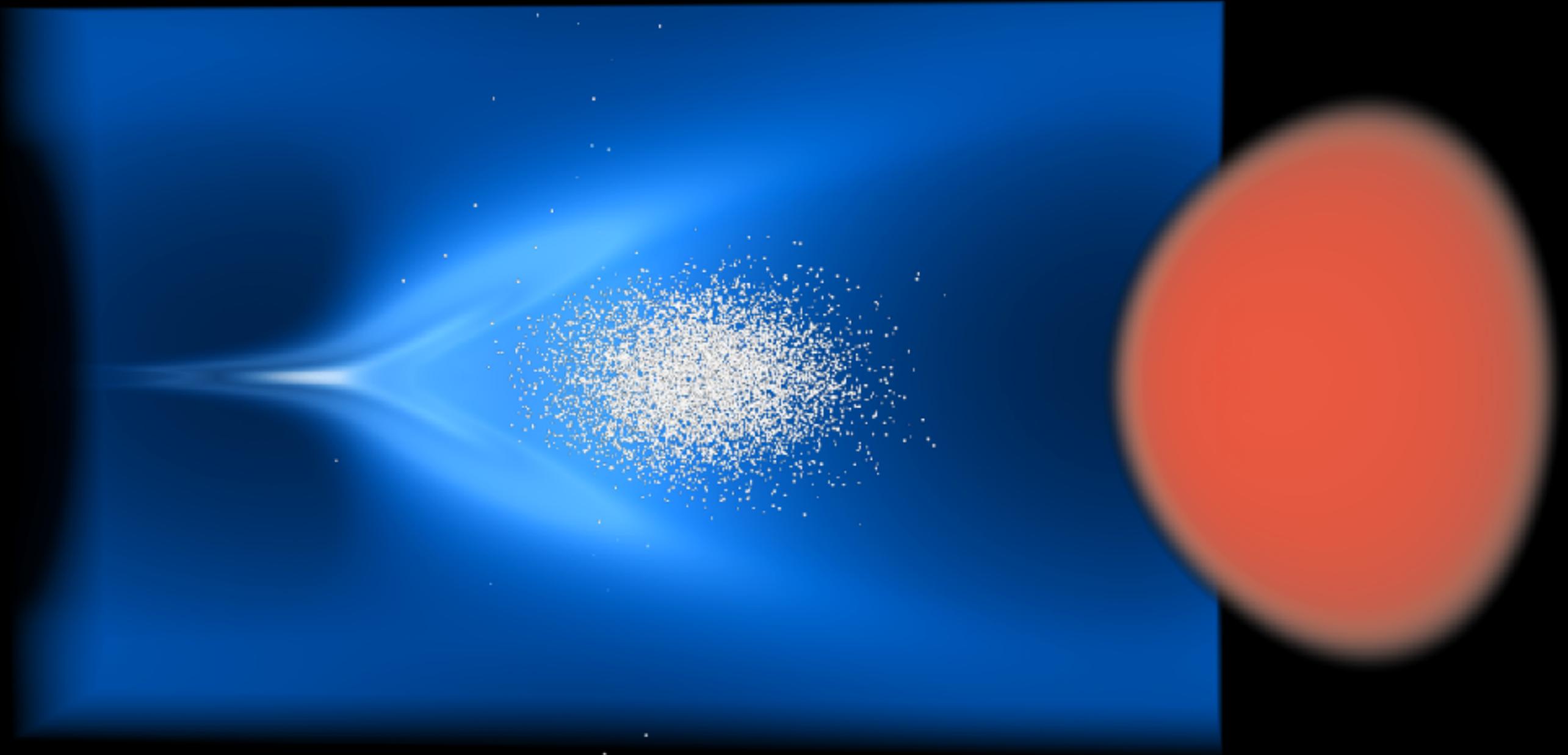


# Simulation of Laser Wakefield Acceleration



Francesco Massimo, LPGP

1-4 Dec 2023



# Outline

- Why plasma accelerators?
- Basics of laser wakefield acceleration
- Numerical simulation of plasma acceleration: PIC codes
- Introduction to the case study and the practical

# Outline

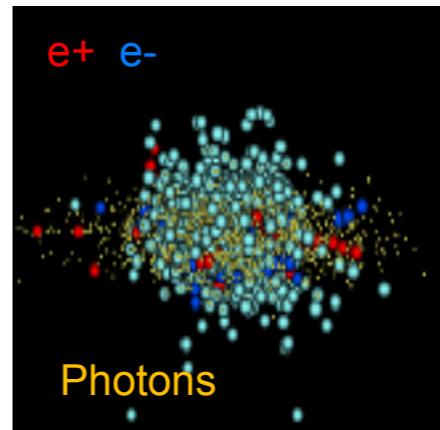
- Why plasma accelerators?
- Basics of laser wakefield acceleration
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- Introduction to the case study and the practical

# Application examples of relativistic electrons (just a few)

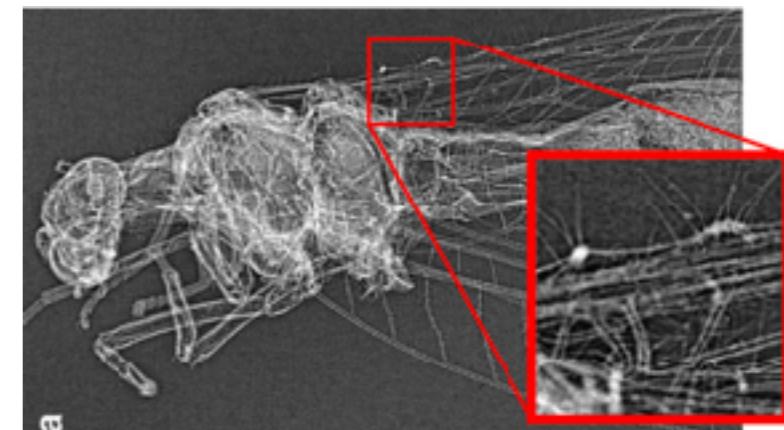
## Fundamental Research:

- QED, Particle physics
- Matter Physics
- Biology

Smilei Simulation  
of QED  $e^+e^-$  pairs creation  
from photons



J. Wenz et al., Nat Comm 2014  
Imaging through Betatron radiation

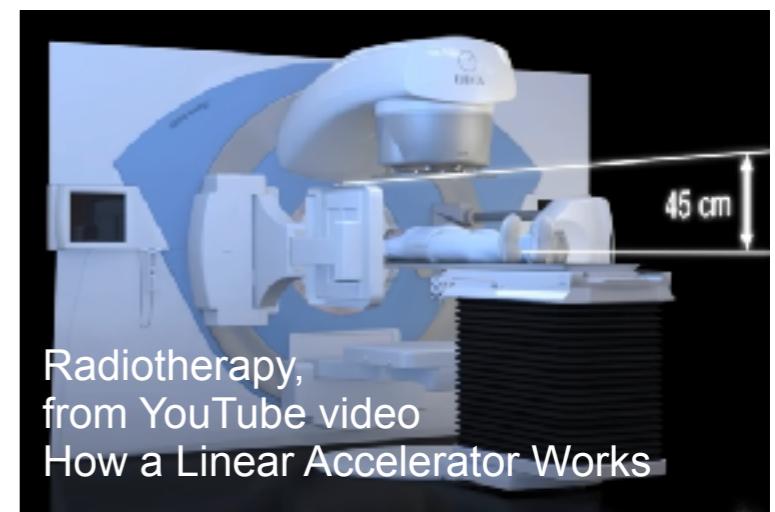


## Medicine, Industry, Heritage, ...:

- Cancer treatment,
- Medical imaging
- Electronic industry
- Study of materials
- Authentication of artwork
- ...



Hidden Archimede's Palimpsest,  
revealed by SLAC's synchrotron radiation  
in 2005 (U. Bergmann)



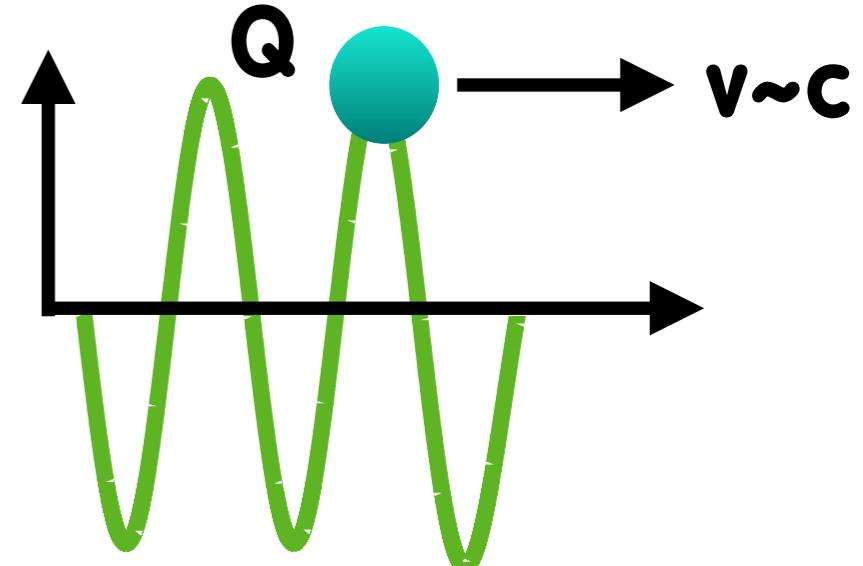
# Particle accelerators size and cost

Relativistic charge **Q**

In an accelerating cavity of length **L**

With peak accelerating field **E**

**Electric  
Field**



$$\text{Maximum energy gain} = Q E L$$

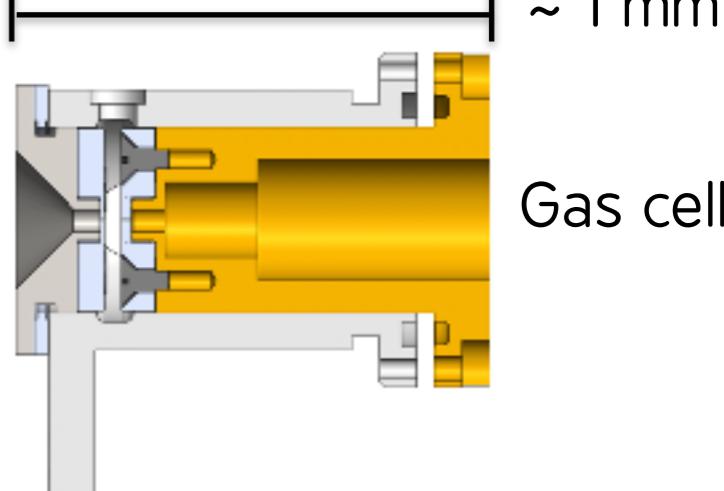
Given a target energy,

if **E** is limited by technology, **L** increases

→ need more metallic cavities

→ the accelerator size and cost increase

# Accelerating E-field of Laser Wakefield Acceleration

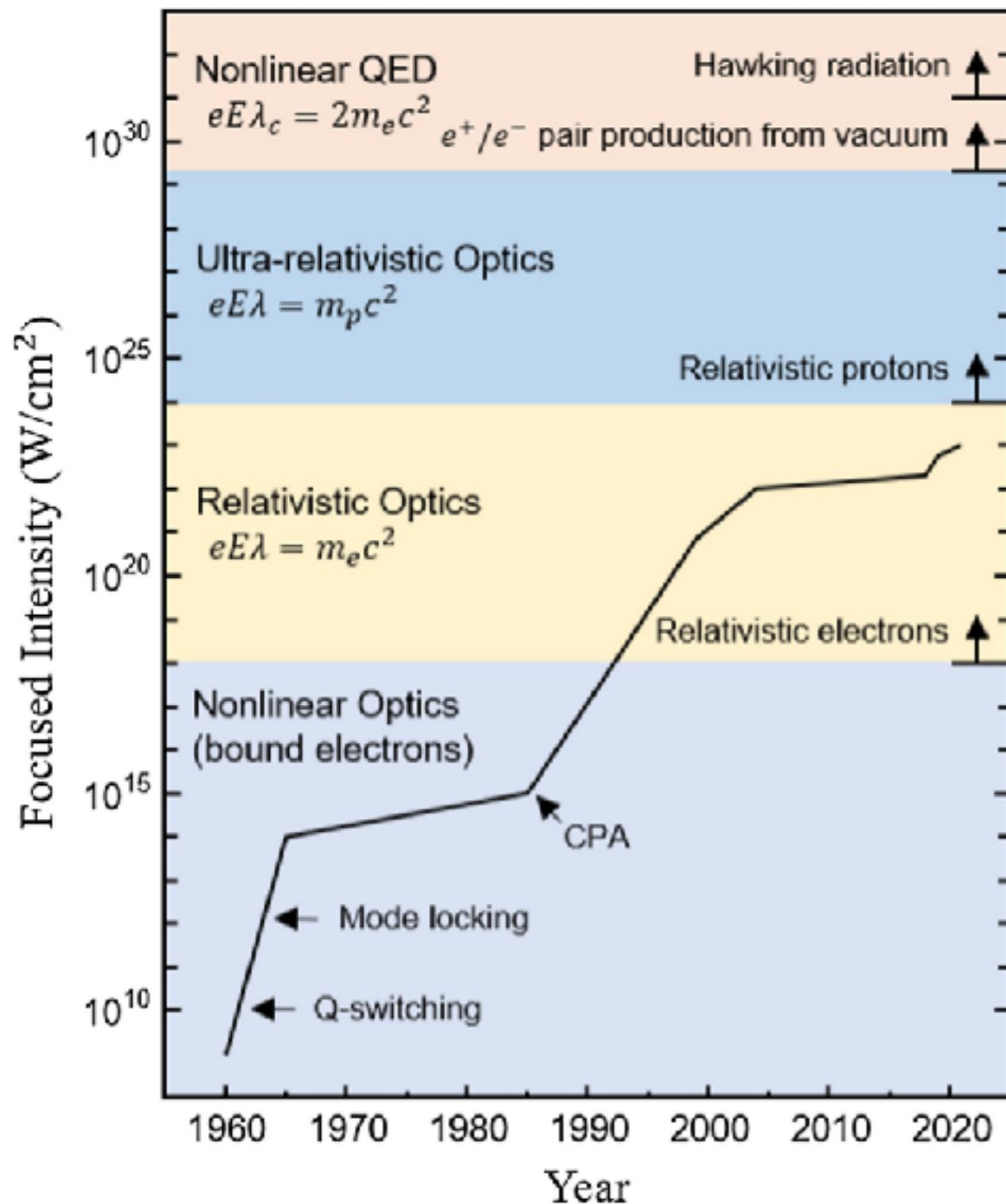
Accelerator technology	Peak Accelerating Field	Acceleration length to gain 100 MeV	~1 m H Accelerating Cavity
<b>Radiofrequency metallic cavities</b>	Max: $\sim 10^2$ MV/m Typical: $\sim 10$ MV/m	Min: $\sim 1$ m Typical: $\sim 10$ m	
<b>Laser Wakefield Acceleration (LWFA)*</b>	Max: $\sim 10^5$ MV/m Typical: $\sim 10^4$ MV/m	Min: $\sim 0.001$ m Typical: $\sim 0.01$ m	

**\*Open challenge: improve performances  
of Laser Wakefield Acceleration.  
Numerical modeling is necessary!**

# Outline

- Why plasma accelerators?
- Basics of laser wakefield acceleration
- Numerical simulation of plasma acceleration: PIC codes
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# High Intensity Lasers and their interaction with matter



## Techniques:

- Chirped Pulse Amplification
- Femtosecond Lasers
- Nonlinear Optics
- ...

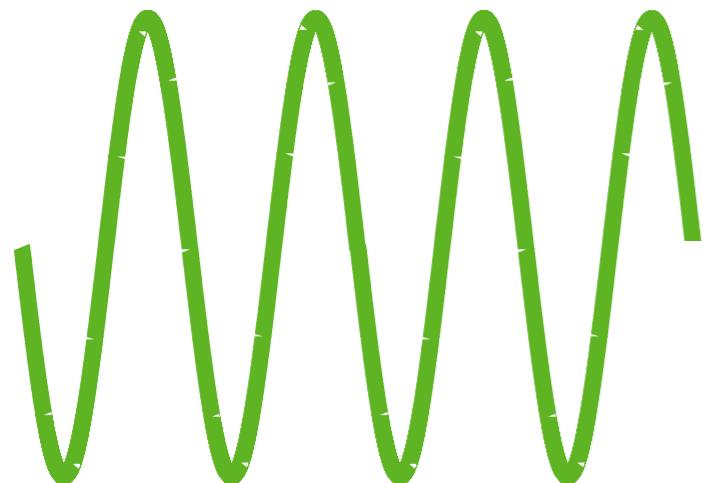
## Physics Domains:

- Relativistic Fluids
- Physics of Relativistic Plasmas
- Relativistic Optics
- Laboratory Astrophysics

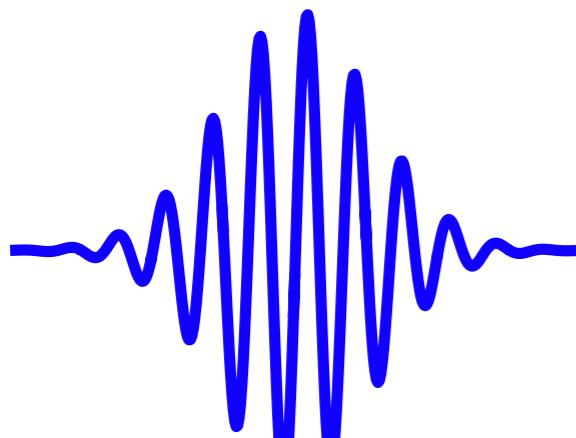
## Applications:

- Electron and ion sources
- Acceleration of particles
- Radiation sources (UV, X,  $\gamma$ )
- Novel, high resolution diagnostics
- Pump-probe measurements

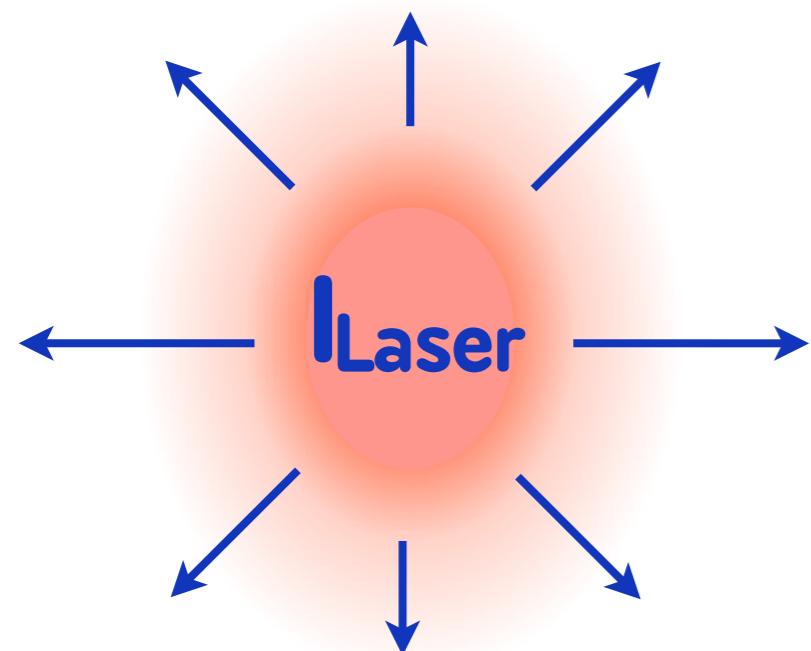
# Ponderomotive Force: the trigger for laser wakefield excitation in plasmas



