

Interpolation

Gauss' Forward And Backward Difference Formula

Theory

Gauss' forward formula is considered that the following difference table in which the central ordinate is taken for convenience as y_0 corresponding to $x = x_0$. The differences used in this formula lie on the line shown in the following table –

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
x_{-3}	y_{-3}						
		Δy_{-3}					
x_{-2}	y_{-2}		$\Delta^2 y_{-3}$				
		Δy_{-2}		$\Delta^3 y_{-3}$			
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		$\Delta^4 y_{-3}$		
		Δy_{-1}		$\Delta^3 y_{-2}$		$\Delta^5 y_{-3}$	
x_0	y_0	Δy_0	$\Delta^2 y_{-1}$	$\Delta^3 y_{-1}$	$\Delta^4 y_{-2}$	$\Delta^5 y_{-2}$	$\Delta^6 y_{-3}$
x_1	y_1	Δy_1	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_{-1}$		
x_2	y_2	Δy_2	$\Delta^2 y_1$				
x_3	y_3						

The formula is therefore of the form,

$$y_p = G_1 \Delta y_0 + G_2 \Delta^2 y_{-1} + G_3 \Delta^3 y_{-1} + G_4 \Delta^4 y_{-2} + \dots \quad (1)$$

Where G_1, G_2, G_3, G_4, h, p are ,

$$h = (x_1 - x_0)$$

$$p = \frac{x - x_0}{h}$$

$$G_1 = p$$

$$G_2 = \frac{p(p-1)}{2!}$$

$$G_3 = \frac{(p+1)p(p-1)}{3!}$$

$$G_4 = \frac{(p+1)p(p-1)(p-2)}{4!}$$

Gauss' backward formula uses the difference which lie on the line that shown in the table below-

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
\vdots	\vdots						
x_{-1}	y_{-1}						
x_0	y_0	Δy_{-1}	$\Delta^2 y_{-1}$	$\Delta^3 y_{-2}$	$\Delta^4 y_{-2}$	$\Delta^5 y_{-3}$	$\Delta^6 y_{-3}$
x_1	y_1	Δy_0	$\Delta^2 y_{-1}$	$\Delta^3 y_{-1}$	$\Delta^4 y_{-2}$	$\Delta^5 y_{-2}$	
\vdots	\vdots						

Gauss' backward formula can therefore be assumed to be of the form,

$$y_p = G'_1 \Delta y_{-1} + G'_2 \Delta^2 y_{-1} + G'_3 \Delta^3 y_{-2} + G'_4 \Delta^4 y_{-2} + \dots \quad (2)$$

