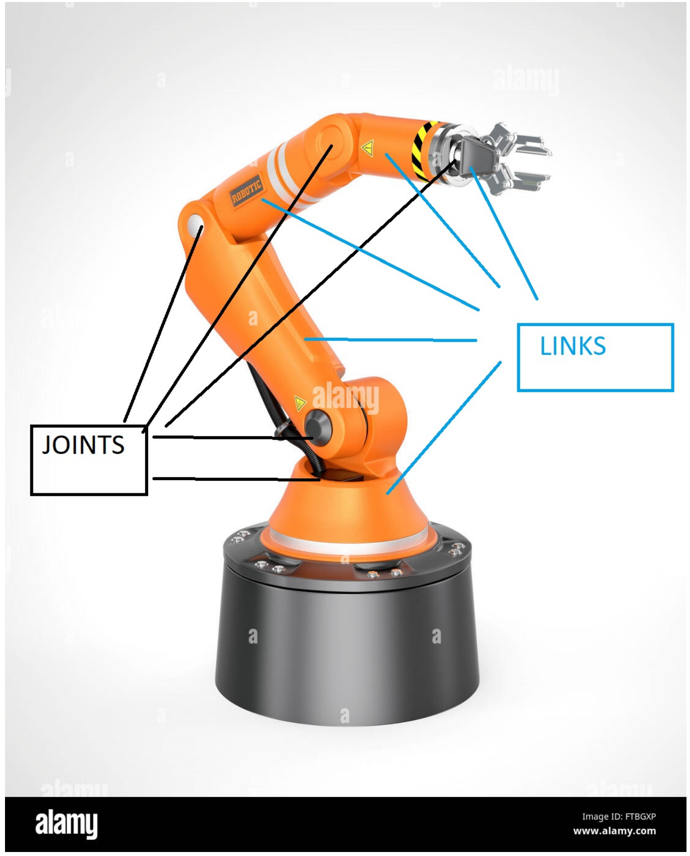
1. Problem 1: Go online and find any robot that you like and has the ability to move itself or parts of its body. You can find a collection of robots at https://robots.ieee.org/. For your chosen robot

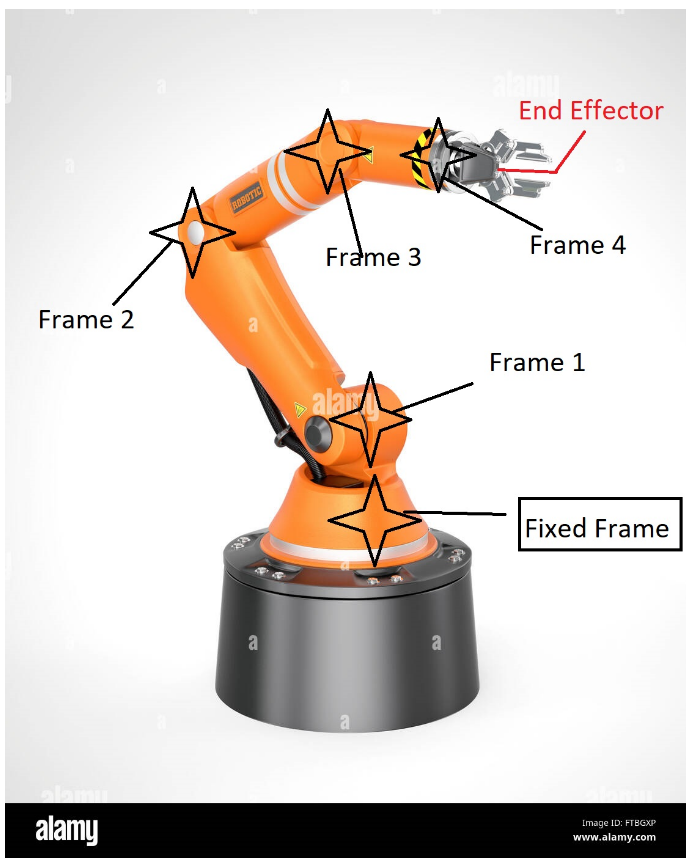
   (a) The ROBOT name is KUKA KR IONTIC Industrial Robotic Arm, KR IONTEC: a robot for a wide variety of applications in the medium payload category Whether on the floor, on the wall, or inclined, the KR IONTEC combines compact design with the largest working envelope in its class for optimal use of space with a small footprint. Equipped with a waterproof and dustproof in-line wrist and protected motors, the robot is suitable for almost every area of application. A Foundry option also enables use in extremely hot environments with an expanded temperature range from 0° to 55°C.

