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& Syed Muhammad Hussain.

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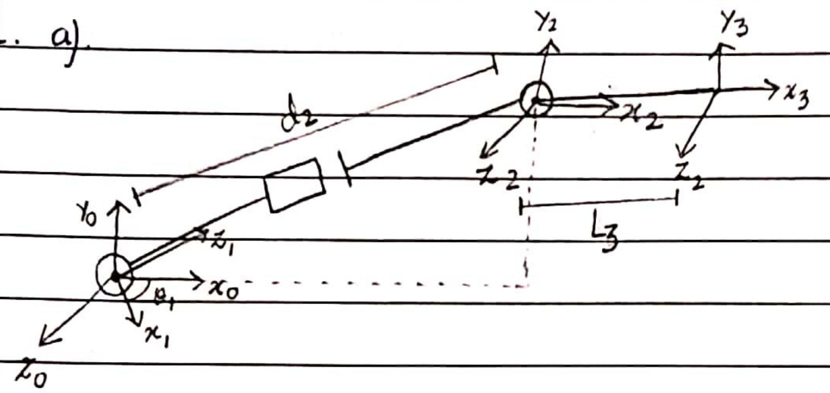
Robotics HW 2

Q 1. a).



From the above drawn diagram, we can say that the coordinates of x_2 and y_2 will be

$\cancel{x_2 = d_2 \cos\theta_1}$ $\qquad$ $\cancel{x_2 = -d_2 \sin\theta_1}$

$\cancel{y_2 = d_2 \sin\theta_1}$ $\qquad$ $\cancel{y_2 = d_2 \cos\theta_1}$

$$x_2 = d_2 \left(\cos \theta_1 + \pi/2\right)$$

$$y_2 = d_2 \left(\sin \theta_1 + \pi/2\right)$$

From angle identities, we deduce that

$$x_2 = -d_2 \sin\theta_1$$

$$y_2 = d_2 \cos\theta_1$$

Now to write x and y in Terms of θ_1, d_2 and θ_3, by inspection we can write

$$x = x_2 + L_3 \cos(\theta_1 + \theta_3)$$

$$y = y_2 + L_3 \sin(\theta_1 + \theta_3)$$

Thus

$$x = L_3 \cos(\theta_1 + \theta_3) - d_2 \sin\theta_1$$

$$y = L_3 \sin(\theta_1 + \theta_3) + d_2 \cos\theta_1$$

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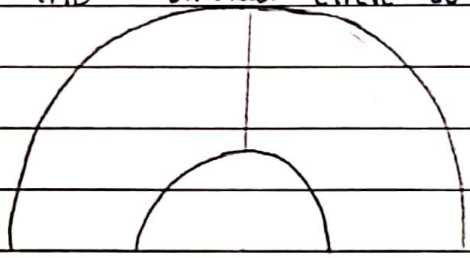
b) It's reachable workspace would be like. When all the joints will be straight and we move the first revolute joint, then it will be making a semicircle in the outer edge. Inside, the arm will never be able to touch the smaller circle so it is not a part of reachable workspace.



c) If the desired position is in the interior of the workspace then the number of solutions can be 2. i.e. there are 2 ways in which the desired position can be achieved. As there are 2 equations and 3 unknowns. Where as if the given x, y position is on the boundary of the workspace then there can be only one possible solution.

d) If the orientation is also specified then it is possible for ~~multiple~~ only one solutions to exist, if the end effector position is inside the reachable workspace. As then, we will be having 3 equations and 3 unknowns. As $\phi = \theta_1 + \theta_3$ will also be a ~~solut~~ equation.
Incase the position is at the end of the workspace or on the boundary then there can be ~~two~~ no solutions or one solution, depending on the geometry of the robot.

