## **fminunc**

Find minimum of unconstrained multivariable function

Nonlinear programming solver.

Finds the minimum of a problem specified by

```
\min_{\mathbf{r}} f(\mathbf{x})
```

where f(x) is a function that returns a scalar.

x is a vector or a matrix; see Matrix Arguments.

## **Syntax**

```
x = fminunc(fun,x0)
x = fminunc(fun,x0,options)
x = fminunc(problem)

[x,fval] = fminunc(__)

[x,fval,exitflag,output] = fminunc(__)

[x,fval,exitflag,output,grad,hessian] = fminunc(__)
example
```

## **Description**

x = fminunc(fun, x0) starts at the point x0 and attempts to find a local minimum x of the function described in fun. The point x0 can be a scalar, vector, or matrix.

example

**Note:** Passing Extra Parameters explains how to pass extra parameters to the objective function and nonlinear constraint functions, if necessary.

fminunc is for nonlinear problems without constraints. If your problem has constraints, generally use fmincon. See Optimization Decision Table.

x = fminunc(fun, x0, options) minimizes fun with the optimization options specified in options. Use optimoptions to set these options.

example

x = fminunc(problem) finds the minimum for problem, where problem is a structure described in Input Arguments. Create the problem structure by exporting a problem from Optimization app, as described in Exporting Your Work.

example

 $[x, fval] = fminunc(\_)$ , for any syntax, returns the value of the objective function fun at the solution x.

example

 $[x,fval,exitflag,output] = fminunc(\__)$  additionally returns a value exitflag that describes the exit condition of fminunc, and a structure output with information about the optimization process.

example

[x,fval,exitflag,output,grad,hessian] = fminunc(\_\_\_) additionally returns:

- grad Gradient of fun at the solution x.
- hessian Hessian of fun at the solution x. See fminunc Hessian.

# Examples

collapse all

Minimize a Polynomial

Minimize the function  $f(x) = 3x_1^2 + 2x_1x_2 + x_2^2 - 4x_1 + 5x_2$ .

Write an anonymous function that calculates the objective.

fun = 
$$@(x)3*x(1)^2 + 2*x(1)*x(2) + x(2)^2 - 4*x(1) + 5*x(2);$$

Call fminunc to find a minimum of fun near [1,1].

```
x0 = [1,1];
[x,fval] = fminunc(fun,x0);
```

After a few iterations, fminunc returns the solution, x, and the value of the function at x, fval.

```
x,fval
x =
    2.2500 -4.7500

fval =
    -16.3750
```

Supply the Gradient

fminunc can be faster and more reliable when you provide derivatives.

Write an objective function that returns the gradient as well as the function value. Use the conditionalized form described in Including Gradients and Hessians. The objective function is Rosenbrock's function,

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

which has gradient

$$\nabla f(x) = \begin{bmatrix} -400(x_2 - x_1^2)x_1 - 2(1 - x_1) \\ 200(x_2 - x_1^2) \end{bmatrix}.$$

```
function [f,g] = rosenbrockwithgrad(x)
% Calculate objective f
f = 100*(x(2) - x(1)^2)^2 + (1-x(1))^2;

if nargout > 1 % gradient required
    g = [-400*(x(2)-x(1)^2)*x(1)-2*(1-x(1));
        200*(x(2)-x(1)^2)];
end
```

Save this code as a file on your MATLAB® path, named rosenbrockwithgrad.m.

Create options to use the objective function's gradient. Also, set the algorithm to 'trust-region'.

```
options = optimoptions('fminunc','Algorithm','trust-region','SpecifyObjectiveGradient',true);
```

Set the initial point to [-1,2]. Then call fminunc.

```
x0 = [-1,2];
fun = @rosenbrockwithgrad;
x = fminunc(fun,x0,options)
```

Local minimum found.

Optimization completed because the size of the gradient is less than the default value of the function tolerance.

<stopping criteria details>

x =

1.0000 1.0000

Use a Problem Structure

Solve the same problem as in Supply the Gradient using a problem structure instead of separate arguments.

