Natural Language Processing for Simple Word Problems

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Presented for
International Conference on Natural Language Processing (ICON)
Trivandrum, India
2015

The Problem

Solve an algebraic word problem that is expressed in English.

Sample Input:

John has 3 apples. He gave 1 apple to Mary. How many apples does John have now?
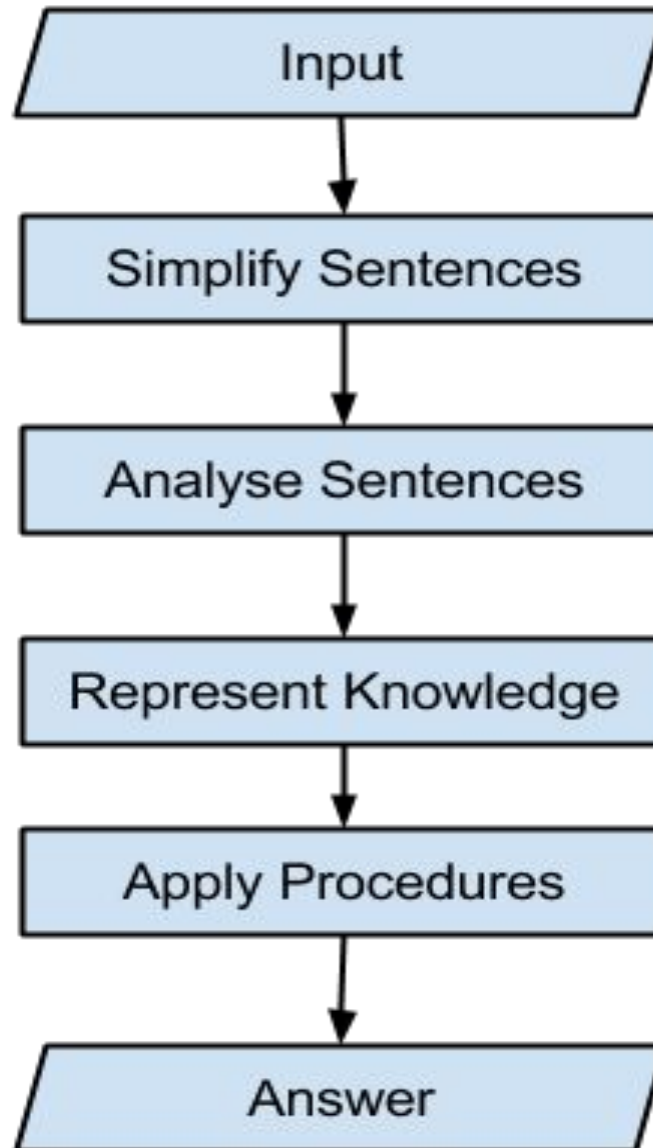
Sample Output:

John has 2 apples

Motivation

▷ Good alternative for obtaining answers for word problems - Google is not adequate

▷ First step to building systems that can generate an explanation and help students understand the method to solve a problem

Problem Solving Process

Schemas

Schemas* are templates that describe how entities interact.

"John had 3 apples. He forfeited 1 apple. How many apples does he have now?"

* Fletcher (1985)

(owner) had (X) (object)

(owner) (transfer-out-ownership) (Y) (object)

(owner) has (Z) (object)

$Y + Z = X.$

(owner) = John, (X) = 3, (Y) = 1, (object) = apple, (transfer-out-ownership) = forfeit

Disadvantages of Schemas for Math Problem Solving

▷ There have not been attempts to introduce some assumptions, especially for initial conditions.

▷ Importance for ordering of events had not been addressed

Temporal Schemas

John buys 11 peaches. He eats 9 peaches. How many does John have now?"

John buys 11 peaches.

t0
John : has : peaches : x1
unknown0 : has : peaches : x2

t1
John : buy : peaches : 11

t2
John : has : peaches : x1 + 11
unknown0 : has : peaches : x2 − 11

Temporal Schemas

John buys 11 peaches. He eats 9 peaches. How many does John have now?"

He eats 9 peaches.

t0
John : has : peaches : x1
unknown0 : has : peaches : x2

t1
John : buy : peaches : 11

t2
John : has : peaches : x1 + 11
unknown0 : has : peaches : x2 −11

t3
John : eat : peaches : 9

t4
John : has : peaches : x1 + 11−9
unknown0 : has : peaches : x2 −11

Temporal Schemas

John buys 11 peaches. He eats 9 peaches. How many does John have now?"