   (b) By definition, the robot's task space is the space in which the robot's task is naturally expressed. The decision of how to define the task space is driven by the task independently of the robot. For instance, a pick-and-place task may require only 3 DOFs while the robot arm has 6 DOFs. Task space is defined by the position and orientation of the end-effector of a robot. Joint space is defined by a vector whose components are the translational and angular displacements of each joint of a robotic link. The Below Picture is an example of task space like a robot picking the box in the industry.

   (c) Identify all the links and joints of this robot. You can indicate the links and joints on a picture of your chosen robot.



(d) Indicate where you would place frames for describing the motion of this robot appropriately.



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Q2.

a) The rotation of the square is about the z axis so we can write the rotation matrix as:

$$ {}^{R}R_{B} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

The translation of the square is along the point $\begin{bmatrix} 2 \\ 1.5 \\ 0 \end{bmatrix}$

So putting these matrices to make the homogenous transformation

$$ {}^{R}T_{B} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 2 \\ \sin\theta & \cos\theta & 0 & 1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

b) Done on Matlab

c) The pose of this frame is the same as the homogenous transformation. So pose $= \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 2 \\ \sin\theta & \cos\theta & 0 & 1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

d) ${}^{B}P_{2} = {}^{R}P_{1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ Frame 1 in the red ~~axis~~ frame has these points. The same is the case for ${}^{B}P_{2}$. ~~as~~

To find ${}^{B}P_{1}$ and ${}^{R}P_{2}$ we need to multiply the matrices.

We have ${}^{B}P_{2}$, ${}^{R}P_{1}$, ${}^{R}T_{B}$ and (taking its inverse) ${}^{B}T_{R}$

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So $\quad {}^{R}P_2 = {}^{R}T_B \times {}^{B}P_2$

and

$\quad {}^{B}P_1 = {}^{B}T_R \times {}^{R}P_1$

As shown in the matlab file.

