A Guide to the Physics GRE

Version 1.1

Sam Moore

November 2019

Quick note: The following is a LaTeX version of my GRE physics guide. These notes are a work in progress in two ways. First, the notes are not yet complete, but shall be completed within the next month (I hope). Second, it is my hope to transfer these notes into website form, making the notes easier to read.

It is here that I shall give my acknowledgements.

Chapter 1

Introduction

The Physics GRE is currently one of the major parts of the application to graduate programs throughout the US and at certain other institutions. Whilst there is a current trend for some schools to move away from the GRE and focus on other aspects of the application, it is a mistake commonly made to neglect to study for the test. Many graduate programs see the test as a standardised way to prove that you are capable of passing a relatively simple practice exam, as well as proof that you have a general understanding of some of the fundamental aspects of physics.
And yet, studying for the GRE can feel difficult. There are very few resources for the study of this particular exam, which can lead to one trawling through countless textbooks to review a plethora of topics when many of these textbooks go into too much detail. Likewise, the four tests that ETS have currently released are useful to work through, but these tests are old enough that the current GRE has a very different feel. Finally, unlike a standard undergraduate physics exam, the GRE works by testing brevity, including quick ballpark estimates, the memorisation of essential or difficult to derive equations, and the elimination of multiple-choice answers by simple reasoning.
This website is designed as a study guide for the GRE, to fill a gaping hole in the resources for this test. It is currently a work in progress. I confess right away that I have erred on the side of including more detail rather than less. Additionally, I have sought to incorporate questions to build and test your knowledge. Some of these are conceptual questions - they exist to check your understanding and give you confidence. The others are GRE-style questions - they exist to give you a feel for the actual test, but more importantly, a feel for whether you can do these questions quickly enough to be comfortable when taking the GRE.
About me, I am a former student of Astrophysics at Princeton University. I have designed this guide motivated by my pure loathing for this standardised test.

1.1 About the Exam

There are three important things to know about the GRE:

1. No calculator allowed - get comfortable with mental arithmetic and estimation!

2. Minimal Equations - start memorising those essential and hard-to-derive equations.

3. It's 100 questions in 170 minutes - you need to be quick at the questions.

1.1.1 The Content

Taken straight from the horse's mouth. Take note of the percentages if you're pushed for time to study; some knowledge is more valuable!

- CLASSICAL MECHANICS — 20% (such as kinematics, Newton's laws, work and energy, oscillatory motion, rotational motion about a fixed axis, dynamics of systems of particles, central forces and celestial mechanics, three-dimensional particle dynamics, Lagrangian and Hamiltonian formalism, noninertial reference frames, elementary topics in fluid dynamics).
- ELECTROMAGNETISM — 18% (such as electrostatics, currents and DC circuits, magnetic fields in free space, Lorentz force, induction, Maxwell's equations and their applications, electromagnetic waves, AC circuits, magnetic and electric fields in matter).
- OPTICS AND WAVE PHENOMENA — 9% (such as wave properties, superposition, interference, diffraction, geometrical optics, polarization, Doppler effect).
- THERMODYNAMICS AND STATISTICAL MECHANICS — 10% (such as the laws of thermodynamics, thermodynamic processes, equations of state, ideal gases, kinetic theory, ensembles, statistical concepts and calculation of thermodynamic quantities, thermal expansion and heat transfer).
- QUANTUM MECHANICS — 12% (such as fundamental concepts, solutions of the Schrödinger equation (including square wells, harmonic oscillators, and hydrogenic atoms), spin, angular momentum, wave function symmetry, elementary perturbation theory).
- ATOMIC PHYSICS — 10% (such as properties of electrons, Bohr model, energy quantization, atomic structure, atomic spectra, selection rules, black-body radiation, x-rays, atoms in electric and magnetic fields).
- SPECIAL RELATIVITY — 6% (such as introductory concepts, time dilation, length contraction, simultaneity, energy and momentum, four-vectors and Lorentz transformation, velocity addition).
- LABORATORY METHODS — 6% (such as data and error analysis, electronics, instrumentation, radiation detection, counting statistics, interaction of charged particles with matter, lasers and optical interferometers, dimensional analysis, fundamental applications of probability and statistics).
- SPECIALIZED TOPICS — 9% Nuclear and Particle physics (e.g., nuclear properties, radioactive decay, fission and fusion, reactions, fundamental properties of elementary particles), Condensed Matter (e.g., crystal structure, x-ray diffraction, thermal properties, electron theory of metals, semiconductors, superconductors), Miscellaneous (e.g., astrophysics, mathematical methods, computer applications)

This website tries to go through these topics in order, that said, it is my opinion that some of these topics are better taught before others, and so I have reordered where appropriate. The ETS website also sets out a disclaimer stating that this list isn't exhaustive, so other topics are fair game.

Chapter 2

Classical Mechanics

Let us begin by reviewing some of the fundamental basics of classical mechanics. You probably know this already but this is fundamental. If anything feels unfamiliar or rusty, then this is a sign that some serious review may be needed. Additionally, don't feel bound to reading the 'Basic Introduction' if it's too easy; for most of you I suspect this is material you know better than the back of your hand. This section exists solely to make this guide a more comprehensive and accessible one.

2.1 Basic Introduction

To begin with, you should be aware of Newton's Three Laws, which are fundamental to the study of kinematics. As general advice for the GRE, you should also be aware of which one is which; occasionally questions can reference the names of laws as answers to questions.

Newton's First Law: *An object with no resultant force will move at a constant velocity (or remain at rest).*

To spell this out, the resultant force is the net force acting on an object. If all the forces on an object cancel out, then the object moves at the same speed (i.e: zero acceleration). Remember that an object at rest is at a constant velocity of $0ms^{-1}$, so the parentheses above are actually redundant, but it's good to emphasise this. Consider the following example:

Newton's Second Law: *An object acted upon by a resultant force will accelerate in the direction of that force.*

You can think of this as a natural extension from Newton's First Law. You may be more familiar with the law written as $F = ma$. In words, the resultant (or overall) force is equal to the product of mass and acceleration. If we assume that the mass is constant[1], then it is simple to calculate the acceleration given the resultant force. Furthermore, the resultant force and acceleration are both vector quantities that point in the same direction.

Newton's Third Law: *Every reaction has an equal and opposite reaction.*

Think of it this way: if I stand next to a wall and push into it, I am exerting a force onto the wall. The wall

[1]It may seem peculiar for me to spell out that we assume that mass is constant. However, in many real life cases, the mass of the accelerating object is decreasing (for example, in a car, the fuel is being used up). This is important when it comes to certain systems (e.g: rockets), but not important for the GRE.

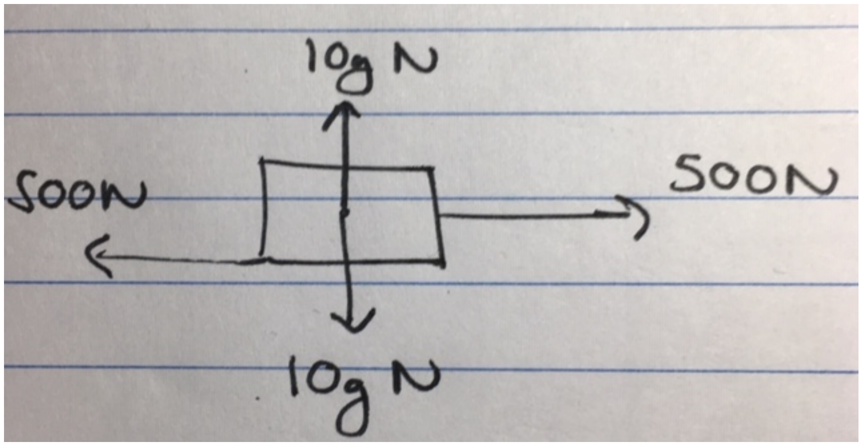

Figure 2.1: Caption

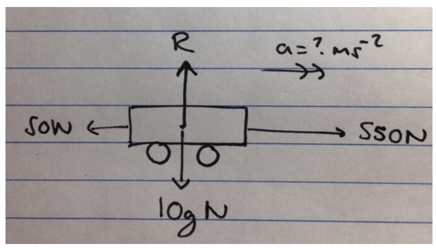

Figure 2.2: Caption

exerts an opposite but equal force into me. This is actually critical, for this allows for us to both remain stationary, or for collisions to occur as we see them, etc.

As a quick review of Newton's Laws, let us consider a block on a level surface.[2]
Suppose that the following block has a force of 500N acting to the right of it. There also exists a friction force acting in the opposite direction acting to the left, also of 500N. We note that there are no other forces acting in the horizontal direction. Let us arbitrarily take right to be the positive direction here, then the resultant force is:

$F = 500N + (-500N) = 0$

The resultant force in this direction is zero, and thus from $F = ma$, the acceleration in the horizontal direction is zero.

Let us now consider the vertical direction. First, gravity acts on the slope producing a force equal to mg, where g is gravitational acceleration (typically equal to 9.81 on Earth). This should come as no surprise: it's just $F = ma$ where $a = g$. This force (known as 'weight') acts straight downwards as shown. There is an additional force known as the reaction force. In this case, it is equal to the weight of the block. This is necessary to avoid vertical acceleration, or else the block would accelerate into the Earth.

Let us keep going with the block examples: Suppose that a car has the following forces acting on it as shown in the diagram.

[2] In all likelihood, this is probably overkill for you if you've studied physics even at a basic level. I include it merely for completeness, but always feel free to skip ahead.

