Title

Contents

Abstract

A

Chapter 1

Introduction

Predictions of the grawitational wave signal from early Universe cosmoliogical phase trnsition depend on the shape of effective potential of the theory. In this thesis we will investigate how different renormalisations schemes can change form of that potential.

Chapters 1-4 are mean as an introduction. New results are in 5 onward.

Chapter 2

Technical introduction

2.1 Models

2.1.1 Toy model

This model will be used throughout the ehole thesis, unless stated otherwise.
For a toy model we choose theory of scalar electrodynamics, described by the Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\Phi D^\mu\Phi^\dagger - \lambda\Phi^4, \tag{2.1.1.0.1}$$

where Φ is a complex scalar field and the vector field present is $U(1)$ gauge boson.

Writing operator D more explicitly it reads:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial_\mu\Phi + ieA_\mu\Phi)(\partial^\mu\Phi^\dagger - ieA^\mu\Phi^\dagger) - \lambda\Phi^4, \tag{2.1.1.0.2}$$

For the reasons that will be clear in **??** we will write Φ field as two real scalar fields φ_1 and φ_2, such that:

$$\Phi = \frac{1}{\sqrt{2}}(\varphi_1 + \varphi_2) \tag{2.1.1.0.3}$$

Then Lagrangian takes form:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\varphi_1 - eA_\mu\varphi_2)(\partial^\mu\varphi_1 - eA^\mu\varphi_2) \\ &+\frac{1}{2}(\partial_\mu\varphi_2 + eA_\mu\varphi_1)(\partial^\mu\varphi_2 + eA^\mu\varphi_1) - \frac{1}{4}\lambda(\varphi_1^2 + \varphi_2^2)^2,\end{aligned} \tag{2.1.1.0.4}$$

which we will write for brevity as:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\varphi_1 - eA_\mu\varphi_2)^2 \\ &+\frac{1}{2}(\partial_\mu\varphi_2 + eA_\mu\varphi_1)^2 - \frac{1}{4}\lambda(\varphi_1^2 + \varphi_2^2)^2.\end{aligned} \tag{2.1.1.0.5}$$

For a better track of what is independent of numerical convention, we will also write:

$$\begin{aligned}\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\varphi_1 - eA_\mu\varphi_2)^2 \\ +\frac{1}{2}(\partial_\mu\varphi_2 + eA_\mu\varphi_1)^2 - c_\lambda\lambda(\varphi_1^2+\varphi_2^2)^2,\end{aligned} \tag{2.1.1.0.6}$$

but $c_\lambda = \frac{1}{4}$ everywhere in the thesis if not stated otherwise.

2.1.2 Real model

$U(2) \times U(2)$ coṡtam coṡtam

2.2 Renormalisation schemes

2.2.1 $\overline{\mathrm{MS}}$

The minimal-substraction scheme

2.2.2 On-shell

Zero momentum limit version

2.2.3 Half $\overline{\mathrm{MS}}$ -Half On-shell

2.3 Effective potential

TO DO: some statements about effective potential in general

Chapter 3

Calculation of the unrenormalized effective potential

Tree level potential in our model is:

$$V_T = \frac{1}{4}\lambda(\varphi_1^2 + \varphi_2^2)^2 \tag{3.0.0.0.1}$$

The one loop correction to the effective potential is calculated as a sum of the following diagrams:

$$i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{2n}\left(\frac{\lambda\frac{1}{2}\varphi_1^2}{k^2+i\varepsilon}\right)^n \tag{3.0.0.0.2}$$

$$i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{2n}\left(\frac{1}{3}\frac{\lambda\frac{1}{2}\varphi_1^2}{k^2+i\varepsilon}\right)^n \tag{3.0.0.0.3}$$

$$i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{2n}\left(\frac{2e^2\frac{1}{2}\varphi_1^2}{k^2+i\varepsilon}\right)^n (g^\mu{}_\mu - 1) \tag{3.0.0.0.4}$$

Summing all the diagrams in series it gives:

$$i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{1}{2n}\left(\frac{\lambda\frac{1}{2}\varphi_1^2}{k^2+i\varepsilon}\right)^n \tag{3.0.0.0.5}$$

$$i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{1}{2n}\left(\frac{1}{3}\frac{\lambda\frac{1}{2}\varphi_1^2}{k^2+i\varepsilon}\right)^n \tag{3.0.0.0.6}$$

$$i\int \frac{\mathrm{d}^4k}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{1}{2n}\left(\frac{2e^2\frac{1}{2}\varphi_1^2}{k^2+i\varepsilon}\right)^n (g^\mu{}_\mu - 1) \tag{3.0.0.0.7}$$