Write an objective function that returns the gradient as well as the function value. Use the conditionalized form described in Including Gradients and Hessians. The objective function is Rosenbrock's function,

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

which has gradient

$$\nabla f(x) = \begin{bmatrix} -400(x_2 - x_1^2)x_1 - 2(1 - x_1) \\ 200(x_2 - x_1^2) \end{bmatrix}.$$

```
function [f,g] = rosenbrockwithgrad(x)
% Calculate objective f
f = 100*(x(2) - x(1)^2)^2 + (1-x(1))^2;

if nargout > 1 % gradient required
    g = [-400*(x(2)-x(1)^2)*x(1)-2*(1-x(1));
        200*(x(2)-x(1)^2)];
end
```

Save this code as a file on your MATLAB path, named rosenbrockwithgrad.m.

Create options to use the objective function's gradient. Also, set the algorithm to 'trust-region'.

```
options = optimoptions('fminunc','Algorithm','trust-region','SpecifyObjectiveGradient',true);
```

Create a problem structure including the initial point x0 = [-1, 2].

```
problem.options = options;
problem.x0 = [-1,2];
problem.objective = @rosenbrockwithgrad;
problem.solver = 'fminunc';
```

Solve the problem.

```
x = fminunc(problem)
```

Local minimum found.

Optimization completed because the size of the gradient is less than the default value of the function tolerance.

<stopping criteria details>

x =

1.0000 1.0000

Obtain the Optimal Objective Function Value

Find both the location of the minimum of a nonlinear function and the value of the function at that minimum.

The objective function is

$$f(x) = x(1)e^{-\|x\|_2^2} + \|x\|_2^2/20.$$

```
fun = @(x)x(1)*exp(-(x(1)^2 + x(2)^2)) + (x(1)^2 + x(2)^2)/20;
```

Find the location and objective function value of the minimizer starting at x0 = [1,2].

```
x0 = [1,2];
[x,fval] = fminunc(fun,x0)
```

Local minimum found.

Optimization completed because the size of the gradient is less than the default value of the function tolerance.

<stopping criteria details>

x =

-0.6691 0.0000

fval =

-0.4052

**Examine the Solution Process** 

Choose fminunc options and outputs to examine the solution process.

Open This Example

Set options to obtain iterative display and use the 'quasi-newton' algorithm.

```
options = optimoptions(@fminunc,'Display','iter','Algorithm','quasi-newton');
```

The objective function is

$$f(x) = x(1)e^{-\|x\|_2^2} + \|x\|_2^2 / 20.$$

```
fun = @(x)x(1)*exp(-(x(1)^2 + x(2)^2)) + (x(1)^2 + x(2)^2)/20;
```

Start the minimization at x0 = [1,2], and obtain outputs that enable you to examine the solution quality and process.

```
x0 = [1,2];
[x,fval,exitflag,output] = fminunc(fun,x0,options)
```

			First-order
Func-count	f(x)	Step-size	optimality
3	0.256738		0.173
6	0.222149	1	0.131
9	0.15717	1	0.158
18	-0.227902	0.438133	0.386
21	-0.299271	1	0.46
30	-0.404028	0.102071	0.0458
33	-0.404868	1	0.0296
36	-0.405236	1	0.00119
39	-0.405237	1	0.000252
42	-0.405237	1	7.97e-07
	3 6 9 18 21 30 33 36 39	3 0.256738 6 0.222149 9 0.15717 18 -0.227902 21 -0.299271 30 -0.404028 33 -0.404868 36 -0.405236 39 -0.405237	3 0.256738 6 0.222149 1 9 0.15717 1 18 -0.227902 0.438133 21 -0.299271 1 30 -0.404028 0.102071 33 -0.404868 1 36 -0.405236 1 39 -0.405237 1

Local minimum found.

Optimization completed because the size of the gradient is less than the default value of the optimality tolerance.

```
x =
    -0.6691    0.0000

fval =
    -0.4052

exitflag =
    1

output =
    iterations: 9
    funcCount: 42
    stepsize: 2.9343e-04
    lssteplength: 1
    firstorderopt: 7.9721e-07
        algorithm: 'quasi-newton'
        message: 'Local minimum found....'
```

- The exit flag 1 shows that the solution is a local optimum.
- The output structure shows the number of iterations, number of function evaluations, and other information.
- The iterative display also shows the number of iterations and function evaluations.

