Matrix Computations

CPSC 5006-EL

Assignment3

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Q1

3.1.5

(a)

$$\begin{bmatrix} 1 & 1 \\ 1 & 1.5 \\ 1 & 2 \\ 1 & 2.5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1.1 \\ 1.2 \\ 1.3 \\ 1.3 \\ 1.4 \end{bmatrix}$$

(b)

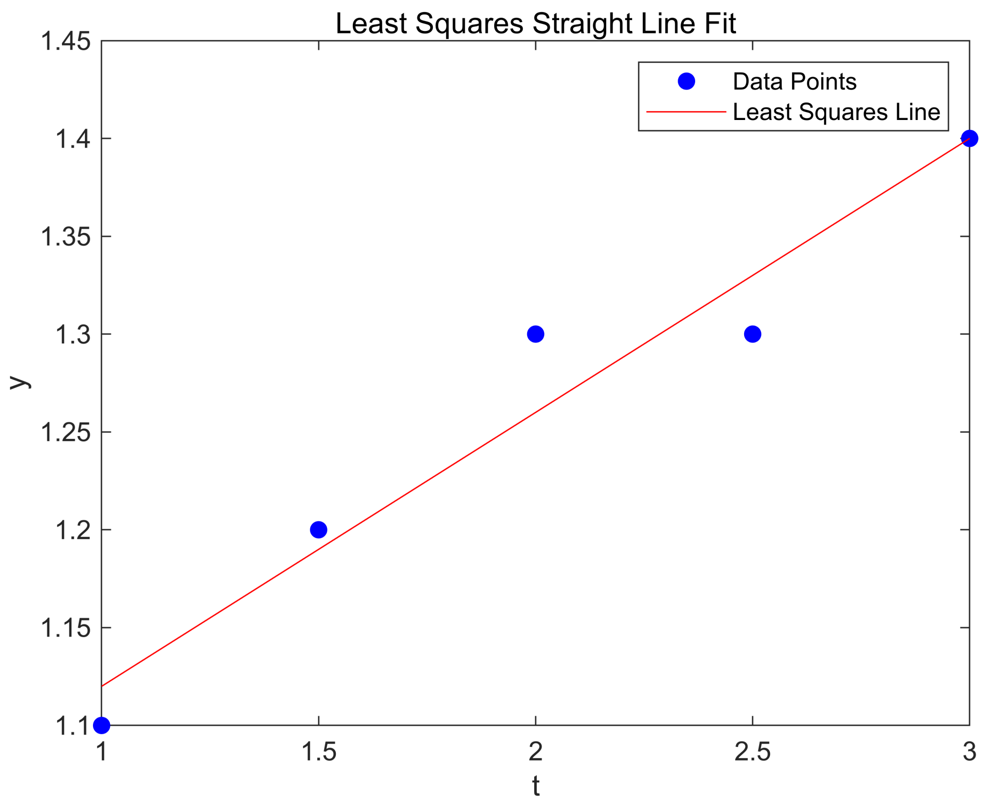
```
t = 1:.5:3; t = t';
s = ones(5, 1); A = [s t];
y = [1.1 1.2 1.3 1.3 1.4]';
x = A \ y
```

```
x = 2×1
    0.9800
    0.1400
```

(c)

```
% Prepare for plotting
figure;
% Plot the data points
plot(t, y, 'bo', 'MarkerFaceColor', 'b', 'DisplayName', 'Data Points');
hold on; % Keep the plot open to add the least squares line

% Plot the least squares straight line
t_fine = linspace(min(t), max(t), 100); % finer division for line plot
y_fine = x(1) + x(2) * t_fine;
plot(t_fine, y_fine, 'r-', 'DisplayName', 'Least Squares Line');

% Set plot labels and title
xlabel('t');
ylabel('y');
title('Least Squares Straight Line Fit');

% Show legend
legend show;
```

```
% Show the plot
hold off;
```



(d)

```
norm(y - A*x)
```

```
ans = 0.0548
```

3.1.6

(a)

$$\begin{bmatrix} 1 & 1 & 1^2 \\ 1 & 1.5 & 1.5^2 \\ 1 & 2 & 2^2 \\ 1 & 2.5 & 2.5^2 \\ 1 & 3 & 3^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1.1 \\ 1.2 \\ 1.3 \\ 1.3 \\ 1.4 \end{bmatrix}$$

(b)