How many does John have now?
x1 + 11 - 9
0 + 11 - 9
2
John has 2 peaches

t0
John : has : peaches : x1
unknown0 : has : peaches : x2

t1
John : buy : peaches : 11

t2
John : has : peaches : x1 + 11
unknown0 : has : peaches : x2 −11

t3
John : eat : peaches : 9

t4
John : has : peaches : x1 + 11−9
unknown0 : has : peaches : x2 −11

Properties

▷ Initial unknown values are taken to be zero

▷ Equations are created and solved as per requirement

▷ Joan has 24 seashells. She gave some seashells to Sam. Now she has 14 seashells, how many did she give?

t0

Joan : has : 24 : seashells

t1

Joan : give : x : seashells

t2 :

Joan : has : 24 - x : seashells

$24 - x = 14$

Properties

▷ Initial unknown values are taken to be zero

▷ Equations are created and solved as per requirement

▷ Joan has 24 seashells. She gave some seashells to Sam. Now she has 14 seashells, how many did she give?

24 - x = 14
- Keep the expression with x on the left
- Compute LHS and RHS using Javascript engine ignoring x
- If the sign of x is positive, return RHS - LHS, else LHS - RHS

Schemas Used

Name	Schema	Update
Change In	Owner1 has X objects. Owner2 has Y objects. Z objects were transferred from Owner2 to Owner1	Owner1 has (X+Z) objects. Owner2 has (Y-Z) objects.
Change Out	Owner1 has X objects. Owner2 has Y objects. Z objects were transferred from Owner1 to Owner2	Owner1 has (X-Z) objects. Owner2 has (Y+Z) objects.

Schemas Used

Name	Schema	Update
Combine	Owner1 had X objects. Owner2 had Y objects. Together, they have Z objects.	Z = X + Y
Compare Plus	Owner1 had X objects. Owner2 had Y objects more than Owner1.	Owner2 has (X+Y) objects.

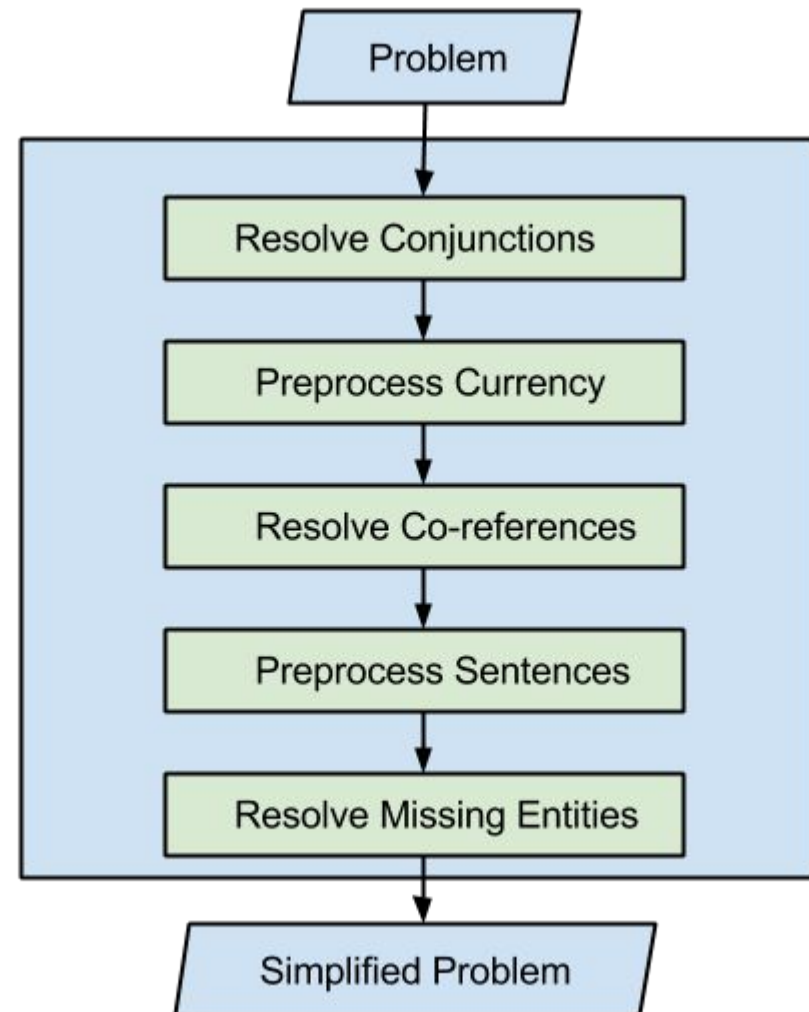# Schemas Used

Name	Schema	Update
Compare Minus	Owner2 had X objects. Owner1 had Y objects less than Owner2.	Owner1 has (X-Y) objects.
Increase	Owner1 had X objects. Owner1 got Y objects more.	Owner1 has (X+Y) objects.
Reduction	Owner1 had X objects. Owner1 loses Y objects.	Owner1 has (X-Y) objects