## **Related Examples**

• Unconstrained Optimization

Input Arguments collapse all

**fun** — Function to minimize function handle | function name

Function to minimize, specified as a function handle or function name. fun is a function that accepts a vector or array x and returns a real scalar f, the objective function evaluated at x.

Specify fun as a function handle for a file:

```
x = fmincon(@myfun, x0, A, b)
where myfun is a MATLAB function such as
```

function 
$$f = myfun(x)$$

 $f = \dots$  % Compute function value at x

You can also specify fun as a function handle for an anonymous function:

```
x = fmincon(@(x)norm(x)^2,x0,A,b);
```

If you can compute the gradient of fun and the SpecifyObjectiveGradient option is set to true, as set by

options = optimoptions('fminunc','SpecifyObjectiveGradient',true)

then fun must return the gradient vector g(x) in the second output argument.

If you can also compute the Hessian matrix and the HessianFcn option is set to 'objective' via options = optimoptions('fminunc','HessianFcn','objective') and the Algorithm option is set to 'trust-region-reflective', fun must return the Hessian value H(x), a symmetric matrix, in a third output argument. fun can give a sparse Hessian. See Hessian for fminunc trust-region or fmincon trust-region-reflective algorithms for details.

The trust-region-reflective algorithm allows you to supply a Hessian multiply function. This function gives the result of a Hessian-times-vector product without computing the Hessian directly. This can save memory. See Hessian Multiply Function.

Example: fun =  $@(x)\sin(x(1))*\cos(x(2))$ 

Data Types: char | function handle

### x0 — Initial point

real vector | real array

Initial point, specified as a real vector or real array. Solvers use the number of elements in, and size of, x0 to determine the number and size of variables that fun accepts.

**Example:**  $\times 0 = [1, 2, 3, 4]$ 

Data Types: double

### options — Optimization options

output of optimoptions | structure such as optimset returns

Optimization options, specified as the output of optimoptions or a structure such as optimset returns.

Some options apply to all algorithms, and others are relevant for particular algorithms. See Optimization Options Reference for detailed information.

Some options are absent from the optimoptions display. These options are listed in italics. For details, see View Options.

## **All Algorithms**

Algorithm Choices are 'trust-region' (default) or 'quasi-newton'.

If you use optimset (not recommended, see Choose Between optimoptions and optimset), use LargeScale instead of Algorithm.

The 'trust-region' algorithm requires you to provide the gradient (see the description of fun), or else fminunc uses the 'quasi-newton' algorithm. For information on choosing the algorithm, see Choosing the Algorithm.

CheckGradients

Compare user-supplied derivatives (gradient of objective) to finite-differencing derivatives.

Choices are false (default) or true.

Diagnostics

Display diagnostic information about the function to be minimized or solved. Choices are 'off'

(default) or 'on'.

DiffMaxChange

Maximum change in variables for finite-difference gradients (a positive scalar). The default is

Int

DiffMinChange

Minimum change in variables for finite-difference gradients (a positive scalar). The default is  $\emptyset$ .

Display

Level of display (see Iterative Display):

- 'off' or 'none' displays no output.
- 'iter' displays output at each iteration, and gives the default exit message.
- 'iter-detailed' displays output at each iteration, and gives the technical exit message.
- 'notify' displays output only if the function does not converge, and gives the default exit message.
- 'notify-detailed' displays output only if the function does not converge, and gives the technical exit message.
- 'final' (default) displays only the final output, and gives the default exit message.
- 'final-detailed' displays only the final output, and gives the technical exit message.

FiniteDifferenceStepSize

Scalar or vector step size factor for finite differences. When you set FiniteDifferenceStepSize to a vector v, forward finite differences steps delta are

where sign'(x) = sign(x) except sign'(0) = 1. Central finite differences are

delta = v.\*max(abs(x),TypicalX);

Scalar FiniteDifferenceStepSize expands to a vector. The default is sqrt(eps) for forward finite differences, and eps^(1/3) for central finite differences.