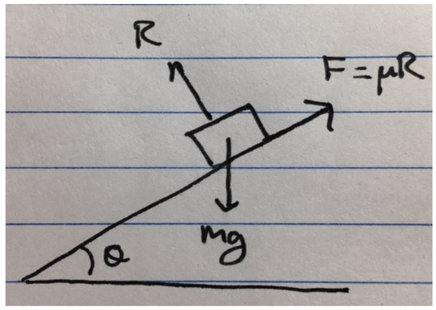

Figure 2.3: Caption

1. What is the reaction force, R?

2. What is the acceleration of the car?(specify both the magnitude and the direction)

For the first question, consider that the vertical velocity of the car should be zero (it shouldn't fly off the road into space, or down into the core of the Earth!). As such, the reaction force upwards must equal the gravitational force downwards. This yields $R = 10gN$.
For the second question, consider the horizontal forces. Taking right to be positive:
$F = 550N + (-50N) = 500N$
Now given that the weight $= mg = 10g$, one can deduce that $m = 10$kg. Rearranging $F = ma$, we find $a = 5ms^{-2}$, acting to the right.

Now that we have reviewed Newton's Laws, let us move on and consider a standard set-up in Classical Mechanics: a box on a slope.

The following set up is standard: we have a block held at rest on a slope of angle θ. The block is released, and we now see to determine certain things about the block, such as its acceleration. Note carefully that we are using a block here because this block moves down the slope without rolling (unlike, say, a sphere placed on a slope). The reason is because rolling objects should be treated slightly differently in these types of questions. We shall come to this later, but for now, think of blocks.
The block will have at least two forces acting on it, possibly three. First, gravity acts on the slope again. This force acts straight downwards as shown. Using some basic geometry and trigonometry, it is possible to decompose this force into two components: a sin term that acts parallel to the slope, and a cos term that acts perpendicular to it. Since most forces tend to act parallel or perpendicular to the slope, these components are more useful than considering the downwards force mg on its own. The second force is the reaction force, which we already saw earlier on. In this case, it acts perpendicular to the slope, pointing outwards. Finally, if the surface of the slope is smooth, then there is no friction force to be considered. However, in most cases on the GRE (and in real physics) you shall consider the surface to be rough, meaning that there is friction.

Finally, let us consider for a moment what happens when θ goes to zero. The system should reduce to that of the block on the level surface we discussed earlier.

Remember that you will not have a calculator on the GRE. As such, the angles used tend to be limited to values x such that sin(x) and cos(x) give nice numbers. For a review of this, see the mathematical methods section.

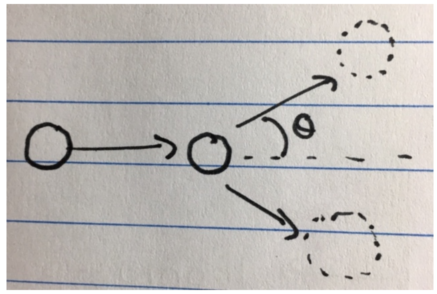

Figure 2.4: Caption

2.1.1 Collisions

Collision questions are a common topic on the GRE, so it generally pays off to be well acquainted with the topic. That said, these questions tend to be a touch time-consuming, and making a careless error in the algebra is a sure way of both wasting time and potentially missing an easy point. My advice here is to review the following, and then practise the questions that follow, focusing on speed.

The most fundamental concept in any collision question is the conservation of momentum, which states that $\rho_i = \rho_f$ provided that the system isn't subject to external forces. Linear momentum is defined as $\rho = mv$. To see how this works, let us look at an example.

[Insert Example here]

Let us extend the following framework to three dimensions.

The GRE lists 'particle dynamics in 3D' as a topic to learn for classical mechanics, and a standard collisions question usually covers this topic. However, I would be negligent not to point out that this doesn't give the complete picture. To do so, one requires use of energy-momentum vectors, which shall be covered later on in the Special Relativity section.

2.2 Energy and Work

The concepts of energy and work are essential in Classical Mechanics, and the study of physics as a whole. More importantly here, they are fundamental topics for the GRE, commonly tested as questions in their own right.
To begin, let us define work:

$$W = Fdcos(\theta) \tag{2.1}$$

2.2.1 Kinematics

There are four key equations that you should know:

$$v = u + at \tag{2.2}$$

$$s = ut + \frac{1}{2}at^2 \tag{2.3}$$

$$v^2 = u^2 + 2as \tag{2.4}$$

2.3 Energy

2.4 Fluids

Here, let us take a quick break to review what the GRE calls 'elementary topics in fluid mechanics'. This may seem like a tangent, but it is a nice way of reviewing some of the material already seen in a slightly different light.

Bernoulli's Equation

$$\frac{\rho_1 v_1^2}{2} + \rho_1 g h_1 + P_1 = \frac{\rho_2 v_2^2}{2} + \rho_2 g h_2 + P_2 \tag{2.5}$$

This is effectively a conservation of energy equation.

Archimedes Principle

The famous story is that Archimedes was asked by Hiero II to determine whether a golden crown was made of pure gold, or whether it was made of a mixture of gold and silver. It was on a trip to the baths that Archimedes realised that the answer could be determined by fluid displacement, shouting "Eureka!"
Here is the equation that you need to know:

$$F = \rho V d \tag{2.6}$$

2.4.1 Rotational Motion

Let us continue our discussion, moving from the realm of linear motion to that of rotational motion. Questions involving rotational motion are very popular on the GRE, so go through this section with care.

2.4.2 Oscillatory Motion

To begin, let us review the concept of a spring.

2.4.3 Lagrangian and Hamiltonian Formalism

Lagrangian and Hamiltonian mechanics encompasses a new way of thinking about Classical Physics that can fill a whole undergraduate course. The methodology is actually quite simple, but understanding why if works takes a bit more time.

I fully intend on expanding this section for people looking for a more detailed review of Lagrangian mechanics, but for now I shall keep this short for the purposes of quick GRE review.

Consider how problems in Newtonian mechanics work. In the most involved problems, you are typically given a bunch of forces, and then using various initial conditions can solve for acceleration, velocity and displacement. However, these are all vector quantities, which can be irksome. One effectively has to keep track of both the magnitude and the direction of each vector at all times. Sometimes this can be simplified by using a more convenient coordinate system, but this can only take you so far.

Enter Lagrangian mechanics. Instead of formulating a problem using forces, we shall do so using energies. We are going from a series of vectors to a series of scalars. Let us look at how this works:

Consider a system with total kinetic energy T, and total potential energy V. The Lagrangian of the system is then defined as:

$$L = T - V \tag{2.7}$$

With this, we can then use the Euler-Lagrange equations to solve our problems:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q} \tag{2.8}$$

Solving the Euler-Lagrange equation will give you the equations of motion.

[Insert Example]

2.4.4 Orbits

Now that we've reviewed Lagrangian mechanics, it will be much easier to understand the formalism of orbits here. That said, I would recommend simply memorising the necessary equations for the GRE, because the derivations are too long to be worth doing.

2.4.5 Non-inertial Reference Frames

This is the last topic explicitly referenced on the GRE. I have never actually seen this come up so I would personally advise against focusing on this material if there are other areas where you should practise more.

First of all, some definitions: An inertial reference frame is a reference frame that is moving at a constant velocity. For example, a person in a lift moving downwards at $5ms^{-1}$ is standing in an inertial reference frame. These are important because the laws of physics are the same in all inertial frames. A non-inertial frame is therefore moving at a non-constant velocity (I.e: is accelerating or decelerating). An example here could be a lift in free-fall.

In a non-inertial frame, the laws of motion change in a way that is specific to the non-inertial frame. This seems daunting on the face of it: It seems as though I am effectively saying that $F = ma$ won't hold inside this non-inertial frame. Well, luckily that's not quite true, so long as I include some fictitious forces (or pseudo-forces) along the way.

Let us start with the most famous "fake" force: centrifugal force.

$$F = -m\Omega \times (\Omega \times r) \tag{2.9}$$

The second fictitious force that you should know is the Coriolis force.

$$F = -2m\Omega \times v \tag{2.10}$$

There is a third common fictitious force known as the Euler force which is caused by azimuthal acceleration (I.e: a change in the angular velocity) of the rotating frame. I can never see this coming up on the GRE, but you never know, so here is the mathematical description of it:

$$F = -m\frac{d\Omega}{dt} \times r \tag{2.11}$$

Wikipedia gives the nicely intuitive example of a Merry-go-round: as it's angular velocity increases, a person is pushed to the back of the horse. Additionally, a person nearer to the outside experiences a bigger force than one nearer the axis of rotation.

Chapter 3

Electromagnetism

3.1 Maxwell's Equations

Electromagnetism is one of the larger section on the GRE, so it pays to be highly familiar with it. Let us begin with the fundamental equations of Electromagnetism: Maxwell's Equations. We shall work on using them a bit later, but these are so fundamental that it is worth spending some time understanding them.

Let us begin with the integral forms of Maxwell's Equations. They are slightly more intuitive and more useful for the GRE than the differential forms. For clarify, I am working in SI units rather than Gaussian units, which may explain why certain factors may seem slightly different. Get used to SI units for the GRE!

$$\int E \cdot dS = \frac{1}{\epsilon_0} \int \rho dV \tag{3.1}$$

The above equation is called Gauss's Law.

$$\int B \cdot dS = 0 \tag{3.2}$$

The above equation is called Gauss's Law for magnetism.

$$\int E \cdot dl = -\frac{d}{dt} \int B \cdot dS \tag{3.3}$$

The above equation is called Faraday's law (of induction)

$$\int B \cdot dl = \mu_0 (\int J \cdot dS + \epsilon_0 \frac{d}{dt} \int E \cdot dS) \tag{3.4}$$

The above equation is called Ampere's Law.