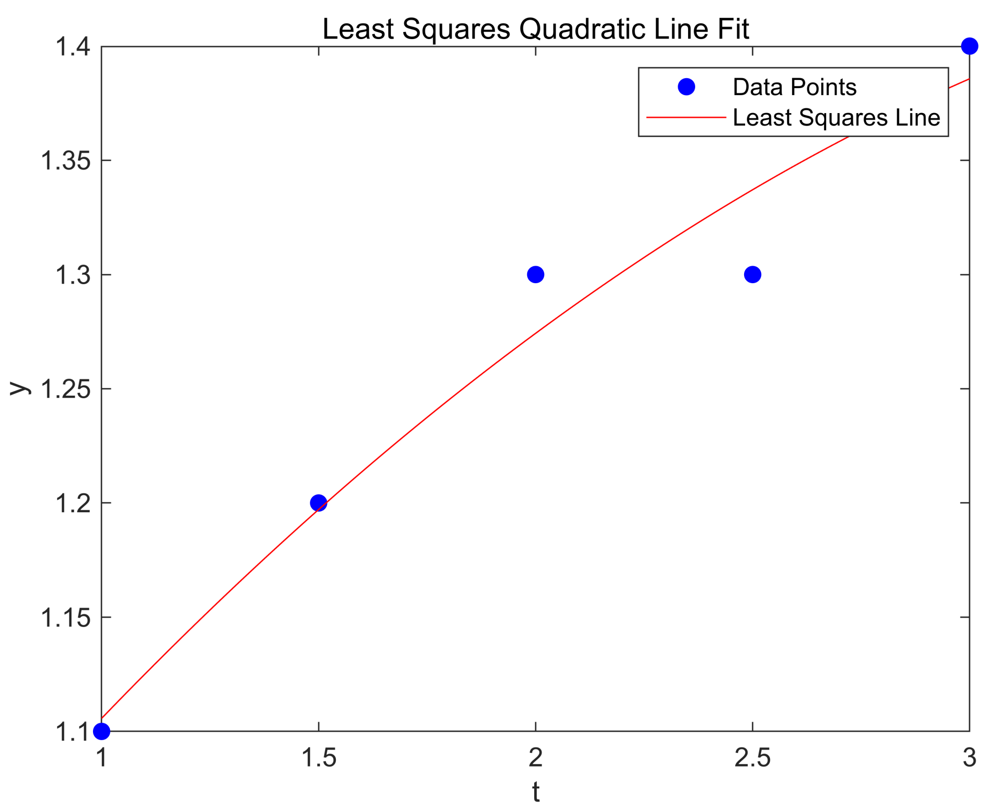
```
t = 1:.5:3; t = t';
s = ones(5, 1); A = [s t power(t,2)];
y = [1.1 1.2 1.3 1.3 1.4]';
x = A \ y
```

```
x = 3×1
    0.8800
```

```
    0.2543
   -0.0286
```

(c)

```
% Prepare for plotting
figure;
% Plot the data points
plot(t, y, 'bo', 'MarkerFaceColor', 'b', 'DisplayName', 'Data Points');
hold on; % Keep the plot open to add the least squares line

% Plot the least squares straight line
t_fine = linspace(min(t), max(t), 100); % finer division for line plot
y_fine = x(1) + x(2) * t_fine + x(3) * power(t_fine, 2);
plot(t_fine, y_fine, 'r-', 'DisplayName', 'Least Squares Line');

% Set plot labels and title
xlabel('t');
ylabel('y');
title('Least Squares Quadratic Line Fit');

% Show legend
legend show;

% Show the plot
hold off;
```



(d)

```
norm(y - A*x)
```

```
ans = 0.0478
```

Q2

Q is orthogonal, then $QQ^T = QQ^{-1} = I$

$\langle Q\,x, Q\,y\rangle = (Q\,y)^T(Q\,x) = y^T Q^T Q\,x = y^T x = \langle x, y\rangle$

$\langle Q\,x, Q\,x\rangle = \|Q\,x\|_2^2 = \langle x, x\rangle = \|x\|_2^2$, so $\|Q\,x\|_2 = \|x\|_2$

