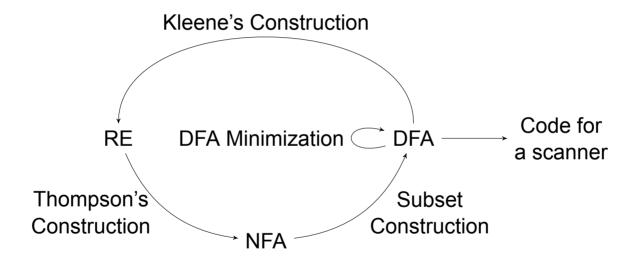
From Regular Expression to Scanner



From Regular Expression to Scanner

The construction of a Deterministic FA (DFA) from an RE follows three steps.

- 1. Thompson's construction derives a Nondeterministic FA (NFA) from a RE
- 2. Subset construction builds a DFA that simulates the NFA
- 3. **Hopcroft's algorithm** minimizes the DFA

Nondeterministic Finite Automata

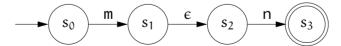
A Nondeterministic Finite Automaton (NFA) is an FA that allows...

- transitions on the empty string, i.e., ϵ -transitions that do not advance the input, and
- states that have multiple transitions on the same character.

We can use transitions on ϵ to combine FAs and form FAs for more complex REs.

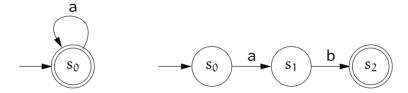


For example, using a ϵ -transition, the FAs for the REs m and n can be combined into an FA for the RE mn as follows.

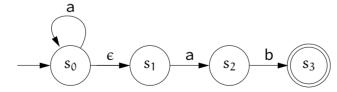


Nondeterministic Finite Automata

Merging two FAs with an ϵ -transition can complicate our model of how FAs work. Consider the FAs for the languages a^* and ab.



We can combine them with an ϵ -transition to form a FA for a*ab.



Nondeterministic Finite Automata

The ϵ -transition gives the FA two distinct transitions out of s_0 on the letter a.

- it can take the transition $s_0 \stackrel{a}{\rightarrow} s_0$, or
- the two transitions $s_0 \xrightarrow{\epsilon} s_1$ and $s_1 \xrightarrow{a} s_2$

Which one is correct? Consider the strings aab and ab, which the NFA should both accept.

aab
$$s_0 \xrightarrow{a} s_0, s_0 \xrightarrow{\epsilon} s_1, s_1 \xrightarrow{a} s_2, s_2 \xrightarrow{b} s_3$$

ab $s_0 \xrightarrow{\epsilon} s_1, s_1 \xrightarrow{a} s_2, s_2 \xrightarrow{b} s_3$

 \longrightarrow The correct transition out of s_0 depends on the characters that follow a.

Nondeteministic Finite Automata

To understand how a NFA operates, we need a set of rules that describe its behavior. Historically, two distinct models have been given for the behavior of a NFA.

1. NFA guesses correct transition at each point

- NFA is omniscient
- follow transition that leads to an accepting state, if such a transition exists

2. NFA pursues all paths concurrently

- NFA clones itself to pursue each possible transition
- specific set of states in which the NFA is active is called its configuration
- NFA accepts the string if reaches a configuration in which it has exhausted the input and one or more of the clones has reached an accepting state

Equivalence of NFA and DFA

NFAs and DFAs are equivalent in their expressive power

- → any DFA is a special case of an NFA, *i.e.*, an NFA is at least as powerful as a DFA
- ← any NFA can be simulated by a DFA by subset construction (*cf.* Slide 80)

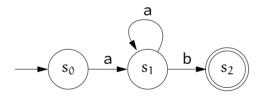
The intuition behind **subset construction** is simple: to simulate the behavior of the NFA, we need a DFA with a state for each configuration of the NFA.

- under the second model of NFA behavior, the NFA has a finite number of clones
- this number can be bounded: an NFA with $\mathfrak n$ states produces at most $|\Sigma|^n$ configurations
- to simulate the NFA, we need a DFA with a state for each configuration of the NFA

Note The DFA that simulates the NFA still runs in time proportional to the length of the input string, *i.e.*, we have a potential space problem, but not a time problem.