Electron in infinite plane wave:  
Oscillating Force



Electron in finite laser pulse:  
Oscillating Force + Ponderomotive Force



$F_{\text{pond}} \propto -\nabla I_{\text{Laser}}$   
a.k.a. radiation pressure  
(Relativistic formula is more complex)

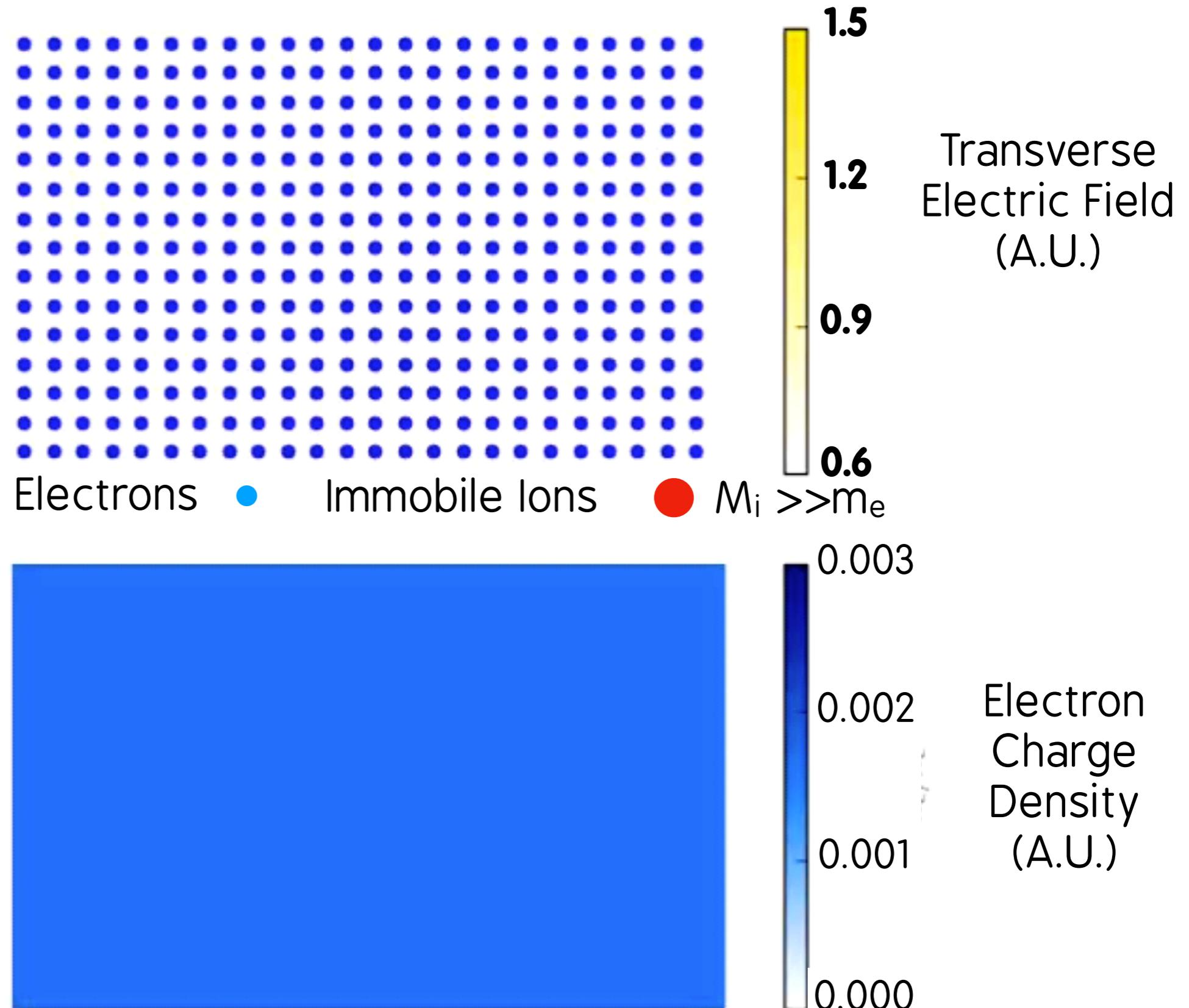
# Laser Wakefield Acceleration (LWFA): plasma wave excitation by the laser ponderomotive force

Laser Beam  
Duration: 28 fs

Ponderomotive  
Force:  
 $F = -\nabla I_{\text{Laser}}$

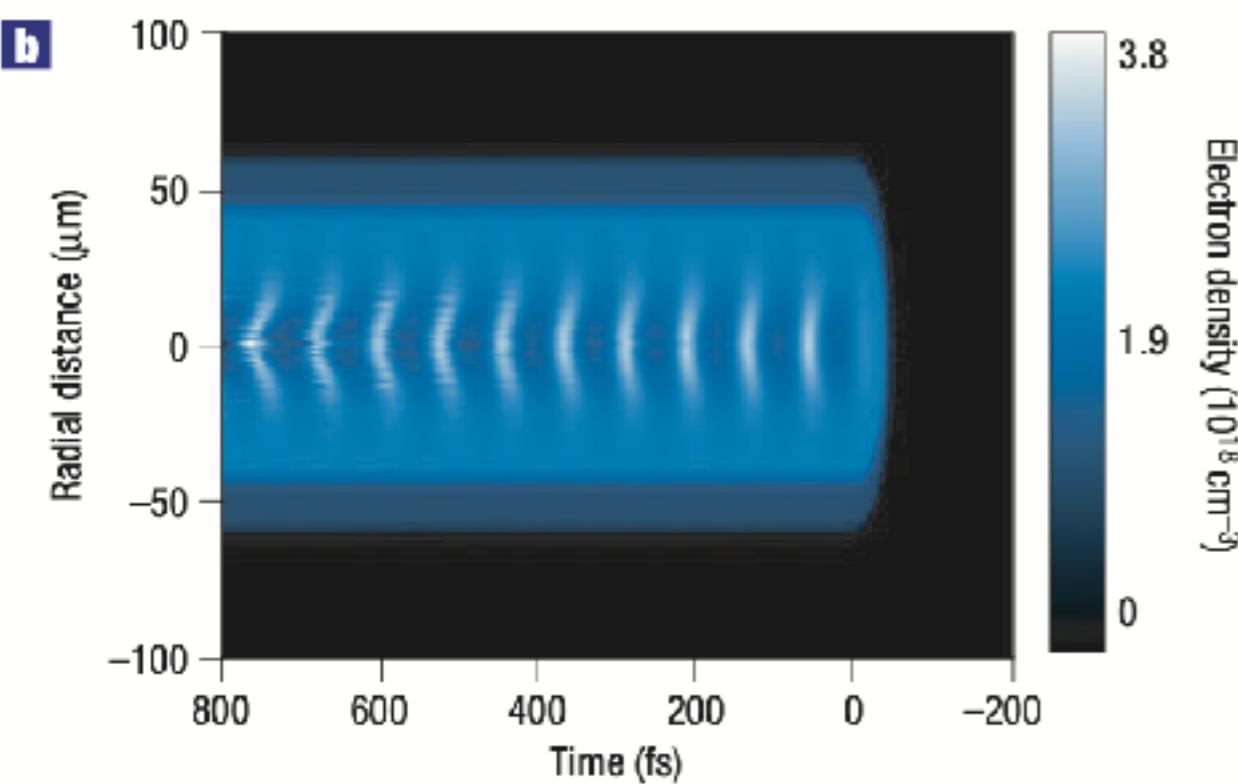
Plasma density:  
 $3 \cdot 10^{18} \text{ cm}^{-3}$

Plasma  
wavelength:  
 $\sim 20 \mu\text{m}$

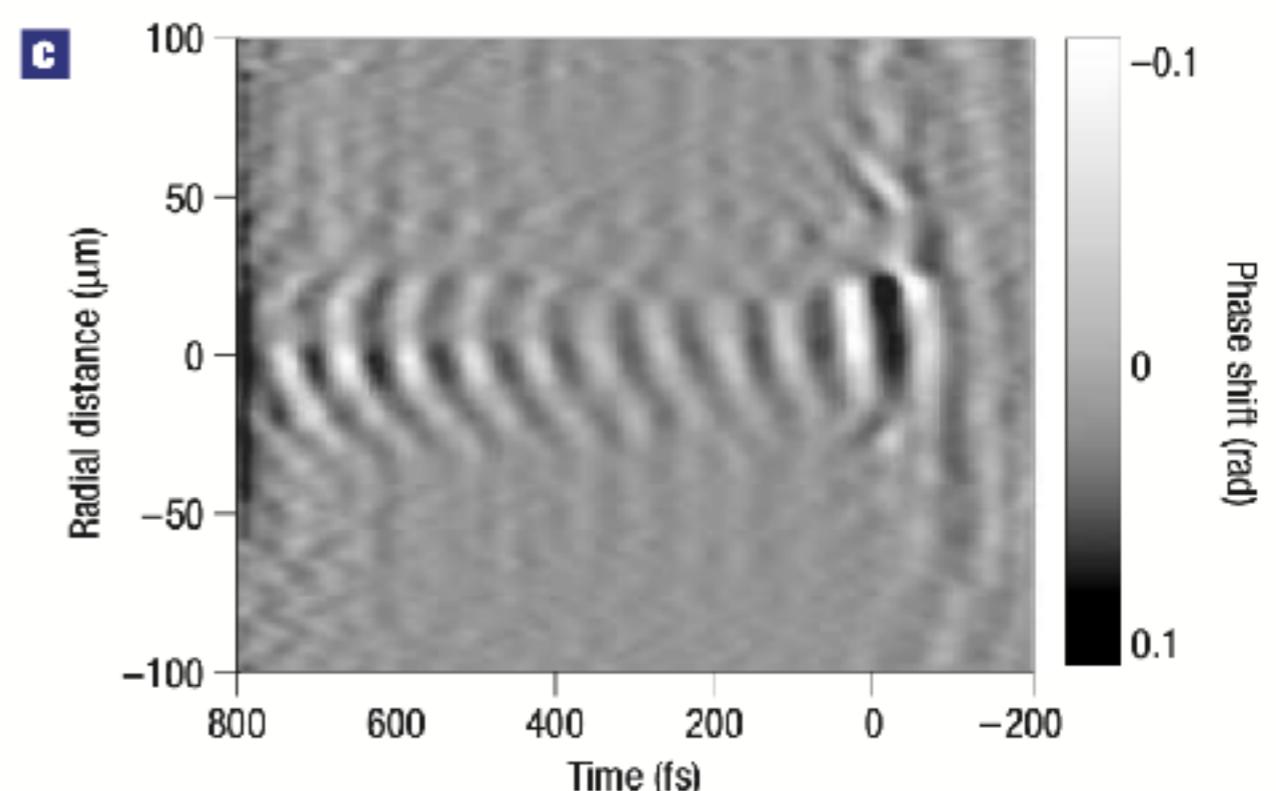


# Laser Wakefield Acceleration (LWFA): visualising the plasma waves

Simulation



Experiment

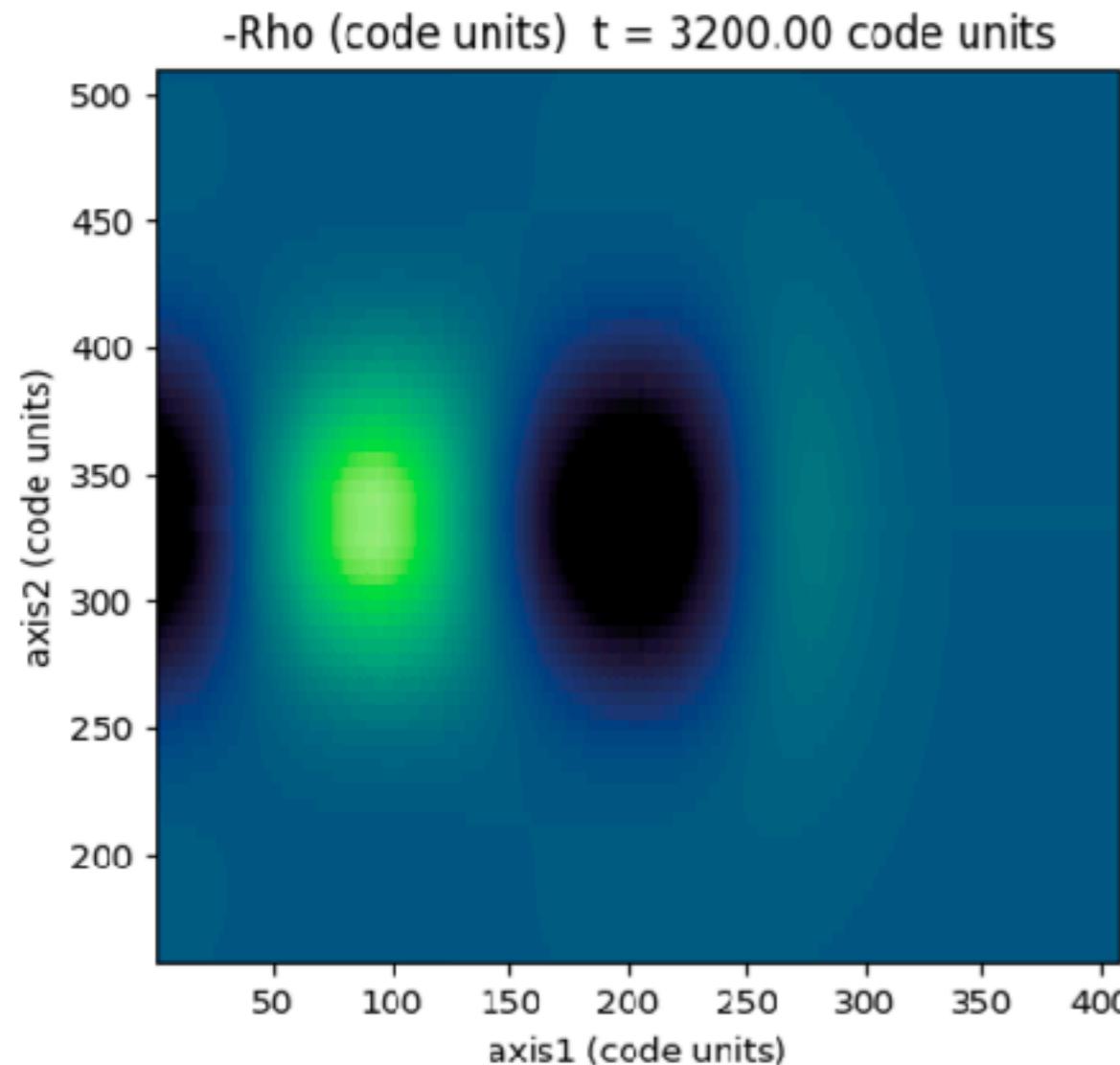


N. Matlis et al., Snapshots of laser wakefields, Nature Physics (2006)

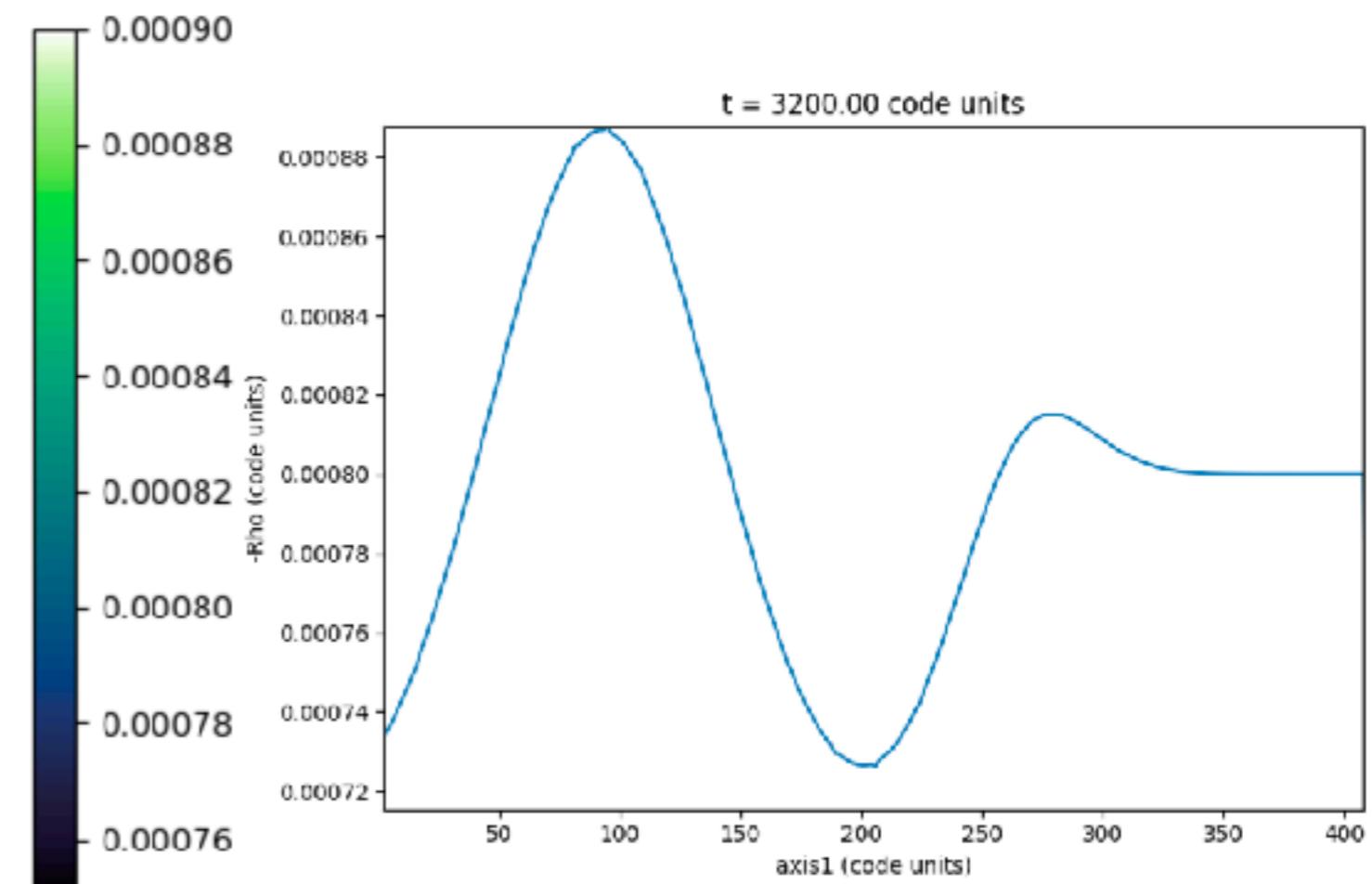
# Laser wakefield excitation: linear regime

$a_0$  = normalized laser peak field =  $0.86 \lambda_0[\mu\text{m}] (I [10^{18} \text{W/cm}^2])^{1/2}$