Since $\|Q\|_2 = \max\limits_{\|x\|_2=1} \|Q\,x\|_2$, for all x

So, $\|Q\|_2 = \max\limits_{\|x\|_2=1} \|x\|_2 = 1$

Since Q is orthognal, $Q^T = Q^{-1}$also preserves vector lengths, $\|Q^{-1}\|_2 = 1$

then $\kappa_2(Q) = \|Q\|_2\,\|Q^{-1}\|_2 = 1$

Q3

3.2.14

$$c = \frac{2}{\sqrt{2^2 + 5^2}} = \frac{2}{\sqrt{29}}$$

$$s = \frac{5}{\sqrt{29}}$$

$$Q = \begin{bmatrix} c & -s \\ s & c \end{bmatrix} = \frac{1}{\sqrt{29}}\begin{bmatrix} 2 & -5 \\ 5 & 2 \end{bmatrix}$$

$$R = Q^T A = \frac{1}{\sqrt{29}}\begin{bmatrix} 2 & 5 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \frac{1}{\sqrt{29}}\begin{bmatrix} 29 & 41 \\ 0 & -1 \end{bmatrix}$$

$\text{solving } Q\,c = b$

$$c = Q^T b = \frac{1}{\sqrt{29}}\begin{bmatrix} 2 & 5 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 12 \\ 29 \end{bmatrix} = \frac{1}{\sqrt{29}}\begin{bmatrix} 169 \\ -2 \end{bmatrix}$$

$\text{solving } R\,x = c$

$$x_2 = \frac{-2}{-1} = 2$$

$$x_1 = \frac{169 - 41 * 2}{29} = 3$$

$$x = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

3.2.47

(a)

$\tau = \|y\|_2 = \|1^2 + 1^2\|_2 = \sqrt{2}$

Suppose, $y = \begin{bmatrix} -\tau \\ 0 \end{bmatrix} = \begin{bmatrix} -\sqrt{2} \\ 0 \end{bmatrix}$

$$u = \frac{(x-y)}{\|x-y\|_2} = \frac{\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} -\sqrt{2} \\ 0 \end{bmatrix}}{\left\| \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} -\sqrt{2} \\ 0 \end{bmatrix} \right\|} = \frac{\begin{bmatrix} 1+\sqrt{2} \\ 1 \end{bmatrix}}{\sqrt{\left(1+\sqrt{2}\right)^2 + 1}} = \frac{1}{\sqrt{4+2\sqrt{2}}} \begin{bmatrix} 1+\sqrt{2} \\ 1 \end{bmatrix}$$

$$\widehat{Q} = I - 2u\,u^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 2\left(\frac{1}{\sqrt{4+2\sqrt{2}}}\right)^2 \begin{bmatrix} 1+\sqrt{2} \\ 1 \end{bmatrix} \begin{bmatrix} 1+\sqrt{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{1}{2+\sqrt{2}} \begin{bmatrix} 3+2\sqrt{2} & 1+\sqrt{2} \\ 1+\sqrt{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{(3+2\sqrt{2})(2-\sqrt{2})}{2} & \frac{(1+\sqrt{2})(2-\sqrt{2})}{2} \\ \frac{(1+\sqrt{2})(2-\sqrt{2})}{2} & \frac{2-\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{2+\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{2-\sqrt{2}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\widehat{R} = \widehat{Q}^T A = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -2 & -5 \\ 0 & 1 \end{bmatrix}$$

(b)

$$D = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$Q = \widehat{Q}D = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$R = D\widehat{R} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} -2 & -5 \\ 0 & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 2 & 5 \\ 0 & 1 \end{bmatrix}$$

Q4

3.2.37

(a)

$$u\,v^T = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} = \begin{bmatrix} u_1v_1 & u_1v_2 & \cdots & u_1v_2 \\ u_2v_1 & u_2v_2 & \cdots & u_2v_n \\ \vdots & \vdots & \ddots & \vdots \\ u_nv_1 & u_nv_2 & \cdots & u_nv_n \end{bmatrix}$$

The outer product like $u\,v^T$ results in nxn matrix.

