

# Numerical simulation of laser wakefield acceleration with a Particle in Cell code



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# Outline

- Why plasma accelerators?
- Basics of plasma acceleration
- Numerical simulation of plasma acceleration: PIC codes
- The PIC code Smilei
- Introduction to the case study and the practical

# Outline

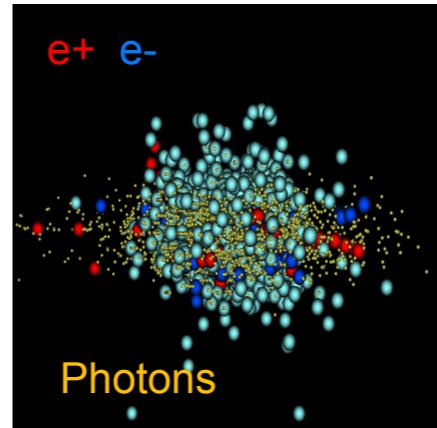
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# Applications of high energy electrons ( just a few )

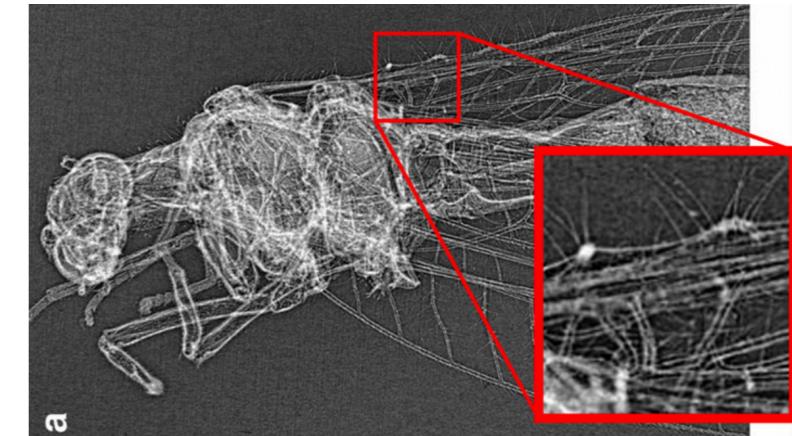
## Fundamental Research:

- QED, Particle physics
- Matter Physics
- Biology

Smilei Simulation  
of QED e+e- pairs creation  
from photons



J. Wenz et al., Nat Comm 2014  
Imaging through Betatron radiation



## Medicine, Industry, Heritage, ....:

- Cancer treatment,
- Medical imaging
- Electronic industry
- Study of materials
- Authentication of artwork
- ...



Hidden Archimede's Palimpsest,  
revealed by SLAC's synchrotron radiation  
in 2005 (U. Bergmann)



Radiotherapy,  
from YouTube video  
How a Linear Accelerator Works

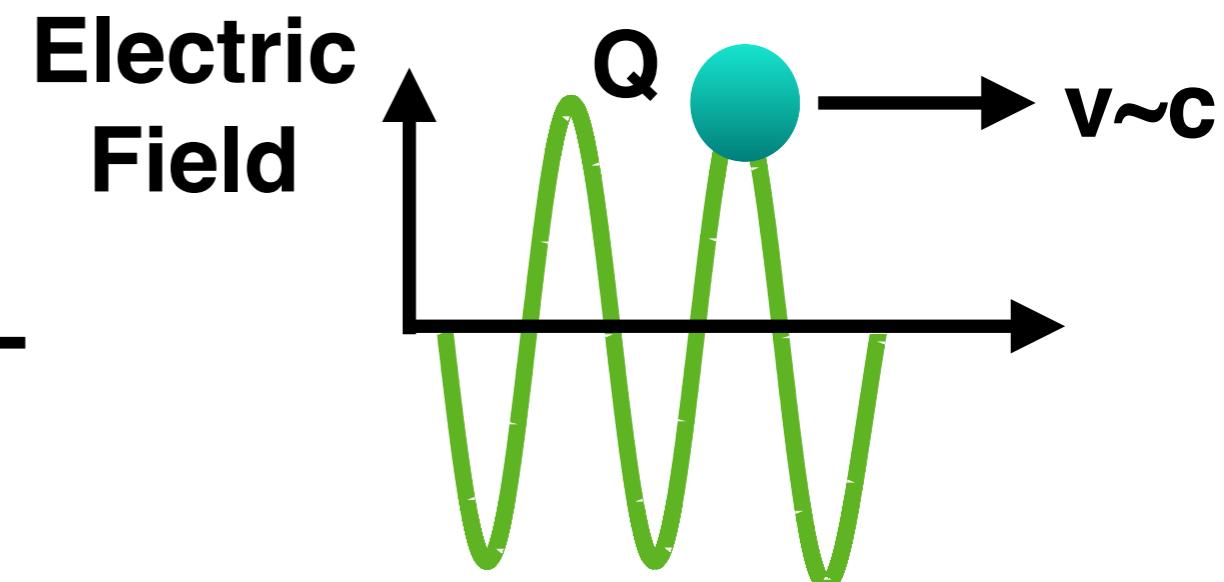
**Can we have  
more compact, cheaper  
electron accelerators in the future?**

# Particle accelerators size and cost

Relativistic charge **Q**

In an accelerating cavity of length **L**

With peak accelerating field **E**



$$\text{Maximum energy gain} = Q E L$$

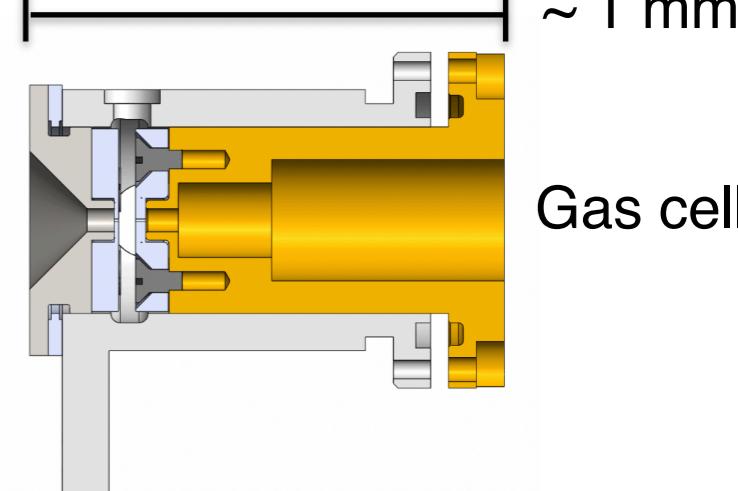
Given a target energy,

if **E** is limited by technology, **L** increases

→ need more cavities

→ the accelerator size (and cost) increases

# Particle accelerators size and cost

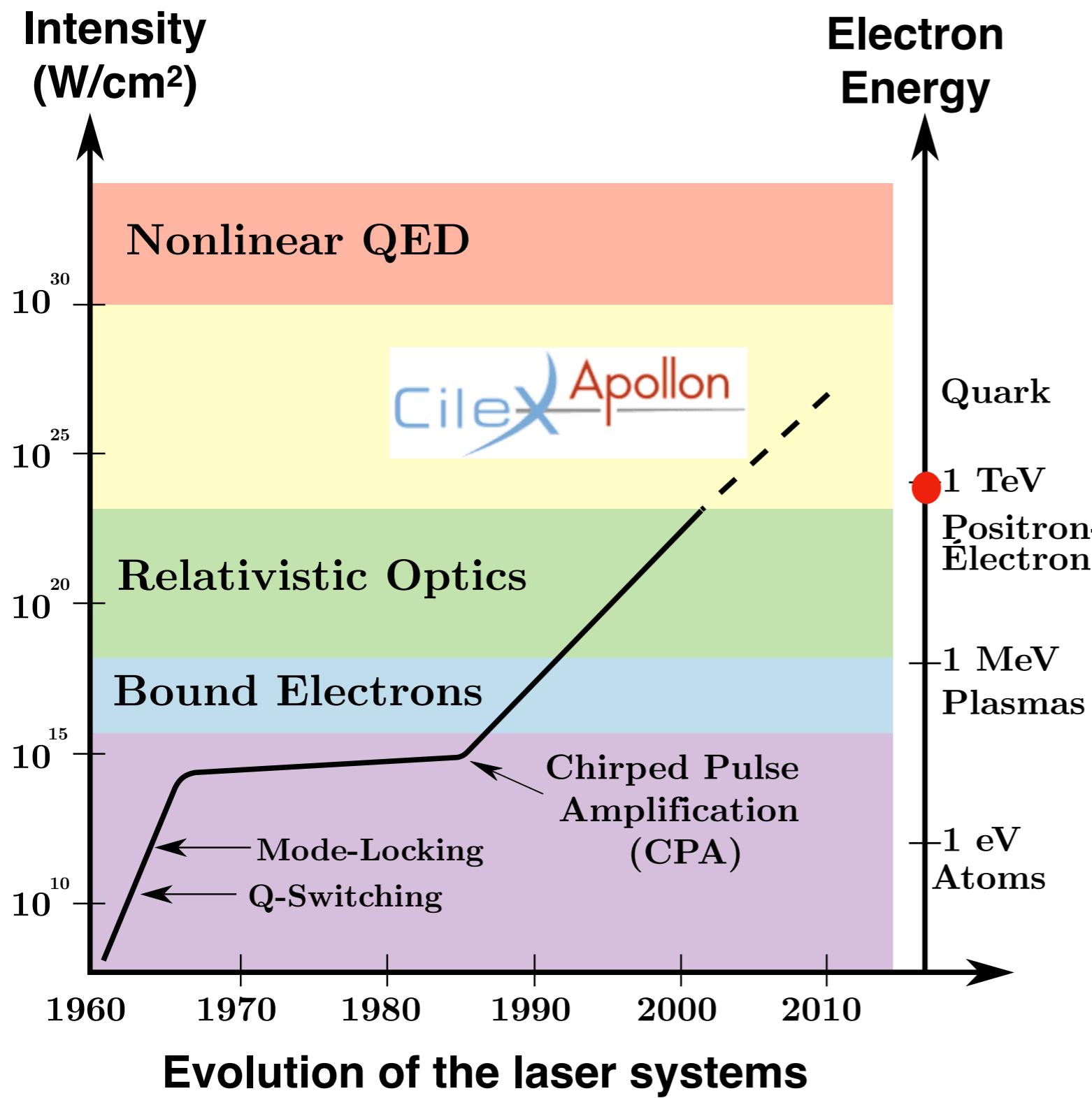
Accelerator technology	Peak Accelerating Field	Acceleration length to gain 100 MeV	~1 m — Accelerating Cavity
Radiofrequency metallic cavities	$\sim 10^2$ MV/m	1 m	
Laser Wakefield Acceleration (LWFA)*	$\sim 10^4$ MV/m	10 mm	

\*Open challenge: improve performances  
of Laser Wakefield Acceleration.  
Numerical modeling is necessary!

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# High Intensity Laser-Matter Interaction



## Techniques:

- Chirped Pulse Amplification
- Femtosecond Lasers
- Nonlinear Optics
- ...

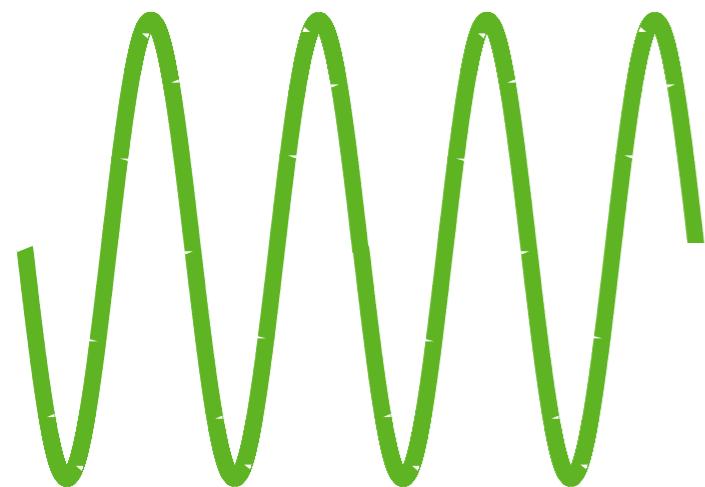
## Physics Domains:

- Relativistic Fluids
- Physics of Relativistic Plasmas
- Relativistic Optics
- Laboratory Astrophysics

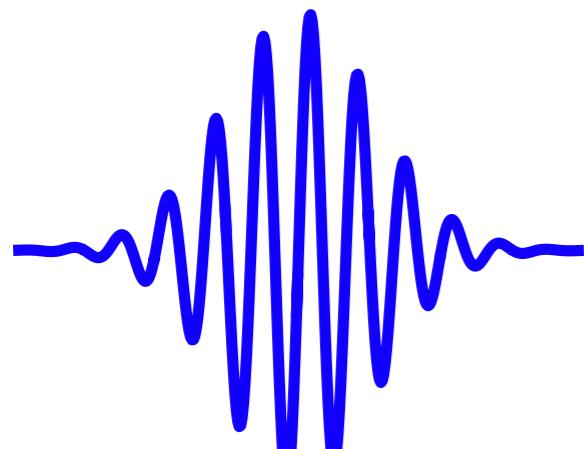
## Applications:

- Electron and ion sources
- Acceleration of particles
- Radiation sources (UV, X,  $\gamma$ )
- Novel, high resolution diagnostics
- Pump-probe measurements

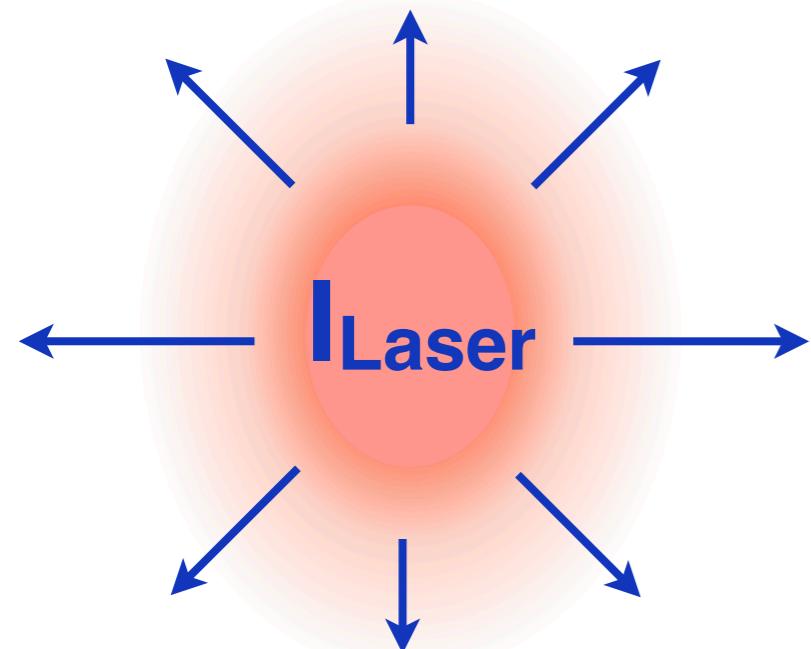
# Ponderomotive Force



Electron in infinite plane wave:  
Oscillating Force



Electron in finite laser pulse:  
Oscillating Force + Ponderomotive Force



$$F_{\text{pond}} \propto -\nabla I_{\text{Laser}} \quad (\text{a.k.a. radiation pressure})$$

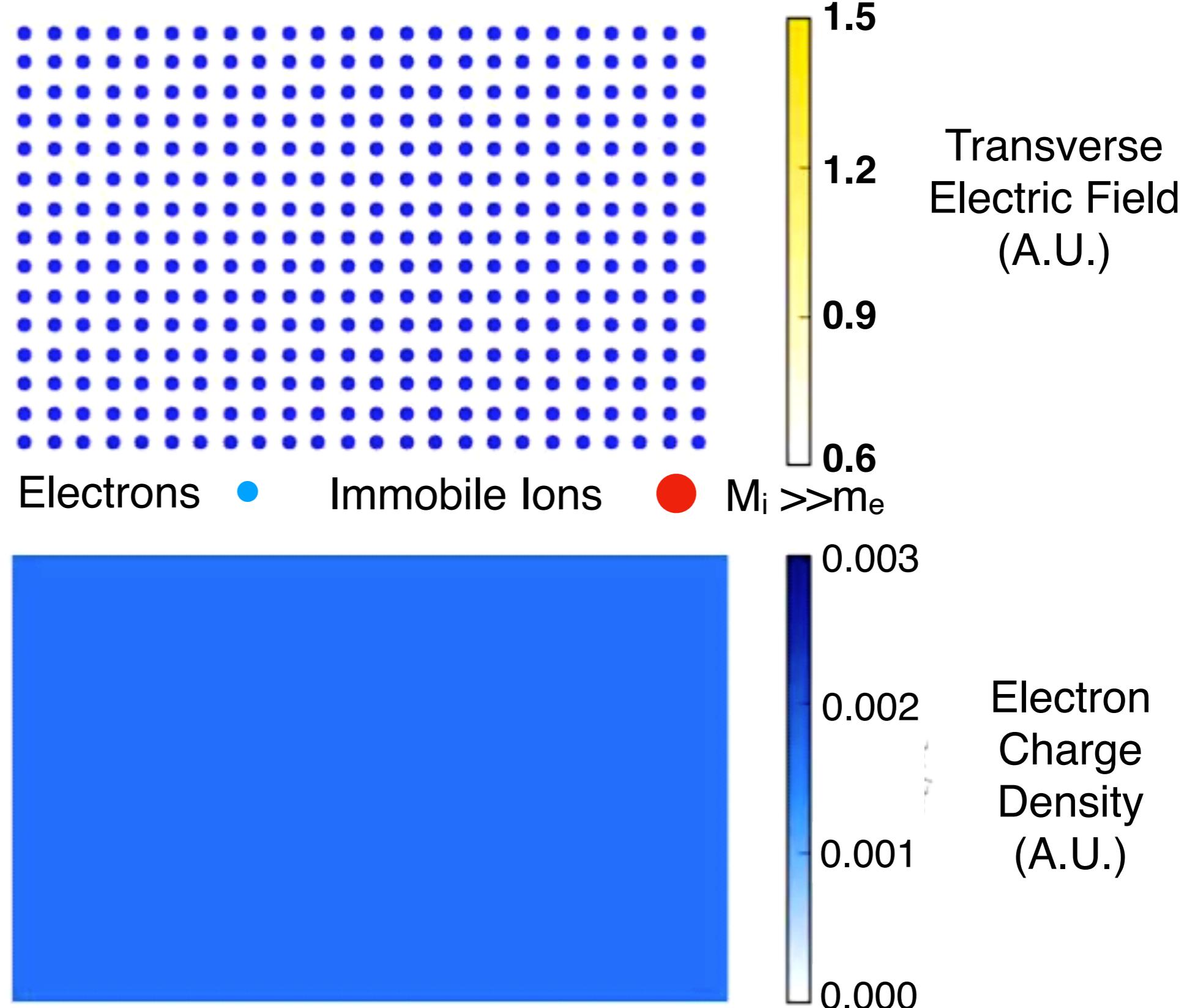
# Laser Wakefield Acceleration (LWFA): plasma wave excitation

