

# DETERMINING THE PARAMETERS OF EXOPLANETARY CANDIDATES FROM TRANSIT TIMING VARIATIONS

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## Observation

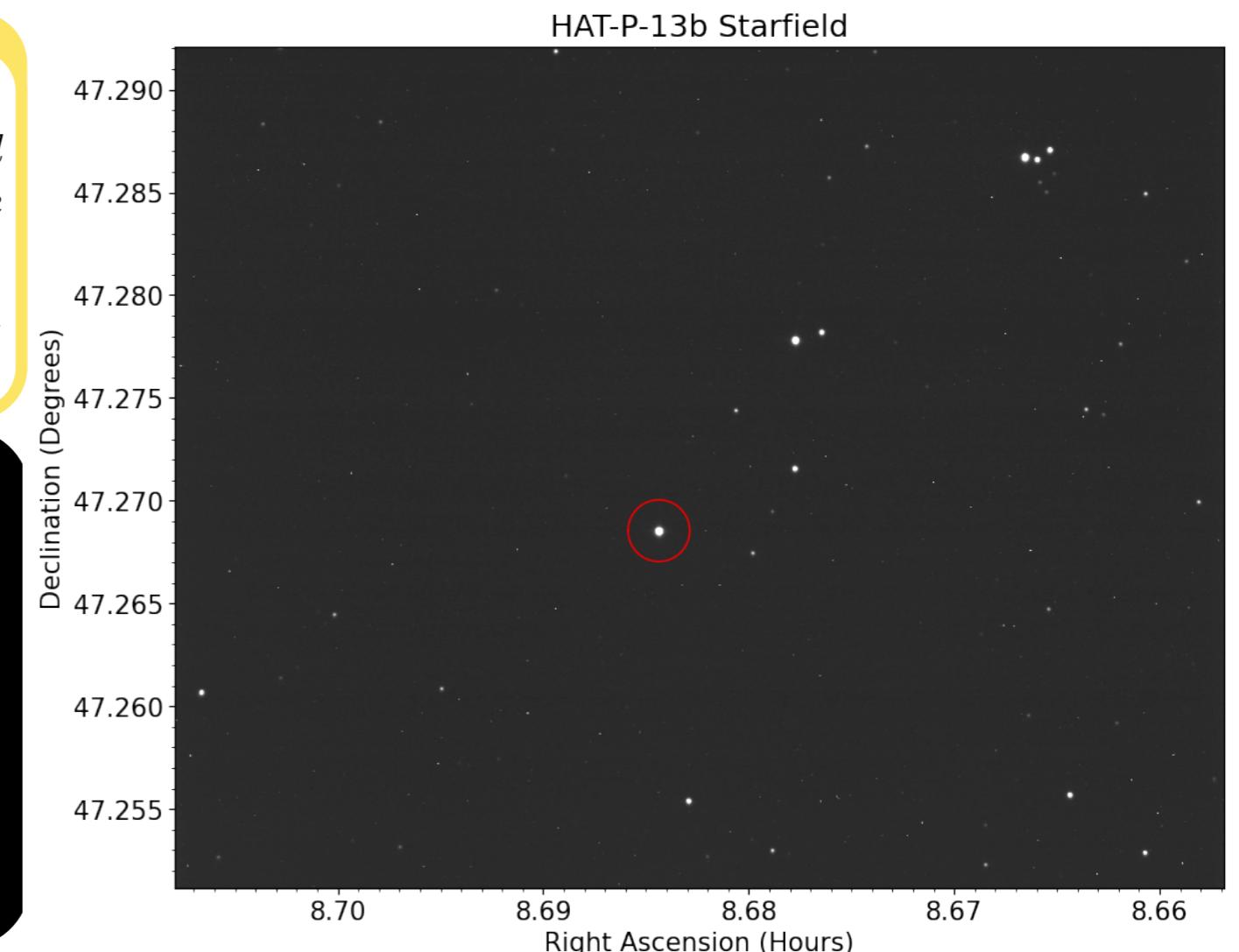
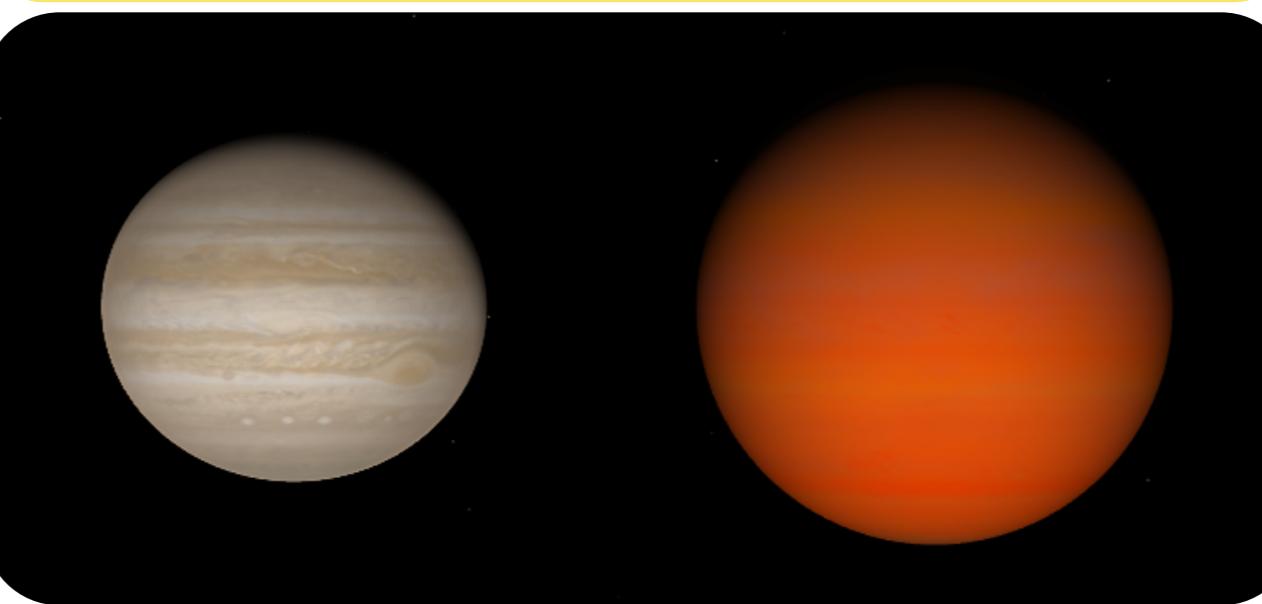
To collect astronomical data, I have used the 24" Ritchey-Chrétien telescope at Clanfield observatory. The imaging system is a Starlight-Xpress SX-46 CCD with a 16.7 megapixel sensor, which -with the focal length of the telescope of 4810mm- gives a resolution of 257.29 milli-arcseconds per pixel. For exoplanet observation a Johnson-Cousins Red filter is used, as it decreases stellar scintillation and improves the definition of transit ingress and egress.

