On the Electrodynamics of Moving Bodies

Zur Elektrodynamik bewegter Körper
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On the Electrodynamics of moving bodies

It is well known that Maxwell's electrodynamics—as usually understood at present—when applied to moving bodies, leads to asymmetries that do not seem to attach to the phenomena. Let us recall, for example, the electrodynamic interaction between a magnet and a conductor. The observable phenomenon depends here only on the relative motion of conductor and magnet, while according to the customary conception the two cases, in which, respectively, either the one or the other of the two bodies is the one in motion, are to be strictly differentiated from each other. For if the magnet is in motion and the conductor is at rest, there arises in the surroundings of the magnet an electric field endowed with a certain energy value that produces a current in the places where parts of the conductor are located. But if the magnet is at rest and the conductor is in motion, no electric field arises in the surroundings of the magnet, while in the conductor an electromotive force will arise, to which in itself there does not correspond any energy, but which, provided that the relative motion in the two cases considered is the same, gives rise to electrical currents that have the same magnitude and the same course as those produced by the electric forces in the first-mentioned case.

Examples of a similar kind, and the failure of attempts to detect a motion of the earth relative to the "light medium", lead to the conjecture that not only in mechanics, but in electrodynamics as well, the phenomena do not have any properties corresponding to the concept of absolute rest, but that in all coordinate systems in which the mechanical equations are valid, also the same electrodynamic and optical laws are valid, as has already been shown for quantities of the first order. We shall raise this conjecture (whose content will be called "the principle of relativity" hereafter) to the status of a postulate and shall introduce, in addition, the postulate, only seemingly incompatible with the former one, that in empty space light is always propagated with a definite velocity V which is independent of the state of motion of the emitting body. These two postulates suffice for arriving at a simple and consistent electrodynamics of moving bodies on the basis of Maxwell's theory for bodies at rest. The introduction of a "light ether" will prove superfluous, inasmuch as in accordance with the concept to be developed here, no "space at absolute rest" endowed with special properties will be introduced, nor will a velocity vector be assigned to a point of empty space at which electromagnetic processes are taking place.

Like every other electrodynamics, the theory to be developed is based on the kinematics of the rigid body, since assertions of each and any theory concern the relations between rigid bodies (coordinate systems), clocks, and electromagnetic processes. Insufficient regard for this circumstance is at the root of the difficulties with which the electrodynamics of moving bodies must presently grapple.

I. Kinematic Part

§1. Definition of simultaneity

Consider a coordinate system in which the Newtonian mechanical equations are valid. To distinguish it verbally from the coordinate systems that will be introduced later on, and to visualize it more precisely, we will designate this system as the "system at rest."

If a material point is at rest relative to this coordinate system, its position relative to the latter can be determined by means of rigid measuring rods using the methods of Euclidean geometry and can be expressed in Cartesian coordinates.

If we want to describe the *motion* of a material point, we give the values of its coordinates as a function of time. However, we should keep in mind that for such a mathematical description to have physical meaning, we first have to clarify what is to be understood here by "time." We have to bear in mind that all our propositions involving time are always propositions about *simultaneous events*. If, for example, I say that "the train arrives here at 7 o'clock," that means, more or less, "the pointing of the small hand of my clock to 7 and the arrival of the train are simultaneous events."[1]

It might seem that all difficulties involved in the definition of "time" could be overcome by my substituting "position of the small hand of my clock" for "time." Such a definition is indeed sufficient if time has to be defined exclusively for the place at which the clock is located; but the definition becomes insufficient as soon as series of events occurring at different locations have to be linked temporally, or—what amounts to the same—events occurring at places remote from the clock have to be evaluated temporally.

To be sure, we could content ourselves with evaluating the time of the events by stationing an observer with the clock at the coordinate origin, and having him assign the corresponding clock-hand position to each light signal that attests to an event to be evaluated and reaches him through empty space. But as we know from experience, such an assignment has the drawback that it is not independent of the position of the observer equipped with the clock. We arrive at a far more practical arrangement by the following consideration.

If there is a clock at point A of space, then an observer located at A can evaluate the time of the events in the immediate vicinity of A by finding the clock-hand positions that are simultaneous with these events. If there is also a clock at point B—we should add, "a clock of exactly the same constitution as that at A"—then the time of the events in the immediate vicinity of B can likewise be evaluated by an observer located at B. But it is not possible to compare the time of an event at A with one at B without a further stipulation; thus far we have only defined an "A-time" and a "B-time" but not a "time" common to A and B. The latter can now be determined by establishing *by*

[1]We shall not discuss here the imprecision that is inherent in the concept of simultaneity of two events taking place at (approximately) the same location and that also must be surmounted by an abstraction.

definition that the "time" needed for the light to travel from A to B is equal to the "time" it needs to travel from B to A. For, suppose a ray of light leaves from A toward B at "A-time" t_A, is reflected from B toward A at "B-time" t_B, and arrives back at A at "A-time" t'_A. The two clocks are synchronous by definition if

$$t_B - t_A = t'_A - t_B.$$

We assume that it is possible for this definition of synchronism to be free of contradictions, and to be so for arbitrarily many points, and that the following relations are therefore generally valid:

1. If the clock in B is synchronous with the clock in A, then the clock in A is synchronous with the clock in B.

2. If the clock in A is synchronous with the clock in B as well as with the clock in C, then the clocks in B and C are also synchronous relative to each other.

With the help of some physical (thought) experiments, we have thus laid down what is to be understood by synchronous clocks at rest that are situated at different places, and have obviously obtained thereby a definition of "synchronous" and of "time." The "time" of an event is the reading obtained simultaneously with the event from a clock at rest that is located at the place of the event and that for all time determinations is in synchrony with a specified clock at rest.

Based on experience, we also postulate that the quantity

$$\frac{2\overline{AB}}{t'_A - t_A} = V$$

is a universal constant (the velocity of light in empty space).

It is essential that we have defined time by means of clocks at rest in a system at rest; because it belongs to the system at rest, we designate the time just defined as "the time of the system at rest."

§2. On the relativity of lengths and times

The considerations that follow are based on the principle of relativity and the principle of the constancy of the velocity of light, two principles that we define as follows:

1. The laws governing the changes of the state of any physical system do not depend on which one of two coordinate systems in uniform translational motion relative to each other these changes of the state are referred to.

2. Each ray of light moves in the coordinate system "at rest" with the definite velocity V independent of whether this ray of light is emitted by a body at rest or a body in motion. Here,

$$\text{velocity} = \frac{\text{light path}}{\text{time interval}},$$

where "time interval" should be understood in the sense of the definition in §1.