Schemas and Keywords

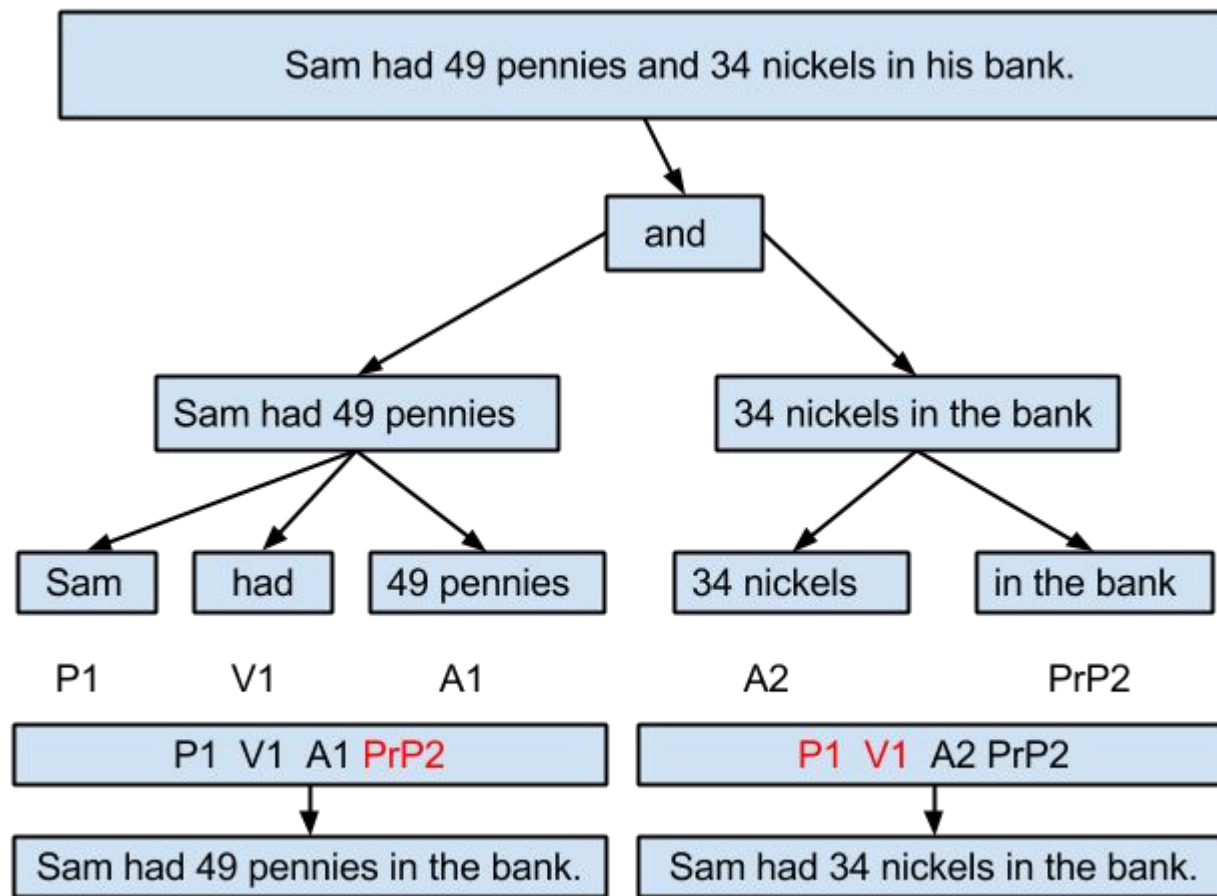| Schema	Keywords
Change Out	put, place, plant, add, sell, distribute, load, give
Change In	take from, get, pick, buy, borrow, steal
Reduction	eat, destroy, spend, remove, decrease
Compare Plus	more than, taller than, longer than, etc.
Compare Minus	less than, fewer than, shorter than, etc.
Combine	together, in all, combined
Increase	more, carry, find