Equivalence of NFA and DFA

Since NFAs and DFAs are equivalent, we can construct the following DFA for a*ab.



It relies on the observation that a*ab specifies the same regular language as aa*b.

From RE to NFA: Thompson's Construction

As a first step in moving from an RE to an implemented scanner, we must derive an NFA from the RE.

Thompson's construction builds an NFA from an RE in a straightforward way

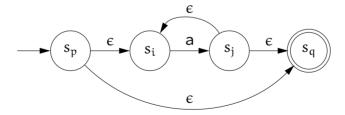
- one template for building each NFA that corresponds to a single-letter RE
- NFA transformations for RE operators: concatenation, alternation, and closure



From RE to NFA: Thompson's Construction



NFA for a | b

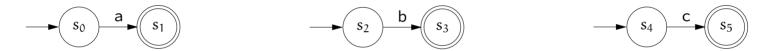


NFA for a*

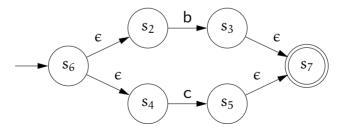
Example: Thompson's Construction

As an example, we use Thompson's Construction to derive the NFA for the RE a ($b \mid c$)*.

1. The construction begins by first building NFAs for a, b, and c

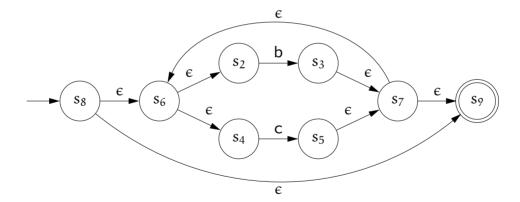


2. Since parentheses have highest precedence, it then builds the NFA for the expression enclosed in parentheses, $b \mid c$



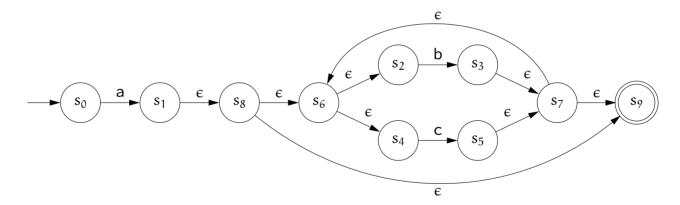
Example: Thompson's Construction

 Since closure has higher precedence than concatenation, the construction next builds the closure, (b | c)*



Example: Thompson's Construction

4. Finally, it concatenates the NFA for a to the NFA for (b | c)*

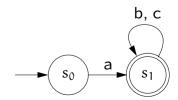


From RE to NFA: Thompson's Construction

The NFAs derived from Thompson's construction have **several specific** properties that simplify an implementation.

- each NFA has one start state and one accepting state
- no transition, other than the initial transition, enters the start state
- no transition leaves the accepting state
- an ϵ -transition always connects two states that were, earlier in the process, the start state and the accepting state of NFAs for some component REs
- each state has at most two entering and two exiting ϵ -moves, and at most one entering and one exiting move on a symbol in the alphabet

Not to rain on your parade, but in general Thompson's construction builds very large NFAs with many ϵ -moves that are unneeded.



From NFA to DFA: Subset Construction

Since DFA execution is much easier to simulate than NFA execution, we convent the NFA built by Thompson's construction into a DFA that recognizes the **same** language.

The **Subset Construction** takes as input an NFA $(N, \Sigma, \delta_N, n_0, N_A)$ and produces a DFA $(D, \Sigma, \delta_D, d_0, D_A)$

- the NFA and the DFA use the same alphabet Σ
- the DFA's start state d_0 and its accepting states D_A will emerge from the construction
- the complex part of the construction is
 - the derivation of the set of DFA states D from the NFA states N, and
 - the derivation of the DFA transition function $\delta_{\rm D}$

From NFA to DFA: Subset Construction

```
1 q_0 \leftarrow \epsilon-closure(\{n_0\})
2 Q \leftarrow q_0
3 WorkList \leftarrow {q₀}
4 while WorkList \neq \emptyset do
     remove q from WorkList
     foreach character c \in \Sigma do
        t \leftarrow \epsilon-closure(Delta(q,c))
        T[q,c] \leftarrow t
         if t \notin Q then
            add t to Q
10
            add t to WorkList
11
```

