The 13th Northeast Collegiate Programming Contest Editorial

By Claris & quailty

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Overview

	Easiest Hardest
Idea	J B C E G H F D I A
Coding	J E G H B C F A D I
Summary	J B E C G H F D A I

J. Time Limit

Author: Claris

Shortest Judge Solution: 319 Bytes

Description

Given a problem's standard length and the running time of several verification codes, set a time limit according to the rules.

- n ÿ 10ÿ
- ai ÿ 10ÿ

Solution

- Simulate according to the question.

B. Balanced Diet

Author: Claris

Shortest Judge Solution: 948 Bytes

Description

There are n candies, each of which has a value of ai and a type of bi. Select some

candies so that the maximum value of the candies is the maximum value divided by the number of candies of the same type.

big.

If the i-th type of candy is selected, then at least li must be selected .

- n, m ÿ 100000ÿ

Solution

- Let fi represent the maximum value of each candy choice not exceeding i, then

  ans = max(Each $\frac{}{be}$)ÿ

- candy is sorted by value from largest to smallest, and its contribution to f is a suffix.
- Parts that contribute the same are implemented using prefix sum.
- Time complexity O(n log n + m).

E. Minimum Spanning Tree

Author: quailty

Shortest Judge Solution: 548 Bytes

Description

Given a tree G of n nodes, each edge has a weight.

The edge weight connecting (u, v) in the line graph L(G) of G is the sum of the edge weights of the two edges in their corresponding trees.

Find the sum of the edge weights of the minimum spanning tree of L(G).

- n ÿ 100000ÿ

Solution

- Consider the contribution of each point i in the tree to the answer.
- If the degree of i is less than 2, then there is no contribution.
- Otherwise, the edges connected to i form a clique in the line graph.
- The minimum spanning tree of a clique is an edge connecting each point to the point with the smallest weight.
- Time complexity O(n).

C. Line-line Intersection

Author: quailty

Shortest Judge Solution: 1166 Bytes

Description

Given n lines on a plane, count how many pairs of lines have common points.

- n ÿ 100000ÿ

Solution

- Two lines have a common point if and only if they are coincident or non-parallel.
- Overlapping straight lines: Find the straight line expression ax+by+c = 0, and calculate the result after reducing it.
- Parallel or overlapping lines: Set c in the expression to zero, reduce the fraction, and then calculate the result.
- The answer is the total number of solutions minus the number of parallel or overlapping solutions plus the number of overlapping solutions.
- Time complexity O(n log n).

G. Radar Scanner

Author: Claris

Shortest Judge Solution: 605 Bytes

Description

Given n rectangles on a plane with sides parallel to the coordinate axes, each rectangle covers a number

of

squares. Move these rectangles up, down, left, and right with the least number of operations so that there is a

square covered by all the rectangles.

- n ÿ 100000ÿ

Solution

- The two dimensions are independent, so let's consider the case of one dimension.
- Assuming the final intersection point is x, the cost of moving the line segment [l,r] to x is |

  lÿx|+|rÿx|ÿ(rÿl)
  ———————————
  2
  ÿ

- The division by 2 and the subtraction of r ÿ l are independent of x and only require minimizing ÿ(|

  l ÿ x| + |r ÿ x|).

- Classic problem, the optimal value is when x is the median of all l and r.
- Time complexity O(n).

H. Skyscraper

Author: Claris

Shortest Judge Solution: 940 Bytes

Description

There are n skyscrapers arranged in a row from left to right. The estimated number of floors of skyscraper i is

eat.

m operations: 1.

Add k to all the numbers in a certain interval [l,r]. 2. Given [l,r], how many stages

are needed if only the skyscrapers in [l,r] are constructed from scratch? In each stage, a range can be

selected and all skyscrapers in

that range can be built upwards.

Build one floor.

- n, m ÿ 100000ÿ

Solution

- Consider the static case, let a0 = 0, bi = ai ÿ aiÿ1.
- If bi > 0, then it will take at least bi more stages to complete.
- If bi ÿ 0, then the construction of i ÿ 1 can also repair i.
- Let ci = bi [bi > 0], for the interval [l,r], the minimum number of stages is

  al + cl+1 + cl+2 + · · · + crÿ
- ÿ (b1 + b2 + · · · + bl) + (cl+1 + cl+2 + · · · + cr)ÿ

Solution

- Considering the interval addition operation, the impact on b and c is two single-point modifications at l and $r + 1$.
- The tree array maintains the prefix sum of b and c.
- Time complexity $O((n + m)\log n)$.

F. Mini-game Before Contest

Author: Claris & quality

Shortest Judge Solution: 1674 Bytes

Description

Two teams play a game on a directed graph with n points and m edges.

Take turns moving the chess piece to make a move. The team of the person who cannot move loses.

Normal people want to win, and if they can't win, they want a draw; but actors want to lose, and if they can't lose, they want a draw.

Hope for a draw.

For each point, calculate the final game result when the chess piece is placed at that point to start the game.