One can rewrite these integral equations into their differential forms:

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \tag{3.5}$$

$$\nabla \cdot B = 0 \tag{3.6}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{3.7}$$

$$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \tag{3.8}$$

3.2 Electrostatics

With this understood, let us move to the realm of electrostatics. In electrostatics, we look at non-moving charges. With that, there is no magnetic field, so Maxwell's Equations simplify as follows:

3.3 Magnetostatics

3.4 Fields in Matter

3.5 EM waves

3.6 DC Circuits

One part of the section on electromagnetism is DC circuits. The concepts involved are simple enough, and typically involve a very limited range of components: resistors, capacitors, and inductors. We shall look at problems involving each of these.

3.6.1 Resistors

A resistor is an object with a resistance that opposes the flow of current.
Resistors in series add as follows:

$$R_{Total} = \sum_{i=1}^{n} R_i = R_1 + R_2 + R_3 + ... \tag{3.9}$$

Resistors in parallel add as follows:

$$R_{Total} = (\sum_{i=1}^{n} R_i^{-1})^{-1} \tag{3.10}$$

or in a more intuitive form:

$$\frac{1}{R_{Total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... \tag{3.11}$$

3.6.2 Capacitors

Capacitors add in the opposite way to resistors. Capacitors in series add as:

$$\frac{1}{C_{Total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... \tag{3.12}$$

while capacitors in parallel add as:

$$C_{Total} = \sum_{i=1}^{n} C_i = C_1 + C_2 + C_3 + ... \tag{3.13}$$

3.7 AC Circuits

3.8 Optics and Waves

The next section that we shall review is titled 'optics and waves'. You can rest easy, for this is far easier than the previous section on electromagnetism. In fact, you will have never probably seen a large chunk of this section before stepping foot in an undergraduate physics class. Let us begin with the simplest section, geometric optics:

In geometric optics, we assume that
There are two equations that are integral to geometric optics:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \tag{3.14}$$

$$\frac{1}{f} = (n-1)(\frac{1}{R_1} - \frac{1}{R_2}) \tag{3.15}$$

That's it. We will spend some time practising them, understanding the sign conventions associated with them, etc, but this is the bulk of the material. In fact, if you've studied optics in the past, you probably recall the rather monotonous drawing of ray diagrams. We will review this just in case, but for the most part you can eschew these diagrams in favour of understanding the use of these equations.
To begin, let us look at a standard ray diagram. Typically, they feature either a lens or a mirror. There is an object on the left of the lens or mirror, and light shines from the left of the page (as is convention). The goal of ray diagrams is then to work out where the image appears, and what properties the image has.

Each mirror or lens has a property known as the focal length (f)
Now, let's go back to that first equation. The first equation is known as the thin lens equation.

3.9 Wave Properties

Now that geometric optics has been reviewed, let us look at the more mathematical side of waves and optics in this section. Formally, a wave $f(x,t)$ follows the wave equation:

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v}\frac{\partial^2 f}{\partial t^2} \tag{3.16}$$

where this generalises to multiple dimensions as expected. For clarity when trying to recall this equation, note that it is dimensionally correct (reducing in units to $x = vt \rightarrow m = ms^{-1}s$).
[Add extension of wave definition] [Add standing waves]

3.10 Interference

3.11 Diffraction

3.12 Polarisation

3.13 Doppler Effect

The Doppler Effect is a common occurrence on the GRE, so it pays to be deeply familiar with it. The Doppler Effect is the change in frequency of a wave due to relative movement. The (very bad) textbook example is an ambulance moving towards a stationary observer, such as yourself. The frequency (or pitch) of the sound increases as the ambulance speeds towards you, and decreases as it speeds away from you. This

is actually a dreadful example because the frequency is constantly changing as a function of time, whereas this is not a required condition for the Doppler effect. To clear up this confusion, let's look at the equation and do an example:
Thus the example shows a constant shift in the frequency of the wave.
As a final point, you might be wondering why the frequency of the sound is constantly changing in the example of the ambulance. The reason is because the angle of the source relative to the observer is constantly changing, causing $u = u_0 \cos\theta$ to change. Thus the problem is clarified!

Chapter 4

Thermodynamics

4.1 Microstates and Macrostates

Most courses in thermodynamics tend to start here with a brief review of microstates and macrostates. I will too, because although this is very basic knowledge, it helps us to understand why thermal physics has evolved as a way of doing things. You should at this point be familiar with the general principles of classical mechanics, which usually involves using F = ma to determine positions and velocities given some initial conditions. This process is relatively easy when you have a couple of bodies, such as two masses joined by a spring; or the orbits of the solar system (although there can be some subtleties that make these problems harder). This process becomes far more time-consuming as you increase the number of particles involved. What happens in the case of all the molecules of a gas in your lecture hall? The problem would take too much time and memory to solve, even on a supercomputer, for all practical purposes [1]. It is here that we enter the world of thermal physics. The notion of thermal physics is simple. We define a system that includes all of the particles that we care about. Examples might include all of the particles in a piston, or the air in your lecture hall. Everything else in the universe is described as the surroundings. We seek to describe the system by just a few quantities that describe the macro state of the system. The best example of such a macroscopic quantity is temperature: we can describe the temperature of particles in your lecture hall as T = 20C. The other two big quantities are pressure and volume. We might think of a beaker having a temperature of 300K, a pressure of 3Pa, and a volume of $50cm^3$, and we are effectively describing the system without any reference to the molecules within the beaker.

Microstates are the other side of this coin; it refers to the atoms within the system. For example, I could list the position and momentum for atom 1, 2, ..., N, and I would be describing the microstate of the system. There is a key link between microstates and statistical mechanics that will become clearer when we start to express macrostates like internal energy and entropy in terms of the probabilities of occupiation in a particular microstate. Let's slow down for a second though, and digest some of the more basic ideas in thermal physics before we dive into statistical mechanics.

4.2 Ensembles

To extend the idea of macrostates, let us consider the various statistical ensembles that you should be acquainted with.

[1] It should be noted that this is of course not how thermal physics evolved in practice, coming from a time that predates modern computing and even empirical evidence for the atom itself. The general principle that it is simple to describe a system using a few qualities still holds though, and remains true today even with computational advances.

1. Microcanonical (NVE) - a set of systems with constant N, V and E. Think of a closed beaker with insulation; there is no change in particle number, volume, or energy.

2. Canonical (NVT) - in this case, the energy can change but the temperature must remain the same. Think of that closed beaker without insulation.

3. Grand Canonical (μVT) - the particle number can now also change, but the chemical potential is fixed. Think of an open-top beaker.

For the GRE, all work seems to be done with the canonical ensemble. There are more types of ensembles, but I haven't seen them used even in an actual thermal physics course, so I'm ignoring them here.

4.3 Laws of Thermodynamics

Leading us into our next section on thermodynamics are the fundamental laws of thermodynamics. These are essential for both the GRE and understanding thermodynamics as a whole:

First Law: Energy cannot be created not destroyed.

This is quite a simple law, and means that although energy can be transformed from one form to another, the total energy in the system and the surroundings must remain the same. If the system is completely isolated such that there is no energy transfer to the surroundings, then we can make the even stronger claim that the total energy in the system must be constant. Thinking of this in a slightly different context, you are already familiar with the example of the pendulum with no friction, drag, etc. The pendulum sees transfer of kinetic energy to gravitional potential energy and vice versa, but the total energy is always the same. It is likewise in a thermodynamic system. We can express this with the following equation:

$$\Delta U = \delta Q - \delta W \tag{4.1}$$

where ΔU is the internal energy of the system, δQ is the heat transfer into the system, and δW is the work done by the system. This equation needs some further explaining. First of all, the sign convention can be confusing to some; you may have seen in the past a plus instead of that minus. In that case, δW would be representing the work done on the system. Just think of it intuitively; if work is being done by the system, then energy is moving somewhere else and the internal energy of the system must be decreasing. Second of all, you may be wondering what the difference is between Δ and δ in this case, as in both cases they represent a change in a quantity. The difference is unimportant for the GRE so feel free to skip the explanation, but in short, Δ represents a state function that is independent of the path taken, whereas δ represents a process function that is dependent on the path taken. For example, δQ is a process function because it depends on whether heat is being transferred from the system to the surroundings (one path) or from the surroundings into the system (another path). There are more subtle intricacies regarding the difference between a process and a state function, but that is the gist of it.

Second Law: Entropy increases.

There are actually quite a number of different ways of expressing the second law of thermodynamics, and most of them provide a very useful perspective on thermodynamics, even for the purposes of the GRE.

$$\Delta S \geq \int \frac{\delta Q}{T} \tag{4.2}$$

Third Law: No entropy at a temperature of absolute zero.

The third law can be thought of as an extension of the second. Looked at in an alternative light, consider that in practice, it is impossible to have a process that has zero entropy. As such, it is in practice impossible to reach a temperature of absolute zero. The current world record for cooling is at 100pK.

These laws were thought up before someone realised that there is a more fundamental law. Rather than renumber everything, I present to you the zeroth law:

Zeroth Law: if body 1 is in equilibrilum with body 2, and body 2 in equilibrium with body 3, then body 1 and body 3 are in equilibrium.

As a quick example, think of three objects touching each other for long enough without external heating or cooling. Then $T_1 = T_2 = T_3$. This is intuitive, and I've never seen it appear on the GRE. Oh well.

