

# Title



# Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Technical introduction</b>	<b>3</b>
2.1	Scalar QED . . . . .	3
2.2	Effective potential . . . . .	4
2.2.1	Calculation of the unrenormalized effective potential . . . . .	4
2.3	Renormalisation of the effective potential . . . . .	7
2.3.1	$\overline{\text{MS}}$ renormalisation of the effective potential . . . . .	7
2.3.2	On-shell renormalisation of the effective potential . . . . .	9
<b>3</b>	<b>Literature summary</b>	<b>10</b>
3.1	Coleman Weinberg . . . . .	10
3.2	$\lambda\varphi^4$ theory . . . . .	10
3.2.1	Renormalisation . . . . .	11
3.3	Scalar electrodynamics . . . . .	12
3.3.1	Renormalisation . . . . .	13
3.3.2	The mass ratio . . . . .	15
3.4	Possible approaches . . . . .	16
<b>4</b>	<b>Zero momentum approach</b>	<b>17</b>
4.1	Analogues to Coleman-Weinberg . . . . .	18
4.2	Vanishing derivatives . . . . .	20
4.2.1	Comparison . . . . .	21
4.3	Half $\overline{\text{MS}}$ -Half Onshell scheme . . . . .	21
4.4	On shell with $\varphi^4$ potential . . . . .	23
4.5	Passing between schemes . . . . .	25
4.6	Summary table . . . . .	26
<b>5</b>	<b>On shell renormalisation of the effective potential – finite momentum approach</b>	<b>28</b>
5.1	Second derivative condition . . . . .	28
5.2	Introducing explicit VEV . . . . .	30
5.3	Contributions to $\Sigma$ and $T$ . . . . .	30
5.4	Second derivative condition – explicit VEV . . . . .	31
5.5	Counterterms . . . . .	32
5.6	Renormalization conditions . . . . .	32

5.7	Partition by order in $e$ . . . . .	33
5.8	Finiteness of the renormalized potential . . . . .	33
5.9	Resulting effective potential . . . . .	33
5.10	Downsides of this approach . . . . .	34
<b>6</b>	<b>Summary</b>	<b>35</b>
<b>7</b>	<b>Conclusions</b>	<b>36</b>
<b>A</b>	<b>Introduction – mass term case</b>	<b>37</b>
A.1	On-shell finite momentum approach . . . . .	37
A.2	"Zero momentum" approach . . . . .	38

## Abstract

Aqężćńłóś

# Chapter 1

## Introduction

Predictions of the gravitational wave signal from early Universe cosmological phase transition depend on the shape of effective potential of the theory. In this thesis we will investigate how different renormalisations schemes can change form of that potential.

Chapters 1-4 are meant as an introduction. New results are in 5 onward.

# Chapter 2

## Technical introduction

### 2.1 Scalar QED

This model will be used throughout the whole thesis, unless stated otherwise.

For a toy model we choose theory of scalar electrodynamics, described by the Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\Phi D^\mu\Phi^\dagger - \lambda\Phi^4, \quad (2.1.0.0.1)$$

where  $\Phi$  is a complex scalar field and the vector field present is  $U(1)$  gauge boson.

Writing operator  $D$  more explicitly it reads:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial_\mu\Phi + ieA_\mu\Phi)(\partial^\mu\Phi^\dagger - ieA^\mu\Phi^\dagger) - \lambda\Phi^4, \quad (2.1.0.0.2)$$

For the reasons that will be clear in reffinite momentum we will write  $\Phi$  field as two real scalar fields  $\varphi_1$  and  $\varphi_2$ , such that:

$$\Phi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2) \quad (2.1.0.0.3)$$

Then Lagrangian takes form:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\varphi_1 - eA_\mu\varphi_2)(\partial^\mu\varphi_1 - eA^\mu\varphi_2) \\ & + \frac{1}{2}(\partial_\mu\varphi_2 + eA_\mu\varphi_1)(\partial^\mu\varphi_2 + eA^\mu\varphi_1) - \frac{1}{4}\lambda(\varphi_1^2 + \varphi_2^2)^2, \end{aligned} \quad (2.1.0.0.4)$$

which we will write for brevity as:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\varphi_1 - eA_\mu\varphi_2)^2 \\ & + \frac{1}{2}(\partial_\mu\varphi_2 + eA_\mu\varphi_1)^2 - \frac{1}{4}\lambda(\varphi_1^2 + \varphi_2^2)^2. \end{aligned} \quad (2.1.0.0.5)$$

For a better track of what is independent of numerical convention, we will also write:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\varphi_1 - eA_\mu\varphi_2)^2 \\ & + \frac{1}{2}(\partial_\mu\varphi_2 + eA_\mu\varphi_1)^2 - c_\lambda\lambda(\varphi_1^2 + \varphi_2^2)^2, \end{aligned} \quad (2.1.0.0.6)$$

but  $c_\lambda = \frac{1}{4}$  everywhere in the thesis if not stated otherwise.

## 2.2 Effective potential

### TO DO:

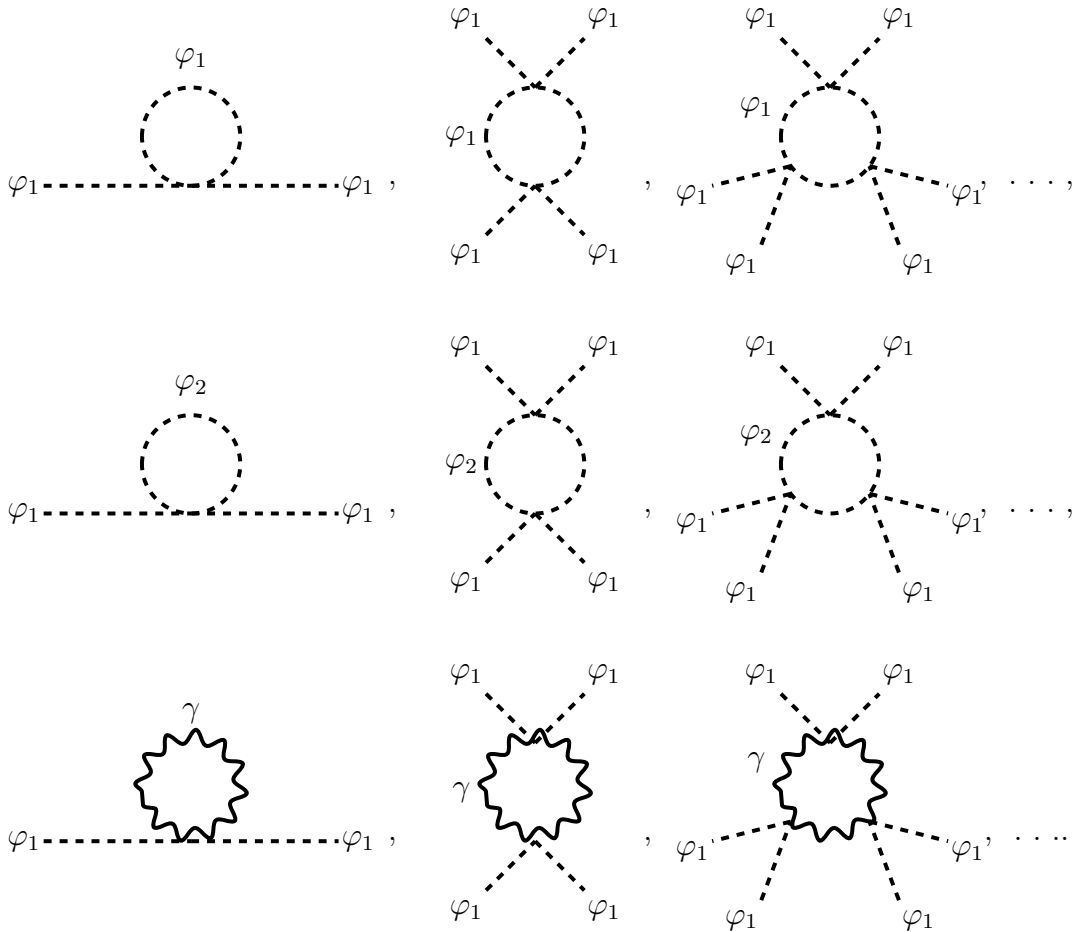
Some words about what it is and why we are interested in it.

#### 2.2.1 Calculation of the unrenormalized effective potential

Tree level potential in our model is:

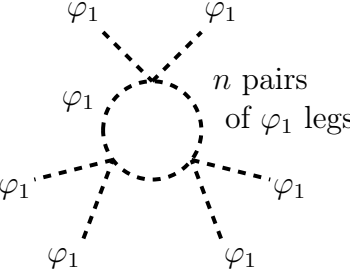
$$V_T = c_\lambda \lambda (\varphi_1^2 + \varphi_2^2)^2. \quad (2.2.1.0.1)$$

The one loop correction to the effective potential is calculated as a sum of the following diagrams:

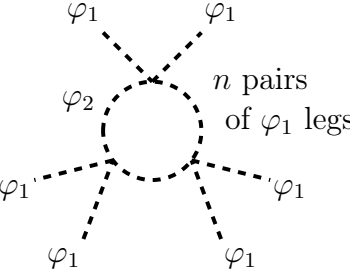


where:

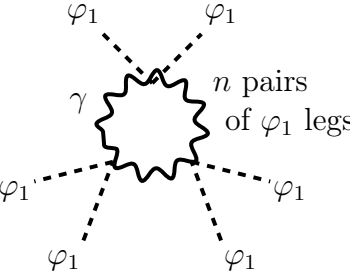




$$= i \int \frac{d^4 k}{(2\pi)^4} \frac{1}{2n} \left( \frac{12c_\lambda \lambda \varphi_1^2}{k^2 + i\varepsilon} \right)^n \quad (2.2.1.0.2)$$



$$= i \int \frac{d^4 k}{(2\pi)^4} \frac{1}{2n} \left( \frac{1}{3} \frac{12c_\lambda \lambda \varphi_1^2}{k^2 + i\varepsilon} \right)^n \quad (2.2.1.0.3)$$



$$= i \int \frac{d^4 k}{(2\pi)^4} \frac{1}{2n} \left( \frac{2e^{\frac{1}{2}} \varphi_1^2}{k^2 + i\varepsilon} \right)^n (g^\mu{}_\mu - 1). \quad (2.2.1.0.4)$$

Summing all the diagrams in series gives:

$$V_{1L} = i \int \frac{d^4 k}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{1}{2n} \left( \frac{12c_\lambda \lambda \varphi_1^2}{k^2 + i\varepsilon} \right)^n + \quad (2.2.1.0.5)$$

$$i \int \frac{d^4 k}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{1}{2n} \left( \frac{1}{3} \frac{12c_\lambda \lambda \varphi_1^2}{k^2 + i\varepsilon} \right)^n + \quad (2.2.1.0.6)$$

$$i \int \frac{d^4 k}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{1}{2n} \left( \frac{2e^{\frac{1}{2}} \varphi_1^2}{k^2 + i\varepsilon} \right)^n (g^\mu{}_\mu - 1). \quad (2.2.1.0.7)$$

These integrals seems to be hideously infrared divergent. We can, however Wick rotate them to the Euklidean space and perform summation to get:

$$V_{1L} = \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \ln \left( 1 + \frac{12c_\lambda \lambda \varphi_1^2}{k^2} \right) + \quad (2.2.1.0.8)$$

$$\frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \ln \left( 1 + \frac{4c_\lambda \lambda \varphi_1^2}{k^2} \right) + \quad (2.2.1.0.9)$$

$$\frac{3}{2} \int \frac{d^4 k}{(2\pi)^4} \ln \left( 1 + \frac{e^2 \varphi_1^2}{k^2} \right) \quad (2.2.1.0.10)$$

Now, integrals have only singularity at  $\varphi_1 = 0$ .

