



# Estimating Noise Transition Matrix with Label Correlations for Noisy Multi-Label Learning

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# Introduction



Figure: Example of image with noisy multi-labels. (C. O. Pene *et al.*)

Clean Data  $(\mathbf{X}, \mathbf{Y})$ , where  $\mathbf{Y} = \{Y^1, Y^2, \dots, Y^q\} \in \{0, 1\}^q$

Noisy Data  $(\mathbf{X}, \bar{\mathbf{Y}})$ , where  $\bar{\mathbf{Y}} = \{\bar{Y}^1, \bar{Y}^2, \dots, \bar{Y}^q\} \in \{0, 1\}^q$

Transition Matrix  $T_{ik}^j(\mathbf{x}) = P(\bar{Y}^j = k \mid Y^j = i, \mathbf{X} = \mathbf{x}), j = 1, 2, \dots, q$

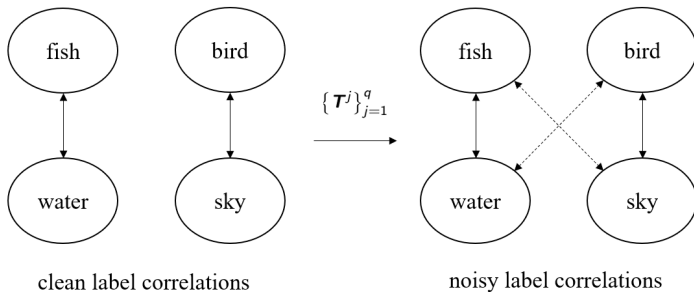
The transition matrix bridges the class posterior probabilities for noisy and clean data, i.e.  $P(\bar{Y} = k \mid \mathbf{X} = \mathbf{x}) = \sum_{i=0}^1 T_{ik} P(Y = i \mid \mathbf{X} = \mathbf{x})$ . Thus, it has been exploited to achieve many **statistically consistent algorithms** in noisy multi-class learning, which also can be applied in noisy multi-label learning. As the effectiveness of these algorithms heavily relies on estimating the transition matrix, a series of methods have been proposed to achieve estimation.

## Problem:

- Most of estimation methods assume **the existence of anchor points** (T. Liu *et al.*), while the assumption is strong and hard to check.
- The methods need to **accurately fit the noisy class posterior** of anchor points, which is rather difficult in multi-label cases, due to the severe positive-negative imbalance.

## Intuitive Solution:

To address the problems, we consider utilizing **label correlations among noisy multiple labels**. At a high level, we can utilize the **mismatch of label correlations** to identify the transition matrix without neither anchor points nor accurate fitting of noisy class posterior.



**Figure:** The mismatch of label correlations.

# Our Method

As the instance-dependent transition matrix is non-identifiable without any additional assumption (X. Xia *et al.*), we assume that the transition matrix is class-dependent and instance-independent, i.e.  $P(\bar{Y}^j = k \mid Y^j = i, \mathbf{X} = \mathbf{x}) = P(\bar{Y}^j = k \mid Y^j = i) = T_{ik}^j$ .

## Theorem

*If  $P(\bar{Y}^i \mid Y^j)$  is known, two noisy labels  $\{\bar{Y}^j, \bar{Y}^i\}$  are sufficient to identify  $\mathbf{T}^j$ .*

This theorem theoretically guarantees that the identifiability of the class-dependent transition matrix can be achieved by utilizing the occurrence probabilities  $P(\bar{Y}^i, \bar{Y}^j)$  and  $P(\bar{Y}^i \mid Y^j)$ . Note that  $P(\bar{Y}^i, \bar{Y}^j)$  can represent **noisy label correlations**, and  $P(\bar{Y}^i \mid Y^j) = \sum_{Y^i} P(\bar{Y}^i \mid Y^i)P(Y^i \mid Y^j)$ , which can imply **clean label correlations**.

At a high level, **the mismatch of label correlations** implied in these occurrence probabilities can achieve the identifiability.

# Our Method

Based on the above discussions, we propose to estimate transition matrices  $\{\mathbf{T}^j\}_{j=1}^q$  by following two stages.

**First Stage:** We utilize **sample selection** to obtain the extra information that implies clean label correlations, which can be used to estimate  $\hat{P}(\bar{Y}^i | Y^j)$ .

