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Abstract

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Chapter 1

Introduction

Predictions of the grawitational wave signal from early Universe cosmoliogical phase trnsition depend on the shape of effective potential of the theory. In this thesis we will investigate how

1.1 Toy model

For a toy model we choose theory described by the Lagransian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_{\mu}\Phi D^{\mu}\Phi^{\dagger} - \lambda\Phi^{4}, \tag{1.1.0.0.1}$$

where Φ is a comlex scalar field and the vector field present is $U(1)$ gauge boson.

Writing operator D more explicitly it reads:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial_{\mu}\Phi + ieA_{\mu}\Phi)(\partial^{\mu}\Phi^{\dagger} - ieA^{\mu}\Phi^{\dagger}) - \lambda\Phi^{4}, \tag{1.1.0.0.2}$$

For the reasons that will be clear in **??** we will write Φ field as two real scalar fields φ_1 and φ_2, such that:

$$\Phi = \frac{1}{\sqrt{2}}(\varphi_1 + \varphi_2) \tag{1.1.0.0.3}$$

Then Lagransian take form:

$$\begin{aligned}\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_{\mu}\varphi_1 - eA_{\mu}\varphi_2)(\partial^{\mu}\varphi_1 - eA^{\mu}\varphi_2)\\ +\frac{1}{2}(\partial_{\mu}\varphi_2 + eA_{\mu}\varphi_1)(\partial^{\mu}\varphi_2 + eA^{\mu}\varphi_1) - \frac{1}{4}\lambda(\varphi_1^2 + \varphi_2^2)^2,\end{aligned} \tag{1.1.0.0.4}$$

which we will write for brevity as:

$$\begin{aligned}\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_{\mu}\varphi_1 - eA_{\mu}\varphi_2)^2\\ +\frac{1}{2}(\partial_{\mu}\varphi_2 + eA_{\mu}\varphi_1)^2 - \frac{1}{4}\lambda(\varphi_1^2 + \varphi_2^2)^2.\end{aligned} \tag{1.1.0.0.5}$$

For a better track of what is independent of numerical convention, we will also write:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\varphi_1 - eA_\mu\varphi_2)^2 \\ +\frac{1}{2}(\partial_\mu\varphi_2 + eA_\mu\varphi_1)^2 - c_\lambda\lambda(\varphi_1^2 + \varphi_2^2)^2, \tag{1.1.0.0.6}$$

but $c_\lambda = \frac{1}{4}$ everywhere in the thesis if not stated otherwise.

1.2 Real model

$U(2) \times U(2)$ cośtam cośtam

Chapter 2

MS bar renormalisation of the effective potential

For now we will be working with our toy model described by Lagransian 1.1.0.0.6 In this model tree level effective potensial is equal to:

$$V_T = \frac{1}{4}\lambda(\varphi_1^2 + \varphi_2^2)^2 \tag{2.0.0.0.1}$$

The one loop correction to the effective potential is calculated as a sum of the following diagrams:

$$i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{2n}\left(\frac{\lambda\frac{1}{2}\varphi_1^2}{k^2+i\varepsilon}\right)^n \tag{2.0.0.0.2}$$

$$i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{2n}\left(\frac{1}{3}\frac{\lambda\frac{1}{2}\varphi_1^2}{k^2+i\varepsilon}\right)^n \tag{2.0.0.0.3}$$

$$i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{2n}\left(\frac{2e^2\frac{1}{2}\varphi_1^2}{k^2+i\varepsilon}\right)^n (g^\mu{}_\mu - 1) \tag{2.0.0.0.4}$$

Summing all the diagrams in series it gives:

$$i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{1}{2n}\left(\frac{\lambda\frac{1}{2}\varphi_1^2}{k^2+i\varepsilon}\right)^n \tag{2.0.0.0.5}$$

$$i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{1}{2n}\left(\frac{1}{3}\frac{\lambda\frac{1}{2}\varphi_1^2}{k^2+i\varepsilon}\right)^n \tag{2.0.0.0.6}$$

$$i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{1}{2n}\left(\frac{2e^2\frac{1}{2}\varphi_1^2}{k^2+i\varepsilon}\right)^n (g^\mu{}_\mu - 1) \tag{2.0.0.0.7}$$

After passing to $D = 4 - 2\epsilon$ dimentions and using dimentional regularisation we have:

$$V_{1L} = \frac{1}{4}\frac{(\frac{1}{2}\lambda\varphi_1^2)^2}{(4\pi)^2}\left(-\frac{2}{\epsilon}+\gamma_E-\frac{3}{2}+\log\frac{(\frac{1}{2}\lambda\varphi_1^2)^2}{4\pi\mu^2}\right)+ \tag{2.0.0.0.8}$$

$$\frac{1}{4}\frac{(\frac{1}{6}\lambda\varphi_1^2)^2}{(4\pi)^2}\left(-\frac{2}{\epsilon}+\gamma_E-\frac{3}{2}+\log\frac{(\frac{1}{6}\lambda\varphi_1^2)^2}{4\pi\mu^2}\right)+ \tag{2.0.0.0.9}$$

$$\frac{1}{4}\frac{3(e^2\varphi_1^2)^2}{(4\pi)^2}\left(-\frac{2}{\epsilon}+\gamma_E-\frac{5}{6}+\log\frac{e^2\varphi^2}{4\pi\mu^2}\right) \tag{2.0.0.0.10}$$

