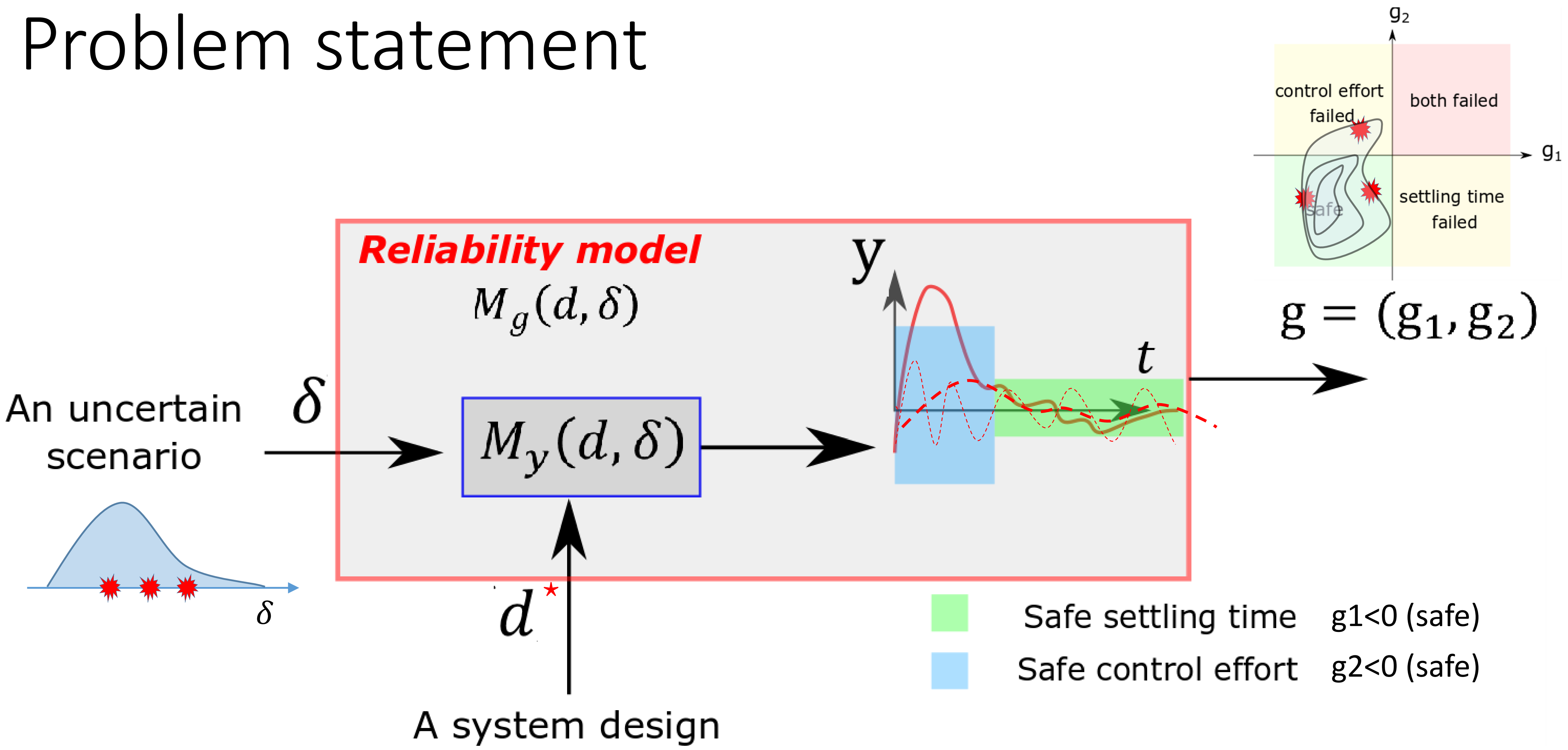

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Robustness &Reliability Metrics

(to compare different designs)

Worst-case performance score:

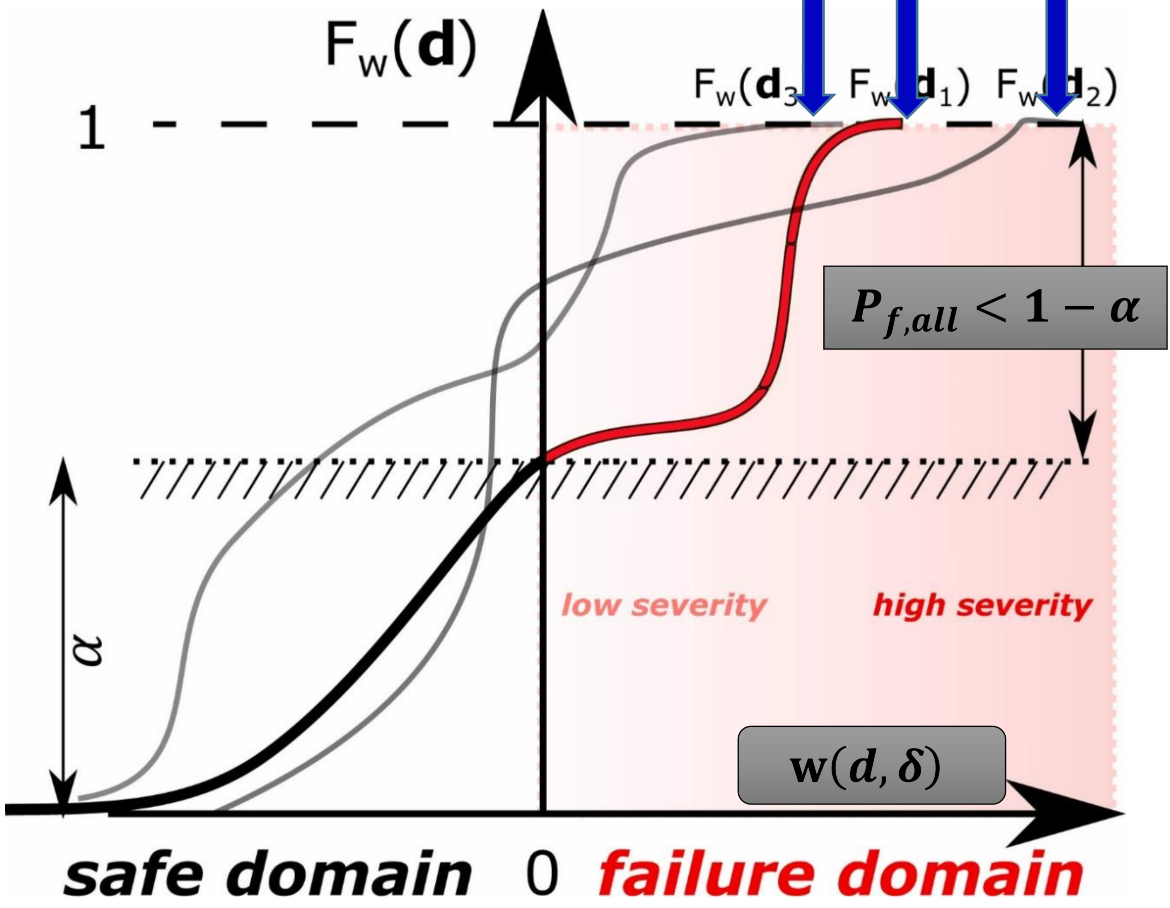
$$\mathrm{w}(d,\delta) = \max_{i=1,..,n_g} g_i\,(d,\delta);$$

Worst-case scenario score:

$$\bar{g}_i(d) = \max_{k=1,..,N} g_i\,(d,\delta_k)$$

$$\bar{\boldsymbol{w}}(\boldsymbol{d}) = \max_{k=1,..,N} \boldsymbol{w}\,(\boldsymbol{d},\boldsymbol{\delta}_k)$$

Empirical cumulative distribution function



Note: *$\mathbb{P}[\cdot]$ and $\mathbb{E}[\cdot]$ require numerical integration, e.g., via sampling-based methods (but we only have a few samples here)*

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$$\bar{w}(d) = \max_{k=1,..,N} w\,(d,\delta_k)$$