Of the thousands of exoplanet systems discovered, only a limited number need to be considered for observation for this project. Those to be considered must be visible at night, contain at least two planets for which at least one was discovered by transit photometry, and orbit about a single central star with a brightness higher than the telescopes limit of magnitude 15.

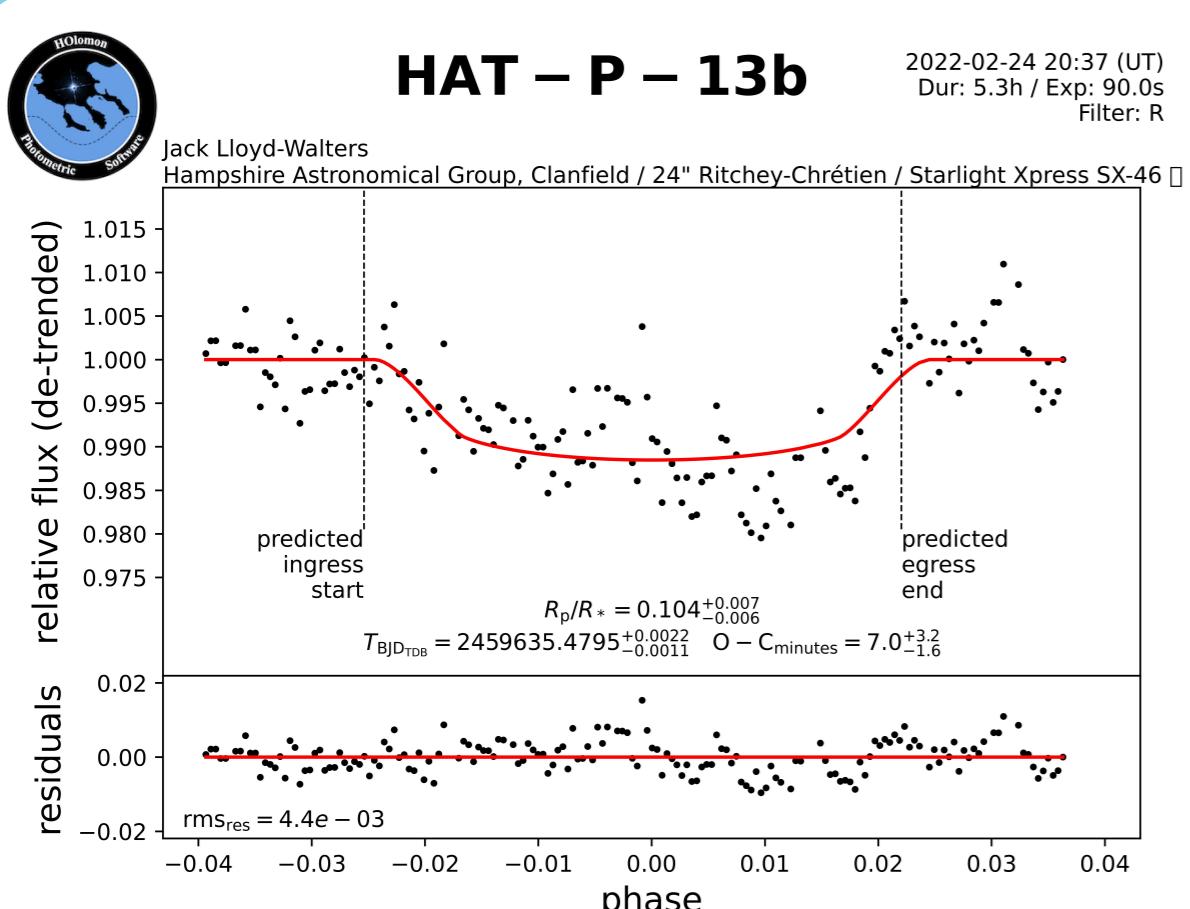
One of the candidate planets for this project is HAT-P-13b, a hot Jupiter 700 light years distant in the direction of Ursa major [4]. Within the system is a non-transiting sub-stellar object, HAT-P-13c, the presence of which I will attempt to prove by way of transit timing variations as a first step in the development of this project!



Left: The 24" telescope used for this project at Clanfield.  
Below: Side-by-side comparison of Jupiter and a hypothetical visualisation of HAT-P-13b created from known properties of the planet. Sourced from NASA's "Eyes on Exoplanets" [3].  
Right: The star-field around HAT-P-13, taken from an observation on 2022-02-27.