**$a_0 \ll 1$ : Linear regime**  
**Sinusoidal plasma waves at the plasma frequency**



2D charge density

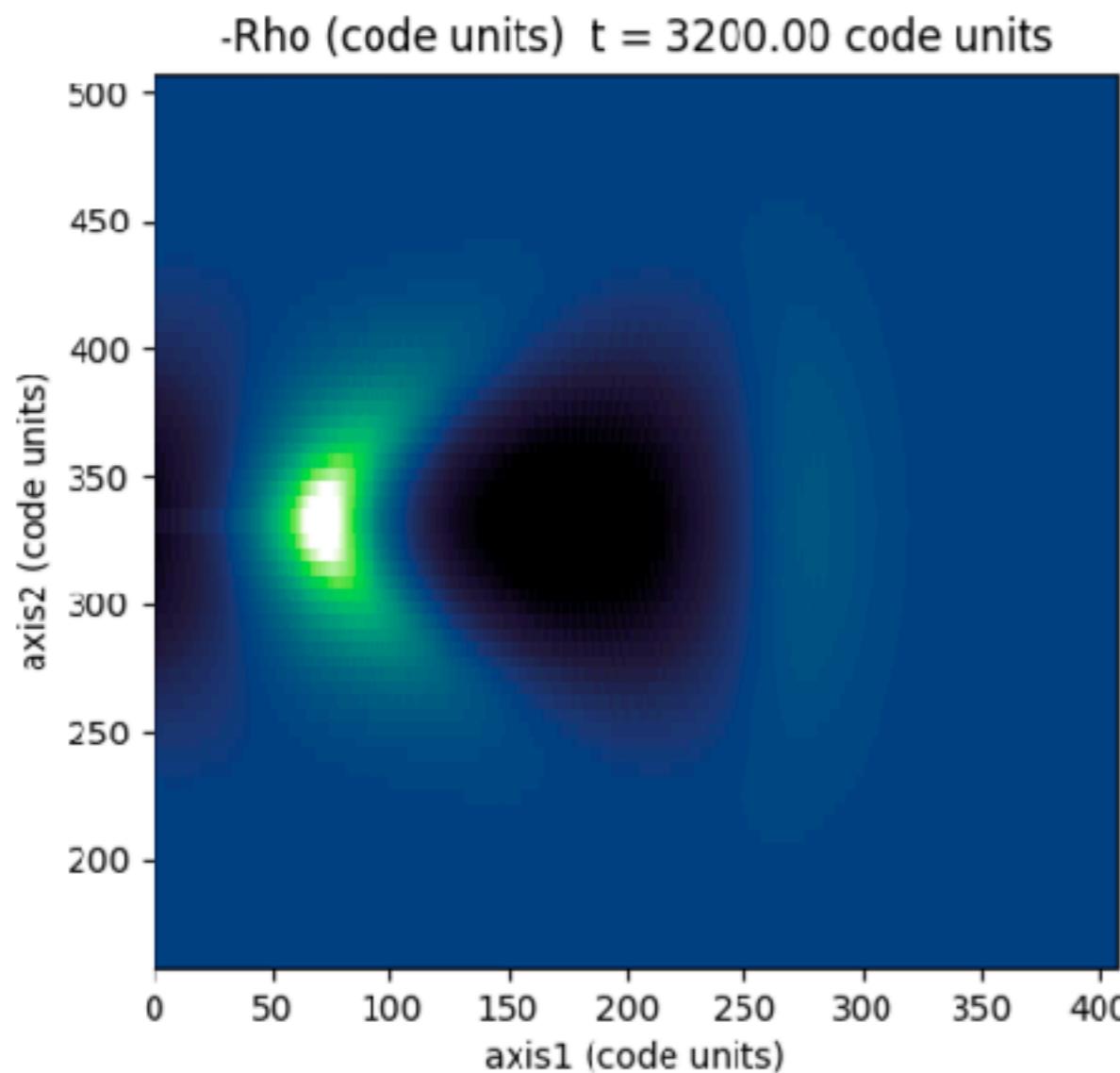


1D charge density  
on propagation axis

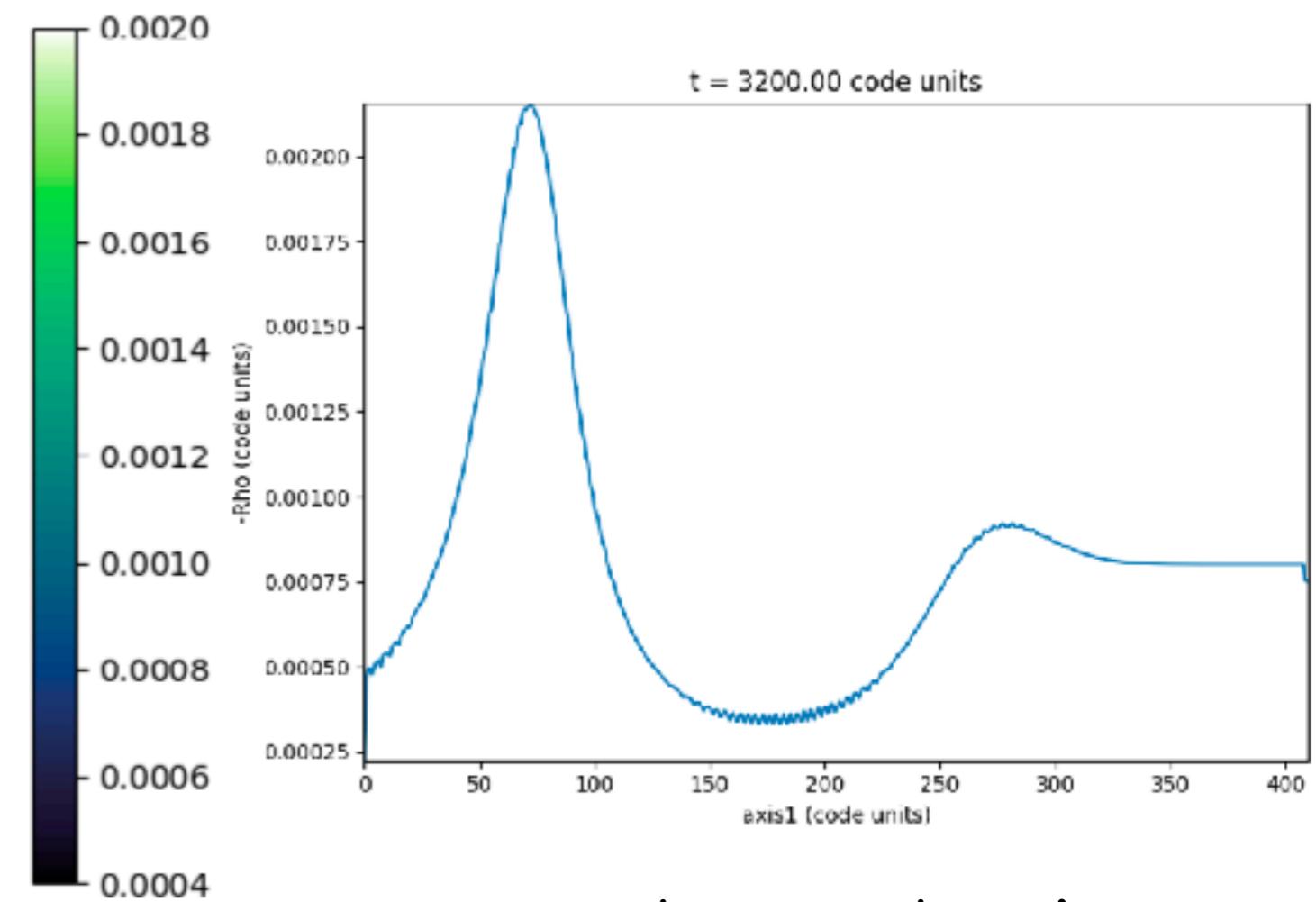
# Laser wakefield excitation: weakly linear regime

$a_0$  = normalized laser peak field =  $0.86 \lambda_0[\mu\text{m}] (I [10^{18} \text{W/cm}^2])^{1/2}$

$a_0 \lesssim 1$  : Weakly nonlinear regime



2D charge density

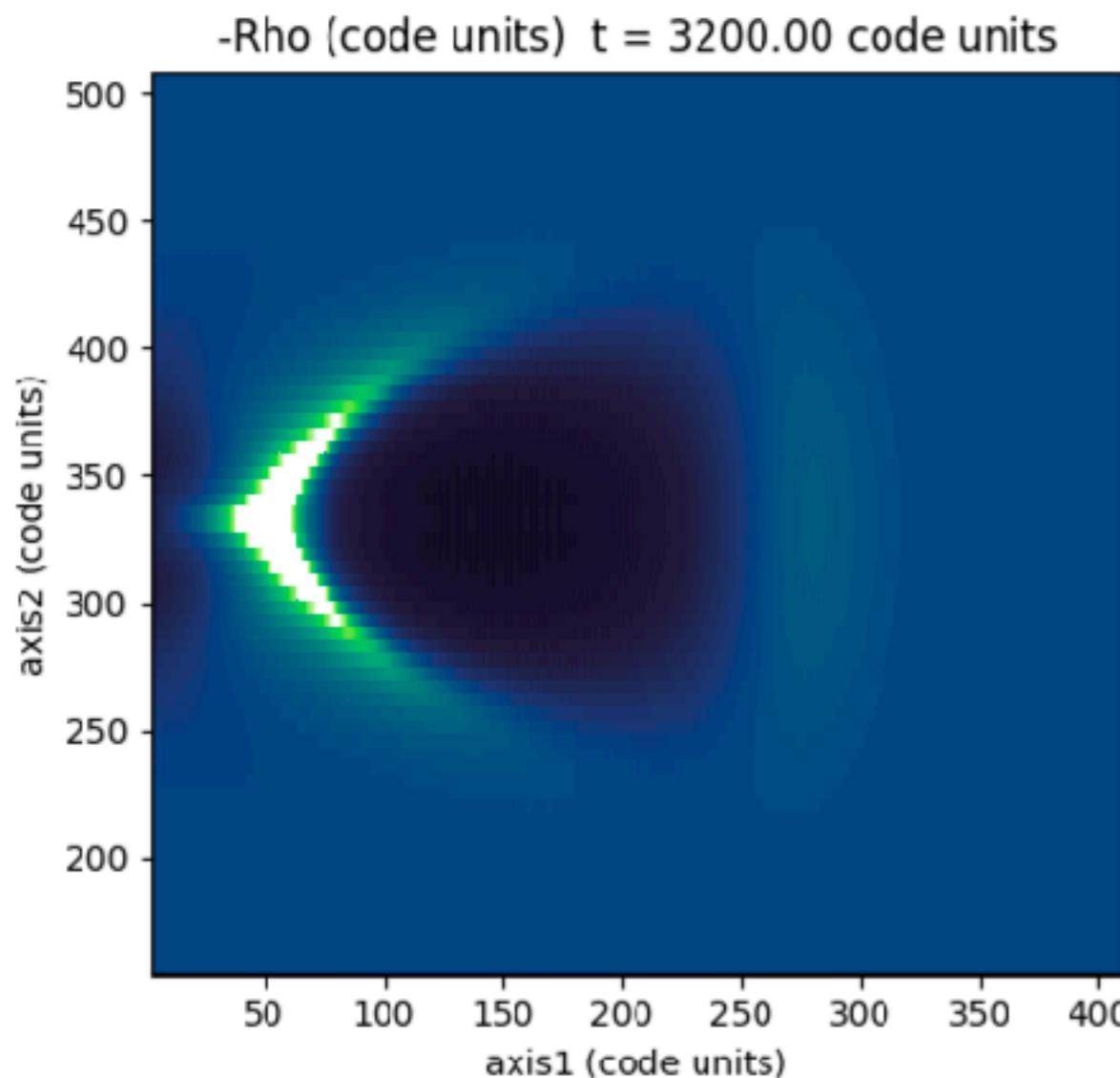


1D charge density  
on propagation axis

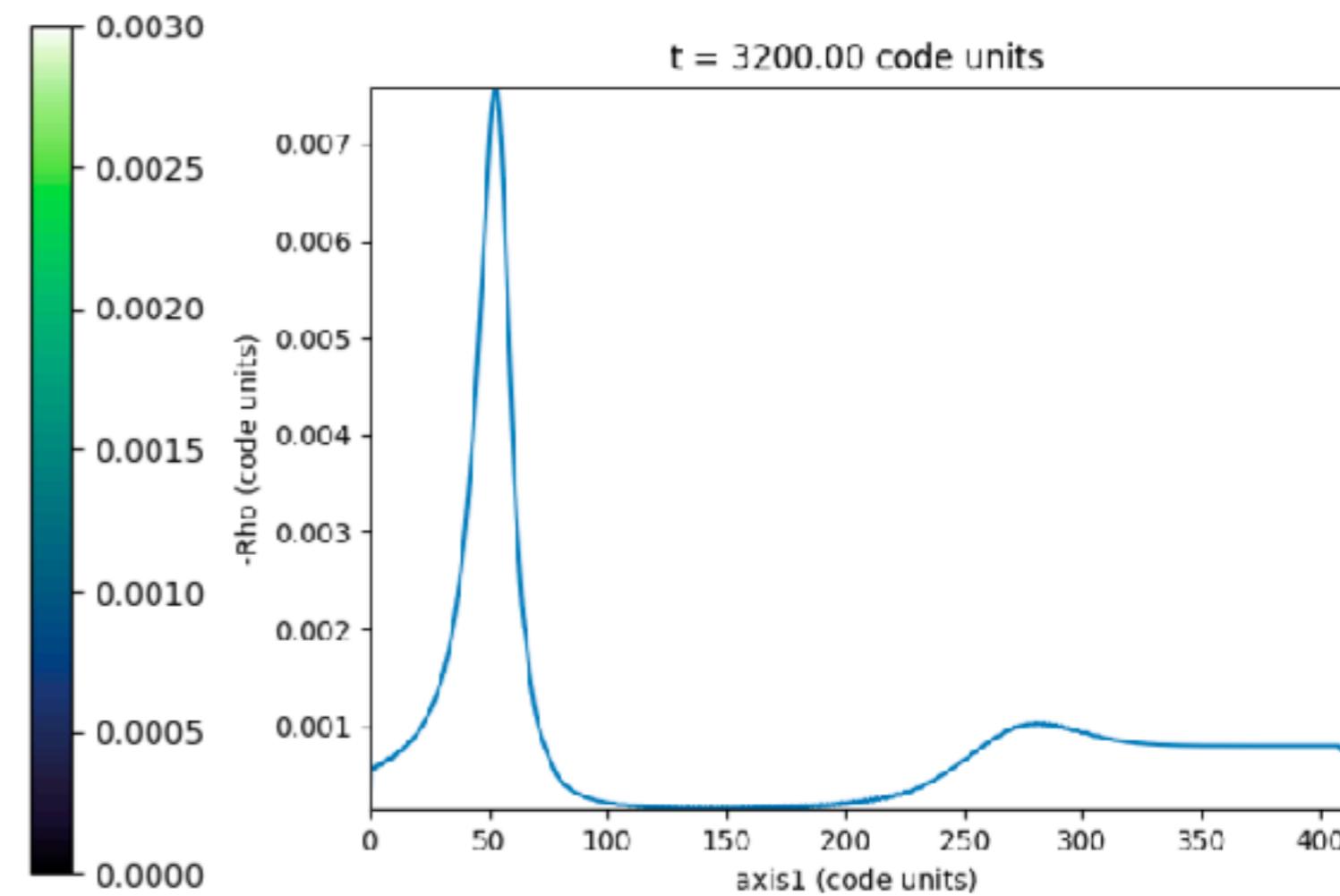
# Laser wakefield excitation: nonlinear regime

$a_0$  = normalized laser peak field =  $0.86 \lambda_0[\mu\text{m}] (I [10^{18} \text{W/cm}^2])^{1/2}$

**$a_0 > 1$ : Nonlinear regime  
“Bubble”-like waves**



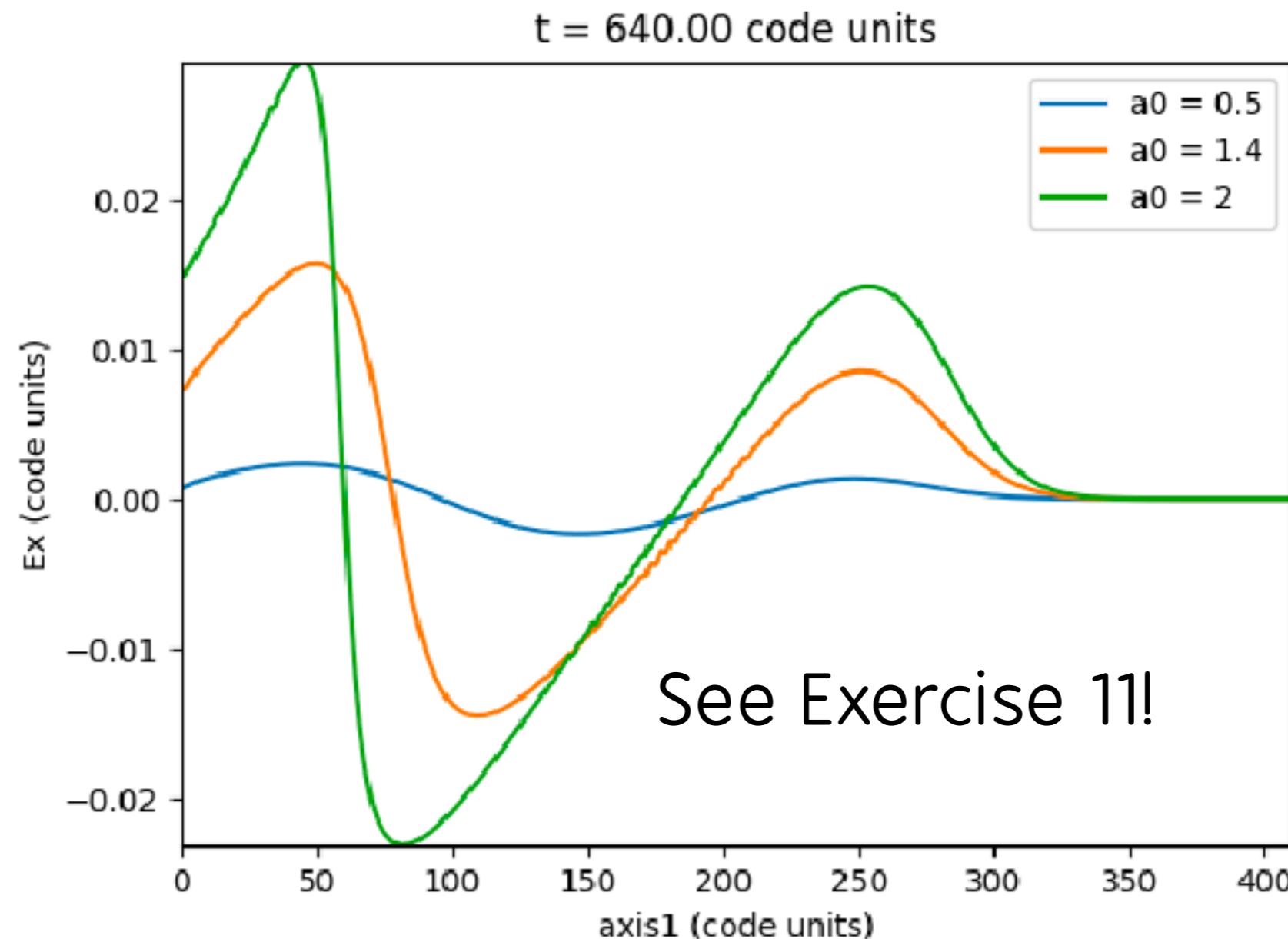
2D charge density



1D charge density  
on propagation axis

# Laser wakefield excitation regimes: Ex on axis

```
import happy
S1 = happy.Open("path/to/sim1"); Ex1 = S1.Probe.Probe0("Ex",timesteps=1000)
S2 = happy.Open("path/to/sim2"); Ex2 = S2.Probe.Probe0("Ex",timesteps=1000)
S3 = happy.Open("path/to/sim3"); Ex3 = S3.Probe.Probe0("Ex",timesteps=1000)
happy.multiPlot(Ex1,Ex2,Ex3,figure=3)
```