$$
\left(u\,v^T\right)B=\begin{bmatrix} u_1v_1 & u_1v_2 & \cdots & u_1v_n \\ u_2v_1 & u_2v_2 & \cdots & u_2v_n \\ \vdots & \vdots & \ddots & \vdots \\ u_nv_1 & u_nv_2 & \cdots & u_nv_n \end{bmatrix}\begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1m} \\ B_{21} & B_{22} & \cdots & B_{1m} \\ \vdots & \vdots & \ddots & \vdots \\ B_{\mathrm{n}1} & B_{\mathrm{n}2} & \cdots & B_{\mathrm{nm}} \end{bmatrix}=\begin{bmatrix} \sum_{i=1}^{n} u_1v_iB_{\mathrm{i}1} & \sum_{i=1}^{n} u_1v_iB_{\mathrm{i}2} & \cdots & \sum_{i=1}^{n} u_1v_iB_{\mathrm{im}} \\ \sum_{i=1}^{n} u_2v_iB_{\mathrm{i}1} & \sum_{i=1}^{n} u_2v_iB_{\mathrm{i}2} & \cdots & \sum_{i=1}^{n} u_2v_iB_{\mathrm{im}} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{n} u_nv_iB_{\mathrm{i}1} & \sum_{i=1}^{n} u_nv_iB_{\mathrm{i}2} & \cdots & \sum_{i=1}^{n} u_nv_iB_{\mathrm{im}} \end{bmatrix}
$$

```
Computing uvT

for i from 1 to n:

for j from 1 to n:

uvT[i, j] = u[i] * v[j]

Computing (uvT)B

for i from 1 to n:

for j from 1 to m:

C[i, j] = 0

for k from 1 to n:

C[i, j] = C[i, j] + uvT[i, k] * B[k, j]
```

$$
\mathrm{flops}=\sum_{i=1}^{n}\sum_{j=1}^{n}1+\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{n}2=n^2+2n^2m\approx 2n^2m
$$

(b)

$$
v^TB=\begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix}\begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1m} \\ B_{21} & B_{22} & \cdots & B_{1m} \\ \vdots & \vdots & \ddots & \vdots \\ B_{\mathrm{n}1} & B_{\mathrm{n}2} & \cdots & B_{\mathrm{nm}} \end{bmatrix}=\begin{bmatrix} \sum_{i=1}^{n} v_iB_{\mathrm{i}1} & \sum_{i=1}^{n} v_iB_{\mathrm{i}2} & \cdots & \sum_{i=1}^{n} v_iB_{\mathrm{im}} \end{bmatrix}
$$

The product like v^TB results in 1xm matrix.

$$u(v^TB) = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{n} v_i B_{\text{i1}} & \sum_{i=1}^{n} v_i B_{\text{i2}} & \cdots & \sum_{i=1}^{n} v_i B_{\text{im}} \end{bmatrix} = \begin{bmatrix} u_1 \sum_{i=1}^{n} v_i B_{\text{i1}} & u_1 \sum_{i=1}^{n} v_i B_{\text{i2}} & \cdots & u_1 \sum_{i=1}^{n} v_i B_{\text{im}} \\ u_2 \sum_{i=1}^{n} v_i B_{\text{i1}} & u_2 \sum_{i=1}^{n} v_i B_{\text{i2}} & \cdots & u_2 \sum_{i=1}^{n} v_i B_{\text{im}} \\ \vdots & \vdots & \ddots & \vdots \\ u_n \sum_{i=1}^{n} v_i B_{\text{i1}} & u_n \sum_{i=1}^{n} v_i B_{\text{i2}} & \cdots & u_n \sum_{i=1}^{n} v_i B_{\text{im}} \end{bmatrix}$$

```
Computing v^T B

for j from 1 to m:

vTB[j] = 0

for i from 1 to n:

vTB[j] = vTB[j] + v[i] * B[i, j]

Computing u(vTB)

for i from 1 to n:

for j from 1 to m:

C[i, j] = u[i] * vTB[j]
```

$$\text{flops} = \sum_{j=1}^{m} \sum_{i=1}^{n} 2 + \sum_{i=1}^{n} \sum_{j=1}^{m} 1 = 2\text{nm} + \text{nm} = 3\text{nm}$$

(c)

$$Q\,B = B - u\,v^T B = B - u\left(v^T B\right)$$

We know that $u\left(v^T B\right)$ needs 3nm flops. The minus operator takes 1nm flops

So the total flops =3nm + 1nm = 4nm

(d)

$$QB = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ q_{\text{n1}} & q_{\text{n2}} & \cdots & q_{\text{nn}} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1m} \\ B_{21} & B_{22} & \cdots & B_{1m} \\ \vdots & \vdots & \ddots & \vdots \\ B_{\text{n1}} & B_{\text{n2}} & \cdots & B_{\text{nm}} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} q_{1i} B_{\text{i1}} & \sum_{i=1}^{n} q_{1i} B_{\text{i2}} & \cdots & \sum_{i=1}^{n} q_{1i} B_{\text{im}} \\ \sum_{i=1}^{n} q_{2i} B_{\text{i1}} & \sum_{i=1}^{n} q_{2i} B_{\text{i2}} & \cdots & \sum_{i=1}^{n} q_{2i} B_{\text{im}} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{n} q_{\text{ni}} B_{\text{i1}} & \sum_{i=1}^{n} q_{\text{ni}} B_{\text{i2}} & \cdots & \sum_{i=1}^{n} q_{\text{ni}} B_{\text{im}} \end{bmatrix}$$

```
Computing QB
```

```
for i from 1 to n:

for j from 1 to m:

C[i, j] = 0

for k from 1 to n:

C[i, j] = C[i, j] + q[i, k] * B[k, j]
```