## Lightcurve data

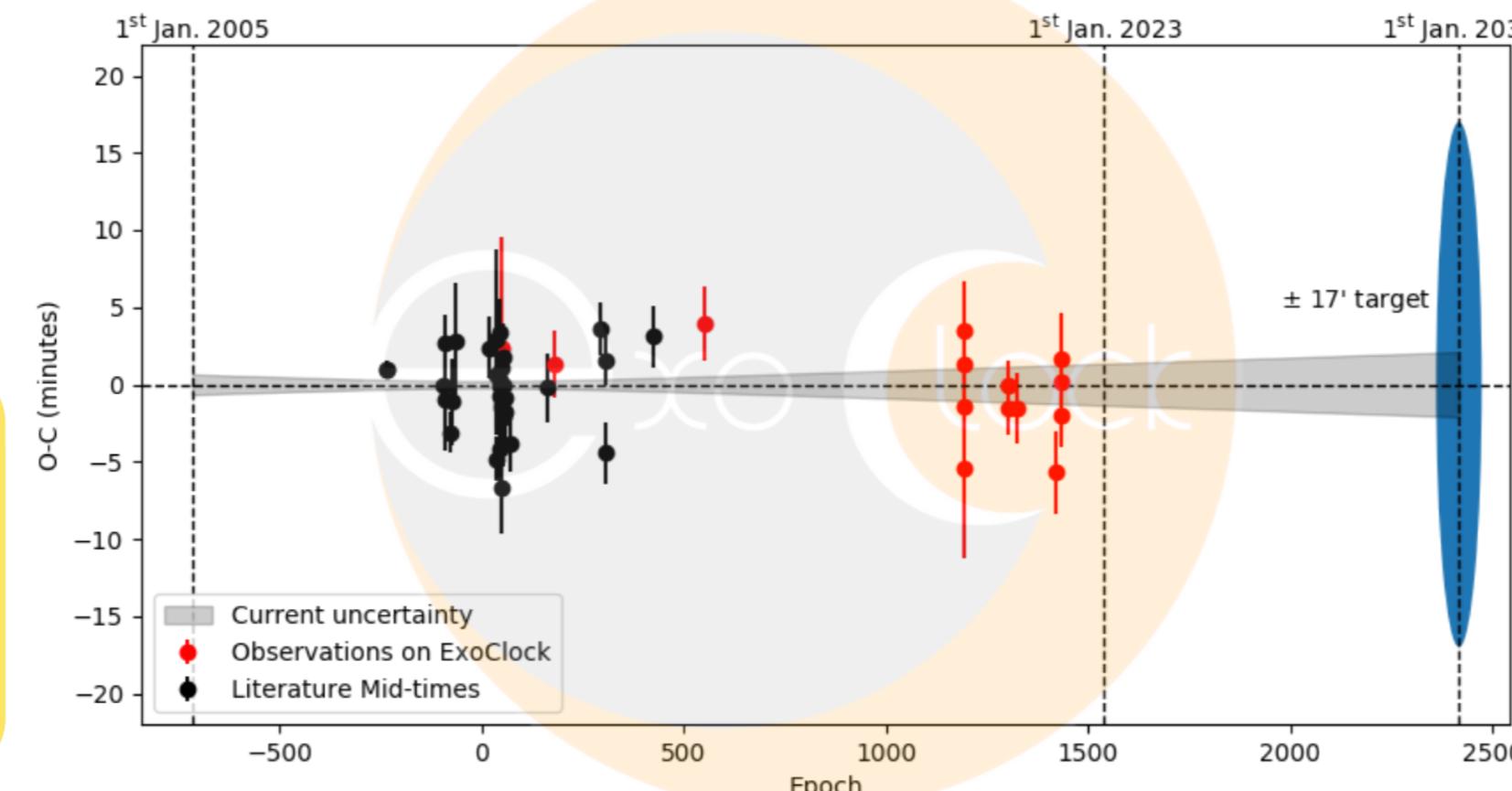


Above: Sample output from HOPS for an observation of HAT-P-13b on 2022-02-24, showing both the model lightcurve and the residuals of the model and data.

Right: ExoClock "O-C" diagram for HAT-P-13b, including mid-transit times and error bars from both literature and observations uploaded to ExoClock.