After passing to $D = 4 - 2\epsilon$ dimensions and using dimensional regularisation we have:

$$V_{1L} = \frac{1}{4}\frac{(\frac{1}{2}\lambda\varphi_1^2)^2}{(4\pi)^2}\left(-\frac{2}{\epsilon}+\gamma_E-\frac{3}{2}+\log\frac{(\frac{1}{2}\lambda\varphi_1^2)^2}{4\pi\mu^2}\right)+ \tag{3.0.0.0.8}$$

$$\frac{1}{4}\frac{(\frac{1}{6}\lambda\varphi_1^2)^2}{(4\pi)^2}\left(-\frac{2}{\epsilon}+\gamma_E-\frac{3}{2}+\log\frac{(\frac{1}{6}\lambda\varphi_1^2)^2}{4\pi\mu^2}\right)+ \tag{3.0.0.0.9}$$

$$\frac{1}{4}\frac{3(e^2\varphi_1^2)^2}{(4\pi)^2}\left(-\frac{2}{\epsilon}+\gamma_E-\frac{5}{6}+\log\frac{e^2\varphi^2}{4\pi\mu^2}\right) \tag{3.0.0.0.10}$$

Chapter 4

MS bar renormalisation of the effective potential

$\overline{\text{MS}}$ renormalisation gives:

$$V_R = c_\lambda \lambda \varphi_1^4 + \frac{1}{4}\frac{(\frac{1}{2}\lambda\varphi_1^2)^2}{(4\pi)^2}\left(-\frac{3}{2} + \log\frac{(\frac{1}{2}\lambda\varphi_1^2)^2}{\mu^2}\right) + \tag{4.0.0.0.1}$$

$$\frac{1}{4}\frac{(\frac{1}{6}\lambda\varphi_1^2)^2}{(4\pi)^2}\left(-\frac{3}{2} + \log\frac{(\frac{1}{6}\lambda\varphi_1^2)^2}{\mu^2}\right) + \tag{4.0.0.0.2}$$

$$\frac{1}{4}\frac{3(e^2\varphi_1^2)^2}{(4\pi)^2}\left(-\frac{5}{6} + \log\frac{e^2\varphi_1^2}{\mu^2}\right) \tag{4.0.0.0.3}$$

Here, as will be apparent in the second, we are interested only in e^4 part, so from now on, we will write:

$$V_R = c_\lambda \lambda \varphi_1^4 + \frac{1}{4}\frac{3(e^2\varphi_1^2)^2}{(4\pi)^2}\left(-\frac{5}{6} + \log\frac{e^2\varphi_1^2}{\mu^2}\right) \tag{4.0.0.0.4}$$

We can bind e to λ and v at the loop level, demanding that VEV does not change due to one loop corrections, stating that:

$$\left.\frac{\partial V_R}{\partial \varphi_1}\right|_v = 0 \tag{4.0.0.0.5}$$

This gives the condition:

$$4c_\lambda \lambda v^3 - \frac{e^4 v^3}{16\pi^2} - \frac{3e^4 v^3}{16\pi^2}\log\frac{e^2 v^2}{\mu^2} = 0 \tag{4.0.0.0.6}$$

Setting scale parameter μ to the effective mass of the vector, namely ev, we have simpler form of:

$$4c_\lambda \lambda v^3 - \frac{e^4 v^3}{16\pi^2} = 0 \tag{4.0.0.0.7}$$

Which gives:

$$\lambda = \frac{e^4}{64c_\lambda \pi^2} \tag{4.0.0.0.8}$$

Writing potential with this substitutions yields:

$$V_R = \frac{3e^4}{64\pi^2}\Big(-\frac{1}{2} + \log\frac{\varphi_1^2}{v^2}\Big)\varphi_1^4 \tag{4.0.0.0.9}$$

Equations 4.0.0.0.8 is very important in this discussion as it states, that λ is of order e^4 in our model. This post factum justifies our choice in taking only e^4 part, as other part was of order e^8.

We can now also bind square if the physical mass M_P^2 as the second derivative of the renormalised effective potential at VEV.

$$M_P^2 = \frac{\partial^2 V_R}{\partial \varphi_1^2}\Big|_v = \frac{3e^4}{16\pi^2}\Big(-\frac{1}{2}\Big)v^2 + \frac{e^4v^2}{8\pi^2} + \frac{3e^4v^2}{32\pi^2} \tag{4.0.0.0.10}$$

This gives that:

$$M_P^2 = \frac{e^4v^2}{8\pi^2} \tag{4.0.0.0.11}$$

From this we have that the ratio between scalar mass M_P^2 and vector mass $m(V)^2$ is:

$$\frac{M_P^2}{m(V)^2} = \frac{\frac{e^4v^2}{8\pi^2}}{e^2v^2} = \frac{e^2}{8\pi^2} \tag{4.0.0.0.12}$$

