

CCCC Interviews 2024

Coding Club Competitive Coding

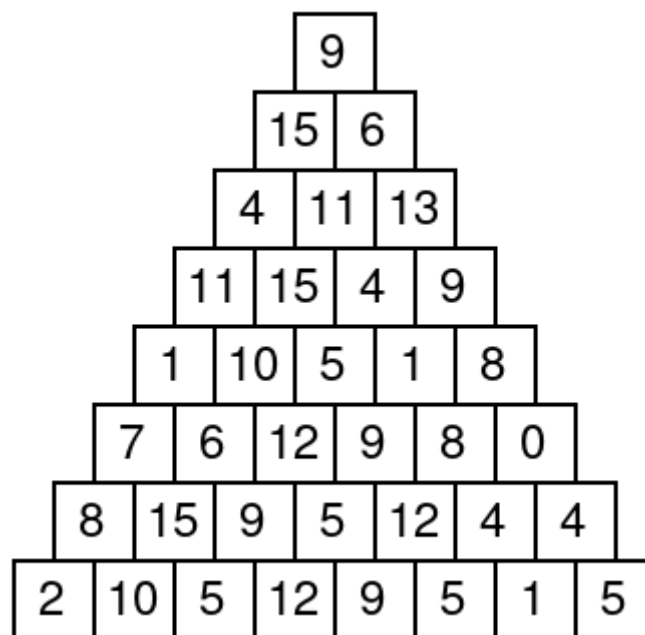
February 3, 2025

Instructions

- There are 6 questions in this paper, some of which have multiple subparts
 - After solving a question/subpart, you must explain your solution to one of the CCCC members listed at the end of the question
 - The number of points allotted to each subpart is inversely proportional to the number of people who solve it
 - Duration: 3 hours.
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Question 1

Consider a **XOR** pyramid where each number is the **XOR** of lower-left and lower-right numbers. Here is an example pyramid:



Given the bottom row of the pyramid, your task is to find:

- a) The topmost number of the pyramid
- b) Any arbitrary number specified by coordinates (i, j) which represent the j_{th} number on the i_{th} row
- c) How to calculate the values if **OR** or **AND** were used instead of **XOR**

[Answer to Kanav/Abheek]

Question 2

You are in charge of a counting shipping containers at a wharf. This wharf contains N stacks of containers all of which are initially empty. A total of K ships come to the wharf to unload their containers. The i_{th} ($1 \leq i \leq K$) ship unloads l_i shipping containers. These shipping containers are picked up by a crane and placed in l_i different stacks. In particular, for the i_{th} ship, the first container is placed at position a_i and every subsequent container is placed d_i stacks to the right of the previous, i.e. at positions $a_i, a_i + d_i, \dots, a_i + (l_i - 1)d_i$. It is guaranteed that all l_i containers can be placed in the n stacks ($a_i + (l_i - 1)d_i \leq N$). Find the number of shipping containers in each of the N stacks after all K ships have been unloaded.

Sub parts:

- a) Solve this in time complexity $O(NK)$
- b) Solve this in time complexity $O(N + K)$ if it given that $d_1 = d_2 = \dots = d_k$
- c) Solve this in time complexity $O((N + K) \cdot \sqrt{N})$

For clarification on time complexity and big O notation, contact an invigilator.

[Answer to Ashish/Siddhant]

Question 3

N people came to a birthday party. Then those, who had no friends among people at the party, left. Then those, who had exactly 1 friend among those who stayed, left as well. Then those, who had exactly 2, 3, \dots , $N - 1$ friends among those who stayed by the moment of their leaving, did the same.

Devise an algorithm to calculate the maximum possible people that could be left in the end.

[Answer to Vansh/Siddhant]

Question 4

- a) You are climbing a staircase with N steps. At each step, you can either take 1 step or 2 steps at a time. Find the number of distinct ways to reach the top.
- b) Now, you are given an additional option: You can also take a jump of exactly 3 steps at a time. However the jump of exactly 3 steps can only be taken if the previous jump wasn't also a 3 step jump Find the new number of ways to reach the top.
- c) Consider a more general case where you have a staircase with N steps, and at each step, you can take either 1, 2, or any of the first K natural number steps. Find the number of distinct ways to reach the top. Note: there is no restriction on when you can take a certain like in the part (b)

For all subtasks you are expected to find an algorithm which has a time complexity of $O(N)$. For clarification on time complexity and big O notation, contact an invigilator.

[Answer to Abheek/Vansh]

Question 5

There are N slimes standing on a number line. The i_{th} slime from the left is at position X_i . It is guaranteed that the positions of the slimes form a strictly increasing sequence, i.e. $1 \leq X_1 < X_2 < \dots < X_N$.

Gautam will perform $N - 1$ operations. The i_{th} operation consists of the following procedures:

- Choose an integer k between 1 and $N - i$ (inclusive) with equal probability.
- Move the k_{th} slime from the left, to the position of the neighboring slime to the right.
- Fuse the two slimes at the same position into one slime.

Find the total distance traveled by the slimes multiplied by $(N - 1)!$. If a slime is born by a fuse and that slime moves, we count it as just one slime.

Sub parts:

- a) Solve this in time complexity $O(N^2)$
- b) Solve this in time complexity $O(N)$

For clarification on time complexity and big O notation, contact an invigilator.

[Answer to Gautam/Kanav]

Question 6

You are given n numbers a_1, a_2, \dots, a_n . Find a value of x that minimizes the sum

$$\sum_1^n |a_i - x|^c = |a_1 - x|^c + |a_2 - x|^c + \dots + |a_n - x|^c$$

Sub parts:

- a) Solve this for the case where $c = 1$
- b) Solve this for the case where $c = 2$
- c) Solve this for the general case where $c \in \mathbb{N}$

[Answer to Gautam/Nikhil]

Best of Luck