$$\text{flops} = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{n} 2 = 2n^2m$$

Q5

$Q = Q_1Q_2 \cdots Q_{n-1}$

$$Q_i = \begin{bmatrix} I_{i-1} & 0 \\ 0 & I - \gamma_i u^{(i)} u^{(i)T} \end{bmatrix}$$

where I_{i-1} denotes the identity matrix of dimension $i-1$.

According to Q4(c),

if $(Q_2Q_3 \cdots Q_{n-1})$ is computed, $Q_1(Q_2Q_3 \cdots Q_{n-1})$ takes $4n^2$ flops

if $(Q_3Q_4 \cdots Q_{n-1})$ is computed, $Q_2(Q_3Q_4 \cdots Q_{n-1})$ takes $4(n-1)^2$ flops

...

Q_{n-1} is computed

So, computing Q takes:

$$\text{flops} = 4n^2 + 4(n-1)^2 + \cdots + 1 \approx \frac{4}{3}n^3.$$

Q6

$$Q_i = \begin{bmatrix} I_{i-1} & 0 \\ 0 & I - \gamma_i u^{(i)} u^{(i)T} \end{bmatrix}$$

where I_{i-1} denotes the identity matrix of dimension $i-1$.

for k = 1, ..., n-1

Determine $Q_k = I - \gamma_k u^{(k)} u^{(k)T}$

$a_{k:n,\,k+1:n} \leftarrow Q_k\, a_{k:n,\,k+1:n}$

$a_{\mathrm{kk}} \leftarrow -\tau_k$

Computing $a_{k:n,\,k+1:n} \leftarrow Q_k\, a_{k:n,\,k+1:n}$ costs a lot more than computing $u^{(k)}$.

So $\text{flops} \approx \text{Compute } a_{k:n,\,k+1:n} \leftarrow Q_k\, a_{k:n,\,k+1:n}$

Since $Q_k\, a_{k:n,\,k+1:n} = \left(I - \gamma_k u^{(k)} u^{(k)T}\right) a_{k:n,\,k+1:n}$

According to Q4(c), it takes $4(n-k-1)^2$ flops, k=1,...,n

So, $\text{flops} = 4n^2 + 4(n-1)^2 + \cdots + 1 \approx \frac{4}{3}n^3.$

Q7

3.2.39

(a)

$$\tau = \|y\|_2 = \sqrt{3^2+4^2+1^2+3^2+1^2} = 6$$
$$y = \begin{bmatrix} -6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},\ u = x - y = \begin{bmatrix} 3 \\ 4 \\ 1 \\ 3 \\ 1 \end{bmatrix} - \begin{bmatrix} -6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ 1 \\ 3 \\ 1 \end{bmatrix}$$
$$\gamma = \frac{2}{\|u\|_2^2} = \frac{2}{9^2+4^2+1^2+3^2+1^2} = \frac{1}{54}$$

(i)

$$Q = I - \gamma u\, u^T = \begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ 0 & & & & 1 \end{bmatrix} - \frac{1}{54}\begin{bmatrix} 9 \\ 4 \\ 1 \\ 3 \\ 1 \end{bmatrix}[9 \quad 4 \quad 1 \quad 3 \quad 1]$$