Algorithm constructs a set Q whose elements q_i are each a **subset** of N, *i.e.*, each $q_i \in 2^N$

 ϵ -closure(S) examines each state $s_i \in S$ and adds to S any state reachable by following one or more ϵ -transitions from s_i

Delta (q, c) applies the transition function of the NFA to each state s in q and returns

$$\bigcup_{s\in\mathfrak{q}}\delta_N(\mathfrak{q},c)$$

From NFA to DFA: Subset Construction

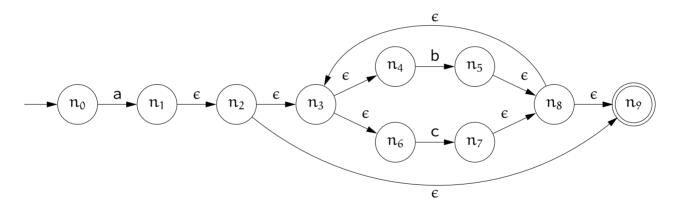
Notice

- each q represents a valid configuration of the original NFA
- the set Q grows **monotonically** and can become as large as $|2^N|$ distinct states
- when the algorithm halts...
 - Q contains all of the valid configurations of the NFA
 - each $q_i \in Q$ corresponds to **one** state $d_i \in D$ in the DFA
 - T holds **all** of the transitions between them

Building the DFA from Q and T is therefore straightforward

- each $q_i \in Q$ needs a state $d_i \in D$ to represent it
- if q_i contains an accepting state of the NFA, then d_i is an accepting state of the DFA
- the transition function δ_D is defined by the mapping from q_i to d_i in T
- the state constructed from q_0 becomes d_0 , the initial state of the DFA

Consider the NFA built for a (b | c)* and shown below with its states renumbered.



The algorithm takes the following steps

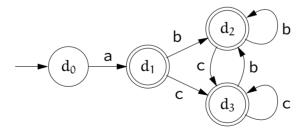
- 1. The initialization sets q_0 to ϵ -closure($\{n_0\}$), which is just n_0
- 2. The 1st iteration computes ϵ -closure(Delta(q_0, α)), which contains six NFA states, and ϵ -closure(Delta(q_0, b)) and ϵ -closure(Delta(q_0, c)), which are empty
- 3. The 2^{nd} iteration examines q_1 and produces two configurations, named q_2 and q_3
- 4. The 3^{rd} iteration examines q_2 and constructs two configurations, which are identical to q_2 and q_3
- 5. The 4th iteration examines q_3 and it reconstructs q_2 and q_3 (like the 3rd iteration)

The following table sketches the steps that the subset construction algorithm follows.

Set Name	DFA States	NFA States	ϵ -closure(Delta (q,\circ))		
			а	b	С
q_0	d_0	n_0	$\left\{\begin{array}{l} n_{1}, n_{2}, n_{3}, \\ n_{4}, n_{6}, n_{9} \end{array}\right\}$	Ø	Ø
q_1	d_1	$\left\{\begin{array}{l} n_{1}, n_{2}, n_{3}, \\ n_{4}, n_{6}, n_{9} \end{array}\right\}$	Ø	$\left\{\begin{array}{l} n_5, n_8, n_9, \\ n_3, n_4, n_6 \end{array}\right\}$	$\left\{ \begin{array}{l} n_7, n_8, n_9, \\ n_3, n_4, n_6 \end{array} \right\}$
q_2	d_2	$\left\{\begin{array}{l}n_5,n_8,n_9,\\n_3,n_4,n_6\end{array}\right\}$	Ø	q_2	q_3
q_3	d_3	$\left\{\begin{array}{l}n_7,n_8,n_9,\\n_3,n_4,n_6\end{array}\right\}$	Ø	q ₂	q_3

Finally, we can construct the resulting DFA as follows

- the states correspond to the DFA states from the table
- the transitions are given by the Delta operations that generate those states
- since the sets q_1 , q_2 , and q_3 all contain n_9 (the accepting state of the NFA), all three become accepting states in the DFA



Digression: Fix-Point Computation

Subset construction is an example of a **fixed-point computation**, a style of computation that arises regularly in computer science.

