Scheduling Problem and Local Search

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Outline

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Problem Background

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    - Assistants should not work at multiple places in the meantime.
    - Work times among students should not vary widely.

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- Method: construct a matrix, with assistants as rows, time periods as column.

Matrix Notation

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

- A matrix M representing the free time period.
- OPERATION: change a certain entries from 1 to 0.
- TARGET: $\forall j \in [1, m], \sum_{i=1}^{n} M_{ij}$ and $\forall i \in [1, n], \sum_{j=1}^{m} M_{ij}$

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What will you probably do to solve this problem?

Evaluation

How to evaluate an algorithm?

- Efficiency?

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- Flexibility

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How to evaluate an algorithm?

- Efficiency?
- Flexibility
- Extendability

First Attempt: Genetic Algorithm

- An algorithm adapting Darwin's evolution theory by natural selection
- Steps
  - Initialization
  - Evaluation
  - Selection
  - Crossover
  - Mutation
  - Termination
- Pros & Cons

Second Attempt: Encoding into Constraint Satisfaction Problem

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- Typically, There are two type of algorithms.
  - complete search
  - incomplete search, or local search

Local Search

A general framework of local search:

- Randomly generate a solution.
- find a possible better variable to flip.
- if stuck in a local optima, restart the procedure.

Local search has cycling problem!

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Extensions

- Nurse Scheduling Problem

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- Course Scheduling Problem