Failure probability for individual and all requirements:

$$P_{f,i}(d) = \mathbb{P}[g_i(d,\delta) > 0]$$

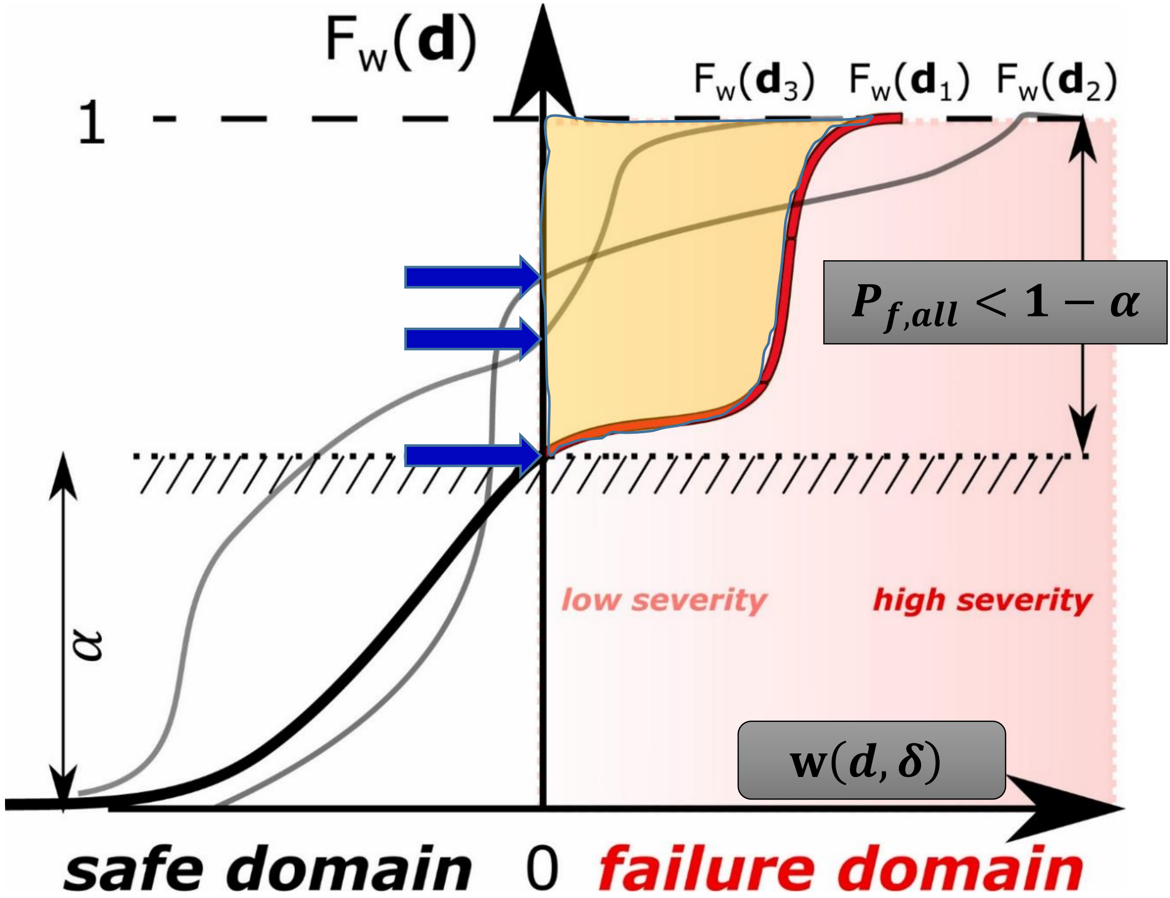
$$\boldsymbol{P_{f,all}(d) = \mathbb{P}[w(d,\delta) > 0]}$$

Severity for individual and all requirements:

$$S_i(d) = \mathbb{E}[g_i(d,\delta) \mid g_i(d,\delta) > 0]$$

$$\boldsymbol{S_{all}(d) = \mathbb{E}[w(d,\delta) \mid w(d,\delta) > 0]}$$

Empirical cumulative distribution function



Note: *$\mathbb{P}[\cdot]$ and $\mathbb{E}[\cdot]$ require numerical integration, e.g., via sampling-based methods (but we only have a few samples here)*

The challenge problem: Reliability-based design and reliability assessment of a controller

Goals:

- Optimize the controller design for a two-mass spring system from given data.
- Verify system reliability and robustness, e.g., reachability of failure domains and "severe" failure regions.

