Interpolation

Newton's Forward And Backward Difference Formula

Theory

If $y_0, y_1, y_2, \ldots\ldots y_n$ denote a set of values of y, then $y_1 - y_0$, $y_2 - y_1$, ….. $y_n - y_{n-1}$ are called the first forward differences of y. Denoting these differences by $\Delta y_0, \Delta y_1, \ldots.. \Delta y_{n-1}$ respectively we have,

$\Delta y_0 = \Delta y_1 - \Delta y_0$, $\Delta y_1 = \Delta y_2 - \Delta y_1$, $\Delta y_2 = \Delta y_3 - \Delta y_2$, …… $\Delta y_{n-1} = \Delta y_n - \Delta y_{n-1}$

$h = x_0 - x_1$, $P = \frac{x - x_0}{h}$

Where Δ is called the Forward Difference Operator. And Δy_1 , Δy_0 are called first forward differences. The differences of first forward differences are called second forward differnces and are denoted by $\Delta^2 y_0$, $\Delta^2 y_1$ …… $\Delta^2 y_n$, Similarly third, fourth, fivth ,…… nth forward differences can be determined. Forward Difference formula for finding the unknown value of y(x) for the value of x is :

$$y_n(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0 + \cdots + \frac{P(P-1)(P-2)\ldots\ldots(P-n+1)}{n!}\Delta^n y_0 \quad \ldots\ldots\ldots (1)$$

If y_0, y_1, y_2 , ……. yn denote a set of values of y, then $y_1 - y_0$, $y_2 - y_1$, ….. $y_n - y_{n-1}$ are called the first backward differences of y. Denoting these differences by $\blacktriangledown y_0$, $\blacktriangledown y_1$,….. $\blacktriangledown y_{n-1}$ respectively we have,

$\blacktriangledown y_0 = \blacktriangledown y_1 - \blacktriangledown y_0$, $\blacktriangledown y_1 = \blacktriangledown y_2 - \blacktriangledown y_1$, $\blacktriangledown y_2 = \blacktriangledown y_3 - \blacktriangledown y_2$, …….. $\blacktriangledown y_{n-1} = \blacktriangledown y_n - \blacktriangledown y_{n-1}$

$h = x_0 - x_1$, $P = \frac{(x - x_0)}{h}$

Where $\blacktriangledown$ is called the backward Difference Operator. And $\blacktriangledown y1$, $\blacktriangledown y0$ are called first backward differences. The differences of first backward differences are called second backward differnces and are denoted by $\blacktriangledown^2 y_0$, $\blacktriangledown^2 y_1$ …… $\blacktriangledown^2 y_n$, Similarly third, fourth, fivth ,…… nth backward differences can be determined. backward Difference formula for finding the unknown value of y(x) for the value of x is :

$$y_n(x) = y_n + P\blacktriangledown y_n + \frac{P(P+1)}{2!}\blacktriangledown^2 y_n + \frac{P(P+1)(P+2)}{3!}\blacktriangledown^3 y_n + \ldots\text{-.-} + \frac{P(P+1)(P+2)\ldots\ldots(P+n+1)}{n!}\blacktriangledown^n y_n \quad \ldots\ldots (2)$$