Let there be given a rigid rod at rest; its length, measured by a measuring rod that is also at rest, shall be l. We now imagine that the axis of the rod is placed along the X-axis of the coordinate system at rest, and that the rod is then set in uniform parallel translational motion (velocity v) along the X-axis in the direction of increasing x. We now seek to determine the length of the *moving* rod, which we imagine to be obtained by the following two operations:

(a) The observer co-moves with the above-mentioned measuring rod and the rod to be measured, and measures the length of the rod directly, by applying the measuring rod exactly as if the rod to be measured, the observer, and the measuring rod were at rest.

(b) Using clocks at rest that are set up in the system at rest and are synchronous in the sense of §1, the observer determines the points of the system at rest at which the beginning and the end of the rod to be measured are found at some given time t. The distance between these two points, measured by the rod used before, which in the present case is at rest, is also a length, which can be designated as the "length of the rod."

According to the principle of relativity, the length to be found in operation (a), which we shall call "the length of the rod in the moving system," must equal the length l of the rod at rest.

We will determine the length to be found in operation (b), which we shall call "the length of the (moving) rod in the system at rest," on the basis of our two principles, and will find it to be different from l.

The commonly used kinematics tacitly assumes that the lengths determined by the two methods mentioned are exactly identical, or, in other words, that in the time epoch t a moving rigid body is totally replaceable, as far as geometry is concerned, by the same body when it is at rest in a particular position.

Further, we imagine that the two ends (A and B) of the rod are equipped with clocks that are synchronous with the clocks of the system at rest, i.e., whose readings always correspond to the "time of the system at rest" at the locations they happen to occupy; hence, these clocks are "synchronous in the system at rest."

We further imagine that each clock has an observer co-moving with it, and that these observers apply to the two clocks the criterion for synchronism formulated in §1. Suppose a ray of light starts out from A at time[2] t_A, is reflected from B at time t_B, and arrives back at A at time t'_A. Taking into account the principle of the constancy of the velocity of light, we find that

$$t_B - t_A = \frac{r_{AB}}{V - v}$$

and

$$t'_A - t_B = \frac{r_{AB}}{V + v},$$

[2]Time" here means both "time of the system at rest" and "the position of the hands of the moving clock located at the place in question.

where r_{AB} denotes the length of the moving rod, measured in the system at rest. The observers co-moving with the moving rod would thus find that the two clocks do not run synchronously while the observers in the system at rest would declare them synchronous.

Thus we see that we must not ascribe *absolute* meaning to the concept of simultaneity; instead, two events that are simultaneous when observed from some particular coordinate system can no longer be considered simultaneous when observed from a system that is moving relative to that system.

§3. Theory of transformation of coordinates and time from a system at rest to a system in uniform translational motion relative to it

Let there be given two coordinate systems in the space "at rest," i.e., two systems of three mutually perpendicular rigid material lines issuing from one point. Let the X-axes of the two systems coincide and their Y- and Z-axes be parallel. Each system shall be supplied with a rigid measuring rod and a number of clocks, and the two measuring rods and all the clocks of the two systems should be exactly alike.

The origin of one of the two systems (k) shall now be imparted a (constant) velocity v in the direction of increasing x of the other system (K), which is at rest, and this velocity shall also be imparted to the coordinate axes, the corresponding measuring rod, and the clocks. To each time t of the system at rest K there corresponds then a definite position of the axes of the moving system, and for reasons of symmetry we may right- fully assume that the motion of k can be such that at time t ("t" always denotes a time of the system at rest) the axes of the moving system are parallel to the axes of the system at rest.

We now imagine the space to be measured both from the system at rest K by means of the measuring rod at rest and from the moving system k by means of the measuring rod moving along with it, and that the coordinates x, y, z and ξ, η, ζ are obtained in this way. Further, by means of the clocks at rest in the system at rest and using light signals in the manner described in §1, the time t of the system at rest is determined for all its points where there is a clock; likewise, the time τ of the moving system is determined for all the points of the moving system having clocks that are at rest relative to this system, applying the method of light signals described in §1 between the points containing these clocks.

To every system of values x, y, z, t that determines completely the place and time of an event in the system at rest, there corresponds a system of values ξ, η, ζ, τ that fixes this event relative to the system k, and the problem to be solved is to find the system of equations connecting these quantities.

First of all, it is clear that these equations must be *linear* because of the properties of homogeneity that we attribute to space and time.

If we put $x' = x - vt$, then it is clear that a point at rest in the system k has a definite, time-independent system of values x', y, z belonging to it. We first determine τ as a function of x', y, z, and t. To this end, we must express in equations that τ is in fact the aggregate of the readings of the clocks at rest in the system k, which have been synchronized according to the rule given in §1

Suppose that at time τ_0 a light ray is sent from the origin of the system k along the X-axis to x' and is reflected from there at time τ_1 toward the origin, where it arrives at time τ_2; we then must have

$$\frac{1}{2}(\tau_0 + \tau_2) = \tau_1,$$

or, if we write out the arguments of the function τ and apply the principle of the constancy of the velocity of light in the system at rest,

$$\frac{1}{2}\left[\tau(0,0,0,t) + \tau\left(0,0,0,\left\{t + \frac{x'}{V-v} + \frac{x'}{V+v}\right\}\right)\right] = \tau\left(x',0,0,t + \frac{x'}{V-v}\right).$$

From this we get, if x' is chosen infinitesimally small,

$$\frac{1}{2}\left(\frac{1}{V-v} + \frac{1}{V+v}\right)\frac{\partial\tau}{\partial t} = \frac{\partial\tau}{\partial x'} + \frac{1}{V-v}\frac{\partial\tau}{\partial t},$$

or

$$\frac{\partial\tau}{\partial x'} + \frac{v}{V^2 - v^2}\frac{\partial\tau}{\partial t} = 0.$$

t should be noted that, instead of the coordinate origin, we could have chosen any other point as the starting point of the light ray, and the equation just derived therefore holds for all values of x' , y, z.

Analogous reasoning—applied to the H and Z axes—yields, if we consider that light always propagates along these axes with the velocity $\sqrt{V^2 - v^2}$ when observed from the system at rest,

$$\frac{\partial\tau}{\partial y} = 0$$

$$\frac{\partial\tau}{\partial z} = 0.$$

These equations yield, since τ is a *linear* function,

$$\tau = a\left(t - \frac{v}{V^2 - v^2}x'\right),$$

where a is a function $\varphi(v)$ as yet unknown, and where we assume for brevity that at the origin of k we have $t = 0$ when $\tau = 0$.