There are now several ways to perform these integrals. We will present here briefly two of them: cut-off and dimensional regularisation (DR).

For all of our further calculations in the thesis, we will use DR, but cut-off will appear in 3.1 as a method used in [1].

### Cut-off

The idea of this method is to restrict the integration to  $k^2 \leq \Lambda$ , for a parameter  $\Lambda$ . Integrating 2.2.1.0.8 up to  $\Lambda$  gives:

$$\begin{aligned} V_{1L} = & \frac{3c_\lambda \lambda \Lambda^2}{8\pi^2} \varphi_c^2 + \frac{9c_\lambda \lambda^2 \varphi_c^4}{4\pi^2} \left( \ln \frac{12c_\lambda \lambda \varphi_c^2}{\Lambda^2} - \frac{1}{2} \right) \\ & + \frac{c_\lambda \lambda \Lambda^2}{8\pi^2} \varphi_c^2 + \frac{c_\lambda \lambda^2 \varphi_c^4}{4\pi^2} \left( \ln \frac{4c_\lambda \lambda \varphi_c^2}{\Lambda^2} - \frac{1}{2} \right) \\ & + \frac{3e^2 \Lambda^2}{32\pi^2} \varphi_c^2 + \frac{3e^4 \varphi_c^4}{64\pi^2} \left( \ln \frac{e^2 \varphi_c^2}{\Lambda^2} - \frac{1}{2} \right). \end{aligned} \quad (2.2.1.0.11)$$

### Dimentional regularisation

The idea of this method is to observe, that the divergence is only in four dimentions and if we formally change expresions to  $4 - 2\epsilon$  dimentions. It is performed by treating the integral as a linear functional depending on two parameters – function and dimention given by some specific formula. Then we can calculate it with our function of interest and  $D = 4 - 2\epsilon$  as arguments.

After passing to  $D = 4 - 2\epsilon$  dimensions the integrals take form:

$$V_{1L} = \frac{\mu^\epsilon}{2} \int \frac{d^D k}{(2\pi)^D} \ln \left( 1 + \frac{12c_\lambda \lambda \varphi_1^2}{k^2} \right) \quad (2.2.1.0.12)$$

$$+ \frac{\mu^\epsilon}{2} \int \frac{d^D k}{(2\pi)^D} \ln \left( 1 + \frac{4c_\lambda \lambda \varphi_1^2}{k^2} \right) \quad (2.2.1.0.13)$$

$$+ \frac{(D-1)\mu^\epsilon}{2} \int \frac{d^D k}{(2\pi)^D} \ln \left( 1 + \frac{e^2 \varphi_1^2}{k^2} \right). \quad (2.2.1.0.14)$$

This introduces the parameter  $\mu$ , **To do.**

After calculating the integrals in  $D$  dimentions we have:

$$V_{1L} = \frac{1}{4} \frac{(12c_\lambda \lambda \varphi_1^2)^2}{(4\pi)^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{3}{2} + \log \frac{(12c_\lambda \lambda \varphi_1^2)^2}{4\pi \mu^2} \right) \quad (2.2.1.0.15)$$

$$+ \frac{1}{4} \frac{(4c_\lambda \lambda \varphi_1^2)^2}{(4\pi)^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{3}{2} + \log \frac{(4c_\lambda \lambda \varphi_1^2)^2}{4\pi \mu^2} \right) \quad (2.2.1.0.16)$$

$$+ \frac{1}{4} \frac{3(e^2 \varphi_1^2)^2}{(4\pi)^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 \varphi^2}{4\pi \mu^2} \right). \quad (2.2.1.0.17)$$

## 2.3 Renormalisation of the effective potential

The task that is still left, is to manage the infinities that come when we pass to the limit and:

- take  $\Lambda \rightarrow \infty$ , allowing for arbitrary large  $k^2$  in the case of the cut-off
- take  $\epsilon \rightarrow 0$  and returning to 4 dimensions in the case of DR.

This is the scope of renormalisation and is the main interest of this thesis. Different renormalisation techniques should ...

**TO DO: how this is exactly**

It is the goal of this thesis to show whether two of them –  $\overline{\text{MS}}$  and On shell give same results.

### 2.3.1 $\overline{\text{MS}}$ renormalisation of the effective potential

Although the goal of our thesis is to renormalise effective potential on-shell  $\overline{\text{MS}}$  renormalisation gives:

$$\begin{aligned}
 V_R = & c_\lambda \lambda \varphi_1^4 + \\
 & + \frac{1}{4} \frac{(12c_\lambda \lambda \varphi_1^2)^2}{(4\pi)^2} \left( -\frac{3}{2} + \log \frac{(12c_\lambda \lambda \varphi_1^2)^2}{\mu^2} \right) \\
 & + \frac{1}{4} \frac{(4c_\lambda \lambda \varphi_1^2)^2}{(4\pi)^2} \left( -\frac{3}{2} + \log \frac{(4c_\lambda \lambda \varphi_1^2)^2}{\mu^2} \right) \\
 & + \frac{1}{4} \frac{3(e^2 \varphi_1^2)^2}{(4\pi)^2} \left( -\frac{5}{6} + \log \frac{e^2 \varphi_1^2}{\mu^2} \right). \tag{2.3.1.0.1}
 \end{aligned}$$

Here, as will be apparent in the second, we are interested only in  $e^4$  part, so from now on, we will write:

$$V_R = c_\lambda \lambda \varphi_1^4 + \frac{3e^4}{64\pi^2} \left( -\frac{5}{6} + \log \frac{e^2 \varphi_1^2}{\mu^2} \right) \varphi_1^4. \tag{2.3.1.0.2}$$

We can bind  $e$  to  $\lambda$  and  $v$  at the loop level from the VEV definition:

$$\left. \frac{\partial V_R}{\partial \varphi_1} \right|_v = 0. \tag{2.3.1.0.3}$$

This gives the condition:

$$4c_\lambda \lambda v^3 - \frac{e^4 v^3}{16\pi^2} - \frac{3e^4 v^3}{16\pi^2} \log \frac{e^2 v^2}{\mu^2} = 0. \tag{2.3.1.0.4}$$

Setting scale parameter  $\mu$  to the effective mass of the vector, namely  $ev$ , we have simpler form of:

$$4c_\lambda \lambda v^3 - \frac{e^4 v^3}{16\pi^2} = 0. \tag{2.3.1.0.5}$$

Which gives:

$$\lambda = \frac{e^4}{64c_\lambda\pi^2}. \quad (2.3.1.0.6)$$

Writing potential with this substitutions yields:

$$V_R = \frac{3e^4}{64\pi^2} \left( -\frac{1}{2} + \log \frac{\varphi_1^2}{v^2} \right) \varphi_1^4. \quad (2.3.1.0.7)$$

This is exactly the same potential as obtained in [1] with different renormalisation. Equations 4.1.0.0.12 is very important in this discussion as it states, that  $\lambda$  is of order  $e^4$  in our model. This post factum justifies our choice in taking only  $e^4$  part, as other part was of order  $e^8$ .

### Physical mass

## TO DO: Zrobić tu audyt merytoryczny

We can now also bind square of the physical mass  $M_P^2$  as the second derivative of the renormalised effective potential at VEV.

$$M_P^2 = \left. \frac{\partial^2 V_R}{\partial \varphi_1^2} \right|_v = \frac{9e^4}{16\pi^2} \left( -\frac{1}{2} \right) v^2 + \frac{3e^4 v^2}{8\pi^2} + \frac{9e^4 v^2}{32\pi^2}. \quad (2.3.1.0.8)$$

This gives that:

$$M_P^2 = \frac{3e^4 v^2}{8\pi^2}. \quad (2.3.1.0.9)$$

From this we have that the ratio between scalar mass  $M_P^2$  and vector mass  $m(V)^2$  is:

$$\frac{M_P^2}{m(V)^2} = \frac{\frac{3e^4 v^2}{8\pi^2}}{e^2 v^2} = \frac{3e^2}{8\pi^2}. \quad (2.3.1.0.10)$$

We can now also express  $e^4$  and  $\lambda$  in terms of  $M_P^2$  and  $v$ :

$$e^4 = \frac{8M_P^2\pi^2}{3v^2} \quad (2.3.1.0.11)$$

$$\lambda = \frac{M_P^2}{24c_\lambda v^2}. \quad (2.3.1.0.12)$$

Writing potential in these terms gives:

$$V_R = \frac{M_P^2}{8v^2} \left( -\frac{1}{2} + \log \frac{\varphi_1^2}{v^2} \right) \varphi_1^4. \quad (2.3.1.0.13)$$

## TO DO: COŚ o stałym renormalizacji tu napisać

### 2.3.2 On-shell renormalisation of the effective potential

TO DO: dac tutaj motywacje do on-shell i napisac ze to jest glowne zagadnienie i ze to bedzie wlasnie zaraz omowione

# Chapter 3

## Literature summary

### TO DO: omówić literaturę

#### 3.1 Coleman Weinberg

As a founding paper for this considerations we present:

### TO DO: zrobić jakieś wprowadzenie

Here, we will present a summary of the parts of [1] that are of interest to us.

#### 3.2 $\lambda\varphi^4$ theory

Coleman and Weinberg in [1] start with " $\lambda\varphi^4$ " model, namely, with Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{\lambda}{4!}\varphi^4 + \frac{1}{2}A(\partial_\mu\varphi)^2 - \frac{1}{2}B\varphi^2 - \frac{1}{4!}C\varphi^4, \quad (3.2.0.0.1)$$

where  $A$ ,  $B$ ,  $C$  are renormalisation constants.

Then there are calculations of the renormalised effective potential in this theory.