We can now also express e^4 and λ in terms of M_P^2 and v:

$$e^4 = \frac{8M_P^2\pi^2}{v^2} \tag{4.0.0.0.13}$$

$$\lambda = \frac{M_P^2}{8c_\lambda v^2} \tag{4.0.0.0.14}$$

Writing potential in these terms gives:

$$V_R = \frac{3M_P^2}{8v^2}\Big(-\frac{1}{2} + \log\frac{\varphi_1^2}{v^2}\Big)\varphi_1^4 \tag{4.0.0.0.15}$$

Chapter 5

On shell renormalisation of the effective potential

One of the main topic of this thesis is to show a coherent way to renormalise a conformal theory without explicit mass term in the on-shell scheme. Along the way, we will also discuss the case with explicit mass term, for comparison.

5.1 Finite momentum approach

To calculate on-shell renormalisation we need to calculate self energy. However, it turns out, that simple calculation of self energy fails the test of comparison between the zero-momentum limit of the self energy and the second derivative of the effective potential.
Namely, it schould be safisfied that:

$$\lim_{p^2 \to 0} \Sigma(p^2) = \frac{\partial^2 V_{eff}}{\partial \varphi_1^2}, \tag{5.1.0.0.1}$$

TO DO: napisać ile wychodzi

but it is not the case.

However, from the $\overline{\text{MS}}$ considerations, we know that Φ have non-zero VEV, let us call it v. Let us rotate Φ in such a way, that $\langle\varphi_1\rangle = v$ and $\langle\varphi_2\rangle = 0$, where now v is real.
Keeping this in mind, we can rewrite Lagrangian in terms of shifted fields φ_1, φ_2 which have both zero VEV, now VEV is explicitly in the Lagrangian:

$$\begin{aligned}\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu(\varphi_1 + v) - eA_\mu\varphi_2)^2 \\ +\frac{1}{2}(\partial_\mu\varphi_2 + eA_\mu(\varphi_1 + v))^2 - c_\lambda\lambda((\varphi_1 + v)^2 + \varphi_2^2)^2.\end{aligned} \tag{5.1.0.0.2}$$

This breaks the symmetry, but now there are more interaction terms in the Lagrangian and this leads to different self energy, now consistent with the second derivative of the effective potential, as will be shown in **??**.

Following [1] we put the mass counterterm even though initialy the mass term was not present in the Lagrangian. It will turn out to be crutial in **??**.
The Lagrangian with δZ, $\delta\lambda$ and δm counterterms looks like this:

$$\begin{aligned}\mathcal{L}_{\mathcal{R}} =& -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} && (5.1.0.0.3)\\ &+(1+\delta Z)(\frac{1}{2}(\partial_\mu(\varphi_1+v)-eA_\mu\varphi_2)^2+\frac{1}{2}(\partial_\mu\varphi_2+eA_\mu(\varphi_1+v))^2)\\ &-(1+\delta Z)^2c_\lambda(\lambda+\delta\lambda)((\varphi_1+v)^2+\varphi_2^2)^2\\ &-c_m\delta m((\varphi_1+v)^2+\varphi_2^2). && (5.1.0.0.4)\end{aligned}$$

Separating the terms with the first power of renormalisation constants and second power of φ_1, we obtain correction to the self energy equal to:

$$\delta\Sigma = -12c_\lambda v^2(2\lambda\delta Z_\varphi+\delta\lambda)-2c_m\delta m-p^2\delta Z_\varphi, \tag{5.1.0.0.5}$$

where $p^2=-\partial_\mu\varphi_1\partial^\mu\varphi_1$.
Separating the terms with the first power of renormalisation constants and first power of φ_1, we obtain correction to the tadpole equal to:

$$\delta T = -4c_\lambda v^3(2\lambda\delta Z_\varphi+\delta\lambda)-2c_m\delta m v \tag{5.1.0.0.6}$$

First approach is to impose renormalisation conditions resembling classical on-shell. Here, Σ' stands for $\frac{\mathrm{d}\Sigma}{\mathrm{d}p^2}$ and, if not stated otherwise, Σ, $\delta\Sigma$ and Σ' are evaluated at $p^2=M_P^2$, where M_P stands for physical mass. We denote real part as $\Re\left(\right)$.

$$T+\delta T=0 \tag{5.1.0.0.7}$$
$$\Re(\Sigma)+\Re(\delta\Sigma)=0 \tag{5.1.0.0.8}$$
$$\Re(\Sigma')=0 \tag{5.1.0.0.9}$$

This gives us:

$$\delta m = \frac{-1}{4c_m}\left(\Re(\Sigma)-\frac{3}{v}T-M_P^2\Re(\Sigma')\right) \tag{5.1.0.0.10}$$
$$\delta\lambda = \frac{1}{8c_\lambda v^2}\left(\Re(\Sigma)-\frac{1}{v}T-(16c_\lambda\lambda v^2+M_P^2)\Re(\Sigma')\right) \tag{5.1.0.0.11}$$
$$\delta Z = \Re(\Sigma') \tag{5.1.0.0.12}$$

We define M_P^2 as the second derivative of the tree potential evaluated at VEV, so:

$$M_P^2 = \frac{\partial^2}{\partial\varphi_1^2}c_\lambda\lambda\varphi_1^4\Big|_v = 12c_\lambda\lambda v^2 \tag{5.1.0.0.13}$$

and we impose, that it does not change after one loop contributions.