(ii)

$$Q = \begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ 0 & & & & 1 \end{bmatrix} - \frac{1}{54}\begin{bmatrix} 81 & 36 & 9 & 27 & 9 \\ 36 & 16 & 4 & 12 & 4 \\ 9 & 4 & 1 & 3 & 1 \\ 27 & 12 & 3 & 9 & 3 \\ 9 & 4 & 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ 0 & & & & 1 \end{bmatrix} - \begin{bmatrix} \frac{3}{2} & \frac{2}{3} & \frac{1}{6} & \frac{1}{2} & \frac{1}{6} \\ \frac{2}{3} & \frac{8}{27} & \frac{2}{27} & \frac{2}{9} & \frac{2}{27} \\ \frac{1}{6} & \frac{2}{27} & \frac{1}{54} & \frac{1}{18} & \frac{1}{54} \\ \frac{1}{2} & \frac{2}{9} & \frac{1}{27} & \frac{1}{6} & \frac{1}{18} \\ \frac{1}{6} & \frac{2}{27} & \frac{1}{54} & \frac{1}{18} & \frac{1}{54} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2} & -\frac{2}{3} & -\frac{1}{6} & -\frac{1}{2} & -\frac{1}{6} \\ -\frac{2}{3} & \frac{19}{27} & -\frac{2}{27} & -\frac{2}{9} & -\frac{2}{27} \\ -\frac{1}{6} & -\frac{2}{27} & \frac{53}{54} & -\frac{1}{18} & -\frac{1}{54} \\ -\frac{1}{2} & -\frac{2}{9} & -\frac{1}{27} & \frac{5}{6} & -\frac{1}{18} \\ -\frac{1}{6} & -\frac{2}{27} & -\frac{1}{54} & -\frac{1}{18} & \frac{53}{54} \end{bmatrix}$$

(b)

$$\tau = \|y\|_2 = \sqrt{0^2 + 2^2 + 1^2 + (-1)^2 + 0^2} = \sqrt{6}$$

$$y = \begin{bmatrix} -\sqrt{6} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, u = a - y = \begin{bmatrix} 0 \\ 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} -\sqrt{6} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{6} \\ 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\gamma = \frac{2}{\|u\|_2^2} = \frac{2}{6 + 4 + 1 + 1 + 0} = \frac{1}{6}$$

(i)

$$Q\,a = (I - \gamma u\,u^T)a = \begin{bmatrix} 0 \\ 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \frac{1}{6}\begin{bmatrix} \sqrt{6} \\ 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}\left(\begin{bmatrix} \sqrt{6} & 2 & 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \frac{1}{6} * 6 * \begin{bmatrix} \sqrt{6} \\ 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\sqrt{6} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(ii)

$$
Q = \begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ 0 & & & & 1 \end{bmatrix} - \frac{1}{6}\begin{bmatrix} 6 & 2\sqrt{6} & \sqrt{6} & -\sqrt{6} & 0 \\ 2\sqrt{6} & 4 & 2 & -2 & 0 \\ \sqrt{6} & 2 & 1 & -1 & 0 \\ -\sqrt{6} & -2 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ 0 & & & & 1 \end{bmatrix} - \begin{bmatrix} 1 & \frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & 0 \\ \frac{\sqrt{6}}{3} & \frac{2}{3} & \frac{1}{3} & -\frac{1}{3} & 0 \\ \frac{\sqrt{6}}{6} & \frac{1}{3} & \frac{1}{6} & -\frac{1}{6} & 0 \\ -\frac{\sqrt{6}}{6} & -\frac{1}{3} & -\frac{1}{6} & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
= \begin{bmatrix} 0 & -\frac{\sqrt{6}}{3} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & 0 \\ -\frac{\sqrt{6}}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & 0 \\ -\frac{\sqrt{6}}{6} & -\frac{1}{3} & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{\sqrt{6}}{6} & \frac{1}{3} & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
Q\,a = \begin{bmatrix} 0 & -\frac{\sqrt{6}}{3} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & 0 \\ -\frac{\sqrt{6}}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & 0 \\ -\frac{\sqrt{6}}{6} & -\frac{1}{3} & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{\sqrt{6}}{6} & \frac{1}{3} & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\sqrt{6} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

3.3.7

(a)

I guess $x = \frac{(9+5)}{2} = 7$

(b)

$c = \frac{1}{\sqrt{2}}, s = \frac{1}{\sqrt{2}}$

$Q = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$R = Q^T A = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$c = Q^T b = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ 5 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 14 \\ -4 \end{bmatrix}$

So, $\hat{c} = 14, \hat{R} = 2, d = -4$

Since $\hat{R}x = \hat{c}$, so 2x = 14, x = 7

Since $\|s\|_2^2 = \|\hat{c} - \hat{R}x\|_2^2 + \|d\|_2^2$ and $\|\hat{c} - \hat{R}x\|_2^2 = 0$

then $\|s\|_2 = \|d\|_2 = 4$.