Fix-Point Computation

- characterized by the iterated application of a monotone function to some collection of sets drawn from a domain whose structure is known
- computation terminates when they reach a state where further iteration produces the **same answer**, *i.e.*, if a "fixed point" is reached
- Fixed-point computations play an important and recurring role in compiler construction

DFA to Minimal DFA: Hopcroft's Algorithm

The DFA that emerges from the subset construction can have a **large** set of states

- we can use an algorithm to minimize the number of states in the DFA
- this does not decrease the time needed to scan a string
- but it does decrease the size of the recognizer in memory

To minimize the number of states in a DFA $(D, \Sigma, \delta, d_0, D_A)$, we need an algorithm to detect if two states are **equivalent**, *i.e.*, if they produce the same behavior on any input string.

DFA to Minimal DFA: Hopcroft's Algorithm

The algorithm constructs a set partition $P = \{p_1, p_2, \dots, p_m\}$ of the DFA states by grouping together DFA states that have the **same behavior**.

Two DFA states $d_i, d_j \in p_s$ have the same behavior in response to all input characters

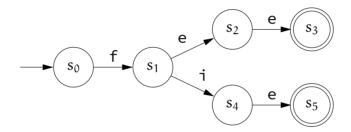
$$\forall p_s \in P; d_i, d_j \in p_s; c \in \Sigma : d_i \xrightarrow{c} d_x, d_j \xrightarrow{c} d_y \implies d_x = d_y$$

DFA to Minimal DFA: Hopcroft's Algorithm

Observations

- To minimize a DFA, each set $p_s \in P$ should be as large as possible, within the constraint of behavioral equivalence
- The initial partition contains two sets: $p_0 \in D_A$ and $p_1 \in \{D D_A\}$
- Since the algorithm never combines two partitions, this separation ensures that no set in the final partition contains both accepting and nonaccepting states
- To construct the new DFA from the final partition P, we
 - create a single state to represent each set p ∈ P
 - add the appropriate transitions between these new representative states
- The resulting DFA is minimal (the proof is beyond our scope)
- This algorithm is another example of a fixed-point computation
- The worst-case behavior occurs when each state in the DFA has different behavior

As a first example, consider a DFA that recognizes the language *fee* | *fie*, shown below.



By inspection, we can see that states s_3 and s_5 serve the same purpose

- both are accepting states entered only by a transition on the letter e
- neither has a transition that leaves the state
- We would expect the DFA minimization algorithm to discover this fact and replace them with a single state.

The algorithm takes the following steps

- 1. Initialization separates accepting from nonaccepting states: $\{\{s_3, s_5\}, \{s_0, s_1, s_2, s_4\}\}$
- 2. Step 1 examines $\{s_3, s_5\}$: neither state has an exiting transition, *i.e.*, no split occurs
- 3. Step 2 examines $\{s_0, s_1, s_2, s_4\}$: on charachter e, it splits $\{s_2, s_4\}$ out of the set
- 4. Step 3 examines $\{s_0, s_1\}$: on charachter f, it splits $\{s_1\}$ out of the set
- 5. Step 4 makes a final pass over the current partition $\{\{s_3, s_5\}, \{s_0\}, \{s_1\}, \{s_2, s_4\}\}$: no more splits occur and, therefore, the fix-point is reached

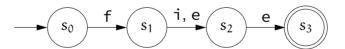
Note We assume that the various loops in the algorithm iterate over the sets of P and over the characters in $\Sigma = \{e, f, i\}$ in order.

The following summarizes the significant steps that occur in minimizing this DFA.