Contributions to Σ and T constitutes of the following diagrams:
TO DO: diagrams

The values of diagrams are as follows: Contributing to Σ:

$$-i\frac{e^2}{M_V}\Big[M_V^2 a(M_2) + (-p^2 - M_V^2 + M_2^2)a(M_V) - (p^2 + M_2^2)^2 b_0(p, 0, M_2) + (p^2 + M_2^2 - M_V^2)^2 b_0(p, M_V, M_2)\Big] \tag{5.1.0.0.14}$$

$$-i\frac{e^4 v^2}{2M_V^4}\Big[2M_V^2 a(M_V) + p^4 b_0(p, 0, 0) - 2(p^2 - M_V^2)^2 b_0(p, M_V, 0) + 16M_V^4 b_0^b(p, M_V, M_V) + (p^4 - 4p^2 M_V^2 - 4M_V^4) b_0(p, M_V, M_V)\Big] \tag{5.1.0.0.15}$$

$$-i3e^2 a_b(M_V) \tag{5.1.0.0.16}$$
$$-i12c_\lambda \lambda a(M_1) \tag{5.1.0.0.17}$$
$$-i4c_\lambda \lambda a(M_2) \tag{5.1.0.0.18}$$
$$-i288c_\lambda^2 \lambda^2 v^2 b_0(p, M_1, M_1) \tag{5.1.0.0.19}$$
$$-i32c_\lambda^2 \lambda^2 v^2 b_0(p, M_2, M_2) \tag{5.1.0.0.20}$$
$$\tag{5.1.0.0.21}$$

Contributing to T:

$$-i3e^2 v a^b(M_V) \tag{5.1.0.0.22}$$
$$-i12c_\lambda \lambda v a(M_1) \tag{5.1.0.0.23}$$
$$-i4c_\lambda \lambda v a(M_2) \tag{5.1.0.0.24}$$
$$-i4c_\lambda \lambda v^3 \tag{5.1.0.0.25}$$

Where

$$a(M) = \tag{5.1.0.0.26}$$
$$b_0(p, M_1, M_2) = \tag{5.1.0.0.27}$$
$$a^b(M) = \tag{5.1.0.0.28}$$
$$b_0^b(p, M_1, M_2) = \tag{5.1.0.0.29}$$

Here, we will be interested in only contributions up to order e^4. From our $\overline{\text{MS}}$ considerations we can see, that λ should be of order e^4, that M_V^2 should be of order e^2 and that M_1, M_2 should be of order e^4. As that we are intersted only in following parts of contributions to Σ and T:

$$\begin{aligned}
\Sigma_{e^0} =& -\frac{e^2}{M_V^2}\Big[-p^4 b_0(p,0,M_2)+p^4 b_0(p,M_V,M_2)\Big]- \\
& \frac{e^4 v^2}{M_V^4}\Big[p^4 b_0(p,0,0)-2p^4 b_0(p,M_V,0)+p^4 b_0(p,M_V,M_V)\Big] && (5.1.0.0.30)\\
\Sigma_{e^2} =& -\frac{e^2}{M_V^2}\Big[-p^2 a(M_V)-2p^2 M_V^2 b_0(p,M_V,M_2)\Big]- \\
& \frac{e^4 v^2}{2M_V^4}\Big[4p^2 M_V^2 b_0(p,M_V,0)-4p^2 M_V^2 b_0(p,M_V,M_V)\Big] && (5.1.0.0.31)\\
\Sigma_{e^4} =& -\frac{e^2}{M_V^2}a(M_V)\Big[-M_V^2 a(M_V)+M_V^4 b_0(p,M_V,M_2)\Big]- \\
& \frac{e^4 v^2}{2M_V^4}\Big[2M_V^2 a(M_V)-2M_V^4 b_0(p,M_V,0)+ \\
& 16M_V^4 b_0^b(p,M_V,M_V)-4M_V^4 b_0(p,M_V,M_V)\Big]- \\
& 3e^2 a^b(M_V)- \\
& \frac{e^2}{M_V^2}\Big[-2p^2 M_2^2 b_0(p,0,M_2)+2p^2 M_2^2 b_0(p,M_V,M_2)\Big] && (5.1.0.0.32)\\
T_{e^4} =& -3e^2 v a^b(M_V)-4c_\lambda \lambda v^3 && (5.1.0.0.33)
\end{aligned}$$