- n ÿ 100000ÿ
- m ÿ 200000ÿ

Solution

- State: fi,j indicates the final game result when the chess piece is at position i and it is the jth person's turn to act.
- Define ÿ1 to indicate A wins, 0 to indicate a draw, and 1 to indicate B wins, then the transition of each state is either min or max.
- Initially, we assume that the values of all states are 0, and maintain a cnt array to indicate the number of ÿ1, 0, and 1 in the successor state of each state, which is used to calculate the value of a state in O(1).
- For all states that cannot act, reset its state value and add it to the queue.

Solution

- Use SPFA to relax the predecessor state of each state whose value changes in turn, and update the cnt array.
- The value of each state switches between ÿ1, 0, and 1 a constant number of
- times. The time complexity is $O(n + m)$.

D. Master of Data Structure

Author: Claris

Shortest Judge Solution: 2181 Bytes

Description

Given a tree of n points, each point has a weight wi. At the beginning, each point

The weights are all 0.

It needs to support m operations, each of which is one of the given 7 path point weight

operations.

- n ÿ 500000ÿ
- m ÿ 2000ÿ

Solution

- Note that m is very small.
- Define the two endpoints u and v of the chain of each operation as key points.
- There are O(m) key points in total. Find the
- virtual tree of these key points. The number of points in the virtual tree is only O(m), and each virtual tree edge represents a number of compressed points.
- For each operation, brute force simulation can be performed on the virtual tree, and pay attention to the case where the edge length is 0.
- Time complexity $O(n + m^2)$.

A. Apple Business

Author: Claris

Shortest Judge Solution: 1694 Bytes

Description

Given a complete binary tree with n nodes, each node has ai apples. Given m

orders, each order specifies a path from u to v, with at most

Buy c apples from these apple trees and pay w yuan for each apple.

Please find a sales method that maximizes total revenue.

- n ÿ 100000, m ÿ 100000ÿ
- It is guaranteed that u is an ancestor of v.

Solution

- Sort all orders by w from largest to smallest and consider each order in turn.
- If the current order can be fulfilled, then fulfill it.
- If the current order cannot be satisfied, the only way to satisfy it is to abandon the previous order, but the previous order costs more than it pays, so it is not cost-effective.
- You only need to find out how many current orders can be fulfilled without changing the sales of all previous orders.

Solution

- Consider violence judgment.
- Create a bipartite graph with m points on the left and n points on the right. Each order on the left is connected to the apple tree on the corresponding path on the right.
- S is connected to each order on the left, and the capacity is the actual sales volume.
- Each apple tree on the right is connected to T with a capacity equal to the number of apples.
- A sales plan is legal if and only if the edges of S to every order are fully flowed.

Solution

- Hall's Theorem: A complete matching exists on the left side of a bipartite graph if and only if for any subset U of k points on the left side, U is connected to at least k points on the right side.
- In the graph we built earlier, Hall's theorem can be expressed as:
- Full flow occurs if and only if for any subset of orders U, assuming that the sum of their actual sales is A and the union of their corresponding paths is V, then the sum of the number of apples in all apple trees in V, B, cannot be less than A.

Solution

- The union of several paths is multiple connected blocks on the tree.
- We only need to consider the tightest restrictions: the case where the union of the paths is a connected block.
- That is: for any connected block S on the tree, the sum of the number of apples in S cannot be less than the sum of the actual sales of orders whose end points are in S.

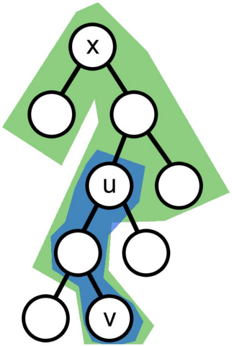
Solution

- Consider how to calculate the actual sales volume d of the current order (u, v, c).
- Let fx denote the minimum value of the sum of the number of apples in the connected block containing the subtree with x as the root minus the sum of the actual sales of orders whose two endpoints are in the connected block.
- Let gx,y denote the minimum value of the sum of the number of apples in the connected block containing the node y minus the sum of the actual sales of orders whose endpoints are both in the connected block, considering the subtree rooted at y (x is an ancestor of y) in order to compute fx.
- In the case of a complete binary tree, the total number of states is O(n log n).

Solution

- $f_x = g_{x,x}$. Consider how to calculate $g_{x,y}$.
- Let $b_{x,y}$ represent the weight of point y in order to calculate f_x. Initially, $b_{x,y}$ = the number of apples at point y.

Solution

- 

- For a previous order (u, v), the actual sales volume is d. Then, if and only if x is the ancestor of u and the connected block contains v, both endpoints of the order are included in the connected block.

Solution

- Enumerate u's ancestors x and subtract d from the corresponding $b_{x,v}$.
- Then $g_{x,y}$ only needs to find the minimum sum of b's of the connected blocks containing y, so $g_{x,y}$ = $b_{x,y} + \min(g_{x,2y}, 0) + \min(g_{x,2y+1}, 0)$. While subtracting d from
- $b_{x,v}$, it is necessary to recalculate the DP values $g_{x,y}$ of all points y on the path from v to x. A total of $O(\log n)$ x's need to be
- enumerated, and $O(\log n)$ points need to be enumerated within each x.

  and.
- The time complexity of adding the impact of each order is $O(\log^2 n)$.