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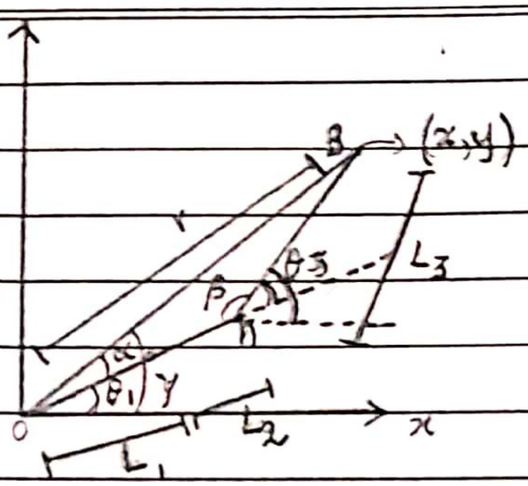
Q1 e)

When only (x, y) is given:
In the given figure, θ_1 and θ_3 are the angles formed by the two revolute joints and the prismatic joint is incorporated in $L_1 + L_2$.



$$\gamma = \tan^{-1}\left(\frac{y}{x}\right)$$

Applying the cosine Law on the Δ OAB

$$\cos\alpha = \frac{r^2 + (L_1+L_2)^2 - L_3^2}{2r(L_1+L_2)}, \quad \cos\beta = \frac{(L_1+L_2)^2 + L_3^2 - r^2}{2(L_1+L_2)L_3}$$

where $r^2 = x^2 + y^2$

$$\theta_1 = \gamma - \alpha, \quad \theta_3 = 180^\circ - \beta$$

Here, $L_1 + L_2 = d_2$

So $$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \cos^{-1}\left(\frac{r^2 + d_2^2 - L_3^2}{2 r d_2}\right)$$

and

$$\theta_3 = 180^\circ - \cos^{-1}\left(\frac{d_2^2 + L_3^2 - r^2}{2 d_2 L_3}\right)$$

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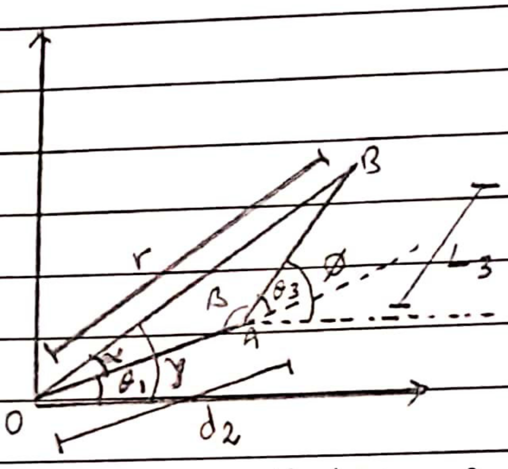
e) ii) When x, y, ϕ are given:



Here, in addition to the previous parameters (coordinates) we also know the angle $\emptyset$ as shown in the figure. So we can write

$$\emptyset = \theta_1 + \theta_3 \quad \cdot \text{eq(i)}$$

$$\cancel{\emptyset = \left(\tan^{-1}\left(\frac{y}{x}\right) - \cos^{-1}\left(\frac{r^2 + d_2^2 - L_3^2}{2rd_2}\right)\right) + 180^\circ - \left(\cos^{-1}\left(\frac{d_2^2 + L_3^2 - r^2}{2d_2L_3}\right)\right)}$$

Using cosine law on ΔOAB

$\gamma = \tan^{-1}\left(\frac{y}{x}\right)$ and $\cos\alpha = \frac{r^2 + d_2^2 - L_3^2}{2rd_2}$.

We find θ_1 by

$$\theta_1 = \gamma - \alpha \Rightarrow \tan^{-1}\left(\frac{y}{x}\right) - \cos^{-1}\left(\frac{r^2 + d_2^2 - L_3^2}{2rd_2}\right)$$

and using eq(i) we can write

$\theta_3 = \emptyset - \theta_1$. We found θ_1 and $\emptyset$ is already known so

$$\theta_3 = \emptyset - \tan^{-1}\left(\frac{y}{x}\right) + \cos^{-1}\left(\frac{r^2 + d_2^2 - L_3^2}{2rd_2}\right)$$