The divergent part of T, Σ and Σ' are:

$$\text{div}T = -\frac{3e^4v^4}{16\pi^2}\Big(-\frac{2}{\epsilon}\Big) \qquad (5.1.0.0.34)$$

$$\text{div}\Sigma = \frac{6e^2(M_P^2-3e^2v^2)}{32\pi^2}\Big(-\frac{2}{\epsilon}\Big) \qquad (5.1.0.0.35)$$

$$\text{div}\Sigma' = \frac{3e^2}{16\pi^2}\Big(-\frac{2}{\epsilon}\Big) \qquad (5.1.0.0.36)$$

After subsituing to $\delta\lambda$, δm, δZ and then to V_R we see that div$V_R = 0$, thus renormalisation procedure succeeds in canceling divergences.

5.2 Zero momentum approach

We are concerned with the theory described by 5.1.0.0.2. Here, as well, we will consider potential up to order e^4.
We start with the 1-loop level potential with counterterms:

$$V_R^{\text{1-loop}} = \frac{3e^4\varphi^4}{64\pi^2}\Big(-\frac{2}{\epsilon}+\gamma_E-\frac{5}{6}+\log\frac{e^2\varphi_1^2}{4\pi\mu^2}\Big)+c_\lambda\delta\lambda\varphi_1^4+c_m\delta m\varphi_1^2 \qquad (5.2.0.0.1)$$

As renormalisation conditions we impose that:

$$\frac{\partial^2}{\partial\varphi_1^2}V_R^{\text{1-loop}}\Big|_v = 0 \tag{5.2.0.0.2}$$

$$\frac{\partial^4}{\partial\varphi_1^4}V_R^{\text{1-loop}}\Big|_v = 0 \tag{5.2.0.0.3}$$

Codesponding derivatives are:

$$\frac{\partial^2}{\partial\varphi_1^2}V_R^{\text{1-loop}} = \frac{9e^4\varphi_1^2}{16\pi^2}\Big(-\frac{2}{\epsilon}+\gamma_E-\frac{5}{6}+\log\frac{e^2\varphi_1^2}{4\pi\mu^2}\Big)+\frac{21e^4}{32\pi^2}\varphi_1^2+12c_\lambda\delta\lambda\varphi_1^2+2c_m\delta m \tag{5.2.0.0.4}$$

$$\frac{\partial^4}{\partial\varphi_1^4}V_R^{\text{1-loop}} = \frac{9e^4\varphi_1^2}{8\pi^2}\Big(-\frac{2}{\epsilon}+\gamma_E-\frac{5}{6}+\log\frac{e^2\varphi_1^2}{4\pi\mu^2}\Big)+\frac{75e^4}{16\pi^2}+24c_\lambda\delta\lambda. \tag{5.2.0.0.5}$$

So the conditions take form:

$$\frac{9e^4v^2}{16\pi^2}\Big(-\frac{2}{\epsilon}+\gamma_E-\frac{5}{6}+\log\frac{e^2v^2}{4\pi\mu^2}\Big)+\frac{21e^4}{32\pi^2}v^2+12c_\lambda\delta\lambda v^2+2c_m\delta m = 0 \tag{5.2.0.0.6}$$

$$\frac{9e^4v^2}{8\pi^2}\Big(-\frac{2}{\epsilon}+\gamma_E-\frac{5}{6}+\log\frac{e^2v^2}{4\pi\mu^2}\Big)+\frac{75e^4}{16\pi^2}+24c_\lambda\delta\lambda = 0. \tag{5.2.0.0.7}$$

Solving for $\delta\lambda$ and δm we have:

$$\delta\lambda = \frac{-e^4}{64\pi^2c_\lambda}\Big(3\Big(-\frac{2}{\epsilon}+\gamma_E-\frac{5}{6}+\log\frac{e^2v^2}{4\pi\mu^2}\Big)+\frac{25}{2}\Big) \tag{5.2.0.0.8}$$

$$\delta m = \frac{27e^4v^2}{32\pi^2c_m}. \tag{5.2.0.0.9}$$

Then, the renormalised potential is:

$$V_R = c_\lambda\lambda\varphi_1^4+\frac{e^4\varphi_1^4}{64\pi^2}\Big(3\log\frac{\varphi_1^2}{v^2}-\frac{25}{2}\Big)+\frac{27e^4v^2\varphi_1^2}{32\pi^2} \tag{5.2.0.0.10}$$