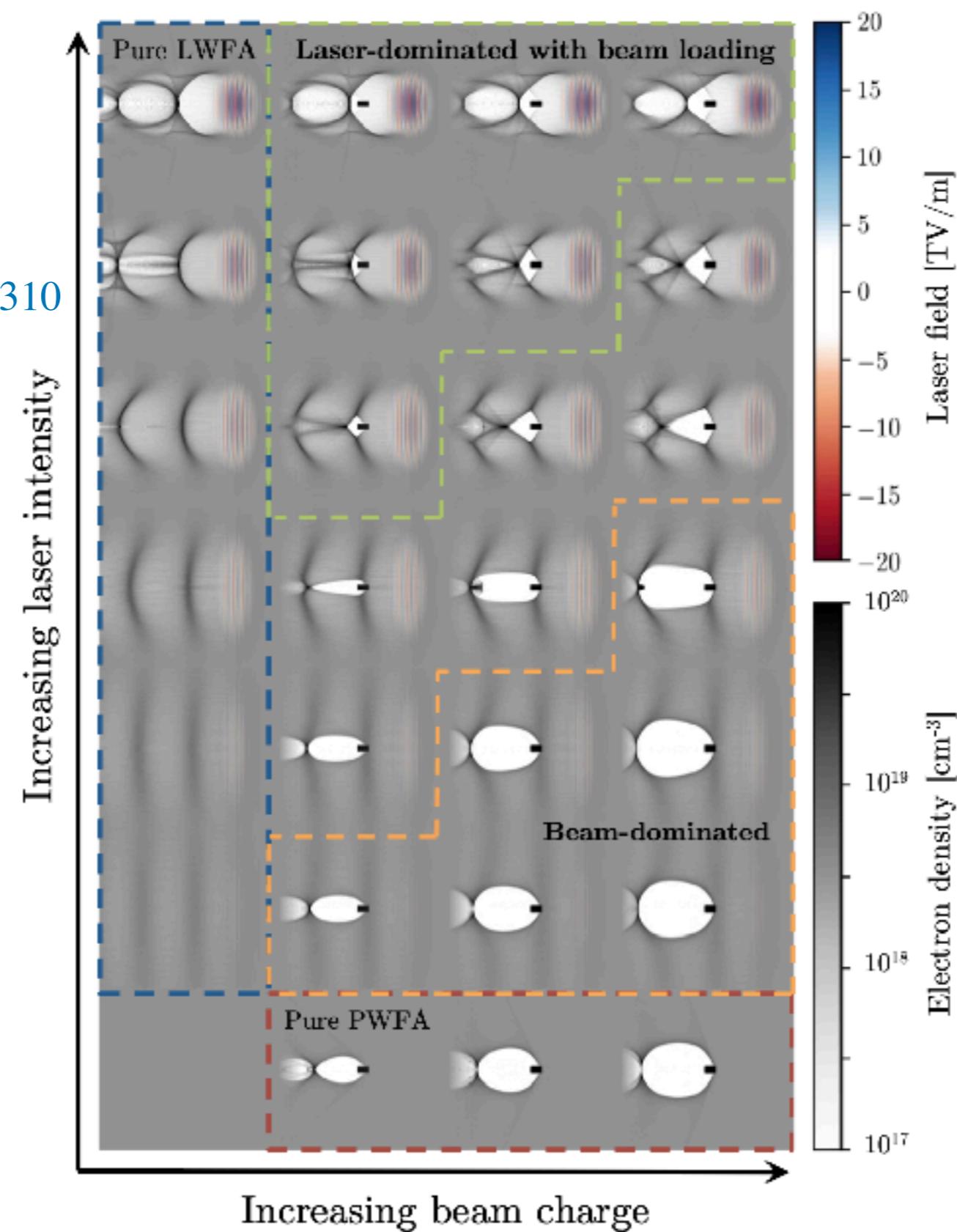


# Plasma wakefields can be driven by electron beams too

J. Götzfried et al.,

Phys. Rev. X (2020)

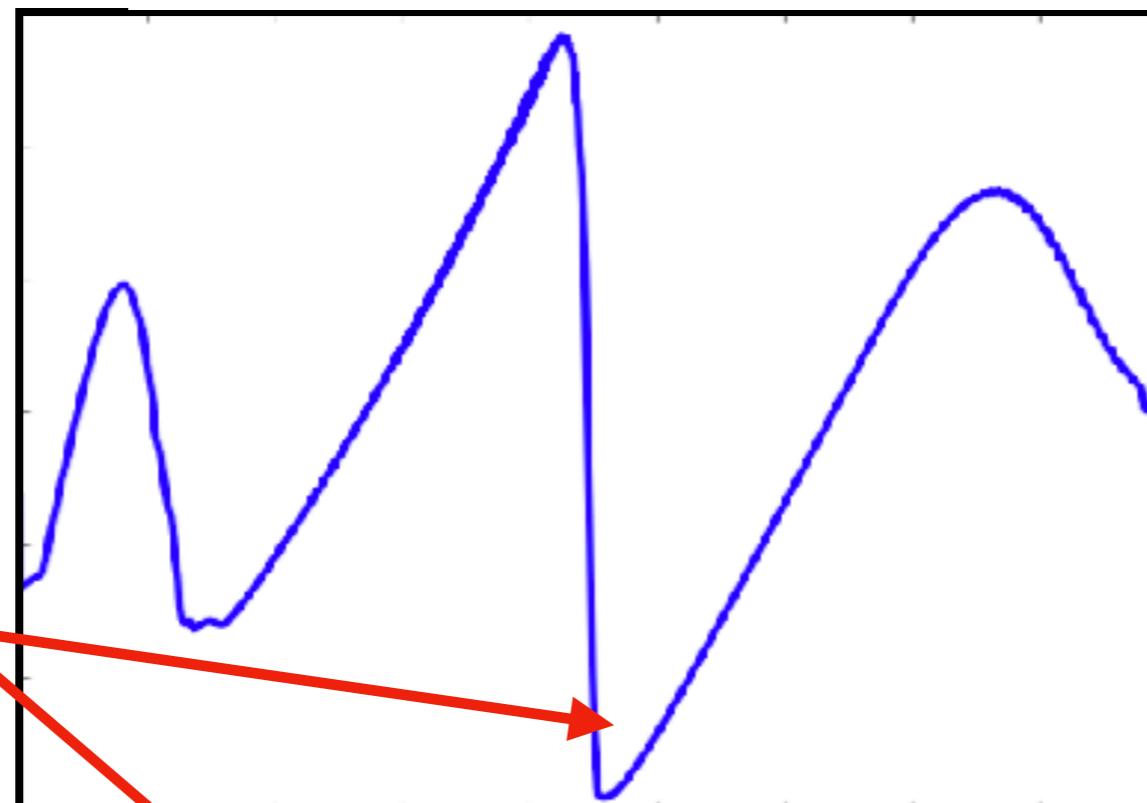
<https://arxiv.org/pdf/2004.10310>



# Laser Wakefield Acceleration (LWFA): accelerating electric field

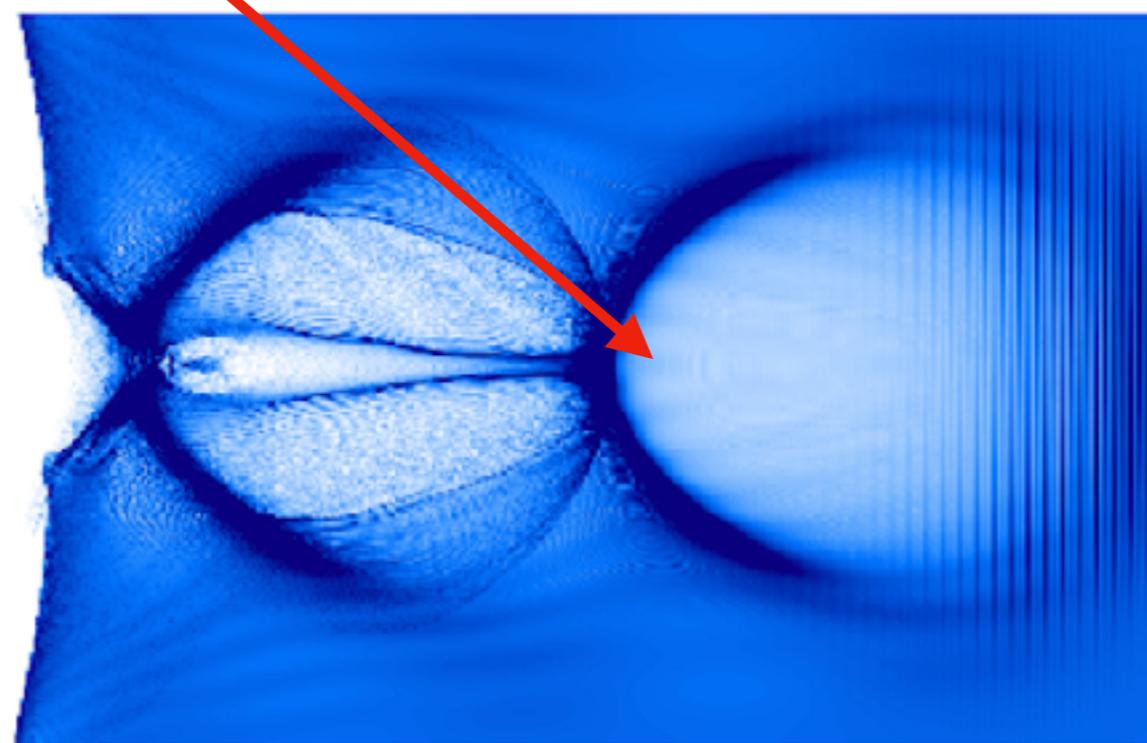
$E > 100 \text{ GV/m}$

Relativistic electrons injected here are accelerated towards the right



Longitudinal Electric Field (A.U.)  
on propagation axis

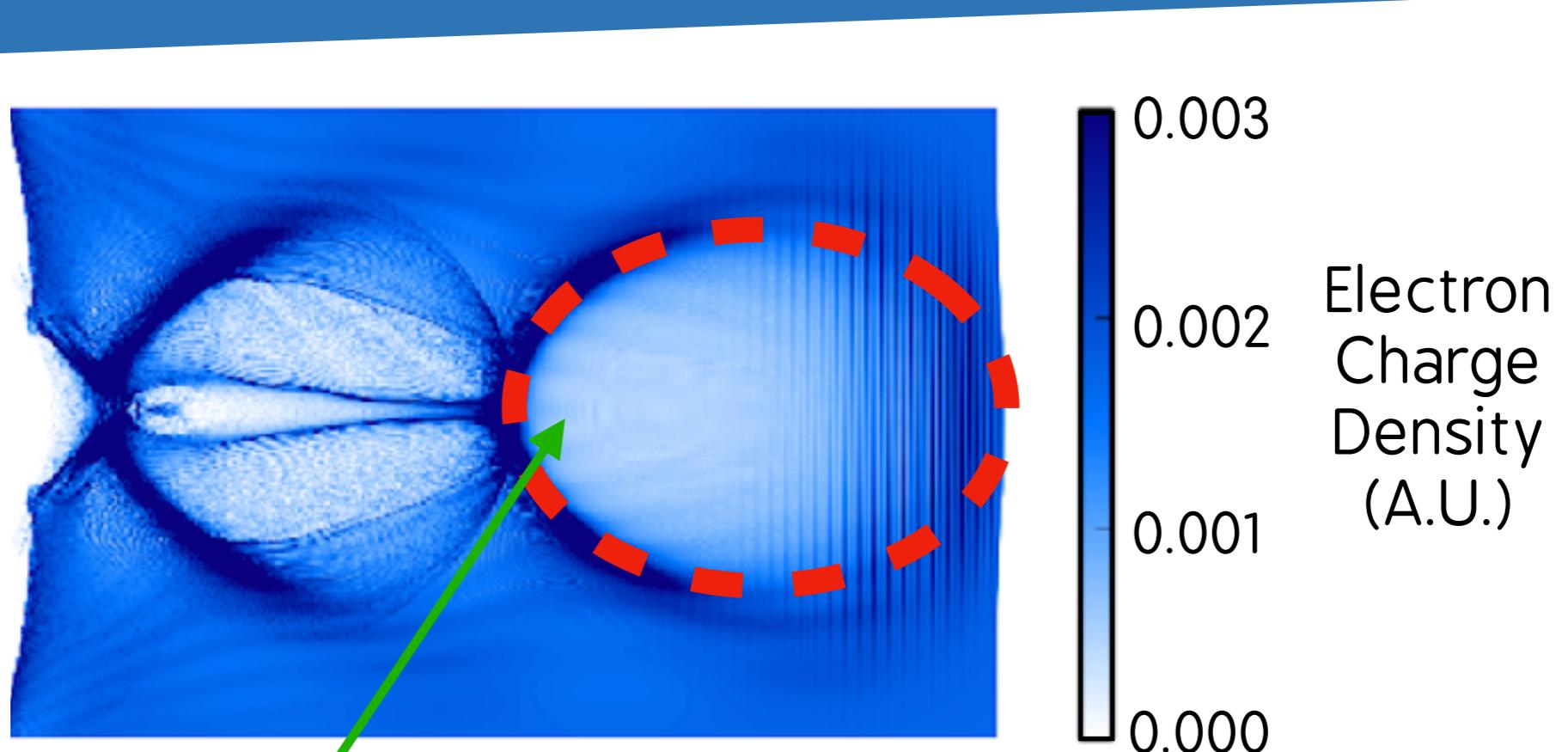
Nonlinear regime here



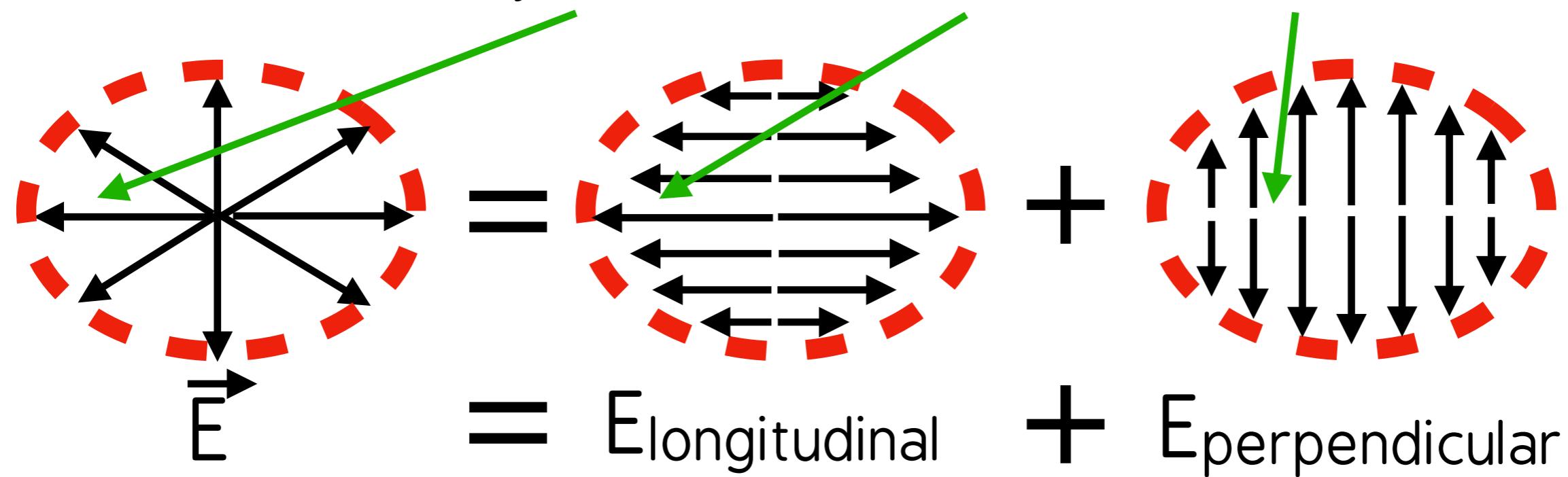
0.000  
0.001  
0.002  
0.003  
Electron Charge Density (A.U.)