Tree level potential is

$$V = \frac{\lambda}{4!}\varphi_c^4, \quad (3.2.0.0.2)$$

and up to the one loop level is:

$$V = \frac{\lambda}{4!}\varphi_c^4 + \frac{1}{2}B\varphi_c^2 + \frac{1}{4!}C\varphi_c^4 + i \int \frac{d^4k}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{1}{2n} \left( \frac{\frac{1}{2}\lambda\varphi_c^2}{k^2 + i\epsilon} \right)^n. \quad (3.2.0.0.3)$$

As mentioned in [1] and 2.2.1, this expression seems "hideously infrared divergent". It is thus transformed into one in Euclidean space with apparent infrared divergence

turned into logarithmic singularity at  $\varphi_c = 0$ :

$$V = \frac{\lambda}{4!}\varphi_c^4 + \frac{1}{2}B\varphi_c^2 + \frac{1}{4!}C\varphi_c^4 + \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} \ln\left(1 + \frac{\lambda\varphi_c^2}{2k^2}\right). \quad (3.2.0.0.4)$$

This is then calculated using cut-off method, with cut off at  $k^2 = \Lambda^2$ . The result is:

$$V = \frac{\lambda}{4!}\varphi_c^4 + \frac{1}{2}B\varphi_c^2 + \frac{1}{4!}C\varphi_c^4 + \frac{\lambda\Lambda^2}{64\pi^2}\varphi_c^2 + \frac{\lambda^2\varphi_c^4}{256\pi^2}\left(\ln\frac{\lambda\varphi_c^2}{2\Lambda^2} - \frac{1}{2}\right). \quad (3.2.0.0.5)$$

### 3.2.1 Renormalisation

Then, the potential is renormalised. First renormalisation condition is for the renormalised mass to vanish (eq. (3.5) in [1]).

It is written as:

$$\left.\frac{d^2V}{d\varphi_c^2}\right|_0 = 0. \quad (3.2.1.0.1)$$

The second derivative of the potential evaluated at zero is equal to:

$$\left.\frac{d^2V}{d\varphi_c^2}\right|_0 = B + \frac{\lambda\Lambda^2}{32\pi^2}, \quad (3.2.1.0.2)$$

so condition 3.2.1.0.1 gives us, that

$$B = -\frac{\lambda\Lambda^2}{32\pi^2}, \quad (3.2.1.0.3)$$

Note that, if the derivative would be evaluated at some non zero  $\langle\varphi\rangle$ , this relation will vastly change.

We would like to have an analogous condition for the fourth derivative, but the fourth derivative does not exist at 0, because of the logarithmic infrared singularity. To resolve this problem, we can introduce a parameter  $M$ , with the dimension of the mass and take the derivative at it. The condition takes the form:

$$\left.\frac{d^4V}{d\varphi_c^4}\right|_M = \lambda. \quad (3.2.1.0.4)$$

The parameter  $M$  taken is arbitrary. Let us note, that taking different  $M'$  will result in different  $\lambda'$ , but the form of the final potential will remain unchanged.

The resulting  $C$  is:

$$C = -\frac{3\lambda^2}{32\pi^2}\left(\ln\frac{\lambda M^2}{2\Lambda^2} + \frac{11}{3}\right). \quad (3.2.1.0.5)$$

The resulting potential is:

$$V = \frac{\lambda}{4!}\varphi_c^4 + \frac{\lambda^2\varphi_c^4}{256\pi^2}\left(\ln\frac{\varphi_c^2}{M^2} - \frac{25}{6}\right). \quad (3.2.1.0.6)$$

Then, the discussion proceeds, that, at  $\varphi_c = 0$  we have now maximum, not minimum, and that the minimum of the potential occurs at the value of:  $\varphi_c$  determined by:

$$\lambda \ln \frac{\langle \varphi \rangle^2}{M^2} = -\frac{32}{3}\pi^2 + O(\lambda), \quad (3.2.1.0.7)$$

which is very far outside the expected range of validity of the one-loop approximation and must be rejected as superficial.

### 3.3 Scalar electrodynamics

Now, we will present treatment of the theory of our main interest conducted in [1]. It starts with the theory with the lagrangian [1](4.1):

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}(\partial_\mu\varphi_1 - eA_\mu\varphi_2)^2 + \frac{1}{2}(\partial_\mu\varphi_2 + eA_\mu\varphi_1)^2 - \frac{\lambda}{4!}(\varphi_1^2 + \varphi_2^2)^2 + \text{counterterms}. \quad (3.3.0.0.1)$$

Then, the resulting renormalised potential is presented [1](4.5):

$$V = \frac{\lambda}{4!}\varphi_c^4 + \left( \frac{5\lambda^2}{1152\pi^2} + \frac{3e^4}{64\pi^2} \right) \varphi_c^4 \left( \ln \frac{\varphi_c^2}{M^2} - \frac{25}{6} \right), \quad (3.3.0.0.2)$$

with only a brief note, that it is obtained after "straightforward computation" .

We will now investigate more carefully this omitted step .

It is implied, that the procedure was the same as in the  $\lambda\varphi^4$  case and we shall see, whether it was indeed the same, as well as, whether applied procedure is eligible for this theory.

Let us start with the effective potential with not yet calculated integrals and not yet evaluated renormalisation constants:

$$\begin{aligned} V = & \frac{\lambda}{4!}\varphi_c^4 - \frac{1}{2}B\varphi_c^2 - \frac{1}{4!}C\varphi_c^4 \\ & + i \int \frac{d^4k}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{1}{2n} \left( \frac{\frac{1}{2}\lambda\varphi_c^2}{k^2 + i\epsilon} \right)^n \\ & + i \int \frac{d^4k}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{1}{2n} \left( \frac{\frac{1}{6}\lambda\varphi_c^2}{k^2 + i\epsilon} \right)^n \\ & + i \int \frac{d^4k}{(2\pi)^4} 3 \sum_{n=1}^{\infty} \frac{1}{2n} \left( \frac{e^2\varphi_c^2}{k^2 + i\epsilon} \right)^n. \end{aligned} \quad (3.3.0.0.3)$$

This can be transformed as previously from this infrared divergent form, to the form



with singularity only at  $\varphi_c = 0$ :

$$\begin{aligned}
V = & \frac{\lambda}{4!} \varphi_c^4 - \frac{1}{2} B \varphi_c^2 - \frac{1}{4!} C \varphi_c^4 \\
& + \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \ln \left( 1 + \frac{\lambda \varphi_c^2}{2k^2} \right) \\
& + \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \ln \left( 1 + \frac{\lambda \varphi_c^2}{6k^2} \right) \\
& + \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} 3 \ln \left( 1 + \frac{e^2 \varphi_c^2}{k^2} \right), \tag{3.3.0.0.4}
\end{aligned}$$

and then calculated using cut-off method at  $k^2 = \Lambda^2$ :

$$\begin{aligned}
V = & \frac{\lambda}{4!} \varphi_c^4 + \frac{1}{2} B \varphi_c^2 + \frac{1}{4!} C \varphi_c^4 \\
& + \frac{\lambda \Lambda^2}{64\pi^2} \varphi_c^2 + \frac{\lambda^2 \varphi_c^4}{256\pi^2} \left( \ln \frac{\lambda \varphi_c^2}{2\Lambda^2} - \frac{1}{2} \right) \\
& + \frac{\lambda \Lambda^2}{3 \cdot 64\pi^2} \varphi_c^2 + \frac{\lambda^2 \varphi_c^4}{9 \cdot 256\pi^2} \left( \ln \frac{\lambda \varphi_c^2}{6\Lambda^2} - \frac{1}{2} \right) \\
& + \frac{3e^2 \Lambda^2}{32\pi^2} \varphi_c^2 + \frac{3e^4 \varphi_c^4}{64\pi^2} \left( \ln \frac{e^2 \varphi_c^2}{\Lambda^2} - \frac{1}{2} \right). \tag{3.3.0.0.5}
\end{aligned}$$

### 3.3.1 Renormalisation

The first imposed renormalisation condition in the previous  $(\lambda\varphi^4)$  case was:

$$\left. \frac{d^2 V}{d\varphi_c^2} \right|_0 = 0, \tag{3.3.1.0.1}$$

where, it was stated, is equivalent, to renormalised mass being zero.

What is important, this "renormalized mass" is not meant to be a physical mass, rather, the mass parameter, the constant that stand beside second power of the field, which can be defined as " $\left. \frac{d^2 V}{d\varphi_c^2} \right|_0$ " itself.

The same renormalisation condition was used as the first one in this case.

The second derivative evaluated at zero is:

$$\left. \frac{d^2 V}{d\varphi_c^2} \right|_0 = \frac{1}{2} B \varphi_c^2 + \frac{\lambda \Lambda^2}{64\pi^2} \varphi_c^2 + \frac{\lambda \Lambda^2}{3 \cdot 64\pi^2} \varphi_c^2 + \frac{3e^2 \Lambda^2}{32\pi^2} \varphi_c^2. \tag{3.3.1.0.2}$$

Thus:

$$B = -2 \left( \frac{\lambda \Lambda^2}{64\pi^2} + \frac{\lambda \Lambda^2}{3 \cdot 64\pi^2} + \frac{3e^2 \Lambda^2}{32\pi^2} \right). \tag{3.3.1.0.3}$$

The second renormalisation condition is:

$$\left. \frac{d^4 V}{d\varphi_c^4} \right|_M = \lambda, \tag{3.3.1.0.4}$$

We have that:

$$\begin{aligned}
\left. \frac{d^4 V}{d\varphi_c^4} \right|_M &= \lambda + C \\
&+ \frac{11\lambda^2}{32\pi^2} + \frac{3\lambda^2}{32\pi^2} \ln \frac{\lambda M^2}{2\Lambda^2} \\
&+ \frac{11\lambda^2}{288\pi^2} + \frac{\lambda^2}{96\pi^2} \ln \frac{\lambda M^2}{6\Lambda^2} \\
&+ \frac{(75-18\alpha)e^4}{16\pi^2} + \frac{9e^4}{8\pi^2} \ln \frac{e^2 M^2}{\Lambda^2}.
\end{aligned} \tag{3.3.1.0.5}$$

From this, for  $\left. \frac{d^4 V}{d\varphi_c^4} \right|_M = \lambda$ , we conclude that:

$$\begin{aligned}
C &= - \left( \frac{11\lambda^2}{32\pi^2} + \frac{3\lambda^2}{32\pi^2} \ln \frac{\lambda M^2}{2\Lambda^2} \right. \\
&+ \frac{11\lambda^2}{288\pi^2} + \frac{\lambda^2}{96\pi^2} \ln \frac{\lambda M^2}{6\Lambda^2} \\
&\left. + \frac{(75-18\alpha)e^4}{16\pi^2} + \frac{9e^4}{8\pi^2} \ln \frac{e^2 M^2}{\Lambda^2} \right).
\end{aligned} \tag{3.3.1.0.6}$$

Substituting these results to the potential result in:

$$\begin{aligned}
V &= \frac{\lambda}{4!} \varphi_c^4 - \left( \frac{\lambda\Lambda^2}{64\pi^2} + \frac{\lambda\Lambda^2}{3 \cdot 64\pi^2} + \frac{3e^2\Lambda^2}{32\pi^2} \right) \varphi_c^2 \\
&- \frac{1}{4!} \left( \frac{11\lambda^2}{32\pi^2} + \frac{3\lambda^2}{32\pi^2} \ln \frac{\lambda M^2}{2\Lambda^2} \right. \\
&+ \frac{11\lambda^2}{288\pi^2} + \frac{\lambda^2}{96\pi^2} \ln \frac{\lambda M^2}{6\Lambda^2} \\
&+ \left. \frac{(75-18\alpha)e^4}{16\pi^2} + \frac{9e^4}{8\pi^2} \ln \frac{e^2 M^2}{\Lambda^2} \right) \varphi_c^4 \\
&+ \frac{\lambda\Lambda^2}{64\pi^2} \varphi_c^2 + \frac{\lambda^2 \varphi_c^4}{256\pi^2} \left( \ln \frac{\lambda \varphi_c^2}{2\Lambda^2} - \frac{1}{2} \right) \\
&+ \frac{\lambda\Lambda^2}{3 \cdot 64\pi^2} \varphi_c^2 + \frac{\lambda^2 \varphi_c^4}{9 \cdot 256\pi^2} \left( \ln \frac{\lambda \varphi_c^2}{6\Lambda^2} - \frac{1}{2} \right) \\
&+ \frac{3e^2\Lambda^2}{32\pi^2} \varphi_c^2 + \frac{3e^4 \varphi_c^4}{64\pi^2} \left( \ln \frac{e^2 \varphi_c^2}{\Lambda^2} - \frac{1}{2} \right),
\end{aligned} \tag{3.3.1.0.7}$$

and after canceling the renormalised potential is indeed:

$$V = \frac{\lambda}{4!} \varphi_c^4 + \left( \frac{5\lambda^2}{1152\pi^2} + \frac{3e^4}{64\pi^2} \right) \varphi_c^4 \left( \ln \frac{\varphi_c^2}{M^2} - \frac{25}{6} \right). \tag{3.3.1.0.8}$$

Let us note, that 3.3.1.0.1 is precesuily the condition, that guarantees the  $\varphi^4$  form of the potential.