We would like to write V_R in terms of M_P – the physical mass and v. First relation is $\lambda = \frac{M_P^2}{12c_\lambda v^2}$. Potential writen with this substitution becames:

$$V_R = \frac{M_P^2\varphi_1^4}{12v^2}+\frac{e^4\varphi_1^4}{64\pi^2}\Big(3\log\frac{\varphi_1^2}{v^2}-\frac{25}{2}\Big)+\frac{27e^4v^2\varphi_1^2}{32\pi^2} \tag{5.2.0.0.11}$$

To write e in terms of M_P and v we use the condition that

$$\frac{\partial}{\partial\varphi_1}V_R\Big|_v = 0 \tag{5.2.0.0.12}$$

as v is by definition minimum of the potential.
It gives the relation:

$$\frac{M_P^2v}{3}+\frac{e^4v^3}{16\pi^2}\Big(-\frac{25}{2}\Big)+\frac{3e^4v^3}{32\pi^2}+\frac{27e^4v^3}{16\pi^2} = 0 \tag{5.2.0.0.13}$$

Thus, we conclude that:

$$e^4 = \frac{-M_P^2\pi^2}{3v^2}. \tag{5.2.0.0.14}$$

This, unfortunatelly, is unacceptable, as then e is no longer a real number, which is unphysical. Thus, we conclude, that presented renormalisation method is not working and we need to search for another. One of possible ways is to expand the method to finite momentum.
We will now investigate, whether this has some chance of working by comparing above "potential only" method, only with first and second derivative, "self energy and tadpole" method and it's zero momentum limit.
With the conditions:

$$\frac{\partial}{\partial\varphi_1} V_R^{\text{1-loop}}\Big|_v = 0 \tag{5.2.0.0.15}$$

$$\frac{\partial^2}{\partial\varphi_1^2} V_R^{\text{1-loop}}\Big|_v = 0 \tag{5.2.0.0.16}$$

renormalisation constants $\delta\lambda$ and δm take form:

$$\delta\lambda = -\frac{e^4}{8\pi^2} - \frac{3e^4}{16\pi^2}\Big(-\frac{2}{\epsilon} - \gamma_E + \log\frac{e^2v^2}{4\pi\mu^2}\Big) \tag{5.2.0.0.17}$$

$$\delta m = \frac{3e^4v^2}{16\pi^2}. \tag{5.2.0.0.18}$$

And the renormalised potential is equal to:

$$V_R = \frac{M_P^2}{48c_\lambda v^2} + \frac{e^4\varphi_1^4}{128\pi^2}\Big(6\log\frac{\varphi_1^2}{v^2} - 9\Big) + \frac{3e^4v^2\varphi_1^2}{32\pi^2}. \tag{5.2.0.0.19}$$

Now, however, the condition for v is meaningless ass we already used it in renormalisation conditions. Nethertheless, we want to investigate how values of $\delta\lambda$ and δm are compared to other approaches.

Chapter 6

Half $\overline{\text{MS}}$ -Half Onshell scheme

Due to (so far) lack of experimenttal data of coupling λ in considered theories, the on shell condition for that constatnt renders itself meaningless.
Thus, we propose mixed scheme, where we demand that the physical mass remain unchanged due to the one-loop corrections, but for the coupling case, we demand only that the $\delta\lambda$ counterterm is such that the fourth derivative of the renormalzsed effective potential is finite – in the $\overline{\text{MS}}$ manner.

6.1 Finite momentum

6.2 Zero momentum

Here we impose following renormalisation conditions:

$$\frac{\partial^2}{\partial \varphi_1^2} V_R^{\text{1-loop}}\Big|_v = 0 \tag{6.2.0.0.1}$$

$$\frac{\partial^4}{\partial \varphi_1^4} V_R^{\text{1-loop}}\Big|_v = \frac{9e^4}{8\pi^2}\Big(-\frac{5}{6} + \log\frac{e^2v^2}{\mu^2}\Big) + \frac{75e^4}{16\pi^2} \tag{6.2.0.0.2}$$

Written in the full form these conditions take form:

$$\begin{aligned}&\frac{9e^4}{16\pi^2}\Big(-\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log\frac{e^2v^2}{4\pi\mu^2}\Big)v^2 + \\ &12c_\lambda\delta\lambda v^2 + \frac{21e^4}{32\pi^2}v^2 + 2c_m\delta m = 0\end{aligned} \tag{6.2.0.0.3}$$

$$\begin{aligned}&\frac{9e^4}{8\pi^2}\Big(-\frac{2}{\epsilon} + \gamma_E - \frac{5}{6} + \log\frac{e^2v^2}{4\pi\mu^2}\Big) + 24c_\lambda\delta\lambda + \frac{75e^4}{16\pi^2} = \\ &\frac{9e^4}{8\pi^2}\Big(-\frac{5}{6} + \log\frac{e^2v^2}{\mu^2}\Big) + \frac{75e^4}{16\pi^2}\end{aligned} \tag{6.2.0.0.4}$$