# LWFA: Electric fields inside the “bubble”

Plasma wavelength:  
~20 μm

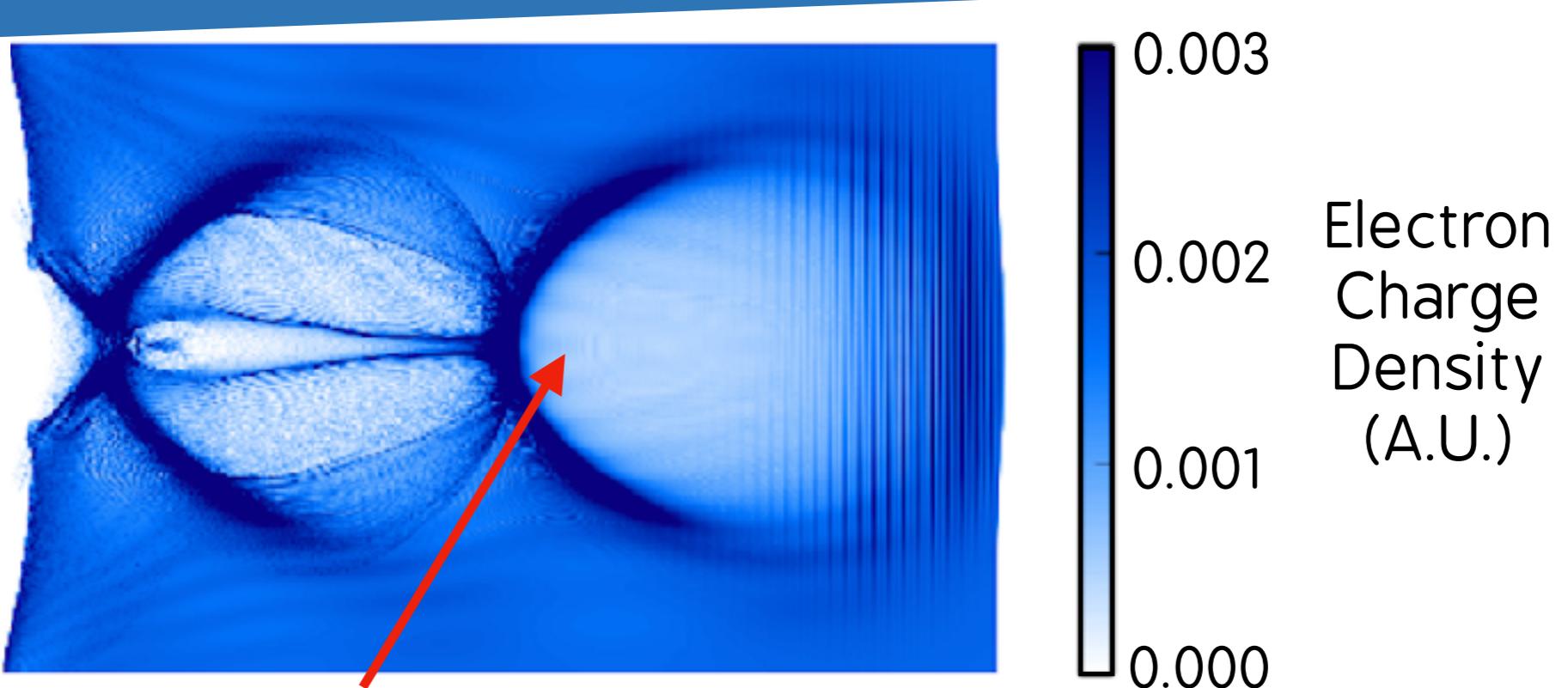


Relativistic electrons injected here are both accelerated and focused

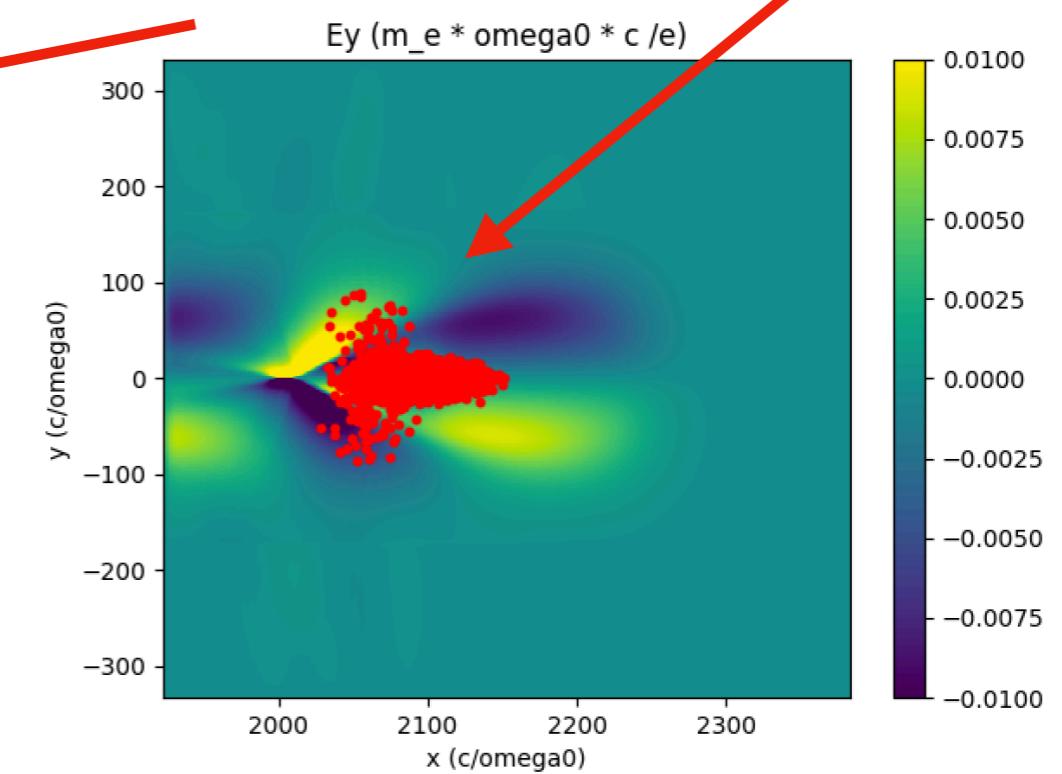
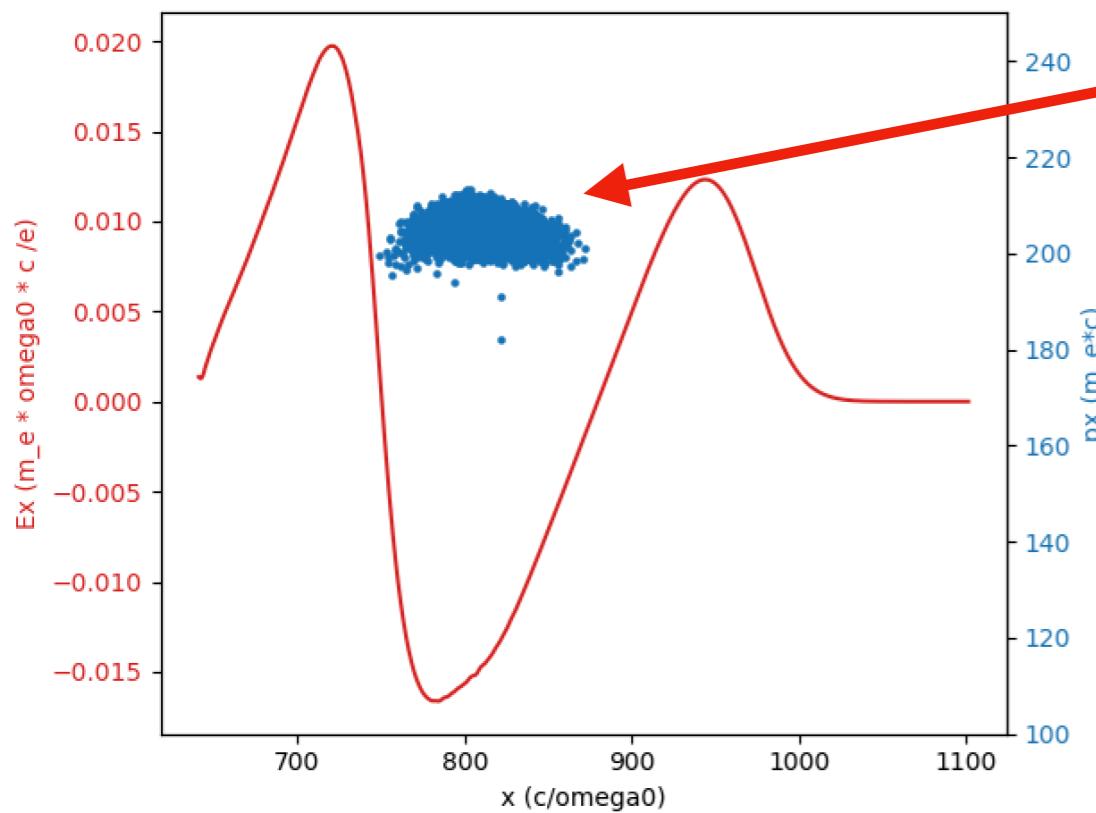


# Fields inside the “bubble”: effects on an electron beam

Plasma wavelength:  
 $\sim 20 \mu\text{m}$



Relativistic electrons injected **here** are both **accelerated** and **focused**

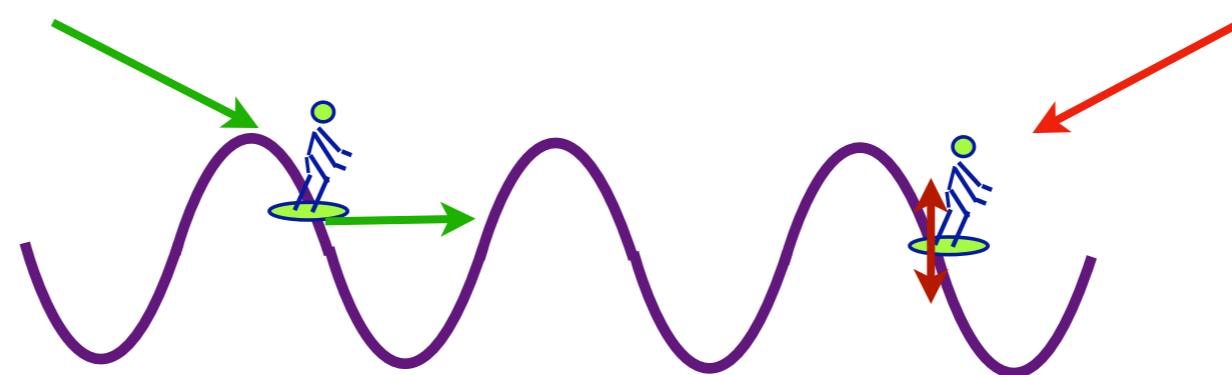


# Laser Wakefield Acceleration (LWFA) challenges: injection

## Surfer with

- sufficient initial speed,
- injected in the proper phase

## Surfer with Zero initial speed



**Surfer: electron**

**wave: electric field of the plasma wave in the wake of the laser**

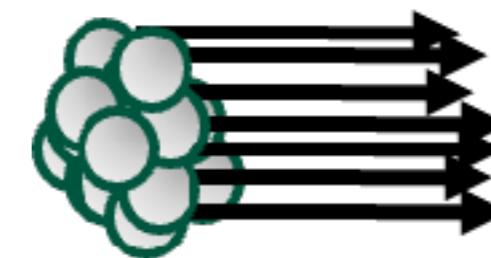
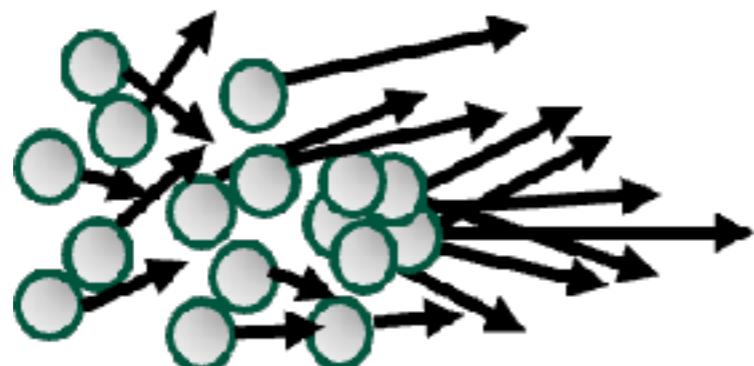
Plasma wavelength  $\sim 10\text{s } \mu\text{m}$ , Duration of the electron beam  $< \sim 10 \text{ fs} \sim 3 \text{ } \mu\text{m}$

# Laser Wakefield Acceleration with ionization injection



Plasma wave  
Intense Laser Pulse  
High energy electrons

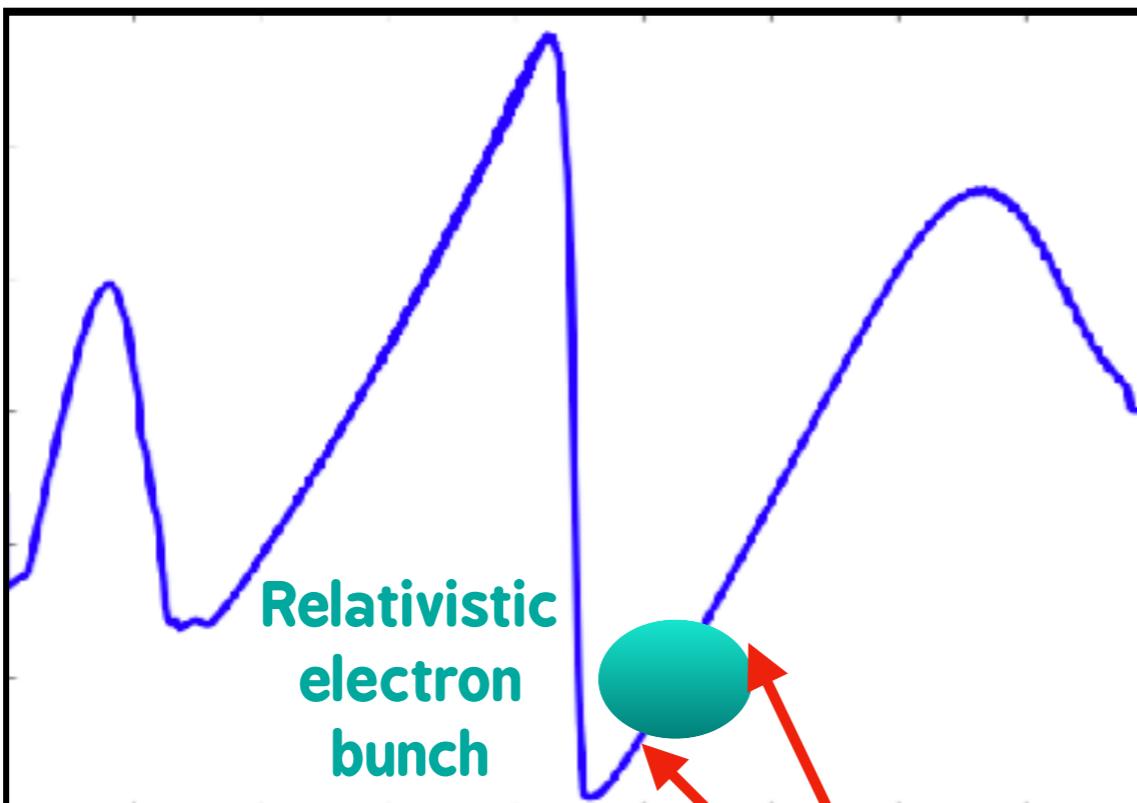
# LWFA objective (not the only one): realize compact electron accelerators for applications



- Few electrons or many dispersed in space
- High divergence
- Different Energies (high energy spread)
- Many electrons in a small volume
- Low divergence
- Similar Energies (low energy spread)



# LWFA challenges: reduce the energy spread



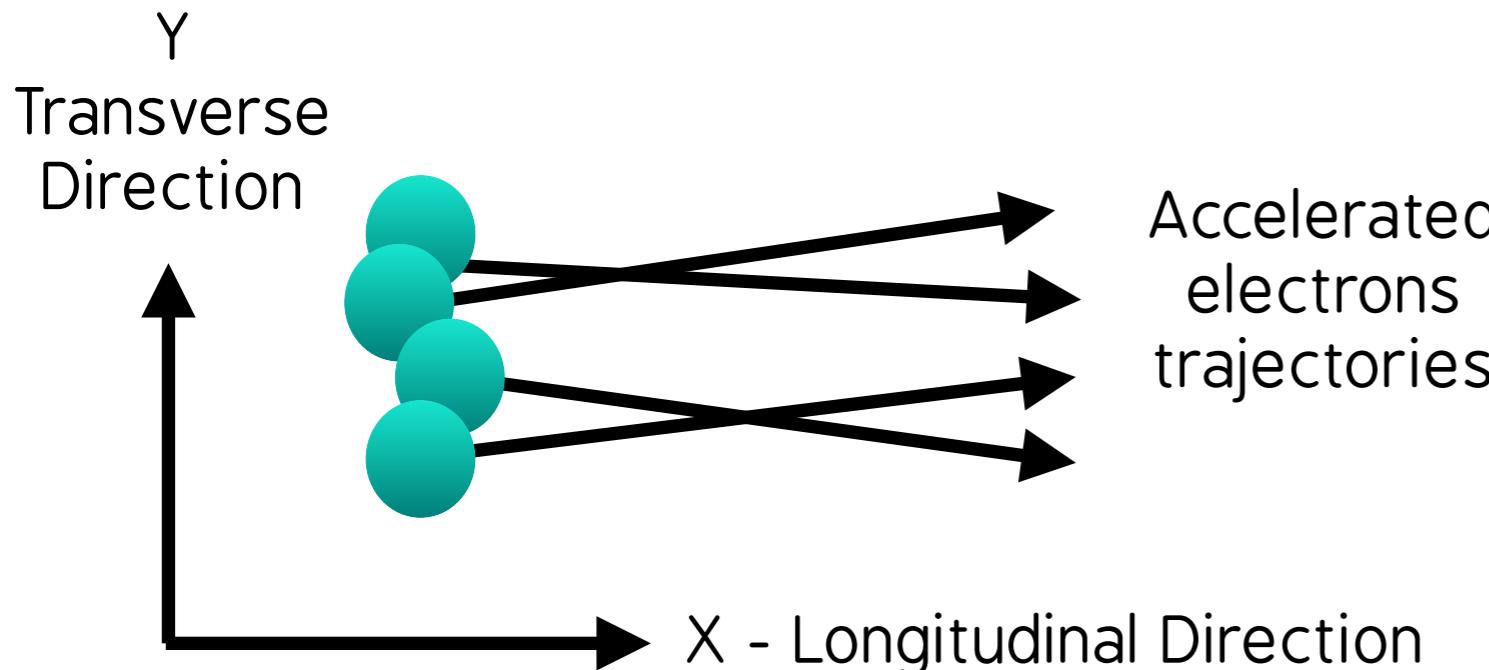
Longitudinal  
Electric Field  
(A.U.)  
on propagation axis  
(negligible beam loading)

Electrons injected  
in different phases of the wave  
will experience a different accelerating field



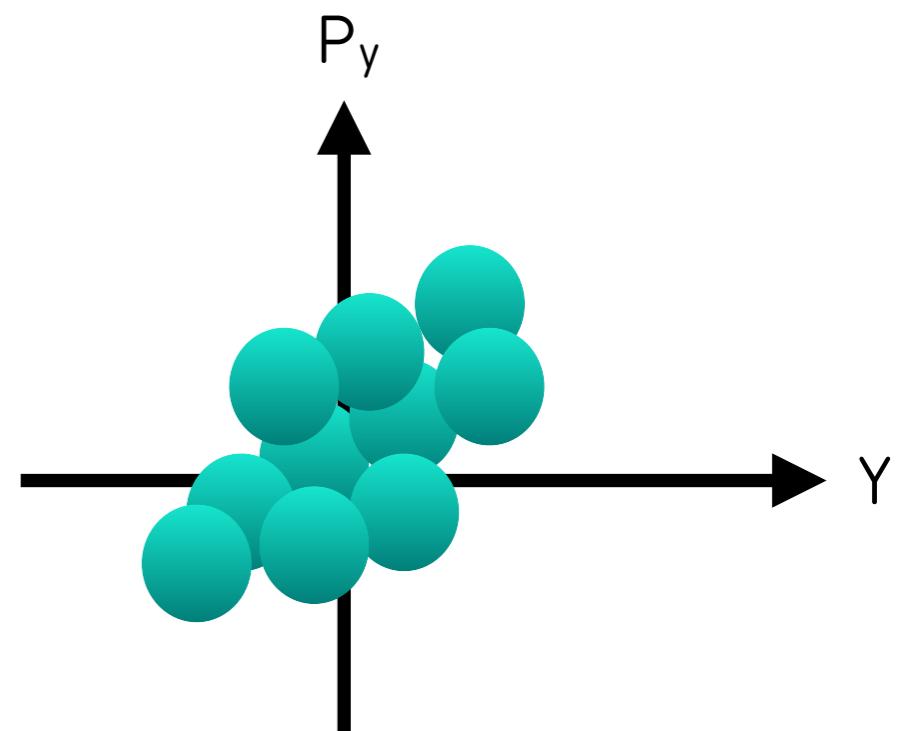
Energy spread increase

# LWFA challenges: lowering the emittance



Accelerated  
electrons  
trajectories

Bunch distribution  
in the transverse phase space



Most applications of accelerated beams require:

- Small transverse size (i.e. small  $\sigma_y$ )
- Small divergence [i.e. small  $\sigma_{\arctan(p_y/p_x)}$ ]