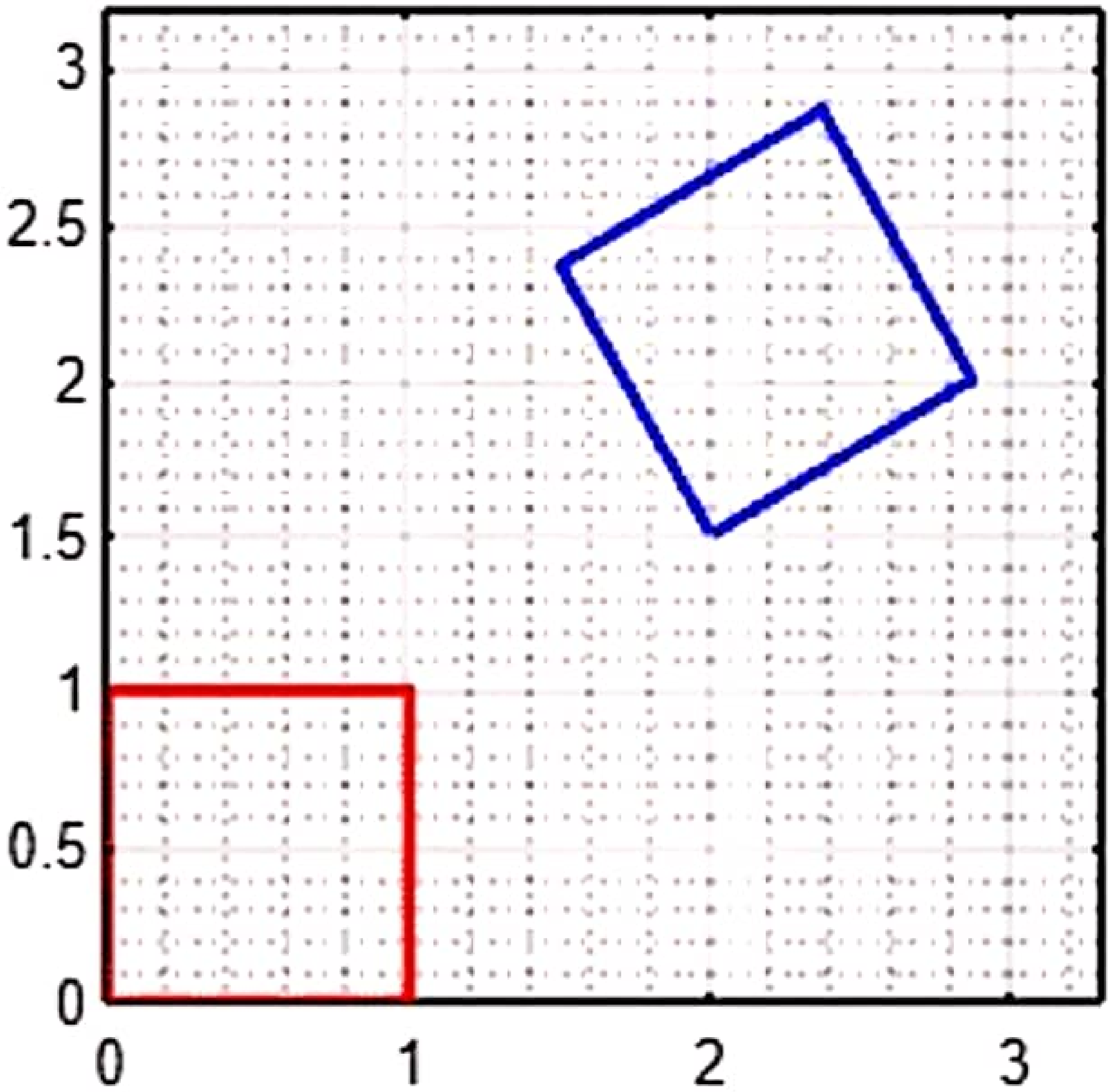
$${}^{R}P_2 = \begin{bmatrix} 2.866 \\ 2 \\ 0 \\ 1 \end{bmatrix} \quad \text{and} \quad {}^{B}P_1 = \begin{bmatrix} -1.61 \\ -0.79 \\ 0 \\ 1 \end{bmatrix}$$

e). We get ${}^{B}P_1$ by simply multiplying two matrices

$${}^{B}P_1 = {}^{B}T_R \times {}^{R}P_1$$

OR $\quad {}^{B}P_1 = {}^{B}T_R \times {}^{B}P_2$

f). As done in part d and shown in the matlab code to confirm we get ${}^{B}P_2$ by multiplying ${}^{B}T_R$ and ${}^{R}P_2$ together.

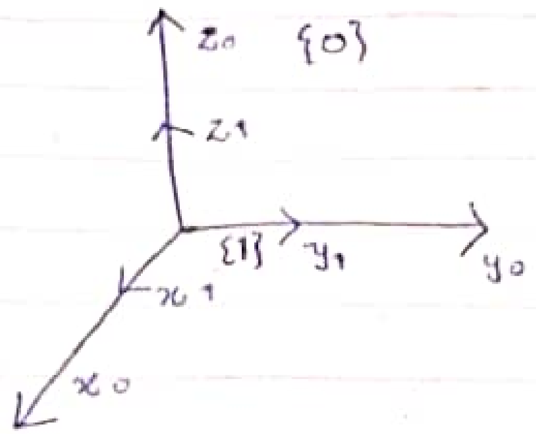

3
2.5
2
1.5
1
0.5
0
0
1
2
3

```
B_P_1 = 4x1

   -1.6160
   -0.7990
         0
    1.0000


B_P_2 = 4x1

    1.0000
    0.0000
         0
    1.0000
```

R_P_2 = 4×1

```
    2.8660
    2.0000
         0
    1.0000
```

Question # 03
Solution
Part (a)



frame {0} and {1} initially allinged with each other frame

1- $${}^{0}R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

2- $${}^{0}R_2 = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

3- $${}^{0}R_3 = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

∵ Due to the rotation about fixed frame we multiplied on the left

$$^{0}R_4 = Rot(\hat{z},\gamma)\ Rot(\hat{y},\beta)\ Rot(\hat{x},\alpha)$$

$$^{0}R_4 = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

Part (b)