Using this result, we can easily determine the quantities ξ, η, ζ by expressing in equations that (as demanded by the principle of the constancy of the velocity of light in conjunction with the principle of relativity) light propagates with velocity V also when measured in the moving system. For a light ray emitted at time $\tau = 0$ in the direction of increasing ξ, we will have

$$\xi = V\tau,$$

or

$$\xi = aV\left(t - \frac{v}{V^2 - v^2}x'\right).$$

But as measured in the system at rest, the light ray propagates with velocity $V - v$ relative to the origin of k, so that

$$\frac{x'}{V - v} = t.$$

Substituting this value of t in the equation for ξ, we obtain

$$\xi = a\frac{V^2}{V^2 - v^2}x'.$$

Analogously, by considering light rays moving along the two other axes, we get

$$\eta = V\tau = aV\left(t - \frac{v}{V^2 - v^2}x'\right),$$

where

$$\frac{y}{\sqrt{V^2 - v^2}} = t; \qquad x' = 0;$$

hence

$$\eta = \frac{V}{\sqrt{V^2 - v^2}}y$$

and

$$\zeta = \frac{V}{\sqrt{V^2 - v^2}}z.$$

If we substitute for x' its value, we obtain

$$\begin{aligned}
\tau &= \varphi(v)\beta\left(t - \frac{v}{V^2}x\right), \\
\xi &= \varphi(v)\beta(x - vt), \\
\eta &= \varphi(v)y, \\
\zeta &= \varphi(v)z,
\end{aligned}$$

where

$$\beta = \frac{1}{\sqrt{1 - \left(\frac{v}{V}\right)^2}}$$

and φ is a function of v that is as yet unknown. If no assumptions are made regarding the initial position of the moving system and the zero point of τ, then an additive constant must be attached to the right-hand sides of these equations.

Now we have to prove that every light ray measured in the moving system propagates with the velocity V, if it does so, as we have assumed, in the system at rest; for we have not yet provided the proof that the principle of the constancy of the velocity of light is compatible with the relativity principle.

Suppose that at time $t = \tau = 0$ a spherical wave is emitted from the coordinate origin, which is at that time common to the two systems, and that this wave propagates in the system K with the velocity V. Hence, if (x, y, z) is a point just reached by this wave, we will have

$$x^2 + y^2 + z^2 = V^2 t^2.$$

We transform these equations using our transformation equations, and, after a simple calculation, obtain

$$\xi^2 + \eta^2 + \zeta^2 = V^2 \tau^2.$$

Thus, the wave under consideration is a spherical wave of propagation velocity V also when it is observed in the moving system. This proves that our two fundamental principles are compatible.

The transformation equations we have derived also contain an unknown function φ of v, which we now wish to determine.

To this end we introduce a third coordinate system K', which relative to the system k is in parallel-translational motion parallel to the axis Ξ such that its origin moves along the Ξ-axis with velocity -v. Let all three coordinate origins coincide at time $t = 0$, and let the time t' of the system K' be zero at $t = x = y = z = 0$. We denote the coordinates measured in the system K' by x',y' ,z' and, by twofold application of our transformation equations, we get

$$\begin{aligned}
t' &= \varphi(-v)\beta(-v)\left\{\tau + \frac{v}{V^2}\xi\right\} &&= \varphi(v)\varphi(-v)t, \\
x' &= \varphi(-v)\beta(-v)\{\xi + v\tau\} &&= \varphi(v)\varphi(-v)x, \\
y' &= \varphi(-v)\eta &&= \varphi(v)\varphi(-v)y, \\
z' &= \varphi(-v)\zeta &&= \varphi(v)\varphi(-v)z.
\end{aligned}$$

Since the relations between x' ,y',z' and x,y,z do not contain the time t, the systems K and K' are at rest relative to each other, and it is clear that the transformation from K to K' must be the identity transformation. Hence,

$$\varphi(v)\varphi(-v) = 1.$$

Let us now explore the meaning of $\varphi(v)$. We shall focus on that portion of the H-axis of the system k that lies between $\xi = 0$, $\eta = 0$, $\zeta = 0$, and $\xi = 0$, $\eta = l$, $\zeta = 0$. This portion of the H-axis is a rod that moves perpendicular to its axis with a velocity v relative to the system K and whose ends possess in K the coordinates

$$x_1 = vt, \qquad y_1 = \frac{l}{\varphi(0)}, \quad z_1 = 0$$

and

$$x_2 = vt, \qquad y_2 = 0, \qquad z_2 = 0.$$

The length of the rod, measured in K, is thus $l/p(v)$; this establishes the meaning of the function φ. For reasons of symmetry it is obvious that the length of a rod measured in the system at rest and moving perpendicular to its own axis can depend only on its velocity and not on the direction and sense of its motion. Thus, the length of the moving rod measured in the system at rest does not change when v is replaced by $-v$. From this we arrive at

$$\frac{l}{\varphi(v)} = \frac{l}{\varphi(-v)},$$

or

$$\varphi(v) = \varphi(-v).$$

It follows from this relation and the one found before that $\varphi(v)$ must equal 1, so that the transformation equations obtained become

$$\begin{aligned}
\tau &= \beta\left(t - \frac{v}{V^2}x\right), \\
\xi &= \beta(x - vt), \\
\eta &= y, \\
\zeta &= z,
\end{aligned}$$

where

$$\beta = \sqrt{1 - \left(\frac{v}{V}\right)^2}.$$

§4. The physical meaning of the equations obtained concerning moving rigid bodies and moving clocks

We consider a rigid sphere[3] of radius R that is at rest relative to the moving system k and whose center lies at the origin of k. The equation of the surface of this sphere, which moves with velocity v relative to the system k, is

$$\xi^2 + \eta^2 + \zeta^2 = R^2.$$

Expressed in x, y, z, the equation of this surface at time $t = 0$ is

$$\frac{x^2}{\left(1 - \left(\frac{v}{V}\right)^2\right)^2} + y^2 + z^2 = R^2.$$

[3] I.e., a body possessing the shape of a sphere when investigated at rest.

A rigid body that has a spherical shape when measured in the state of rest thus in the state of motion—observed from a system at rest—has the shape of an ellipsoid of revolution with axes

$$R\sqrt{1-\left(\frac{v}{V}\right)^2}, R, R.$$

Thus, while the Y and Z dimensions of the sphere (and hence also of every rigid body, whatever its shape) do not appear to be altered by motion, the X dimension appears to be contracted in the ratio $1 : \sqrt{1-(v/V)^2}$, i.e., the greater the value of v, the greater the contraction. At $v = V$, all moving objects—observed from the system "at rest"—shrink into plane structures. For superluminal velocities our considerations become meaning-less; we shall see in the considerations that follow that in our theory the velocity of light physically plays the part of infinitely great velocities.