Laser Beam  
Duration: 28 fs

Ponderomotive  
Force:  
 $F = -\nabla I_{\text{Laser}}$

Plasma density:  
 $3 \cdot 10^{18} \text{ cm}^{-3}$

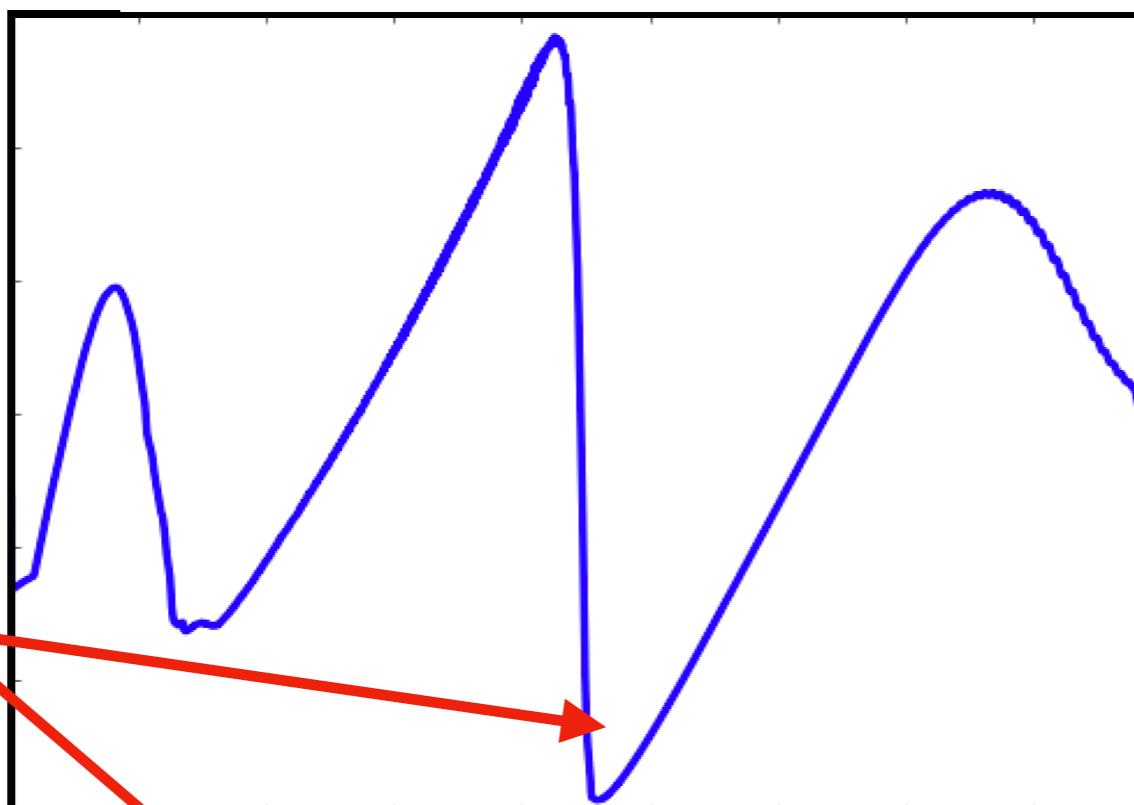
Plasma  
wavelength:  
 $\sim 20 \mu\text{m}$



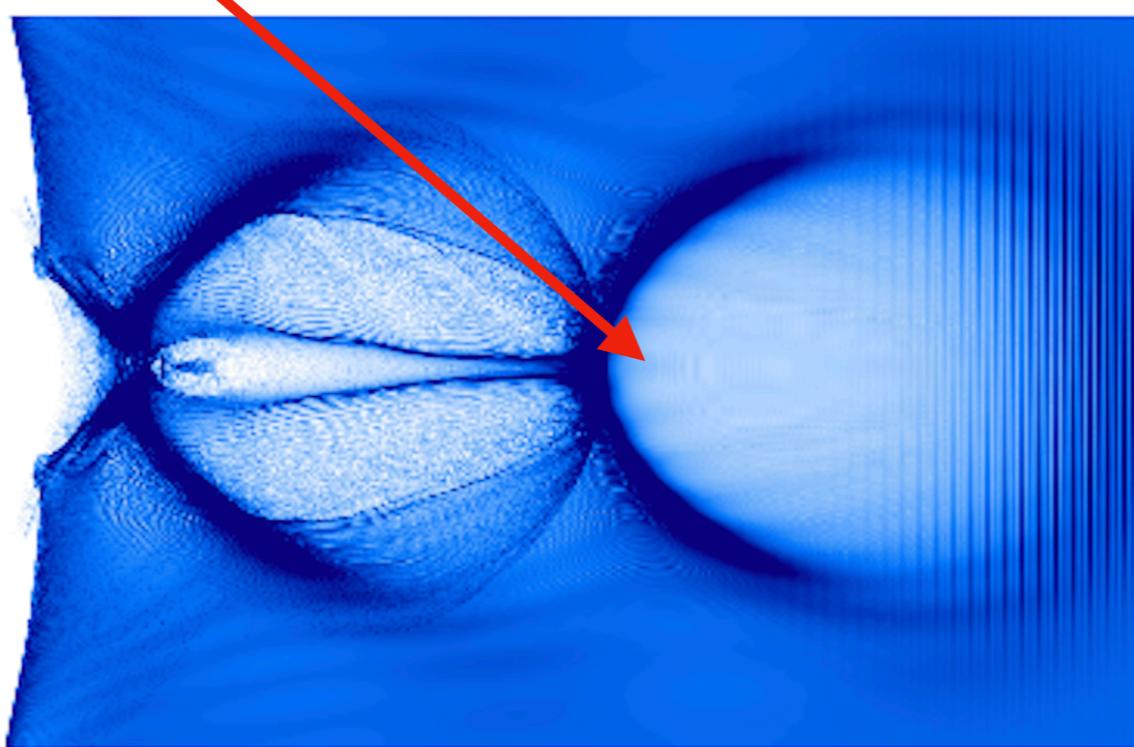
# Laser Wakefield Acceleration (LWFA): accelerating electric field

$E > 100 \text{ GV/m}$

Relativistic  
electrons  
injected here  
are accelerated  
towards the right

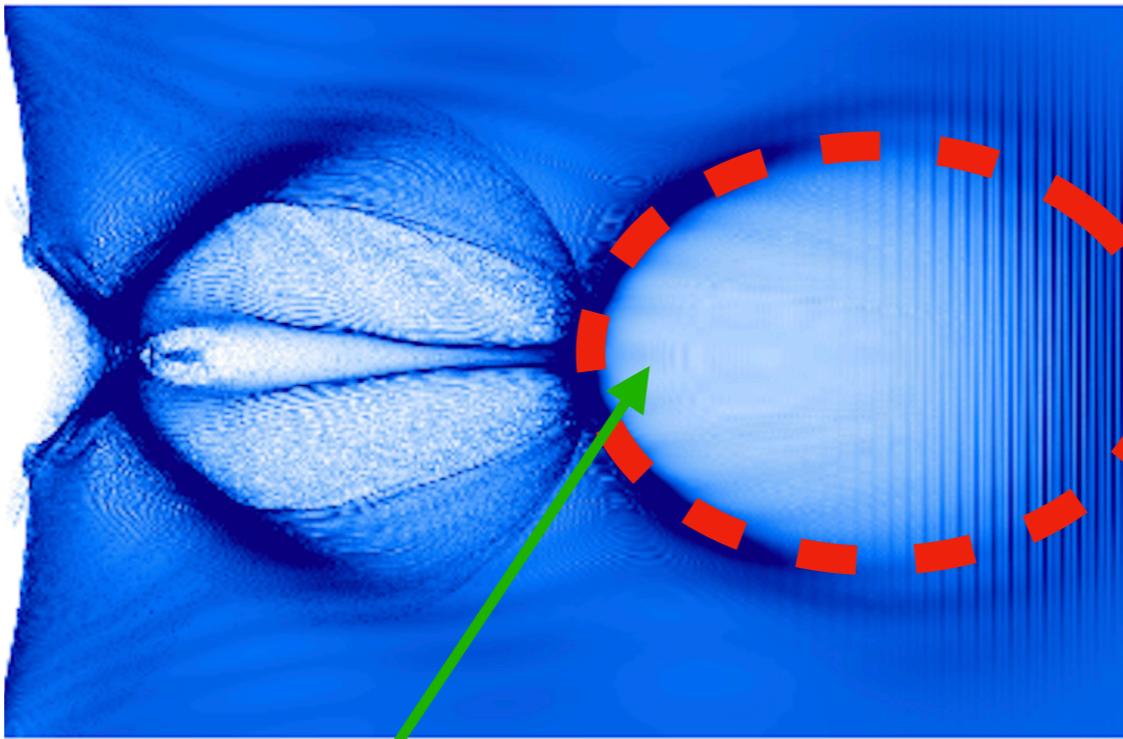


Longitudinal  
Electric Field  
(A.U.)  
on propagation axis



0.003  
0.002  
0.001  
0.000  
Electron  
Charge  
Density  
(A.U.)

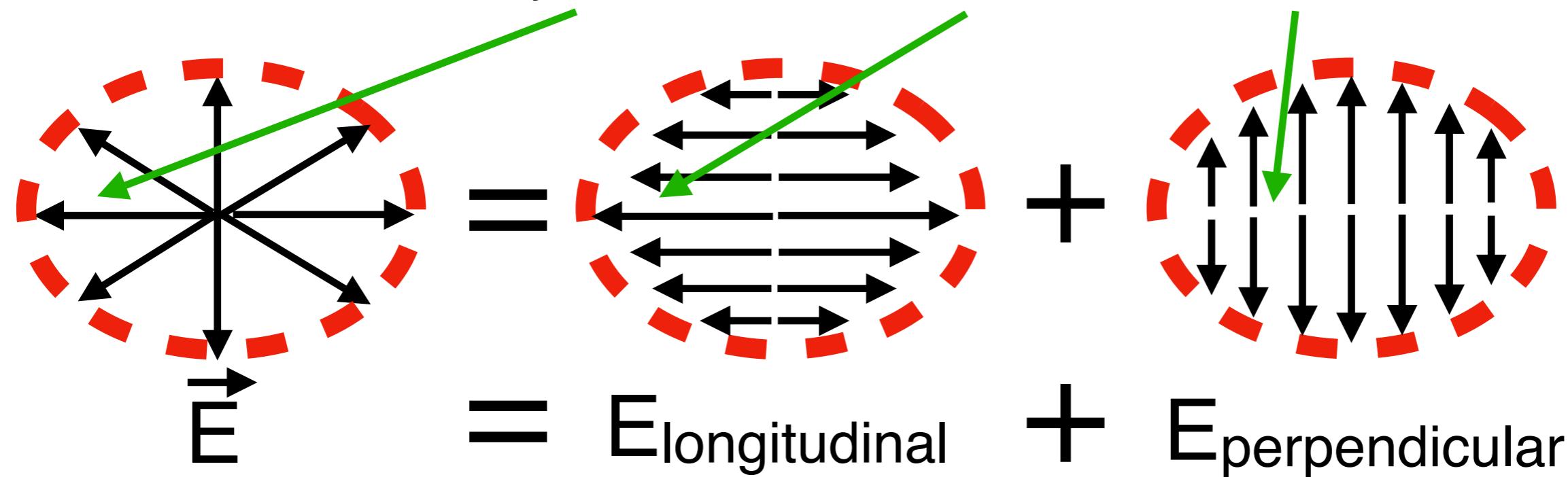
# Laser Wakefield Acceleration (LWFA): fields inside the bubble



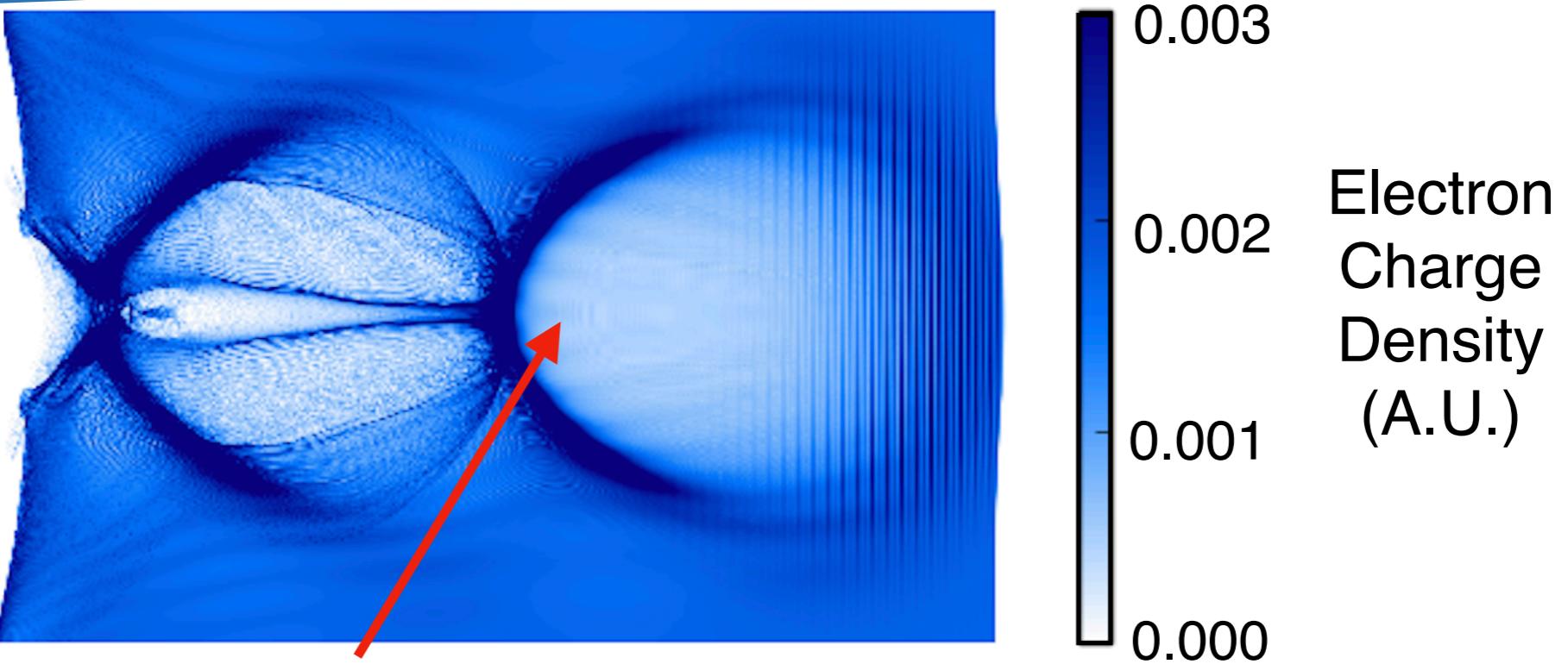
0.003  
0.002  
0.001  
0.000

Electron Charge Density (A.U.)