Step	Current		Examines	
	Partition	Set	Char	Action
0	$\{\{s_3, s_5\}, \{s_0, s_1, s_2, s_4\}\}$		_	
1	$\{\{s_3, s_5\}, \{s_0, s_1, s_2, s_4\}\}$	$\{s_3,s_5\}$	all	none
2	$\{\{s_3, s_5\}, \{s_0, s_1, s_2, s_4\}\}$	$\{s_0, s_1, s_2, s_4\}$	е	<i>split</i> {s ₂ , s ₄]
3	$\{\{s_3, s_5\}, \{s_0, s_1\}, \{s_2, s_4\}\}$	$\{s_0, s_1\}$	f	split {s₁}
4	$\{\{s_3, s_5\}, \{s_0\}, \{s_1\}, \{s_2, s_4\}\}$	all	all	none

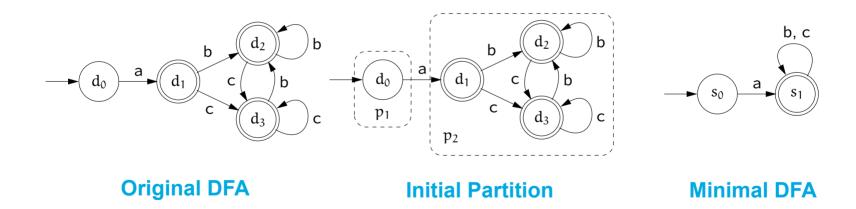
To construct the new DFA, we must build...

- a state to represent each set in the final partition,
- the appropriate transitions from the original DFA, and
- designate initial and accepting state(s)



Final DFA with renumbered states

As a second example, consider the DFA for a ($b \mid c$)* that we built earlier using Thompson's construction and the subset construction.



Using a DFA as a Recognizer

We now have the techniques to construct a DFA implementation from a **single** RE. But we need a **recognizer** that can handle **all** the REs for the language's microsyntax.

Given the REs for the various syntactic categories, $r_1, r_2, r_3, \ldots, r_k$, we can construct a single RE for the entire collection by forming $(r_1|r_2|r_3|\ldots|r_k)$.

However, since most real programs contain more than one word, we need to transform either the language or the recognizer.

- at the language level, we can insist that each word ends with an recognizable delimiter, e.g., a blank or a tab
- at the recognizer level, we can change the implementation of the DFA and its notion of acceptance
- As the first option is cumbersome, *e.g.*, "1+2" would not be recognized, whereas "1 + 2" would, we focus on the second option.

Using a DFA as a Recognizer

To find the **longest word** that matches one of the REs, the DFA should run until it reaches the point where the current state s has no outgoing transition on the next character.

Which RE was matched at this point?

- 1. if s is an **accepting state**: report the recognized word and its syntactic category
- 2. if s is an **nonaccepting state**
 - if possible, roll back to most recent accepting state to match longest valid prefix
 - otherwise, report an error since no prefix of the input string is a valid word

Note An accepting state in the DFA may represent several accepting states of the NFA. A keyword such as new might match two REs: one for keywords and one for identifiers.

A recognizer must have a strategy to break such ties, *e.g.*, prioritize REs.

Using a DFA as a Recognizer

A compiler writer must also specify REs for parts of the input stream that do not form words in the program text.

Whitespace

- include an RE that matches blanks, tabs, end-of-line characters, etc.
- action on accepting whitespace: invoke the scanner, recursively, and return its result

Comments

- if comments are discarded, they are handled in a similar fashion as whitespace

Implementing Scanners

Scanner construction is a problem where the theory of formal languages has produced tools, so-called **scanner generators**, that can automate implementation.

Scanner generator

- creates NFA for each RE defined by compiler writer for syntactic classes
- 2. joins all resulting NFAs by ϵ -transitions
- translates NFA into a corresponding DFA
- minimizes DFA
- converts DFA into executable code

Having covered Steps 1 to 4 already, we now look at three implementation strategies for Step 5: **table-driven scanners**, **direct-coded scanners**, and **hand-coded scanners**.

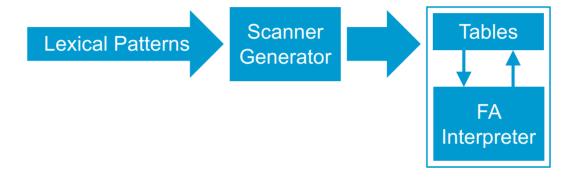
Implementing Scanners

The three implementation strategies, table-driven, direct-coded, and hand-coded, differ in

- how they encode the DFA's transition structure
- how they simulate its operation

Those differences, in turn, produce **different runtime costs**. However, they all have the **same asymptotic complexity**—constant cost per character, plus the cost of roll back.