The trust-region algorithm uses FiniteDifferenceStepSize only when CheckGradients is set to true.

FiniteDifferenceType

Finite differences, used to estimate gradients, are either 'forward' (the default), or 'central' (centered). 'central' takes twice as many function evaluations, but should be more accurate. The trust-region algorithm uses FiniteDifferenceType only when CheckGradients is set to true.

FunctionTolerance

Termination tolerance on the function value, a positive scalar. The default is 1e–6. See Tolerances and Stopping Criteria.

FunValCheck

Check whether objective function values are valid. The default setting, 'off', does not perform a check. The 'on' setting displays an error when the objective function returns a value that is complex, Inf, or NaN.

LargeScale

Choose the algorithm. When set to the default 'on', use the 'trust-region' algorithm if possible. When set to 'off', use the 'quasi-newton' algorithm.

If you use optimoptions (recommended, see Choose Between optimoptions and optimset), use Algorithm instead of LargeScale.

The 'trust-region' algorithm requires you to provide the gradient (see the description of fun). Otherwise, fminunc uses the 'quasi-newton' algorithm. For more information, see Choosing the Algorithm.

MaxFunctionEvaluations

Maximum number of function evaluations allowed, a positive integer. The default value is 100\*numberOfVariables. See Tolerances and Stopping Criteria and Iterations and Function Counts.

MaxIterations

Maximum number of iterations allowed, a positive integer. The default value is 400. See Tolerances and Stopping Criteria and Iterations and Function Counts.

OptimalityTolerance

Termination tolerance on the first-order optimality, a positive scalar. The default is 1e-6. See First-Order Optimality Measure.

OutputFcn

Specify one or more user-defined functions that an optimization function calls at each iteration, either as a function handle or as a cell array of function handles. The default is none ([]). See Output Function.

PlotFcn

Plot various measures of progress while the algorithm executes. Select from predefined plots or write your own. Pass a function handle or a cell array of function handles. The default is none ([]).

- @optimplotx plots the current point.
- @optimplotfunccount plots the function count.
- @optimplotfval plots the function value.
- @optimplotstepsize plots the step size.
- @optimplotfirstorderopt plots the first-order optimality measure.

For information on writing a custom plot function, see Plot Functions.

SpecifyObjectiveGradient

Gradient for the objective function defined by the user. See the description of fun to see how to define the gradient in fun. Set to true to have fminunc use a user-defined gradient of the objective function. The default false causes fminunc to estimate gradients using finite differences. You must provide the gradient, and set SpecifyObjectiveGradient to true, to use the trust-region algorithm. This option is not required for the quasi-Newton algorithm.

StepTolerance

Termination tolerance on x, a positive scalar. The default value is 1e-6. See Tolerances and Stopping Criteria.

TypicalX

Typical x values. The number of elements in Typical X is equal to the number of elements in x0, the starting point. The default value is ones (number of variables, 1). fminunc uses TypicalX for scaling finite differences for gradient estimation.

The trust-region algorithm uses TypicalX only for the CheckGradients option.

### trust-region Algorithm

HessianFcn

If set to [] (default), fminunc approximates the Hessian using finite differences.

If set to 'objective', fminunc uses a user-defined Hessian for the objective function. The Hessian is the third output of the objective function (see fun).

Function handle for Hessian multiply function. For large-scale structured problems, this function computes the Hessian matrix product H\*Y without actually forming H. The function is of the form

W = hmfun(Hinfo,Y)

where Hinfo contains the matrix used to compute H\*Y.

The first argument is the same as the third argument returned by the objective function fun, for example

[f,q,Hinfo] = fun(x)

Y is a matrix that has the same number of rows as there are dimensions in the problem. The matrix W = H\*Y, although H is not formed explicitly. fminunc uses Hinfo to compute the preconditioner. For information on how to supply values for any additional parameters hmfun needs, see Passing Extra Parameters.