It is clear that the same results apply for bodies at rest in a system "at rest" that are observed from a uniformly moving system.

We further imagine that one of the clocks that is able to indicate time t when at rest relative to the system at rest and time τ when at rest relative to the system in motion, is placed in the origin of k and set such that it indicates the time τ. What is the rate of this clock when observed from the system at rest?

The quantities x, t, and τ, which refer to the position of this clock, are obviously related by the equations

$$\tau = \frac{1}{\sqrt{1-\left(\frac{v}{v}\right)^2}}(t - \frac{v}{V^2}x)$$

and

$$x = vt.$$

We thus have

$$\tau = t\sqrt{1-\left(\frac{v}{V}\right)^2} = t - \left(1-\sqrt{1-\left(\frac{v}{V}\right)^2}\right)t,$$

which shows that the clock (observed in the system at rest) is retarded each second by $(1-\sqrt{1-(v/V^2)})$ sec or, with quantities of the fourth and higher orders neglected, by $\frac{1}{2}(v/V)^2$ sec.

This yields the following peculiar consequence: If at the points A and B of K there are located clocks at rest which, observed in a system at rest, are synchronized, and if the clock in A is transported to B along the connecting line with velocity v, then upon arrival of this clock at B the two clocks will no longer be synchronized; instead, the clock that has been transported from A to B will lag $\frac{1}{2}tv^2/V^2$ sec (up to quantities of the fourth and higher orders) behind the clock that has been in B from the outset, if t is the time needed by the clock to travel from A to B.

We see at once that this result holds even when the clock moves from A to B along any arbitrary polygonal line, and even when the points A and B coincide.

If we assume that the result proved for a polygonal line holds also for a continuously curved line, then we arrive at the following proposition: If there are two synchronous clocks in A, and one of them is moved along a closed curve with constant velocity until it has returned to A, which takes, say, t sec, then this clock will lag on its arrival at A $\frac{1}{2}(v/V)^2$ sec behind the clock that has not been moved. From this we conclude that a balance-wheel clock that is located at the Earth's equator must be very slightly slower than an absolutely identical clock, subjected to otherwise identical conditions, that is located at one of the Earth's poles.

§5. The addition theorem of velocities

In the system k moving with velocity v along the X-axis of the system K let there be a point moving according to the equation

$$\begin{aligned}\xi &= w_\xi \tau,\\ \eta &= w_\eta \tau,\\ \zeta &= 0,\end{aligned}$$

where w_ξ and w_η denote constants.

We seek the motion of the point relative to the system K. Introducing the quantities x, y, z, t into the equations of motion of the point by means of the transformation equations derived in §3, we obtain

$$\begin{aligned}x &= \frac{w_\xi + v}{1 + \frac{v w_\xi}{V^2}} t,\\ y &= \frac{\sqrt{1 - \left(\frac{v}{V}\right)^2}}{1 + \frac{v w_\xi}{V^2}} w_\eta t,\\ z &= 0.\end{aligned}$$

Thus, according to our theory, the law of the parallelogram of velocities holds only in first approximation. We put

$$\begin{aligned}U^2 &= \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2,\\ w^2 &= w_\xi^2 + w_\eta^2\end{aligned}$$

and

$$\alpha = arctg \frac{w_y}{w_x};$$

α should then be considered as the angle between the velocities v and w. After a simple calculation, we obtain

$$U = \frac{\sqrt{(v^2 + w^2 + 2vw\cos\alpha) - \left(\frac{vw\sin\alpha}{V}\right)^2}}{1 + \frac{vw\cos\alpha}{V^2}}.$$

It is noteworthy that v and w enter the expression for the resultant velocity in a symmetric fashion. If w too has the direction of the X-axis (Ξ-axis), we obtain

$$U = \frac{v + w}{1 + \frac{vw}{V^2}}.$$

It follows from this equation that the composition of two velocities that are smaller than V always results in a velocity that is smaller than V. For if we put $v = V - \kappa$, and $w = V - \lambda$, where κ and λ are positive and smaller than V, we get

$$U = V\frac{2V - \kappa - \lambda}{2V - \kappa - \lambda + \frac{\kappa\lambda}{V}} < V.$$

It follows further that the velocity of light V cannot be changed by compounding it with a "subluminal velocity." For this case we get

$$U = \frac{V + w}{1 + \frac{w}{v}} = V.$$

For the case that v and w have the same direction, the formula for U could also have been obtained by compounding two transformations according to §3. If in addition to the systems K and k, which figure in §3, we also introduce a third coordinate system k', which moves parallel to k and whose origin moves with velocity w along the axis Ξ, we obtain relations between the quantities x, y, z, t and the corresponding quantities of k1 that differ from those found in §3 only insofar as "v" is being replaced by the quantity

$$\frac{v + w}{1 + \frac{vw}{V^2}};$$

from this we see that such parallel transformations form a group-as they indeed must.

We have now derived the required propositions of the kinematics that corresponds to our two principles, and will now proceed to show their application in electrodynamics.

II. Electrodynamic Part

§6. Transformation of the Maxwell-Hertz equations for empty space. On the nature of the electromotive forces that arise upon motion in a magnetic field

Let the Maxwell-Hertz equations for empty space be valid for the system at rest K, so that we have

$$\begin{aligned}
\frac{1}{V}\frac{\partial X}{\partial t} &= \frac{\partial N}{\partial y} - \frac{\partial M}{\partial z}, & \frac{1}{V}\frac{\partial L}{\partial t} &= \frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y},\\
\frac{1}{V}\frac{\partial Y}{\partial t} &= \frac{\partial L}{\partial z} - \frac{\partial N}{\partial x}, & \frac{1}{V}\frac{\partial M}{\partial t} &= \frac{\partial Z}{\partial x} - \frac{\partial X}{\partial z},\\
\frac{1}{V}\frac{\partial Z}{\partial t} &= \frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}, & \frac{1}{V}\frac{\partial N}{\partial t} &= \frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x},
\end{aligned}$$

where (X,Y,Z) denotes the vector of the electric force and (L,M,N) that of the magnetic force.