∵ Now the Rotation is about moving frame instead of fixed frame so we multiplied on the Right side

$$^{0}R_4 = {}^{0}R_3\ Rot(\hat{z},\gamma)$$
$$= Rot(\hat{y},\beta)\ Rot(\hat{x},\alpha)\ Rot(\hat{z},\gamma)$$

$$
{}^{0}R_{4} = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}
$$

$$
\begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

Part (C)

1- Rotate {1} about {1} frame $\underline{\hat{y}}$ axis by $\underline{\beta}$

2- Rotate {2} about {2} frame $\underline{\hat{x}}$ axis by $\underline{\alpha}$

3- Rotate {3} about {3} frame $\underline{\hat{z}}$ axis by $\underline{\gamma}$

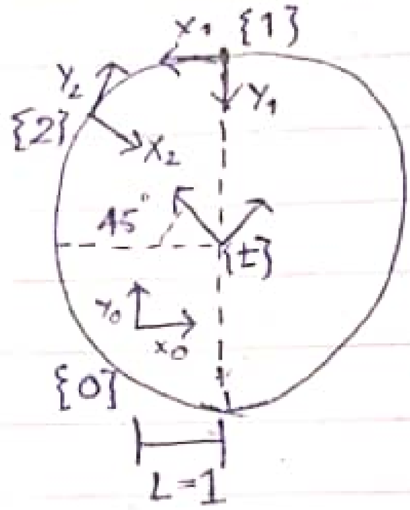
Answer

Question # 04

Solution

⇒ Car 1 moves at a constant speed V_1 along the Circumference of table

⇒ Car 2 moves at a constant speed V_2 along the radius, the Position of two Vehicles at t = 0



The Origin is at the center of the table and whose orientation is $Rot(\hat{z}, \theta)$ with respect to the frame {0}

Assume

→ $\theta = 45°$ to fram t_2

→ $\theta = V_1 t / r$ to frame t_1

As shown in above figure

Part (a)

Inorder to find ${}^0T_1$ and ${}^0T_2$ transformation Matrix, first we need to find the transformation Matrix of frame [0], from [1] and frame [2]

⇒ Length and Height is use to obtain the Position Vector and insert into its corresponding transformation frame Matrix as a function of t

General Form of Transformation Matrix

$$\therefore \begin{bmatrix} \begin{bmatrix} & {}^0_1R(t) & \end{bmatrix} & \begin{bmatrix} {}^0_1O(t) \end{bmatrix} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

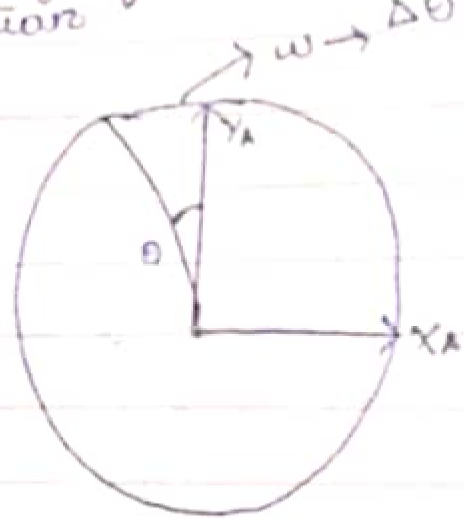
L = 1
H = 2
R = 2

$$\Rightarrow T_{0t} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & L \\ \sin\theta & \cos\theta & 0 & L \\ 0 & 0 & 1 & H \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow T_{t_1 1} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & R \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow T_{t_2 2} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & R - V_1 t \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now we can find the required transformation



$V = \omega r$

$$\text{i)} \; - {}^{0}T_1 = T_{0t_1} T_{t_1 1} \quad \left(\text{subtitute } \theta = V_1 t / r\right)$$

$$\because A = \theta = V_1 t / r$$

$$= \begin{bmatrix} -\cos A & \sin A & 0 & 1 - 2\sin A \\ -\sin A & -\cos A & 0 & 1 + 2\cos A \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

After Multiplication we got the above ${}^{0}T_1$ Matrix

$\therefore$ ${}^0T_2 = T_{0t_2} T_{t_2 2}$ (Subtitute $\theta = 45°$)

$$= \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 & 1-\sqrt{2}+\frac{\sqrt{2}}{2} v_2 t \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 & 1+\sqrt{2}-\frac{\sqrt{2}}{2} v_2 t \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Part (b) ✓

find ${}^1T_2$ as a function of t.