After solving equations for δm and $\delta\lambda$ we obtain:

$$\delta m = -\frac{3e^4v^2}{32c_m\pi^2}\Big(1+3\log\frac{e^2v^2}{\mu^2}\Big) \tag{6.2.0.0.5}$$

$$\delta\lambda = \frac{3}{64c_\lambda\pi^2}\Big(\frac{2}{\epsilon}-\gamma_E+\log(4\pi)\Big) \tag{6.2.0.0.6}$$

The renormalized potential is then:

$$\begin{aligned} V_R = & c_\lambda\lambda\varphi_1^4+ \\ & \frac{3e^4}{64\pi^2}\Big(-\frac{5}{6}+\log\frac{e^2\varphi_1^2}{\mu^2}\Big)\varphi_1^4+ \\ & \frac{3e^4v^2}{32\pi^2}\Big(-1-3\log\frac{e^2v^2}{\mu^2}\Big)\varphi_1^2 \end{aligned} \tag{6.2.0.0.7}$$

From the tree level potential we have, as usual, the relation $\lambda = \frac{M_P^2}{12c_\lambda v^2}$. Written in these terms we have:

$$\begin{aligned} V_R = & \frac{M_P^2}{12v^2}\varphi_1^4+ \\ & \frac{3e^4}{64\pi^2}\Big(-\frac{5}{6}+\log\frac{e^2\varphi_1^2}{\mu^2}\Big)\varphi_1^4+ \\ & \frac{3e^4v^2}{32\pi^2}\Big(-1-3\log\frac{e^2v^2}{\mu^2}\Big)\varphi_1^2 \end{aligned} \tag{6.2.0.0.8}$$

We can bind e to M_P and v at the loop level, demanding that VEV does not change due to one loop corrections, stating that:

$$\frac{\partial V_R}{\partial\varphi_1}\Big|_v = 0 \tag{6.2.0.0.9}$$

This gives the condition:

$$-\frac{e^4v^3}{4\pi^2}-\frac{3e^4v^3}{8\pi^2}\log\frac{e^2v^2}{\mu^2}+\frac{M_P^2v}{3} = 0 \tag{6.2.0.0.10}$$

Setting scale parameter μ to the effective mass of the vector, namely ev, we have simpler form of:

$$-\frac{e^4v^3}{4\pi^2}+\frac{M_P^2v}{3} = 0 \tag{6.2.0.0.11}$$

Which gives:

$$e^4 = \frac{4M_P^2\pi^2}{3v^2} \tag{6.2.0.0.12}$$

Writing potential with this substitutions yields asdad:

$$V_R = \frac{M_P^2}{16v^2}\Big(\frac{1}{2}+\log\frac{\varphi_1^2}{v^2}\Big)\varphi_1^4-\frac{M_P^2}{8}\varphi_1^2 \tag{6.2.0.0.13}$$

Chapter 7

Mass term case

For comparison, we present here results for above methods used in the case with explicit mass term.

7.0.1 On-shell finite momentum

For the comparison, we will present the same calculation, performed on the analoguos theory with explicit mass term. Similary as in the 5.1.0.0.2 we need to shift fields for 5.1.0.0.1 to be satisfied. The Lagrangian in this case is:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu(\varphi_1 + v) - eA_\mu\varphi_2)^2 \\ &+ \frac{1}{2}(\partial_\mu\varphi_2 + eA_\mu(\varphi_1 + v))^2 - c_m m^2((\varphi_1 + v)^2 + \varphi_2^2) - c_\lambda\lambda((\varphi_1 + v)^2 + \varphi_2^2)^2.\end{aligned} \tag{7.0.1.0.1}$$

With renormalisation constatnts:

$$\begin{aligned}\mathcal{L}_\mathcal{R} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &+ (1+\delta Z)(\frac{1}{2}(\partial_\mu(\varphi_1 + v) - eA_\mu\varphi_2)^2 + \frac{1}{2}(\partial_\mu\varphi_2 + eA_\mu(\varphi_1 + v))^2) \\ &- (1+\delta Z)^2 c_\lambda(\lambda + \delta\lambda)((\varphi_1 + v)^2 + \varphi_2^2)^2 \\ &- (1+\delta Z)c_m(m+\delta m)((\varphi_1 + v)^2 + \varphi_2^2).\end{aligned} \tag{7.0.1.0.2-7.0.1.0.3}$$