If we apply the transformations derived in §3 to these equations in that we refer the electromagnetic processes to the coordinate system introduced there, which moves with velocity v, we obtain the following equations:

$$\begin{aligned}
\frac{1}{V}\frac{\partial X}{\partial \tau} &= \frac{\partial \beta\left(N - \frac{v}{V}Y\right)}{\partial \eta} - \frac{\partial \beta\left(M + \frac{v}{V}Z\right)}{\partial \zeta},\\
\frac{1}{V}\frac{\partial \beta\left(Y - \frac{v}{V}N\right)}{\partial \tau} &= \frac{\partial L}{\partial \zeta} - \frac{\partial \beta\left(N - \frac{v}{V}Y\right)}{\partial \xi},\\
\frac{1}{V}\frac{\partial \beta\left(Z + \frac{v}{V}M\right)}{\partial \tau} &= \frac{\partial \beta\left(M + \frac{v}{V}Z\right)}{\partial \xi} - \frac{\partial L}{\partial \eta},\\
\frac{1}{V}\frac{\partial L}{\partial \tau} &= \frac{\partial \beta\left(Y - \frac{v}{V}N\right)}{\partial \zeta} - \frac{\partial \beta\left(Z + \frac{v}{V}M\right)}{\partial \eta},\\
\frac{1}{V}\frac{\partial \beta\left(M + \frac{v}{V}Z\right)}{\partial \tau} &= \frac{\partial \beta\left(Z + \frac{v}{V}M\right)}{\partial \xi} - \frac{\partial X}{\partial \zeta},\\
\frac{1}{V}\frac{\partial \beta\left(N - \frac{v}{V}Y\right)}{\partial \tau} &= \frac{\partial X}{\partial \eta} - \frac{\partial \beta\left(Y - \frac{v}{V}N\right)}{\partial \xi},
\end{aligned}$$

where

$$\beta = \frac{1}{\sqrt{1 - \left(\frac{v}{V}\right)^2}}.$$

The relativity principle demands that the Maxwell-Hertz equations for empty space also be valid in the system k if they are valid in the system K, i.e., that the vectors of the electric and the magnetic force (X',Y',Z') and (L',M',N')) of the moving system k, which are defined in this system by their ponderomotive effects on the electric and magnetic masses, respectively, satisfy the equations

$$\begin{aligned}
\frac{1}{V}\frac{\partial X'}{\partial \tau} &= \frac{\partial N'}{\partial \eta} - \frac{\partial M'}{\partial \zeta}, & \frac{1}{V}\frac{\partial L'}{\partial \tau} &= \frac{\partial Y'}{\partial \zeta} - \frac{\partial Z'}{\partial \eta}, \\
\frac{1}{V}\frac{\partial Y'}{\partial \tau} &= \frac{\partial L'}{\partial \zeta} - \frac{\partial N'}{\partial \xi}, & \frac{1}{V}\frac{\partial M'}{\partial \tau} &= \frac{\partial Z'}{\partial \xi} - \frac{\partial X'}{\partial \zeta}, \\
\frac{1}{V}\frac{\partial Z'}{\partial \tau} &= \frac{\partial M'}{\partial \xi} - \frac{\partial L'}{\partial \eta}, & \frac{1}{V}\frac{\partial N'}{\partial \tau} &= \frac{\partial X'}{\partial \eta} - \frac{\partial Y'}{\partial \xi}.
\end{aligned}$$

Obviously, the two systems of equations found for the system k must express exactly the same thing, since both are equivalent to the Maxwell-Hertz equations for the system K. Further, since the equations of the two systems coincide apart from the symbols representing the vectors, it follows that the functions occurring in the systems of equations at corresponding places must coincide up to a possibly v-dependent factor $\psi(v)$, which is common to all functions of one system of equations and is independent of ξ, η, ζ, and τ. The following relations will therefore be valid:

$$\begin{aligned}
X' &= \psi(v)X, & L' &= \psi(v)L, \\
Y' &= \psi(v)\beta\left(Y - \frac{v}{V}N\right), & M' &= \psi(v)\beta\left(M + \frac{v}{V}Z\right), \\
Z' &= \psi(v)\beta\left(Z + \frac{v}{V}M\right), & N' &= \psi(v)\beta\left(N - \frac{v}{V}Y\right).
\end{aligned}$$

If we now invert this system of equations, first, by solving the equations just obtained and, second, by applying the equations to the inverse transformation (from k to K) which is characterized by the velocity $-v$, we obtain, if we take into account that the two systems of equations so obtained must be identical,

$$\psi(v) \cdot \psi(-v) = 1.$$

Further, it follows for reasons of symmetry[4] that

$$\psi(v) = \psi(-v);$$

thus

$$\psi(v) = 1,$$

and our equations take the form

$$\begin{aligned}
X' &= X, & L' &= L, \\
Y' &= \beta\left(Y - \frac{v}{V}N\right), & M' &= \beta\left(M + \frac{v}{V}Z\right), \\
Z' &= \beta\left(Z + \frac{v}{V}M\right), & N' &= \beta\left(N - \frac{v}{V}Y\right).
\end{aligned}$$

[4]If, e.g., $X = Y = Z = L = M = 0$ and $N \neq 0$, then it is clear for reasons of symmetry that if v changes its sign without changing its numerical value, then Y' too must change its sign without changing its numerical value.

By way of interpreting these equations, we shall add the following: Imagine a pointlike quantity of electricity whose magnitude, measured in the system at rest K, is "one," i.e., which, when at rest in the system at rest, exerts a force of 1 dyne on an equal quantity of electricity at a distance of 1 cm. According to the principle of relativity this electric mass is also of magnitude "one" if measured in a moving system. If this quantity of electricity is at rest relative to the system at rest, the vector (X,Y,Z) equals the force exerted on it by definition. If this quantity of electricity is at rest relative to the moving system (at least at the instant considered), the force exerted on it, and measured in the moving system, will equal the vector (X',Y',Z'). Hence, the first three of the above equations can be expressed in words in the following two ways:

1. If a pointlike unit electric pole is in motion in an electromagnetic field, there will act on it, in addition to the electric force, an "electromotive force" which, if we neglect terms multiplied by the second and higher powers of v/V, equals the vector product of the velocity of motion of the unit pole and the magnetic force, divided by the velocity of light. (Old mode of expression.)

2. If a pointlike unit electric pole is in motion in an electromagnetic field, the force acting on it equals the electric force present at the location of the unit pole, which is obtained by transforming the field to a coordinate system that is at rest relative to the unit electric pole. (New mode of expression.)

Analogous propositions apply for "magnetomotive forces." We can see that in the theory developed, the electromotive force merely plays the role of an auxiliary concept, whose introduction is due to the circumstance that the electric and magnetic forces do not have an existence independent of the state of motion of the coordinate system.

It is further clear that the asymmetry mentioned in the Introduction when considering the currents produced by the relative motion of a magnet and a conductor, disappears. Questions as to the "seat" of the electrodynamic electromotive forces (unipolar machines) also become pointless.