Solution

- Back to the previous question, how to calculate the actual sales volume d of the current order (u, v, c)?
- Enumerate the ancestors x of u, and use similar DP to calculate the minimum value tmp of the sum of b of the connected blocks whose highest point is x and which contain both x and v.
- It is legal if tmp ÿ d ÿ 0, that is, d ÿ tmp.
- Therefore, d = min(c,tmp). The
- time complexity of calculating the actual sales volume for each order is O(log2 n). The
- total time complexity is O(n log n + m log m + m log2 n).

I. Temperature Survey

Author: quailty

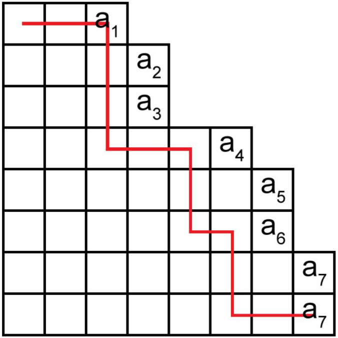
Shortest Judge Solution: 3368 Bytes

Description

Given a non-decreasing sequence a1, a2, . . . , an, count how many non-decreasing positive integer sequences b1, b2, . . . , bn satisfy bi ÿ ai.

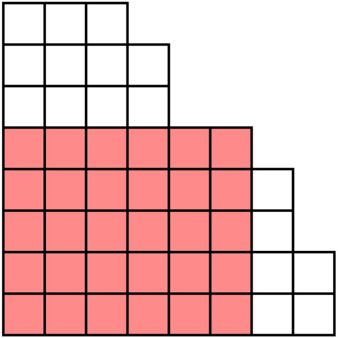
- n ÿ 200000ÿ
- ai ÿ n.

Solution

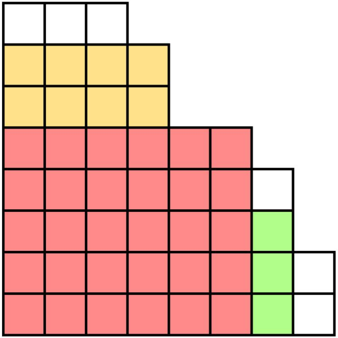


- Let an+1 = an. The number of options is equal to the number of paths from the upper left corner to the right and down to the lower right corner in the above figure.

Solution

- 
- By continuously taking $a_{\frac{l+r}{2}}$ as the upper right corner of the rectangle, O(n) rectangles can be recursively divided.

Solution

- 
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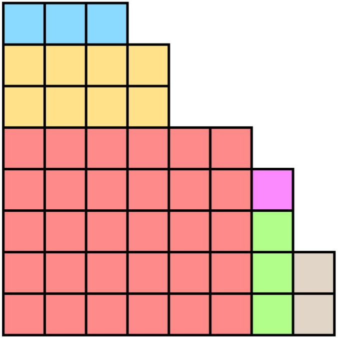
- 
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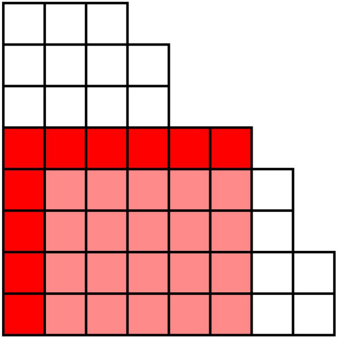
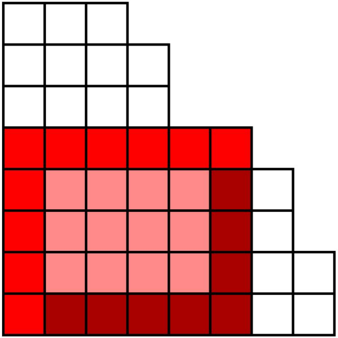
- 

- Let $f_{i,j}$ denote the number of ways to walk from the upper left corner to (i, j). The left and upper boundaries of each rectangle can be found by brute force transfer: $f_{i,j} = f_{i-1,j} + f_{i,j-1}$.

Solution

- 

- Considering the contribution of the left and upper boundaries to the right and lower boundaries, which is in the form of the convolution DP value of the combination number, the DP values of the right and lower boundaries can be obtained by NTT.

Solution

- A rectangle with side length $w \times h$ requires $O((w + h)\log(w + h))$ cost to NTT.

- Conclusion: The sum of the side lengths of all rectangles is $O(n \log n)$, and the total time complexity is

  $O(n \log2 n)$.

- Proof: The recursive process of dividing the rectangle has only $O(\log n)$ layers. Rectangles in the same layer do not intersect in two dimensions. The sum of the side lengths of each layer is $O(n)$, so the total sum of the side lengths is $O(n \log n)$.

Thank you!