

J3D: using moments to quantify the shape of molecular clouds in 3D

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Summary

Molecular clouds are the birthplace of stars. In these chaotic environments, the interplay of gravity, pressure, turbulence, chemistry, radiation, and feedback from young stars creates a wide variety of structures. We can observe these projected on the sky in two dimensions (position-position, PP), and, using chemical tracers to infer a velocity (position-position-velocity, PPV), separate structures which are moving towards or away from us at the same speed and are therefore assumed to be related. We can also simulate many of these processes and create theoretical molecular clouds in two (PP) or three spatial dimensions (PPP), and then use our understanding of the physics and chemistry of the interstellar medium to create synthetic observations of these clouds, which can then be compared with real observations. In all of these studies a key step is to compare pixelated greyscale images in two or three dimensions.

J plots is a tool to quantify the shape of pixelated structures using the moment of inertia. Written in Python, this allows users to import their data set from any format Python can understand (including the commonly used FITS format using the [pyfits package](#)) convert to a numpy array, and segment it using a method of their choice into meaningful structures (dendrograms are used in the example scripts, relying on the [astrodendro](#) package, but other techniques such as [clumpfind](#) (Williams, de Geus, & Blitz, 1994) or simple thresholding could equally be used). These structures can be individually or collectively analysed with J plots in 2D or 3D to reveal trends in their shape with environment, observational or simulated constraints, or segmentation method. This allows astronomers to quantify these chaotic structures and compare them in a statistical sense.

The J plots method

The 2D J plots method, as described in Jaffa, Whitworth, Clarke, & Howard (2018), takes as input a set of real-valued pixels representing a part of a greyscale image (called a 'structure'). For each structure, we calculate the area, A (number of pixels), mass M (sum of pixel values), centre of mass and principle moments of inertia, I_i . We also calculate what the moments would be for a reference shape of the same mass and area, I_0 . The J moments are then defined as

$$J_i = I_0 + \frac{I_i}{I_0 - I_i}$$

The two-dimensional case

In two dimensions the reference shape is a circle of constant surface density, so $I_0 = AM/4\pi$. If the shape is centrally concentrated, such as a collapsing core, both principle moments will

be smaller than I_0 so both J values will be positive. For a hollow ring shape such as bubble blown in the cloud by stellar feedback, both principle moments will be greater than I_0 so both J values will be negative. For elongated shapes such as filaments, which are prevalent in molecular clouds, one moment will be larger and one smaller, so J_1 will be positive and J_2 will be negative. This gives us a simple diagnostic of these common shapes and allows us to place quantitative restrictions on shapes that fall between these categories.