§7. Theory of Doppler's principle and of aberration

Imagine in the system K, very far from the coordinate origin, a source of electrodynamic waves, which in a part of space containing the coordinate origin is represented with sufficient accuracy by the equations

$$\begin{aligned} X &= X_0 \sin\Phi, & L &= L_0 \sin\Phi, & & \\ Y &= Y_0 \sin\Phi, & M &= M_0 \sin\Phi, & \Phi &= \omega\left(t - \frac{ax+by+cz}{V}\right) \\ Z &= Z_0 \sin\Phi, & N &= N_0 \sin\Phi. & & \end{aligned}$$

Here (X_0,Y_0,Z_0) and (L_0,M_0,N_0) are the vectors determining the amplitude of the wave train, and a,b,c are the direction cosines of the wave normals.

We ask, what characterizes these waves when investigated by an observer who is at rest in the moving system k? — Applying the transformation equations for electric and magnetic forces found in §6 and those for coordinates and time found in §3, we obtain directly

$$X' = X_0 \sin\Phi', \qquad L' = L_0 \sin\Phi',$$
$$Y' = \beta\left(Y_0 - \frac{v}{V}N_0\right)\sin\Phi', \qquad M' = \beta\left(M_0 + \frac{v}{V}Z_0\right)\sin\Phi',$$
$$Z' = \beta\left(Z_0 + \frac{v}{V}M_0\right)\sin\Phi', \qquad N' = \beta\left(N_0 - \frac{v}{V}Y_0\right)\sin\Phi',$$
$$\Phi' = \omega'\left(t - \frac{a'\xi + b'\eta + c'\zeta}{V}\right),$$

where we have put

$$\omega' = \omega\beta\left(1 - a\frac{v}{V}\right),$$
$$a' = \frac{a - \frac{v}{V}}{1 - a\frac{v}{V}},$$
$$b' = \frac{b}{\beta\left(1 - a\frac{v}{V}\right)},$$
$$c' = \frac{c}{\beta\left(1 - a\frac{v}{V}\right)}.$$

From the equation for ω' it follows that if an observer moves with velocity v relative to an infinitely distant source of light of frequency ν, such that the connecting line "light source - observer" forms an angle φ with the observer's velocity, where this velocity is referred to a coordinate system that is at rest relative to the light source, then ν', the frequency of the light perceived by the observer, is given by the equation

$$\nu' = \nu\frac{1 - \cos\varphi\frac{v}{V}}{\sqrt{1 - \left(\frac{v}{V}\right)^2}}.$$

This is Doppler's principle for arbitrary velocities. For $\Phi = 0$ the equation takes the simple form

$$\nu' = \nu\sqrt{\frac{1 - \frac{v}{V}}{1 + \frac{v}{V}}}.$$

We see that, contrary to the usual conception, when $v = -\infty$, then $v = \infty$.

If φ' denotes the angle between the wave normal (the direction of the ray) in the moving system and the connecting line "light source - observer," the equation for a' takes the form

$$\cos\varphi' = \frac{cos\varphi - \frac{v}{V}}{1 - \frac{v}{V}\cos\varphi}.$$

This equation expresses the law of aberration in its most general form. If $(\varphi = \pi/2$, the equation takes the simple form

$$\cos\varphi' = -\frac{v}{V}.$$

It remains now to find the amplitude of the waves the way it appears in the moving system. If A and A' denote the electric or magnetic force in the system at rest and in motion, respectively, we get

$$A'^2 = A^2 \frac{1 - \frac{v}{V}}{1 + \frac{v}{V}}.$$

It follows from the equations derived above that to an observer approaching a light source with velocity V, this source would appear to have infinite intensity.

§8. Transformation of the energy of light rays. Theory of the radiation pressure exerted on perfect mirrors.

Since $A^2/8\pi$ equals the energy of light per unit volume, according to the principle of relativity we have to consider $A'^2/8\pi$ as the light energy in the moving system. Hence A'^2/A^2 would be the ratio of the energy of a given light complex "measured in motion" and the same energy "measured at rest," if the volume of a light complex were the same whether measured in K or k. However, this is not the case. If a,b,c are the direction cosines of the wave normal of the light in the system at rest, then the surface elements of the spherical surface

$$(x - Vat)^2 + (y - Vbt)^2 + (z - Vct)^2 = R^2,$$

which moves with the velocity of light, are not traversed by any energy; we may therefore say that this surface permanently encloses the same light complex. We ask for the quantity of energy enclosed by this surface as observed in the system k, i.e., the energy of the light complex relative to the system k.

Observed in the moving system, the spherical surface is an ellipsoidal surface whose equation at time $\tau = 0$ is

$$\left(\beta\xi - \alpha\beta\frac{v}{V}\xi\right)^2 + \left(\eta - b\beta\frac{v}{V}\xi\right)^2 + \left(\zeta - c\beta\frac{v}{V}\right)^2 = R^2,$$

If S denotes the volume of the sphere and S' that of the ellipsoid, then a simple calculation shows that

$$\frac{S'}{S} = \frac{\sqrt{1 - \left(\frac{v}{V}\right)^2}}{1 \frac{v}{V} \cos\varphi},$$

If the energy of the light enclosed by the surface under consideration is denoted by E when measured in the system at rest and by E' when measured in the moving system, we obtain

$$\frac{E'}{E} = \frac{\frac{A'^2}{8\pi}S'}{\frac{A^2}{8\pi}S} = \frac{1 \frac{v}{V} \cos\varphi}{\sqrt{1 - \left(\frac{v}{V}\right)^2}}.$$

which for $\varphi = 0$ reduces to the simpler formula

$$\frac{E'}{E} = \sqrt{\frac{1-\frac{v}{V}}{1+\frac{v}{V}}}.$$

It is noteworthy that the energy and the frequency of a light complex vary with the observer's state of motion according to the same law.

Let the coordinate plane $\xi = 0$ be a completely reflecting surface at which the plane waves considered in the last section are getting reflected. We ask for the light pressure exerted on the reflecting surface and the direction, frequency, and intensity of the light after reflection.

