CONTROLLING THE POSITION OF A BALL ON A BEAM

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I. Objectives:

The objective of this project is to control the position of a ball on a beam rotating about a central axis, under these specifications:

Tracking: Settling time 5 (s). Zero steady-state error for a step reference.

Regulation: Rejection of a step disturbance (1^0) on the angle of the beam in a steady state.

Different controllers shall be analyzed, including two of type PID and a state-feedback controller with integral action.

II. Proposed model:

The system studied consists of a ball that rolls without friction on a beam rotating a small angle about a central axis, under the action of a DC motor.

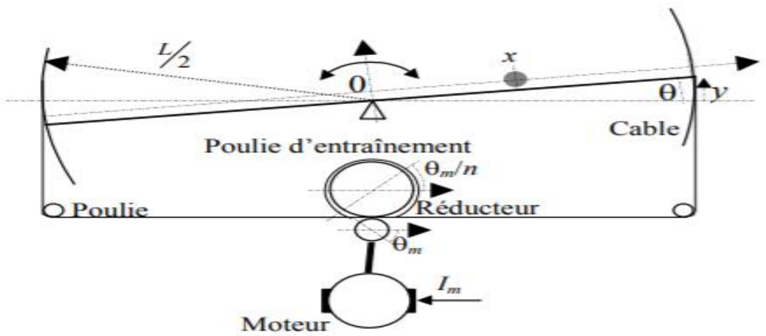

Figure 1: The physical system studied

Our system is governed by the following equations:

The equations of the motor:

$$J\dot{\Omega}_m = \Phi I_m - f\Omega_m$$

$$U_m = \Phi\Omega_m + RI_m + L\frac{dI_m}{dt}$$

The equation of the ball:

$$\frac{d^2x}{dt^2} = K_b\theta$$

Measurements of variables through measurements of voltages:

$$U_x = k_1 x, \quad U_\theta = k_2\theta, \quad U_\Omega = k_3\Omega_m, \quad U_{\text{Im}} = \frac{1}{k_i}\text{I}_m$$

Given that we have implemented suitable internal controllers for the motor's speed and the beam angle, which are in our case a PI and an output feedback controller respectively. The system's parameters are as follow:

Parameters	*Values*	*Units*
T_s	0.2	s
J	0.46×10^{-4}	$kg.m^2$
f	0.7×10^{-4}	Nms
R	1/3	Ω
L	1	mH
Φ	0.026	$Nm/A, Vs/rad$
nm	960	No unit
K_b	6.1	$ms^{-2}rad^{-1}$
k_1	7	V/m
k_2	28.65	V/rad
k_3	0.0159	Vs/rad
k_i	1.6	A/V

Table 1: Our parameters

III. RS structure PID control approach

The first controller implemented is discrete PID of type RS, with the following structure:

$$G(z) = \frac{R(z)}{S(z)} = \frac{r_0 + r_1 z^{-1} + r_2 z^{-2}}{(1 + s_1' z^{-1})(1 - z^{-1})}$$

By implementing the specifications, the controller's coefficients are:

$$r_0 = 25.6073, r_1 = -44.6561, r_2 = 19.5510, s_1' = 0.5828$$

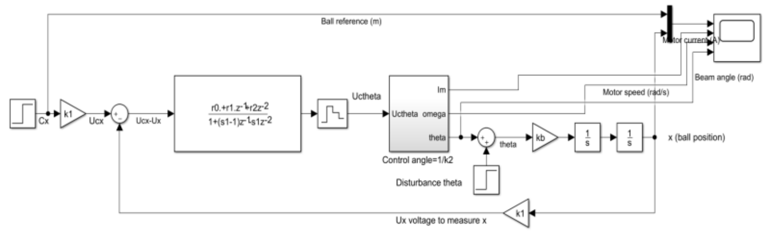

Figure 2: The Simulink model of the RS structure PID controller

This controller gives the following results:

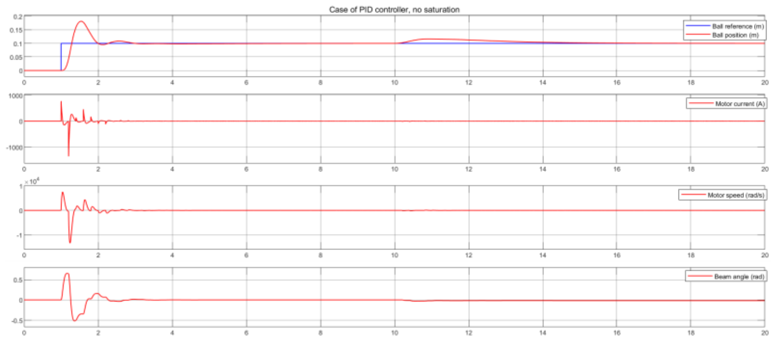

Figure 3: Case of RS structure PID controller, without saturation

Analysis: With a settling time of 1.770 (s), this controller well satisfies all the specifications we set previously. However, we notice a large overshoot of 81.3% for the position. The fast response and the overshoot are explained by the additional zeros introduced by R(z).