4.4 Equations of State

An equation of state is an equation that relates the state variables of a particular system in the form

$$f(P, V, T, ...) = 0 \tag{4.3}$$

If this seems somewhat abstract, let us look at some examples of equations of state below. I am certain that you will have seen some of these before.

- Boyle's Law

$$pV = constant \tag{4.4}$$

- Charles Law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \tag{4.5}$$

- Ideal Gas Law

$$pV = nRT \tag{4.6}$$

- Van der Waals Equation

$$(p + \frac{a}{V^2})(V - b) = RT \tag{4.7}$$

4.5 Ideal Gases

An ideal gases is a gas which obeys the equation of state $pV = nRT$. Because this equation is so important in thermal physics and engineering, let's analyse the assumptions going on behind it.

1. The gas contains a large number of particles in random motion obeying Newton's laws.
2. The size of the particles is negligible compared to the size of the gas as a whole.
3. There are no interactive forces between the particles of the gas. (This of course ignores the existence of Van der Waals' forces, and so on).

4.6 Kinetic Theory

As a slight distraction from all the information just presented, let's briefly discuss the elementary ideas of kinetic theory. Consider a bunch of atoms in gaseous from in a three-dimensional container. Perhaps it's a canister of neon gas. The basic idea is that the atoms move around at a speed that correlates to their temperature. This is the basis of the equipartition theorem:

(4.8)

4.7 Thermodynamic Processes

To begin, let us review the idea of a thermodynamic cycle.

4.7.1 Carnot Cycle

The Carnot cycle is probably the cycle that you should be most acquainted with. To begin with, you should understand that the Carnot cycle is an ideal heat engine in which there are two reservoirs, one hot at temperature T_H, and one cold at temperature T_C. Heat is drawn from the hot reservoir to the cold reservoir, and

It is considered a perfect thermodynamic engine because there is no increase in net entropy after one cycle. As such, the Carnot cycle is not actually seen in the real world, but close approximations of it do exist. Because the Carnot cycle is important both for the GRE, and for the field of thermal physics in general, let us look at it a bit more closely:

1. Isothermal Expansion - in this stage one sees an increase in the entropy of the gas, $\Delta S_1 = \frac{Q_1}{T_H}$
2. Isentropic Expansion $\Delta S_2 = 0$
3. Isothermal Compression $\Delta S_3 = \frac{Q_2}{T_C}$
4. Isentropic Compression $\Delta S_4 = 0$

4.8 Heat Transfer

There are three major methods of heat transfer that you should be acquainted with.

- Conduction - conduction is the major form of heat transfer between and within solids. A simple way to think of conduction is by using the collision theory approach: atoms in a solid bump into the adjacent atoms causing them to move (and thus transferring their temperature through the solid). The major equation of conduction across a solid (such as a wall) is as follows:
- Convection - convection is the form of heat transfer between and within fluids (liquids, gases, and plasmas). The common saying 'heat rises' is a half-correct way of thinking about this (although the expression is partly true as a rule of thumb for different reasons). It can be 'free' (happens naturally) or 'forced' (caused by an external mechanism, such as a fan).

- Radiation - radiation is the third major form of heat transfer. By making use of EM radiation, this form of heat transfer is unique as it doesn't require a medium to disperse/transfer heat. As such, the Sun transfers energy to the Earth across the vacuum solely through radiation. For the purposes of the GRE, when a question comes up regarding radiative transfer, consider the radiating object to be a black body.

4.9 Statistical Mechanics

We shall close this section on thermal physics with the slightly lengthy section on statistical mechanics. Let us recall the notion of an ensemble, which is effectively the probability distribution for a system with certain fixed states. In the case of the canonical ensemble, those states are N, V, T.

$$p_i = \frac{e^{-\beta E_i}}{{}_i e^{-\beta E_i}} \tag{4.9}$$

where β is defined as:

$$\beta = \frac{1}{k_B T} \tag{4.10}$$

Chapter 5

Quantum Mechanics

Quantum mechanics is where physics starts to become less intuitive for most people. This section is more different solely because you need to have a firm grasp of the mathematical formalism of the subject, and then have to rely on it to carry you through the subject in many cases. This section is not designed to teach you quantum mechanics comprehensively from scratch; for that I would recommend a textbook like Griffiths. This section instead will review the fundamentals that are required for the GRE.

5.1 Wavefunctions

QM is built around the concept of a wavefunction. A wavefunction is a function that relates to the probability of a particle being found at a particular point in space. More formally:

$$P = \Psi^2 \tag{5.1}$$

We know from basic statistics that probabilities are normalised to 1. Therefore, if we integrate over all space, we should find that:

$$\int_{-\infty}^{\infty} \Psi^2 = 1 \tag{5.2}$$

Note that Ψ is a complex number, which means that squaring works slightly differently. For a complex number z, $z = a + ib$. The conjugate of z is $z^* = a - ib$, and $z^2 = z^* z = (a - ib)(a + ib) = a^2 + b^2$. As such, $\Psi^2 = \Psi^* \Psi$. This returns a real number. This is important; probabilities have to be real numbers!

[Add examples of normalisation]

5.2 Examples of Wavefunctions

5.2.1 Infinite Square Well

The infinite square well is one of the first systems that you encounter in QM. The setup of the one-dimensional version is as follows.

5.2.2 Harmonic Oscillator

5.2.3 Dirac Delta Function

5.2.4 Finite Square Well

5.2.5 Hydrogen Atom

5.3 Formalism

5.4 Spin

Chapter 6

Atomic Physics

Atomic physics is a natural continuation of the quantum physics section, although there are some topics here that don't require a strong knowledge of the quantum theory. Either way, proceed with some caution if quantum confuses you.

6.0.1 Properties of Electrons

To begin with, let us review some information regarding the properties of the electron. Historically there has been major debate over whether the electron was a particle or a wave. Although you probably picture the electron as a particle (usually flying around a nucleus), the reality is far more complex.

6.0.2 The Bohr model

The Bohr model is a slightly outdated model of the atom. Despite being outdated, it gets a number of things right that make it far better to understand atomic physics. In the Bohr model, electrons orbit the nucleus (made up of protons and neutrons) at discrete energy levels.
The fact that these energy levels are discrete is hugely important, we say that they are quantised. This will become highly useful when we talk about energy transitions in a minute, because they are always[1] the same between two orbits for a certain type of atom. For example, for the hydrogen atom, the energy transition between the closest orbit to the nucleus and the second-closest orbit is XeV.
Because we are treating the electrons as being in circular orbits, another way to think about this is to say that the angular momentum is quantised. One can derive the relation as the following:

$$L = n\hbar \text{ where } L = mvr \tag{6.1}$$

It is also worth thinking for a second about the intricacies of such an orbit. Note that the negatively charged electron doesn't at any point fall into the positively charged nucleus: it is a stable orbit. This is even weirder when we think back to EM: the electron is accelerating, so we would typically expect it to produce radiation as per the Larmor equation, lose energy, and have the orbital distance decrease. This does not happen!

For completeness, let me re-emphasise that the Bohr model is an imperfect model by noting some of the features of the atom not encompassed by it. The main distinction is that now, instead of thinking of orbits, we think of electrons existing in 'clouds', a probabilistic region of space where the electron is most likely to be. This gets even more complicated when we include other quantum effects including tunnelling that allows electrons to jump around.

[1] Actually, there are factors that can alter the energy levels, such as external magnetic and electric fields, that we shall learn about later.

6.0.3 Energy Quantisation

Now we have a knowledge of the Bohr model, we are aware of the fact that if the angular momentum is quantised, the orbits are at fixed energy levels.
However, it is possible for electrons to jump energy levels, either by the absorption or release of a photon.

(6.2)

6.0.4 Selection Rules

The selection rules are a set of rules that govern electron transitions. To call them selection 'rules' is a little bit misleading, because they hold in the dipole approximation. Thus if for some reason that approximation is inappropriate, these rules need not hold. A better way of putting it is that rules are made to be broken, and indeed one finds examples of 'forbidden' transitions found in physics. Regardless, this is useful to know in atomic physics, and extremely useful to know for the GRE. Here they are in all their glory:

$$\Delta l = \pm 1 \tag{6.3}$$

$$\Delta m = 0, \pm 1 \tag{6.4}$$

The most notable one is Δl. If you remember nothing else from this section, just remember that l must increase or decrease by 1. l cannot remain the same!

6.0.5 Blackbody Radiation

A blackbody is a perfect emitter (and absorber) of EM radiation. Of course, a perfect blackbody doesn't exist in practice, but many things come close. A common example of a blackbody is a star like our Sun. The power radiated by a blackbody is modelled by the Stefan-Boltzmann law:

$$P = A\sigma T^4 \tag{6.5}$$

where A is the surface area, σ is the Stefan-Boltzmann constant, and T is the temperature.

6.0.6 The EM spectrum and X-rays

The syllabus of the GRE explicitly lists x-rays as a topic worthy of discussion. Why? Well, there are a few things worth noting:
X-rays are commonly produced through the rapid deceleration of electrons; these x-rays are called brehmsstrahlung (or braking radiation).
[insert diagram]
Another useful piece of trivia to know is various values for x-rays, such as wavelength and energy. For completeness, I have done this for the entire EM spectrum.
Learning these orders of magnitude is important, for it can be useful (if not necessary) for certain types of questions on the GRE. This is especially true for the wavelengths of the EM spectrum; once you know the wavelengths, you can quickly calculate other values such as frequency and energy. Just in case you're shaky on this topic, I include an example below:

6.0.7 Atoms in Electric and Magnetic Fields

We have discussed how energy transitions work for electrons in an atom, using the Bohr model as a method for understanding this. Importantly in this entire approach, we think of each orbit ($n = 1$, $n = 2$, etc) corresponding to a fixed orbit with an associated angular momentum, and as such, the energy transitions are fixed. However, it is actually possible to shift these orbits (and energy levels) with the application of an external electric or magnetic field. This is known as the Stark effect and the Zeeman effect respectively. Questions rarely go into much detail, but for completeness, I shall do so here.