Code

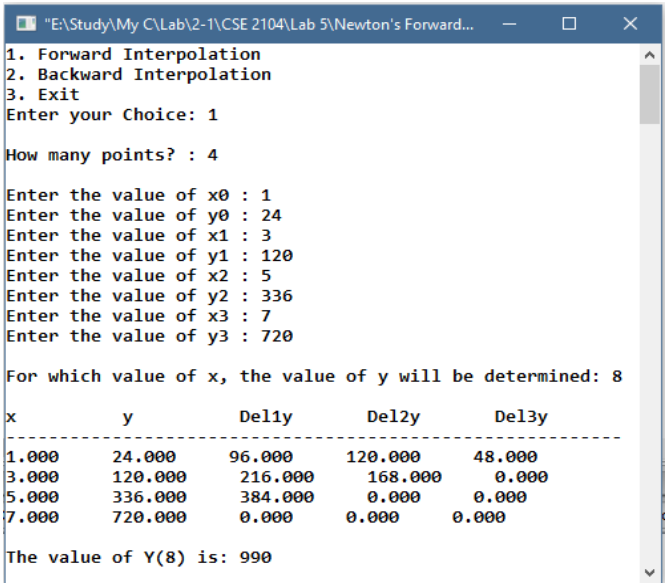
```
#include<bits/stdc++.h>
using namespace std;

int fact(int n)
{
   if(n==0 || n==1)
      return 1;
   else if(n>1)
      return n*fact(n-1);
}

void PrintForTable(double x[],double y[],double Del1[], double Del2[], double Del3[])
{

   int i,j;

   cout<<"x          y          Del1y       Del2y       Del3y"<<endl;
   cout<<"-------------------------------------------------------"<<endl;

   for(i=0 ; i<4 ; i++)
   {
      printf("%0.3lf     %0.3lf     %0.3lf     %0.3lf     %0.3lf\n",x[i],y[i],Del1[i],Del2[i],Del3[i]);
   }
   printf("\n");
}

void PrintBackTable(double x[],double y[],double Del1[], double Del2[], double Del3[], double
Del4[], double Del5[])
{

   int i,j;

   cout<<"x                  y          Del1y           Del2y           Del3y        Del4y
Del5y"<<endl;
   cout<<"-------------------------------------------------------------------------------------------------
--"<<endl;

   for(i=0 ; i<6 ; i++)
   {
      printf("%0.7lf      %0.7lf      %0.7lf      %0.7lf      %0.7lf      %0.7lf
%0.7lf\n",x[i],y[i],Del1[i],Del2[i],Del3[i],Del4[i],Del5[i]);
   }
   printf("\n");
}
```

```
void Forward()
{
   int points,flag=0,i,j=0,k=0,l=0;
   double x[10],y[10],h,p,X,Del1[10],Del2[10],Del3[10],Y;
   for(int x=0;x<10;x++)
   {
      Del1[x] = 0; Del2[x] = 0; Del3[x] = 0;
   }
   cout<<"\nHow many points? : ";
   cin>>points;  printf("\n");
   for(i=0;i<points;i++)
   {
      cout<<"Enter the value of x"<<i<<" : " ;
      cin>>x[i];
      cout<<"Enter the value of y"<<i<<" : ";
      cin>>y[i];
   }
   printf("\n");
   i = 0;
   while(i<points-1)
   {
      Del1[i] = y[i+1] - y[i];
      i++;
   }
   i = 0;
   while(i<points-2)
   {
      Del2[i] = Del1[i+1] - Del1[i];
      i++;
   }
   i = 0;
   while(i<points-3)
   {
      Del3[i] = Del2[i+1] - Del2[i];
      i++;
   }
   cout<<"For which value of x, the value of y will be determined: ";
   cin>>X;
   h = x[1] - x[0];
   p = (X - x[0])/h;
   printf("\n");
   PrintForTable(x,y,Del1,Del2,Del3);
   Y = y[0] + (p/fact(1))*Del1[0] + ((p*(p-1))/fact(2))*Del2[0] + ((p*(p-1)*(p-
2))/fact(3))*Del3[0];
   cout<<"The value of Y("<<X<<") is: " <<Y<<endl;
}
```

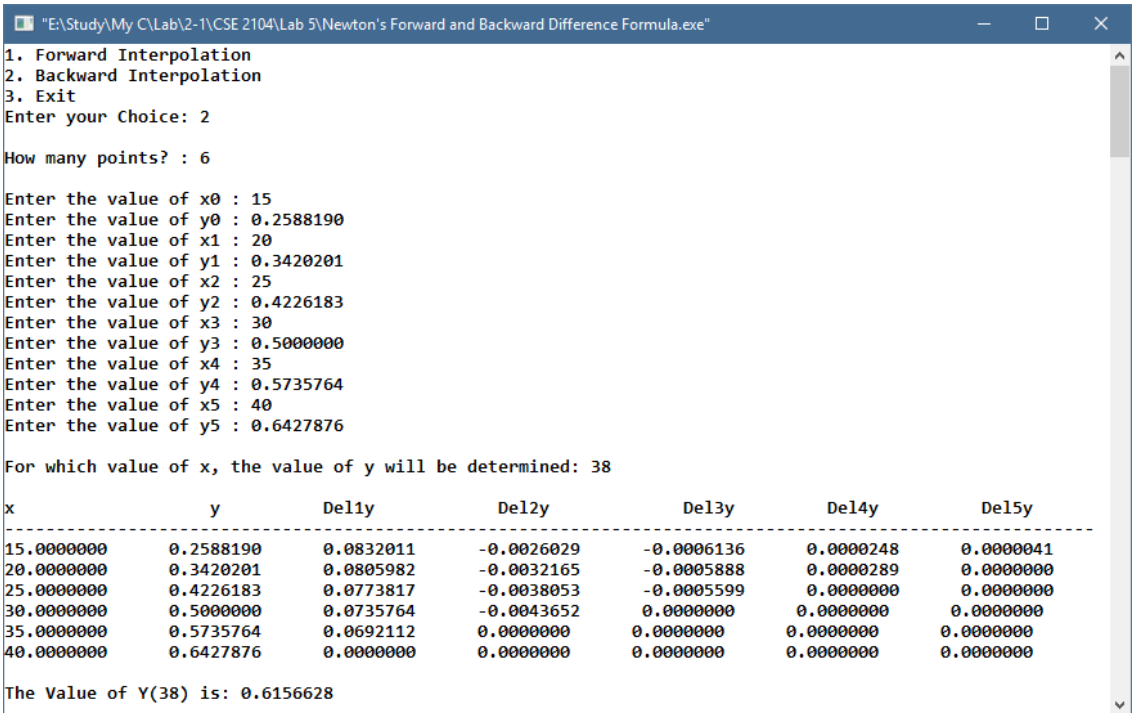
```
void Backward()
{
   int points,flag=0,i,j=0,k=0,l=0;
   double x[10],y[10],h,p,X,Del1[10],Del2[10],Del3[10],Del4[10],Del5[10],Y;
   for(int x=0;x<10;x++)
   {
      Del1[x] = 0;
      Del2[x] = 0;
      Del3[x] = 0;
      Del4[x] = 0;
      Del5[x] = 0;
   }
   cout<<"\nHow many points? : ";
   cin>>points;
   printf("\n");
   for(i=0;i<points;i++)
   {
      cout<<"Enter the value of x"<<i<<" : " ;
      cin>>x[i];
      cout<<"Enter the value of y"<<i<<" : ";
      cin>>y[i];
   }
   printf("\n");
   h = x[1] - x[0];
   cout<<"For which value of x, the value of y will be determined: ";
   cin>>X;
   p = (X - x[points-1])/h;
   i = 0;
   while(i<points-1)
   {
      Del1[i] = y[i+1] - y[i];
      i++;
   }
   i = 0;
   while(i<points-2)
   {
      Del2[i] = Del1[i+1] - Del1[i];
      i++;
   }
   i = 0;
   while(i<points-3)
   {
      Del3[i] = Del2[i+1] - Del2[i];
      i++;
   }
```

```
    i = 0;
    while(i<points-4)
    {
        Del4[i] = Del3[i+1] - Del3[i];
        i++;
    }
    i = 0;
    while(i<points-5)
    {
        Del5[i] = Del4[i+1] - Del4[i];
        i++;
    }
    printf("\n");
    PrintBackTable(x,y,Del1,Del2,Del3,Del4,Del5);
    int m = (int) X;
    Y = y[--points] + (p/fact(1))*Del1[--points] + ((p*(p+1))/fact(2))*Del2[--points] +
((p*(p+1)*(p+2))/fact(3))*Del3[--points];
    printf("The Value of Y(%d) is: %0.7lf\n",m,Y);
}


void menu()
{
    int choice;
    cout<<"1. Forward Interpolation\n2. Backward Interpolation\n3. Exit\nEnter your Choice: ";
    cin>>choice;
    switch(choice)
    {
        case 1:
            Forward(); break;
        case 2:
            Backward(); break;
        case 3:
            cout<<"The program ended successfully" <<endl; break;
        default:
            cout<<"wrong Input"<<endl; break;
    }
}

int main()
{
    menu();
    return 0;
}
```