To analyse collected data we use **H**olomon **P**hotometric **S**oftware [7], written in Python by Angelos Tsiaras. HOPS takes the raw .fit files that our imager collects, reduces and aligns them, and plots the relative brightness of our target star compared to other stars in the image of similar spectral type.

This gives a transit lightcurve, for which HOPS will fit a model using various regression methods. This model is then uploaded to the ExoClock database [1] [2], and can be analysed to compute properties of the exoplanet.



## Exoplanet properties from transit observables

$$\Delta L = \frac{A_{\text{planet}}}{A_{\star}} = \frac{\delta^2_{\text{planet}}}{\delta^2_{\star}} \approx \frac{R^2_{\text{planet}}}{R^2_{\star}} \quad (1)$$

Transit depth,  $\Delta L$ , relates the angular area  $A$ , angular diameter  $\delta$ , and true radius  $R$  of the star to the planet.

$$R_\beta = \sqrt{(R_{\star} + R_{\text{planet}})^2 - (a \cos i \sin \Omega)^2} \quad (2)$$

$$T_{\text{transit}} = \frac{2a}{v_{\text{planet}}} \arcsin \frac{R_\beta}{a} \quad (3)$$

The duration of the transit,  $T_{\text{transit}}$ , relates the radius  $R$  of the bodies to the semi-major axis  $a$ , inclination  $i$ , longitude of ascending node  $\Omega$  and velocity  $v$  of the transiting planet.

$$P_{\text{transit}} = 2\pi \sqrt{\frac{a^3}{\mu}} \quad (4)$$

The period between each transit,  $P_{\text{transit}}$ , is equal to the orbital period of the planet (If there are no other planets in the system), and is dependent on the semi-major axis  $a$  and gravitational parameter  $\mu$  of the orbit.

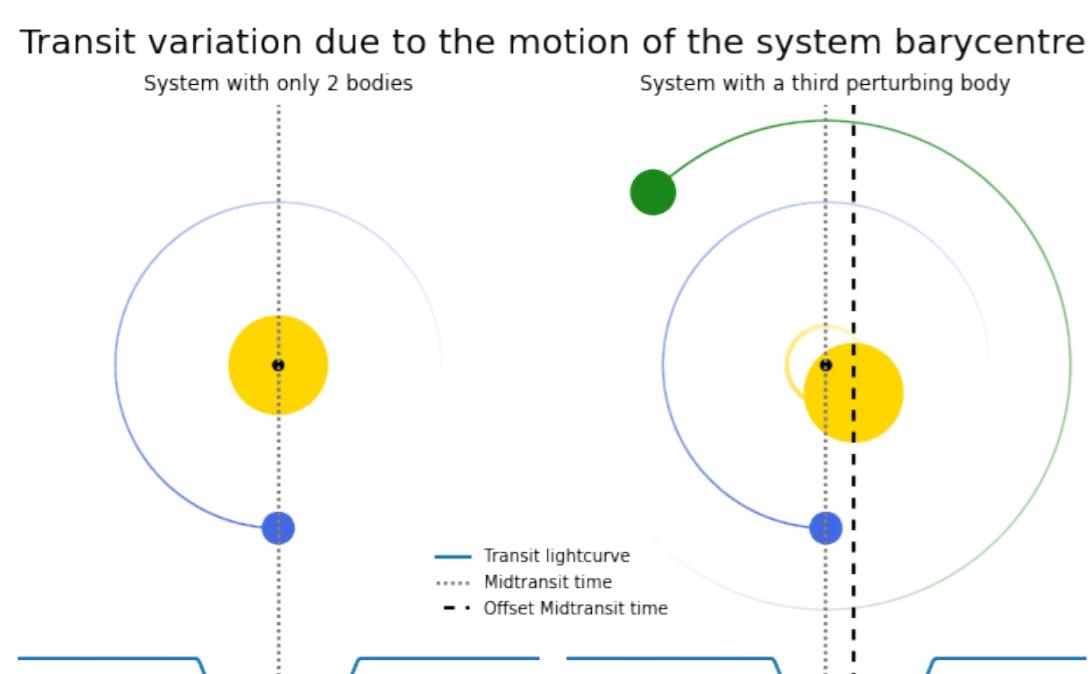
$$v = \sqrt{\mu \left( \frac{2}{r} - 1a \right)} \approx \sqrt{\frac{\mu}{a}} \approx \frac{2\pi a}{P_{\text{transit}}} \quad (5)$$