The Stark Effect - the Stark effect is the shift in the spectral lines of an atom due to an external electric field.

The Zeeman Effect - the Zeeman effect is the shift in the spectral lines of an atom due to an external magnetic field.

A far more rigorous and informative approach to these effects may be found in a standard Quantum Mechanics textbook (e.g: Griffiths). However, I have found that the GRE only tends to assess the basics.

Chapter 7

Special Relativity

Special Relativity was proposed in 1905 as a way of reconciling Newtonian mechanics with electromagnetism. We shall look more closely at some of the historic problems that prompted Einstein and others to propose SR. To begin with though, let us come to grips with some of the features of SR.

SR is based on two postulates:

1. The laws of physics are invariant in all inertial frames of reference.
2. The speed of light (in a vacuum) is the same for all observers, independent of their reference frame.

7.1 Introduction to Relativity

7.2 Lorentz Transformation

Imagine that we have two frames, S and S', where S' oves at velocity v in the x-direction w.r.t S. Define some event to have coordinates (t, x, y, z) in S, and (t'. x', y', z') in S'. Then we can relate these coordinates to each other with the following Lorentz transformations:

$$t' = \gamma(t - \frac{vx}{c^2}) \tag{7.1}$$
$$x' = \gamma(x - vt) \tag{7.2}$$
$$y' = y \tag{7.3}$$
$$z' = z \tag{7.4}$$

where we note the define of the Lorentz factor, γ as:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{7.5}$$

We will look more closely at the Lorentz factor in a moment. For reference, the inverse Lorentz transformations are:

$$t = \gamma(t' + \frac{vx'}{c^2}) \tag{7.6}$$
$$x = \gamma(x' + vt') \tag{7.7}$$
$$y = y' \tag{7.8}$$
$$z = z' \tag{7.9}$$

i.e: substituting velocity v with -v.

7.3 Tests of Relativity

There are some very important experimental tests for both SR and GR, and I shall give a brief summary of some of the big tests. Feel free to read more about these tests online, for this is physics in the making!

The Michelson-Morley experiment - this is the big experiment. It was believed in the 19th century that there existed this universal medium called the 'aether'. This was conjured up by some simple logic: light was believed to be a wave, and all waves need a medium to propagate through, so there must be a medium throughout the whole universe that allows (for example) light to travel from the Sun to the Earth. This logic may seem funny to us today for a number of reasons, but it's actually quite reasonable as the basis for a hypothesis.
Michelson and Morley sought to investigate the effect that this aether had on the speed of light, using the idea of aether wind. In the two major models of the aether, the aether and the Earth are in relative motion, resulting in this 'aether wind'. As such, one would expect the wind to be different as the direction of the Earth changes as it orbits the Sun. From this, one would expect the speed of light to change by a small amount over the course of a year.
To investigate whether this was true, Michelson and Morley set up an interferometer as set up below:
We shall discuss the workings of this device later on in the Specialised Topics section
The experiment returned a negative result: there was no difference in the speed of light, suggesting that there was no aether.
If some of this seems a bit convoluted, it should simply be noted that a) the Michelson–Morley experiment has been replicated with extreme precision since the original experiment and b) there have been other experiments that support the idea that there is no aether/GR is a better model.

- The Ives-Stilwell experiment - this test demonstrated time dilation, a major part of SR.
- The Kennedy-Thorndike experiment - the Michelson–Morley experiment showed that the speed of light is independent of the orientation of the apparatus, while the Kennedy–Thorndike experiment showed that it is also independent of the velocity of the apparatus in different inertial frames.
- The Rebka-Pound experiment - in this experiment, gamma rays were fired downwards from the top of a building to the bottom. It showed that photons gain energy when traveling toward a gravitational source, which is a major test of GR.

Chapter 8

Lab Methods

8.1 Error Analysis

Error analysis is a highly important aspect of experimental physics, and indeed statistics as a whole. In order to quantify error, we look at the variance of a dataset. The variance of a dataset $(x_1, x_2, ..., x_n)$ is given as follows:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2 \tag{8.1}$$

This equation holds when the entire dataset is used. If, on the other hand, a sample of the complete dataset is used, then the variance is calculated by:

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2 \tag{8.2}$$

The square root of the variance is known as the standard deviation. When we read a measurement (such as the length of an object in the lab), we usually give it in the form $\bar{X} \pm \sigma$. For example, the measurement $5mm \pm 1mm$ has mean $\bar{X} = 5mm$ and standard deviation $\sigma = 1mm$.

This is relatively straightforward. One of the more complicated parts of error analysis (that is routinely tested on the GRE) is calculating the error of some function. For example, suppose that there exists some calculated variable $f(z) = xy$, where $x = \bar{X} + \sigma_x$ and $y = \bar{Y} + \sigma_y$. What is the standard deviation, $\sigma_{f(z)}$? Given that the uncertainty of each variable is independent of each other, we note that errors add in quadrature. This yields the following rule:

$$\sigma^2_{f(z)} = \sum_{i=1}^{n}(\frac{\partial z}{\partial x_i})^2\sigma^2_{x_i} \tag{8.3}$$

The other error analysis question that I have seen come up that requires you to be aware of a specific formula is a weighted averages type of question. If you have two variables and wish to calculate the weighted average, then use:

$$w_{avg} = (\frac{x}{\sigma_x^2} + \frac{y}{\sigma_y^2})\sigma^2_{Total} \tag{8.4}$$

where $sigma_{Total}$ is defined as a harmonic mean:

$$\sigma^2_{Total} = \frac{1}{\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2}} \tag{8.5}$$

8.2 Electronics

The electronics aspect here and the circuits section in EM are effectively partners. Go and review the electronics section there first, for that is far more comprehensive and useful. This section sweeps such related topics that is useful, but perhaps not essential.
To begin, let us consider some other circuit components that are useful in physics:

- Diode - a diode is a circuit component that only allows for current to travel in one direction. This can be useful in a number of cases, for example, one can build a rectifier circuit to convert AC into DC. Diodes also can be used to prevent backflow of current into sensitive circuit components. A special kind of diode is a Light-Emitting Diode (or LED), which is commonly used in lighting displays.

- Thermistor - a thermistor is a type of variable resistor, where it's resistance changes an a function of the temperature. While this is technically true for all resistors, this effect is dramatic in thermistors so that it can be exploited. One can use them as temperature sensors.

- Light Dependent Resistor (LDR) - similar to a thermistor, but now the resistance varies as a function of light input. One can use them as light sensors, and proximity sensors.

- Op-Amp - An operational amplifier may be used to amplify voltage. More specifically, it typically amplifies the difference in voltage between two inputs.

- Integrated Circuit (IC) - An IC is just a circuit in a chip. If you're familiar with electronics as a hobby, you might already know ICs like the 555, 4017, and 4026. If not, don't worry too much about it.

- Logic Gates - logic gates are devices used to give a binary output depending on the binary input. This is best understood through the examples of different logic gates, which we shall go through now:

The AND gate has two inputs and one output. If both inputs are True (i.e: there is a current running into each input such that we might say that the input is 'on'), then the output is also True. For all other input combinations (both False/'off', or one True and one False) the output is off. One can think of this with the following circuit with both switches in series. Both switches must be closed or current cannot flow. We can represent this more succinctly in a logic table, where we assign a 0 to False/'off', and a 1 to True/'on'.

There are more types of logic gates that you should be aware of. If you want this information in a concise (and perhaps more understandable) manner, zoom ahead to the logic gate tables below. The OR gate gives an input of 1 if A or B is 1, or if both A and B are 1. This may seem weird if you haven't seen this before in electronics (or in logic), but just think of a circuit with two switches in parallel.

The NOT gate is just an inverter: it takes one input (0 or 1) and gives one output (1 or 0). A NAND gate is logically the combination of a NOT gate and an AND gate; the output is 1 when both inputs are 0. The output is 0 for all other cases. A NOR gate is the combination of a NOT gate and an OR gate. A XOR gate is an exclusive OR gate. See the table below for details. You should also be aware of the circuit symbols for these logic gates:
As a quick memory aid, my old electronics teacher used to say that the N (in NOT, NAND, NOR) actually stood for nipple.

8.2.1 Counting Statistics

When looking at discrete data, it is important to know it's statistical distribution. From this, it is possible to calculate other quantities very easily, such as the mean and variance.
The main one that comes up often is the Poisson distribution.

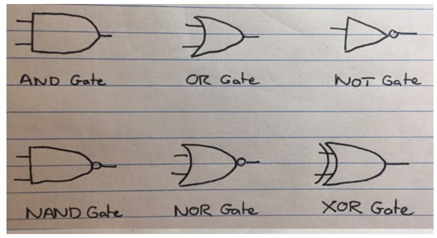

Figure 8.1: Caption

Suppose you have a series of discrete events that occur over a constant time period (e.g: 3 events per day) and the events are independent of each other. Then the data can be modelled by a Poisson distribution. The classic example is the number of car crashes that occur on a highway in the course of an hour. More useful examples of this in physics include the number of radioactive particles that decay over a given time period, the number of photons hitting a detector in one night, and so on.
If a dataset can be modelled by a Poisson distribution, then:

$$P(k) = \frac{\lambda^k e^{-\lambda}}{k!} \tag{8.6}$$

where k is the number of events in a set interval, and λ is the average number of events in this interval.
The following statistics may be calculated:

$$x = \lambda \tag{8.7}$$

$$\sigma^2 = \lambda \tag{8.8}$$

That was easy enough!