### 3.3.2 The mass ratio

The parameter  $M$  taken was arbitrary, so we can as well set it to  $\langle\varphi\rangle$ . Then, the potential takes form:

$$V = \frac{\lambda}{4!}\varphi_c^4 + \left(\frac{5\lambda^2}{1152\pi^2} + \frac{3e^4}{64\pi^2}\right)\varphi_c^4\left(\ln\frac{\varphi_c^2}{\langle\varphi\rangle^2} - \frac{25}{6}\right). \quad (3.3.2.0.1)$$

As  $\langle\varphi\rangle$  is defined to be the minimum of the potential, we have that:

$$\left.\frac{dV}{d\varphi_c}\right|_{\langle\varphi\rangle} = 0. \quad (3.3.2.0.2)$$

Thus:

$$\left(\frac{\lambda}{6} - \frac{11e^4}{16\pi^2}\right)\langle\varphi\rangle^3 = 0. \quad (3.3.2.0.3)$$

The actual location of the minimum is non-zero, so:

$$\lambda = \frac{33}{8\pi^2}e^4. \quad (3.3.2.0.4)$$

From this, we can rewrite potential as:

$$V = \frac{3e^4}{64\pi^2}\varphi^4\left(\ln\frac{\varphi^2}{\langle\varphi\rangle^2} - \frac{1}{2}\right). \quad (3.3.2.0.5)$$

This gives parametrisation of potential with  $e$  and  $\langle\varphi\rangle$  alone. With different  $M$  choosen, the  $\lambda$  would change and intermediate expressions would change, but 3.3.2.0.5 would remain the same. From this we can express masses of the scalar and the vector in terms of  $e$  and  $\varphi$ . The mass of the scalar is:

$$m^2(S) = \left.\frac{d^2V}{d\varphi_c^2}\right|_{\langle\varphi\rangle} = \frac{3e^4}{8\pi^2}\langle\varphi\rangle^2. \quad (3.3.2.0.6)$$

The mass of the vector is:

$$m^2(V) = e^2\left.\frac{d^2V}{d\varphi_c^2}\right|_0. \quad (3.3.2.0.7)$$

From this, the mass ratio is obtained:

$$\frac{m^2(S)}{m^2(V)} = \frac{3}{2\pi}\frac{e^2}{4\pi} = \frac{3e^2}{8\pi^2}. \quad (3.3.2.0.8)$$

**TO DO: napisać koniecznie o radiacyjnym łamaniu symetrii w tym rozdziale**

**TO DO: napisać o tym jak to oni biorą tą pochodną w zerze**

### 3.4 Possible approaches

TO DO: wprowadzenie do tego, co my chcemy zrobić

# Chapter 4

## Zero momentum approach

Here, as well, we will consider potential up to order  $e^4$ .

We start with the 1-loop level potential with counterterms:

$$V_R^{1\text{-loop}} = \frac{3e^4}{64\pi^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 \varphi_1^2}{4\pi\mu^2} \right) \varphi_1^4 + c_\lambda \delta\lambda \varphi_1^4 + c_m \delta m \varphi_1^2. \quad (4.0.0.0.1)$$

Note, that when we will write  $v$ , meaning VEV, the vacuum expectation value, we will mean 1-loop level VEV.

Here we will present the approach more similar to one from [1]. The renormalisation conditions will be of the form:

$$\left. \frac{d^k}{d\varphi_1^k} V \right|_\alpha = \beta \quad (4.0.0.0.2)$$

where  $V$  is some part of the renormalized effective potential and  $\alpha, \beta$  are some parameters that will vary between the versions we will present here.

In this way renormalisation conditions are stated only in terms of the effective potential itself without referring to self energy. As a on-shell condition we treat:

$$\left. \frac{d^2}{d\varphi_1^2} V_R^{1\text{-loop}} \right|_v = 0 \quad (4.0.0.0.3)$$

as it is, in a sense, a zero-momentum version of the condition of preservation of the tree level mass at the one-loop level.

In [1] the conditions used were (see 3.1):

$$\left. \frac{d^2}{d\varphi_1^2} V_R \right|_0 = 0 \quad (4.0.0.0.4)$$

$$\left. \frac{d^4}{d\varphi_1^4} V_R \right|_v = 24c_\lambda \lambda. \quad (4.0.0.0.5)$$

Note, that, in [1], as mentioned in 3.1, the convention is used, that  $c_\lambda = 4!$ .

We will present a few different versions of possible renormalisation conditions.

## 4.1 Analogues to Coleman-Weinberg

As renormalisation conditions we impose that:

$$\left. \frac{\partial^2}{\partial \varphi_1^2} V_R \right|_0 = 0 \quad (4.1.0.0.1)$$

$$\left. \frac{\partial^4}{\partial \varphi_1^4} V_R \right|_v = 24c_\lambda \lambda. \quad (4.1.0.0.2)$$

These are exactly the same conditions as presented in [1]. We calculate it here once again for two reasons:

Firstly – we use different regularisation, so, although we expect for the resulting potential to be the same, the intermediate steps may (and will) differ. We think, that presenting them here can give a better inside what is going on and better reference to compare with other presented versions of the scheme.

Secondly – we will now write it with our convention of naming things, so it will be a better for reference to compare with other versions of the scheme for this reason too. Codesponding derivatives are:

$$\frac{\partial^2}{\partial \varphi_1^2} V_R^{1\text{-loop}} = \frac{9e^4 \varphi_1^2}{16\pi^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 \varphi_1^2}{4\pi\mu^2} \right) + \frac{21e^4}{32\pi^2} \varphi_1^2 + 12c_\lambda \delta\lambda \varphi_1^2 + 2c_m \delta m \quad (4.1.0.0.3)$$

$$\frac{\partial^4}{\partial \varphi_1^4} V_R^{1\text{-loop}} = \frac{9e^4 \varphi_1^2}{8\pi^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 \varphi_1^2}{4\pi\mu^2} \right) + \frac{75e^4}{16\pi^2} + 24c_\lambda \delta\lambda. \quad (4.1.0.0.4)$$

So the conditions take form:

$$2c_m \delta m = 0 \quad (4.1.0.0.5)$$

$$\frac{9e^4 v^2}{8\pi^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 v^2}{4\pi\mu^2} \right) + \frac{75e^4}{16\pi^2} + 24c_\lambda \delta\lambda = 0. \quad (4.1.0.0.6)$$

Solving for  $\delta\lambda$  and  $\delta m$  we have:

$$\delta m = 0 \quad (4.1.0.0.7)$$

$$\delta\lambda = -\frac{3e^4}{64\pi^2 c_\lambda} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 v^2}{4\pi\mu^2} \right) - \frac{25e^4}{128\pi^2 c_\lambda}. \quad (4.1.0.0.8)$$

Then, the renormalised potential is:

$$V_R = c_\lambda \lambda + \frac{3e^4 \varphi_1^4}{64\pi^2} \left( -\frac{25}{6} + \log \frac{\varphi_1^2}{v^2} \right) \varphi_1^4. \quad (4.1.0.0.9)$$

Which is identical to the one from [1], as described in 3.3.1 and all the results for expressing  $\lambda$  as  $e$  and  $v$ . As well as relations between  $\lambda$ ,  $e$  and the pshysical mass would be the same as in [1].

We can bind  $\lambda$  to  $e$  and  $v$  at the loop level from the VEV definition:

$$\left. \frac{\partial V_R}{\partial \varphi_1} \right|_v = 0. \quad (4.1.0.0.10)$$

This gives the condition:

$$4c_\lambda \lambda v^3 - \frac{25e^4 v^3}{32\pi^2} + \frac{3e^4 v^3}{32\pi^2} = 0. \quad (4.1.0.0.11)$$

Which gives:

$$\lambda = \frac{11e^4}{64c_\lambda \pi^2}. \quad (4.1.0.0.12)$$

Writing potential with this substitution yields:

$$V_R = \frac{3e^4}{64\pi^2} \left( -\frac{1}{2} + \log \frac{\varphi_1^2}{v^2} \right) \varphi_1^4. \quad (4.1.0.0.13)$$

This is, as expected, exactly same potential as in [1].