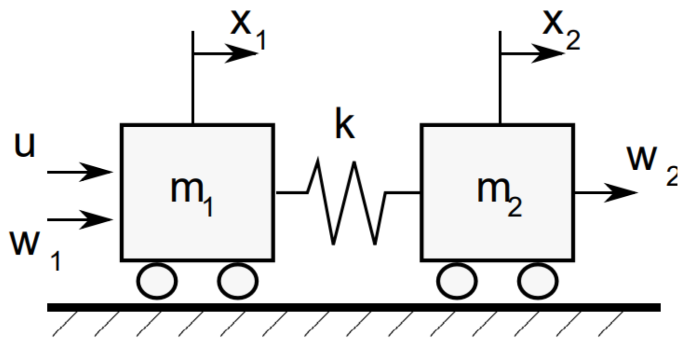


FIGURE 2: THE TWO-MASS SPRING SYSTEM BENCHMARK FOR ROBUST CONTROL DESING.

https://github.com/Roberock/ControllerRobust

The challenge problem:
Reliability and system model

Reliability model $g = M_g(d, \delta)$

g_1: Stability.
g_2: Settling time. position of the mass 2 must fall between ±0.1 from the original state after 15 seconds;
g_3: Control effort. The control signal, u, must fall between ±1.

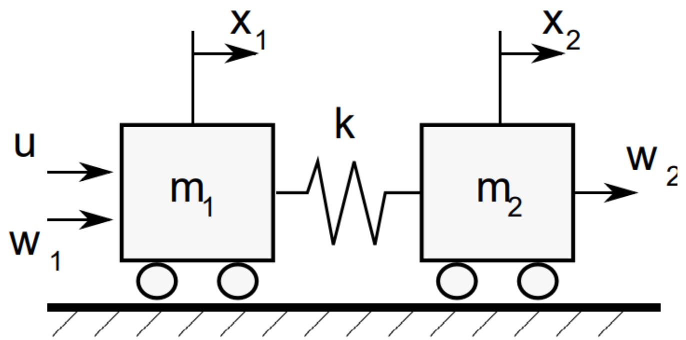


FIGURE 2: THE TWO-MASS SPRING SYSTEM BENCHMARK FOR ROBUST CONTROL DESING.

Note that the time domain requirements require simulating the time response of the system by numerical integration.

Dynamic model $M_y(d, \delta)$: $\begin{aligned} \dot{x}_c &= Ax_c + By \\ u_c &= Cx_c + Dy \end{aligned}$ $\xrightarrow[\text{transfer-function}]{\text{From canonical form to}}$ $H(s) = \dfrac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$

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The challenge problem:
Design parameters and uncertain factors

Design parameters : $d = [b_0, b_1, b_2, b_3, a_0, a_1, a_2, a_3, a_4] \in \mathbb{R}^9;$

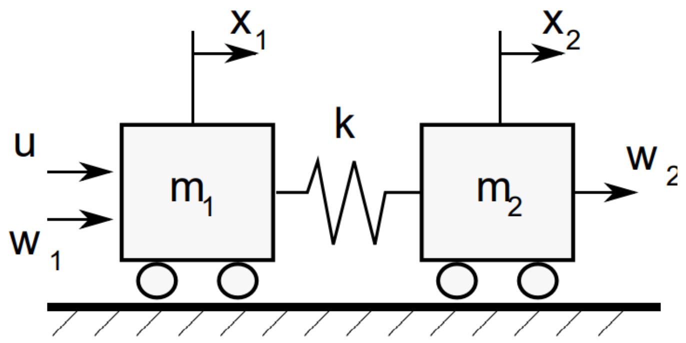
(coefficient of the system transfer function)

Uncertain parameters : $\delta = (m_1, m_2, k_l, k_n, \lambda, \tau) \in \mathbb{R}^6$;

(masses, springs, time delay, control loop lag)

The system-state matrices depend on the uncertain factor:

$$A(\delta), B(\delta), C(\delta), D(\delta).$$



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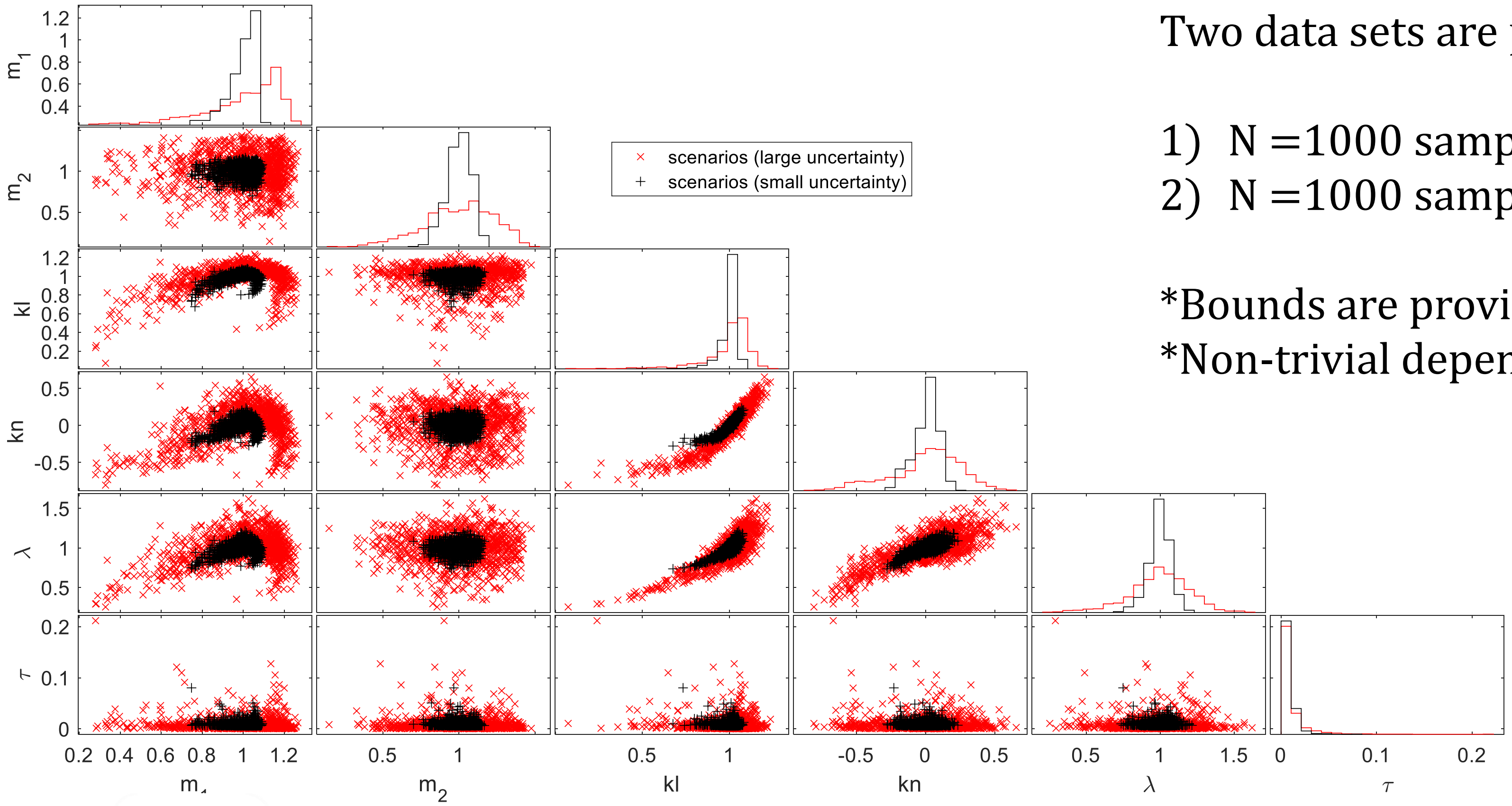
The data set: $D = \{\delta_i\}_{i=1,..,N}$



Two data sets are provided:

1) N =1000 samples, large uncertainty
2) N =1000 samples, small uncertainty

*Bounds are provided
*Non-trivial dependency structures

```
δnames=["mass1" "mass2" "kl" "kn" "λ" "τ" ]
# admissible ranges
δlims=[0.1    2;       0.1    2;
       0.05   1.75;    -1     1;
       0.2    1.8;     0.0001 0.3];
```

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Four designs from [1]:

$\boldsymbol{SP_1(D,0)}$:

Minimizes the worst-case scenario score, $\overline{w}(d)$

$\mathbf{SP_1(D,0.5)}$:

Minimizes 95th quantile of the $w(d,\delta)$ distribution

$\mathbf{SP_2(D)}$:

Minimize $P_{f,all}(d)$

Nominal:

A baseline design

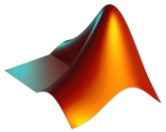
Compromise between severity and reliability optimization! Competing reliability requirements!