Simplifying Sentences

▷ **Preprocess Currency - Change Rs. 5 to 5 rupees such that dependency parser retrieves information in a uniform manner**
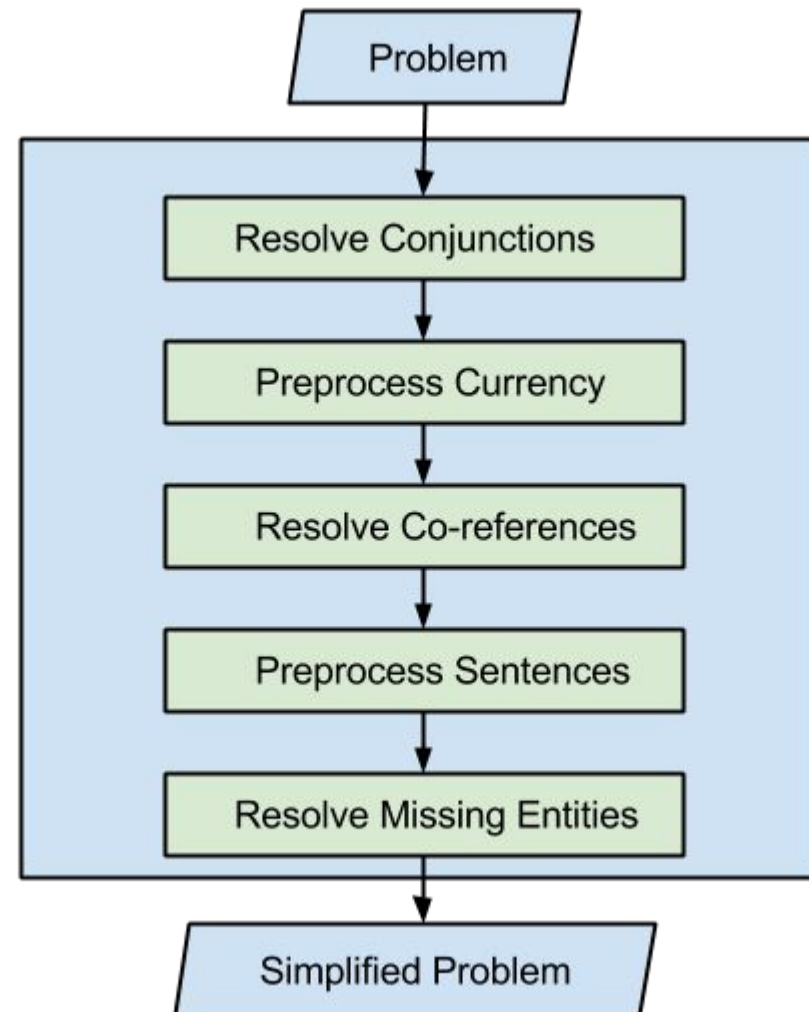
▷ **Used Stanford Core NLP suite**