We can now also bind square of the physical mass  $M_P^2$  as the second derivative of the renormalised effective potential at VEV. The result for expressing  $M_P^2$  in terms of  $e$ , as well as the ratio between scalar and vector mass will be the same as in the 2.3.1 and [1] as they depend only on the form of the potential as in 4.1.0.0.13.

$$M_P^2 = \left. \frac{\partial^2 V_R}{\partial \varphi_1^2} \right|_v = \frac{9e^4}{16\pi^2} \left( -\frac{1}{2} \right) v^2 + \frac{3e^4 v^2}{8\pi^2} + \frac{9e^4 v^2}{32\pi^2}. \quad (4.1.0.0.14)$$

This gives that:

$$M_P^2 = \frac{3e^4 v^2}{8\pi^2}. \quad (4.1.0.0.15)$$

From this we have that the ratio between scalar mass  $M_P^2$  and vector mass  $m(V)^2$  is:

$$\frac{M_P^2}{m(V)^2} = \frac{\frac{3e^4 v^2}{8\pi^2}}{e^2 v^2} = \frac{3e^2}{8\pi^2}. \quad (4.1.0.0.16)$$

We can now also express  $e^4$  and  $\lambda$  in terms of  $M_P^2$  and  $v$  (this part will be different than in 4.1.0.0.13 or [1] for  $\lambda$ ):

$$e^4 = \frac{8M_P^2 \pi^2}{3v^2} \quad (4.1.0.0.17)$$

$$\lambda = \frac{11M_P^2}{24c_\lambda v^2}. \quad (4.1.0.0.18)$$

Writing potential in these terms gives:

$$V_R = \frac{M_P^2}{8v^2} \left( -\frac{1}{2} + \log \frac{\varphi_1^2}{v^2} \right) \varphi_1^4. \quad (4.1.0.0.19)$$

## 4.2 Vanishing derivatives

As renormalisation conditions we impose that:

$$\left. \frac{\partial^2}{\partial \varphi_1^2} V_R^{1\text{-loop}} \right|_v = 0 \quad (4.2.0.0.1)$$

$$\left. \frac{\partial^4}{\partial \varphi_1^4} V_R^{1\text{-loop}} \right|_v = 0. \quad (4.2.0.0.2)$$

Corresponding derivatives are:

$$\frac{\partial^2}{\partial \varphi_1^2} V_R^{1\text{-loop}} = \frac{9e^4 \varphi_1^2}{16\pi^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 \varphi_1^2}{4\pi \mu^2} \right) + \frac{21e^4}{32\pi^2} \varphi_1^2 + 12c_\lambda \delta \lambda \varphi_1^2 + 2c_m \delta m \quad (4.2.0.0.3)$$

$$\frac{\partial^4}{\partial \varphi_1^4} V_R^{1\text{-loop}} = \frac{9e^4 \varphi_1^2}{8\pi^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 \varphi_1^2}{4\pi \mu^2} \right) + \frac{75e^4}{16\pi^2} + 24c_\lambda \delta \lambda. \quad (4.2.0.0.4)$$

So the conditions take form:

$$\frac{9e^4 v^2}{16\pi^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 v^2}{4\pi \mu^2} \right) + \frac{21e^4}{32\pi^2} v^2 + 12c_\lambda \delta \lambda v^2 + 2c_m \delta m = 0 \quad (4.2.0.0.5)$$

$$\frac{9e^4 v^2}{8\pi^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 v^2}{4\pi \mu^2} \right) + \frac{75e^4}{16\pi^2} + 24c_\lambda \delta \lambda = 0. \quad (4.2.0.0.6)$$

Solving for  $\delta \lambda$  and  $\delta m$  we have:

$$\delta m = \frac{27e^4 v^2}{32\pi^2 c_m} \quad (4.2.0.0.7)$$

$$\delta \lambda = -\frac{3e^4}{64\pi^2 c_\lambda} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 v^2}{4\pi \mu^2} \right) - \frac{75e^4}{128\pi^2 c_\lambda}. \quad (4.2.0.0.8)$$

Then, the renormalised potential is:

$$V_R = c_\lambda \lambda \varphi_1^4 + \frac{3e^4}{64\pi^2} \left( -\frac{25}{6} + \log \frac{\varphi_1^2}{v^2} \right) \varphi_1^4 + \frac{27e^4 v^2}{32\pi^2} \varphi_1^2. \quad (4.2.0.0.9)$$

We would like to write  $V_R$  in terms of  $M_P$  – the physical mass and  $v$ . First relation is  $\lambda = \frac{M_P^2}{12c_\lambda v^2}$ . Potential written with this substitution becomes:

$$V_R = \frac{M_P^2}{12v^2} \varphi_1^4 + \frac{3e^4}{64\pi^2} \left( -\frac{25}{6} + \log \frac{\varphi_1^2}{v^2} \right) \varphi_1^4 + \frac{27e^4 v^2}{32\pi^2} \varphi_1^2. \quad (4.2.0.0.10)$$

To write  $e$  in terms of  $M_P$  and  $v$  we use the condition that

$$\left. \frac{\partial}{\partial \varphi_1} V_R \right|_v = 0, \quad (4.2.0.0.11)$$

as  $v$  is by definition minimum of the potential.

It gives the relation:

$$\frac{M_P^2 v}{3} + \frac{e^4 v^3}{16\pi^2} \left( -\frac{25}{2} \right) + \frac{3e^4 v^3}{32\pi^2} + \frac{27e^4 v^3}{16\pi^2} = 0. \quad (4.2.0.0.12)$$



Thus, we conclude that:

$$e^4 = \frac{-M_P^2 \pi^2}{3v^2}. \quad (4.2.0.0.13)$$

This, unfortunately, is unacceptable, as then  $e$  is no longer a real number, which is unphysical. Thus, we conclude, that presented renormalisation method is not working and we need to search for another. One of possible ways is to expand the method to finite momentum.

## TO DO: czy zostawić, to co poniżej

### 4.2.1 Comparison

We will now investigate, whether this has some chance of working by comparing above "zero momentum" method, only with first and second derivative, with "finite momentum" method and it's zero momentum limit.

With the conditions:

$$\left. \frac{\partial}{\partial \varphi_1} V_R^{1\text{-loop}} \right|_v = 0 \quad (4.2.1.0.1)$$

$$\left. \frac{\partial^2}{\partial \varphi_1^2} V_R^{1\text{-loop}} \right|_v = 0 \quad (4.2.1.0.2)$$

renormalisation constants  $\delta\lambda$  and  $\delta m$  take form:

$$\delta\lambda = -\frac{e^4}{8\pi^2} - \frac{3e^4}{16\pi^2} \left( -\frac{2}{\epsilon} - \gamma_E + \log \frac{e^2 v^2}{4\pi\mu^2} \right) \quad (4.2.1.0.3)$$

$$\delta m = \frac{3e^4 v^2}{16\pi^2}. \quad (4.2.1.0.4)$$

And the renormalised potential is equal to:

$$V_R = \frac{M_P^2}{48c_\lambda v^2} + \frac{e^4 \varphi_1^4}{128\pi^2} \left( 6 \log \frac{\varphi_1^2}{v^2} - 9 \right) + \frac{3e^4 v^2 \varphi_1^2}{32\pi^2}. \quad (4.2.1.0.5)$$

Now, however, the condition for  $v$  is meaningless as we already used it in renormalisation conditions. Nevertheless, we want to investigate how values of  $\delta\lambda$  and  $\delta m$  are compared to other approaches.

### 4.3 Half $\overline{\text{MS}}$ -Half Onshell scheme

Due to (so far) lack of experimental data of coupling  $\lambda$  in considered theories, the on shell condition for that constant renders itself meaningless.

Thus, we propose mixed scheme, where we demand that the physical mass remain unchanged due to the one-loop corrections, but for the coupling case, we demand

only that the  $\delta\lambda$  counterterm is such that the fourth derivative of the renormalised effective potential is finite – in the  $\overline{\text{MS}}$  manner.

Here we impose following renormalisation conditions:

$$\left. \frac{\partial^2}{\partial \varphi_1^2} V_R^{1\text{-loop}} \right|_v = 0 \quad (4.3.0.0.1)$$

$$\left. \frac{\partial^4}{\partial \varphi_1^4} V_R^{1\text{-loop}} \right|_v = \frac{9e^4}{8\pi^2} \left( -\frac{5}{6} + \log \frac{e^2 v^2}{\mu^2} \right) + \frac{75e^4}{16\pi^2}. \quad (4.3.0.0.2)$$

Written in the full form these conditions take form:

$$\begin{aligned} & \frac{9e^4}{16\pi^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 v^2}{4\pi\mu^2} \right) v^2 \\ & + 12c_\lambda \delta\lambda v^2 + \frac{21e^4}{32\pi^2} v^2 + 2c_m \delta m = 0 \end{aligned} \quad (4.3.0.0.3)$$

$$\begin{aligned} & \frac{9e^4}{8\pi^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 v^2}{4\pi\mu^2} \right) + 24c_\lambda \delta\lambda + \frac{75e^4}{16\pi^2} \\ & = \frac{9e^4}{8\pi^2} \left( -\frac{5}{6} + \log \frac{e^2 v^2}{\mu^2} \right) + \frac{75e^4}{16\pi^2}. \end{aligned} \quad (4.3.0.0.4)$$

After solving equations for  $\delta m$  and  $\delta\lambda$  we obtain:

$$\delta m = -\frac{3e^4 v^2}{32c_m \pi^2} \left( 1 + 3 \log \frac{e^2 v^2}{\mu^2} \right) \quad (4.3.0.0.5)$$

$$\delta\lambda = -\frac{3e^4}{64c_\lambda \pi^2} \left( -\frac{2}{\epsilon} + \gamma_E - \log(4\pi) \right). \quad (4.3.0.0.6)$$

The renormalized potential is then:

$$\begin{aligned} V_R = & c_\lambda \lambda \varphi_1^4 + \\ & \frac{3e^4}{64\pi^2} \left( -\frac{5}{6} + \log \frac{e^2 \varphi_1^2}{\mu^2} \right) \varphi_1^4 \\ & + \frac{9e^4 v^2}{32\pi^2} \left( -\frac{1}{3} - \log \frac{e^2 v^2}{\mu^2} \right) \varphi_1^2. \end{aligned} \quad (4.3.0.0.7)$$

From the tree level potential we have the relation  $\lambda = \frac{M_P^2}{12c_\lambda v^2}$ . Written in these terms we have:

$$\begin{aligned} V_R = & \frac{M_P^2}{12v^2} \varphi_1^4 \\ & + \frac{3e^4}{64\pi^2} \left( -\frac{5}{6} + \log \frac{e^2 \varphi_1^2}{\mu^2} \right) \varphi_1^4 \\ & + \frac{3e^4 v^2}{32\pi^2} \left( -1 - 3 \log \frac{e^2 v^2}{\mu^2} \right) \varphi_1^2. \end{aligned} \quad (4.3.0.0.8)$$

We can bind  $e$  to  $M_P$  and  $v$  at the loop level, from the definition of VEV:

$$\left. \frac{\partial V_R}{\partial \varphi_1} \right|_v = 0. \quad (4.3.0.0.9)$$

This gives the condition:

$$\frac{M_P^2 v}{3} - \frac{e^4 v^3}{4\pi^2} - \frac{3e^4 v^3}{8\pi^2} \log \frac{e^2 v^2}{\mu^2} = 0. \quad (4.3.0.0.10)$$

Setting scale parameter  $\mu$  to the effective mass of the vector, namely  $ev$ , we have simpler form of:

$$-\frac{e^4 v^3}{4\pi^2} + \frac{M_P^2 v}{3} = 0. \quad (4.3.0.0.11)$$

Which gives:

$$e^4 = \frac{4M_P^2 \pi^2}{3v^2}. \quad (4.3.0.0.12)$$

Writing potential with this substitutions yields asdad:

$$V_R = \frac{M_P^2}{16v^2} \left( \frac{1}{2} + \log \frac{\varphi_1^2}{v^2} \right) \varphi_1^4 - \frac{M_P^2}{8} \varphi_1^2. \quad (4.3.0.0.13)$$

Potential can be written also in terms of  $v$  and  $e$ :

$$V_R = \frac{3e^4}{64\pi^2} \left( \frac{1}{2} + \log \frac{\varphi_1^2}{v^2} \right) \varphi_1^4 - \frac{3e^2 v^2}{32\pi^2} \varphi_1^2. \quad (4.3.0.0.14)$$

We can also derive direct relation between  $\lambda$  and  $e^4$ , namely:

$$\lambda = \frac{e^4}{16c_\lambda \pi^2}, \quad (4.3.0.0.15)$$

and the ratio of masses of scalar and vector:

$$\frac{M_P^2}{m(V)^2} = \frac{\frac{3e^4 v^2}{4\pi^2}}{e^2 v^2} = \frac{3e^2}{4\pi^2}. \quad (4.3.0.0.16)$$

## 4.4 On shell with $\varphi^4$ potential

Although the previous renormalisation succeeded in making the potential finite and being in agreement with radiative symmetry breaking it has one last problem – the resulting potential have square term in the field.