Corrections then are:

$$\delta\Sigma = -12c_\lambda v^2(2\lambda\delta Z_\varphi + \delta\lambda) - 2c_m\delta m - 2c_m m^2\delta Z - p^2\delta Z \tag{7.0.1.0.4}$$

$$\delta T = -4c_\lambda v^3(2\lambda\delta Z_\varphi + \delta\lambda) - 2c_m\delta m v - 2c_m m^2 v\delta Z. \tag{7.0.1.0.5}$$

This changes the form of renormalisation constants to:

$$\delta m = \frac{-1}{4c_m}\left(\Re(\Sigma) - \frac{3}{v}T - (4c_m m^2 + M_P^2)\Re(\Sigma')\right) \tag{7.0.1.0.6}$$

$$\delta\lambda = \frac{1}{8c_\lambda v^2}\left(\Re(\Sigma) - \frac{1}{v}T - (16c_\lambda\lambda v^2 + M_P^2)\Re(\Sigma')\right) \tag{7.0.1.0.7}$$

$$\delta Z = \Re(\Sigma') \tag{7.0.1.0.8}$$

The only difference is $4c_m m^2$ term in 7.0.1.0.6.

7.1 "Zero momentum" approach

Here we will compare two kinds of "zero momentum" approach. First will be imposing renormalisation conditions in terms of only derivatives of effective potential. This is the zero momentum approach as first and second derivatives are limits of, respectively, taddpole and sef-energy in the zero momentum limit.
qSecond kind will be to calculate approach from 5.1in the zero momentum limit.
Later we will discus some "potential only" version with different conditions and discuss whether adding finite momentum to it will produce satisfying results.

Potential only version

For comparison, we include also a version of this aprouch steming from **??**. Inclusion of the mass term do not change the form of $\delta\lambda$ and δm. The frist difference occurs in the potential.
First we will describe the case with derivatives II and IV used in renormalisation conditions. Then the potential is equal:

$$V_R = c_\lambda \lambda \varphi_1^4 + c_m m^2 \varphi_1^2 + \frac{e^4 \varphi_1^4}{64\pi^2}\left(3\log\frac{\varphi_1^2}{v^2} - \frac{25}{2}\right) + \frac{27 e^4 v^2 \varphi_1^2}{32\pi^2}. \tag{7.1.0.0.1}$$

Now, we have that $M_P = 12 c_\lambda \lambda v^2 + 2 c_m m^2$. No, we will use the condition, that:

$$\left.\frac{\partial}{\partial \varphi_1} V^T \right|_v = 0 \tag{7.1.0.0.2}$$

we have that $4c_\lambda \lambda v^3 + 2c_m m^2 v = 0$, so $\lambda = -\frac{c_m m^2}{2c_\lambda v^2}$, so

$$m^2 = \frac{-M_P^2}{4c_m} \quad \text{and} \tag{7.1.0.0.3}$$

$$\lambda = \frac{M_P^2}{8c_\lambda v^2}. \tag{7.1.0.0.4}$$

Writing V_R with respect to that gives:

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$$V_R = \frac{M_P^2 \varphi_1^4}{8v^2} - \frac{M_P^2 \varphi_1^2}{4} + \frac{e^4 \varphi_1^4}{64\pi^2}\left(3\log\frac{\varphi_1^2}{v^2} - \frac{25}{2}\right) + \frac{27 e^4 v^2 \varphi_1^2}{32\pi^2} \tag{7.1.0.0.5}$$

From this we have, that:

$$\frac{e^4 v^3}{\pi^2} = 0, \tag{7.1.0.0.6}$$

which is also a not safisying result.
However, if we drop the condition, that $\left.\frac{\partial}{\partial \varphi_1} V^T \right|_v = 0$, we have potential in the form:

$$V_R = \frac{M_P^2 - 2c_m m^2}{12 v^2}\varphi_1^4 + c_m m^2 \varphi_1^2 + \frac{e^4 \varphi_1^4}{64\pi^2}\left(3\log\frac{\varphi_1^2}{v^2} - \frac{25}{2}\right) + \frac{27 e^4 v^2 \varphi_1^2}{32\pi^2} \tag{7.1.0.0.7}$$

From thism, using the condition that $\frac{\partial}{\partial \varphi_1} V_R \Big|_v = 0$, we can derive the correspondence between e and M_P, v and m:

$$e^4 = -\frac{(M_P^2 + 4c_m m^2)\pi^2}{3v^2}, \tag{7.1.0.0.8}$$

which is finally a sensible result as it can be realised with real, positive e. However, then it must hold that $m^2 < -\frac{M_P^2}{4c_m}$.

Chapter 8

Conclusions

Bibliography

[1] Sidney Coleman and Erick Weinberg, *Radiative corrections as the origin of spontaneous symmetry breaking*, Phys. Rev. D **7** (1973Mar), 1888–1910.