$\overline{\text{MS}}$ renormalisation gives:

$$V_R = \frac{1}{4}\frac{(\frac{1}{2}\lambda\varphi_1^2)^2}{(4\pi)^2}\left(-\frac{3}{2}+\log\frac{(\frac{1}{2}\lambda\varphi_1^2)^2}{\mu^2}\right)+ \tag{2.0.0.0.11}$$

$$\frac{1}{4}\frac{(\frac{1}{6}\lambda\varphi_1^2)^2}{(4\pi)^2}\left(-\frac{3}{2}+\log\frac{(\frac{1}{6}\lambda\varphi_1^2)^2}{\mu^2}\right)+ \tag{2.0.0.0.12}$$

$$\frac{1}{4}\frac{3(e^2\varphi_1^2)^2}{(4\pi)^2}\left(-\frac{5}{6}+\log\frac{e^2\varphi^2}{\mu^2}\right) \tag{2.0.0.0.13}$$

Chapter 3

On shell renormalisation of the effective potential

One of the main topic of this thesis is to show a coherent way to renormalize a conformal theory without explicit mass term in the on-shell scheme.

To calculate on-shell renormalisation we need to calculate self energy. However, it turns out, that simple calculation of self energy fails the test of comparison between the zero-momentum limit of the self energy and the second derivative of the effective potential.
Namely, it schould be safisfied that:

$$\lim_{p^2 \to 0} \Sigma(p^2) = \frac{\partial^2 V_{eff}}{\partial \varphi_1^2}, \tag{3.0.0.0.1}$$

TO DO: napisać ile wychodzi

but it is not the case.

However, from the $\overline{\text{MS}}$ considerations, we know that Φ have non-zero VEV, let us call it v. Let us rotate Φ in such a way, that $\langle \varphi_1 \rangle = v$ and $\langle \varphi_2 \rangle = 0$, where now v is real.
Keeping this in mind, we can rewrite Lagransian in terms of shifted fields φ_1, φ_2 which have both zero VEV, now VEV is explicitly in the Lagransian:

$$\begin{aligned} \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}(\partial_\mu(\varphi_1 + v) - e A_\mu \varphi_2)^2 \\ + \frac{1}{2}(\partial_\mu \varphi_2 + e A_\mu(\varphi_1 + v))^2 - c_\lambda \lambda((\varphi_1 + v)^2 + \varphi_2^2)^2. \end{aligned} \tag{3.0.0.0.2}$$

This breaks the symmetry, but now there are more interaction terms in the Lagransian and this leads to different self energy, now consistent with the second derivative of the effective potential, as will be shown in **??**.

Following [1] we put the mass counterterm even though initialy the mass term was not present in the Lagransian. It will turn out to be crutial in **??**.
The Lagransian with δZ_φ, δZ_λ and δZ_m counterterms looks like this:

$$\mathcal{L}_{\mathcal{R}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{3.0.0.0.3}$$

$$+(1+\delta Z)(\frac{1}{2}(\partial_\mu(\varphi_1+v)-eA_\mu\varphi_2)^2+\frac{1}{2}(\partial_\mu\varphi_2+eA_\mu(\varphi_1+v))^2)$$
$$-(1+\delta Z)^2c_\lambda(\lambda+\delta\lambda)((\varphi_1+v)^2+\varphi_2^2)^2$$
$$+\delta m c_m((\varphi_1+v)^2+\varphi_2^2). \tag{3.0.0.0.4}$$

Separating the terms with the first power of renormalisation constants and second power of φ_1, we obtain correction to the self energy equal to:

$$\delta\Sigma = -6c_\lambda v^2(2\lambda\delta Z_\varphi+\delta\lambda)+c_m\delta m-p^2\delta Z_\varphi, \tag{3.0.0.0.5}$$

where $p^2=\partial_\mu\varphi_1\partial^\mu\varphi_1$.
Separating the terms with the first power of renormalisation constants and first power of φ_1, we obtain correction to the tadpole equal to:

$$\delta T = -4c_\lambda v^3(2\lambda\delta Z_\varphi+\delta\lambda)+2vc_m\delta m \tag{3.0.0.0.6}$$

First approuch is to impose renormalisation conditions resembling clissical on-shell (M_P stands for physical mass):

$$T+\delta T=0 \tag{3.0.0.0.7}$$
$$\Re\left(\Sigma(p^2=M_P^2)\right)+\Re\left(\delta\Sigma(p^2=M_P^2)\right)=M_P^2 \tag{3.0.0.0.8}$$
$$\frac{\mathrm{d}\Sigma}{\mathrm{d}p^2}(p^2=M_P^2)=0 \tag{3.0.0.0.9}$$

We will denote $\frac{\mathrm{d}\Sigma}{\mathrm{d}p^2}$ as Σ'.

This gives us:

$$\delta m = \frac{1}{4c_m}\left(-\Re\left(\Sigma(M_P^2)\right)+M_P^2+\frac{3}{v}T+M_P^2\Re\left(\Sigma'(M_P^2)\right)\right) \tag{3.0.0.0.10}$$
$$\delta\lambda = \frac{1}{8c_\lambda v^3}\left(v\Re\left(\Sigma(M_P^2)\right)-T-(16c_\lambda\lambda v^3+M_P^2v)\Re\left(\Sigma'(M_P^2)\right)\right) \tag{3.0.0.0.11}$$
$$\delta Z_\varphi = \Re\left(\Sigma'(M_P^2)\right) \tag{3.0.0.0.12}$$

3.1 "Zero momentum" approuch

Chapter 4

Conclusions

Bibliography

[1] Sidney Coleman and Erick Weinberg, *Radiative corrections as the origin of spontaneous symmetry breaking*, Phys. Rev. D **7** (1973Mar), 1888–1910.