**Minimize  
Transverse Emittance**

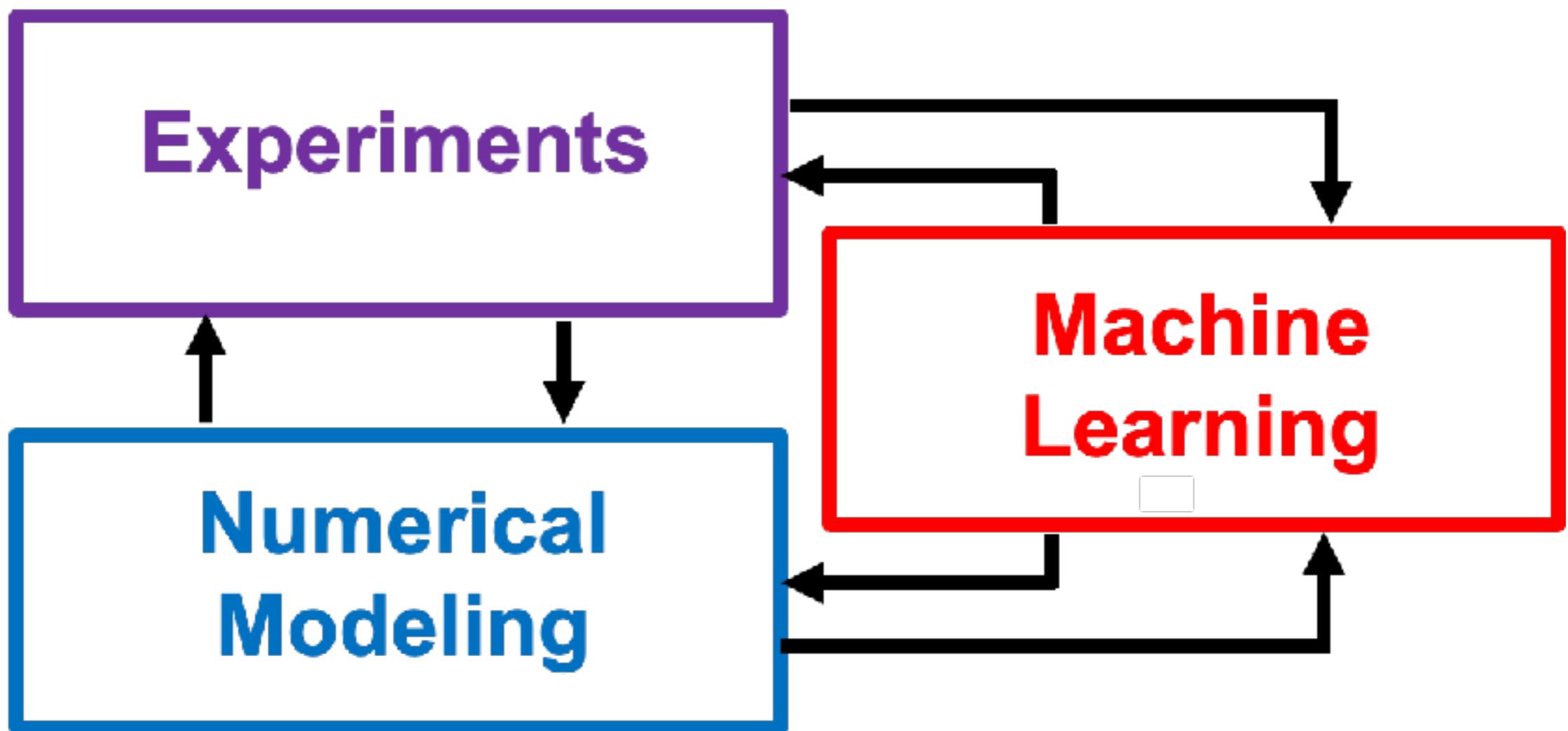
**Transverse  
Normalized  
Emittance**

$$\epsilon_{ny} = \frac{1}{m_e c^2} \sqrt{\sigma_y^2 \sigma_{p_y}^2 - \sigma_{y p_y}^2}$$

# Outline

- Why plasma accelerators?
- Basics of laser wakefield acceleration
- Numerical simulation of plasma acceleration: PIC codes
- Introduction to the case study and the practical

# LWFA investigation techniques



# Why numerical simulation is necessary for LWFA?

- **No 3D analytical solutions are available for the general case**
- **You cannot measure everything simultaneously**
- **Simulations before the experiment:**
  - Study new physical phenomena
  - Conceive new kind of experiments
  - Design experiments (also using Machine Learning)
- **Simulations after the experiment:**
  - Analyze the data (also using Machine Learning)
  - Understand the physics

# Model for Laser-Plasma Interaction in LWFA

## Complete Maxwell-Vlasov system

Plasma distribution function

$f(x, y, z, p_x, p_y, p_z, t) \longrightarrow \text{6 dimensions + time!}$

Coupled to the electromagnetic fields  $\longrightarrow$  **Non-linearity**

$$\frac{\partial f}{\partial t} + \beta c \cdot \nabla_x f - e(E + \beta c \times B) \cdot \nabla_p f = 0 \quad \leftarrow \text{Collisions neglected}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad \begin{matrix} \text{Current density} \\ J \text{ of the plasma} \end{matrix}$$

$$\nabla \times B = -\mu_0 ec \left( \int \beta f d^3 p \right) + \frac{1}{c^2} \frac{\partial E}{\partial t}$$

Explorable Physics:

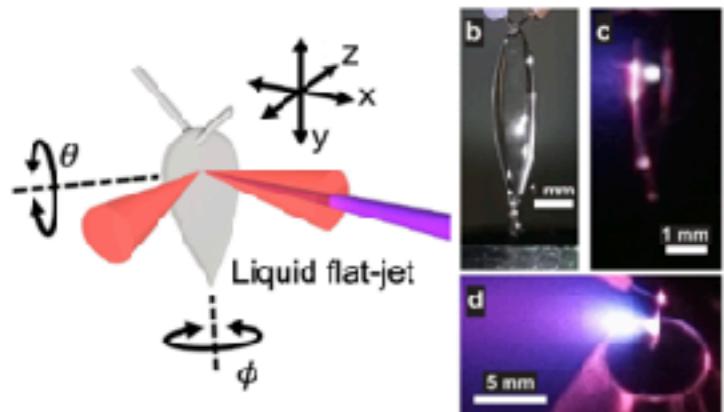
- Relativistic optics (laser self-focusing, ...)
- Nonlinear phenomena
- Wave-breaking
- Injection and acceleration of particles

## Approximate solution: Particle in Cell method

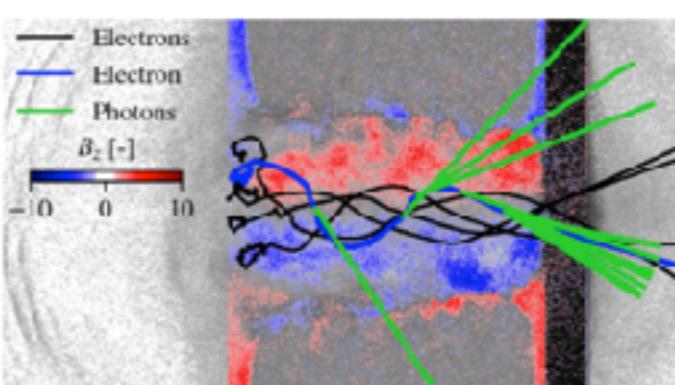
# Particle in Cell (PIC) method : essential kinetic plasma investigation technique

## Applications From Laboratory plasmas to Space and Astrophysical plasmas

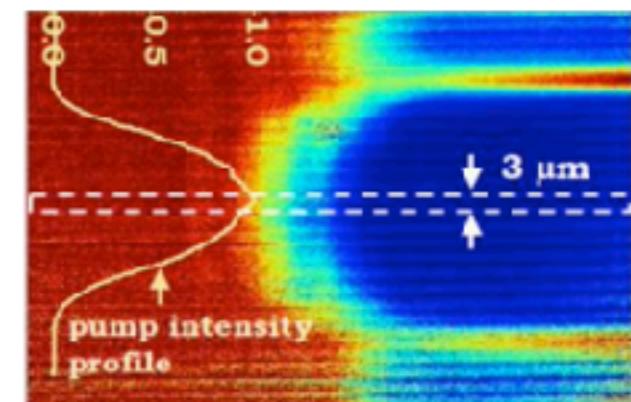
High-Harmonic Generation



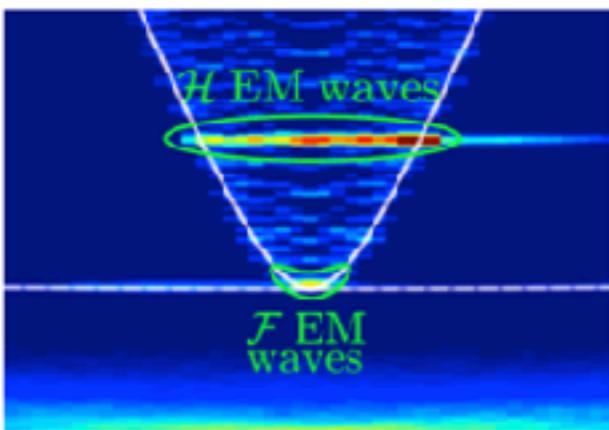
Radiation sources from DLT



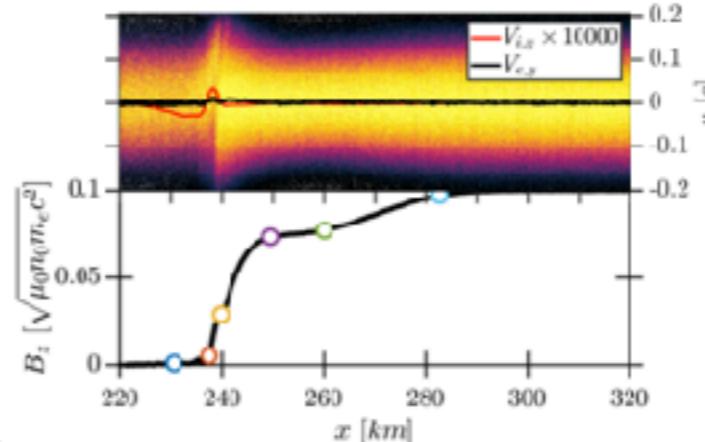
Solid to plasma transition



Solar radio-burst



Comet boundaries



Collisionless shocks & Dark Matter



- Wide range of physics applications
- Conceptually simple
- Efficiently implemented on small or massively parallel supercomputers

Some studies performed with **Smilei**)

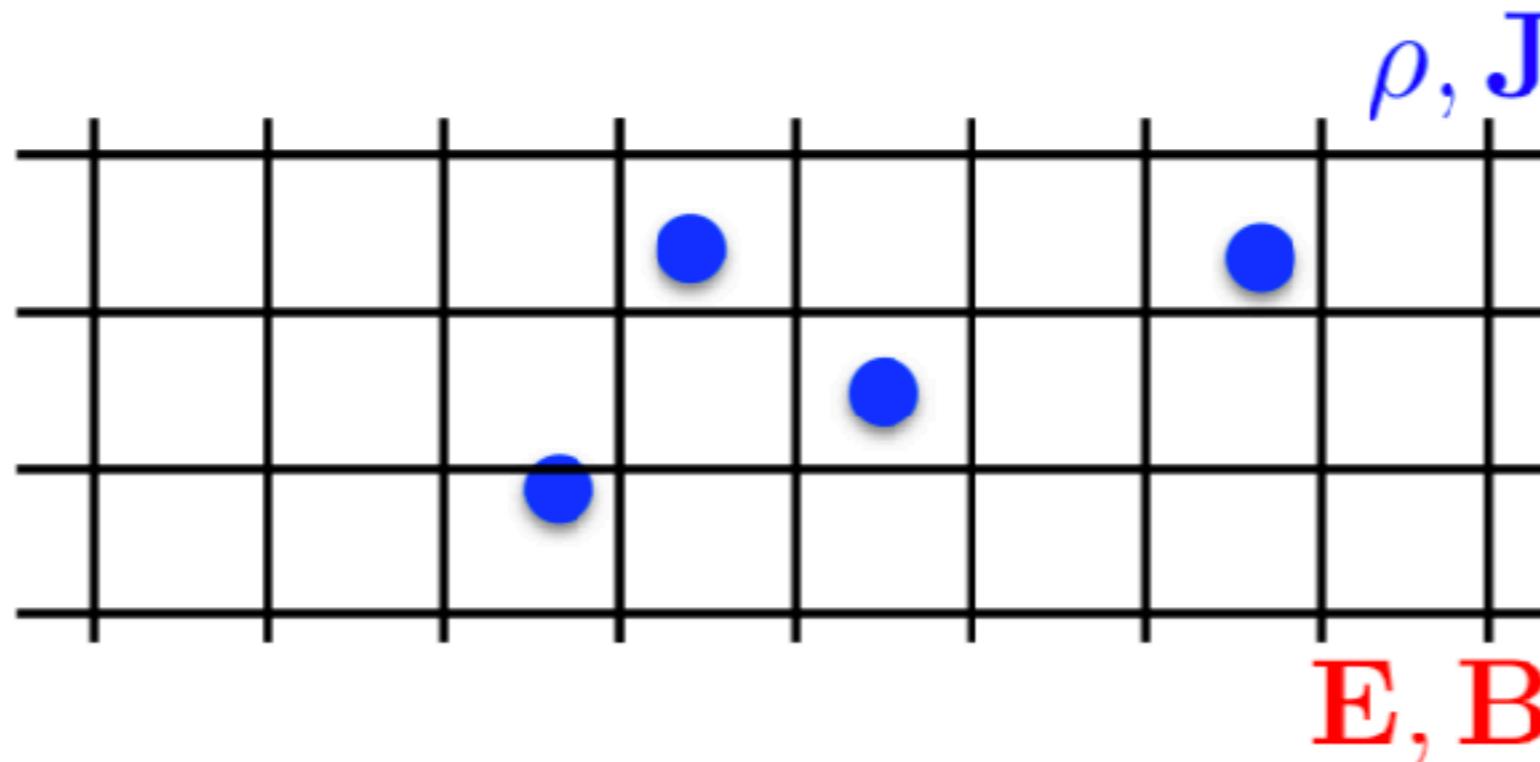
# Particle in Cell concept

Sample Plasma with Macro-Particles  
(1 Macroparticle = position, momentum, charge, ...)

+

Discretize space with computational grid

Define  $E, B, \rho, J$  on the grid cells



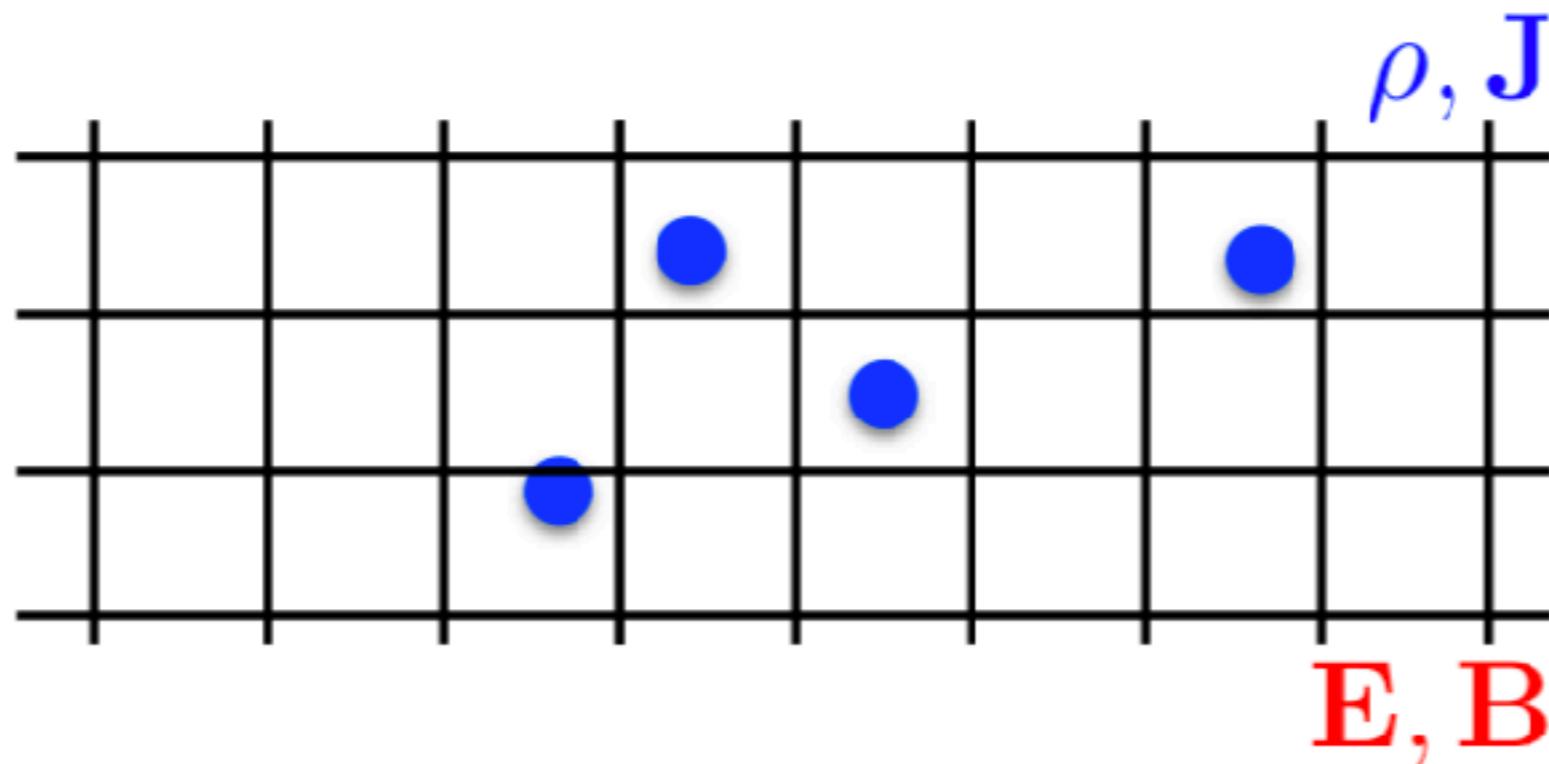
# Particle in Cell concept

## **Macro-Particles:**

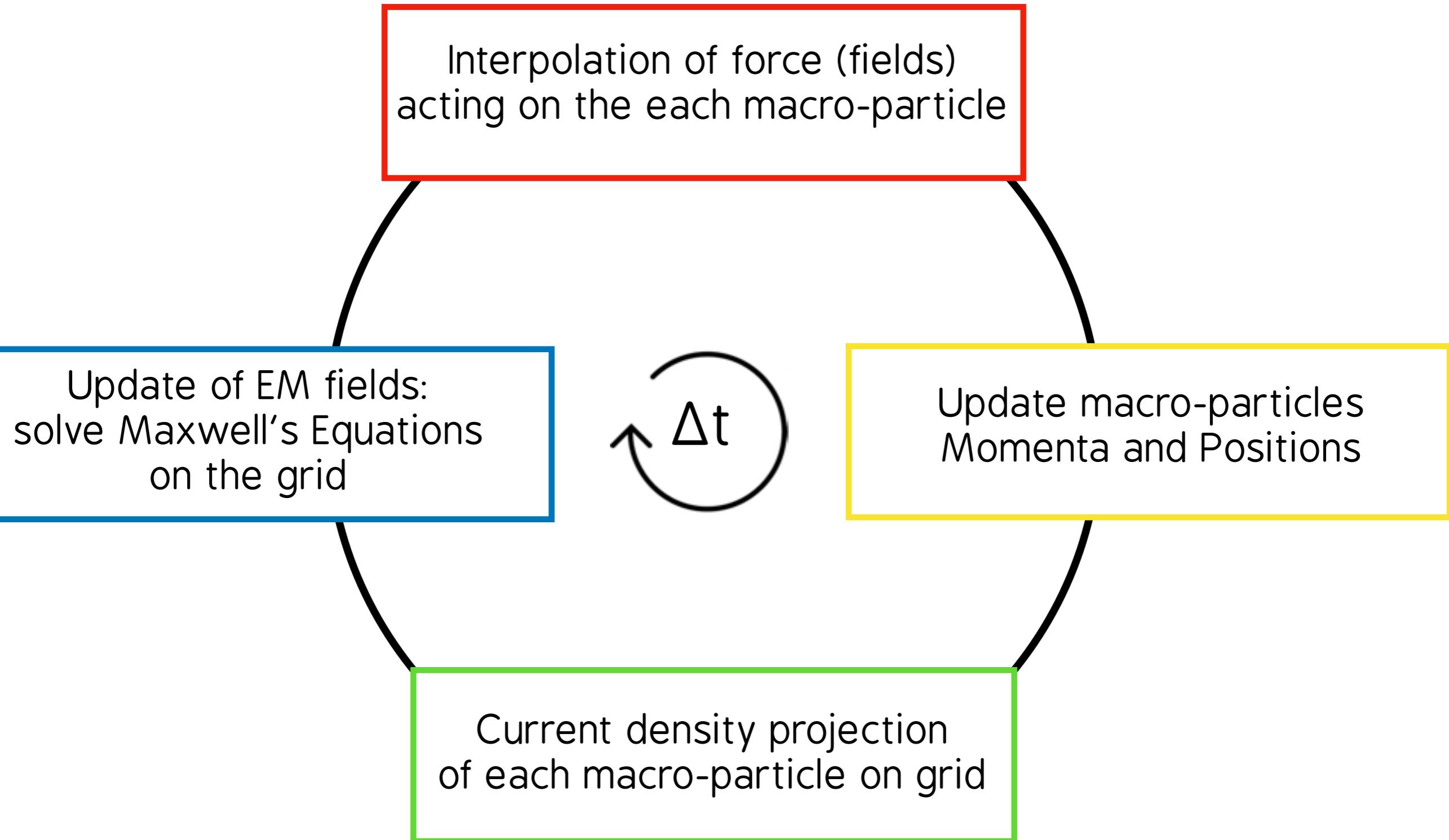
evolve following the characteristics of Vlasov Equation  
(In the PIC method they look like equations of motion)