Q8

3.3.10

(a)

```
n = 6; m = 3;
A = randn(n, m)
```

```
A = 6×3
   -0.0068   -1.0891   -1.4916
    1.5326    0.0326   -0.7423
   -0.7697    0.5525   -1.0616
    0.3714    1.1006    2.3505
   -0.2256    1.5442   -0.6156
    1.1174    0.0859    0.7481
```

```
[Q, R] = qr(A)
```

```
Q = 6×6
   -0.0033   -0.4831   -0.3096    0.6217    0.3933    0.3600
    0.7324    0.0473   -0.4648    0.1693   -0.1742   -0.4316
   -0.3678    0.2285   -0.3728    0.3201   -0.7230    0.2198
    0.1775    0.4960    0.5578    0.6386    0.0563   -0.0174
   -0.1078    0.6799   -0.4781   -0.1212    0.5092    0.1533
    0.5340    0.0620    0.0969   -0.2446   -0.1725    0.7823
R = 6×3
    2.0925   -0.1011    0.7346
         0    2.2551    1.2366
         0         0    2.8805
         0         0         0
         0         0         0
         0         0         0
```

```
Q' * Q
```

```
ans = 6×6
```

```
    1.0000   -0.0000    0.0000    0.0000    0.0000    0.0000
   -0.0000    1.0000   -0.0000    0.0000    0.0000   -0.0000
    0.0000   -0.0000    1.0000    0.0000    0.0000   -0.0000
    0.0000    0.0000    0.0000    1.0000   -0.0000         0
    0.0000    0.0000    0.0000   -0.0000    1.0000   -0.0000
    0.0000   -0.0000   -0.0000         0   -0.0000    1.0000
```

```
norm(eye(n) - Q' * Q)
```

```
ans = 1.1462e-15
```

```
norm(A - Q * R)
```

```
ans = 2.7506e-15
```

(b)

```
% Part (a): Prepare the Vandermonde matrix for quadratic polynomial fit
t_a = [-1, -0.75, -0.5, 0, 0.25, 0.5, 0.75]';
A_a = [ones(length(t_a), 1), t_a, t_a.^2];
y_a = [1.00, 0.8125, 0.75, 1.00, 1.3125, 1.75, 2.3125]';

% Solve for part (a)
[x_a, residual_norm_a] = solve_least_squares(A_a, y_a)
```

```
x_a = 3×1
    1.0000
    1.0000
    1.0000
residual_norm_a = 4.5776e-16
```

```

% Part (b): Prepare the Vandermonde matrix for linear polynomial fit - alpha
t_b_alpha = [1000, 1050, 1060, 1080, 1110, 1130]';
A_b_alpha = [ones(length(t_b_alpha), 1) * 50, t_b_alpha - 1065];
y_b_alpha = [6010, 6153, 6421, 6399, 6726, 6701]';

% Solve for part (b) - alpha
[x_b_alpha, residual_norm_b_alpha] = solve_least_squares(A_b_alpha, y_b_alpha)
```

