V. Example: Benchmark Robust Control Problem

V.A. Problem Statement

The tuning of a controller designed for the robust control challenge problem[17] posed in the 1990 American Control Conference is considered next. The control verification of several solutions to this problem is presented in Reference [14]. The benchmark plant, shown in Figure 4, is a two-mass/spring system with a non-collocated sensor actuator pair. Several design problems were posed

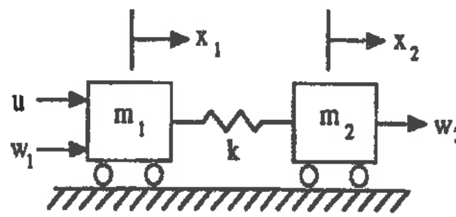

Figure 4. Two-mass spring system.

based on this setting. In all of them, stability and performance requirements in the time domain were prescribed for plants with uncertain masses and stiffness whose values lie within a bounded set. As in Reference [18], additional sources of uncertainty are considered herein to fully exercise the scope of the methodology. We added a non-linear spring with constant k_n, a time delay τ denoting a first order lag between controller command and actuator response, and a loop-gain uncertainty f resulting from multiplicative variation in observation, control gain and/or actuator failure.

The state space plant model is

$$\begin{aligned}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
\dot{x}_3 &= \frac{k}{m_1}(x_2 - x_1) + \frac{k_n}{m_1}(x_2 - x_1)^3 + \frac{fu}{m_1}, \\
\dot{x}_4 &= \frac{k}{m_2}(x_1 - x_2) + \frac{k_n}{m_2}(x_1 - x_2)^3 + \frac{w_2}{m_2}, \\
\tau\dot{u} &= u_c - u.
\end{aligned}$$

While the output $\mathbf{z}$ and the observed variable $\mathbf{y}$ are both equal to x_2, only the disturbance w_2 will be active. The uncertain parameter vector is $\boldsymbol{p} = [m_1, m_2, k, k_n, \tau, f]^T$ whose nominal value is $\bar{\boldsymbol{p}} = [1, 1, 1, 0, 0, 1]^T$. Note that the nominal values of the additional parameters lead to the plant used in the original benchmark problem. In order to prevent deformations leading to infeasible plants, the constraints $m_1 > 0$, $m_2 > 0$, $k > 0$, $\tau > 0$ and $f > 0$ are imposed on the optimization problem used to calculate the CPVs.

The specifications imposed on the closed-loop system are:

1. Local closed-loop stability.
2. Settling time: the response to a unit-impulse must fall between ± 0.1 after 15s.
3. Control saturation: the control signal corresponding to the impulse response must fall between ± 1.

In the context of this paper, the corresponding set of constraints is

$$g = \left[\max_{1 \le i \le n_p} \{\Re(s^i)\}, \quad \max_{t>15}\{|x_2(t)|\} - 0.1, \quad \max_{t>0}\{|u(t)|\} - 1 \right]^T,$$

where s^i is a closed-loop pole of the linearized system and $\Re(\cdot)$ is the real part operator. Eleven controllers were designed for the above problem by several authors. The controllers have been design using several different methods, including robust H_∞, loop-transfer recovery, imaginary-axis shifting, constrained optimization, structured covariance, game theory, the internal model principle[18,19] and μ-synthesis.[20] A Monte Carlo-based analysis of some of these controllers is available.[18]

The state space representation of a controller is given by

$$\dot{\mathbf{x}}_c = \mathbf{A}_c \mathbf{x}_c + \mathbf{B}_c \mathbf{y},$$
$$u_c = \mathbf{C}_c \mathbf{x}_c + \mathbf{D}_c \mathbf{y},$$

where $\mathbf{x}_c$ is the controller state, u_c is the actuator command, and $\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{C}_c$, and $\mathbf{D}_c$ are the controller matrices. The controllers considered here are the ones labeled as A, B, C, D, E, F, and H in Reference [18], and the controllers designed for problems one and two in Reference [19] and Reference [20]. In this paper, the controllers from Reference [19] will be labeled as W_1 and W_2, and those from Reference [20] will be labeled as B_1 and B_2.

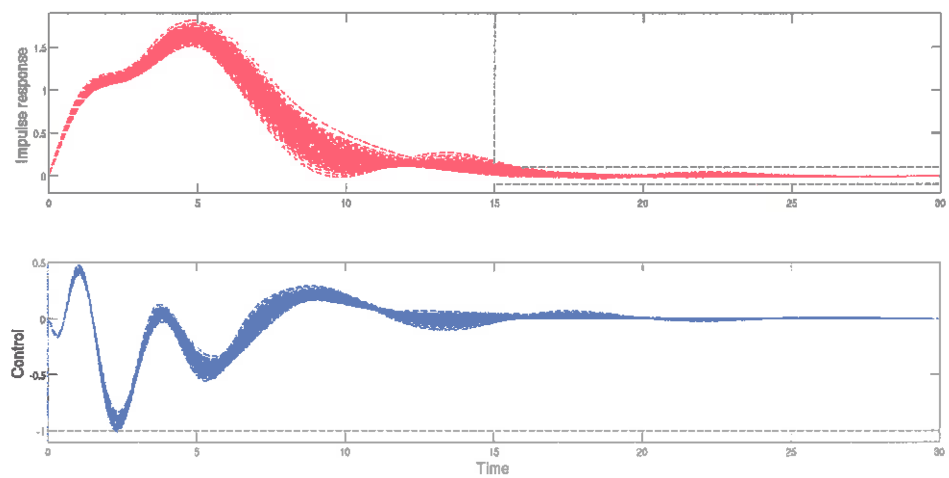

Figure 5. Percentiles 2% apart of the impulse response and control signal for B_2.

V.B. Example: Control Tuning

In this section we search for a controller with improved robustness characteristics by applying the developments of Section IV. For this, we assume that m_1, m_2, k, k_n, τ and f are independent, Beta distributed random variables with shape parameters, $[5,5]$, $[5,5]$, $[2,3.7]$, $[6,6]$, $[0.3,5]$, and $[0.5,1.5]$, having the support sets $[0,2]$, $[0,2]$, $[0.5,2]$, $[-0.5,0.5]$, $[\varepsilon,0.1]$ and $[0.5,1.5]$, respectively. The ranges of variation of the parameters and the shapes of the distributions are assigned according to engineering judgment.