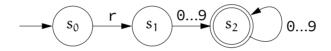
Table-Driven Scanners



The table-driven approach uses a generic **skeleton scanner** for control and a set of **three generated tables** that encode language-specific knowledge

- classifier table CharCat maps each $c \in \Sigma$ to a category
- transition table δ encodes the transitions of the underlying DFA
- token type table Type maps each state s_i of the DFA to a token type

As an example, we study the table-driven scanner for the RE r [0...9]+, which is recognized by the DFA given below.



The code generated for the skeleton scanner, *i.e.*, algorithm NextWord() on Slide 103, is divided into **four** sections.

- initializations
- a scanning loop that models the DFA's behavior
- a roll back loop in case the dfa overshoots the end of the token
- a final section that interprets and reports the results

Algorithm NextWord()

```
1 state \leftarrow s_0
2lexeme ← ""
3 clear stack
4 push (bad)
5 while state \neq s_e do
    NextChar(char)
    lexeme \leftarrow lexeme + char
    if state \in S_A then
     clear stack
    push(state)
10
    cat ← CharCat[char]
    state \leftarrow \delta[state,cat]
```

```
13 while state \notin S_A \land state \neq bad do

14 | state \leftarrow pop()

15 | truncate lexeme

16 | RollBack()

17 if state \in S_A then

18 | return Type[state];

19 else

20 | return invalid
```

Notice the similarity between this code and the fragment shown on Slide 57.

Tables CharCat and δ encode the DFA. To update the state, a **two-step translation** is performed.

- 1. character → category
- 2. current state and category \rightarrow new state This approach lets the scanner use a compressed transition table.

r	0,, 9	EOF	Other
Register	Digit	Digit	Other

Classifier Table CharCat

Larger character sets, *e.g.*, Unicode, may need a more complex data structure to represent CharCat.

	Register	Digit	Other
so	s ₁	s _e	Se
s ₁	s _e	s_2	S _e
s_2	s _e	s_2	Se
s _e	s _e	s _e	s _e

Transition Table δ

s ₀	s ₁	S ₂	s _e
invalid	invalid	register	invalid

Token Type Table Type

In the algorithm shown on Slide 103, some regular expressions can produce **quadratic** calls to roll back.

Example

Consider a scanner built using our approach for the RE $ab \mid (ab)^* c$.

ababababc Scanner reads all characters and returns the entire string as one

single word.

abababab Scanner must scan all characters before it can determine that the

longest prefix is ab. On the next invocation, it will scan ababab to

return ab, etc

The so-called **Maximal Munch Scanner** avoids excess roll back by marking dead-end transitions as they are popped from the stack.

It differs from the scanner on Slide 103 in three important ways.

- 1. a global counter InputPos to record position in the input stream
- 2. a two-dimensional bit-array Failed to record dead-end transitions
- 3. additional routine InitializeScanner() to initialize these two variables
- Over time, the maximal munch scanner records specific \(\)state, input position \(\)-pairs that cannot lead to an accepting state.

Direct-Coded Scanners

What could we do to **improve the performance** of a table-driven scanner?

- read a character
- compute the next DFA transition

Direct-coded scanners reduce the cost of computing DFA transitions by replacing the explicit representation of the DFA's state and transition graph with an implicit one.

- simplifies the two-step, table-lookup computation
- eliminates the memory references entailed in that computation
- allows other specializations
- The resulting scanner has the same functionality as the table-driven scanner, but with a lower overhead per character.

Direct-Coded Scanners

At the heart of a direct-coded scanner is an alternate implementation of the central while loop of the table-driven scanner.

```
1 while state \neq s_e do
2 | ...
3 | cat \leftarrow CharCat[char]
4 | state \leftarrow \delta[state,cat]
```

For each character, the table-driven scanner does **two** table lookups, involing address computations.

- CharCat: @CharCat $_0 + i \times w$
- δ : $\delta_0 + (\text{state} \times |\text{columns in } \delta| + \text{cat}) \times w$

Even though both lookups take O(1) time, they impose **constant-cost overheads** that a direct-coded scanner can avoid.

Rather than representing the current DFA state and the transition diagram explicitly, a direct-coded scanner has a specialized code fragment to implement each state.