Where $G'_1, G'_2, G'_3, G'_4, h, p$ are ,

$$h = (x_1 - x_0)$$

$$p = \frac{x - x_0}{h}$$

$$G'_1 = p$$

$$G'_2 = \frac{p(p+1)}{2!}$$

$$G'_3 = \frac{(p+1)p(p-1)}{3!}$$

$$G'_4 = \frac{(p+1)(p+2)p(p-1)}{4!}$$

Code

```
#include<bits/stdc++.h>
using namespace std;
```

```
int fact(int n)
{
    if(n==0 || n==1)
        return 1;
    else if(n>1)
        return n*fact(n-1);
}
```

```
void PrintForward(double x[],double y[],double Del1[], double Del2[], double Del3[])
{
    int i,j;
    cout<<"\nx      y      Del1y  Del2y  Del3y"<<endl;
    cout<<"-----"<<endl;
    for(i=0 ; i<4 ; i++)
        printf("%0.0lf  %0.5lf  %0.5lf  %0.5lf  %0.5lf\n",x[i],y[i],Del1[i],Del2[i],Del3[i]);
    cout<<"-----"<<endl;
}
```

```
void PrintBackward(double x[],double y[],double Del1[], double Del2[], double Del3[],double
Del4[])
{
    int i,j;
    cout<<"\nx      y      Del1y  Del2y  Del3y  Del4y"<<endl;
    cout<<"-----"<<endl;
    for(i=0 ; i<5 ; i++)
        printf("%0.0lf  %0.5lf  %0.5lf  %0.5lf  %0.5lf
%0.5lf\n",x[i],y[i],Del1[i],Del2[i],Del3[i],Del4[i]);
    cout<<"-----"<<endl;
}
```

```
void Forward()
{
    int points,i,j=0,k=0,l=0;
    double x[10],y[10],h,p,X,Del1[10],Del2[10],Del3[10],Y,G1,G2,G3;
    for(int x=0;x<10;x++)
    {
        Del1[x] = 0; Del2[x] = 0; Del3[x] = 0;
    }

    cout<<"\nHow many points? : ";
    cin>>points;
    cout<<endl;
```

```

for(i=0;i<points;i++)
{
    cout<<"Enter the value of x"<<i<<" : " ;
    cin>>x[i];
    cout<<"Enter the value of y"<<i<<" : " ;
    cin>>y[i];
}
printf("\n");

cout<<"For which value of x, the value of y will be determined: ";
cin>>X;

h = x[1] - x[0];
p = (X - x[1])/h;

for(i=0;i<points-1 ; i++)
    Del1[i] = y[i+1] - y[i];
for(i=0;i<points-2 ; i++)
    Del2[i] = Del1[i+1] - Del1[i];
for(i=0;i<points-3 ; i++)
    Del3[i] = Del2[i+1] - Del2[i];

PrintForward(x,y,Del1,Del2,Del3);

G1 = p;
G2 = (p*(p-1))/fact(2);
G3 = ((p+1)*p*(p-1))/fact(3);

Y = y[1] + G1*Del1[1] + G2*Del2[1] + G3*Del3[0];
cout<<"\nThe value of Y("<<X<<" is: " <<Y<<endl;
}

void Backward()
{
    int points,flag=0,i,j=0,k=0,l=0;
    double x[10],y[10],h,p,X,Del1[10],Del2[10],Del3[10],Del4[10],Y,G1,G2,G3,G4;
    for(int x=0;x<10;x++)
    {
        Del1[x] = 0;
        Del2[x] = 0;
        Del3[x] = 0;
        Del4[x] = 0;
    }

    cout<<"\nHow many points? : ";
    cin>>points;

```

```

cout<<endl;
for(i=0;i<points;i++)
{
    cout<<"Enter the value of x"<<i<<" : " ;
    cin>>x[i];
    cout<<"Enter the value of y"<<i<<" : " ;
    cin>>y[i];
}
printf("\n");

cout<<"For which value of x, the value of y will be determined: ";
cin>>X;

h = x[1] - x[0];
p = (X - x[1])/h;

for(i=0;i<points-1 ; i++)
    Del1[i] = y[i+1] - y[i];
for(i=0;i<points-2 ; i++)
    Del2[i] = Del1[i+1] - Del1[i];
for(i=0;i<points-3 ; i++)
    Del3[i] = Del2[i+1] - Del2[i];
for(i=0;i<points-3 ; i++)
    Del4[i] = Del3[i+1] - Del3[i];

PrintBackward(x,y,Del1,Del2,Del3,Del4);

G1 = p;
G2 = (p*(p+1))/fact(2);
G3 = ((p+1)*p*(p-1))/fact(3);
G4 = ((p+1)*(p+2)*p*(p-1))/fact(4);

Y = y[1] + G1*Del1[1] + G2*Del2[0] + G3*Del3[0] + G4*Del4[0];

cout<<"\nThe value of Y("<<X<<" ) is: " <<Y<<endl;

}

void menu()
{
    int choice;
    cout<<"1.Gauss's Forward Interpolation\n2.Gauss's Backward Interpolation\n3. Exit\nEnter
your Choice: ";
    cin>>choice;