We would like to investigate, whether one can renormalise theory such that tree level mass is the physical mass, namely  $\left. \frac{\partial^2 V_R}{\partial \varphi_1^2} \right|_v$  (on-shell condition) and at the same time square term vanishes.

We will follow, what was done in [1], as described in 3.3.1 and by hand put the condition to square terms to vanish, which is equivalent to the condition for  $\left. \frac{\partial^2 V_R}{\partial \varphi_1^2} \right|_0 = 0$ .

We can see, that in the renormalised potential:

$$V_R = c_\lambda \lambda \varphi_1^4 + \frac{3e^4}{64\pi^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 \varphi_1^2}{4\pi \mu^2} \right) \varphi_1^4 + c_\lambda \delta \lambda \varphi_1^4 + c_m \delta m \varphi_1^2 \quad (4.4.0.0.1)$$

only term square in the fields is  $c_m \delta m \varphi_1^2$ , therefore  $\delta m$  should be zero. Note, that it is not the same as disregarding  $\delta m$  automatically. As stated in [1], the theory has no a priori symmetry for  $\delta m$  to be 0 and we are respectful to that. It just so happens that in our regularisation scheme, if we want to have no square terms in the resulting potential (or to  $\left. \frac{\partial^2 V_R}{\partial \varphi_1^2} \right|_0$  to vanish, which is equivalent), we need to put  $\delta m = 0$ . With different regularisation to satisfy this condition, we would have different  $\delta m$ , as seen in [1], written here at 3.3.1.0.3.

Therefore we impose following renormalising conditions:

$$\left. \frac{\partial^2}{\partial \varphi_1^2} V_R^{1\text{-loop}} \right|_v = 0 \quad (4.4.0.0.2)$$

$$\left. \frac{\partial^2}{\partial \varphi_1^2} V_R \right|_0 = 0. \quad (4.4.0.0.3)$$

Written in the full form these conditions take form:

$$\begin{aligned} & \frac{9e^4 v^2}{16\pi^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 v^2}{4\pi\mu^2} \right) \\ & + 12c_\lambda \delta\lambda v^2 + \frac{21e^4}{32\pi^2} v^2 + 2c_m \delta m = 0 \end{aligned} \quad (4.4.0.0.4)$$

$$\left. \frac{\partial^2}{\partial \varphi_1^2} V_R \right|_0 = 0. \quad (4.4.0.0.5)$$

After solving equations for  $\delta m$  and  $\delta\lambda$  we obtain:

$$\delta m = 0 \quad (4.4.0.0.6)$$

$$\delta\lambda = -\frac{3e^4}{64c_\lambda\pi^2} \left( -\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log \frac{e^2 v^2}{4\pi\mu^2} \right) - \frac{7e^4}{128c_\lambda\pi^2}. \quad (4.4.0.0.7)$$

The renormalized potential is then:

$$V_R = c_\lambda \lambda \varphi_1^4 + \frac{3e^4}{64\pi^2} \left( -\frac{7}{6} + \log \frac{\varphi_1^2}{v^2} \right) \varphi_1^4. \quad (4.4.0.0.8)$$

From the tree level potential we have the relation  $\lambda = \frac{M_P^2}{12c_\lambda v^2}$ . Written in these terms we have:

$$V_R = \frac{M_P^2}{12v^2} \varphi_1^4 + \frac{3e^4}{64\pi^2} \left( -\frac{7}{6} + \log \frac{\varphi_1^2}{v^2} \right) \varphi_1^4. \quad (4.4.0.0.9)$$

We can bind  $e$  to  $M_P$  and  $v$  at the loop level, from the definition of VEV:

$$\left. \frac{\partial V_R}{\partial \varphi_1} \right|_v = 0. \quad (4.4.0.0.10)$$

This gives the condition:

$$\frac{M_P^2 v}{3} + \frac{3e^4 v^3}{16\pi^2} \left( -\frac{7}{6} \right) + \frac{3e^4 v^3}{32\pi^2} = 0. \quad (4.4.0.0.11)$$

Which gives:

$$e^4 = \frac{8M_P^2\pi^2}{3v^2}. \quad (4.4.0.0.12)$$

Writing potential with this substitutions yields:

$$V_R = \frac{M_P^2}{8v^2} \left( -\frac{1}{2} + \log \frac{\varphi_1^2}{v^2} \right) \varphi_1^4. \quad (4.4.0.0.13)$$

Potential can be written also in terms of  $v$  and  $e$ :

$$V_R = \frac{3e^4}{64\pi^2} \left( -\frac{1}{2} + \log \frac{\varphi_1^2}{v^2} \right) \varphi_1^4. \quad (4.4.0.0.14)$$

Note, that, this is exactly same potential as in [1] and 2.3.1.0.7. We can now calculate following quantities: - relation between  $\lambda$  and  $e^4$ :

$$\lambda = \frac{e^4}{32c_\lambda\pi^2}, \quad (4.4.0.0.15)$$

- ratio between masses of scalar and vector:

$$\frac{M_P^2}{m(V)^2} = \frac{\frac{3e^4v^2}{8\pi^2}}{e^2v^2} = \frac{3e^2}{8\pi^2}. \quad (4.4.0.0.16)$$

## 4.5 Passing between schemes

Writing 4.1.0.0.13 is new field  $\varphi'$ , such that  $\varphi'_1 + v := \varphi'$  we have:

$$\frac{3e^4}{64\pi^2} \left( -\frac{1}{2} + \log \left( 1 + \frac{\varphi_1'^2}{v^2} + \frac{2\varphi_1'}{v} \right) \right) (\varphi_1'^4 + 4v\varphi_1'^3 + 6v^2\varphi_1'^2 + 4v^3\varphi_1' + v^4). \quad (4.5.0.0.1)$$

For 4.3.0.0.14 wirtten in terms of  $\varphi'_1$  we have:

$$\begin{aligned} & \frac{3e^4}{64\pi^2} \left( \frac{1}{2} + \log \left( 1 + \frac{\varphi_1'^2}{v^2} + \frac{2\varphi_1'}{v} \right) \right) (\varphi_1'^4 + 4v\varphi_1'^3 + 6v^2\varphi_1'^2 + 4v^3\varphi_1' + v^4) \\ & - \frac{3e^2v^2}{32\pi^2} (\varphi_1'^2 + 2v\varphi_1' + v^2) \end{aligned} \quad (4.5.0.0.2)$$

$$= \frac{3e^4}{64\pi^2} \left( \frac{1}{2} + \log \left( 1 + \frac{\varphi_1'^2}{v^2} + \frac{2\varphi_1'}{v} \right) \right) (\varphi_1'^4 + 4v\varphi_1'^3) \quad (4.5.0.0.3)$$

$$\begin{aligned} & + \left( \frac{9e^4v^2}{32\pi^2} \left( \frac{1}{2} + \log \left( 1 + \frac{\varphi_1'^2}{v^2} + \frac{2\varphi_1'}{v} \right) \right) - \frac{3e^2v^2}{32\pi^2} \right) \varphi_1'^2 \\ & + \left( \frac{3e^4v^3}{16\pi^2} \left( \frac{1}{2} + \log \left( 1 + \frac{\varphi_1'^2}{v^2} + \frac{2\varphi_1'}{v} \right) \right) - \frac{3e^2v^3}{16\pi^2} \right) \varphi_1' \\ & + \frac{3e^4v^4}{64\pi^2} \left( \frac{1}{2} + \log \left( 1 + \frac{\varphi_1'^2}{v^2} + \frac{2\varphi_1'}{v} \right) \right) - \frac{3e^2v^4}{32\pi^2} \end{aligned} \quad (4.5.0.0.4)$$

$$= \frac{3e^4}{64\pi^2} \left( \frac{1}{2} + \log \left( 1 + \frac{\varphi_1'^2}{v^2} + \frac{2\varphi_1'}{v} \right) \right) (\varphi_1'^4 + 4v\varphi_1'^3) \quad (4.5.0.0.5)$$

$$\begin{aligned} & + \frac{9e^4v^2}{32\pi^2} \left( \frac{1}{6} + \log \left( 1 + \frac{\varphi_1'^2}{v^2} + \frac{2\varphi_1'}{v} \right) \right) \varphi_1'^2 \\ & + \frac{3e^4v^3}{16\pi^2} \left( -\frac{1}{2} + \log \left( 1 + \frac{\varphi_1'^2}{v^2} + \frac{2\varphi_1'}{v} \right) \right) \varphi_1' \\ & + \frac{3e^4v^4}{64\pi^2} \left( -\frac{3}{2} + \log \left( 1 + \frac{\varphi_1'^2}{v^2} + \frac{2\varphi_1'}{v} \right) \right) \end{aligned} \quad (4.5.0.0.6)$$

## 4.6 Summary table

Reg. – Regularisation scheme, Ren. – Renormalisation scheme,  $k^2 = \Lambda^2$  – regularisation by cut-off at  $k^2 = \Lambda^2$ .

Reg.	Ren.					
$k^2 = \Lambda^2$	$\frac{d^2}{d\varphi_1^2} V_R \Big _0 = 0$ $\frac{d^4}{d\varphi_1^4} V_R \Big _v = 24c_\lambda \lambda$	$\lambda = \frac{e^4}{\pi^2}$	$\lambda = \frac{M_P^2}{v^2}$	$e^4 = \frac{M_P^2}{\pi^2}$		
Dimensional regularisation	$\frac{d^2}{d\varphi_1^2} V_R \Big _0 = 0$ $\frac{d^4}{d\varphi_1^4} V_R \Big _v = 24c_\lambda \lambda$					

# TO DO: napisać czemu rzeczy tu nie działają tak jak byśmy chcieli

Na przykład, że nie bardzo wiemy jak przechodzić między schematami, a jak uwzględ-  
niami jak kontrybuuje niezerowy pęd, to możliwe, że będziemy wiedzieć, więc:

## Chapter 5

# On shell renormalisation of the effective potential – finite momentum approach

TO DO: wyrzucić wgl wersję z pierwszą pochodną z piedestału, napisać tylko, że w ten sposób, toto nie wyjdzie

One of the goals, was to renormalised potential in the classical on shell scheme. We will present this aprousch here. Following //cittation needed (Japończycy?)//. This approach however, will have one big issue, described in 5.9

### 5.1 Second derivative condition

To calculate on-shell renormalisation we need to calculate self energy. However, it turns out, that simple calculation of self energy fails the test of comparison between the zero-momentum limit of the self energy and the second derivative of the effective potential.