The binomial distribution is the other discrete distribution that you should know about, although it is far less frequently tested. The binomial distribution is used when you have n events with two outcomes, one with probability p, and one with probability $q(= 1 - p)$. For example, if a coin is tossed 10 times with heads $p = 0.5$, tails $q = 0.5$, then we have a binomial distribution. For a binomial distribution, the following statistics may be calculated:

$$x = np \tag{8.9}$$

$$\sigma^2 = np(1 - p) \tag{8.10}$$

For completeness, I will mention that a single binomial trial ($n = 1$) is called a Bernoulli process.

8.2.2 Lasers

The GRE seems to enjoy asking questions about lasers. It also tends to ask very fact-based questions about them, typically testing your knowledge of how specific lasers work and what they should be used for. This is both good and bad news: you'll get to the exam and you'll either know it or you won't. For that reason, memorise the following:

- Solid-state lasers - a solid state laser uses a medium that is a solid. The most common solid-state laser is Nd-YAG.

- Dye lasers - the medium is usually a dye typically in a liquid medium. They are good for producing a large number of wavelengths by changing the dye, particularly in the visible spectrum.
- Gas lasers - a gas laser uses a medium that is a gas. The most common gas lasers are He-Ne and CO2.
- Free-electron lasers - produces radiation by the acceleration and deceleration of electrons (braking radiation). This allows for a continuous range of wavelengths to be produced by simply varying the strength of the external field acting on the electrons.

8.2.3 Dimensional Analysis

Dimensional analysis is one of your greatest tools on the GRE. Not only can it be the direct topic of certain questions, but it can also be a time-saver for other questions. It is very common to see a question on the GRE (about classical mechanics for example) where only one of the answers given has units that make sense. In short, focus on this section, and work on applying it.

As you are aware by now, some constants and variables come with a unit. For example, m has the unit kg, and a has the unit ms^{-2}. Now, in a formula, the units on both sides of the equation have to match. For example:

$$F = ma \rightarrow F = kgms^{-2} \tag{8.11}$$

Here, we see that force has the unit $kgms^{-2}$. In fact, we then define $1N = 1kgms^{-2}$.

Here is a another example:

$$v = u + at \rightarrow ms^{-1} = ms^{-1} + ms^{-2} * s \tag{8.12}$$

In the above example, the RHS reduces to ms^{-1}[1]

[Add practise questions here]

[1]Technically the RHS is $2ms^{-1}$, but constants don't matter here. Just drop the 2!

Chapter 9

Specialised Topics

9.0.1 Nuclear Properties

9.0.2 Radioactive Decay

There are three major types of radiation that you need to be aware of: alpha, beta, and gamma. This is a rather straightforward topic, so we'll jump straight in and review the properties of this radiation:

Review of Radiation					
Type	What is it?	Charge	Mass	Penetrance	Blocked by
Alpha	He-4 nucleus	+2		Lowest	Few sheets of paper
Beta	Electron	-1		Mid	2cm aluminium
Gamma	Gamma wave	0	0	Highest	Reduced by thick lead

Table 9.1:

None of the above properties should be a major surprise once you know what alpha, beta, and gamma radiation actually is. Before we continue to some of the more important parts of this section, here let us take a nice break with some lighter trivia that may or may not be useful.

[Add trivia]
Alpha radiation is used for the working of smoke alarms.
The largest sources of radiation are natural rather than man-made.

9.0.3 Fission and Fusion

We have discussed some of the lighter information regarding the three types of radiation you need to know about. Now, it is time to discuss how they are produced.

An alpha particle is produced by heavy nuclei. In fact, the smallest standard nuclei that undergo alpha decay are isotopes of tellurium (mass number 104 and above).[1] A prime example of a nucleus that undergoes alpha decay is uranium-238:

[1] Actually, beryllium-8 undergoes alpha decay, but this is an exception that bucks the trend, and is also probably useless to know for the GRE. It's interesting though; remember that many rules in observational physics have some flexibility.

$$U \rightarrow Th + \alpha + \gamma \tag{9.1}$$

We'll discuss what the γ is in a second.

Beta decay

Gamma decay
Last and probably least, is gamma radiation. Usually after alpha decay has occurred, the nucleus is left in an excited state. The nucleus will then release this energy by emitting a gamma ray.

9.0.4 Particle Physics

One can almost guarantee that a particle physics question will come up. The one difference with these types of questions compared to the rest of the GRE is that they tend to be fact-based. This means you will either know them or you won't; there is minimal deductive reasoning involved. Let us begin by describing some of the subatomic particles you may encounter. We'll start with two particles that you're already familiar with in the nucleus of an atom: protons and neutrons.

It turns out that protons and neutrons are made up of smaller subatomic particles called quarks. There are six quarks, all with different properties corresponding to properties such as charge and spin. These properties are listed in the table below, with the main points to take away highlighted.

For example, a proton is made from two up quarks and one down quark (written uud). From the table, we see that this corresponds to a charge of +1, as expected.

The neutron, on the other hand, is made from one up quark and two down quarks (written udd). Again, as a quick sanity check in case you forget this composition, check the sum of the charges. It gives zero for the neutrally charged neutron. Good!

We should note that not all particles are made from quarks. The electron isn't, as the best example, and we will discuss these particles later. Particles that are made up of quarks are called hadrons.

The proton and the neutron are made up of three quarks. There are a number of other particles made up of three quarks including sigma and delta particles, but it's unlikely you'll ever need to know about these specifically. The main thing to know is that hadrons made up of three quarks are called baryons. Baryons are also fermions, because they have half-integer spin.

**** One should note that technically a baryon is defined as having an odd number of quarks. However, a particle with five quarks (aptly named the pentaquark) was only experimentally confirmed in 2012, and won't be important for the GRE. Furthermore, there is no such thing as a single quark existing on its own. Why? Keep reading!

The other type of baryon you need to know about is the meson, which is made up of two quarks. Mesons require a bit more explanation, because there's a lot to unpack there. We shall do it as we go!

To begin, let's take a quick tangent to the topic of antimatter. An antiparticle is a particle that has the same mass and spin as its corresponding particle, but the opposite charge.

For example, the antiparticle of the proton is creatively named the antiproton. As shown in the table below, it has the exact same mass and spin, but the complete opposite charge. The antiparticle of the electron is the positron, again, same mass and spin, opposite charge.

A natural question to ask given our prior discussion on the composition of hadrons, is what is the composition of an anti hadron? They are made of antiquarks, which, as you've guessed, have the same mass and spin as their corresponding quarks, but the opposite charge. Also, if you fire a particle into its antiparticle, they will annihilate each other, and release energy as two photons (to be discussed later).

Does the neutron have an antiparticle? Yes! It's composition is anti-up, anti-down, anti-down. The charge is the same, as the antineutron is neutral just like the neutron. It's really a question of semantics as to whether the opposite of a neutral charge can be a neutral charge, but we'll count it here.

As you might have guessed, there is a lot more going on with antimatter that relates it to matter, but this is all we need for the GRE for now.

Why is this important? Rather curiously, the meson consists of a quark and an anti-quark.

Mesons are further divided into pions and kaons. Pions are the lightest mesons, consisting of various combinations of up and down quarks and antiquarks. Write down the possible combinations that work, and you'll see there are four combinations. Counting the two combinations that give a neutral charge as identical, this gives three pions.

Kaons are similar, but they must contain a strange quark or antiquark. In combination with top and down (anti)quarks, there are also four combinations giving three different particles. Particles with a strange quark or antiquark exhibit a property called strangeness, which we will come to in a moment when we talk about particle interactions.

Let us take a moment to consider the various things that are conserved in an interaction.

Energy - we are already familiar that energy cannot be created or destroyed (1st Law is Thermodynamics). As such, energy is a conserved quantity. Momentum - we are also familiar that momentum should be conserved. More specifically, we should be thinking of energy-momentum here (see the Special Relativity section if you're not sure what this means). Charge Baryon Number - this is the Lepton Number - this is the same as the baryon number, but for leptons. Strangeness - in STRONG interactions only!!! This is one of the major differences between the strong and weak interactions.

Let us run through a number of examples to practice this. Some of these examples are good to be familiar with anyway, so look at them carefully.

Feynman Diagrams

I haven't seen this come up yet, but a brief discussion of Feynman diagrams feels both useful to the topic of particle physics, and a feasible topic for a question. Feynman diagrams are a visual representation of particle interactions, with one axis representing time, and the other representing space.

[Add details here]

If this all seems like a rather silly way of representing these decays given that we could more succinctly write the equations as we did above, just be aware that there is a lot more going on here that meets the eye. For more complicated decays, these diagrams are incredibly illuminating and valuable.

Before we finish this section, there are a couple of smaller facts that are possibly good to know for the GRE.

The J/Psi meson was integral in the discovery of the charm quark. This has come up a few times on the GRE.

In the same vein, the upsilon meson was integral in the discovery of the bottom quark.

9.0.5 Condensed Matter

Condensed Matter may well be one of those topics that you haven't formally studied at the undergraduate level, as was true for myself. Alternatively, you could be like my friend who took four different courses in the topic. Either way, some light review here can't go amiss. The material is relatively basic, and GRE questions tend to emphasise fact-based questions here over conceptual knowledge.