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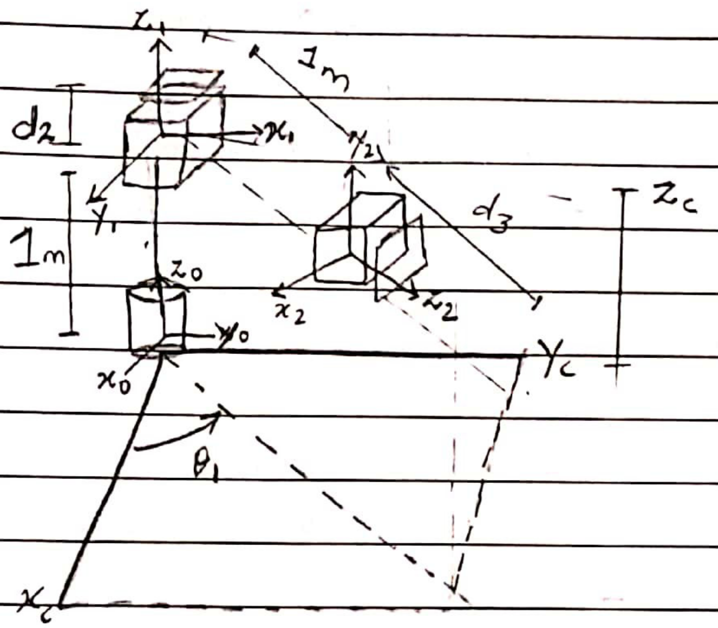
Q3.



Projecting O_c (the line connecting the end effector and the base frame) on the x_0-y_0 plane we get this picture.
Here we can find θ_1 to be
$\theta_1 = \arctan 2(y_c, x_c)$
Another solution for θ_1 can be
$\theta_1 = \arctan(y_c, x_c) + 180°$

We can find d_2 by simply

$d_2 = z - 1$ as we know z and we know that the length from the first prismatic joint and the revolute joint is 1m.

To find d_3 we can see that the line $d_3 + 1$ is parallel to the line on x_0-y_0 plane. So we can write

$$(d_3 + 1)^2 = x_0^2 + y_0^2$$
$$d_3 = \sqrt{x_0^2 + y_0^2} - 1$$

Q4. In the figure, we can see the circle made by putting $\theta_1 = \text{atan}(y, x)$. But if I put π too in θ_1 then the circle is made somewhere outside the frame. This maybe because we have two prismatic joints. The three joints in the robot provide different degree of freedom, and hence we get a ~~diff~~ unique solution (3 unknowns, 3 equations). Thus we can say that there is no singularity.