Note To use the HessianMultiplyFcn option, HessianFcn must be set to [].

For an example, see Minimization with Dense Structured Hessian, Linear Equalities.

Sparsity pattern of the Hessian for finite differencing. Set HessPattern(i,j) = 1 when you can have  $\partial^2 \text{fun}/\partial x(i) \partial x(j) \neq 0$ . Otherwise, set HessPattern(i,j) = 0.

Use HessPattern when it is inconvenient to compute the Hessian matrix H in fun, but you can determine (say, by inspection) when the ith component of the gradient of fun depends on x(j). fminunc can approximate H via sparse finite differences (of the gradient) if you provide the sparsity structure of H as the value for HessPattern. In other words, provide the locations of the nonzeros.

When the structure is unknown, do not set HessPattern. The default behavior is as if HessPattern is a dense matrix of ones. Then fminunc computes a full finite-difference approximation in each iteration. This computation can be expensive for large problems, so it is usually better to determine the sparsity structure.

Maximum number of preconditioned conjugate gradient (PCG) iterations, a positive scalar. The default is max(1, floor(number0fVariables/2)). For more information, see Trust Region Algorithm.

Upper bandwidth of preconditioner for PCG, a nonnegative integer. By default, fminunc uses diagonal preconditioning (upper bandwidth of 0). For some problems, increasing the bandwidth reduces the number of PCG iterations. Setting PrecondBandWidth to Inf uses a direct factorization (Cholesky) rather than the conjugate gradients (CG). The direct factorization is computationally more expensive than CG, but produces a better quality step towards the solution.

Determines how the iteration step is calculated. The default, 'cq', takes a faster but less accurate step than 'factorization'. See fminunc trust-region Algorithm.

Termination tolerance on the PCG iteration, a positive scalar. The default is 0.1.

HessianMultiplyFcn

HessPattern

MaxPCGIter

PrecondBandWidth

SubproblemAlgorithm

**ToIPCG** 

quasi-newton Algorithm

Method for choosing the search direction in the Quasi-Newton algorithm. The choices are:

- · 'bfgs', the default
- 'dfp'
- · 'steepdesc'

See Quasi-Newton Algorithm and Hessian Update for a description of these methods.

#### InitialHessMatrix

This option will be removed in a future release.

Initial quasi-Newton matrix. This option is available only if you set InitialHessType to 'user-supplied'. In that case, you can set InitialHessMatrix to one of the following:

- A positive scalar The initial matrix is the scalar times the identity.
- A vector of positive values The initial matrix is a diagonal matrix with the entries of the vector on the diagonal. This vector should be the same size as the x0 vector, the initial point.

### InitialHessType

This option will be removed in a future release.

Initial quasi-Newton matrix type. The options are:

- 'identity'
- 'scaled-identity', the default
- 'user-supplied' See InitialHessMatrix

ObjectiveLimit

A tolerance (stopping criterion) that is a scalar. If the objective function value at an iteration is less than or equal to 0bjectiveLimit, the iterations halt because the problem is presumably unbounded. The default value is -1e20.

UseParallel

When true, fminunc estimates gradients in parallel. Disable by setting to the default, false. trust-region-reflective requires a gradient in the objective, so UseParallel does not apply. See Parallel Computing.

Example: options = optimoptions('fminunc','SpecifyObjectiveGradient',true)

problem — Problem structure

structure

Problem structure, specified as a structure with the following fields:

Field Name	Entry	
objective	Objective function	
x0	Initial point for x	
solver	'fminunc'	
options	Options created with optimoptions	

The simplest way to obtain a problem structure is to export the problem from the Optimization app.

Data Types: struct

## **Output Arguments**

collapse all

**x** — Solution

real vector | real array

Solution, returned as a real vector or real array. The size of x is the same as the size of x0. Typically, x is a local solution to the problem when exitflag is positive. For information on the quality of the solution, see When the Solver Succeeds.

fval — Objective function value at solution

real number

Objective function value at the solution, returned as a real number. Generally, fval = fun(x).

exitflag — Reason fminunc stopped

integer

Reason fminunc stopped, returned as an integer.