Crystal Structure

A common type of question on the GRE relates to knowing various facts about crystal structure. The smallest repeating structure in a crystal is known as the unit cell. Here are the three major types of unit cells that you need to know about. Simple cubic Body centred cubic Face centred cubic

X-ray diffraction

This is an extension of the optics section, so it is perhaps worth reviewing that first. For the GRE, this relates to Bragg diffraction which occurs when X-rays are incident on a lattice.[2] The equation is as follows:

$$dsin(\theta) = \frac{n\lambda}{2} \tag{9.2}$$

Thermal properties

- Conduction -
- Convection
- Radiation

Electron theory of metals

You may already be somewhat familiar with the conceptual ideas behind the electron theory of metals from your understanding of metallic bonding in chemistry. In this model, a metal is considered to be a lattice of ions in a sea of delocalised electrons. This 'sea' is highly important for understanding the properties of metals: the delocalised electrons can move around easily making metal a good conductor of electricity, while the strong forces this creates give metals a high melting point. Moving into the more rigorous world of physics, let us be more precise and define a Fermi surface.
[To be written at a later date]

Semiconductors

You are already familiar with the idea of conductors and insulators. A semiconductor has the property of having a conductivity value in between the two. Semiconductors are essential to the world of electronics. The most used semiconductor is silicon which is used for a plethora of electronic devices, including transistors, diodes, solar cells, etc. An important fact to know about semiconductors is this:

The resistance of a semiconductor decreases as the temperature increases

[2] This statement is actually a bit strong, it is more generally when EM radiation with wavelength approximately equal to atomic spacings are scattered by the atoms of a crystalline system. Either way, if you see a scattering question involving a lattice/crystal on the GRE, think Bragg diffraction.

That is not a typo; they exhibit the opposite behaviour to regular conductors. In some cases, the conductivity of a semiconductor may need to be increased. Typically, this is done by adding a number of different atoms to the semiconductor in a process known as doping.

A p-type (positive) semiconductor is a material in which there are more 'electron holes'. This can be built by doping a semiconductor with a material with fewer valence electrons. For example, one may take a group 4 semiconductor like silicon and dope it with a group 3 atom like boron.

An n-type (negative) semiconductor is a material in which there are more electrons. This can be built by doping a semiconductor with a material with more valence electrons. For example, one may take a group 4 semiconductor like silicon and dope it with a group 5 atom like arsenic.

Superconductors

A superconductor is a material that exhibits zero resistance when it is below a certain temperature known as the critical temperature.

The primary theory of superconductivity that you must know about is the BCS theory of superconductivity, which involves Cooper pairs. A Cooper pair is a pair of electrons that have coupled to each other. In doing so, they act as a boson and may condense into the ground state. These electrons are coupled together by lattice vibrations in the material, which means that we can think of Cooper pairs being formed by phonon exchange. Think of a electron moving in one electron through a lattice. The lattice is attracted to the electron, resulting in the movement of atoms in the lattice towards the electron. Then, an electron moving in the opposite direction is also attracted to the lattice displacement. This results in a net coupling. This net attraction between the two electrons (which normally repel each other) is very small, but it is enough to keep the electrons bound at extremely low temperatures, resulting in superconductivity. As a final point in this already over-detailed section, there is a correlation between the mass of the isotope used for the material, and the critical temperature. This was a major piece of evidence for BCS theory: it suggests that the electronic structure of the material is not the only factor in determining superconductivity. Instead, the vibrations of the lattice are also important.

The final thing to spend some time on the Meissner effect. The Meissner effect is where the magnetic field is expelled from a material when it reaches and is cooled below its critical temperature. As the interior magnetic field decreases to zero, the external magnetic field decreases due to flux conservation. This is seen in the diagram below, and is how this effect was experimentally seen.

9.0.6 Astrophysics

Astrophysics is a relatively common topic for the GRE, so it is worth being somewhat acquainted with the topic. The field of astrophysics is rather large (so large in fact, that I majored in it), so I have included the most useful topics here for the GRE. That said, astrophysics is interesting, so don't stop here if you're enjoying it!

Cosmological Principle

The cosmological Principle is a relatively key idea in Cosmology. Here it is in short:

On large enough scales, the universe is homogeneous and isotropic.

Let us explain these terms: by homogeneous, we mean that the universe looks the same at all points. A good example is to think of the atoms in a lattice as shown below.

By isotropic, we mean that the universe looks the same in all directions.
As a quick exercise in the difference between these terms, let's think of examples where one applies but not the other.
You might be thinking that this seems false: for example, it is clearly warmer on the Earth than near Neptune, so it isn't homogenous there. That's why we talk about large scales.

Hubble's Law

Back in the 1920s, Edwin Hubble decided to look at a number of stars, and plot the velocity of these stars against their distance away from the Earth. The plot is shown below.

It's a bad graph (although fine for the data at the time): there are few data points, a questionably weak correlation, and as you might be wondering: how did he even measure the velocity and distance of these stars with any accuracy? And yet, this would become a major breakthrough in the field of astrophysics, known as Hubble's Law:

$$v = H_0 d \tag{9.3}$$

There are numerous ways of measuring the value of H_0, some of which give significantly different values. Generally though, we take $H_0 = 70kms^{-1}Mpc$. Take note of these crazy units! Basically we want a distance d in Mpc to give the velocity kms^{-1} by dimensional analysis (if this is making little sense to you, skip ahead to the section on dimensional analysis). As a quick aside here, the term Hubble's constant is a touch misleading. The value is constant in space, but not in time. In short, this value holds from one end of the universe to the other, but doesn't hold when the universe was half its current age. This is because the rate of expansion of the universe is actually accelerating![3]

Redshift

Redshift is the phenomenon where the wavelength of light from an object increases. There are three different types of redshift:

1. The Doppler effect is a type a redshift when the objects are moving away from each other.

2. Cosmological redshift occurs due to the expansion of space as per Hubble's Law. This is subtlety different from the Doppler Effect; it is caused by a change in the scale factor of the metric of the universe.

3. Gravitational redshift occurs due to strong gravitational fields. You should be familiar with this idea from the Pound-Rebka experiment.

Of these three types of redshift, cosmological redshift is the one that physicists (and the GRE) are usually talking about when the term redshift is used. If you're still struggling to comprehend the difference between the Doppler effect and cosmological redshift, I don't blame you. For the latter, you simply need to think of two stationary objects positioned at two points, and the space itself between the objects is expanding.

As a quick aside, the term blueshift is used to describe a decrease in wavelength. This occurs by the Doppler effect. For example, light from Andromeda is blueshifted because it is moving towards the Milkyway.

To quantify redshift, let us define it as follows:

[3] There are lots of other factors at play here that are related. This in my opinion, is entering into one of the most fascinating areas of physics research: the realm of cosmology. Unfortunately, an understanding of this would consume time so the GRE doesn't test it.

$$z = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}} \tag{9.4}$$

Caused by cosmological redshift:

$$1 + z = \frac{a_0}{a(t)} \tag{9.5}$$

Composition of the Universe

Let us talk briefly about the composition of the universe. Think about the materials that we currently know: atoms like carbon, hydrogen, copper, and so on. The bulk of the mass of these objects is the protons and the neutrons of these atoms. In that sense, we call this baryonic matter. There is no doubt that baryonic matter is important: it makes up the whole of the Earth! And yet, only 5% of the universe is made up of baryonic matter. What could the other 95% possibly be?

Another 20% of the universe is made up of dark matter, which is a gravitationally attractive material that has no electromagnetic interaction (which basically means that we can't see it, and haven't even detected it yet). There is a large bulk of evidence that points towards the existence of dark matter. One of the major pieces of evidence can be found looking at galaxy rotation curves.

[insert diagram and explanation]

The final 70% of the universe is made up of dark energy. This is even stranger in my opinion. As we discussed earlier, the universe is expanding as per Hubble's Law. However, as recently as 1997, it was discovered that the universe is also accelerating in its expansion! This has a large number of deep implications in cosmology, but the main point here is that we need a gravitationally repulsive force to counteract the gravitationally attractive forces. We call this dark energy.

With that in mind, let's look in more detail at the baryonic matter. The bulk of baryonic matter in the universe is Hydrogen and Helium.

Star Formation

I don't believe I've seen a question specific to stars that isn't just a fusion question, but it doesn't seem unreasonable to me that a quick trivia question on star formation/evolution could come up.
Below is a quick sketch of the evolution of different stars.
Main-sequence stars are especially important to us, not least because the Sun is a main sequence star. Let us look into main-sequence stars and star evolution in more detail using Hertzsprung-Russell diagrams
The HR diagram in its most common form plots the magnitude of stars against their temperature. First of all, watch out! The temperature is decreasing from left to right! You can see that the diagram forms regions that correspond to the star's evolutionary stage. For example, stars with a high luminosity value at a low temperature are giants and supergiants. As a final note, you might be wondering what OBAFGKM is. This is a system (still used today) for classifying stars based on their temperature. You can remember the order by the phrase "Oh Be A Fine Girl Kiss Me!".

Black Holes

I have split the topic of black holes into its own section because I have seen a couple of questions on this topic. A black hole is the end product of an exceptionally large star. The mass of the star has collapsed into an incredibly small volume, and this results in major gravitational effects. A more complete understanding of this topic is difficult without a long discussion on General Relativity, so we shall stick to the basics.