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Problem
   │
   ▼
┌─────────────────────────────────┐
│  ┌───────────────────────────┐  │
│  │   Resolve Conjunctions    │  │
│  └───────────────────────────┘  │
│              │                  │
│              ▼                  │
│  ┌───────────────────────────┐  │
│  │   Preprocess Currency     │  │
│  └───────────────────────────┘  │
│              │                  │
│              ▼                  │
│  ┌───────────────────────────┐  │
│  │   Resolve Co-references    │  │
│  └───────────────────────────┘  │
│              │                  │
│              ▼                  │
│  ┌───────────────────────────┐  │
│  │   Preprocess Sentences    │  │
│  └───────────────────────────┘  │
│              │                  │
│              ▼                  │
│  ┌───────────────────────────┐  │
│  │  Resolve Missing Entities  │  │
│  └───────────────────────────┘  │
└─────────────────────────────────┘
              │
              ▼
      Simplified Problem
```

Resolving Conjunctions

Sam had 49 pennies and 34 nickels in his bank.

and

Sam had 49 pennies

34 nickels in the bank

Sam | had | 49 pennies

34 nickels | in the bank

P1 | V1 | A1 | A2 | PrP2

P1 V1 A1 PrP2

P1 V1 A2 PrP2

Sam had 49 pennies in the bank.

Sam had 34 nickels in the bank.

Simplifying Sentences

▷ Improved performance of co-reference resolution by using knowledge.

▷ Sam has 8 nickels. His dad gave him 4 nickels. How many nickels does he have now?

▷ 'him' in the second sentence is 'Sam' not 'Sam's dad. By using the schema definition, we can disambiguate.

Problem

Resolve Conjunctions

Preprocess Currency

Resolve Co-references

Preprocess Sentences

Resolve Missing Entities

Simplified Problem