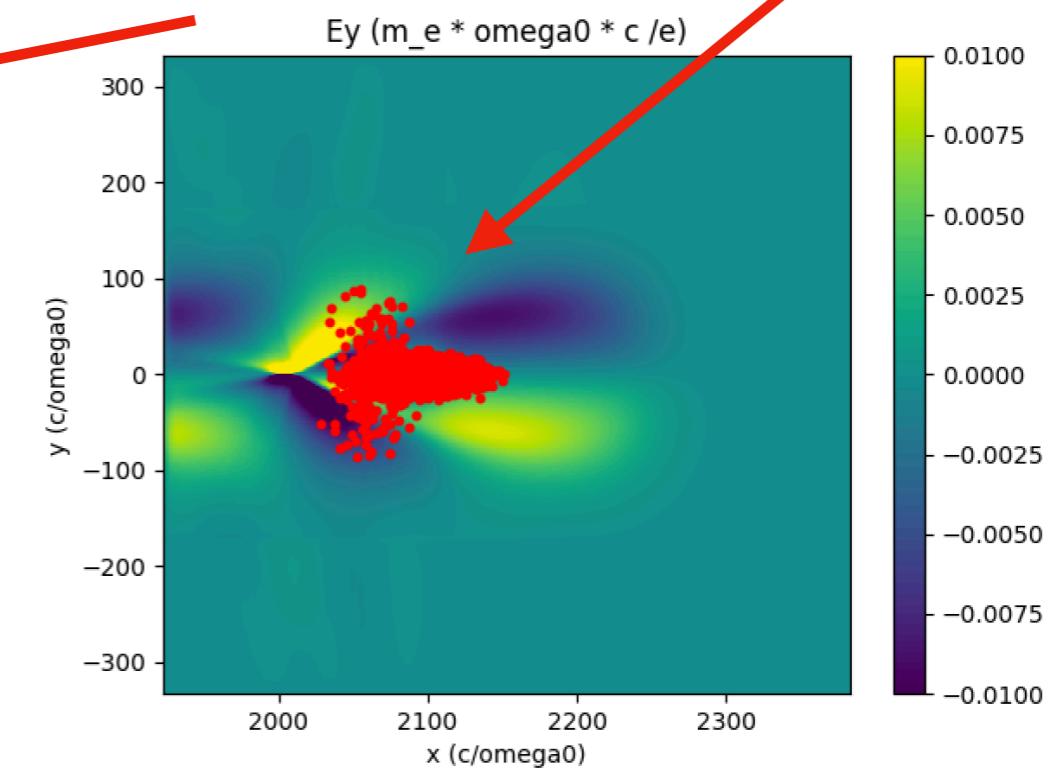
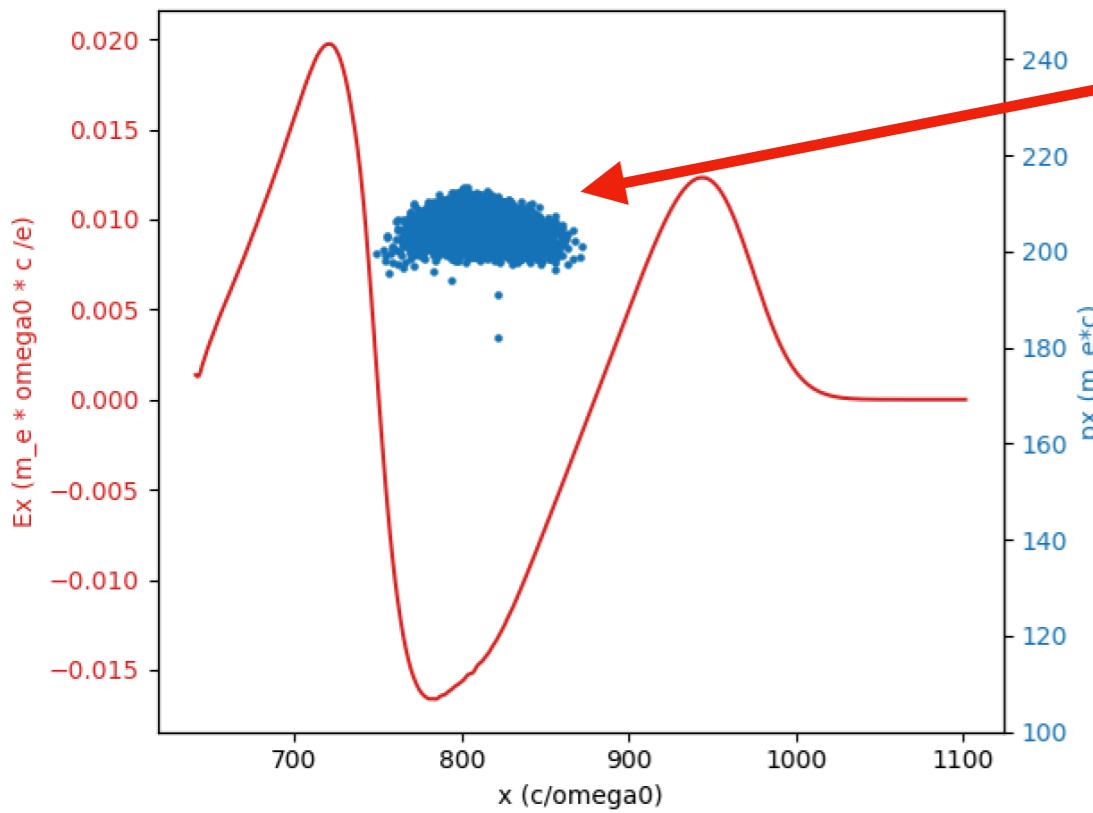
Relativistic electrons injected here are both accelerated and focused



# Laser Wakefield Acceleration (LWFA): fields inside the bubble



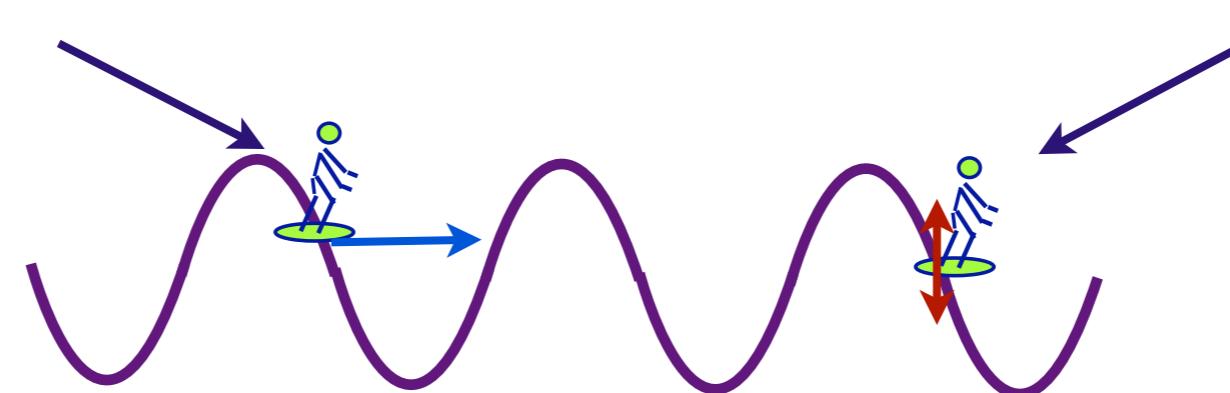
Relativistic electrons injected **here** are both **accelerated** and **focused**



# Laser Wakefield Acceleration (LWFA) challenges: injection

## Surfer with

- sufficient initial velocity,
- injected in the proper phase

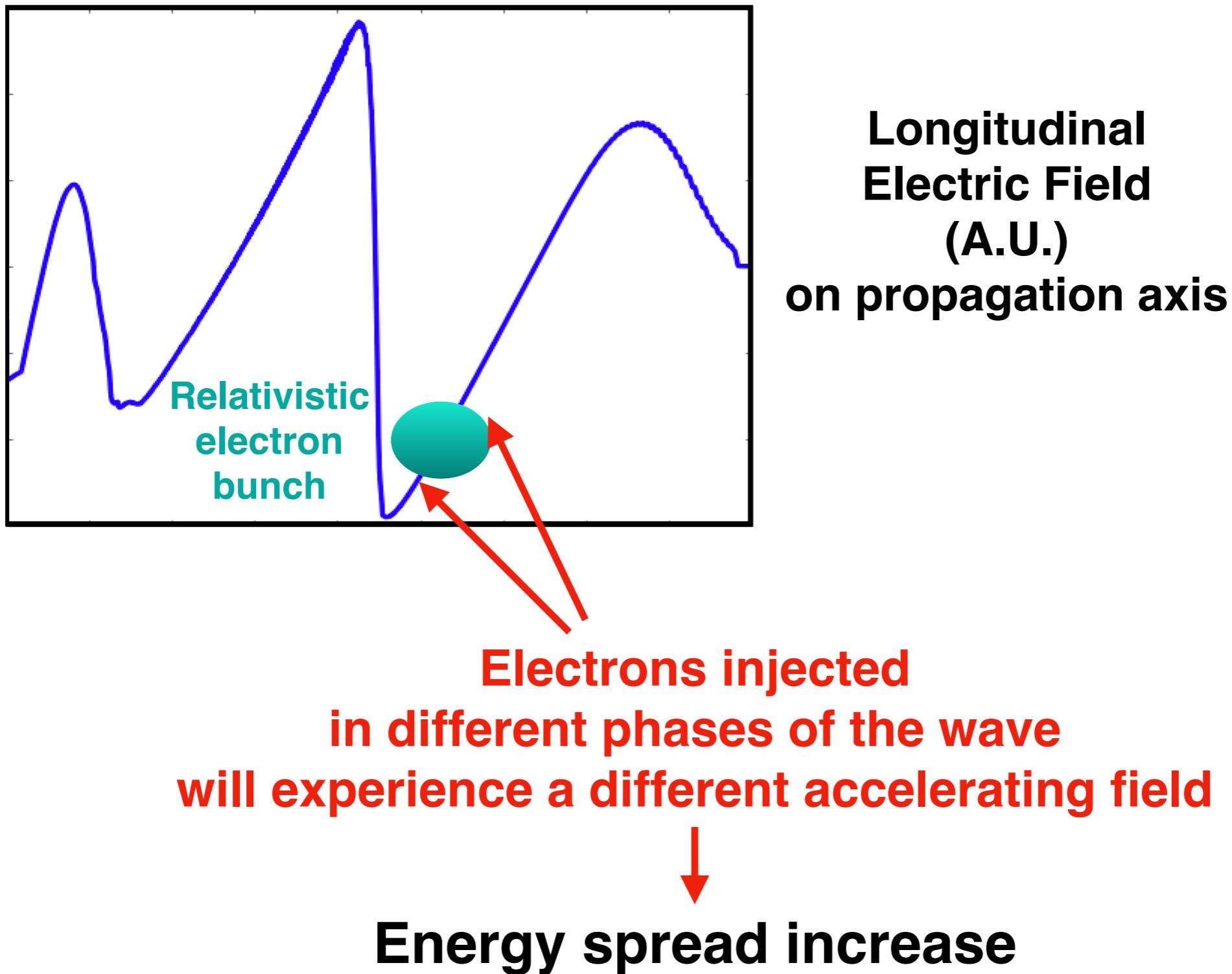


## Surfer with Initial zero velocity

**Surfer: electron**  
**wave: electric field of the plasma wave in the wake of the laser**

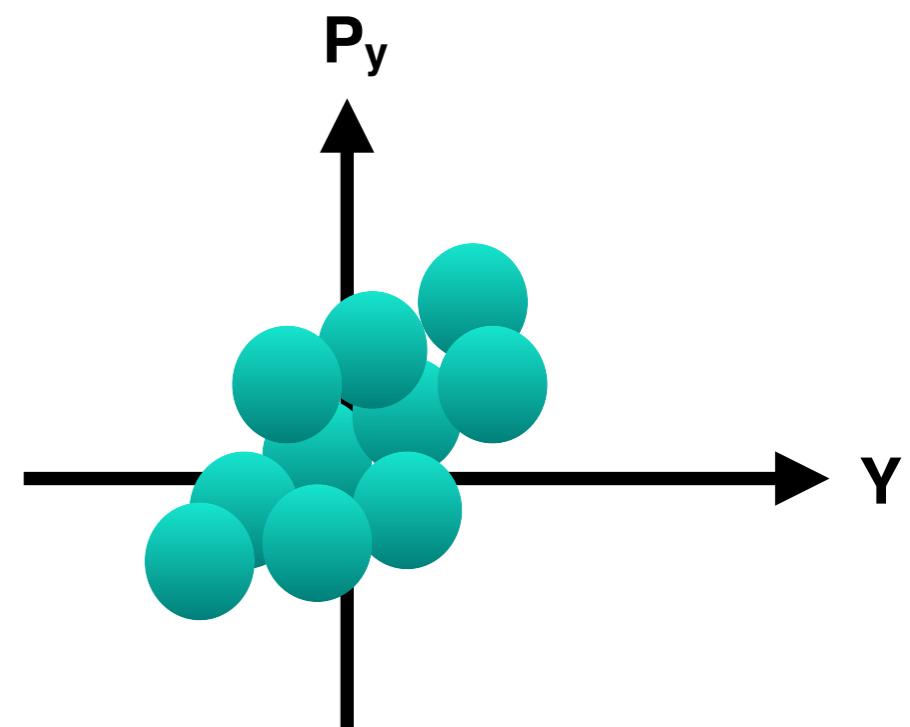
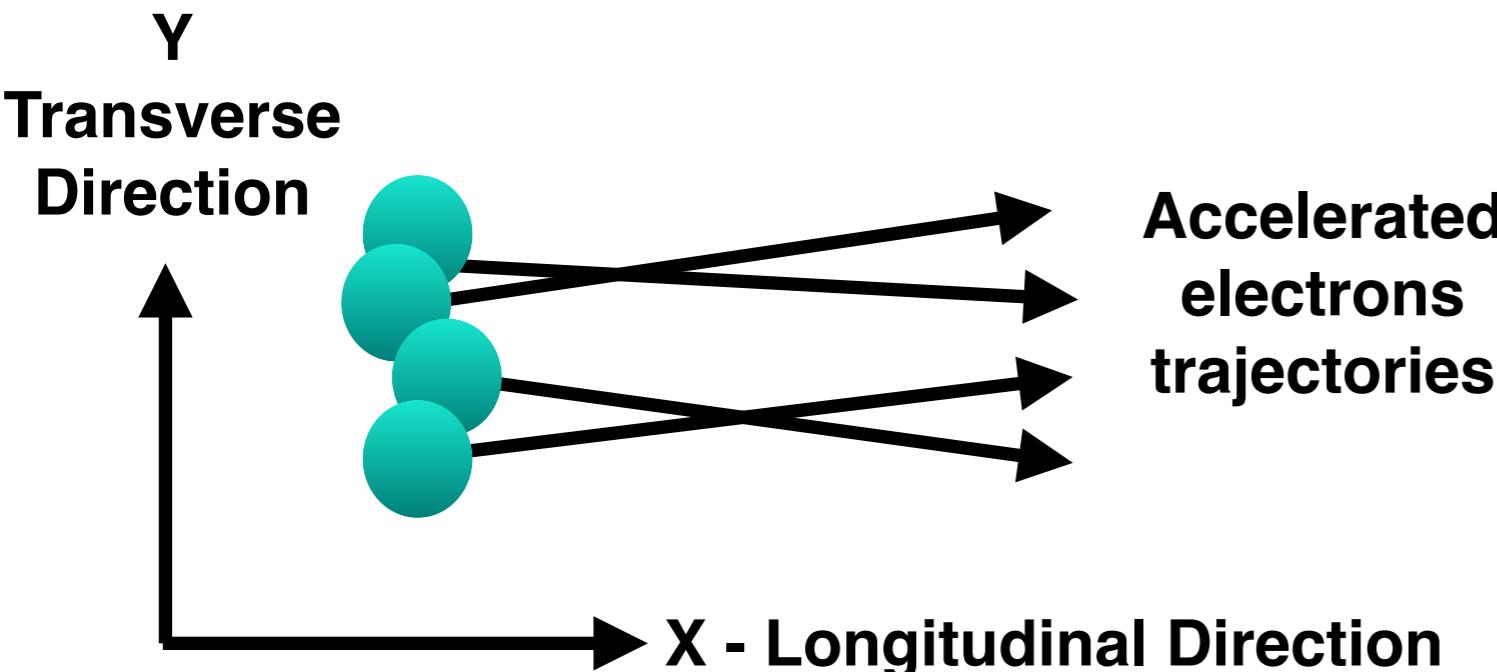
Plasma wavelength < 100 μm, Duration of the electron beam < 10 fs ~3 μm

# LWFA challenges: reduce the energy spread



# LWFA challenges: lowering the emittance

Bunch distribution  
in the transverse phase space



Most applications of accelerated beams require:

- Small transverse size (i.e. small  $\sigma_y$ )
- Small divergence (i.e. small  $\sigma_{p_y}/\text{Energy}$ )



Minimize  
Transverse Emittance

Transverse  
Normalized  
Emittance

$$\epsilon_{ny} = \frac{1}{m_e c^2} \sqrt{\sigma_y^2 \sigma_{p_y}^2 - \sigma_{y p_y}^2}$$

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# Why numerical simulation is necessary for LWFA?

- **No general analytical solutions are available**
- **Often (ok, this happens always) you cannot know everything simultaneously from measurements**
- **Before the experiment:**
  - Study new physical phenomena
  - Conceive new kind of experiments
  - Design experiments (even using Machine Learning)
- **After the experiment:**
  - Analyze the data
  - Understand the physics

# Model for Laser-Plasma Interaction

## Complete Maxwell-Vlasov system

Plasma distribution function

$f(x, y, z, p_x, p_y, p_z, t) \longrightarrow 6 \text{ dimensions + time!}$

Coupled to the electromagnetic fields  $\longrightarrow$  Non-linearity

$$\frac{\partial f}{\partial t} + \beta c \cdot \nabla_x f - e(E + \beta c \times B) \cdot \nabla_p f = 0 \leftarrow \text{Collisions neglected}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad \begin{matrix} \text{Current density} \\ J \text{ of the plasma} \end{matrix}$$

$$\nabla \times B = -\mu_0 ec \left( \int \beta f d^3 p \right) + \frac{1}{c^2} \frac{\partial E}{\partial t}$$

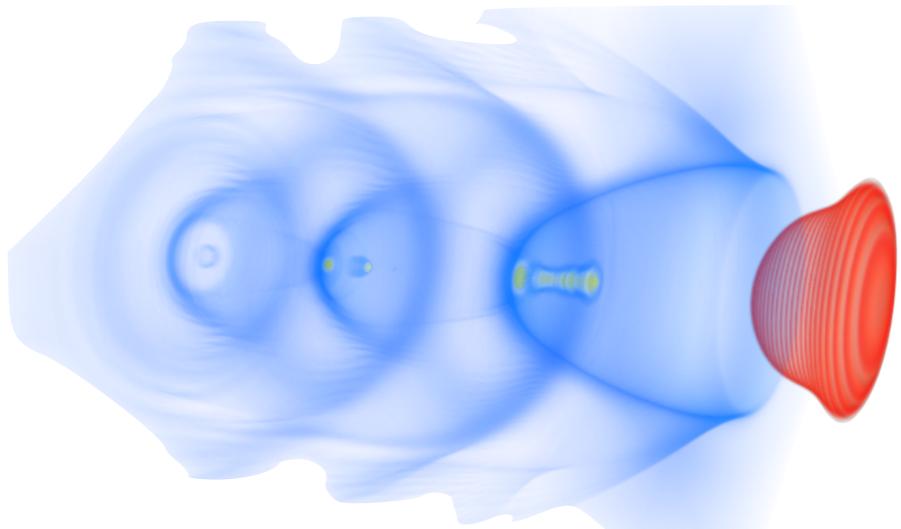
Explorable Physics:

- Relativistic optics ( laser self-focusing, ... )
- Nonlinear phenomena
- Wave-breaking
- Injection and acceleration of particles

