GTN damage model and numerical modeling

Gurson (Gurson, 1977) proposed a continuous damage model for isotropic materials by accounting the hydrostatic stress in order to capture the dilatancy during the void growths in the ductile damage while the previous models neglected the volumetric component. The model was later modified by Tvergaard and Needleman (Needleman and Tvergaard, 1984; Tvergaard and Needleman, 1984) to encapsulates the nucleation, growth and coalescence of voids, consequently addressing the elastic-plastic-damage region. The damage law is represented using the Gurson's potential ϕ, expressed as shown in eq.1.

$$\phi = \frac{\sigma_e^2}{\sigma_y^2} + 2q_1 f^* \cosh\left(\frac{3q_2\sigma_h}{2\sigma_y}\right) + \left(1 + \left(q_1 f^*\right)^2\right) = 0 \qquad (1)$$

Where the macroscopic von mises stress is defined as, $\sigma_e{}^2 = \frac{3}{2}\boldsymbol{\sigma}':\boldsymbol{\sigma}'$, $\boldsymbol{\sigma}' = \boldsymbol{\sigma} - \sigma_h \mathbf{I}$, $\sigma_h = \frac{1}{3}\boldsymbol{\sigma}:\mathbf{I}$

The Cauchy stress tensor is represented by $\boldsymbol{\sigma}$, σ_e is the von mises stress, σ_h is the hydraulic stress, $\boldsymbol{\sigma}'$ is the deviatoric stress and $\mathbf{I}$ is the identity tensor.The parameters q_1 = 1.5 and q_2 = 1 were proposed by Tvergaard and Needleman (Needleman and Tvergaard, 1984) to incorporate the coalescence and damage during the void growth and interactions. GTN model represents the state of the material using a single variable, f the effective porosity. Parameter f^* is the effective volume fraction in the Gurson potential function and it is a piecewise continuous function that substitutes the coalescence and abrupt damage during the necking.

$$f^* = \begin{cases} f, & f < f_c \\ f_c + (f - f_c)/(q_1 - f_c)\left(f_f - f_c\right), & f \geq f_c \end{cases} \qquad (2)$$

Where f_c = 0.12 is the critical porosity and f_f = 0.25 is the failure porosity. GTN model is vulnerable to the shear dominated fracture at low or negative triaxiality, since the damage is evaluated based on the mean stress (eq.3). Nahshon and Huchinson (Nahshon and Hutchinson, 2008) modified the damage law to accommodate the damage contribution from shear since it could play a major role in forming due to the continuous shape changing and load distribution during the deformation. The damage evolution is represented using the rate of porosity $\dot{f}$, in eq.3.

$$\dot{f} = (1 - f)\,tr\left(\dot{\boldsymbol{\varepsilon}}_p\right) + \frac{k_w w(\boldsymbol{\sigma}) f}{\sigma_e}\left(\boldsymbol{\sigma}':\dot{\boldsymbol{\varepsilon}}_p\right) \qquad (3)$$

Where $w(\boldsymbol{\sigma}) = 1 - \left(\frac{27 J_3}{2\sigma_e^3}\right)^2$, $J_3 = \det\left(\boldsymbol{\sigma}'\right)$

Nahshon-Hutchinson parameter, k_w of a material is evaluated by experiments followed by calibration. Nahshon-Hutchinson parameter, k_w is chosen so as 2 to quantify the effect of the shear load in the metal forming. Plastic strain rate tensor is represented as $\dot{\boldsymbol{\varepsilon}}_{\mathbf{p}}$. Yield stress of the material (σ_y) is considered as a dependent variable of plastic strain, strain rate and temperature as given in eq.4.

$$\sigma_y = \sigma_0\left(1 + \varepsilon_p/\varepsilon_{p0}\right)^n \left(\dot{\varepsilon}_p/\dot{\varepsilon}_{p0}\right)^m \left(1 + b_G e^{-c(\theta_0 - 273)}\left(e^{-c(\theta - \theta_0)} - 1\right)\right) \qquad (4)$$

Where, the initial yield stress σ_0 = 375 MPa, ε_p is the plastic strain, ε_{p0} = 0.0034 is the reference plastic strain, power expression n = 0.4, $\dot{\varepsilon}_p$ is the plastic strain rate, $\dot{\varepsilon}_{p0}$ = 1s^{-1} and the power term m = 0.11. The thermal properties of the material, b_G = 0.1406, c = 0.00793, θ is the temperature and θ_0 is the ambient temperature which is taken as 293K.

The porosity contributed from strain nucleation is proportional to the plastic strain rate $\dot{\varepsilon}_p$. The plastic strain porosity is evaluated from the plastic strain rate using eq.5.

$$\dot{f}_{nucl}^{strain} = D\dot{\varepsilon}_p \qquad (5)$$

Where, $D = \frac{f_N^{strain}}{s_N^{strain}\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left[\frac{\varepsilon_p - \varepsilon_N}{s_N^{strain}} \right]^2 \right)$

The void volume fraction of nucleating particles is denoted by $\dot{f}_{nucl}^{strain}$ = 0.04, ε_N = 0.3 is the mean plastic strain at void nucleation and s_N^{strain} = 0.1 is the corresponding standard deviation. The material chosen for this work is ASTM A992 – a widely used material across the industry – and it's elastic thermal properties are tabulated (Table 1). The specific heat capacity, heat conductivity, and work-heat transfer are assumed to be constant with the increment in temperature.

Young's modulus (E)	**Poisson's ratio (ν)**	**Density (ρ)**	**Specific heat capacity (Cp)**	**Thermal conductivity (k)**	**Work-heat transfer (β)**
210 GPa	0.25	7850 Kg/m^3	470 J/kg	51 W/(mK)	0.9

Table 1. Material properties of ASTM A992.

For further reference:

1. K. K Mathur, A. Needleman and V. Tvergaard, '*Three dimensional analysis of dynamic ductile crack growth in a thinplate*',J. Mech. Phys. Solids,44,439{464 (1996).

2. A. Srivastava, L. Ponson, S. Osovski, E. Bouchaud, V. Tvergaard, A. Needleman, *Effect of inclusion density on ductile fracture toughness and roughness*. J. Mech. Phys. Solids, 63 (2014), pp. 62-79