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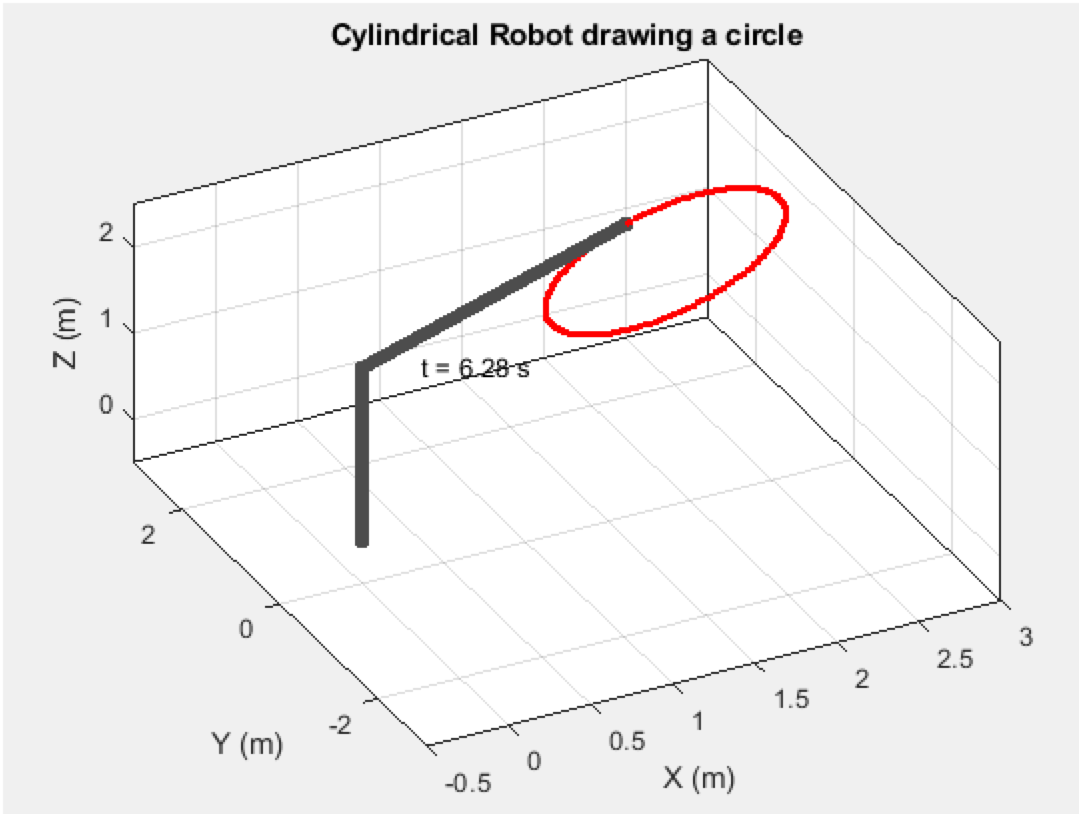
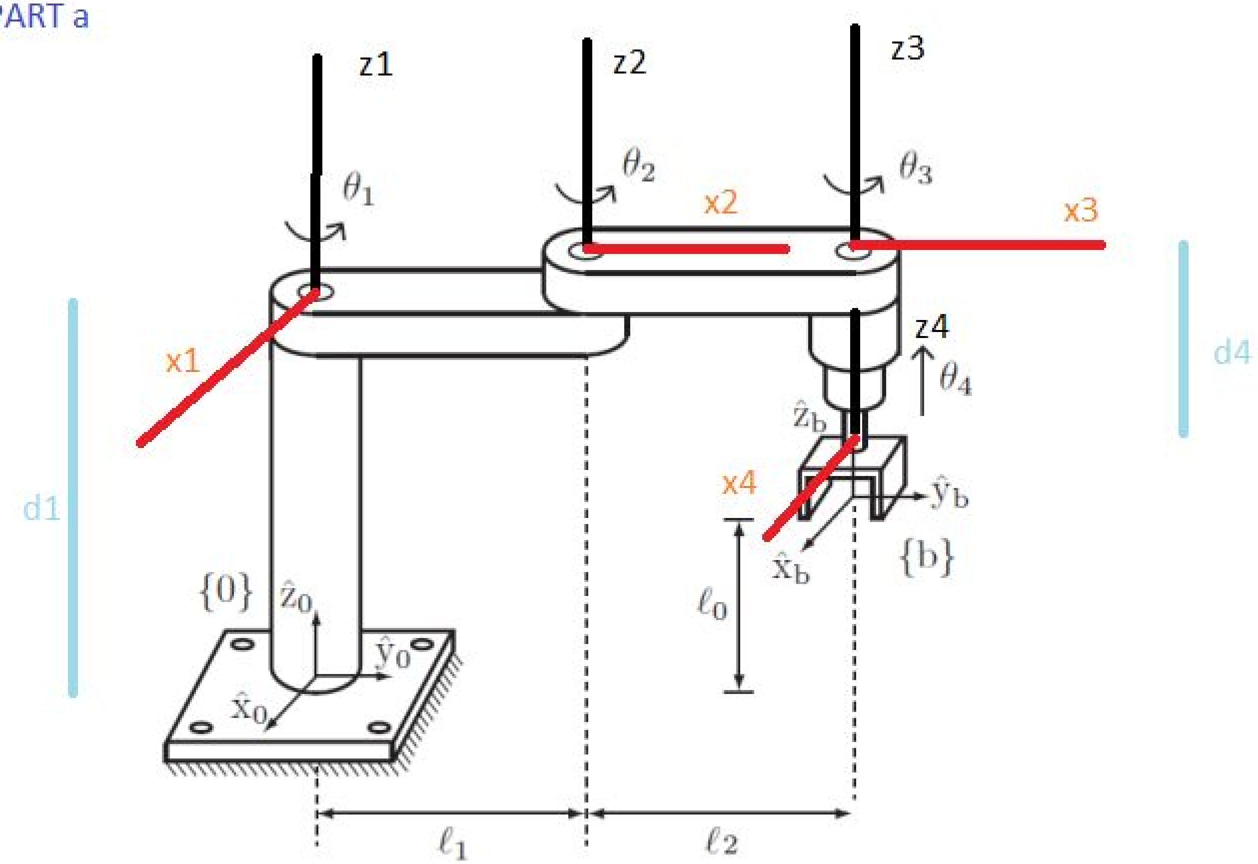