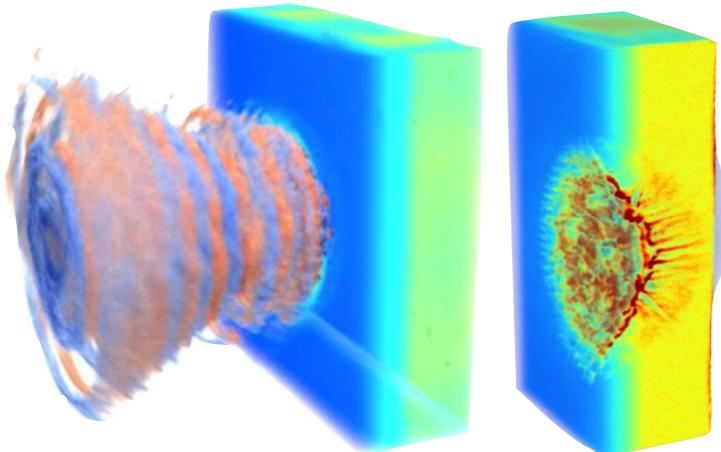
## Approximate solution: Particle in Cell method

# Particle in Cell (PIC) method : essential kinetic plasma investigation technique

## Laser Plasma Interaction

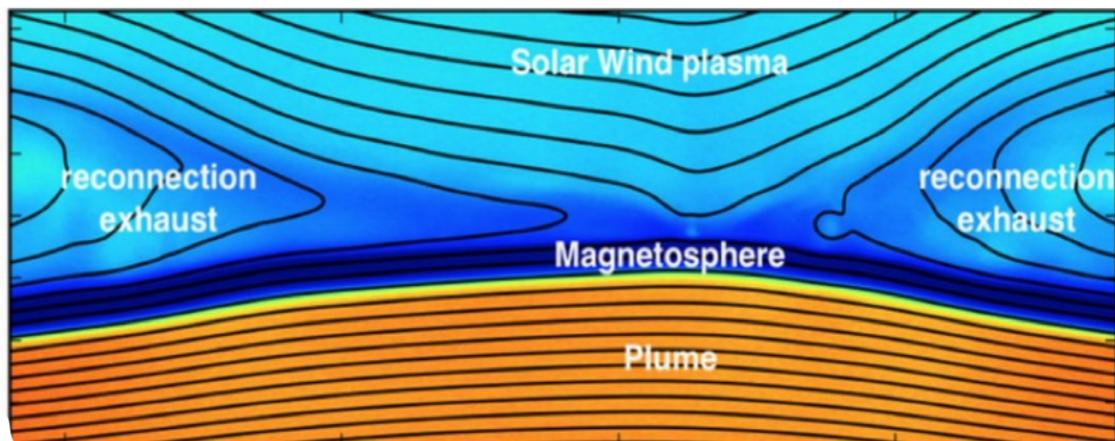


source: Massimo et al. (2020)



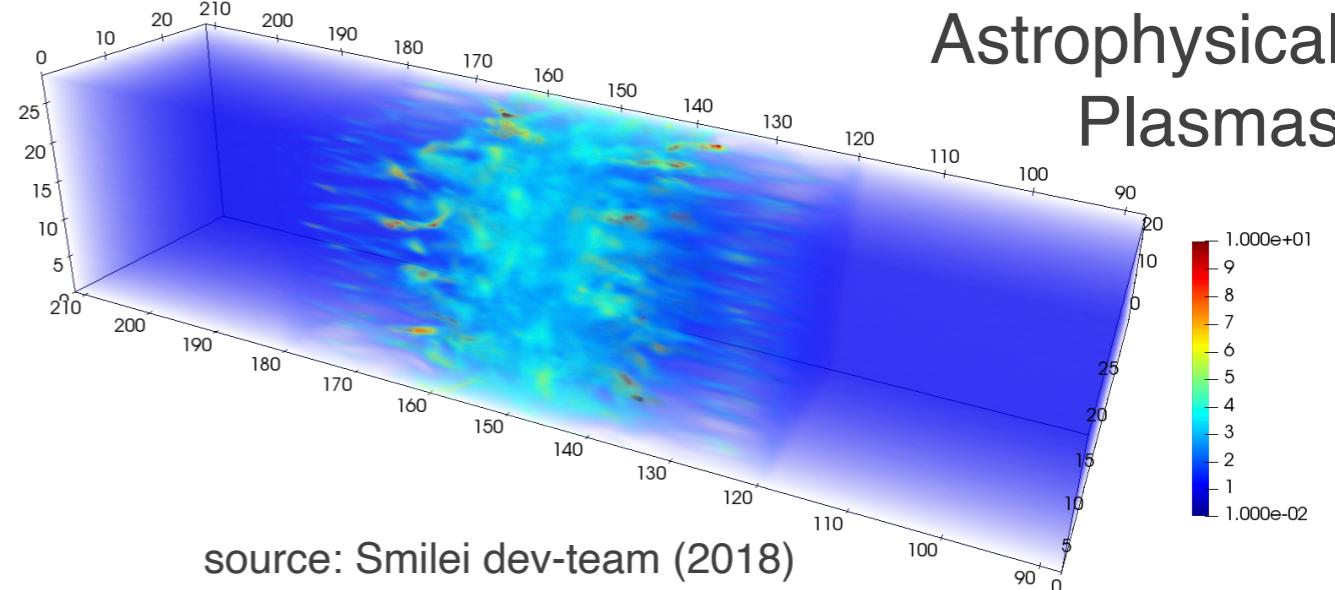
source: Smilei dev-team (2018)

## Space Plasmas



source: Dargent et al. (2017)

## Astrophysical Plasmas



source: Smilei dev-team (2018)

- Wide range of physics applications
- Conceptually simple
- Efficiently implemented on small or massively parallel supercomputers

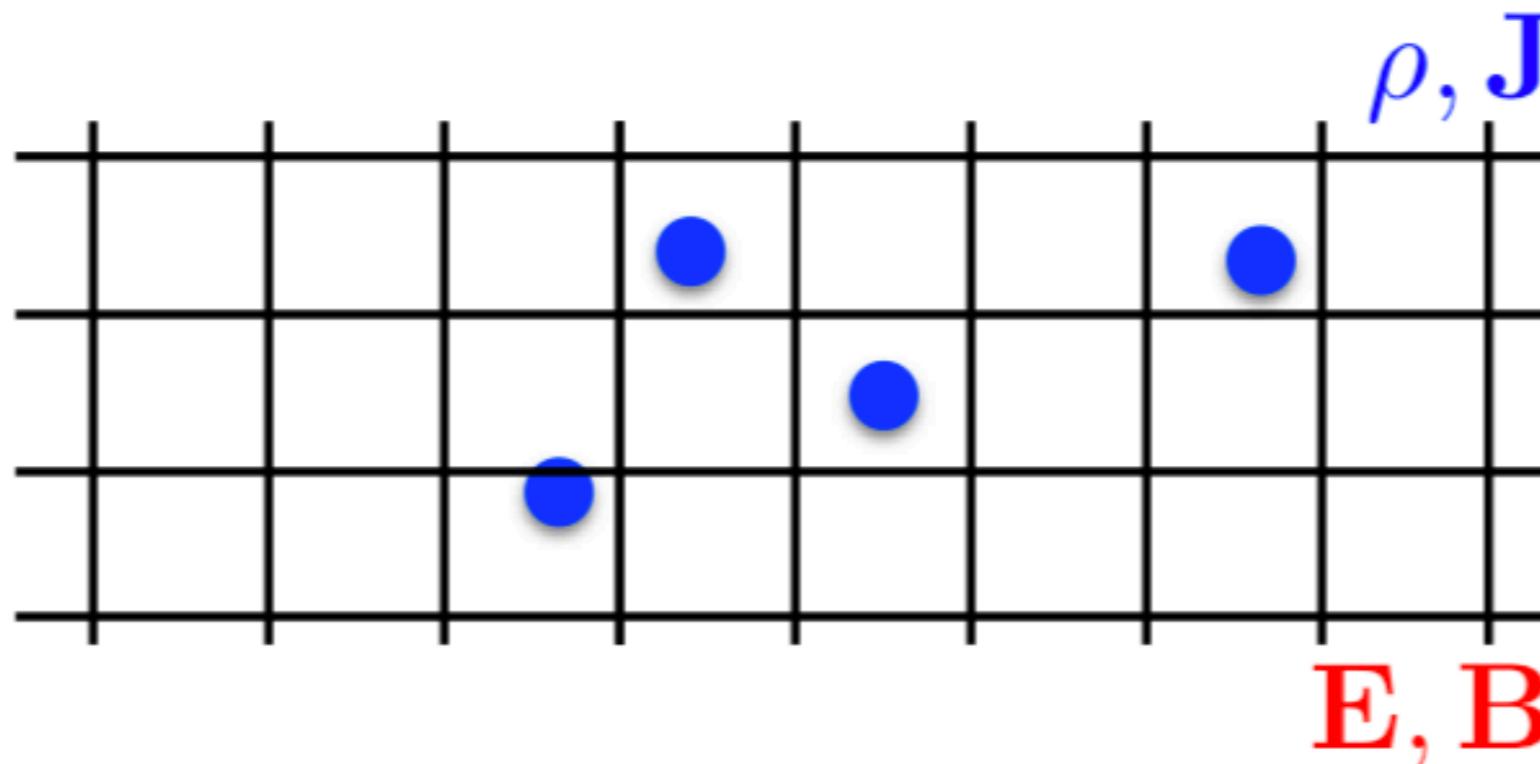
# Particle in Cell concept

Sample Plasma with Macro-Particles  
(1 Macroparticle = position, momentum, charge, ...)

+

Discretize space with computational grid

Define  $E$ ,  $B$ ,  $\rho$ ,  $J$  on the grid cells

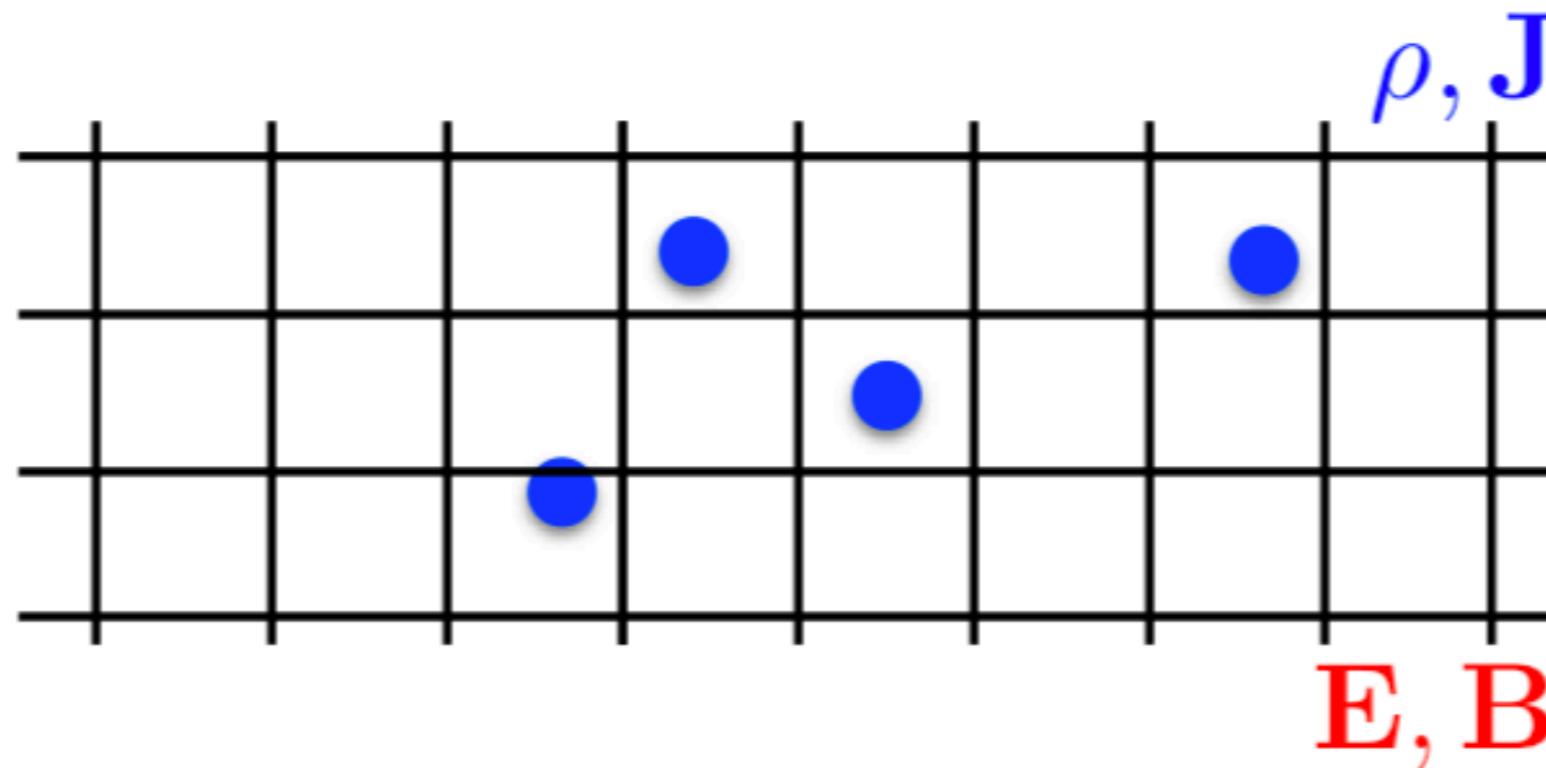


# Particle in Cell concept

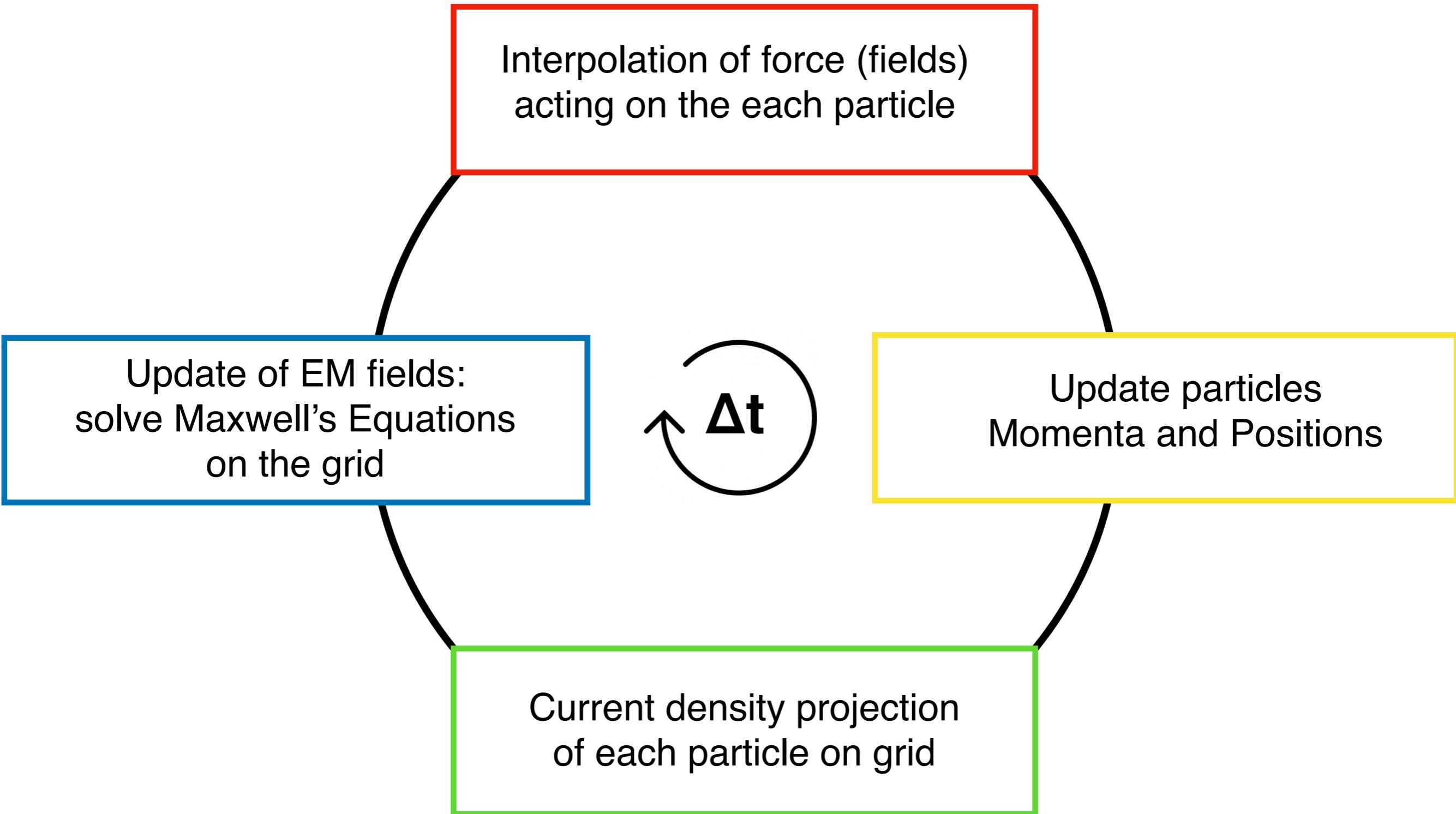
**Particles:** evolve following their equations of motion

+

**Electromagnetic Fields:** evolve following Maxwell's Equations



# Particle in Cell modelling is self-consistent



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A collaborative, open source  
multi-purpose Particle in Cell code  
<https://smileipic.github.io/Smilei/index.html>

## User-friendly

- online: documentation, tutorials
- Python input/output
- quick visualisation library
- teaching platform
- bi-annual training workshop

## High Performance

- MPI + OpenMP parallelization
- dynamic load balancing
- adaptive vectorization

## Multi-physics

- 1D, 2D, 3D, quasi-3D geometries
- ionization, collisions, strong-field QED
- laser envelope model
- relativistic beam field initialization



## High quality

- developers: experts of physics and HPC
- continuously benchmarked
- GitHub bug reporting
- OpenPMD standard



# Additional material

Extensive Documentation  
( Installation, Use, Postprocessing, ...)