Direct-Coded Scanners

```
// State s<sub>1</sub>
  // State s<sub>init</sub>
                                                                     27 push(state)
 1 lexeme ← ""
                                                                     28 if '0' < char < '9' then
                                  14 NextChar(char)
                                  15 lexeme ← lexeme + char
 2 clear stack
                                                                     29 goto 23
 3 push(bad)
                                  16 if state \in S_A then
                                                                     30 else
4 goto 5
                                  17 | clear stack
                                                                     31 goto 32
  // State so
                                                                        // State sout
                                  18 push(state)
 5 NextChar(char)
                                  19 if '0' < char < '9' then
                                                                     32 while state \notin S_A \land
 6 lexeme ← lexeme + char
                                   20 goto 23
                                                                                 state \neq bad do
7 if state \in S_A then
                                   21 else
                                                                      33 | state \leftarrow pop()
 8 | clear stack
                                                                           truncate lexeme
                                  22 | goto 32
 9 push(state)
                                                                           RollBack()
                                     // State s<sub>2</sub>
10 if char = 'r' then
                                  23 NextChar(char)
                                                                     36 if state \in S_A then
11 goto 14
                                  24 lexeme ← lexeme + char
                                                                     37 | return Type[state];
12 else
                                  25 if state \in S_A then
                                                                     38 else
13 | goto 32
                                   26 | clear stack
                                                                      39 return invalid
```

Direct-Coded Scanner

Observations

- Complicated address computations¹ are replaced by simple jumps (goto)
- Scanner can be further optimized
 - stack is not needed, since there is only one accepting state
 - jumps from s_0 and s_1 to s_{out} can be replaced by **return** invalid
 - stack is only needed in a DFA that has a transition from an accepting to a nonaccepting states
- Since all states follow a similar pattern, i.e., assignments followed by branching logic, scanner code can easily be generated
 - unlike the table-driven scanner, the code changes for each set of REs
 - of course, generated code violates many precepts of structured programming

¹Omitting these computations also reduces register demand behind the scenes.

Hand-Coded Scanners

Many compilers use hand-coded scanners. A hand-coded scanner can further reduce the overhead of the interfaces between the scanner and the rest of the system.

In the following, we study two possible optimizations

- buffering the input stream
- generating lexemes on-the-fly

The Real World

Many popular open-source compilers rely on hand-coded scanners. For example, the *flex* scanner generator was ostensibly built to support the *gcc* project, but *gcc 4.0* uses hand-coded scanners in several of its front ends.

Hand-Coded Scanners

In order to reduce the I/O cost per character, we can use buffered I/O

- each read operation returns a longer string of characters or buffer
- the scanner then indexes through the buffer by maintaining a pointer into the buffer
- NextChar fills the buffer and tracks the current location in the buffer

The cost of reading a full buffer of characters has two components

- a large fixed overhead
- and a small per-character cost

A buffer and pointer scheme **amortizes** the fixed costs of the read over many single-character fetches.

Hand-Coded Scanners

The code shown for the table-driven and direct-coded scanners accumulated the input characters into a string lexeme.

However, for some words, the lexeme...

- is implicit and we can avoid building it
- is not a string, but a number or Boolean value

A more efficient way than accumulating the lexeme as a string and then to convert or discard it, is to build the appropriate lexeme one character at a time, if required at all.

Hand-Coded Scanner

Example

In the direct-coded scanner on Slide 109, we could insert the following line (before the goto 23 statement) to compute the register number on-the-fly.

RegNum
$$\leftarrow$$
 RegNum $\times 10 + (char - '0')$

Note In the same way, we could hand-code states that recognize comments to avoid accumulating the lexeme altogether.

Handling Keywords

So far, we have assumed that keywords in the input language are recognized by including **explicit REs** for them in the description that generates the DFA and the recognizer.

Another approach is to recognize keywords as identifiers and then check a **lookup table** (e.g., hash table using perfect hashing) containing all keywords.

Trade-offs

- lookup table can lead to bad performance for nonkeywords, if it performs retry on miss
- extra states for keywords in DFA consume memory, but do not impact compile time

While the lookup table makes sense in the context of a completely hand-implemented scanner, in general it is better to fold keyword recognition into the DFA.