Let the incident light be defined by the quantities A, $\cos\varphi$, and ν (referred to the system K). Observed from k, the corresponding quantities are

$$\begin{aligned}
A' &= A\frac{1-\frac{v}{V}\cos\varphi}{\sqrt{1-\left(\frac{v}{V}\right)^2}},\\
\cos\varphi' &= \frac{\cos\varphi-\frac{v}{V}}{1-\frac{v}{V}\cos\varphi},\\
\nu' &= \nu\frac{1-\frac{v}{V}\cos\varphi}{\sqrt{1-\left(\frac{v}{V}\right)^2}}.
\end{aligned}$$

Referring the process to the system k, we get for the reflected light

$$\begin{aligned}
A'' &= A',\\
\cos\varphi'' &= -\cos\varphi',\\
\nu'' &= \nu'.
\end{aligned}$$

Finally, by transforming back to the system at rest K, we get for the reflected light

$$\begin{aligned}
A''' &= A''\frac{1+\frac{v}{V}\cos\varphi''}{\sqrt{1-\left(\frac{v}{V}\right)^2}} &&= A\frac{1-2\frac{v}{V}\cos\varphi+\left(\frac{v}{V}\right)^2}{1-\left(\frac{v}{V}\right)^2},\\
\cos\varphi''' &= \frac{\cos\varphi''+\frac{v}{V}}{1-\frac{v}{V}\cos\varphi''} &&= -\frac{\left(1+\left(\frac{v}{V}\right)^2\right)\cos\varphi-2\frac{v}{V}}{1-2\frac{v}{V}\cos\varphi+\left(\frac{v}{V}\right)^2},\\
\nu''' &= \nu''\frac{1-\frac{v}{V}\cos\varphi''}{\sqrt{1-\left(\frac{v}{V}\right)^2}} &&= \nu\frac{1-2\frac{v}{V}\cos\varphi+\left(\frac{v}{V}\right)^2}{1-\left(\frac{v}{V}\right)^2}.
\end{aligned}$$

The energy (measured in the system at rest) striking the unit surface of the mirror per unit time is obviously $A^2/8\pi(V\cos\varphi - v)$. The energy leaving the unit surface of the mirror per unit time is $A'''^2/8\pi(-V\cos\varphi''' + v)$.
According to the energy principle, the difference of these two expressions is the work done by the light pressure per unit time. Equating this work with $P\cdot v$,

where P is the pressure of light, we obtain

$$P = 2\frac{A^2}{8\pi}\frac{\left(\cos\varphi - \frac{v}{V}\right)^2}{1 - \left(\frac{v}{V}\right)^2}.$$

n first approximation, in agreement with experience and with other theories, we get

$$P = 2\frac{A^2}{8\pi}\cos^2\varphi.$$

All problems in the optics of moving bodies can be solved by the method employed here. The essential point is that the electric and magnetic forces of light, which is influenced by a moving body, are transformed to a coordinate system that is at rest relative to that body. This reduces every problem in the optics of moving bodies to a series of problems in the optics of bodies at rest.

§9. Transformation of the Maxwell-Hertz equations when convection currents are taken into consideration

We start from the equations

$$\begin{aligned}
&\frac{1}{V}\left\{u_x\varrho + \frac{\partial X}{\partial t}\right\} = \frac{\partial N}{\partial y} - \frac{\partial M}{\partial z}, &\qquad \frac{1}{V}\frac{\partial L}{\partial t} = \frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y},\\
&\frac{1}{V}\left\{u_y\varrho + \frac{\partial Y}{\partial t}\right\} = \frac{\partial L}{\partial z} - \frac{\partial N}{\partial x}, &\qquad \frac{1}{V}\frac{\partial M}{\partial t} = \frac{\partial Z}{\partial x} - \frac{\partial X}{\partial z},\\
&\frac{1}{V}\left\{u_z\varrho + \frac{\partial Z}{\partial t}\right\} = \frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}, &\qquad \frac{1}{V}\frac{\partial N}{\partial t} = \frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x},
\end{aligned}$$

where

$$\varrho = \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z}$$

denotes the 4π-fold density of electricity and (u_x, u_y, u_z) the electricity's velocity vector. If the electric masses are conceived as permanently bound to small, rigid bodies (ions, electrons), then these equations constitute the electromagnetic foundation of Lorentz's electrodynamics and optics of moving bodies.

If, using the transformation equations presented in §3 and §6, we transform these equations, which should be valid in system K, to system k, we get the equations

$$\begin{aligned}
&\frac{1}{V}\left\{u_\xi\varrho' + \frac{\partial X'}{\partial \tau}\right\} = \frac{\partial N'}{\partial \eta} - \frac{\partial M'}{\partial \zeta}, &\qquad \frac{1}{V}\frac{\partial L'}{\partial \tau} = \frac{\partial Y'}{\partial \zeta} - \frac{\partial Z'}{\partial \eta},\\
&\frac{1}{V}\left\{u_\eta\varrho' + \frac{\partial Y'}{\partial \tau}\right\} = \frac{\partial L'}{\partial \zeta} - \frac{\partial N'}{\partial \xi}, &\qquad \frac{1}{V}\frac{\partial M'}{\partial \tau} = \frac{\partial Z'}{\partial \xi} - \frac{\partial X'}{\partial \zeta},\\
&\frac{1}{V}\left\{u_\zeta\varrho' + \frac{\partial Z'}{\partial \tau}\right\} = \frac{\partial M'}{\partial \xi} - \frac{\partial L'}{\partial \eta}, &\qquad \frac{1}{V}\frac{\partial N'}{\partial \tau} = \frac{\partial X'}{\partial \eta} - \frac{\partial Y'}{\partial \xi},
\end{aligned}$$

where

$$\frac{u_x - v}{1 - \frac{u_x v}{V^2}} = u_\xi, \qquad \varrho' = \frac{\partial X'}{\partial \xi} + \frac{\partial Y'}{\partial \eta} + \frac{\partial Z'}{\partial \zeta} = \beta \left(1 - \frac{v u_x}{V^2} \varrho\right).$$
$$\frac{u_y}{\beta\left(1 - \frac{u_x v}{V^2}\right)} = u_\eta,$$
$$\frac{u_z}{\beta\left(1 - \frac{u_x v}{V^2}\right)} = u_\zeta.$$

Since—as follows from the addition theorem of velocities (§5)—the vector $u_\xi, u_\eta, u_\zeta,$) is actually the velocity of the electric masses measured in the system k, we have thus demonstrated that with our kinematic principles taken as a basis, the electrodynamic foundation of Lorentz's theory of the electrodynamics of moving bodies agrees with the principle of relativity.

Let me also briefly add that the following important proposition can easily be deduced from the equations we have derived: If an electrically charged body moves arbitrarily in space without change of its charge, observed from a coordinate system moving with the body, then its charge will also remain constant when observed from the system "at rest" K.

§10. Dynamics of the (slowly accelerated) electron

In an electromagnetic field let a pointlike particle endowed with an electric charge ε (called "electron" in what follows) be in motion; about its law of motion we assume only the following:

If the electron is at rest during a particular epoch, its motion in the next element of time will occur according to the equations

$$\mu \frac{d^2 x}{dt^2} = \varepsilon X,$$
$$\mu \frac{d^2 y}{dt^2} = \varepsilon Y,$$
$$\mu \frac{d^2 z}{dt^2} = \varepsilon Z,$$

where x, y, z denote the coordinates of the electron and ıits mass, as long as the electron moves slowly.