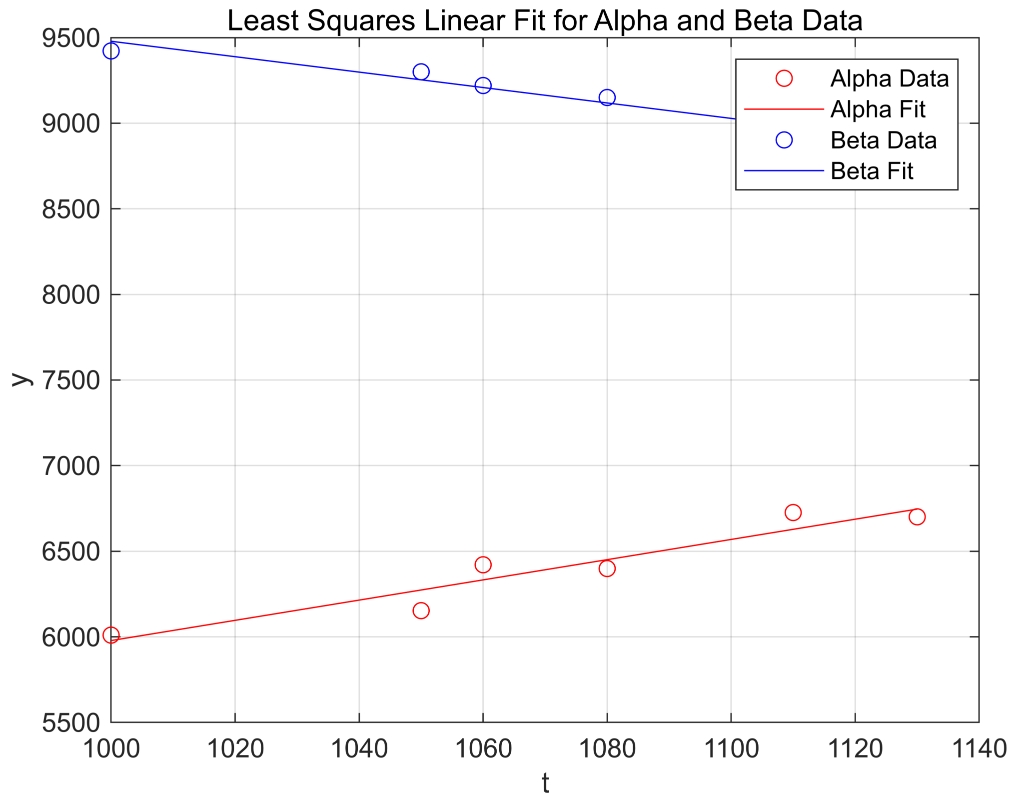
```
x_b_alpha = 2×1
  127.2458
    5.9067
residual_norm_b_alpha = 194.1091
```

```

% Part (b): Prepare the Vandermonde matrix for linear polynomial fit - beta
t_b_beta = [1000, 1050, 1060, 1080, 1110, 1130]';
A_b_beta = [ones(length(t_b_beta), 1) * 50, t_b_beta - 1065];
y_b_beta = [9422, 9300, 9220, 9150, 9042, 8800]';

% Solve for part (b) - beta
[x_b_beta, residual_norm_b_beta] = solve_least_squares(A_b_beta, y_b_beta)
```

```
x_b_beta = 2×1
  183.7139
```

```
    -4.5039
residual_norm_b_beta = 136.5382
```

```
% Plotting for part (b)
figure; % Create a new figure

% Plot the data points for alpha
plot(t_b_alpha, y_b_alpha, 'ro', 'DisplayName', 'Alpha Data');
hold on; % Hold on to plot the fit on the same graph

% Compute the fitted values for alpha and plot
fitted_y_b_alpha = A_b_alpha * x_b_alpha;
plot(t_b_alpha, fitted_y_b_alpha, 'r-', 'DisplayName', 'Alpha Fit');

% Plot the data points for beta
plot(t_b_beta, y_b_beta, 'bo', 'DisplayName', 'Beta Data');

% Compute the fitted values for beta and plot
fitted_y_b_beta = A_b_beta * x_b_beta;
plot(t_b_beta, fitted_y_b_beta, 'b-', 'DisplayName', 'Beta Fit');

% Enhance the plot
title('Least Squares Linear Fit for Alpha and Beta Data');
xlabel('t');
ylabel('y');
legend('show');
grid on;
hold off; % Release the plot hold
```



3.3.15

```
A = randn(5, 3)
```

```
A = 5×3
   -0.1924    0.4882    0.1978
    0.8886   -0.1774    1.5877
   -0.7648   -0.1961   -0.8045
   -1.4023    1.4193    0.6966
   -1.4224    0.2916    0.8351
```

```
A = A * diag([1 3 9])
```

```
A = 5×3
   -0.1924    1.4646    1.7803
    0.8886   -0.5321   14.2893
   -0.7648   -0.5882   -7.2402
   -1.4023    4.2579    6.2696
   -1.4224    0.8748    7.5158
```

```
[Q, R, P] = qr(A)
```

```
Q = 5×5
   -0.0944    0.3011    0.2088   -0.8264   -0.4171
   -0.7578   -0.4255    0.1439    0.1873   -0.4347
    0.3840    0.0172   -0.4872    0.2194   -0.7529
   -0.3325    0.8522    0.0503    0.3952   -0.0675
   -0.3986    0.0433   -0.8342   -0.2788    0.2563
R = 5×3
  -18.8565   -1.7252    0.0843
         0    4.3235   -1.7056
         0         0    1.5763
```

```
         0         0         0
         0         0         0
P = 3×3
     0     0     1
     0     1     0
     1     0     0
```

diag command copy A's first column, 3 times A's second column, 9 times A's third column.

Permutation P is $\begin{bmatrix} & & 1 \\ & 1 & \\ 1 & & \end{bmatrix}$, it swap A's first column and third column.

A's third column has been magnified by 9, which has maximal norm.