Namely, it should be sasisfied that:

$$\lim_{p^2 \rightarrow 0} \Sigma(p^2) = \frac{\partial^2 V_{eff}}{\partial \varphi_1^2}, \quad (5.1.0.0.1)$$

TO DO: napisać ile wychodzi

but it is not the case.

Let us show this.

We have that:

$$V_{eff} = \quad (5.1.0.0.2)$$



So:

$$\frac{\partial^2 V_{eff}}{\partial \varphi_1^2} = \quad (5.1.0.0.3)$$

On the other hand contribution to  $\Sigma$  constituted by following diagrams:

## TO DO: diagrams

The values of these diagrams are as follows:

$$\begin{aligned} & -i \frac{e^2}{M_V} \left[ M_V^2 a(M_2) + (-p^2 - M_V^2 + M_2^2) a(M_V) - (p^2 + M_2^2)^2 b_0(p, 0, M_2) \right. \\ & \left. + (p^2 + M_2^2 - M_V^2)^2 b_0(p, M_V, M_2) \right] \end{aligned} \quad (5.1.0.0.4)$$

$$-i 3e^2 a_b(M_V) \quad (5.1.0.0.5)$$

$$-i 12c_\lambda \lambda a(M_1) \quad (5.1.0.0.6)$$

$$-i 4c_\lambda \lambda a(M_2) \quad (5.1.0.0.7)$$

So:

$$\begin{aligned} \Sigma = & -\frac{e^2}{M_V} \left[ M_V^2 a(M_2) + (-p^2 - M_V^2 + M_2^2) a(M_V) - (p^2 + M_2^2)^2 b_0(p, 0, M_2) \right. \\ & \left. + (p^2 + M_2^2 - M_V^2)^2 b_0(p, M_V, M_2) \right] \\ & - 3e^2 a_b(M_V) \\ & - 12c_\lambda \lambda a(M_1) \\ & - 4c_\lambda \lambda a(M_2) \end{aligned} \quad (5.1.0.0.8)$$

Where:

$$a(M) = \quad (5.1.0.0.9)$$

$$b_0(p, M_1, M_2) = \quad (5.1.0.0.10)$$

$$a^b(M) = \quad (5.1.0.0.11)$$

and

$$b_0(0, M_1, M_2) = \quad (5.1.0.0.12)$$

Then, writing more explicitly we have that:

$$abcd \quad (5.1.0.0.13)$$

So, we see, that indeed:

$$\lim_{p^2 \rightarrow 0} \Sigma(p^2) \neq \frac{\partial^2 V_{eff}}{\partial \varphi_1^2}. \quad (5.1.0.0.14)$$

## 5.2 Introducing explicit VEV

There is a solution to this:

From the  $\overline{\text{MS}}$  considerations, we know that  $\Phi$  have non-zero VEV, let us call it  $v$ . Let us rotate  $\Phi$  in such a way, that  $\langle \varphi_1 \rangle = v$  and  $\langle \varphi_2 \rangle = 0$ , where now  $v$  is real.

Keeping this in mind, we can rewrite Lagrangian in terms of shifted fields  $\varphi_1, \varphi_2$  which have both zero VEV, now VEV is explicitly in the Lagrangian:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu(\varphi_1 + v) - eA_\mu\varphi_2)^2 \\ & + \frac{1}{2}(\partial_\mu\varphi_2 + eA_\mu(\varphi_1 + v))^2 - c_\lambda\lambda((\varphi_1 + v)^2 + \varphi_2^2)^2. \end{aligned} \quad (5.2.0.0.1)$$

This breaks the symmetry, but now there are more interaction terms in the Lagrangian and this leads to different self energy, now consistent with the second derivative of the effective potential, as will be shown in 5.4.

## 5.3 Contributions to $\Sigma$ and $T$

Contributions to  $\Sigma$  and  $T$  constitutes now of the following diagrams:

**TO DO: diagrams**

The values of diagrams are as follows:

Contributing to  $\Sigma$  are:

$$\begin{aligned} & -i\frac{e^2}{M_V} \left[ M_V^2 a(M_2) + (-p^2 - M_V^2 + M_2^2) a(M_V) - (p^2 + M_2^2)^2 b_0(p, 0, M_2) \right. \\ & \left. + (p^2 + M_2^2 - M_V^2)^2 b_0(p, M_V, M_2) \right] \end{aligned} \quad (5.3.0.0.1)$$

$$\begin{aligned} & -i\frac{e^4 v^2}{2M_V^4} \left[ 2M_V^2 a(M_V) + p^4 b_0(p, 0, 0) - 2(p^2 - M_V^2)^2 b_0(p, M_V, 0) \right. \\ & \left. + 16M_V^4 b_0^b(p, M_V, M_V) + (p^4 - 4p^2 M_V^2 - 4M_V^4) b_0(p, M_V, M_V) \right]. \end{aligned} \quad (5.3.0.0.2)$$

$$-i3e^2 a_b(M_V) = \quad (5.3.0.0.3)$$

$$-i12c_\lambda \lambda a(M_1) = \quad (5.3.0.0.4)$$

$$-i4c_\lambda \lambda a(M_2) = \quad (5.3.0.0.5)$$

$$-i288c_\lambda^2 \lambda^2 v^2 b_0(p, M_1, M_1) = \quad (5.3.0.0.6)$$

$$-i32c_\lambda^2 \lambda^2 v^2 b_0(p, M_2, M_2) = \quad (5.3.0.0.7)$$

$$(5.3.0.0.8)$$

Contributing to  $T$  are:

$$-i3e^2 v a^b(M_V) = \quad (5.3.0.0.9)$$

$$-i12c_\lambda \lambda v a(M_1) = \quad (5.3.0.0.10)$$

$$-i4c_\lambda \lambda v a(M_2) = \quad (5.3.0.0.11)$$

$$-i4c_\lambda \lambda v^3 = . \quad (5.3.0.0.12)$$

Where

$$a(M) = \quad (5.3.0.0.13)$$

$$b_0(p, M_1, M_2) = \quad (5.3.0.0.14)$$

$$a^b(M) = \quad (5.3.0.0.15)$$

$$b_0^b(p, M_1, M_2) = . \quad (5.3.0.0.16)$$

## 5.4 Second derivative condition – explicit VEV

Now, we have that:

$$\begin{aligned} \Sigma = & -\frac{e^2}{M_V^2} \left[ M_V^2 a(M_2) + (-p^2 - M_V^2 + M_2^2) a(M_V) - (p^2 + M_2^2)^2 b_0(p, 0, M_2) \right. \\ & \left. + (p^2 + M_2^2 - M_V^2)^2 b_0(p, M_V, M_2) \right] \\ & -\frac{e^4 v^2}{2M_V^4} \left[ 2M_V^2 a(M_V) + p^4 b_0(p, 0, 0) - 2(p^2 - M_V^2)^2 b_0(p, M_V, 0) \right. \\ & \left. + 16M_V^4 b_0^b(p, M_V, M_V) + (p^4 - 4p^2 M_V^2 - 4M_V^4) b_0(p, M_V, M_V) \right] \\ & - 3e^2 a_b(M_V) \\ & - 12c_\lambda \lambda a(M_1) \\ & - 4c_\lambda \lambda a(M_2) \\ & - 288c_\lambda^2 \lambda^2 v^2 b_0(p, M_1, M_1) \\ & - 32c_\lambda^2 \lambda^2 v^2 b_0(p, M_2, M_2), \end{aligned} \quad (5.4.0.0.1)$$

where

$$a(M) = \quad (5.4.0.0.2)$$

$$b_0(p, M_1, M_2) = \quad (5.4.0.0.3)$$

$$a^b(M) = \quad (5.4.0.0.4)$$

$$b_0^b(p, M_1, M_2) = \quad (5.4.0.0.5)$$

and

$$b_0(0, M_1, M_2) = \quad (5.4.0.0.6)$$

$$b_0^b(0, M_1, M_2) = \quad (5.4.0.0.7)$$

So, we have that:

$$\Sigma = \quad (5.4.0.0.8)$$

So indeed, we have now, that:

$$\lim_{p^2 \rightarrow 0} \Sigma(p^2) = \frac{\partial^2 V_{eff}}{\partial \varphi_1^2}, \quad (5.4.0.0.9)$$

## 5.5 Counterterms

Following [1] we put the mass counterterm even though initially the mass term was not present in the Lagrangian. We do so, because our theory has no a priori symmetry that would guarantee disappearing mass term in renormalised Lagrangian. It will turn out to be crucial in our .

The Lagrangian with  $\delta Z$ ,  $\delta\lambda$  and  $\delta m$  counterterms looks like this:

$$\mathcal{L}_{\mathcal{R}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad (5.5.0.0.1)$$

$$\begin{aligned} &+ (1 + \delta Z) \left( \frac{1}{2} (\partial_\mu(\varphi_1 + v) - eA_\mu\varphi_2)^2 + \frac{1}{2} (\partial_\mu\varphi_2 + eA_\mu(\varphi_1 + v))^2 \right) \\ &- (1 + \delta Z)^2 c_\lambda (\lambda + \delta\lambda) ((\varphi_1 + v)^2 + \varphi_2^2)^2 \\ &- c_m \delta m ((\varphi_1 + v)^2 + \varphi_2^2). \end{aligned} \quad (5.5.0.0.2)$$

Separating the terms with the first power of renormalisation constants and second power of  $\varphi_1$ , we obtain correction to the self energy equal to:

$$\delta\Sigma = -12c_\lambda v^2 (2\lambda\delta Z_\varphi + \delta\lambda) - 2c_m \delta m - p^2 \delta Z_\varphi, \quad (5.5.0.0.3)$$

where  $p^2 = -\partial_\mu\varphi_1\partial^\mu\varphi_1$ .

Separating the terms with the first power of renormalisation constants and first power of  $\varphi_1$ , we obtain correction to the tadpole equal to:

$$\delta T = -4c_\lambda v^3 (2\lambda\delta Z_\varphi + \delta\lambda) - 2c_m \delta m v. \quad (5.5.0.0.4)$$

## 5.6 Renormalization conditions

First approach is to impose renormalisation conditions resembling classical on-shell. Here,  $\Sigma'$  stands for  $\frac{d\Sigma}{dp^2}$  and, if not stated otherwise,  $\Sigma$ ,  $\delta\Sigma$  and  $\Sigma'$  are evaluated at  $p^2 = M_P^2$ , where  $M_P$  stands for physical mass. We denote real part as  $\Re()$ .

$$T + \delta T = 0 \quad (5.6.0.0.1)$$

$$\Re(\Sigma) + \Re(\delta\Sigma) = 0 \quad (5.6.0.0.2)$$

$$\Re(\Sigma') = 0. \quad (5.6.0.0.3)$$

This gives us:

$$\delta m = \frac{-1}{4c_m} \left( \Re(\Sigma) - \frac{3}{v} T - M_P^2 \Re(\Sigma') \right) \quad (5.6.0.0.4)$$

$$\delta\lambda = \frac{1}{8c_\lambda v^2} \left( \Re(\Sigma) - \frac{1}{v} T - (16c_\lambda \lambda v^2 + M_P^2) \Re(\Sigma') \right) \quad (5.6.0.0.5)$$

$$\delta Z = \Re(\Sigma'). \quad (5.6.0.0.6)$$

We define  $M_P^2$  as the second derivative of the tree potential evaluated at VEV, so:

$$M_P^2 = \frac{\partial^2}{\partial\varphi_1^2} c_\lambda \lambda \varphi_1^4 \Big|_v = 12c_\lambda \lambda v^2 \quad (5.6.0.0.7)$$

and we impose, that it does not change after one loop contributions.