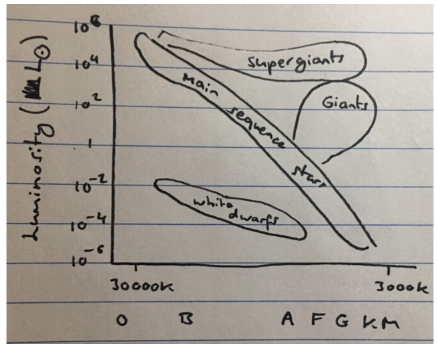

Figure 9.1: Caption

A black hole has a region in which nothing can escape its strong gravitational pull, even light. The boundary of this is known as the event horizon. The distance from the centre of the black hole to the event horizon is known as the Schwarzschild radius, and is calculated by:

Black holes also release Hawking radiation. This means that one can approximate a black hole as a black body with temperature inversely proportional to mass.

Some interesting facts about black holes:

- In 2016, LIGO detected the first gravitational waves from a black hole merger.
- In 2019, the first image of a black hole (shown above) was produced.
- It is believed that there is a supermassive black hole (SMBH) at the centre of our galaxy called Sagittarius A*.

9.1 Mathematical Methods

It is a well known saying that mathematics is the language of physics, and it is a general requirement that if you are studying physics, you need to be proficient enough in the required mathematical topics. The GRE will also occasionally be direct and will ask a mathematics question, such as doing matrix multiplication. Either way, this is useful enough that I have included this section as a way of building the machinery required to do physics problems.

9.1.1 Matrices

Let us begin with some practise of matrices. A matrix has the general form.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Here are some more concrete examples:

$$\begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 4 & 0 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$

Matrix multiplication can feel unusual if you're not well-acquainted with it.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{12} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Here is a healthy check to do. If the first matrix has dimensions m x n, and the second matrix has dimensions n x p, then the dimensions of the product matrix must be m x p. Additionally, the number of columns of the first matrix must equal the number of rows of the second matrix, or matrix multiplication is impossible. One of the things that you need to know how to do is to find the eigenvalues and eigenvectors of a matrix.

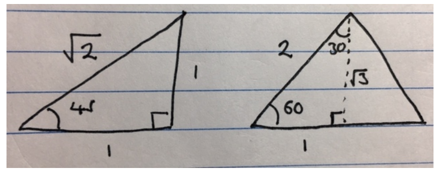

Figure 9.2: Caption

Complex Numbers

A complex number is a number of the form $z = a + ib$, where $i = \sqrt{-1}$. i is called an imaginary number, because it is of course impossible to square root a real number.

9.1.2 Trigonometric Identities

To begin with, let us look at the derivation of certain trigonometric values. It is technically easier just to memorise these values rather than derive them, but it is good to double check when your recall might be hazy.
Additionally, you should be aware that:

$$\frac{sin(x)}{cos(x)} = tan(x) \tag{9.6}$$

$$sin^2(x) + cos^2(x) = 1 \tag{9.7}$$

There are some other trigonometric function that you should be aware of, although they are less useful for the GRE:

$$sec(x) = \frac{1}{cos(x)} \tag{9.8}$$

$$cosec(x) = \frac{1}{sin(x)} \tag{9.9}$$

$$cot(x) = \frac{1}{tan(x)} \tag{9.10}$$

Noting that the sum of $sin^2(x)$ and $cos^2(x)$ is 1, it should be relatively simple to prove that:

$$tan^2(x) + 1 = sec^2(x) \tag{9.11}$$

$$cot^2(x) + 1 = cosec^2(x) \tag{9.12}$$

The next set of useful identities are the double angle identities:

$$sin(2x) = 2sin(x)cos(x) \tag{9.13}$$

$$cos(2x) = cos^2(x) - sin^2(x) \tag{9.14}$$

You can use the identities above to derive more formulae that are similar.

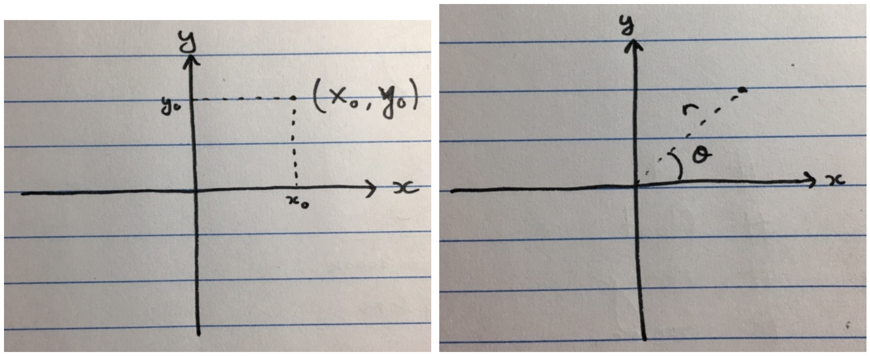

Figure 9.3: Caption

9.1.3 Coordinate Systems

At this stage of your life, you should be very familiar with the cartesian coordinate system, where a point is represented by (x_1, y_1, z_1) in relation to some origin at (x_0, y_0, z_0). From this, we can do a lot of things, such as define length. This is all well and good, but you may be aware that there are some problems in physics that are just hard to solve on such a grid. Sometimes, things are just easier using a new set of coordinates.

To begin with, let us consider polar coordinates.
Next, let us consider the three-dimensional extension of polar coordinates: cylindrical coordinates.
Finally, let us consider a different three-dimensional system of coordinates: spherical coordinates.

9.1.4 Taylor Series

9.1.5 Fourier Series

9.2 Computer Applications

If I'm being completely honest, I have no idea what the GRE gods are looking for when they suggest that 'computer applications' could come up on the test. I have never seen such a question, and imagine that the gain in scouring a computer science textbook for the remote chance of a single question coming up is useless given the opportunity cost. I have included here a brief summary of topics that could possibly be counted here as 'computer applications', but fair warning, this is all conjecture on my behalf. Do not, and I repeat, do not study this page unless you're doing perfectly in every other topic; it's just a waste of time otherwise.

9.2.1 Data Types

A data type is a way of classifying how a certain piece of information is stored and used. You are almost certainly familiar with the integers or 'int'. Here is a list of the major data types:

- Int - this is the data type for the set of integers
- Float - this is a method for storing decimals.
- Double - this is more precise than a float, as it can store 64-bits typically rather than 32.
- Boolean - this is 'True' or 'False'. They are truth statements, and not to be confused with strings.
- String - this is a series of characters, e.g: 'abc123', 'GATACTTAAG', or 'This is an example of a string.'.

Examples:
var1 = 3000 int

var2 = 3000.0 float

var3 = 'Temperature' string

var4 = True boolean

9.2.2 Arrays

There is a technical distinction between lists and arrays in programming languages such as Python, but I am certain that it is unimportant for the GRE. As such, I will (inaccurately) use the terms interchangeably. An array is a collection of items stored together. Here is a simple example[4]:

```
temperature = [10, 20, 40, 100, 150]

dogs = ['Westie', 'Corgi']

print(temperature[0])
>>> 10
print(temperature[1])
>>> 20
print(dogs[1])
>>> Corgi
```

9.2.3 Programming Languages

Below is a list of some of the more common programming languages that you might come across if you do a research project in physics or astrophysics.

- Python - Python is an easy-to-learn, high-level programming language. The syntax is relatively simple, for example, unlike other languages like Java or C, there is no need to declare the data type of variables before using them (i.e: you can directly assign a variable, such as piValue = 3.142. In Java, you would have to do float piValue; piValue = 3.142). In addition, Python has a number of packages that make data analysis easier, such as numpy, scipy, astropy, and pandas. Matplotlib is another package that is good for constructing graphs. This is the language that I would most recommend learning.

- Java - Java is another popular high-level programming language, albeit slightly harder to learn as the syntax is more strict.

- MATLAB - MATLAB is a high-level programming language designed primarily for numerical computing. MATLAB can be particularly good for matrix operations. That said, you do have to pay to download MATLAB. In my opinion, MATLAB is like marmite: you either love it or you hate it.

- Fortran - Fortran is a much older programming language, and you may see it if you work on older projects in physics. It is easily identifiable by its use of specifying line numbers, e.g: GO TO 10, PAUSE 200, etc. I wouldn't recommend starting a project with Fortran today; there are easier programming languages.

- C - C is a low-level language (not to be confused with C+ or C#). It is very popular, and is often used to build compilers.

[4]This example is not quite so simple actually; in Python this would be a list, but the general idea is the same.

- Assembly - a very low-level language, it's one step above writing your code in binary.
- R - this is a language designed for statistical analysis, generating plots, and doing GIS (map) analysis, among other things. I've never seen this used in physics at all actually, but I'm including it here for completeness.

Reading code

Here are some examples of codes in Python.

Examples of codes in Physics/Astrophysics

- N-body simulations - e.g: simulating the motion of planets, galaxies, atoms, etc
- Large scale data analysis - e.g: large galaxy surveys
- Plasma Physics

9.3 GRE Practice Tests and Solutions

Practise makes perfect, and that is at least partially true here. As a matter of fact, it is my opinion that most of these exams are actually harder than what you see on the day. That said, doing well on these harder exams is a good way of showing that you have a strong understanding of the material. Some key advice: take these tests as though they are real. Time yourself. It is easy to forget when reviewing all this material that the GRE is not only designed to assess your understanding of physics. It is also designed to assess your ability to do this physics quickly, without a calculator or outside aids, and over a relatively long stretch of time. Too often I have heard of friends not finishing the GRE when it could make a difference to scores.

On my website are links to LaTeX solutions to each published test (work in progress). These are not the only solutions out there, so feel free to draw upon other sources if you feel that anything is incomplete or unclear. That said, I have tried to be as comprehensive as possible in writing these up. It is my opinion that the 2008 exam and the sample exam warrant the most focus, for they seem to be most like the current GRE tests.