Machine Learning Lab

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(1) Find the derivations of the following formula

$\boxed{1}$ (1) $f(z) = \log_e(1+z)$ where $z = x^T x$, $x \in R^d$

Solution: if $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$ then $x^T = [x_1 \quad x_2 \quad \dots \quad x_d]$

$$x^T x = [x_1^2 + x_2^2 + \dots + x_d^2]$$

Applying Chain Rule,
$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dx}$$

$$= \frac{d}{dz}(\log_e(1+z)) \cdot \frac{d}{dx}(x^T x)$$

$$= \frac{1}{1+z}\frac{d}{dz}(z) \cdot \frac{d}{dx}(x_1^2 + x_2^2 + \dots + x_d^2)$$

$$= \frac{1}{1+z}(2x_1 + 2x_2 + \dots + 2x_d)$$

$$= \frac{1}{1+z} \cdot 2(x_1 + x_2 + \dots + x_d)$$

$$= \frac{2}{1+z}\sum_{i=1}^{d} x_i$$

(Ans.)

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(11) $f(z) = e^{-z/2}$ where $z = g(y)$, $g(y) = y^T s{-1} y$

$y = h(x)$

$h(x) = x - \mu$

Solution using chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

here, $\frac{df}{dz} = \frac{d}{dz}(e^{-z/2}) = -\frac{e^{-z/2}}{2}$

$$\frac{dz}{dy} = \frac{d}{dy}(y^T s{-1} y)$$

$$= \lim_{h \to 0} \frac{g(y+h) - g(y)}{h}$$

$$= \lim_{h \to 0} \frac{(y^T + h) s^{-1} (y+h) - y^T s{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{(y^T s{-1} + h s{-1})(y+h) - y^T s{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{y^T s{-1} y + y^T s{-1} h + h s{-1} y + h^2 s{-1} - y^T s{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{h(y^T s{-1} + s{-1} y + h s{-1})}{h}$$

$$= \lim_{h \to 0} (y^T s{-1} + s{-1} y + h s{-1})$$

$$= y^T s{-1} + s{-1} y$$

$$\frac{dy}{dx} = \frac{d}{dx}(x - \mu)$$

$$= 1$$

$$\therefore \frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= -\frac{e^{-z/2}}{2} \cdot (y^T s{-1} + s{-1} y) \cdot 1$$

$$= -\frac{e^{-z/2}}{2} \cdot \frac{1}{s}(y^T + y)$$

(Ans)