**Second Stage:** We perform **co-occurrence estimation** ( $\hat{P}(\bar{Y}^i, \bar{Y}^j)$  and  $\hat{P}(\bar{Y}^i | Y^j)$ ) by frequency counting, and then estimate the transition matrix  $\mathbf{T}^j$  by **solving the following probability equation**.

$$\hat{P}(\bar{Y}^j, \bar{Y}^i) = \sum_{Y^j} \hat{P}(Y^j) \hat{P}(\bar{Y}^j | Y^j) \hat{P}(\bar{Y}^i | Y^j),$$

where  $\hat{P}(\bar{Y}^j | Y^j)$  represents the transition matrix  $\hat{\mathbf{T}}^j$ .

# Experiments

Multi-label classification datasets:

- VOC2007 (20 classes)
- VOC2012 (20 classes)
- MS-COCO (80 classes)

Generate label noise:

$$\mathbf{T}^j = \mathbf{T} = \begin{pmatrix} 1 - \rho_- & \rho_- \\ \rho_+ & 1 - \rho_+ \end{pmatrix}$$

where  $j = 1, 2, \dots, q$ .

Metric for estimating transition matrices:

- Estimation Error =  $\sum_{j=1}^q \|\mathbf{T}^j - \hat{\mathbf{T}}^j\|_1 / \|\mathbf{T}^j\|_1$

Metric for classification performance:

- mean Average Precision (mAP)
- Overall F1-measure (OF1)
- per-Class F1-measure (CF1)



# Experiments

**Table:** Comparison for estimating transition matrices on Pascal-VOC2007 dataset. The best results are in **bold**.

Noise rates ( $\rho_-, \rho_+$ )	(0,0.2)	(0,0.6)	(0.2,0)	(0.6,0)	(0.1,0.1)	(0.2,0.2)	(0.017,0.2)	(0.034,0.4)
T-estimator max	3.89 $\pm$ 0.0	10.52 $\pm$ 0.5	3.01 $\pm$ 0.1	4.47 $\pm$ 0.2	3.18 $\pm$ 0.2	5.28 $\pm$ 0.2	3.99 $\pm$ 0.1	6.28 $\pm$ 0.4
T-estimator 97%	4.95 $\pm$ 0.1	4.42 $\pm$ 0.1	1.77 $\pm$ 0.0	2.13 $\pm$ 0.1	6.99 $\pm$ 0.1	6.94 $\pm$ 0.1	5.38 $\pm$ 0.1	5.17 $\pm$ 0.0
DualT-estimator max	1.94 $\pm$ 0.1	7.29 $\pm$ 0.1	1.03 $\pm$ 0.0	2.68 $\pm$ 0.1	<b>2.13<math>\pm</math>0.2</b>	4.02 $\pm$ 0.1	<b>1.71<math>\pm</math>0.0</b>	2.67 $\pm$ 0.2
DualT-estimator 97%	12.59 $\pm$ 0.1	7.43 $\pm$ 0.1	1.09 $\pm$ 0.0	2.41 $\pm$ 0.3	14.39 $\pm$ 0.1	11.78 $\pm$ 0.1	13.71 $\pm$ 0.2	11.15 $\pm$ 0.1
Our estimator	<b>1.51<math>\pm</math>0.1</b>	<b>2.30<math>\pm</math>0.1</b>	<b>0.37<math>\pm</math>0.1</b>	<b>1.34<math>\pm</math>0.3</b>	3.06 $\pm$ 0.4	<b>3.21<math>\pm</math>0.3</b>	2.03 $\pm$ 0.2	<b>1.84<math>\pm</math>0.3</b>

**Table:** Summary of the Wilcoxon signed-ranks test for **Reweight-Ours** against other baselines at 0.1 significance level.

Reweight-Ours against	Standard	GCE	CDR	AGCN	CSRA	Reweight-T max	Reweight-T 97%	Reweight-DualT max	Reweight-DualT 97%
mAP	tie[0.29]	tie[0.26]	tie[0.32]	tie[0.18]	tie[0.45]	tie[0.32]	tie[0.23]	tie[0.35]	<b>win</b> [0.02]
OF1	<b>win</b> [0.00]	<b>win</b> [0.05]	<b>win</b> [0.04]	<b>win</b> [0.02]	<b>win</b> [0.05]	<b>win</b> [0.01]	<b>win</b> [0.08]	<b>win</b> [0.09]	<b>win</b> [0.00]
CF1	<b>win</b> [0.02]	<b>win</b> [0.01]	<b>win</b> [0.01]	<b>win</b> [0.01]	<b>win</b> [0.02]	<b>win</b> [0.03]	<b>win</b> [0.02]	<b>win</b> [0.09]	<b>win</b> [0.00]

**Thank you for your hearing!**

Code: `https://github.com/ShikunLi/Estimating\_T\_For\_Noisy\_Mutli-Labels`