The velocity of an exoplanet,  $v$ , is given by the vis-viva equation. If the eccentricity of the orbit is small, this simplifies to the form in the middle, and can be combined with equation 4 to give the form on the right.

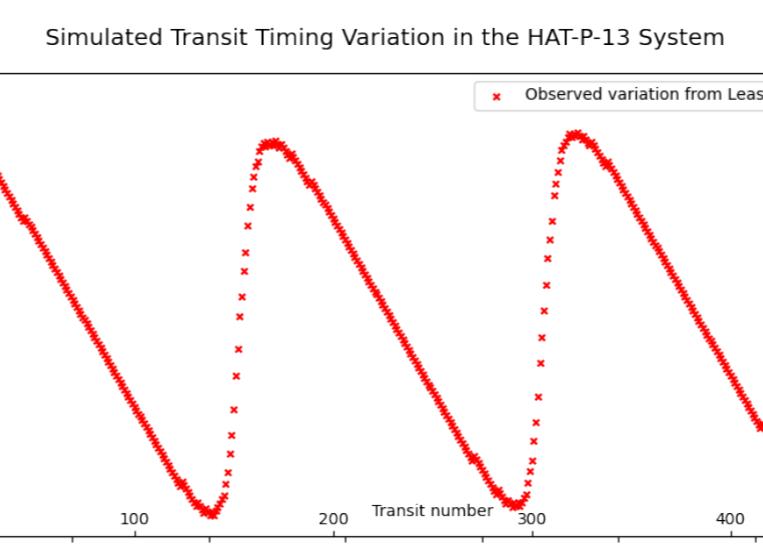
## Transit Variation

All bodies in a planetary system orbit about the centre of mass, or barycentre, of the system, which moves over time as a result of the orbits of each planet. As orbits are periodic, the motion of the barycentre can be described as a linear combination of sinusoids with periods equal to the orbital periods of the planets in the system.

The mid-transit time we measure from earth is highly dependant on the relative positions of the star and transiting planet. As the position of the star and transiting planet are both dependent on the location of the barycentre, it's motion will result in variations of the mid-transit time.