Cylindrical Robot drawing a circle
Z (m)
2
1
0
t = 6.28 s
Y (m)
2
0
-2
X (m)
-0.5
0
0.5
1
1.5
2
2.5
3

Question 2 PART a



Question #02

Solution

→ Part b and c

⇒ Link Parameters

Link	a_i	α_i	d_i	θ_i
1	0	0°	d_1	θ_1
2	L_1	0°	0	θ_2
3	L_2	0°	0	θ_3
4	0	0°	d_4	-90°

Now

$$A_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We have to find A_1, A_2, A_3, A_4 in order to obtain ${}^0T_6$

$$A_1 = {}^0T_1$$
$$A_2 = {}^1T_2$$
$$A_3 = {}^2T_3$$
$$A_4 = {}^3T_6$$

$$A_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & l_1\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & l_1\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & l_2\cos\theta_3 \\ \sin\theta_3 & \cos\theta & 0 & l_2\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad {}^{0}T_b = A_1 A_2 A_3 A_4$

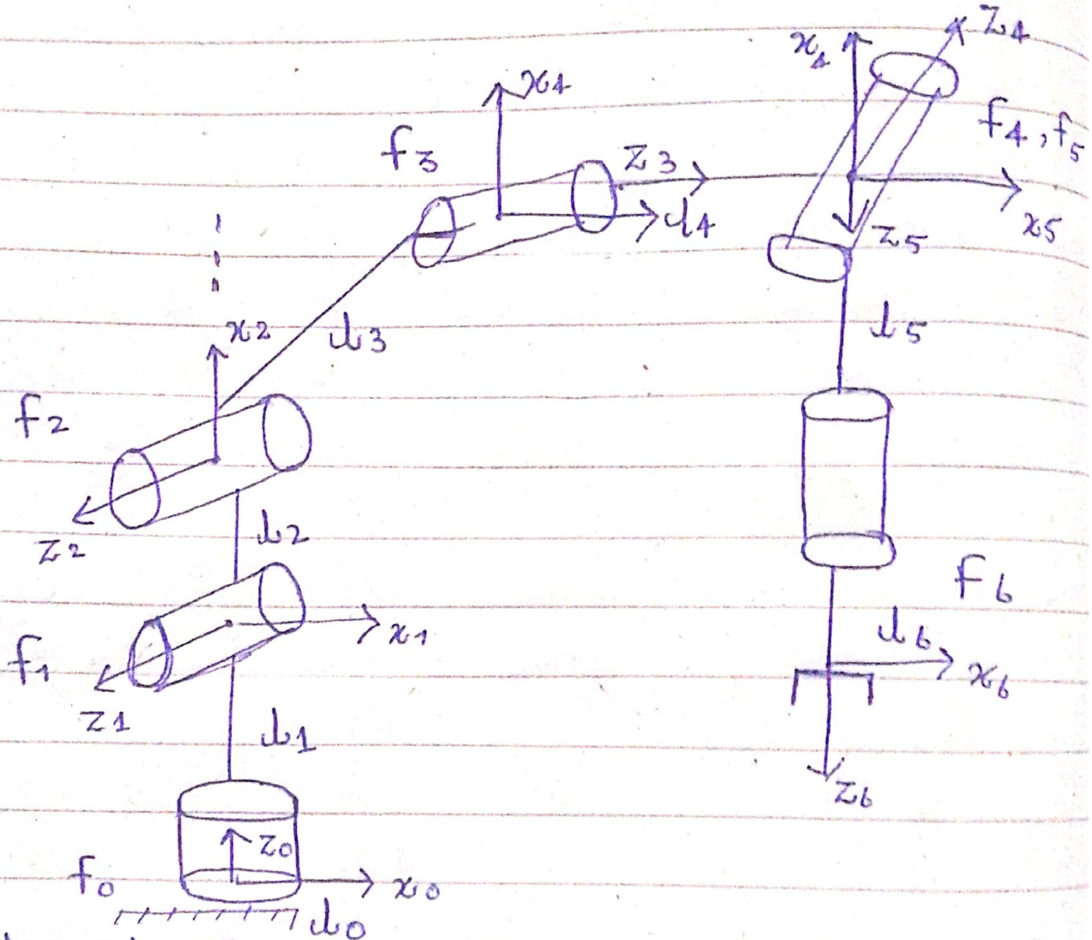
after simplying these we get the final Matrix Transformation