+

**Electromagnetic Fields:** evolve following Maxwell's Equations



# Particle in Cell modelling is self-consistent



# Outline

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# Smilei)

## User-friendly

- online: documentation, tutorials
- Python input / output
- quick visualisation library
- teaching platform
- bi-annual training workshop
- Element chat with the developers

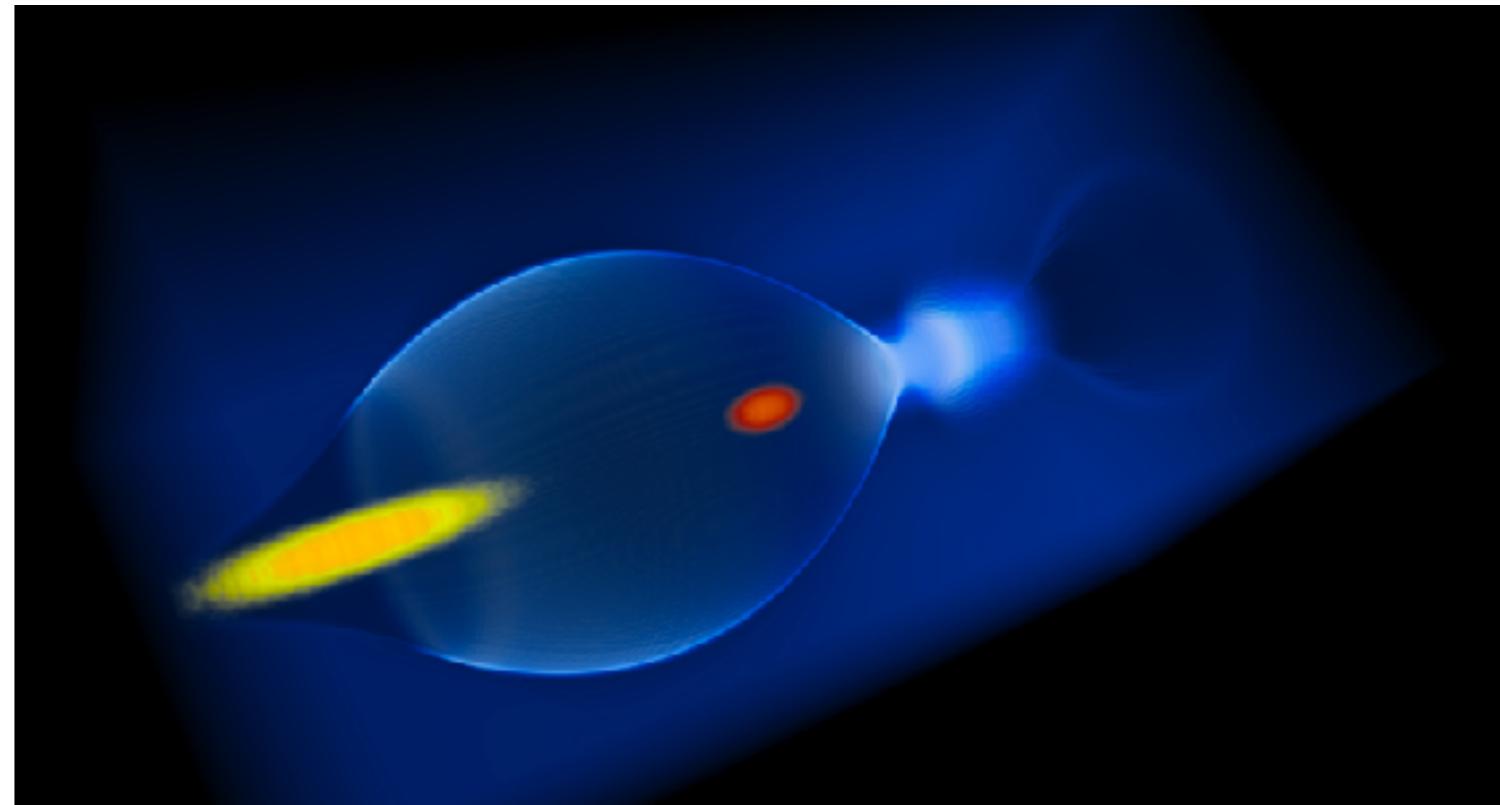
## High Performance

- MPI + OpenMP parallelization
- dynamic load balancing
- adaptive vectorization
- GPU computing on both NVIDIA and AMD architectures

## Multi-physics

- 1D, 2D, 3D, quasi-3D geometries
- ionization, collisions, strong-field QED
- laser envelope model
- relativistic beam field initialization

A collaborative, open source  
multi-purpose Particle in Cell code  
<https://smileipic.github.io/Smilei/index.html>



## High quality

- developers: experts of physics and HPC
- continuously benchmarked
- GitHub bug reporting
- OpenPMD standard
- >200 publications using the code up to 2024

# Additional material

**Extensive Documentation  
( Installation, Use, Postprocessing, ...)**



<https://smileipic.github.io/Smilei/index.html>

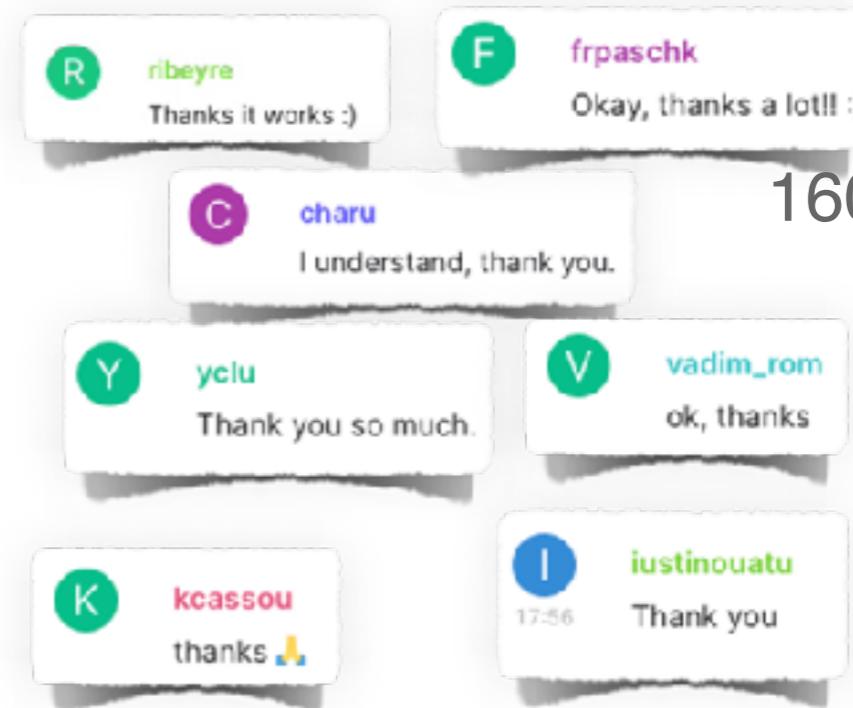
**Presentations from the  
4th User & Training Workshop at ELI-Beamlines**



<https://smileipic.github.io/tutorials/>

<https://indico.math.cnrs.fr/event/9577/>

**Questions?  
We answer on the Element chat**



1600 messages  
in 2021

# Next workshop: March 2025 (Madrid)

<https://indico.math.cnrs.fr/event/13081/>

19–21 Mar 2025 @ ETSIAE (U. P. Madrid) Europe/Madrid timezone

Overview

Venue and transportation

Registration

Call for Abstracts

Contact

 [frederic.perez@polytech...](mailto:frederic.perez@polytech...)

For its 5th iteration, the *Smilei* user & training workshop is **hosted in Madrid** (Spain) by **ETSIAE** from March 19th to 21st, 2025. You will have the opportunity to present your research, review the latest news on Smilei, and for newcomers, learn how to use Smilei on a supercomputer. Coffee and lunches will be provided on site.

**There is no participation fee, but registration is mandatory.**

## Programme

- 1st day: Project status by the Smilei team + presentations from users
- 2nd and 3rd days: Courses and hands-on training sessions with access to a supercomputer (limited to 30 participants)

# Outline

- Why plasma accelerators?
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# Example of input file

```
Main(  
    geometry      = "1Dcartesian",  
    timestep      = 0.009,  
    cell_length   = [0.01],  
    ...  
)
```

```
x_center_plasma = 200.
```

```
def my_density_profile(x):  
    return exp(-(x-x_center_plasma)**2)
```

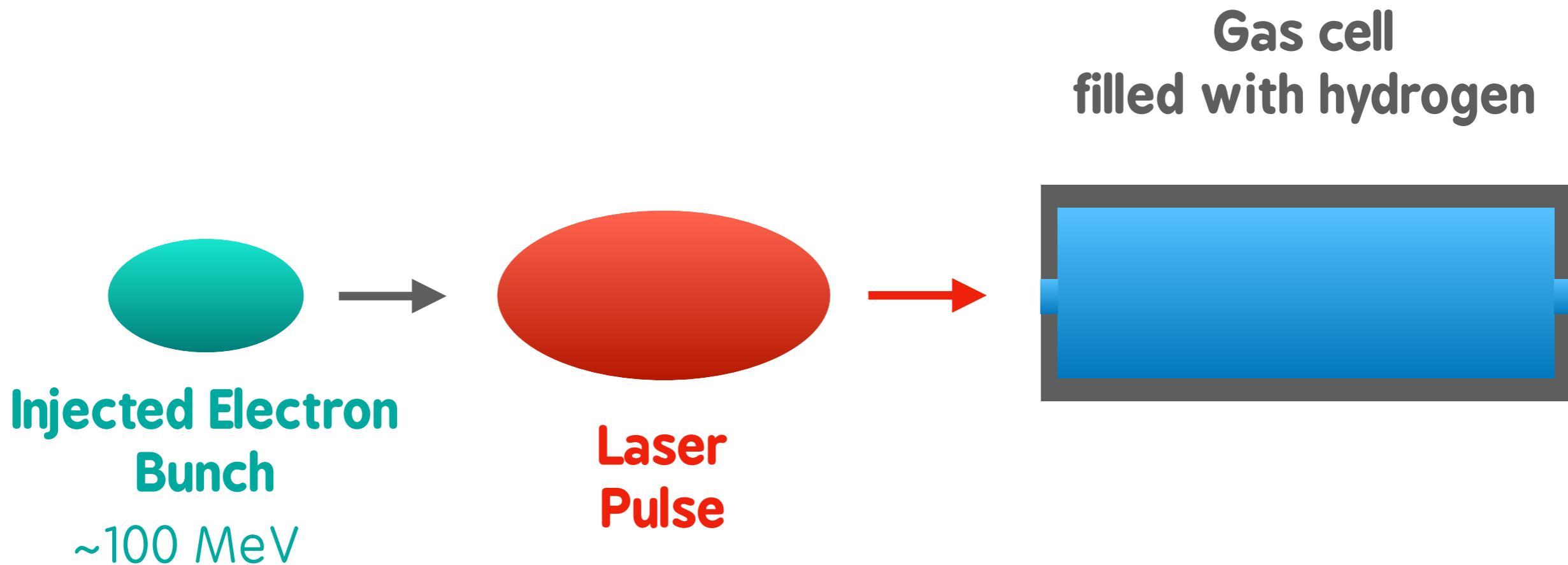
```
Species(  
    name          = "electron",  
    charge        = -1.,  
    mass          = 1.,  
    particles_per_cell = 100,  
    number_density = my_density_profile  
    ...  
)
```

- Normalised units
- Quantities can be computed at runtime

Laser /Plasma profiles = Functions  
(also user-defined)

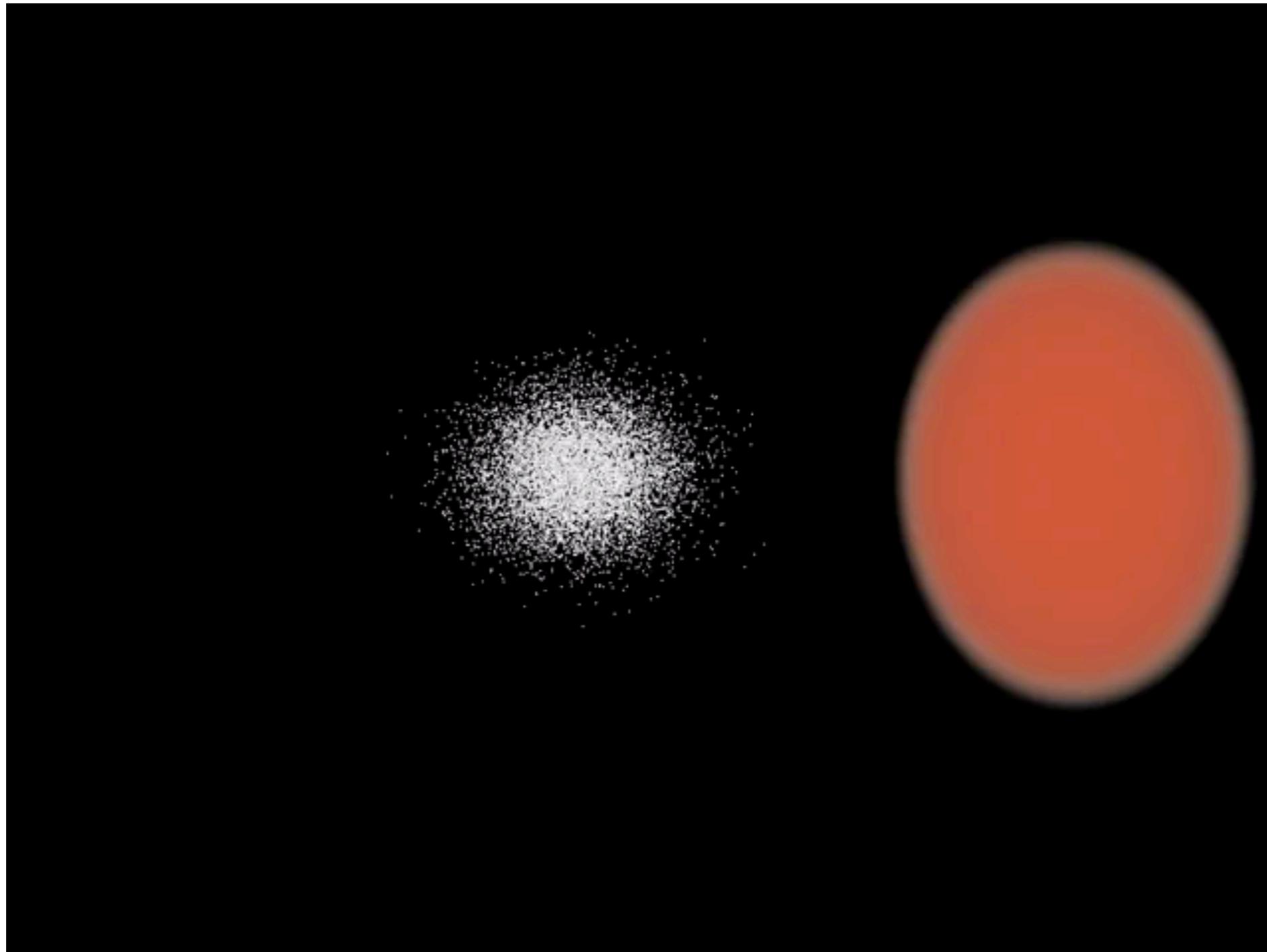
If Python can read it, SMILEI can read it

# Case study: LWFA with external injection



Assumption: plasma already created in the gas cell  
(the laser's repulse ionises the gas in its path)

# 3D rendering of LWFA with external injection



# Practical material

<https://github.com/SmileiPIC/TP-M2-GI/tree/main>

Answer Form for the Exercises   Link to TP documentation and Exercises

The screenshot shows a GitHub repository page for 'TP-M2-GI'. The repository has 2 branches and 0 tags. It contains several files and their commit history:

- .github/workflows
- Answers\_Form (highlighted with an orange border)
- Postprocessing\_Scripts (highlighted with a green border)
- doc
- InputNamelist.py (highlighted with a cyan border)
- LICENSE
- Presentation\_TP.pdf (highlighted with a blue border)
- README.md
- JJ\_submission\_script.sh (highlighted with a purple border)

Commit history (from top to bottom):

- Francesco Massimo and Francesco Massimo change environment
- Update sphinx.yml
- update TP
- update namelist
- correction
- update namelist
- Create LICENSE
- update presentation
- Update README.md
- change environment

Repository details on the right:

- Edit Pins
- Unwatch 7
- Fork 2
- Star 2
- About: Numerical practical for the students of the M2 - Grands Instruments.
- smileipic.github.io/TP-M2-GI (highlighted with a red border)
- Readme
- GPL-3.0 license
- Activity
- 2 stars
- 7 watching
- 2 forks
- Report repository

Annotations below the repository:

- Simulation submission file (points to JJ\_submission\_script.sh)
- These slides (points to Answers\_Form)
- Input Namelist file to run simulations (points to InputNamelist.py)
- Postprocessing Scripts (points to Postprocessing\_Scripts)

# Practical exercises

- Read **in detail** the **TP documentation**  
<https://smileipic.github.io/TP-M2-GI/index.html>
- Solve the exercises in the **TP documentation** progressively, e.g.