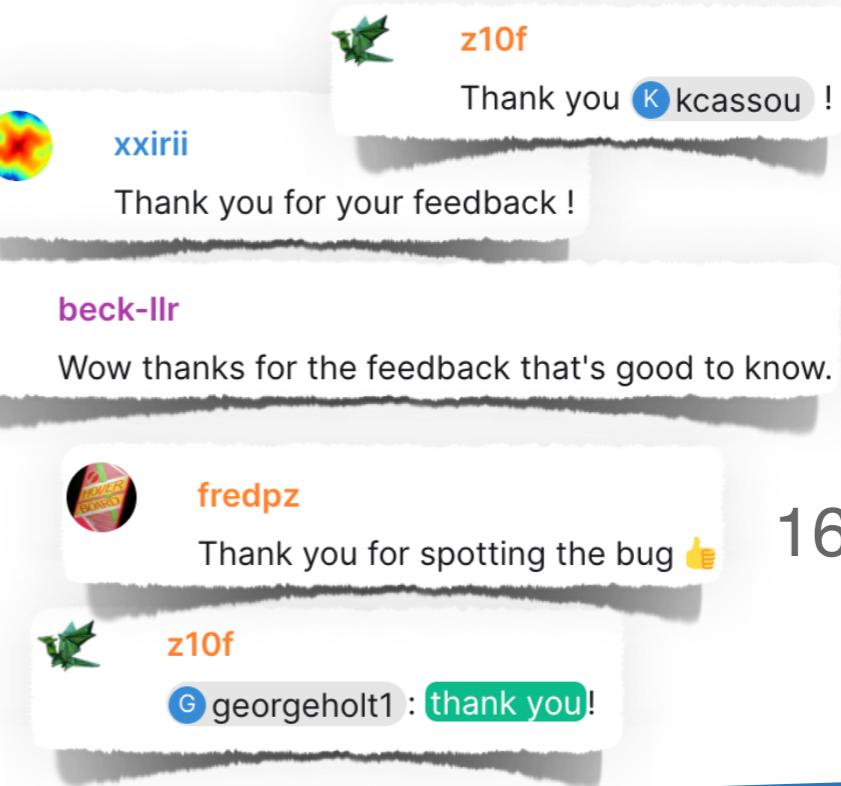
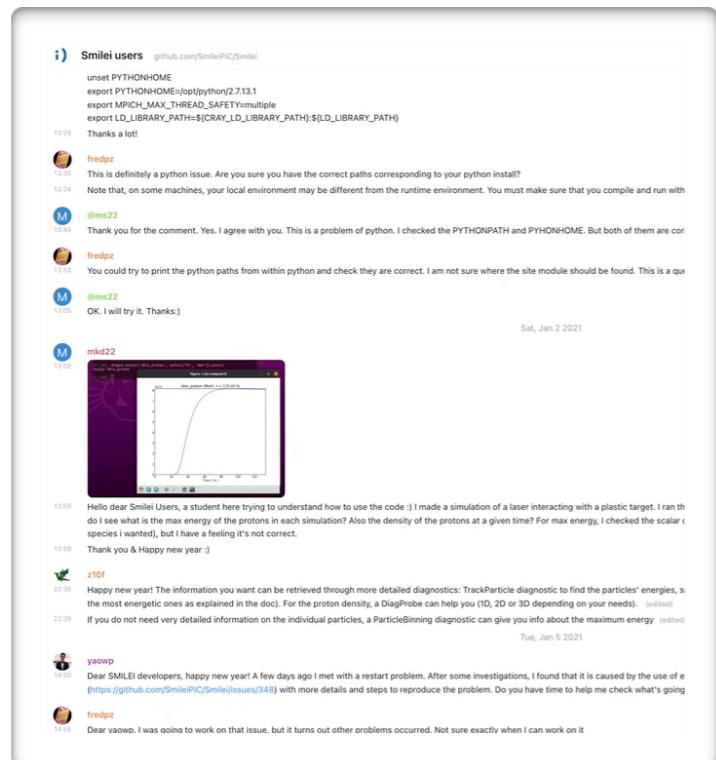


<https://smileipic.github.io/Smilei/index.html>

<https://smileipic.github.io/tutorials/>

Presentations from the 3rd Training Workshop <https://indico.math.cnrs.fr/event/6911/>

Questions? We answer on the Element chat



1600 messages  
in 2021

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# Normalised units for the Vlasov Maxwell system

Velocity

$$c$$

Charge

$$e$$

Mass

$$m_e$$

Momentum

$$m_e c$$

Energy

$$m_e c^2$$

Time

$$1/\omega_r$$

Length

$$c/\omega_r$$

Number Density

$$\epsilon_0 m_e \omega_r^2 / e^2$$

Electric Field

$$m_e \omega_r c / e$$

Magnetic Field

$$m_e \omega_r / e$$

Fixed

Need to  
choose  $\omega_r$

For the practical (input and output), we choose  $\omega_r = 2\pi c/\lambda_0$   
corresponding to  $\lambda_0 = 0.8 \mu\text{m}$  (Ti:Sa laser system)

# Example of input file

```
Main(  
    geometry      = "1Dcartesian",  
    timestep      = 0.009,  
    cell_length   = [0.01],  
    ...  
)
```

```
x_center_plasma = 200.
```

```
def my_density_profile(x):  
    return exp(-(x-x_center_plasma)**2)
```

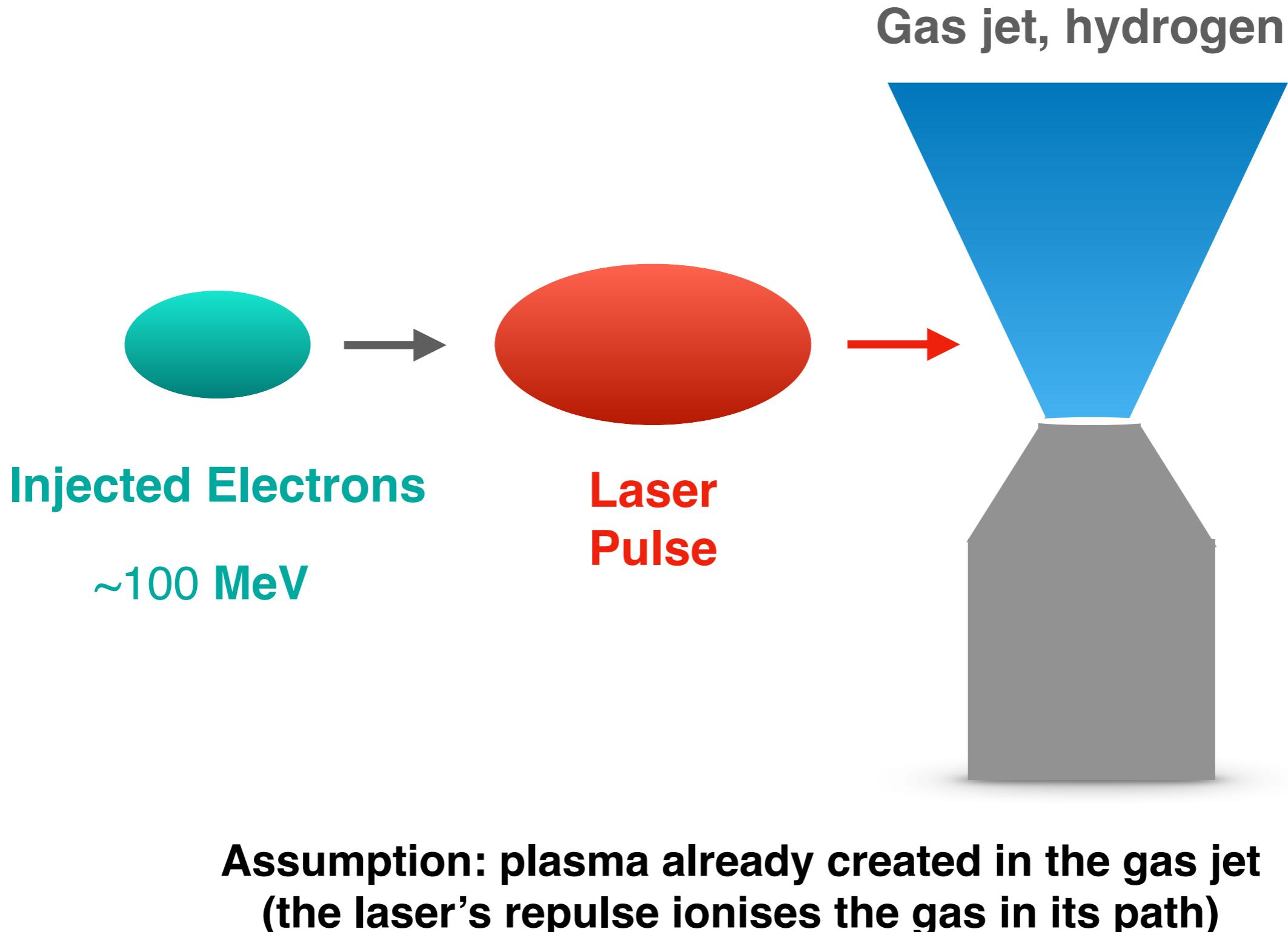
```
Species(  
    name          = "electron",  
    charge        = -1.,  
    mass          = 1.,  
    particles_per_cell = 100,  
    number_density = my_density_profile  
    ...  
)
```

- Normalised units
- Quantities can be computed at runtime

Laser/Plasma profiles = Functions (also user-defined)

If Python can read it, SMILEI can read it

# Case study: LWFA with external injection



# Practical lesson: how does it work?

- Read **in detail** the handouts in **Handouts.pdf**
- Set-up the machine following the instructions in sections 1-3 of **Instructions\_Cluster.pdf**
- Solve the exercises in **Handouts.pdf** progressively
- Run simulations when necessary, following the instructions in **Handouts.pdf** (blocks to uncomment etc.)  
(for a guide to run and analyse simulations, sections 3,4,5 of **Instructions\_Cluster.pdf**)
- Fill the answers report

# Suggestions

- Again read **in detail** the handouts
- **Understand** the physical set-up
- **Any doubts? Ask the instructor**
- Solve the exercises **progressively**
- Create one folder for each simulation asked by the exercises  
**(to avoid losing data)**
- Copy, paste and adapt the post processing commands in the handouts
- When an image is asked for the report, save it or make a screenshot
- **Better do few exercises but understand them at 100%**

# Your resources

- Code [Smilei](#) + Postprocessing library [happi](#)
- Input file for the simulation:
  - [InputNamelist.py](#)
- 5 Postprocessing scripts:
  - [Laser\\_waist\\_theory\\_vs\\_Smilei.py](#)
  - [Ex\\_linear\\_theory\\_vs\\_Smilei.py](#)
  - [Compute\\_bunch\\_parameters.py](#)
  - [Follow\\_electron\\_bunch\\_evolution.py](#)
  - [Export\\_data.py](#) (needed if you want to export output to .txt files)
  - [Save\\_VTK.py](#) (not mandatory for the TP)
- Instructions for the configuration of the machine:
  - [Instructions\\_Cluster.pdf](#)
- Handouts with instructions for the practical ([Handouts.pdf](#)) with:
  - Physical explanations
  - Postprocessing instructions
  - Exercises

# Structure of the input file for the practical

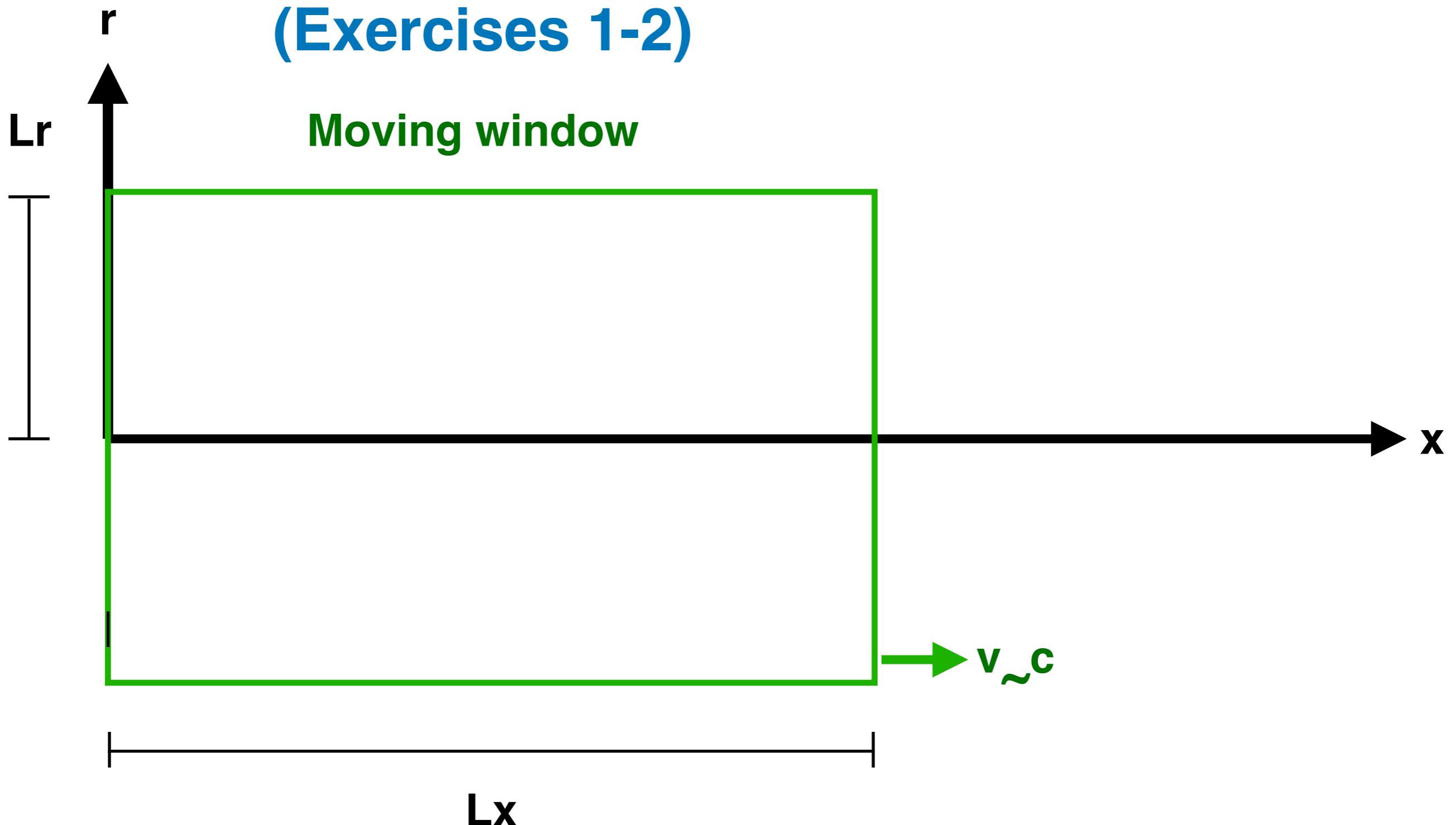
## (Exercises 1-2)

- Main Block
- Moving Window Block
- Laser Envelope Block
- Species Block (for the plasma)
- Species Block (for the relativistic electron bunch)
- Diagnostic Blocks (for Postprocessing)

Blue: Active

Gray: Commented = Inactive

# Simulation setup: simulation window



# Structure of the input file for the practical

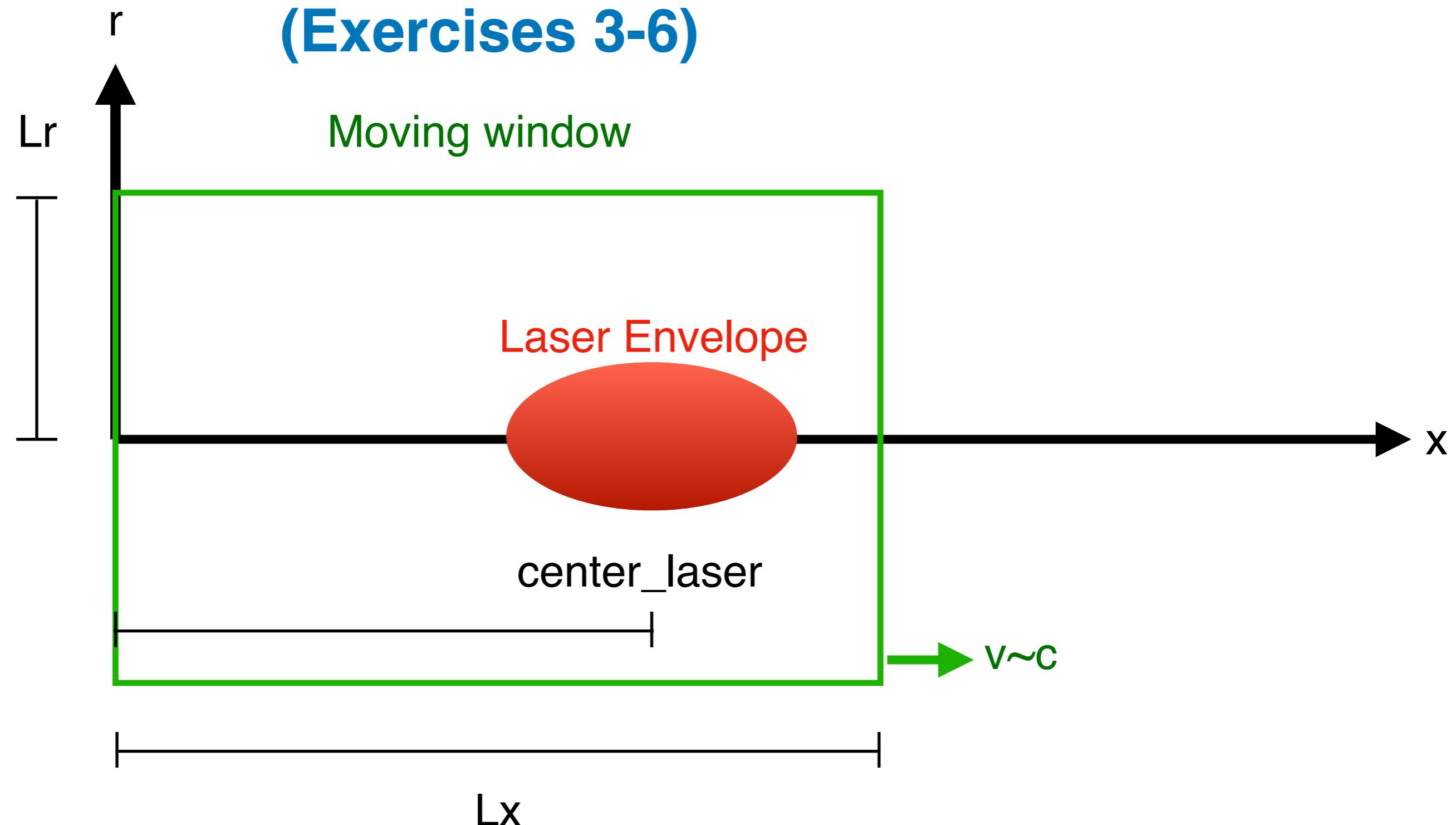
## (Exercises 3-6)