By using transformation Property

$\Rightarrow T = T^{-1} = T^T$

so we can write the above obtain Matrix in ${}^0T_1^{-1}$ which is equal to ${}^1T_0$.

$${}^1T_2 = T_{01}^{-1} T_{02}$$

$$= \begin{bmatrix} R_{01} & P_{01} \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} R_{02} & P_{02} \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ By apply Inverse

$$= \begin{bmatrix} R_{01}^{T} & -R_{01} P_{01} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{02} & P_{02} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\cos V_1 t/2 & -\sin V_1 t/2 & 0 & \sin \frac{V_1 t}{2} + \cos \frac{V_1 t}{2} \\ \sin V_1 t/2 & -\cos V_1 t/2 & 0 & 2 - \sin \frac{V_1 t}{2} + \cos \frac{V_1 t}{2} \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 & 1 - \sqrt{2} + \frac{\sqrt{2}}{2} V_2 t \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 & 1 + \sqrt{2} - \frac{\sqrt{2}}{2} V_2 t \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The Product of these two Matrix give the Transformation of ${}^{1}T_2$

Answer

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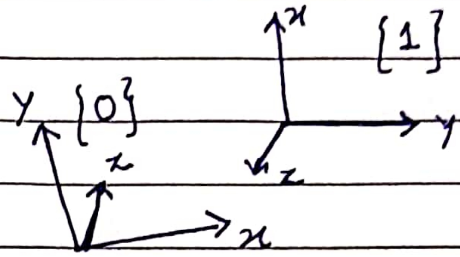
Q5 a). The rotation here is first around the x axis by 180°
Once that is done, its rotated along the z axis by 90°.



For $R_x(180)^\circ$ $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$ and translated by $\begin{bmatrix} r\cos\theta \\ r\sin\theta \\ r\emptyset \end{bmatrix}$

So Homogenous transformation would be

$$T_x \quad {}^{\phi}H_{1} = \begin{bmatrix} 1 & 0 & 0 & r\cos\theta \\ 0 & \cos\theta & -\sin\theta & r\sin\theta \\ 0 & \sin\theta & \cos\theta & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $\theta = 180^\circ$ and $r = 2$

So

$${}^{\emptyset}H_{1} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And about z axis by 90°

$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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M	T	W	T	F	S	S

Now to get the frame multiplying the two matrices

$$^{0}T_1 = \begin{bmatrix} \cos\alpha & -\sin\alpha\cos\theta & -\sin\alpha(-\sin\theta) & r\cos\psi \\ \sin\alpha & \cos\alpha\cos\theta & \cos\alpha(-\sin\theta) & r\sin\psi \\ 0 & \sin\theta & \cos\theta & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This is our first frame. And since every frame is basically multiplied by a rotation of 45° about the z axis, instead of doing all the steps manually we can just add a factor of $45° \times i$. where 'i' is the frame number and every iteration will result in rotation by 45° from the previous frame. ψ in this represents '$45° \times i$' which will also be added in the α factor in the code

) To rotate frame 2 by 45° along the z-axis we'll have to muliply the frame 2 by a rotation matrix along z by 45°.

