CCCC Interviews 2024

Coding Club Competitive Coding

January 31, 2025

Instructions

- There are 6 questions in this paper, some of which have multiple subparts
- After solving a question/subpart, you must explain your solution to one of the CCCC members listed at the end of the question
- The number of points allotted to each subpart is inversely proportional to the number of people who solve it
- Duration: 3 hours.

Question 1

Let's start with a famous problem:

a) Given an array A of integers of size N. Count the number of subarrays that sum to 0.Can you generalize it for any given target sum K?

b) You are given an array $a_1, a_2, \ldots, a_n$ consisting of integers from 0 to 9. A subarray $a_l, a_{l+1}, a_{l+2}, \ldots, a_{r-1}, a_r$ is good if the sum of elements of this subarray is equal to the length of this subarray

$$\sum_{i=l}^{r} a_i = r - l + 1.$$

For example, if $a = [1, 2, 0]$, then there are 3 good subarrays: $a_{1\ldots1} = [1], a_{2\ldots3} = [2, 0], a_{1\ldots3} = [1, 2, 0]$. Calculate the number of good subarrays of the array a.

c) You are given an integer array $a_1, a_2, \ldots, a_n (1 \leq a_i \leq n)$. Find the number of subarrays of a whose XOR has an even number of divisors. In other words, find all pairs of indices $(i \leq j)$ such that $a_i \oplus a_{i+1} \oplus \cdots \oplus a_j$ has an even number of divisors.

For example, numbers 2, 3, 5 or 6 have an even number of divisors, while 1 and 4 — odd. Consider that 0 has an odd number of divisors in this task.

[Answer to Vansh/Ashish]

Question 2

A group of N people are on a hiking trip. It starts raining heavily and the group starts looking for places where they can be protected from the rain. There are 2 enclosures nearby where people can find shelter. Let's call them A and B. Enclosure A can accommodate X people, and Enclosure B can accommodate Y people. It is guaranteed that $(X + Y) \geq N$, and so everyone can be accommodated. The people are numbered from 1 to N . We know how far each person is from each enclosure. More specifically, for person i, $A_{dist}[i]$ denotes how far the ith person is from A and $B_{dist}[i]$ denotes how far the ith person is from B. The group has genuine camaraderie, so they decide that they want to minimize the total distance traveled by their group members altogether to reach their respective enclosures. Given N, X, Y and the distances $(A_{dist}[i], B_{dist}[i]), 1 \leq i \leq N$, your aim is to compute this minimum value.

For example, if $N = 4, X = 2, Y = 2$, and the distances are:

$$A_{dist} = [10, 23, 15, 5]$$
$$B_{dist} = [12, 20, 8, 20]$$

Then the optimal strategy would be for 1 and 4 to go to A and for 2 and 3 to go to B. In this case the total distance traveled is $10 + 5 + 20 + 8 = 43$

[Answer to Siddhant/Ashish]

Question 3

Consider a grid representation of an art gallery, where the layout is defined by a non-increasing sequence $A = [A_1, A_2, \ldots, A_N]$. The ith row from the top contains A_i tiles, aligned to the left.

i. A camera can be placed on any tile.

ii. A camera can face either downward or to the right.

iii. A downward-facing camera monitors the tile it is placed on and all tiles below it in the same column.

iv. A right-facing camera monitors the tile it is placed on and all tiles to the right in the same row.

v. Each tile must be monitored by at least one camera.

vi. No camera should monitor another camera.

vii. Each tile can contain at most one camera.

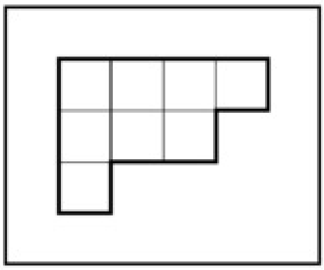
Determine the number of valid placements of cameras modulo $10^9 + 7$.

a) Solve this in time complexity $O(N \cdot max(A_i))$

b) Solve this in time complexity $O(N + max(A_i))$

For clarification on time complexity and big O notation, contact an invigilator.

[Answer to Pramit/Abheek]



Example of art gallery. where $A = [4, 3, 1]$

Question 4

There are N safes and N keys. Each key can open only one safe, and each safe can be opened by only one key. We randomly place one key into each safe. $N - M$ safes are then randomly chosen, and then locked. What is the probability that we can open all the safes with the M keys in the M remaining safes? Note: Once a safe is opened, the key inside the safe can be used to open another safe.
Sub parts:

a) Solve this for the case where $M = 2 \leq N$

b) Solve this for the general case where $M \leq N$ is any integer

[**Answer to Kanav/Siddhant**]

Question 5

You have a number that is formed by concatenating the decimal representations of the first N integers. This number is given to the digit adder machine. The digit adder machines takes a number X as input and outputs the sum of its digits. However today the machine is stuck and will keep feeding the output back into the input until there is only 1 digit left. Find this digit for a given N.

For example, if $N = 12$, we have $X = 123456789101112$. This number after passing into the digit adder machine first becomes 78, 15 and then finally 6.
Sub parts:

a) Solve this in time complexity $O(log^2 N)$

b) Solve this in time complexity $O(1)$

For clarification on time complexity and big O notation, contact an invigilator.

[**Answer to Vansh/Pramit**]

Question 6

a) There is a long hallway which is 2 feet wide and N feet long. You have to tile the entire hallway using 1×2 and 2×1 tiles. Find the number of ways of tiling the entire grid such that the entire hallway is tiles and no two tiles are overlapping.

b) The tile shop you buy tiles from has now introduced a new tile of 1×1 feet^2. Find the new number of ways of tiling the hallway.

c) You have a new hallway which is 3 feet wide and M feet long. Find the number of ways of tiling it using only 1×2 and 2×1 tiles

[**Answer to Abheek/Kanav**]

Best of Luck