- Main Block
- Moving Window Block
- Laser Envelope Block
- Species Block (for the plasma)
- Species Block (for the relativistic electron bunch)
- Diagnostic Blocks (for Postprocessing)

Blue: Active

Gray: Commented = Inactive

# Simulation setup: laser in vacuum



# Structure of the input file for the practical

## (Exercises 7-10)

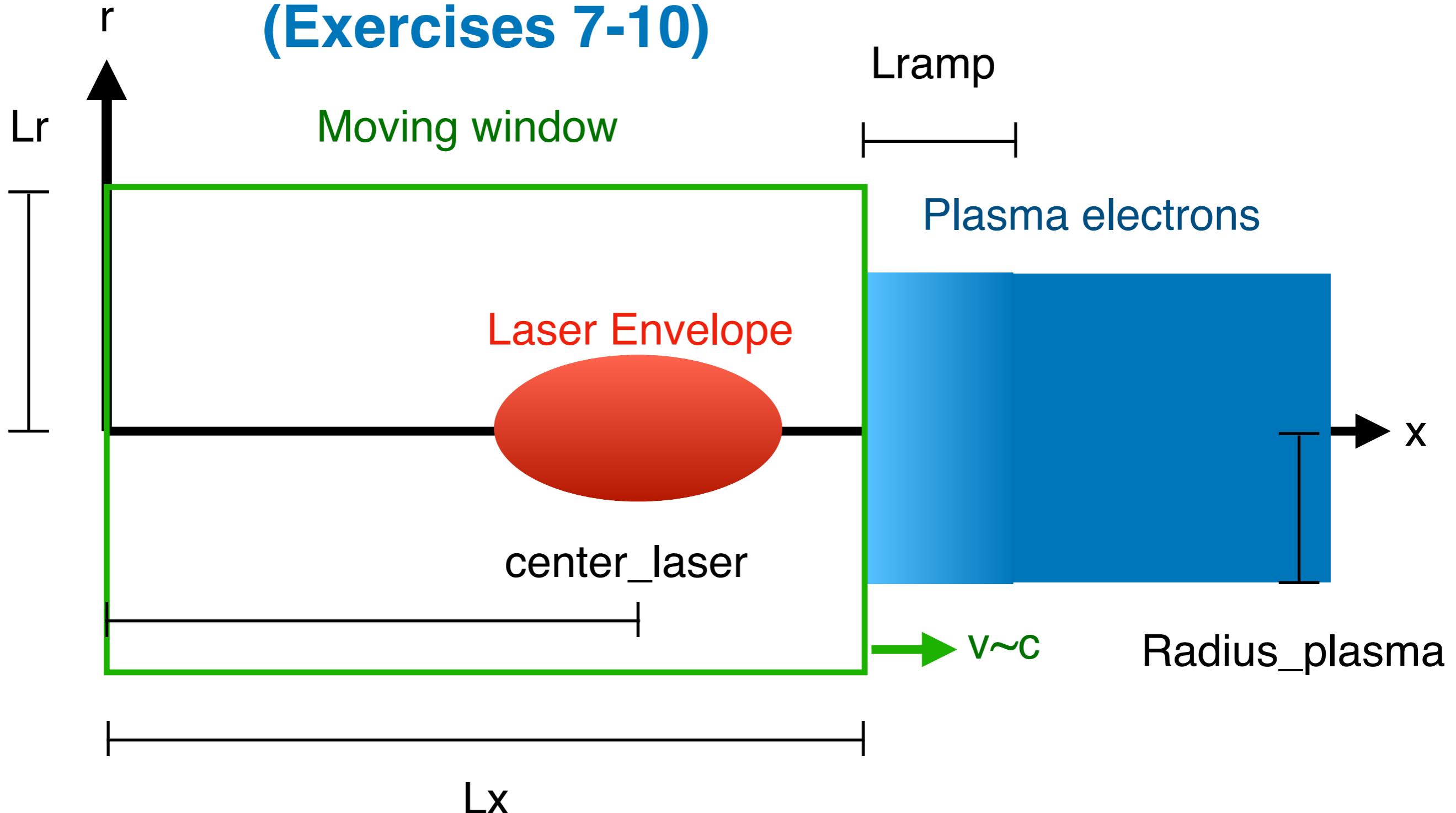
- Main Block
- Moving Window Block
- Laser Envelope Block
- Species Block (for the plasma)
- Species Block (for the relativistic electron bunch)
- Diagnostic Blocks (for Postprocessing)

Blue: Active

Gray: Commented = Inactive

# Simulation setup: laser in plasma

## (Exercises 7-10)



# Structure of the input file for the practical

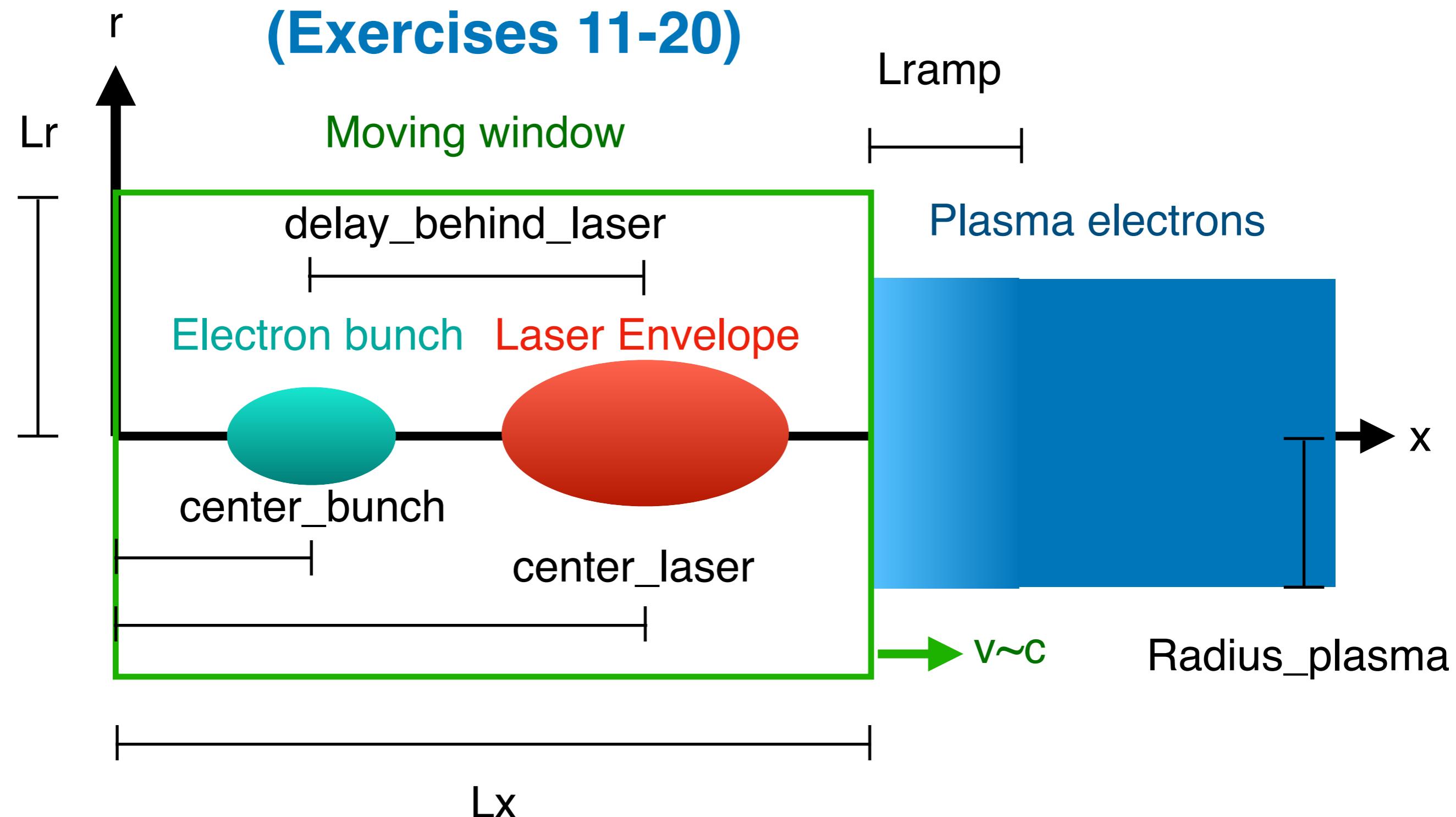
## (Exercises 11-20)

- Main Block
- Moving Window Block
- Laser Envelope Block
- Species Block (for the plasma)
- Species Block (for the relativistic electron bunch)
- Diagnostic Blocks (for Postprocessing)

Blue: Active

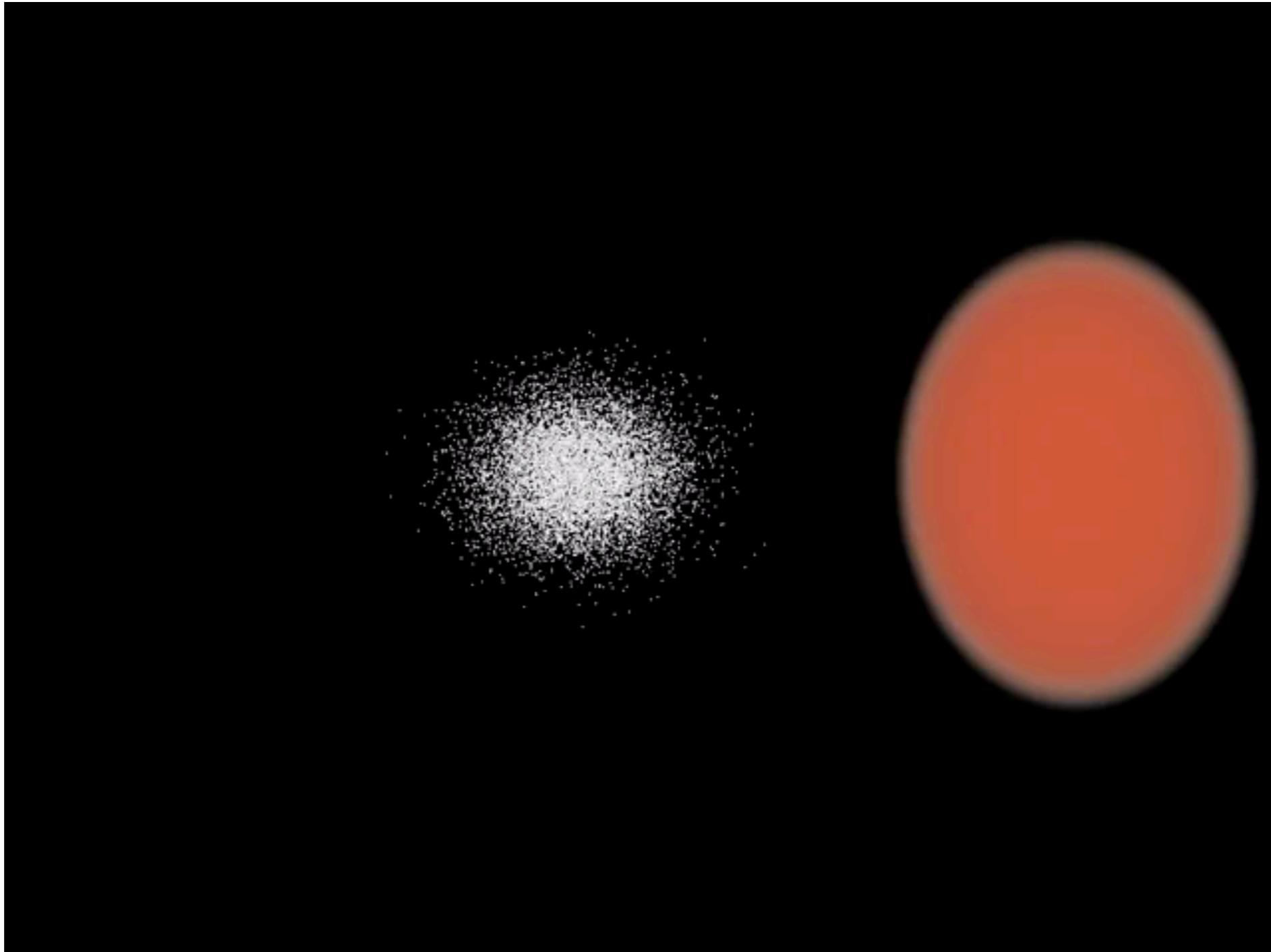
Gray: Commented = Inactive

# Simulation setup: LWFA with external injection



# 3D rendering of LWFA with external injection

**(Exercises 11-20)**



# Postprocessing for the practical

Included in the code you have a Python postprocessing library

```
$ make happy  
$ ipython  
In [1]: import happy
```

**Diagnostics available for this practical:**

- 1D Probe on the x axis (electromagnetic fields, density)
- 2D Probe on the xy plane (electromagnetic fields, density)
- DiagTrackParticles for the bunch electrons (phase space data)

**Tips:**

- Open the Ipython interface
- Copy and paste the happy commands from the handouts
- Adapt them for your purposes
- Use the post processing scripts available (see handouts)

# **Questions?**

# Additional Slides

## Some current LWFA projects and research themes (1)

### LWFA with ionisation injection



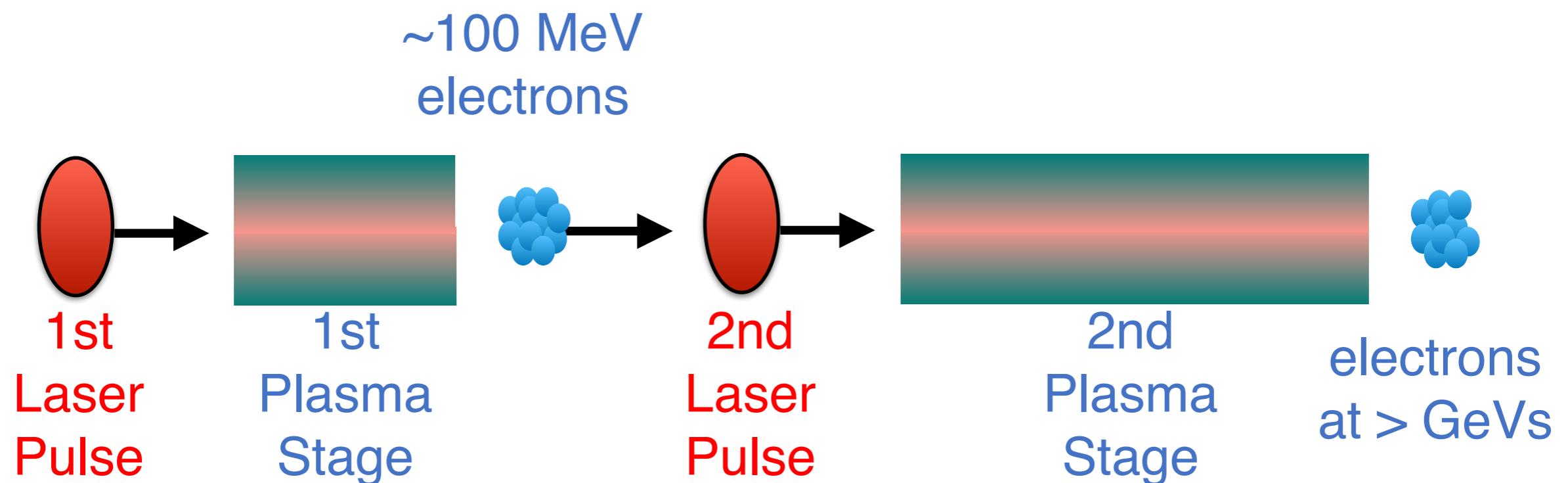
Relativistic plasma waves  
Nitrogen electrons