```

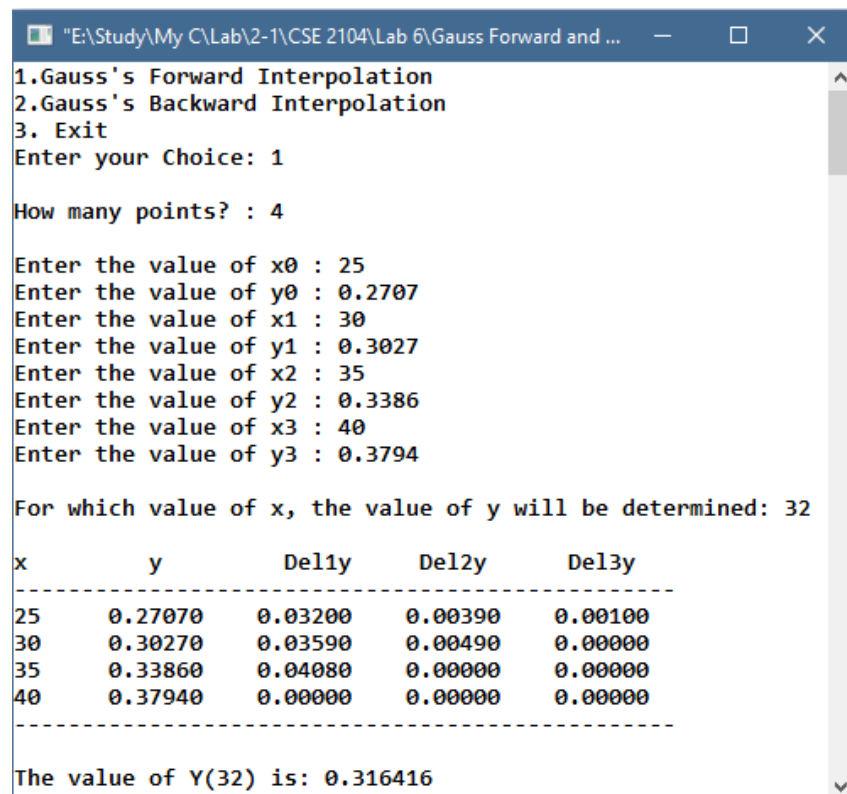
```

switch(choice)
{
    case 1:
        Forward(); break;
    case 2:
        Backward(); break;
    case 3:
        cout<<"The program ended successfully" <<endl; break;
    default:
        cout<<"wrong Input"<<endl; break;
}
}

int main()
{
    menu();
    return 0;
}

```

Output



The screenshot shows a Windows command prompt window titled "E:\Study\My C\Lab\2-1\CSE 2104\Lab 6\Gauss Forward and ...". The program output is as follows:

```

1.Gauss's Forward Interpolation
2.Gauss's Backward Interpolation
3. Exit
Enter your Choice: 1

How many points? : 4

Enter the value of x0 : 25
Enter the value of y0 : 0.2707
Enter the value of x1 : 30
Enter the value of y1 : 0.3027
Enter the value of x2 : 35
Enter the value of y2 : 0.3386
Enter the value of x3 : 40
Enter the value of y3 : 0.3794

For which value of x, the value of y will be determined: 32

```

x	y	Del1y	Del2y	Del3y
25	0.27070	0.03200	0.00390	0.00100
30	0.30270	0.03590	0.00490	0.00000
35	0.33860	0.04080	0.00000	0.00000
40	0.37940	0.00000	0.00000	0.00000

```

-----
The value of Y(32) is: 0.316416

```

```
"E:\Study\My C\Lab\2-1\CSE 2104\Lab 6\Gauss Forward and Backward ...
1.Gauss's Forward Interpolation
2.Gauss's Backward Interpolation
3. Exit
Enter your Choice: 2

How many points? : 5

Enter the value of x0 : 12500
Enter the value of y0 : 111.8034
Enter the value of x1 : 12510
Enter the value of y1 : 111.8481
Enter the value of x2 : 12520
Enter the value of y2 : 111.8928
Enter the value of x3 : 12530
Enter the value of y3 : 111.9375
Enter the value of x4 : 12540
Enter the value of y4 : 111.9829

For which value of x, the value of y will be determined: 12525

x          y          Del1y      Del2y      Del3y      Del4y
-----
12500      111.80340      0.04470      -0.00000      0.00000      0.00070
12510      111.84810      0.04470      0.00000      0.00070      -0.00070
12520      111.89280      0.04470      0.00070      0.00000      0.00000
12530      111.93750      0.04540      0.00000      0.00000      0.00000
12540      111.98290      0.00000      0.00000      0.00000      0.00000
-----

The value of Y(12525) is: 111.915
```

Discussion

Here, in the above code , A menu() function was used to get the Gauss' forward difference table, and backward Difference table which was used to get the required result. In the menu() function, Forward(), Backward() function was called. In the Forward() function, PrintForward() was called to generate the Forward Difference table and similarly PrintBackward() was called in the Backward() function to generate the backward difference table. Thus Using the equation (1) and (2) the result for forward difference and backward difference was shown respectfully