## Exercise 1:

Assuming  $\lambda_{\parallel} = 0.8 \mu m$  (a Ti:Sa laser system), what is the value of the critical density  $n_c$ ?

What is the value of the reference electric field  $E_0 = (2\pi m_e c^2)/(e\lambda_0)$ ?

This choice of  $\lambda_{\parallel}$  will be used throughout all subsequent exercises.

**Hint:** Some lines at the start of the `InputNamelist.py` file can help you in the calculations.

## Exercise 5:

In the next exercise we will check that the Gaussian laser pulse diffracts following the theory for a Gaussian beam [Siegman]:  $w(x) = w_0 \sqrt{1 + x^2/x_R^2}$ , where  $w_0$  is the laser waist size at the focal plane position,  $w(x)$  the laser waist size at propagation distance  $x$ ,  $x_R$  is the Rayleigh length  $x_R = \pi w_0^2/\lambda_{\parallel}$ .

What is the theoretical Rayleigh length  $x_R$ ?

## Exercise 10:

Launch a new simulation with  $a_0=1.8$ . This simulation will be in the nonlinear regime ( $a_0 > 1$ ), so the plasma wave will not be sinusoidal. You can visualize both the normalized absolute value of the envelope of the laser field and the electron number density by defining a transparency for the parts where the latter field is lower than a threshold  $vmin$ :

```
Env_E = S.Probe.Probe1("Env_E_abs",units=["um"],cmap="hot",vmin=0.8,transparent="unde  
Rho = S.Probe.Probe1("-Rho/e",units=["fs","um","1/cm^3"],cmap="Blues_r",vmin=0.,vma  
happi.multiSlide(Rho,Env_E,xmin=0,figure=10, xlabel="x [um]",ylabel="y [um]")
```

Using `timeStep=2500` in the definition of `Env_E` and `Rho`, and then using `multiPlot` instead of `multiSlide`, you should have a plot of the data at half of the propagation length.

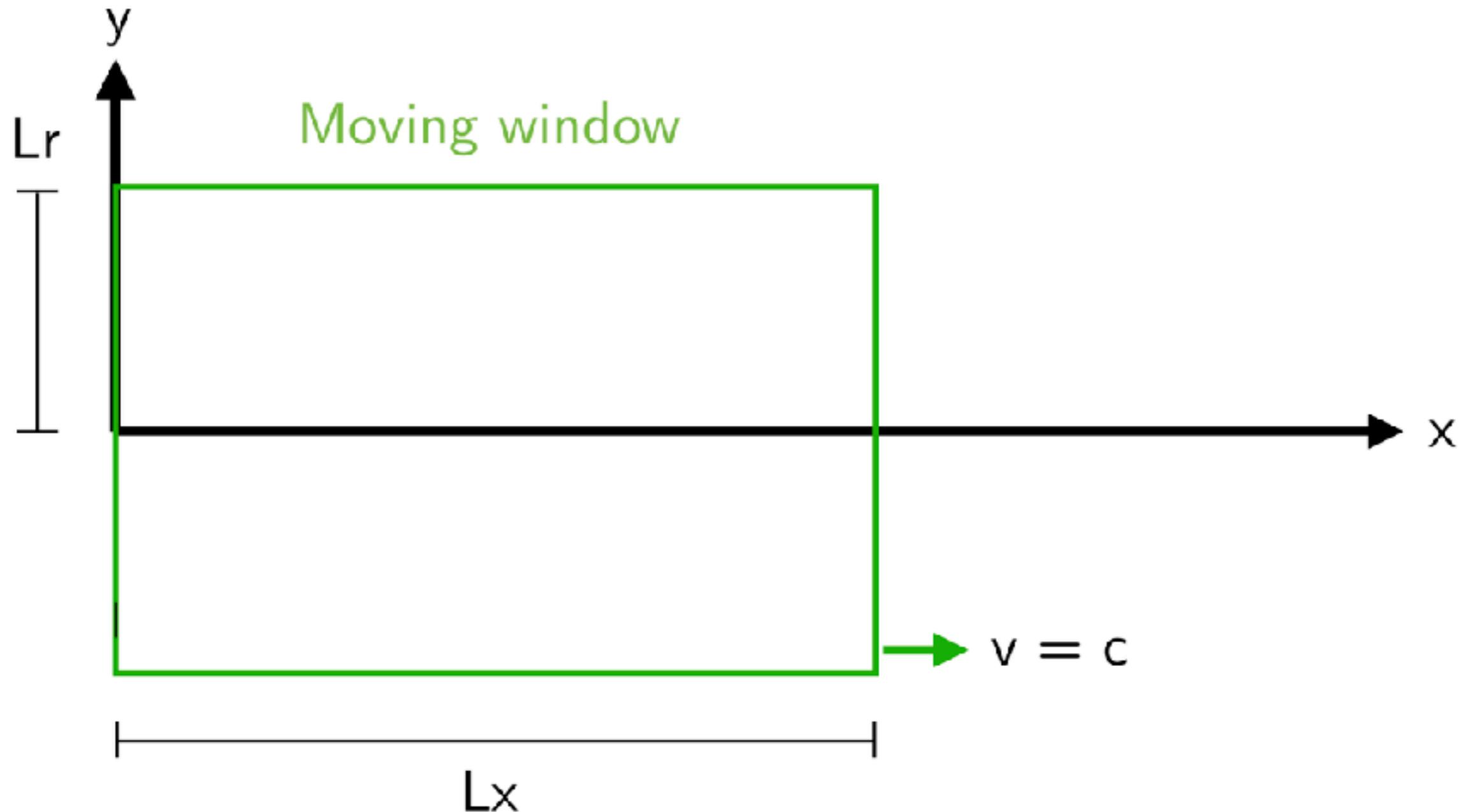
Include this image in your answers.

- Run simulations when necessary, following the instructions in the **TP documentation**
- Fill the answers report **Answers\_Form.docx**  
(You can download it from GitHub  
<https://github.com/SmileiPIC/TP-M2-GI/tree/main>)

# Suggestions

- Again read **in detail** the **TP documentation**
- **Understand** the physical set-up
- **Any doubts? Ask the instructor**
- Solve the exercises **progressively**
- Create one folder for each simulation asked by the exercises  
(to **avoid losing data**)
- Feel free to adapt the commands in the **TP documentation**
- When an image is asked for the report, save it or make a screenshot
- **Better do few exercises but understand them at 100%**

# Part 1: Exploring the Input Namelist (Exercises 1-2)



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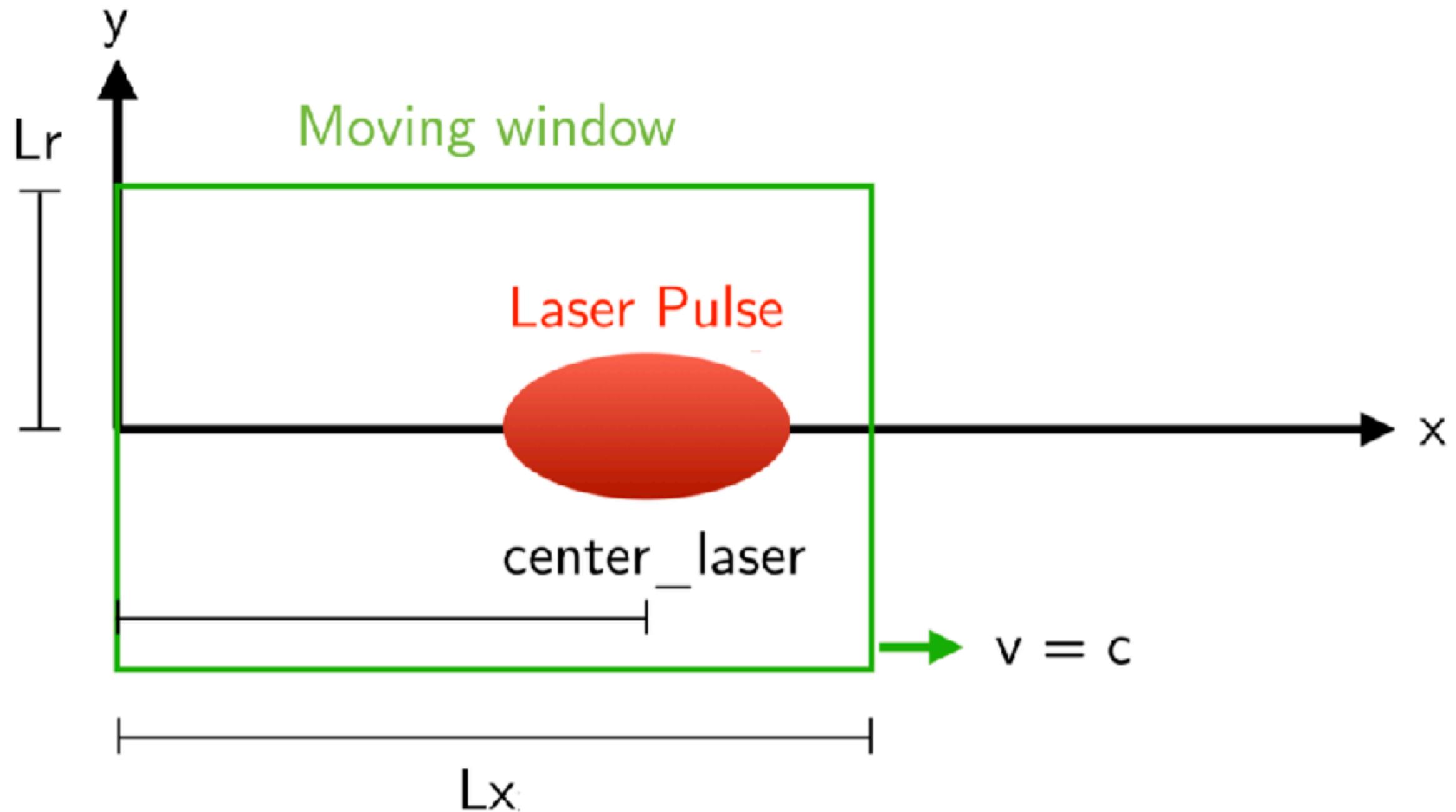
## Input Namelist structure

- Main Block
- Moving Window Block
- Laser Envelope Block
- Species Block (for the plasma)
- Species Block (for the relativistic electron bunch)
- Diagnostic Blocks (for Postprocessing)

Blue: Active

Gray: Commented = Inactive

## Part 2: Laser pulse in vacuum (Exercises 4-6)



## Part 2: Laser pulse in vacuum (Exercises 4-6)

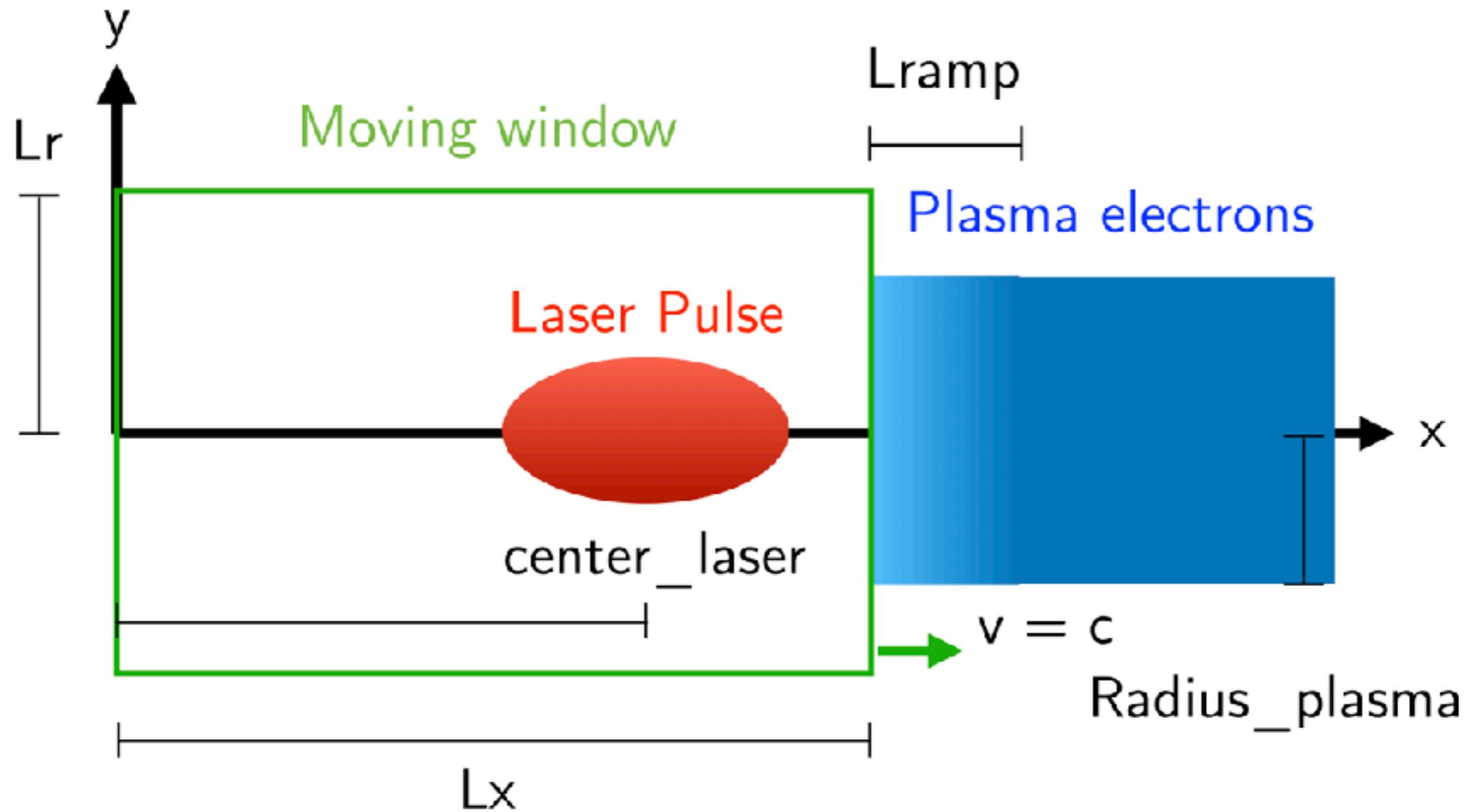
### Input Namelist structure

- Main Block
- Moving Window Block
- Laser Envelope Block
- Species Block (for the plasma)
- Species Block (for the relativistic electron bunch)
- Diagnostic Blocks (for Postprocessing)

Blue: Active

Gray: Commented = Inactive

## Part 3: Laser Wakefield Excitation (Exercises 7-11)



# Part 3: Laser Wakefield Excitation (Exercises 7-11)

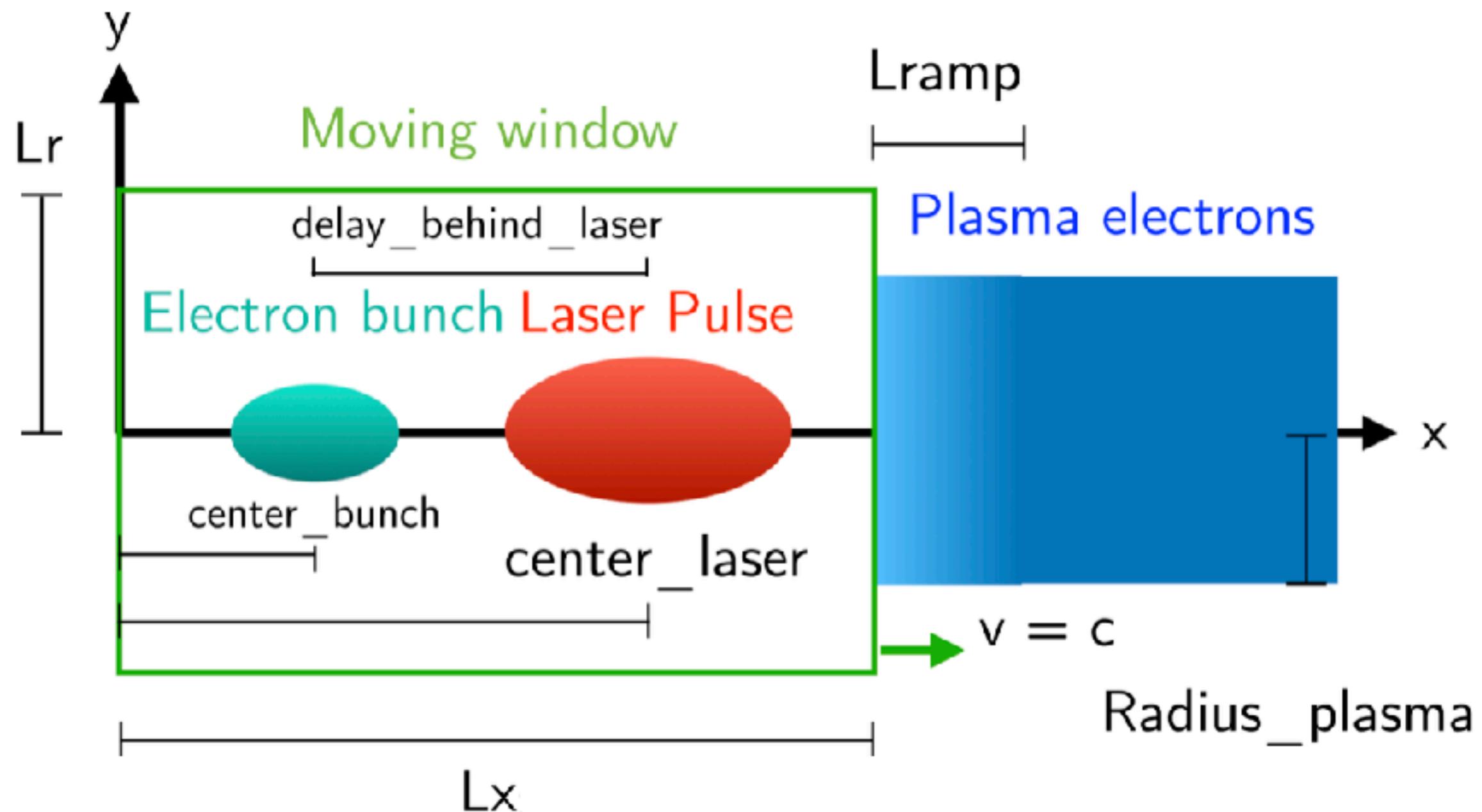
## Input Namelist structure

- Main Block
- Moving Window Block
- Laser Envelope Block
- Species Block (for the plasma)
- Species Block (for the relativistic electron bunch)
- Diagnostic Blocks (for Postprocessing)

Blue: Active

Gray: Commented = Inactive

## Part 4: Laser Wakefield Acceleration of an electron bunch (Exercises 12-20)



## Input Namelist structure

- Main Block
- Moving Window Block
- Laser Envelope Block
- Species Block (for the plasma)
- Species Block (for the relativistic electron bunch)
- Diagnostic Blocks (for Postprocessing)

Blue: Active

Gray: Commented = Inactive

# Postprocessing for the practical

Included in the code: the Python postprocessing library **happi**

```
$ ipython  
In [1]: import happi
```

## Diagnostics available for this practical:

- 1D Probe on the x axis (electromagnetic fields, density)
- 2D Probe on the xy plane (electromagnetic fields, density)
- DiagTrackParticles for the bunch electrons (phase space data)

## Tips:

- Open the **Ipython** interface (command **ipython** in the terminal)
- Copy and paste the happi commands from the handouts
- Adapt them for your purposes
- Use the post processing scripts available (see handouts)

# Questions?