Laser Pulse driving waves  
and ionising nitrogen

## Some current LWFA projects and research themes (2)

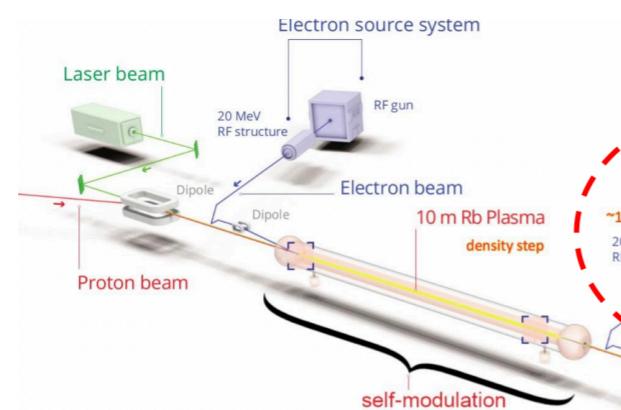
### Making LWFA scalable: Multi-staging

Concept for multi-stage experiments at



# Some current LWFA projects and research themes (3)

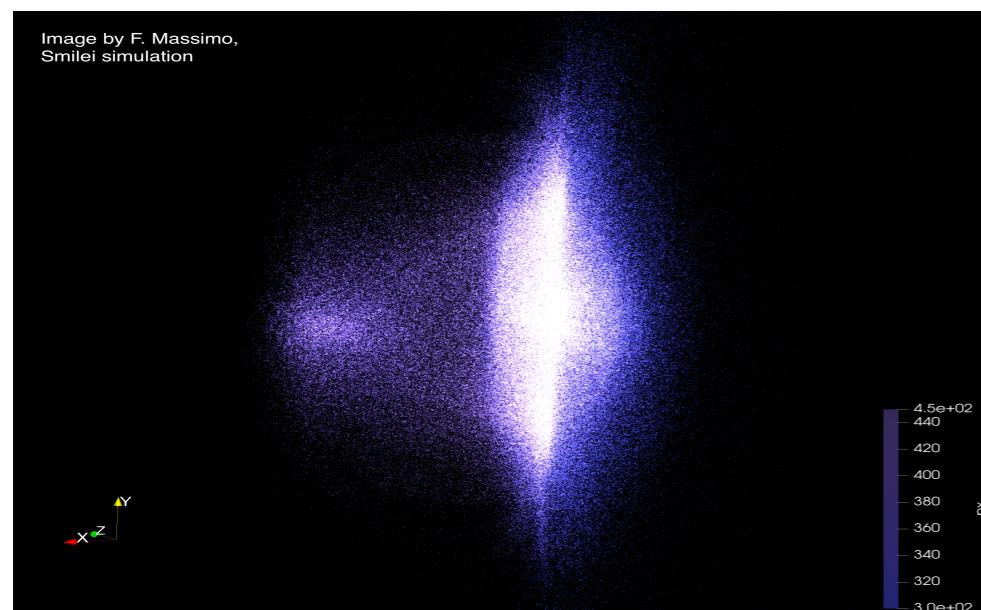
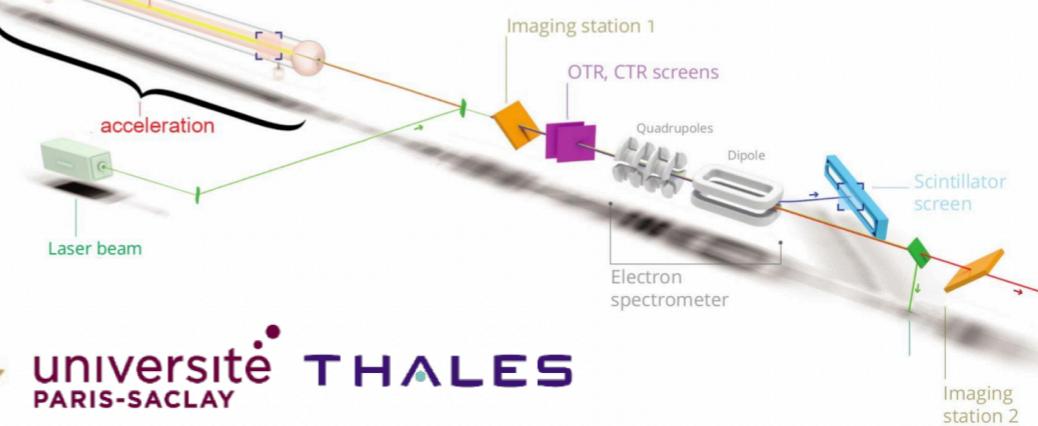
## Electron Accelerator driven by a Reliable Laser for Industrial uses (EARLI)



Can we use  
a laser Wakefield accelerator  
as electron injector in project



Collaboration with



Objective: build a LWFA  
to inject an electron beam  
in proton-driven waves  
propagating in a 10 m plasma

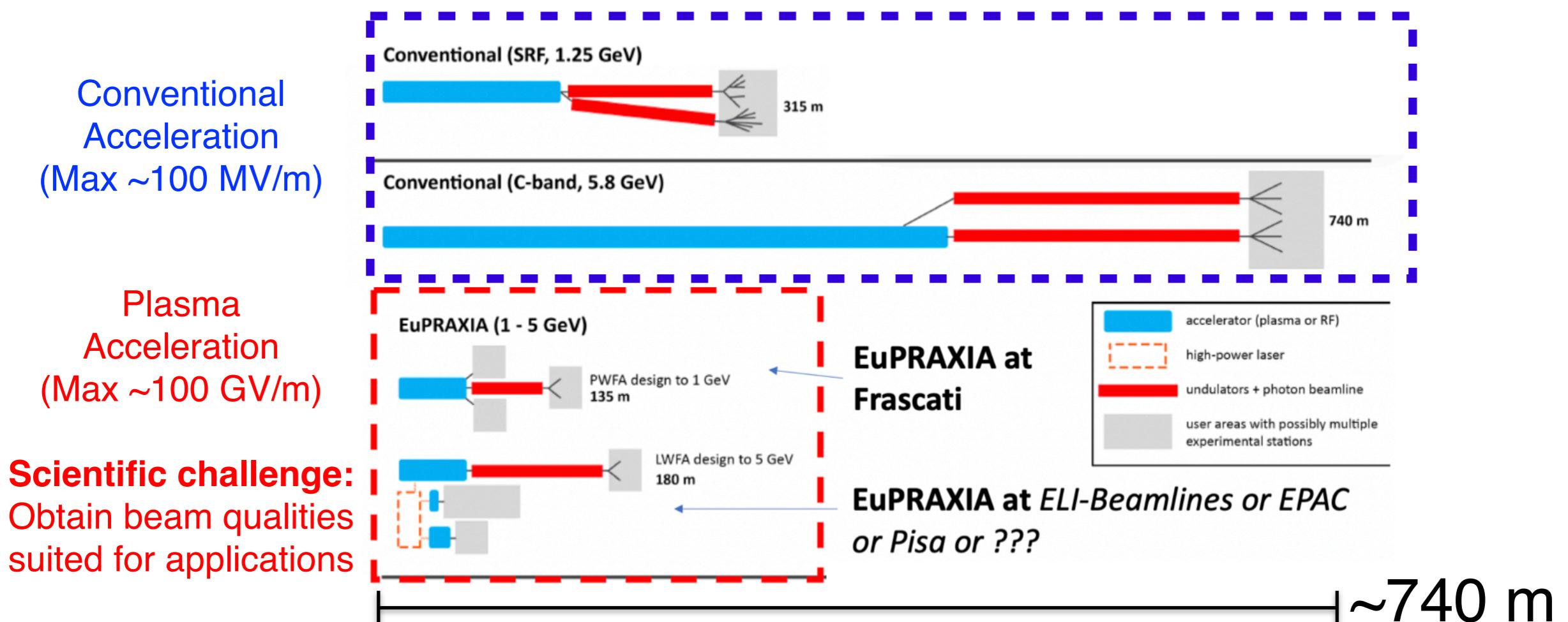
Electron beam at plasma exit, Smilei simulation

# Some current LWFA projects and research themes (4)



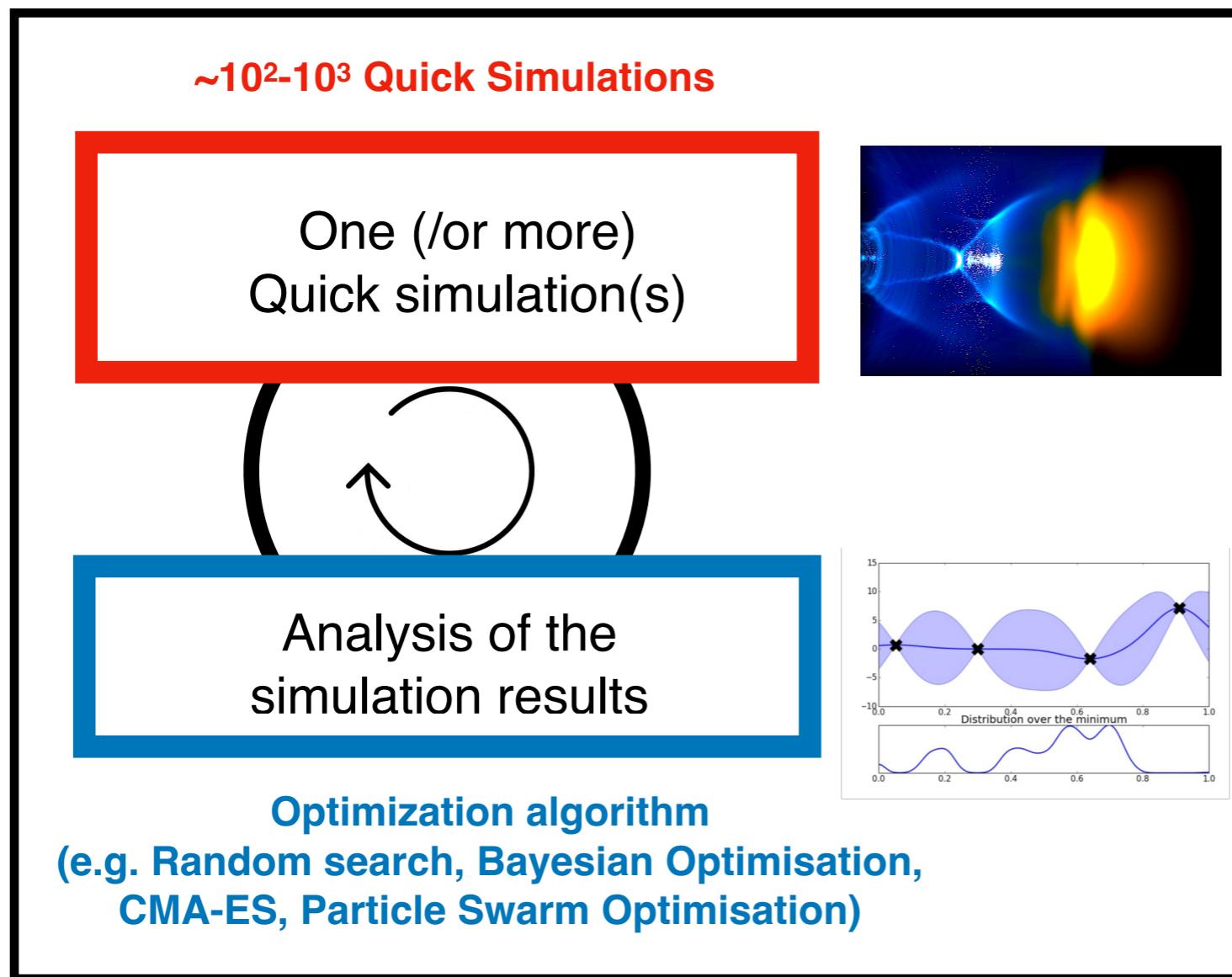
**Objective:** build a European, large scale, distributed plasma acceleration facility for users

R. Assmann, EuPRAXIA Preparatory Phase kick-off meeting (Nov 2022)

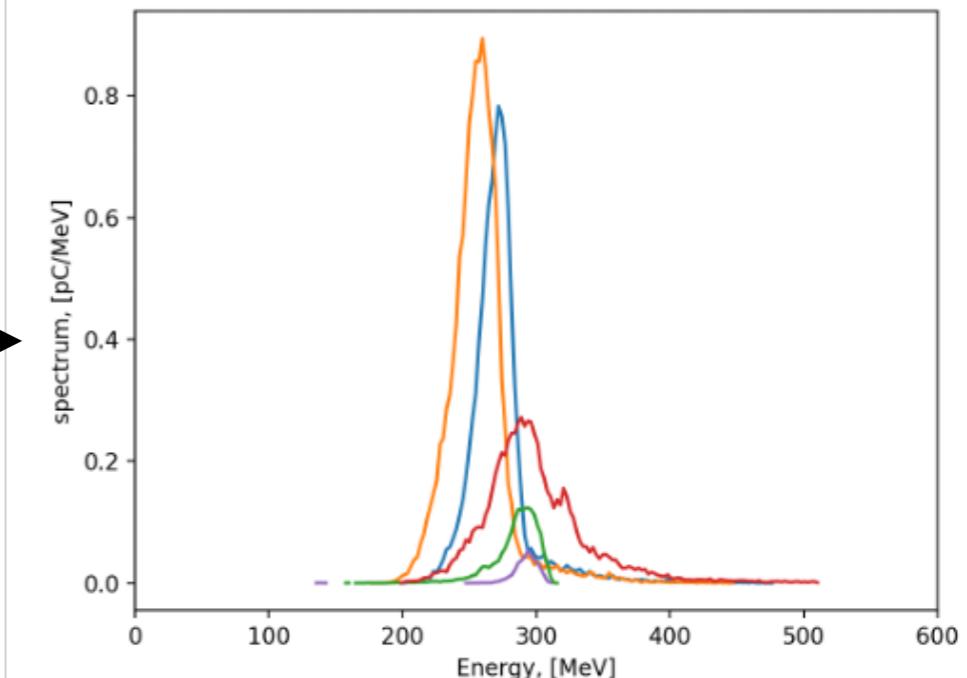


# Some current LWFA projects and research themes (5)

## Machine Learning and AI for LWFA



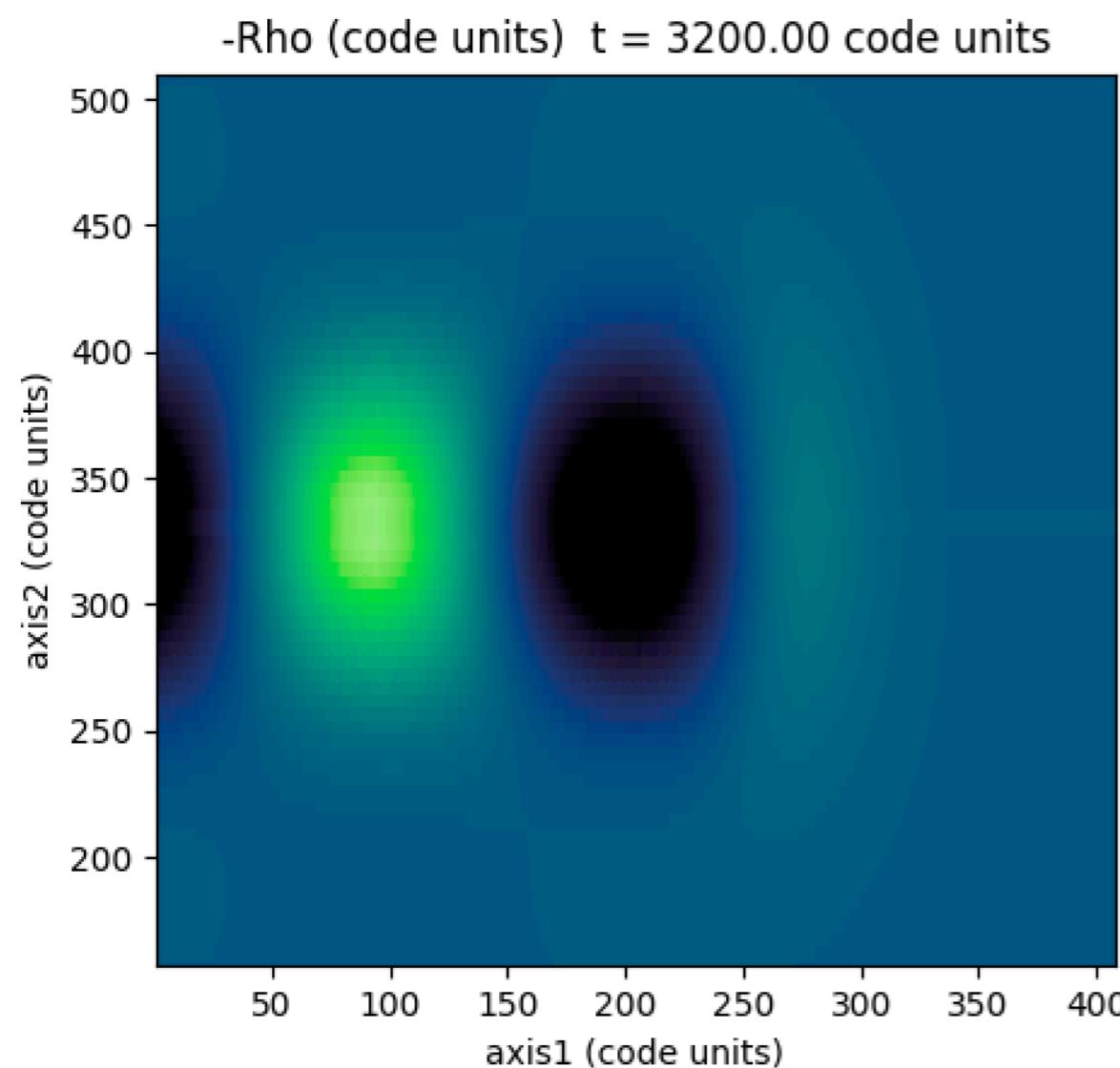
## Optimized Electron Spectra



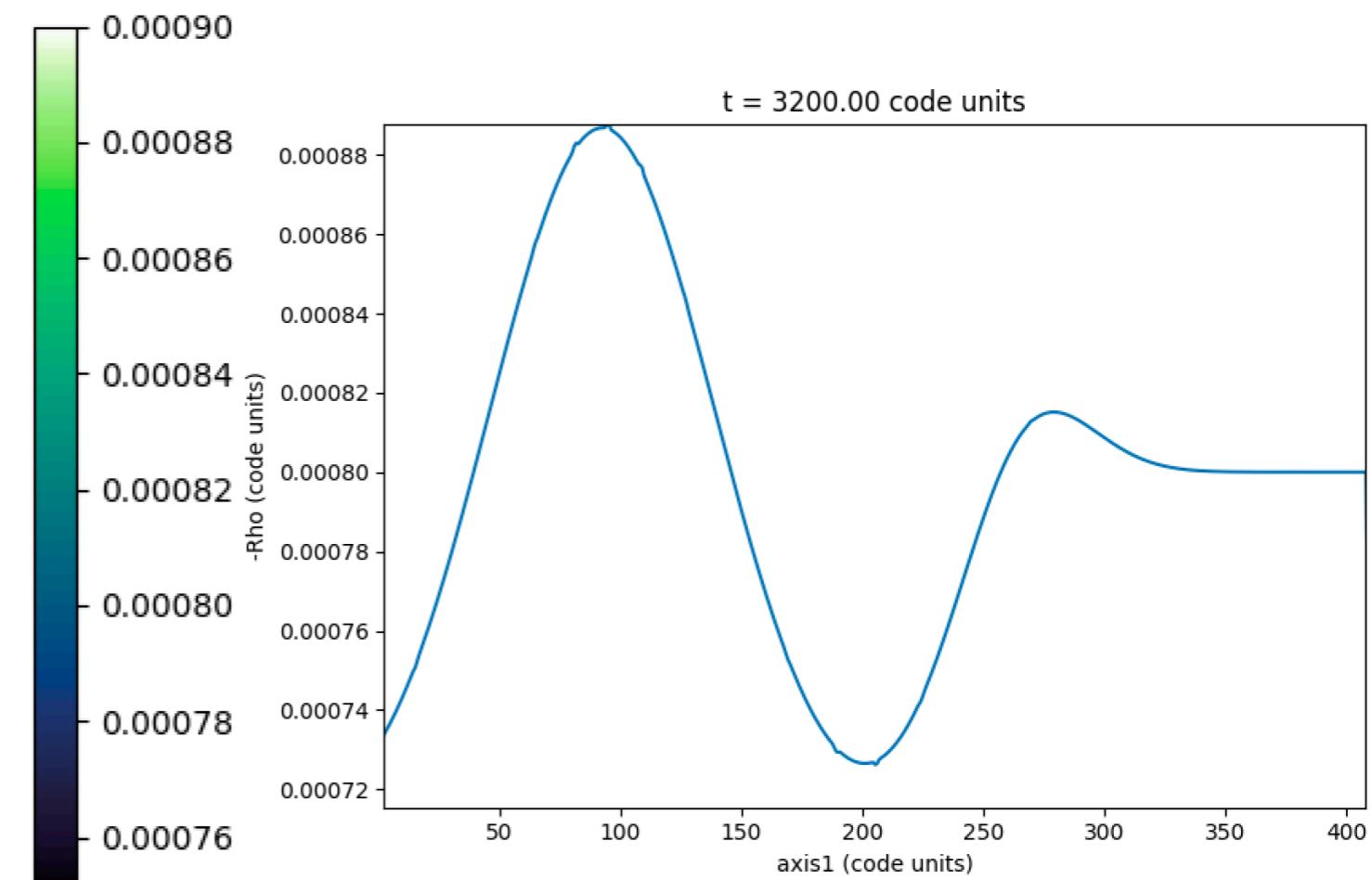
P. Drobniak and IJClab,  
2021

# Exercise 10: Nonlinear plasma waves, density

$a_0 \ll 1$  :  
Linear regime  
(Sinusoidal plasma waves)