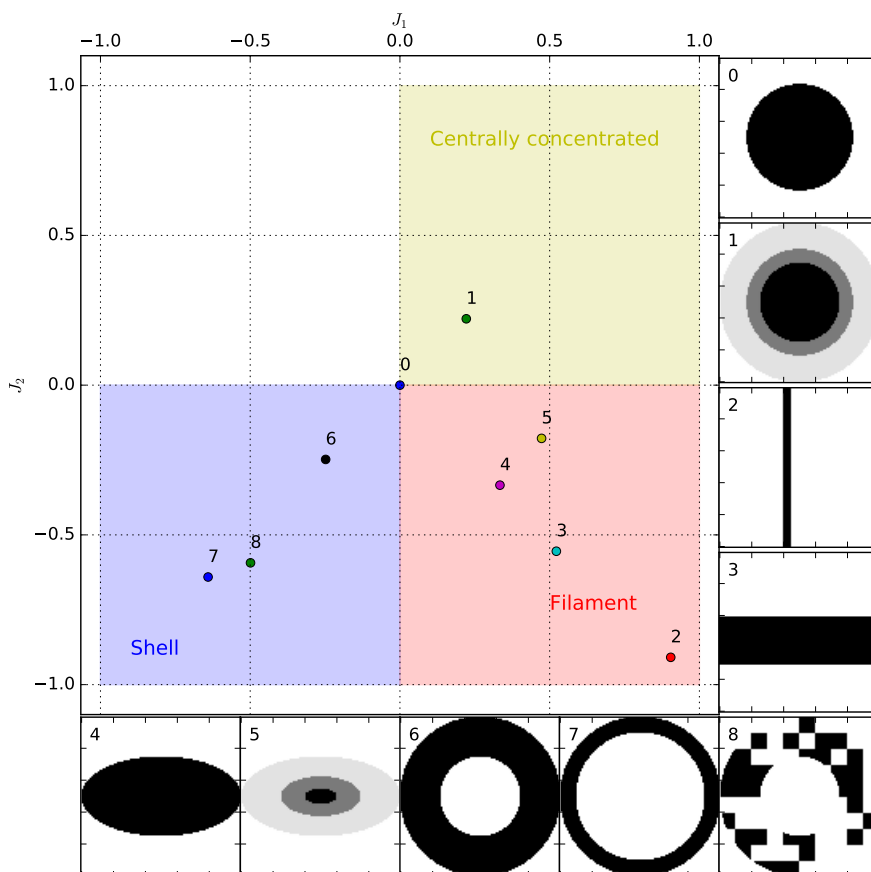


Figure 1: Proof of concept of 2D J plots. The J values of several simple shapes are plotted, representing morphologies observed in molecular clouds. This demonstrates that distinct categories of shape are distributed in different regions on the J plot.

J plots has been used in 2D to analyse the shape of structures within simulated filaments. Clarke et al. (2018) examined the J values of sub-filaments (small elongated structures formed inside the main filament identified in 3D PPP and projected into 2D PP) and fibres (elongated structures inside the main filament identified observationally in PPV, then projected into the PP) and found that the PPV detected fibres did not represent the same gas as the PPP identified sub-filaments. Their distributions of J values showed that the shape of the structures recovered was changed by the nature of the observations, so observed velocity coherent structures should not be taken to represent physically separate objects.

J3D

J3D now allows analysis of 3D objects without projection. This is mainly intended for the analysis of simulations in their native PPP space, but can also be used for observations including velocity information as a third dimension (PPV). In PPP space, the reference shape analogous to a uniform surface density circle becomes a uniform volume density sphere. The moment of this sphere about any set of 3 orthogonal axes is

$$I_0 = \frac{2}{5} M \left(\frac{3V}{4\pi} \right)^{2/3},$$

where M is the mass of the structure (sum of pixel values) and V is the volume (number of pixels).

Again, hollow shapes where the mass is further from the centre than this reference shape will have $J_{1,2,3} < 0$, centrally concentrated shapes will have $J_{1,2,3} > 0$, and shapes that are elongated on one or two dimensions will have one or two positive J values and the others negative. These can be separated into oblate and prolate spheroids.

In this case the deliniation between astrophysically relevant shapes is not as clear because most of the shapes we often discuss are defined in observations and therefore 2D, and the prevalence of filaments, spheroids, and thin sheet-like structures is only discussed theoretically or in simulations. Algorithms to identify these kind of shapes and studies of their relevance to astrophysical phenomena (such as turbulence, feedback, gravitational collapse, etc) are not common. We believe J3D meets an important need for robust quantification of 3D structures in simulations to enable statistical comparison of different theoretical studies.

Velocity dimension

Real observations of molecular lines can provide information on the velocity of observed gas along the line of sight, and this can help us understand the motion of the interstellar medium. Simulations also have velocity information, so can theoretically be analysed in 6-dimensional PPPVVV, or in projected PPV space to be comparable to observations. In such a case we caution against the blind interpretation of velocity as equivalent to a third spatial dimension, as studies have shown that things moving at the same speed may not be spatially colocated (???). Combining spatial and velocity dimensions may provide some interesting insights, but will need much more thoughtful analysis to draw physically meaningful conclusions.

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