Assignment - 1

UG Sem-III (CC-05)

Session: July, 2023 - December, 2023

August 22, 2023

1. For numerical purpose, the recurrence relation among Legendre's polynomials is

$$P_{n+1} = 2xP_n - P_{n-1} - \frac{1}{n+1}(xP_n - P_{n-1}) \tag{1}$$

Write a Python program to generate $P_m(x)$ for any arbitrary $m \geq 2$ on N number of discrete point x_i for $i = 0, 1, 2, ..., (N-1)$ with $x_0 = -1$ and $x_{N-1} = 1$. Initial condition to use the recurrence relation is given by specifying $P_0(x) = 1$ and $P_1(x) = x$.
Plot $P_m(x)$ in $-1 \leq x \leq 1$

2. Write a Python program
(a) To generate pair of $P_m(x)$ and $P_n(x)$ for any arbitrary set $m,\ n$ using equation (1) in the interval $-1 \leq x \leq 1$.
(b) Compute the following integration

$$\int_{-1}^{1} P_m(x)P_n(x)\,dx$$

for two different cases: $m \neq n$ and $m = n$

3. Write a Python program
(a) To generate set of Legendre polynomials $P_2(x)$, $P_3(x)$....., $P_m(x)$ for any arbitrary value of $m \geq 2$ using the recurrence relation (1).
(b) Construct an $(m \times m)$ square matrix say $\boldsymbol{M}$ such that its elements are defined as nearest integer value of the following integral

$$[\boldsymbol{M}]_{i,j} = \left(i + \frac{1}{2}\right)\int_{-1}^{1} P_i(x)P_j(x)\,dx \quad ; \quad i,j = 0, 1, 2, ..., m$$

4. Write a Python program
(a) To generate set of Legendre polynomials $P_2(x)$, $P_3(x)$, $P_4(x)$,....., $P_m(x)$ for any arbitrary value of $m \geq 2$ using the recurrence relation (1).
(b) A function $f(x)$ is defined in the range $-1 \leq x \leq 1$:

$$f(x) = \begin{cases} 0 & ; \quad -1 \leq x < 0 \\ 1 & ; \quad 0 \leq x \leq 1 \end{cases}$$

Obtain the coefficients C_l (for $l = 0, 1, 2, ..., m$) of the Legendre series expansion:

$$f(x) = \sum_{l=0}^{\infty} C_l P_l(x)$$

(c) Define a function:

$$\sigma(x;n) = \sum_{l=0}^{n} C_l P_l(x)$$

Plot this function in the range $-1 \leq x \leq 1$ with $n = 0, 1, 2, 3, ..., m$. Hence, explain your observation.