The system produces large values of motor current – as large as over 1000 (A), and motor speed – more than 10000 rad/s, which can be dangerous or impossible to achieve in practice. To study this problem we shall limit the motor current with a 16 (A) saturation:

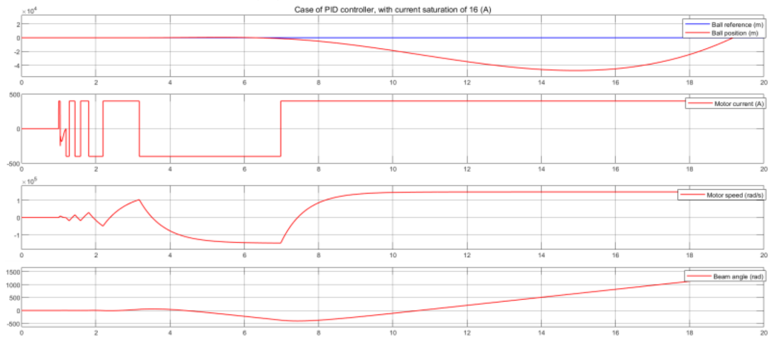

Figure 4: Case of RS structure PID controller, current saturation 16 (A)

Analysis: With the limit on the motor current, the system failed to fulfill any of our specifications. With this controller, our system needs to give large values of motor current as well as motor speed to fulfill our specifications.

IV. RST structure PID control approach

The second controller implemented is a discrete PID of structure RST. We keep the same coefficients for R(z) and S(z) and:

$$T(z) = R(1) = 0.5023$$

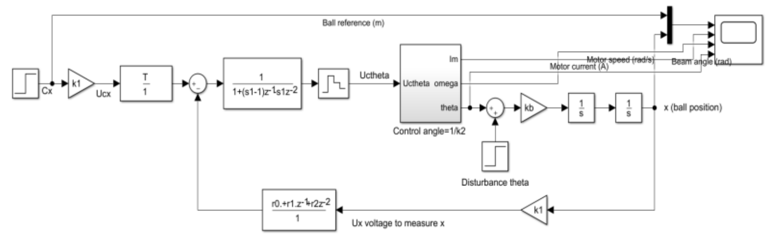

Figure 5: The Simulink model of the RST structure PID controller

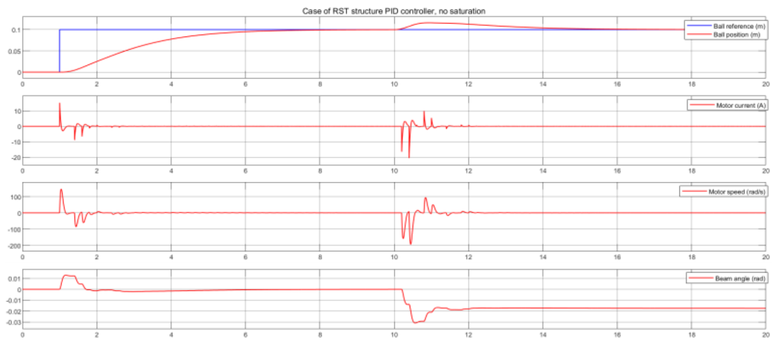

Figure 6: Case of RST structure PID controller, without saturation

Analysis: With a settling time of 5.011 (s), this controller satisfies all the specifications we set previously. This time we have no overshoot, because no additional zero is introduced, and the motor current, as well as motor speed, is not large. We shall test the controller's performance by limiting the motor current with a 16 (A) saturation:

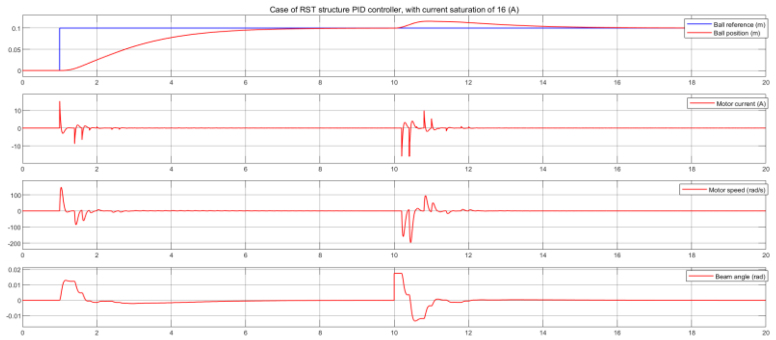

Figure 7: Case of RST structure PID controller, current saturation 16 (A)

Analysis: With saturation, the system this time does not have any significant difference as the current value lies between the limits most of the time. This structure of PID is generally better because with the added T(z) which can be different from R(z), now the controller has more degrees of freedom: R(z) and S(z) for regulation and T(z) for tracking.

V. State-feedback control with integral action approach

The last controller implemented is state-feedback control with integral action. We choose the poles for the system's observer to be 0.3 (double). Then, this is the set of poles for the controller to ensure response time and that the integral pole does not interfere with the system's dynamics:

$$P = \left[e^{\frac{-T_s}{\tau}}, e^{\frac{-T_s}{\tau}}, 0.05 \times e^{\frac{-T_s}{\tau}}\right] = [0.8270, 0.8270, 0.0413]$$

The state-feedback gain is, therefore:

$$F_e = [40.7866 \quad 26.5612 \quad -0.4815]$$

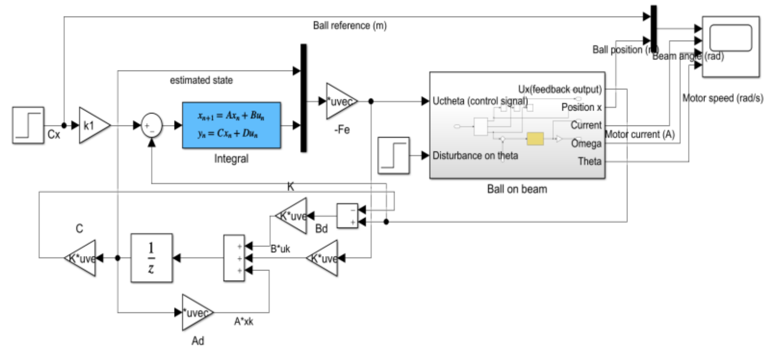

Figure 8: The Simulink model of the state-feedback controller with integral action

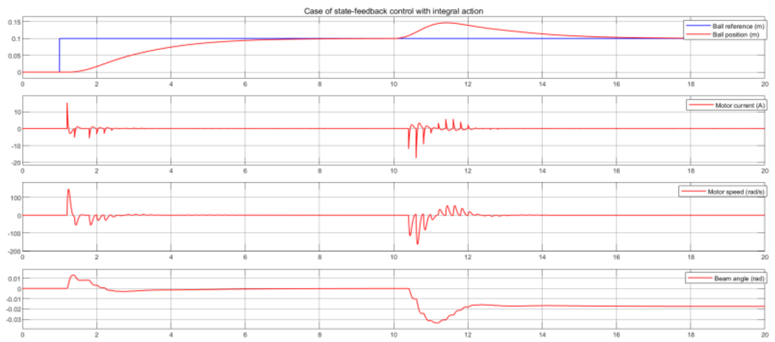

Figure 9: Case of state-feedback control with integral action

Analysis: With a settling time of 5.050 (s), this controller satisfies all the specifications we set previously. We have no overshoot, and the motor current, as well as motor speed, is also not large. We do not need to add saturation to further test it.

VI. Conclusion

Comparison between responses given by the different controllers studied:

	RS structure PID	*RST structure PID*	*State-feedback, integral action*
Rise time (90%)	0.273 (s)	4.110 (s)	4.340 (s)
Overshoot	81.3%	0	0
Settling time (5%)	1.770 (s) *Very fast*	5.011 (s) *Satisfactory*	5.050 (s) *Satisfactory*
Steady-state error	0	0	0
Rejection of disturbances	Yes, if no saturation	Yes	Yes
Saturation	Yes	No	No

Table 2: Comparing performances of controllers

In conclusion, we see that while RS structure PID is fast, RST structure PID control and state-feedback control with integral action are the ones that can fulfill our specifications with little difference while do not need to produce large state values to control the system.

VII. References

1. Hayate Khennouf, 2019, *Digital Control Systems lecture.*

2. Olivier Sename, 2019, *Modelling, Analysis, and Control of Linear Systems using State Space Representations lecture.*