1 Magnitude of gradient is smaller than the OptimalityTolerance tolerance. 2 Change in x was smaller than the StepTolerance tolerance. 3 Change in the objective function value was less than the FunctionTolerance tolerance. 5 Predicted decrease in the objective function was less than the FunctionTolerance tolerance. Number of iterations exceeded MaxIterations or number of function evaluations a exceeded MaxFunctionEvaluations. -1Algorithm was terminated by the output function. -3 Objective function at current iteration went below ObjectiveLimit.

### **output** — Information about the optimization process

structure

Information about the optimization process, returned as a structure with fields:

iterationsNumber of iterations takenfuncCountNumber of function evaluationsfirstorderoptMeasure of first-order optimalityalgorithmOptimization algorithm used

cgiterations Total number of PCG iterations ('trust-region' algorithm only)

lssteplength Size of line search step relative to search direction ('quasi-newton' algorithm only)

stepsize Final displacement in x

message Exit message

### grad — Gradient at the solution

real vector

Gradient at the solution, returned as a real vector. grad gives the gradient of fun at the point x(:).

**hessian** — Approximate Hessian

real matrix

Approximate Hessian, returned as a real matrix. For the meaning of hessian, see Hessian.

More About collapse all

Algorithms

## **Trust Region Algorithm**

The trust-region algorithm requires that you supply the gradient in fun and set SpecifyObjectiveGradient to true using optimoptions. This algorithm is a subspace trust-region method and is based on the interior-reflective Newton method described in [2] and [3]. Each iteration involves the approximate solution of a large linear system using the method of preconditioned conjugate gradients (PCG). See fminunc trust-region Algorithm, Trust-Region Methods for Nonlinear Minimization and Preconditioned Conjugate Gradient Method.

## **Quasi-Newton Algorithm**

The quasi-newton algorithm uses the BFGS Quasi-Newton method with a cubic line search procedure. This quasi-Newton method uses the BFGS ([1],[5],[8], and [9]) formula for updating the approximation of the Hessian matrix. You can select the DFP ([4],[6], and [7]) formula, which approximates the inverse Hessian matrix, by setting the HessUpdate option to 'dfp' (and the Algorithm option to 'quasi-newton'). You can select a steepest descent method by setting HessUpdate to 'steepdesc' (and Algorithm to 'quasi-newton'), although this setting is usually inefficient. See fminunc quasi-newton Algorithm.

- Optimization Problem Setup
- · Unconstrained Nonlinear Optimization Algorithms

## References

- [1] Broyden, C. G. "The Convergence of a Class of Double-Rank Minimization Algorithms." *Journal Inst. Math. Applic.*, Vol. 6, 1970, pp. 76–90.
- [2] Coleman, T. F. and Y. Li. "An Interior, Trust Region Approach for Nonlinear Minimization Subject to Bounds." *SIAM Journal on Optimization*, Vol. 6, 1996, pp. 418–445.
- [3] Coleman, T. F. and Y. Li. "On the Convergence of Reflective Newton Methods for Large-Scale Nonlinear Minimization Subject to Bounds." *Mathematical Programming*, Vol. 67, Number 2, 1994, pp. 189–224.
- [4] Davidon, W. C. "Variable Metric Method for Minimization." A.E.C. Research and Development Report, ANL-5990, 1959.
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- [6] Fletcher, R. "Practical Methods of Optimization." Vol. 1, Unconstrained Optimization, John Wiley and Sons, 1980.
- [7] Fletcher, R. and M. J. D. Powell. "A Rapidly Convergent Descent Method for Minimization." *Computer Journal*, Vol. 6, 1963, pp. 163–168.
- [8] Goldfarb, D. "A Family of Variable Metric Updates Derived by Variational Means." *Mathematics of Computing*, Vol. 24, 1970, pp. 23–26.
- [9] Shanno, D. F. "Conditioning of Quasi-Newton Methods for Function Minimization." *Mathematics of Computing*, Vol. 24, 1970, pp. 647–656.

## See Also

fmincon | fminsearch | optimoptions

Introduced before R2006a