Designs Names

	$\mathcal{SP}_1(\mathcal{D},0)$	$\mathcal{SP}_1(\mathcal{D},0.05)$	$\mathcal{SP}_2(\mathcal{D})$	Nominal
$\hat{P}_f$	0.981	0.057	0.175	0.734
$\overline{g}_1$	-0.0272	-0.013	0.069	$7e-4$
$\overline{g}_2$	0.5925	1.576	3.054	1.974
$\overline{g}_3$	0.5925	0.193	1.406	0.202
a_4	0.2238	0.5375	0.7600	0.5503
a_3	0.6811	1.3346	1.9491	1.4175
a_2	3.1275	2.4206	3.0497	2.6531
a_1	2.3615	2.1689	2.7344	2.4802
a_0	1.1833	0.8084	1.0594	1.0000
b_3	-0.0982	2.4802	-0.0831	-0.1324
b_2	0.4702	0.6146	0.6358	0.3533
b_1	0.5886	0.5265	0.7752	0.6005
b_0	0.0777	0.0716	0.0981	0.0728

Scores

The design parameters



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[1] Rocchetta, R., Crespo, L. G., & Kenny, S. P. (2019). Solution of the benchmark control problem by scenario optimization. In Dynamic Systems and Control Conference, USA.

Design resulting from previous works [1]

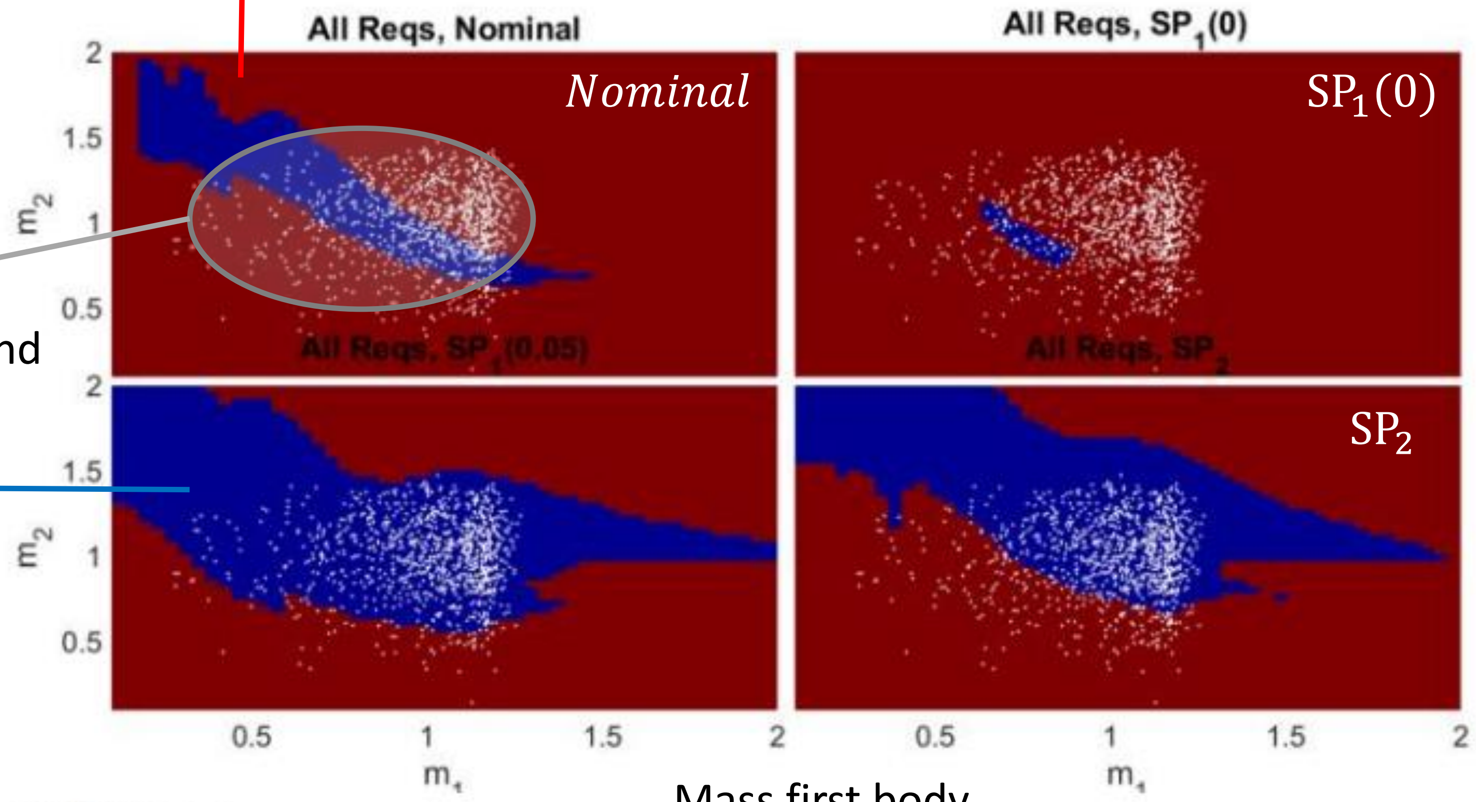
Failure domain,
$w(m_1, m_2, d_{nominal}) > 0$

