ALGORITHM ANALYSIS

Terminology

$\subseteq$	subset	$\cup$	union, or
$\forall$	for all	$\cap$	intersection, and
$\in$	member of	$\mid$	such that
Σ	summation	...	elipsis, so forth

Sets

ϕ empty $\{\ \}$

N Natural $\{1, 2, 3, \ldots\}$

Z integer $\{\ldots -2, -1, 0, 1, 2, \ldots\}$

Q rational $\{ i/d \mid i \in Z, d \in N \}$

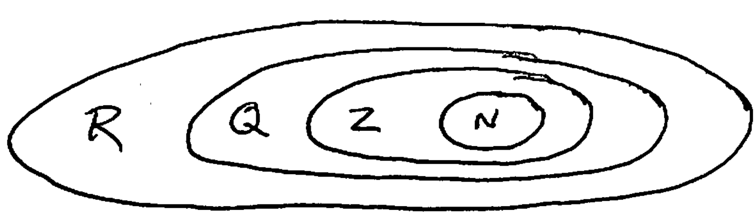
$\mathbb{IR}$ irrational such as $\sqrt{2}, \pi, e, \ldots$

R real $\{ Q \overset{\cup}{\text{and}} \mathbb{IR} \}$

Relationships

$$\phi \subseteq N \subseteq Z \subseteq Q \subseteq R$$

Diagram - Venn



$\mathbb{IR} = \{ (R \cap Q)^c \}$ $\qquad$ $(\ )^c$ compliment

MEK 9/15/11

Useful Summation Rules

$$\sum_{i=M}^{N} a_i = a_M + a_{M+1} + a_{M+2} \cdots + a_{N-2} + a_{N-1} + a_N$$

$$i \in Z, \quad M \le N, \quad a_i \in R$$

Example

Let $M = 3$, $N = 6$, $a_i = 1$

$$\sum_{i=3}^{6} 1 = 1 + 1 + 1 + 1 = 4$$

In general,

1) $\longrightarrow$ $$\sum_{i=M}^{N} a_i = (N - M + 1) \quad \text{where } a_i = 1$$

and

2) $\longrightarrow$ $$\sum_{i=M}^{N} c\, a_i = c \sum_{i=m}^{N} a_i$$

$$= c(N - M + 1) \quad \text{where } a_i = 1$$

3) $\longrightarrow$ $$\sum_{i=M}^{N} a_i = \frac{N(N+1)}{2} - \frac{(m-1)M}{2} \quad \text{where } a_i = i, \quad 0 \le M \le N$$

Example

with $m = 0 \cup 1$ then

3a) $\longrightarrow$ $$\sum_{i=M}^{N} i = \frac{N(N+1)}{2}$$

[signature] 9/15/11

Analyze Mark Sort

Given $a[N] \in R = \{a_0, a_1, \ldots a_{N-1}\}$

```
for (int i = 0; i < N-1; i++) {
    for (int j = i+1; j < N; j++) {
        If (a[i] > a[j]) {
            a[i] = a[i] ^ a[j];
            a[j] = a[i] ^ a[j];
            a[i] = a[i] ^ a[j];
        }
    }
}
```

We can analyze as the following

$$I \longrightarrow C_1 + \sum_{i=0}^{N-2}\left(C_2 + \sum_{j=i+1}^{N-1}\left(C_3 + P C_4\right)\right)$$

where $C_i \in R$

$C_1 = T_= \rightarrow$ clock cycles for $=$

$C_2 = (T_< + T_- + 2T_+ + T_=)$

$C_3 = (T_< + T_> + T_+)$

P = Probabity of a swap

$C_4 = (3T_= + 3T_\wedge)$

[illegible] 9/15/16

Restating + Solve

$$\mathrm{I} \longrightarrow \quad C_1 + \sum_{i=0}^{N-2}\left(C_2 + \sum_{j=i+1}^{N-1}(C_3 + PC_4)\right)$$

a) First solve inside summation

$$\sum_{j=i+1}^{N-1}(C_3 + PC_4) = (C_3 + PC_4)\sum_{j=i+1}^{N-1} 1$$

$$= (C_3 + PC_4)\left((N-1) - (i+1) + 1\right)$$

$$= (N - i - 1)(C_3 + PC_4)$$

define $C_5 = C_3 + PC_4$ then

$$\longrightarrow \quad \sum_{j=i+1}^{N-1}(C_3 + PC_4) = (N-1)C_5 - iC_5$$

b) Reformulate using a)

$$C_1 + \sum_{i=0}^{N-2}\left(\left(C_2 + (N-1)C_5\right) - iC_5\right)$$

$$C_1 + (N-1)^2 C_5 + (N-1)C_2 - \sum_{i=0}^{N-2} i\, C_5$$

$$\downarrow$$

$$\frac{(N-2)(N-1)}{2} C_5$$

[illegible] 9/15/[illegible]

Combining like terms

$$C_1 + (N-1)\left(C_2 + (N-1)C_5 - \frac{(N-2)}{2} C_5\right)$$

$$C_1 + (N-1)\left(C_2 + \frac{2N-2-N+2}{2} C_5\right)$$

$$C_1 + (N-1)C_2 + (N-1)N/2\, C_5$$

$$C_1 + (N-1)C_2 + \frac{N^2-N}{2} C_5$$

$$(C_1 - C_2) + \left(C_2 - \frac{C_5}{2}\right)N + \frac{C_5}{2}N^2$$

where $\boxed{a + bN + cN^2}$

$$a = C_1 - C_2 = T_= - T_< - T_- - 2T_+ - T_=$$

$$b = C_2 - \frac{C_5}{2} = T_< + T_- + 2T_+ + T_= - \frac{T_<}{2} - \frac{T_>}{2} - \frac{T_+}{2} - \frac{3P T_=}{2} - \frac{3P T_\wedge}{2}$$

$$c = \frac{T_< + T_> + T_+ + 3PT_= + 3PT_\wedge}{2}$$

So what is the $O()$

$$f(N) = a + bN + cN^2$$

Choose $g(N) = N^2$

$$\lim_{N \to \infty} \frac{f(N)}{g(N)} \leq C$$

$$\lim_{N \to \infty} \frac{a + bN + cN^2}{N^2} \leq C$$

therefore Mark Sort $= O(N^2)$

We can estimate the maximum # of operations by C_5 using $P = 1$

$C_{5/2} \approx 4.5$ operations

Marksort $\approx 4.5 N^2$ operations

We can estimate P with actual data using $N = 400 \rightarrow 480 \times 10^3$ operations
actual $C_{5/2} \leq 3$ operations

$3 \approx 1.5$ ops $+ 3P$ ops

So Probability of Swap $=$ 50%

[illegible] 9/5/16

Really means

Mark Sort $\approx 3N^2$ operations

So how many clock cycles does the dominate operation take?

$N = 2 \times 10^5 \xrightarrow[\text{sorted}]{\text{data}}$ 122 seconds

Meaning

$3N^2 \xrightarrow{\text{operations}}$ 122 seconds

3.4×10^{10} operations $\rightarrow$ 122 seconds

12×10^{10} ops $= 12 \times 10^{1}$ seconds

$10^9 \dfrac{\text{operations}}{\text{second}}$ the

machine at 4 GHz/sec

each dominate operation takes 4 clock cycles $\nearrow \dfrac{4 \times 10^9 \text{ clock cycles}}{10^9 \text{ operations}}$

9/15/16