Further, let the electron's velocity in some given time epoch be v. We seek to find the law by which the electron is moving in the next element of time.

Without affecting the generality of the consideration, we can and will assume that at the moment when we focus on it, the electron is at the coordinate origin, and is moving with velocity v along the X-axis of the coordinate system K. It is then obvious that at the instant indicated ($t = 0$), the electron is at rest relative to the coordinate system k that moves with constant velocity v parallel to the X-axis.

From the above assumption combined with the relativity principle it is clear that, viewed from the system k, the electron will move during the immediately following time (for small values of t) according to the equations

$$\mu \frac{d^2\xi}{\tau^2} = \varepsilon X',$$
$$\mu \frac{d^2\eta}{d\tau^2} = \varepsilon Y',$$
$$\mu \frac{d^2\zeta}{d\tau^2} = \varepsilon Z',$$

where the symbols ξ, η, ζ, τ, X', Y', Z' refer to the system k. If we also stipulate that for $t = x = y = z = 0$ we should have $\tau = \xi = \eta = \zeta = 0$, then the transformation equations of §§3 and 6 will be valid, so that we get

$$\begin{aligned} \tau &= \beta \left(t - \frac{v}{V^2} x \right), & \qquad X' &= X, \\ \xi &= \beta (x - vt), & Y' &= \beta \left(Y - \frac{v}{V} N \right), \\ \eta &= y, & Z' &= \beta \left(Z + \frac{v}{V} M \right). \\ \zeta &= z, \end{aligned}$$

With the help of these equations we transform the above equations of motion from system k to system K and obtain

$$\text{(A)} \qquad \left\{ \begin{aligned} \tfrac{d^2x}{dt^2} &= \tfrac{\varepsilon}{\mu} \tfrac{1}{\beta^3} X, \\ \tfrac{d^2y}{dt^2} &= \tfrac{\varepsilon}{\mu} \tfrac{1}{\beta} \left(Y - \tfrac{v}{V} N \right), \\ \tfrac{d^2z}{dt^2} &= \tfrac{\varepsilon}{\mu} \tfrac{1}{\beta} \left(Z + \tfrac{v}{V} M \right). \end{aligned} \right.$$

Following the usual approach, we now seek to determine the "longitudinal" and "transverse" masses of the moving electron. We write the equations (A) in the form

$$\mu \beta^3 \frac{d^2x}{dt^2} = \varepsilon X = \varepsilon X',$$
$$\mu \beta^2 \frac{d^2y}{dt^2} = \varepsilon \beta \left(Y - \frac{v}{V} N \right) = \varepsilon Y',$$
$$\mu \beta^2 \frac{d^2z}{dt^2} = \varepsilon \beta \left(Z + \frac{v}{V} M \right) = \varepsilon Z',$$

and note first that $\varepsilon X'$, εY', $\varepsilon Z'$ are the components of the ponderomotive force exerted on the electron, as observed in a system co-moving at this instant with the electron at the latter's speed. (This force could be measured, for example, by a spring balance at rest in the last-mentioned system.) If we simply call this force "the force exerted on the electron," and maintain the equation

Numerical value of mass$\times$numerical value of acceleration $=$ numerical value of force, stipulating, in addition, that the accelerations be measured in the system at rest K, we obtain from the above equations

$$\text{Longitudinal mass} = \frac{\mu}{\left(\sqrt{1-\left(\frac{v}{V}\right)^2}\right)^3},$$

$$\text{Transverse mass} = \frac{\mu}{1-\left(\frac{v}{V}\right)^2}.$$

Of course, with a different definition of force and acceleration we would obtain different numerical values for the masses; this shows that we must proceed with great caution when comparing different theories of the motion of the electron.

It should be noted that these results concerning mass are also valid for ponderable material points, since a ponderable material point can be made into an electron (in our sense) by adding to it an *arbitrarily small* electric charge.

We now determine the kinetic energy of the electron. If an electron starts out from the origin of the system K with an initial velocity 0 and is moving continually along the X-axis under the influence of an electrostatic force X, then it is clear that the energy drawn from the electrostatic field has the value $\int \varepsilon X dx$. Since the electron is supposed to accelerate slowly and will therefore emit no energy in the form of radiation, the energy taken from the electrostatic field must be equated with the energy of motion W of the electron. Bearing in mind that the first of equations (A) holds during the entire process of motion considered, we obtain therefore

$$W = \int \varepsilon X dx = \int_0^\nu \beta^3 v dv = \mu V^2 \left\{ \frac{1}{\sqrt{1-\left(\frac{v}{V}\right)^2}} - 1 \right\}.$$

Thus, W becomes infinitely large when $v = V$. As in our previous results, superluminary velocities have no possibility of existence.

This expression for kinetic energy too must be valid for ponderable masses as well by virtue of the argument presented above.

Let us now enumerate those properties of the motion of the electron that result from the system of equations (A) and are accessible to experiment.

1. It follows from the second equation of the system of equations (A) that an electric force Y and a magnetic force N have an equally strong deflective effect on an electron moving with velocity v if $Y = N \cdot v/V$. Thus we see that according to our theory we can determine the velocity of the electron for any arbitrary velocity from the ratio of the magnetic deflection A_m to the electric deflection A_e by applying the law

$$\frac{A_m}{A_e} = \frac{v}{V}.$$

This relation can be checked experimentally since the velocity of the electron can also be measured directly, e.g., using rapidly oscillating electric and magnetic fields.

2. It follows from the derivation for the kinetic energy of the electron that the potential difference traversed by the electron and the velocity v attained by it must be related by the equation

$$P = \int X dx = \frac{\mu}{\varepsilon} V^2 \left\{ \frac{1}{\sqrt{1 - \left(\frac{v}{V}\right)^2}} - 1 \right\}.$$

3. We calculate the radius of curvature R of the path when a magnetic force N, which acts perpendicular to the velocity of the electron, is present (as the only deflecting force). From the second of equations (A) we obtain

$$-\frac{d^2 y}{dt^2} = \frac{v^2}{R} = \frac{\varepsilon}{\mu} \frac{v}{V} N \cdot \sqrt{1 - \left(\frac{v}{V}\right)^2}$$

or

$$R = V^2 \frac{\mu}{\varepsilon} \cdot \frac{\frac{v}{V}}{\sqrt{1 - \left(\frac{v}{V}\right)^2}} \cdot \frac{1}{N}.$$

These three relations are a complete expression of the laws by which the electron must move according to the theory presented here.

In conclusion, let me note that my friend and colleague M. Besso steadfastly stood by me in my work on the problem here discussed, and that I am indebted to him for many a valuable suggestion.

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