## 5.7 Partition by order in $e$

Here, we will be interested in only contributions up to order  $e^4$ . From our  $\overline{\text{MS}}$  considerations we can see, that  $\lambda$  should be of order  $e^4$ , that  $M_V^2$  should be of order  $e^2$  and that  $M_1, M_2$  should be of order  $e^4$ . As that we are intersted only in following parts of contributions to  $\Sigma$  and  $T$ :

$$\begin{aligned}\Sigma_{e^0} = & -\frac{e^2}{M_V^2} \left[ -p^4 b_0(p, 0, M_2) + p^4 b_0(p, M_V, M_2) \right] \\ & -\frac{e^4 v^2}{M_V^4} \left[ p^4 b_0(p, 0, 0) - 2p^4 b_0(p, M_V, 0) + p^4 b_0(p, M_V, M_V) \right]\end{aligned}\quad (5.7.0.0.1)$$

$$\begin{aligned}\Sigma_{e^2} = & -\frac{e^2}{M_V^2} \left[ -p^2 a(M_V) - 2p^2 M_V^2 b_0(p, M_V, M_2) \right] \\ & -\frac{e^4 v^2}{2M_V^4} \left[ 4p^2 M_V^2 b_0(p, M_V, 0) - 4p^2 M_V^2 b_0(p, M_V, M_V) \right]\end{aligned}\quad (5.7.0.0.2)$$

$$\begin{aligned}\Sigma_{e^4} = & -\frac{e^2}{M_V^2} a(M_V) \left[ -M_V^2 a(M_V) + M_V^4 b_0(p, M_V, M_2) \right] \\ & -\frac{e^4 v^2}{2M_V^4} \left[ 2M_V^2 a(M_V) - 2M_V^4 b_0(p, M_V, 0) \right. \\ & \left. + 16M_V^4 b_0^b(p, M_V, M_V) - 4M_V^4 b_0(p, M_V, M_V) \right] \\ & -3e^2 a^b(M_V) \\ & -\frac{e^2}{M_V^2} \left[ -2p^2 M_2^2 b_0(p, 0, M_2) + 2p^2 M_2^2 b_0(p, M_V, M_2) \right]\end{aligned}\quad (5.7.0.0.3)$$

$$T_{e^4} = -3e^2 v a^b(M_V) - 4c_\lambda \lambda v^3. \quad (5.7.0.0.4)$$

## 5.8 Finitness of the renormalized potential

The divergent part of  $T$ ,  $\Sigma$  and  $\Sigma'$  are:

$$\text{div} T = -\frac{3e^4 v^4}{16\pi^2} \left( -\frac{2}{\epsilon} \right) \quad (5.8.0.0.1)$$

$$\text{div} \Sigma = \frac{6e^2 (M_P^2 - 3e^2 v^2)}{32\pi^2} \left( -\frac{2}{\epsilon} \right) \quad (5.8.0.0.2)$$

$$\text{div} \Sigma' = \frac{3e^2}{16\pi^2} \left( -\frac{2}{\epsilon} \right). \quad (5.8.0.0.3)$$

After subsituing to  $\delta\lambda$ ,  $\delta m$ ,  $\delta Z$  and then to  $V_R$  we see that  $\text{div} V_R = 0$ , thus renormalisation procedure succeeds in canceling divergences.

## 5.9 Resulting effective potential

Although full potential is to long to explicitly write in symbolic form, we will present it with some of the numerical values substituted. What we should note, is that in

this case renormalized potential have both  $\varphi_1^2$  term and  $\varphi_1^4$  term, unlike in [1].

**TO DO: diagramy, równania, takie, takie**

## 5.10 Downsides of this approach

The big downside of this method is that we incorporated the condition of not changing VEV between tree and one-loop level to our renormalization conditions. This results in inability to derive relationships at one-loop level between  $M_P^2$ ,  $v$  and  $e$  from the condition that  $v$  is minimum of the first derivative of renormalised effective potential. Using this condition results in tautology, as we already used this condition for renormalisation.

# Chapter 6

## Summary

## Chapter 7

## Conclusions



# Appendix A

## Introduction – mass term case

For comparison, we present here results for above methods used in the case with explicit mass term.

### A.1 On-shell finite momentum approach

## TO DO: przeredagować to, bo na razie bez sensu, że jeest pierwsze

For the comparison, we will present the same calculation, performed on the analogous theory with explicit mass term. Similarly as in the 5.2.0.0.1 we need to shift fields for 5.4.0.0.9 to be satisfied. The Lagrangian in this case is:

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu(\varphi_1 + v) - eA_\mu\varphi_2)^2 \\ & + \frac{1}{2}(\partial_\mu\varphi_2 + eA_\mu(\varphi_1 + v))^2 - c_m m^2((\varphi_1 + v)^2 + \varphi_2^2) - c_\lambda \lambda((\varphi_1 + v)^2 + \varphi_2^2)^2.\end{aligned}\tag{A.1.0.0.1}$$

With renormalisation constants:

$$\begin{aligned}\mathcal{L}_{\mathcal{R}} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + (1 + \delta Z)\left(\frac{1}{2}(\partial_\mu(\varphi_1 + v) - eA_\mu\varphi_2)^2 + \frac{1}{2}(\partial_\mu\varphi_2 + eA_\mu(\varphi_1 + v))^2\right) \\ & - (1 + \delta Z)^2 c_\lambda (\lambda + \delta\lambda)((\varphi_1 + v)^2 + \varphi_2^2)^2 \\ & - (1 + \delta Z) c_m (m + \delta m)((\varphi_1 + v)^2 + \varphi_2^2).\end{aligned}\tag{A.1.0.0.2}$$

Corrections then are:

$$\delta\Sigma = -12c_\lambda v^2(2\lambda\delta Z_\varphi + \delta\lambda) - 2c_m\delta m - 2c_m m^2\delta Z - p^2\delta Z\tag{A.1.0.0.4}$$

$$\delta T = -4c_\lambda v^3(2\lambda\delta Z_\varphi + \delta\lambda) - 2c_m\delta m v - 2c_m m^2 v\delta Z.\tag{A.1.0.0.5}$$

This changes the form of renormalisation constants to:

$$\delta m = \frac{-1}{4c_m} \left( \Re(\Sigma) - \frac{3}{v}T - (4c_m m^2 + M_P^2) \Re(\Sigma') \right) \quad (\text{A.1.0.0.6})$$

$$\delta \lambda = \frac{1}{8c_\lambda v^2} \left( \Re(\Sigma) - \frac{1}{v}T - (16c_\lambda \lambda v^2 + M_P^2) \Re(\Sigma') \right) \quad (\text{A.1.0.0.7})$$

$$\delta Z = \Re(\Sigma'). \quad (\text{A.1.0.0.8})$$

The only difference is  $4c_m m^2$  term in A.1.0.0.6.

## A.2 "Zero momentum" approach

Here we will compare two kinds of "zero momentum" approach. First will be imposing renormalisation conditions in terms of only derivatives of effective potential. This is the zero momentum approach as first and second derivatives are limits of, respectively, tadpole and self-energy in the zero momentum limit.

Second kind will be to calculate approach from reffinite momentum in the zero momentum limit.

Later we will discuss some "potential only" version with different conditions and discuss whether adding finite momentum to it will produce satisfying results.

### Potential only version

For comparison, we include also a version of this approach stemming from reffmass term. Inclusion of the mass term do not change the form of  $\delta \lambda$  and  $\delta m$ . The first difference occurs in the potential.

First we will describe the case with derivatives II and IV used in renormalisation conditions. Then the potential is equal:

$$V_R = c_\lambda \lambda \varphi_1^4 + c_m m^2 \varphi_1^2 + \frac{e^4 \varphi_1^4}{64\pi^2} \left( 3 \log \frac{\varphi_1^2}{v^2} - \frac{25}{2} \right) + \frac{27e^4 v^2 \varphi_1^2}{32\pi^2}. \quad (\text{A.2.0.0.1})$$

Now, we have that  $M_P = 12c_\lambda \lambda v^2 + 2c_m m^2$ . Now, we will use the condition, that:

$$\left. \frac{\partial}{\partial \varphi_1} V^T \right|_v = 0. \quad (\text{A.2.0.0.2})$$

We have that  $4c_\lambda \lambda v^3 + 2c_m m^2 v = 0$ , so  $\lambda = -\frac{c_m m^2}{2c_\lambda v^2}$ , so

$$m^2 = \frac{-M_P^2}{4c_m} \quad \text{and} \quad (\text{A.2.0.0.3})$$

$$\lambda = \frac{M_P^2}{8c_\lambda v^2}. \quad (\text{A.2.0.0.4})$$

Writing  $V_R$  with respect to that gives:

**TO DO: pytanie**

$$V_R = \frac{M_P^2 \varphi_1^4}{8v^2} - \frac{M_P^2 \varphi_1^2}{4} + \frac{e^4 \varphi_1^4}{64\pi^2} \left( 3 \log \frac{\varphi_1^2}{v^2} - \frac{25}{2} \right) + \frac{27e^4 v^2 \varphi_1^2}{32\pi^2}. \quad (\text{A.2.0.0.5})$$

From this we have, that:

$$\frac{e^4 v^3}{\pi^2} = 0, \quad (\text{A.2.0.0.6})$$

which is also a not satisfying result.

However, if we drop the condition, that  $\left. \frac{\partial}{\partial \varphi_1} V^T \right|_v = 0$ , we have potential in the form:

$$V_R = \frac{M_P^2 - 2c_m m^2}{12v^2} \varphi_1^4 + c_m m^2 \varphi_1^2 + \frac{e^4 \varphi_1^4}{64\pi^2} \left( 3 \log \frac{\varphi_1^2}{v^2} - \frac{25}{2} \right) + \frac{27e^4 v^2 \varphi_1^2}{32\pi^2}. \quad (\text{A.2.0.0.7})$$

From this, using the condition that  $\left. \frac{\partial}{\partial \varphi_1} V_R \right|_v = 0$ , we can derive the correspondence between  $e$  and  $M_P$ ,  $v$  and  $m$ :

$$e^4 = -\frac{(M_P^2 + 4c_m m^2)\pi^2}{3v^2}, \quad (\text{A.2.0.0.8})$$

which is finally a sensible result as it can be realised with real, positive  $e$ . However, then it must hold that  $m^2 < -\frac{M_P^2}{4c_m}$ .

# Bibliography

- [1] Sidney Coleman and Erick Weinberg, *Radiative corrections as the origin of spontaneous symmetry breaking*, Phys. Rev. D **7** (1973Mar), 1888–1910.