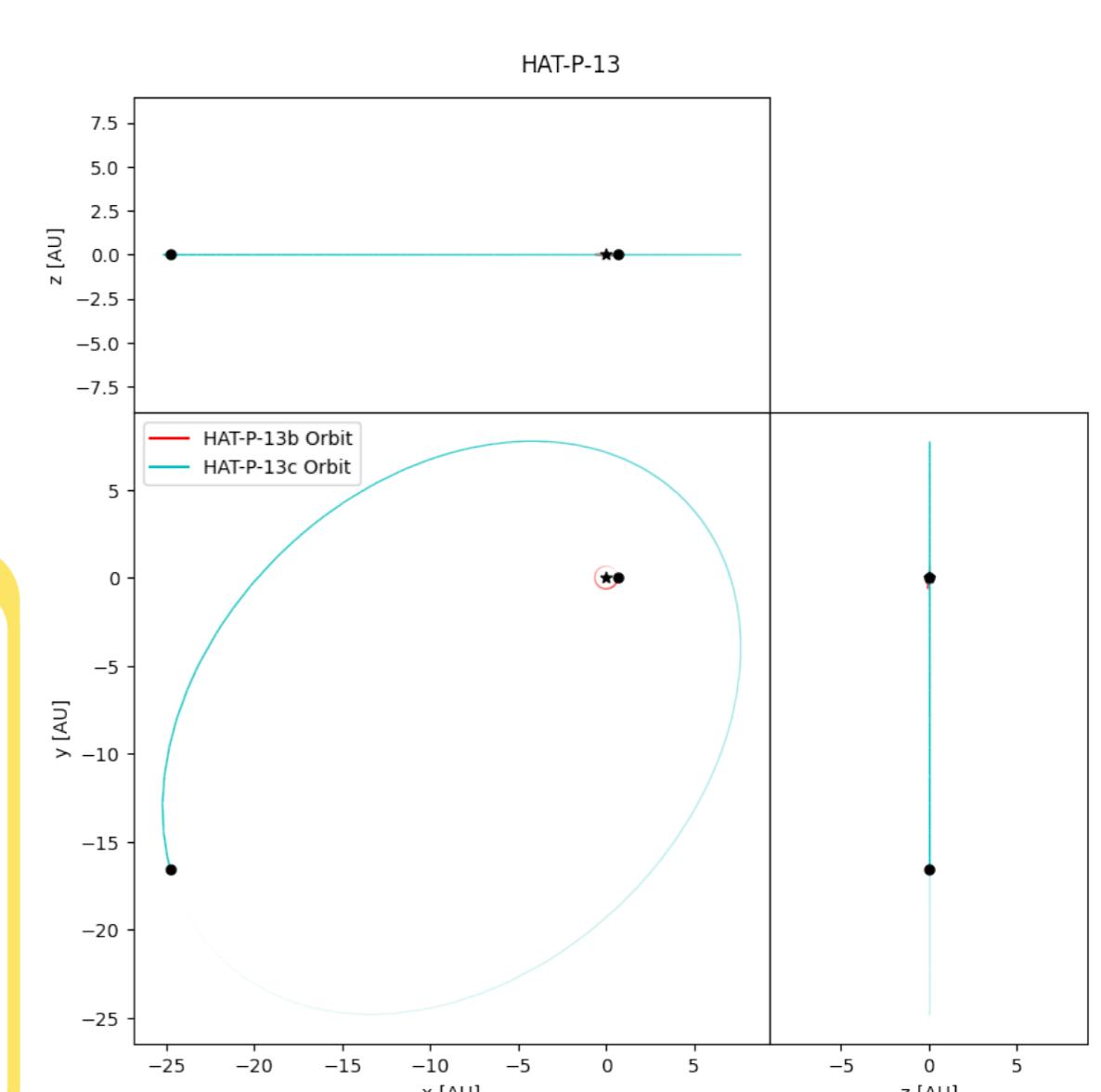
If we measure the timing of many transits and subtract that from the time predicted by a 2-body model of the transiting planet, we obtain an "O-C" curve. As we know that this curve is a linear combination of sinusoids, we can perform Fourier analysis to recover the individual components associated with the orbits of each planet in the system.



Left: Demonstration of how the motion of the barycentre due to a perturbing planet causes transit timing variation.

Above: Transit timing variation for HAT-P-13b, simulated over a timescale of 4.1 years. Note how the variation is cyclic on the order of the orbital period of HAT-P-13c (428 days).

Right: System layout of HAT-P-13. Note the high eccentricity of the perturbing planet (HAT-P-13c), and how that results in the lopsided TTV above.



## References

- [1] A. Kokori et al. "ExoClock Project. II. A Large-scale Integrated Study with 180 Updated Exoplanet Ephemerides". In: *The Astrophysical Journal Supplement Series* 258.2, 40 (2022-02), p. 40. DOI: [10.3847/1538-4365/ac3a10](https://doi.org/10.3847/1538-4365/ac3a10). arXiv: [2110.13863 \[astro-ph.EP\]](https://arxiv.org/abs/2110.13863).
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## Acknowledgements

The simulations in this poster made use of the REBOUND N-body code [5]. The simulations were integrated using IAS15, a 15th order Gauss-Radau integrator [6].