Answer

Question # 05

Solution

=> Part a and b



=> base frame torso

- link = d
- frame = f

⇒ Part C (Link Parameters)

Link	a_i	α_i	d_i	θ_i
1	0	90°	0	θ_1
2	338.5	0°	0	θ_2
3	0	90°	0	θ_3
4	0	-90°	403.3	θ_4
5	0	90	0	θ_5
6	0	0°	0	θ_6

⇒ Length are based on assumptions

Part d (Expression for Inverse Kinematics)

⇒ For θ_1 :-

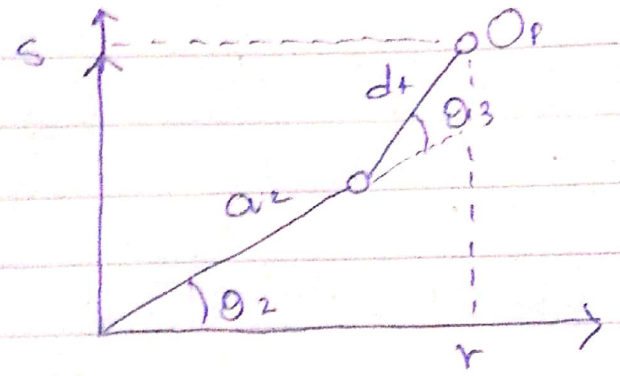
$\Rightarrow \theta_1 = \tan^{-1} y/x$

$\Rightarrow \theta_1 = 180 + \tan^{-1} y/x$

by using the x-y plane projection

⇒ For Spherical Wrist

three frame's Z-axis intersects at one point (Z_3, Z_4, Z_5)



⇒ let consider the intersect Point O_p with its co-ordinate ($x_p . y_p, z_p$)

By using cosine law

we know that by above diagram

$r^2 = x_p^2 + y_p^2$

$s^2 = (z_p - d_1)^2 = z_p^2$

∵

$$\Rightarrow \cos\theta_3 = \frac{r^2+s^2-a_2^2-d_4^2}{2a_2d_4} = D$$

Now $\sin\theta_3 = \sqrt{1-D^2}$

$$\Rightarrow \theta_3 = \tan^{-1} \frac{\sin\theta_3}{\cos\theta_3}$$

$$\Rightarrow \theta_2 = \tan^{-1} \frac{z_p}{r} - \tan^{-1} \frac{d_4 \sin\theta_3}{a_2+d_4\cos\theta_3}$$

$\Rightarrow$ The end effector position is also fixed with respect to θ_6 and θ_6 transformation matrix can be obtained by arbitrary values

$$^0T_6 = \begin{bmatrix} c_6c_4c_5 & -s_6c_4c_5-c_6s_4 & c_4s_5 & 0 \\ s_4c_6c_6 & -s_4c_5c_6+c_3c_6 & s_4s_5 & 0 \\ -s_5c_6 & s_5s_6 & c_5 & 403 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0R_6 = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix}$$

Now

$$\theta_4 = \tan^{-1}\left(\frac{\pm\sqrt{1-t_{33}^2}}{t_{33}}\right), \quad \theta_5 = \tan^{-1}\left(\frac{\pm t_{23}}{t_{13}}\right)$$

$$\theta_6 = \tan^{-1}\left(\frac{t_{32}}{t_{31}}\right)$$

Am

QUESTION # 6

Rida: I spent approximately 12 hours completing my part in this homework (over a span of three days). I completed question 1, 3 and 4 in this homework. This helped me clear my concepts of inverse and forward kinematics and understand how different joints (revolute and prismatic) play their role in determining the parameters and equations for the robot.

Hussain: I spent 4 days on this assignment because first I graps the grip on forward and inverse kinematics strongly and practice through slides then attemp the homework question Question 2 was not too hard but question 5 was very interesting it take my most of the time to research about Boston dynamics arm configurations and chain structure.