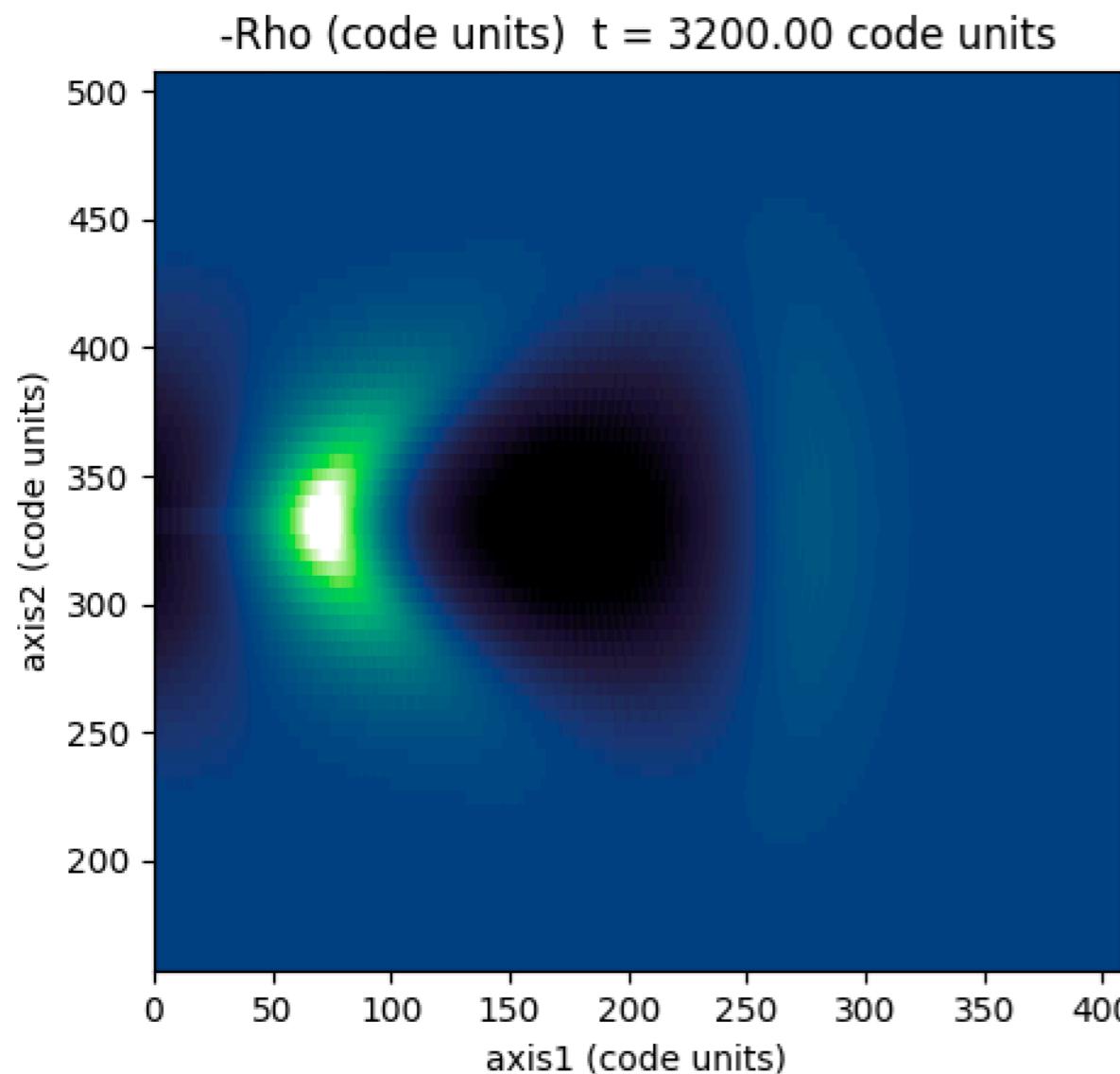
2D charge density



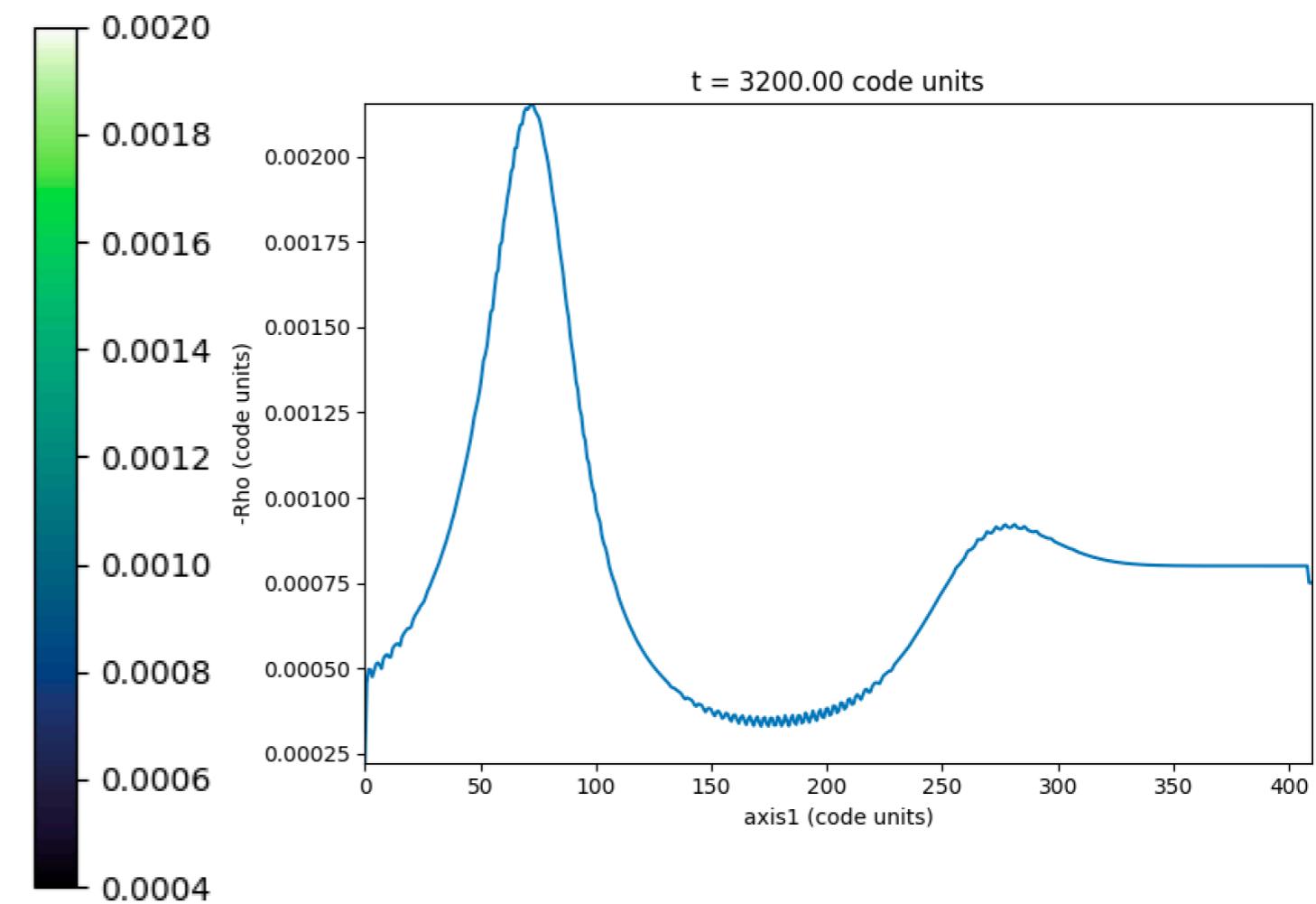
1D charge density  
on propagation axis

# Exercise 10: Plasma waves regimes, density

$a_0 \lesssim 1$  :  
Weakly nonlinear regime



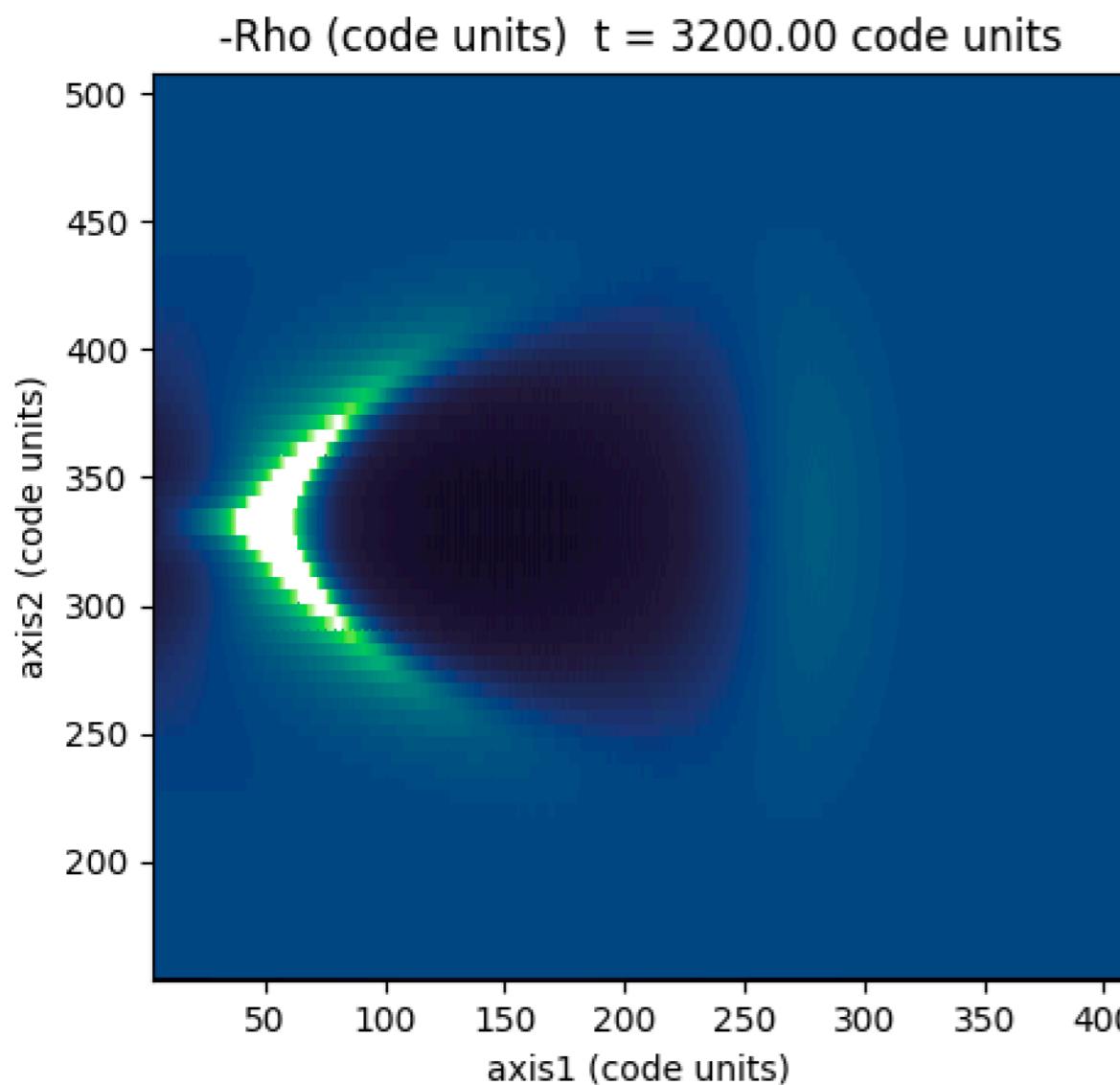
2D charge density



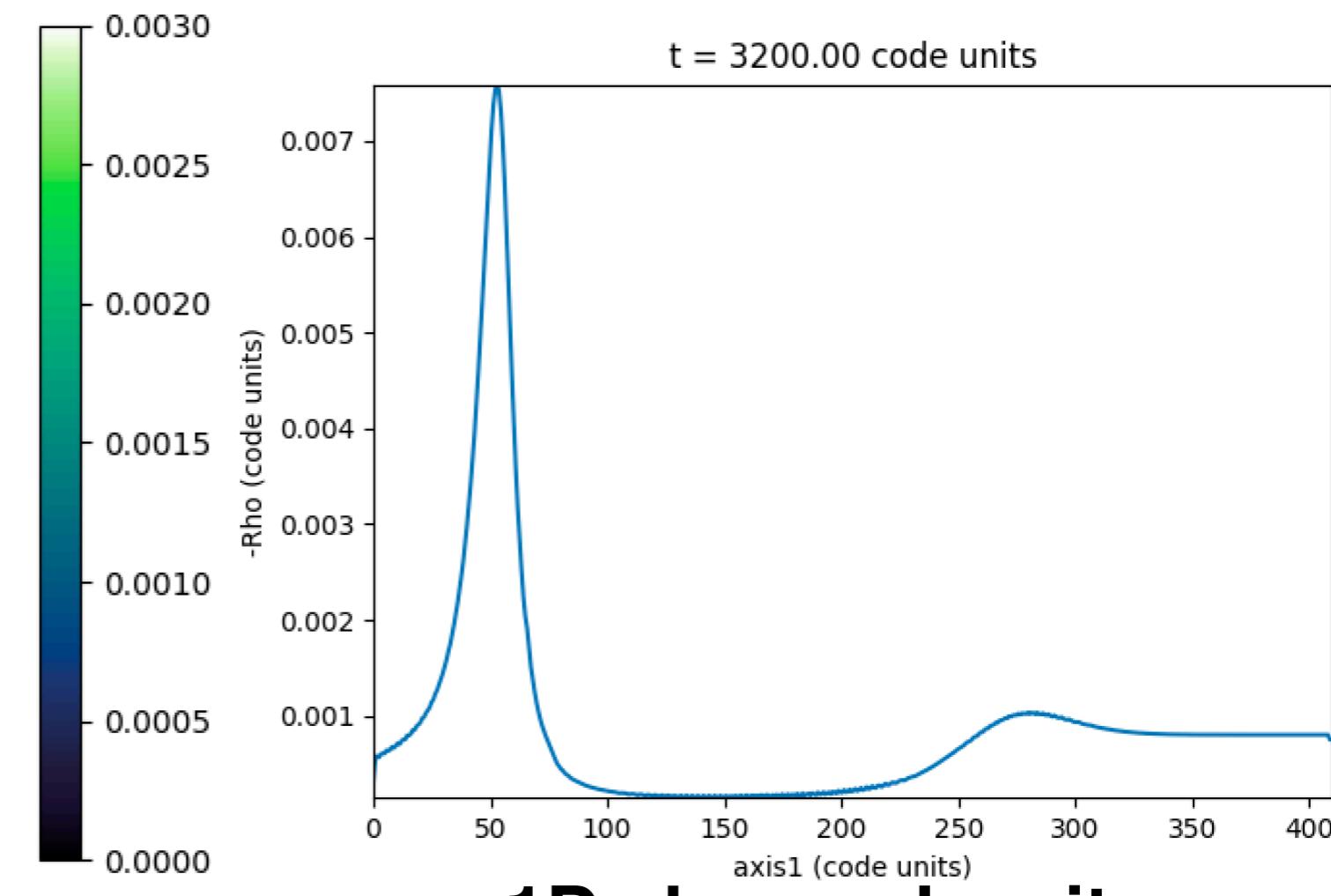
1D charge density  
on propagation axis

# Exercise 10: Plasma waves regimes, density

$a_0 > 1 :$   
**Nonlinear regime  
(Bubble-like waves)**



**2D charge density**



**1D charge density  
on propagation axis**

# Exercise 10: Plasma waves regimes, Ex on axis

```
import happy
S1 = happy.Open("path/to/sim1"); Ex1 = S1.Probe.Probe0("Ex",timesteps=1000)
S2 = happy.Open("path/to/sim2"); Ex2 = S2.Probe.Probe0("Ex",timesteps=1000)
S3 = happy.Open("path/to/sim3"); Ex3 = S3.Probe.Probe0("Ex",timesteps=1000)
happy.multiPlot(Ex1,Ex2,Ex3,figure=3)
```