So our frame 2 was

$$\begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

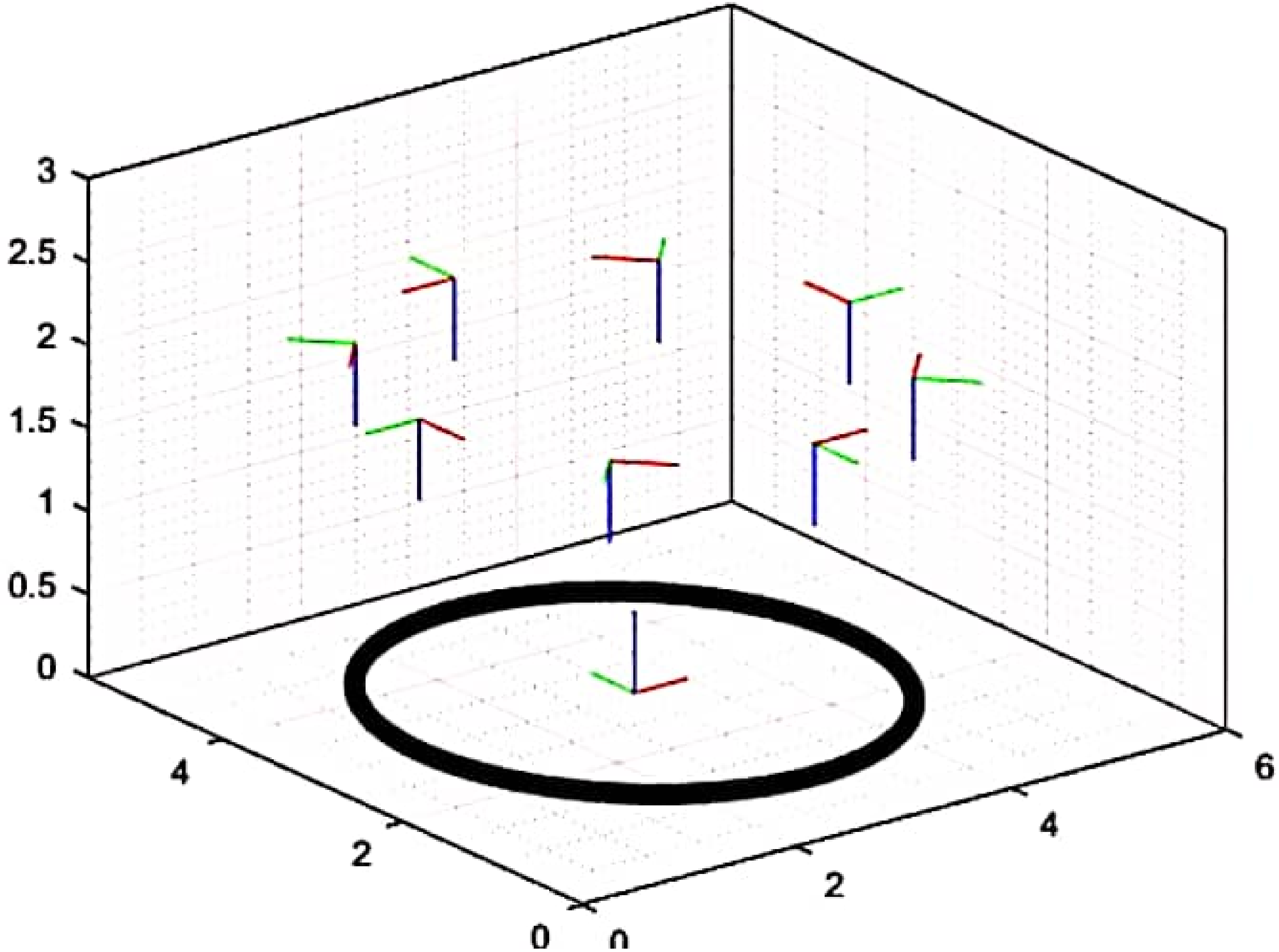
→ Only the rotation matrix part is taken for further multiplication

The 45° rotation around z axis would be

$$\begin{bmatrix} \cos(-45) & -\sin(-45) & 0 \\ \sin(-45) & \cos(-45) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ The resulting matrix is by multiplying the two matrices →

$$\begin{bmatrix} -0.7071 & 0.7071 & 0 \\ 0.7071 & 0.7071 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$


3
2.5
2
1.5
1
0.5
0
4
2
0
0
2
4
6

Comments:

Hussain: This took a whole night and a day to complete and overall I found it very hard. In this Assignment, I learned about the Transformation and Rotation mechanism of robotic arms, and Questions # 3 and 4 took most of the time, and in order to understand the problem, we mutually discussed and solved all the problems in this assignment.

Rida: I spent 3 days completing my part in this assignment (not whole 3 days but small chunks from them). Approximately took me 7 hours to completely understand the questions, its requirements and what we needed to do in it. My part in this assignment was doing question 2 and question 5. For me, these questions were not too hard, it was just understanding the question and applying transformations. This assignment served as a good practice and made me realize on what questions to expect in the future assignments.