Also note that w and g are non-convex functions of d and δ

Samples

Mass second body

Safe domain
$w(m_1, m_2, SP(0.05)) < 0$



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Concluding discussion on challenges a few remarks

Value of information & uncertainty characterization challenges:
How to use the available data in the best way? Shall we prescribe a model for the uncertainty?
How to keep track of the value of information in the initial data set (only 1k samples) ?
The data present some complex dependency structures. Can we account for this?

Reliability challenges:
- ***Competing reliability requirements.***
- ***Robustness and reliability (competing)***
- ***Estimation of failure probability from set-based characterization (reachability of failure sets)***

Robust vs Reliable design optimization challenges:
- ***Need to define an appropriate optimization program (minimize or constrain severity/reliability/worst-cases?)***
- ***Probabilities and expectations are step-wise discontinuous (gradient based optimizer are inapplicable).***

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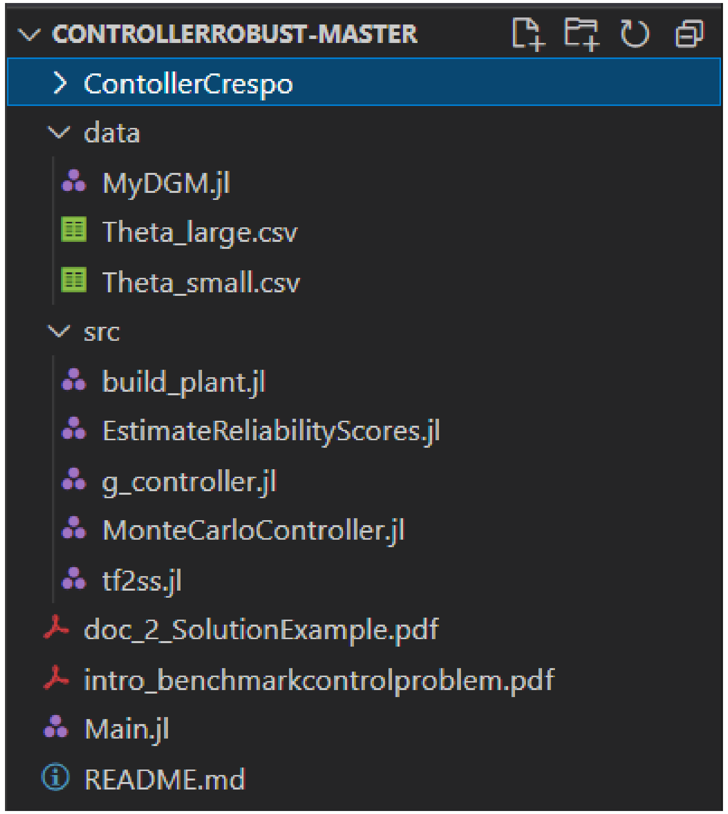
Thank you for listening!



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[2] B. Wie and D. S. Bernstein, "*Benchmark Problems for Robust Control Design*," in 1991 American Control Conference, 1991

[3] Roberto Rocchetta, Luis G. Crespo, Sean P. Kenny, “*A scenario optimization approach to reliability-based design*”, Reliability Engineering & System Safety, Volume 196, 2020, https://doi.org/10.1016/j.ress.2019.106755

[4] R. Rocchetta, Luis G. Crespo, *“A scenario optimization approach to reliability-based and risk-based design: soft-constrained modulation of failure probability bounds”*, Reliability Engineering & System Safety, https://doi.org/10.1016/j.ress.2021.107900