Simplifying Sentences

▷ There are 5 apples in a basket. Ruth picked 2. How many are there in the basket?

▷ Explicitly add "2 apples" such that the problem becomes "There are 5 apples in a basket. Ruth picked **2 apples**. How many are there in the basket?"
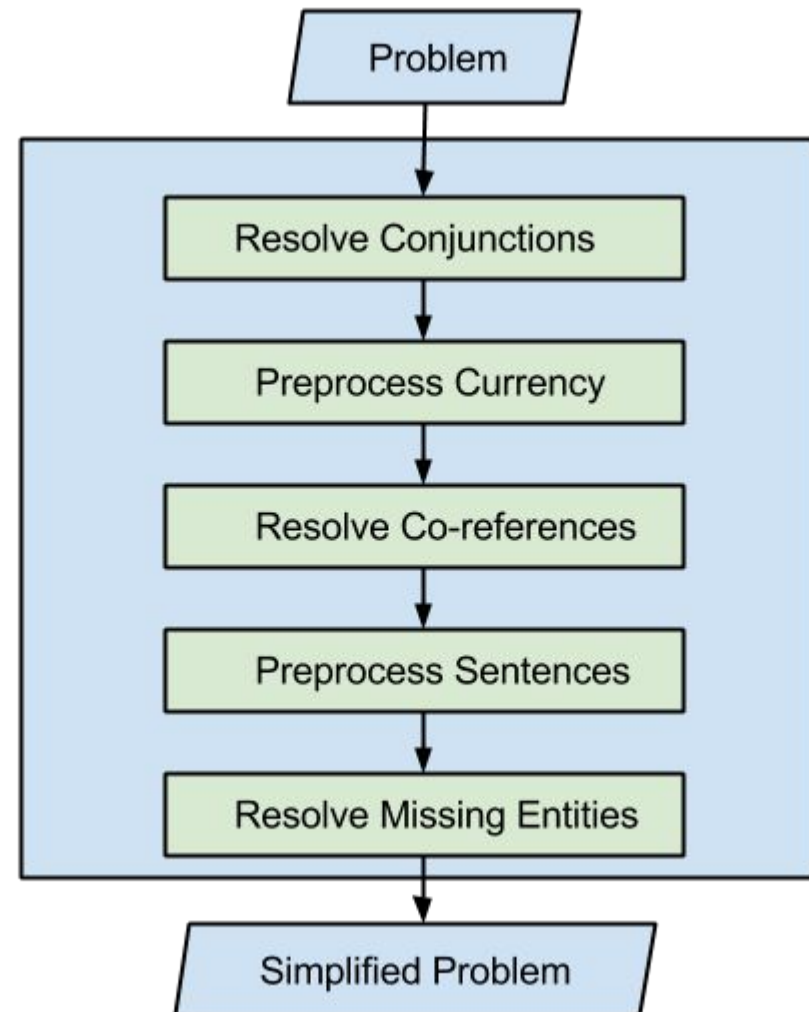
Generating the Answer

▷ The information from the question is analysed and the required quantity is retrieved.

▷ How many apples did Joan eat?
▷ How many apples does Joan have now?
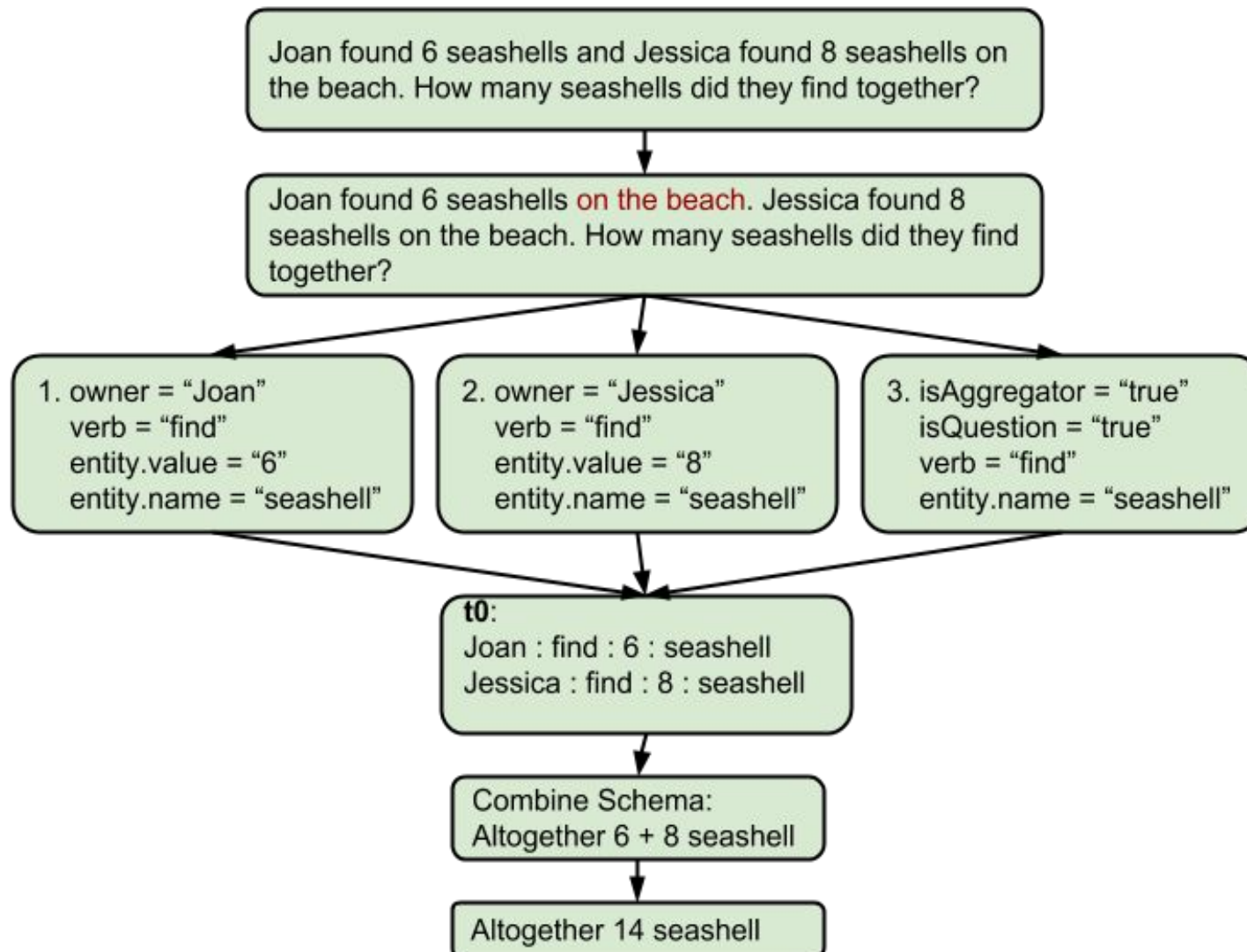
t0

Joan : has : 5 : apples

t1

Joan : eat : 2 : apples

t2

Joan : has : 5-2 : apples

Joan eats 2 apples.

Joan has 3 apples.

Example of the Solving Process

Sample Problems Solved

▷ Fred has 5 yellow balloons , Sam has 6 yellow balloons , and Mary has 7 yellow balloons . The balloons cost 5 dollars . How many yellow balloons do they have in total ?

Sample Problems Solved

▷ Joan has 47 green and 48 red marbles . Fred took 24 of Joan 's green marbles . How many green marbles does Joan now have ?

Performance

▷ Dataset 1 (DS1) has 134 word problems that are straightforward.

▷ Dataset 2 (DS2) has 121 word problems which generally have longer sentences.

▷ Dataset 3 (DS3) has 140 word problems which are similar to those in DS1 but have extra information.

Performance

	DS1 (134)	DS2 (121)	DS3 (140)
Our System	**96.27**	**80.00**	**90.08**
Gold ARIS [1]	94.0	77.1	81.0
ROBUST [2]	12.69	0.71	0
WolframAlpha [3]	5.97	2.14	0.83

- [1] Hosseini et. al. (2014)
- [2] Y. Bakman (2007)
- [3] WolframAlpha. Wolfram Research, Inc (2015)

Summary

▷ Combining state-of-the-art parsing with improvements in schemas

▷ Schemas with time

▷ Exploiting words other than verbs

▷ Generating answer in natural language

▷ Introduced assumptions

Thank You