Output



```
"E:\Study\My C\Lab\2-1\CSE 2104\Lab 5\Newton's Forward...
1. Forward Interpolation
2. Backward Interpolation
3. Exit
Enter your Choice: 1

How many points? : 4

Enter the value of x0 : 1
Enter the value of y0 : 24
Enter the value of x1 : 3
Enter the value of y1 : 120
Enter the value of x2 : 5
Enter the value of y2 : 336
Enter the value of x3 : 7
Enter the value of y3 : 720

For which value of x, the value of y will be determined: 8

x          y          Del1y       Del2y       Del3y
-----------------------------------------------------------
1.000     24.000     96.000     120.000     48.000
3.000     120.000     216.000     168.000     0.000
5.000     336.000     384.000     0.000     0.000
7.000     720.000     0.000     0.000     0.000

The value of Y(8) is: 990
```



```
"E:\Study\My C\Lab\2-1\CSE 2104\Lab 5\Newton's Forward and Backward Difference Formula.exe"
1. Forward Interpolation
2. Backward Interpolation
3. Exit
Enter your Choice: 2

How many points? : 6

Enter the value of x0 : 15
Enter the value of y0 : 0.2588190
Enter the value of x1 : 20
Enter the value of y1 : 0.3420201
Enter the value of x2 : 25
Enter the value of y2 : 0.4226183
Enter the value of x3 : 30
Enter the value of y3 : 0.5000000
Enter the value of x4 : 35
Enter the value of y4 : 0.5735764
Enter the value of x5 : 40
Enter the value of y5 : 0.6427876

For which value of x, the value of y will be determined: 38

x                  y          Del1y           Del2y           Del3y          Del4y          Del5y
-------------------------------------------------------------------------------------------------------
15.0000000      0.2588190      0.0832011      -0.0026029      -0.0006136      0.0000248      0.0000041
20.0000000      0.3420201      0.0805982      -0.0032165      -0.0005888      0.0000289      0.0000000
25.0000000      0.4226183      0.0773817      -0.0038053      -0.0005599      0.0000000      0.0000000
30.0000000      0.5000000      0.0735764      -0.0043652      0.0000000      0.0000000      0.0000000
35.0000000      0.5735764      0.0692112      0.0000000      0.0000000      0.0000000      0.0000000
40.0000000      0.6427876      0.0000000      0.0000000      0.0000000      0.0000000      0.0000000

The Value of Y(38) is: 0.6156628
```

Discussion

Here, in the above code , A menu() function was used to get the forward difference table, and backward Difference table which was used to get the required result. In the menu() function, Forward(), Backward() function was called. In the Forward() function, PrintForTable() was called to generate the Forward Difference table and similarly PrintBackTable() was called in the Backward() functionto generate the backward difference table. Thus Using the equation (1) and (2) the result for forward difference and backward difference was shown respectfully.